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Calculating trading book capital: is risk separation appropriate?

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Non-technical summary

Research Question

Banks trading on their own account are required to hold a minimum amount of capital against the risks in their "trading book". Minimum capital for the trading book's market risk is calculated separately from capital for its credit risk. This method is also common in banks' internal models for economic capital. I analyze whether the separate calculation of capital is appropriate.

Contribution

Previous research focused on non-linear dependencies, and the identified problematic exposures (where separated capital calculation underestimated the true risk) were relatively exotic. I show that, surprisingly, basic exposures and risks are also affected. Furthermore, I enhance a theorem defining a sufficient condition under which risk separation is harmless. The work directly relates to the ongoing reform of regulatory trading book capital.

Results and Policy Recommendations

Focusing on two basic risk factors, namely credit risk (represented by a rating) and the risk premia paid for it (represented by corporate spread indices), I find realistic conditions under which a separate capital calculation substantially underestimates the true risk, even for such ordinary exposures as corporate bonds or credit default swaps. As the *expected shortfall* is equally affected, known shortcomings of the *value-at-risk* are not the issue here. Instead, the cause is separate risk modeling in itself. Doubts are therefore warranted about the reliability of the *incremental risk charge* in current capital requirements. A reformed requirement in which migration risk is segregated from the credit risk engine and incorporated into the market risk model resolves the problems observed under current capital regulation. My results suggest that bank practitioners and supervisors should be warily skeptical of any assumptions of diversification benefits across risk types.

Nichttechnische Zusammenfassung

Fragestellung

Banken, die Handel auf eigene Rechnung betreiben, müssen einen Mindestbetrag an Eigenkapital gegen die Risiken im sogenannten Handelsbuch vorhalten. Dabei wird das Mindesteigenkapital für Marktrisiken im Handelsbuch getrennt vom Kapital für die Kreditrisiken des Handelsbuches berechnet. Dieses Verfahren ist auch in den bankinternen Modellen für ökonomisches Kapital üblich. Ich untersuche, ob die getrennte Kapitalberechnung angemessen ist.

Beitrag

Der Fokus früherer Arbeiten lag bisher auf nichtlinearen Abhängigkeiten, und die gefundenen Problemfälle (bei denen die getrennte Kapitalberechnung das wahre Risiko unterschätzt) betrafen verhältnismäßig exotische Investments. Ich zeige, dass überraschenderweise auch sehr gewöhnliche Investments und Risikoarten betroffen sind. Weiterhin kann ich ein Theorem über eine hinreichende Bedingung verbessern, unter der die getrennte Risikomessung das Gesamtrisiko nicht unterschätzen kann. Die Arbeit hat unmittelbaren Bezug zur laufenden Reform der Eigenkapitalregulierung für das Handelsbuch.

Ergebnisse und Politikempfehlungen

Ich beschränke mich auf zwei grundlegende Risikofaktoren, und zwar Kreditrisiko (in Form eines Ratings) und die dafür gezahlten Risikoprämien (in Form von Spread-Indizes für Firmenanleihen). Für diese Faktoren finde ich realistische Bedingungen, unter denen eine getrennte Kapitalberechnung das wahre Risiko deutlich unterschätzt, selbst für so gewöhnliche Positionen wie Firmenanleihen oder credit default swaps. Das Problem betrifft sowohl den Expected Shortfall als auch den Value-at-Risk. Dessen bekannte Nachteile sind also nicht die Ursache, sondern die getrennte Berechnung an sich. Die Zuverlässigkeit der „Incremental Risk Charge“ in den aktuellen Eigenkapitalregeln steht damit in Frage. Ein Reformvorschlag, nach dem Migrationsrisiken vom Kreditrisikomodell abgespalten und dem Marktrisikomodell zugeschlagen werden sollen, löst die unter den aktuellen Eigenkapitalanforderungen beobachteten Probleme. Meine Ergebnisse unterstreichen, dass Bankpraktiker und -aufseher skeptisch gegenüber jedweden Diversifikationsannahmen über verschiedene Risikoarten hinweg sein sollten.

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Abstract

Regulatory capital for trading book positions includes two components that cover different risks but apply to the same portfolio, one for market risk and one for credit risk. Similar approaches are common in banks' internal models for economic capital. Although it is known that joint market and credit risk of certain investments can be larger than the sum of risks, the problematic cases identified so far have been relatively exotic. I show that very common investments – corporate bond holdings or CDS portfolios – are also affected. There are realistic conditions under which credit risk (represented by ratings and default) and spread risk (represented by rating specific spread indices) combine to a total value-at-risk (VaR) 50 percent larger than the sum of spread and credit VaR; this effect is even stronger for the expected shortfall. If migration risk is segregated from default risk and incorporated into spread risk, as recently put forward by the Basel Committee, total risk is no longer underestimated. Furthermore, I improve a theoretic result of Breuer et al. (2010) that defines a sufficient condition under which risk separation is harmless.

Keywords: Economic capital; Bank capital requirements; Risk measures; Risk aggregation; Trading book

JEL classification: G32, G21, C15.

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1 Introduction

The business of banks is obviously complex, and so are the risks involved. The regulatory and bank-internal calculation of economic capital includes rules stipulating how a multitude of risks are to be transferred into a single amount of capital that a bank should hold as a risk buffer. While a holistic treatment of all facets of a bank's risks would be desirable, the task is so difficult that it makes sense to reduce complexity by splitting risk measurement into blocks. To make the block-specific approach work, proper definitions of risk blocks are needed, and the aggregation of block-specific risks must not fall short of the risks quantified in a sensible unified framework.

Blocks of risks are normally defined either by risk types (leading to notions such as market risk, credit risk, operational risk etc.) or by portfolios (such as securities or loan portfolios).

The portfolio approach has the advantage that fairly simple fundamental conditions are known that assure separate risk measurement for two portfolios, guaranteeing that errors are made on the safe side only. The conditions include the following criteria: the risk measure must be coherent (Artzner, Delbaen, Eber, and Heath, 1999) and the risk (expressed as a random variable) arising from holding two portfolios together equals the sum of the risks arising from each portfolio.[1]

If, by contrast, blocks are defined by risk type, while these risks apply to the same portfolio, no such simple conditions are known in general, which is mainly the case because risks of two different types normally do not add up. In general, it is unsafe to assign risk factors to different blocks and to calculate capital separately if there is a chance that an unfortunate combination of factor realizations from different blocks can generate particularly large losses (this will be defined precisely in Section 2).

Regulation defines that banks which trade securities or derivatives on their own account must hold a minimum amount of capital against the risks in their *trading book*, which conveys positions intended to be held for a relatively short period. The so-called Pillar-1 rules of how this regulatory capital must be calculated have undergone a number of changes over the past few years, including the introduction of the *incremental risk charge* (IRC), which covers credit risks in the trading book that were unaddressed before. As suggested by its name, the IRC is calculated separately from market risk capital. Whether this separation works correctly is one of the questions addressed by this paper.

Current trading book rules are once again being reformed under the headline *fundamental review of the trading book*. In recent regulatory consultative documents (BCBS, 2013, 2014), risk separation still exists, but the risks are now differently grouped. The main difference is that credit risk, currently captured by the IRC, is limited to the pure risk of defaults, while the risk of credit migrations to non-default states (which trigger the revaluation of default-sensitive positions) is now combined with market risk. I will also investigate this type of risk separation.

Aside from adhering to Pillar 1, banks calculate economic capital for internal risk controlling and performance measurement. While the methods used are not prescribed by regulators, they are subject to regulatory scrutiny under the part of the Basel framework known as Pillar 2. My results also matter in the more general context of economic capital as it is often calculated as an aggregate of risk measures originating from separate risk engines. Aggregation, however, is not always performed by addition but by the so-called square-root approach which gives a diversification discount on the sum of two capital figures.[2] Hence, when I find the sum of market

[1] The latter condition is much less trivial than it might appear at first glance. For instance, it often does not hold in the context of credit counterparty risk.

[2] The popular square-root approach determines total capital as if two risks were be given by two correlated bivariate normal variables, and total risk were the sum of them. For instance, assuming a correlation of 0.7 between market and credit risk (Brockmann and Kalkbrener, 2010, p. 54) and equal size for market risk VaR and credit risk VaR, total risk according to the square-root approach would equal $\sqrt{1+1+2\times 0.7}VaR$, which

and credit risk capital to be just about equal to total capital, the square-root approach would still underestimate actual risk. This finding supports the EBA guidelines on the supervisory review and evaluation process (SREP), which preclude the recognition of inter-risk diversification in the assessment of SREP capital (EBA, 2014, Par. 329).[3]

Capital rules for the trading book partly reflect potential problems of risk separation, in that securitization exposures and related positions are exempt from it. Capital for securitizations is calculated as if these were part of the banking book.

The applied part of my analysis focuses on the remaining portfolio of non-securitization positions, assuming a bank has chosen the option to use an internal model for market risk capital. As the portfolio's credit risk capital (i.e. the IRC) is calculated separately from market risk capital, the question arises of whether this component-based capital calculation is appropriate.[4] There would not be any problem from a prudential perspective if the sum of the two capital figures were guaranteed to exceed or equal the capital resulting from a joint model. However, the opposite can (and does) happen, even though any potential diversification between market and credit risk is neglected when separate risk measures are added. I call such cases *underestimation by separation* (UbS).

Breuer, Jandačka, Rheinberger, and Summer (2010) (henceforth BJRS) were the first to bring up this issue in the context of economic capital. They use a set-up where either market or credit risk is understood as one risk factor that varies under its correct marginal distribution while the other factor is kept fixed. Separating risk measures in this way, it transpires that just whether UbS can occur hinges largely on the value function of a position, i.e. on the way in which risk factors map into the value of the position. While their findings were new in the context of market and credit risk on high aggregation levels, derivatives traders are well aware of such effects: given a derivatives position with non-zero *cross gamma*[5], a bundle of separate hedges – one for each risk factor – cannot hedge all risks, especially if the position is sensitive to unfavorable *combinations* of risk factors.

Although similar effects can clearly also appear outside derivatives trading, they do not seem to be reflected in bank risk management and regulation so well. It is possible that an implicit conjecture exists meaning that the effects behind UbS are in any case dominated by diversification if large portfolios are concerned, and that all exceptions are either of pathological nature or generated by the well-known flaws of the value-at-risk (VaR) used in capital requirements.

That being said, the portfolio aspect is not of central importance here for the simple reason that *there is no portfolio of risks*. Moreover, the non-subadditivity of the VaR is not of consequence, as I can show that risk underestimation is even more pronounced if the expected shortfall (ES) is used as a risk measure, despite its advantages over the VaR. In fact, the problem is due to the way in which risks (i.e. random variables) are modeled *before* they are assigned measures by the VaR or ES.

BJRS give the striking example of foreign currency loans where the sum of the credit risk measure and the market risk measure (capturing interest rate and exchange rate risk) can fall short of the total risk measure by a dramatic margin. The primary explanation for this is the strong non-linearity of the loans' valuation function. BJRS also present an example of UbS with

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is 8% less than of the sum of VaRs. Other aggregations use non-normal copulas, but total capital never exceeds the sum of individual capital figures under all approaches.

[3]Banks may still assume inter-risk diversification effects for internal purposes (e.g. in risk-adjusted performance measurement) but not when it comes to assessing whether a bank has enough capital to cover risks as internally calculated by the bank.

[4]Basel rules include another type of trading book capital, the *stressed VaR*. It is neglected in this paper not only for simplicity but also because a risk measure conditional on a bad economic state does not fit into the concept of risk separation.

[5]Cross gamma is the mixed partial derivative of the value function to two of its inputs.

home-currency loans, which at first glance would appear to be very similar to what I do here. However, the example combines an exotic type of contract with an unusual definition of the risky position.[6]

Motivated by the general concept of BJRS and the at times exotic nature of their examples, I endeavored to find more general conditions that would ensure the appropriateness of risk separation, looking for types of trading book positions with problems similar to those associated with foreign currency loans. As a result, I make three contributions to the literature.

First, I generalize the main theoretic result of BJRS: to guarantee that risk separation is conservative, it is sufficient to exclude *malign risk interaction* (Definition 1) in the loss tail, rather than excluding it throughout. This result makes it easier to prove that a certain risk separation is safe.

Second, I provide a thorough analysis of the risks associated with holding portfolios of corporate bonds or single-name credit default swaps (CDS). While these are very basic investments, and despite seemingly harmless valuation functions, I find substantial UbS where total risk is up to 1.5 times as large as the sum of separate market and credit risk. Whether UbS actually occurs depends on a number of factors, such as the bond rating, the value effect that credit migrations have, the correlation between a deterioration of credit and the widening of spreads, the dependency of spread volatility on ratings, and the risk of downgrades.

Third, I show that the reform of trading book capital currently under discussion would eliminate the problem observed with the IRC, at least for the positions considered in this paper.

In addition to the work of BJRS, which serves as the main reference point of my paper, there is of course other research available that deals with the interaction of market and credit risk.

Initial works in this field focus primarily on the question of how spread and other market risks should be integrated with credit risk in a sensible and reliable manner. Concentrating on work closest to my approach, Kiesel, Perraudin, and Taylor (2003) present a model framework similar to one of the two model versions employed in this paper. They build a classic factor model for spread risks into a ratings-based credit portfolio model similar to the industry model CreditMetrics. As in my base case analysis, they concentrate on spreads, leaving default-free interest rate risk aside. Spread changes are assumed to be independent of rating transitions. Whether independence holds makes a big difference when it comes to analyzing UbS, as I will show below. This is why I have put much effort into estimating this dependence. While Kiesel et al. compare pure credit risk with joint credit and spread risk, they do not report measures for pure spread risks, with the result that I cannot categorically state whether UbS is present in their model.

Grundke (2005) extends the model of Kiesel et al. by interest rate risk and correlations between the systematic component of credit risk, default-free interest rate risk and spread risk. Since he does not determine isolated market risk (spread and/or IR risk) either, UbS is not identified.

Barnhill Jr. and Maxwell (2002) investigate the same risk factors as I do but start from a slightly different set-up: rating changes are derived from a multivariate Merton-type model driven by equity index returns. The equity indices (and hence ratings) are correlated with Treasury interest rates, which feed into bond valuation together with ratings. Credit spreads change as well, however their covariance structure is different from mine and, more importantly, spreads seem to change independently of secure interest and credit risk.[7] Barnhill Jr. and Maxwell focus

[6] BJRS consider a one-year loan contract with a lending rate equal to a default-free interest rate r_h plus a fixed spread. This rate is assumed to be a risk factor. In other words, the contractual lending rate would have to be fixed at the end of the horizon, which is not even common with floating-rate loans. Furthermore, and also contrary to practice, the risky total position is assumed to be the loan value *net of funding*, which takes place at the same rate r_h. If either of these unusual assumptions is omitted, UbS is impossible in the model used.

[7] Only the volatility of spread changes is estimated from time series. Barnhill and Maxwell define spreads as

on the benefits to be had from diversification over the number of sectors and bonds, which they find to be strong, but also provide information on the size of total risk vs. the sum of market (spread and interest) risk and credit risk. When measured in standard deviations for a portfolio of 24 B-rated bonds, total risk is only 67% of the sum of separate risks, indicating that separate risk measurement is conservative in the reported case.[8] This finding is consistent with my results insofar as I do not observe UbS for that rating either (but I observe it for higher ratings not considered by Barnhill Jr. and Maxwell).

Kupiec (2007) considers an asymptotic (infinitely diversified) portfolio of corporate loans and uses an integrated model of credit and interest rate risk that is qualitatively the same as the one used by Grundke (2005) and myself. He finds that the sum of separate market and credit risk undershoots integrated risk under many circumstances, but I estimate the result to be an artifact; see Section A.1 for an explanation.

Notable examples of less related analyses of the interaction of credit and market risk and proposals for its modeling are Jarrow and Turnbull (2000), Marsala, Pallotta, and Zenti (2004), Medova and Smith (2005), Böcker and Hillebrand (2008), Alessandri and Drehmann (2010), and Grundke (2013). UbS does not appear in any of the cases where joint and separate risk measures are compared.

Further work directly relates to the current regulation of trading book capital under Pillar 1. Wilkens, Brunac, and Chorniy (2013) develop a rich modeling set-up for the IRC and the CRM, which capital for the "correlation trading activities" is based on. While the framework would be well suited for an analysis of UbS, the authors have other questions in mind, applying each model (IRC vs CRM) exactly to the positions it is designated to so that no model comparison takes place. There are also other analyses of the IRC which, however, do not investigate the relationship to market risk (Martin, Lutz, and Wehn, 2011; Skoglund and Chen, 2011).

To summarize previous work, BJRS seem to be the only researchers who have identified a case of UbS in the context of economic capital. Their work serves as the starting point of my paper.

The next section provides a formal treatment of separate and integrated risk measurement; it presents a new theorem on the appearance of UbS and applies the framework to corporate bonds and CDS. Section 3 outlines the risk model, which is calibrated to data in Section 4. Section 5 contains the results, which are tested for robustness in Section 6. A further Section 7 tests the *revised market risk framework* (BCBS, 2013) for UbS. In Section 8 I set out my conclusions.

2 Separate vs. integrated risk measurement

2.1 The general concept

This section provides a formal framework for the concept of separate risk measurement, as introduced by BJRS. One of their main results, quoted here in Proposition 1, defines conditions under which separate risk measurement is conservative. I generalize it in Proposition 2.

Let Z be a real random variable defined on a probability space $[\Omega, \mathscr{F}, \mathbf{P}]$ and κ be a *risk measure*, which is a mapping from Z to a real number $\kappa(Z)$. I assume κ to be *law-invariant*,

increments from a rating to the next-worse and seem to assume them to be independent of each other and of secure interest and credit migrations (the paper is not explicit on this). Changes of spreads as I understand them (increments to the secure interest rate) are therefore correlated, however in a manner that is not inferred from data. In my paper, these interdependencies and even more those with systematic credit risk factors are found to be important so that they should rely on data.

[8] Barnhill Jr. and Maxwell (2002) make a similar comparison for values-at-risk of a single bond, also finding total risk to be smaller than the sum of separate risks. VaR is not a very informative risk measure for a single bond, however.

which means that two random variables with identical distributions must be assigned the same risk measure. Losses are meant to be low realizations of Z, compared to a non-random current value Z_0. For the purpose of this paper it is sufficient to think of Z as a P/L, which is the motivation to assume $Z_0 = 0$. Losses are then simply negative values of Z.

The propositions below will presume that κ is *coherent* (Artzner et al., 1999), which includes subadditivity (see (2)) and monotonicity. Coherence implies $\kappa(0) = 0$ and $\kappa(Z) \leq 0$ for $\mathbf{P}$-almost surely nonnegative Z.

Despite its well-known flaws, value-at-risk (VaR) is still the most important risk measure both in bank regulation and bank risk management. It is defined as $VaR_\alpha(Z) \equiv -Q_\alpha(Z)$, where Q_α is the quantile at the (typically low) level α, and law-invariant. Except under fairly special conditions, the VaR is not coherent, in contrast to the second important risk measure, the expected shortfall (ES). Following Acerbi and Tasche (2002), the ES is defined as[9]

$$ES_\alpha(Z) \equiv -\mathbf{E}(Z|Z \leq Q_\alpha(Z)) + \frac{\mathbf{P}(Z \leq Q_\alpha(Z)) - \alpha}{\alpha}(Q_\alpha(Z) - \mathbf{E}(Z|Z \leq Q_\alpha(Z))).$$

Coherence is the main reason why the Basel Committee is about to replace the VaR in the capital regulation of the trading book by the ES (BCBS, 2013). However, as the problems identified in this paper apply to ES and VaR equally, coherence is not necessarily a remedy if risks are split up into blocks other than portfolios.

To define separate risk measurement, consider two risk factors X and Y which take on values in two sufficiently well-behaved[10] vector spaces $\mathfrak{X}$ and $\mathfrak{Y}$. At least one of them will typically be multidimensional. In Section 3.2, for example, $\mathfrak{X}$ will be the space of continuous functions on the unit interval so that it has even infinite dimensions.[11] I do not make a conceptual difference between X and Y, except that they may represent different risk types, and that risk managers may wish to measure the total risk in a way that neglects the relationship between X and Y.

The function $V : \mathfrak{X} \times \mathfrak{Y} \to \mathbb{R}$ determines the value of a trading position, which can also represent a portfolio. While V commonly depends on X and Y, leading to the *exact* or *total* measure given by $\kappa(V(X,Y))$, separate risk measurement is executed in three steps. First, a risk measure is calculated where V is an exclusive function of risk X while Y is kept fixed; second, the same exercise is applied to Y while X is kept fixed; third, the risk measures resulting from both exercises are added. More formally, let X_0 and Y_0 be the non-random values of X and Y today. As explained above, all values are set in relation to the position's current value $V(X_0, Y_0)$, meaning that I assume

$$V(X_0, Y_0) = 0.$$

It is natural to define the *X-risk* as $\kappa(V(X, Y_0))$ and the *Y-risk* as $\kappa(V(X_0, Y))$. The *separated* risk measure of V is defined as

$$\kappa_s(V(X,Y)) \equiv \kappa(V(X,Y_0)) + \kappa(V(X_0,Y)). \tag{1}$$

If the risk measure κ is *subadditive*, i.e., if

$$\kappa(Z_1 + Z_2) \leq \kappa(Z_1) + \kappa(Z_2) \tag{2}$$

holds for arbitrary positions Z_1 and Z_2 (assuming all parts of (2) are well defined), κ_s is subadditive as well. However, subadditivity does not guarantee that separate risk measurement is

[9]The second term is zero in many cases so that the ES coincides with the *tail-conditional expectation* of Z. If, however, the cdf of Z jumps when crossing the level α, the second term is necessary to ensure coherence.

[10]Requiring $\mathfrak{X}$ and $\mathfrak{Y}$ to be complete normed spaces would be an appropriate assumption. It probably covers all practically relevant cases.

[11]The probability measure is then defined on the measurable space of Borel sets in $C([0,1])$.

equally or more conservative than exact measurement. To see this more clearly, purely formal steps yield

$$\kappa_s\left(V\left(X,Y\right)\right)=\kappa_s\left(V\left(X,Y_0\right)+V\left(X_0,Y\right)\right),$$

which motivates a closer look at the gap between the arguments of κ_s in this equation. The *approximation error* of the function V is defined as

$$D\left(x,y\right) \equiv V\left(x,y\right)-\left[V\left(x,Y_0\right)+V\left(X_0,y\right)\right]. \tag{3}$$

Note that this formula makes sense only if $V\left(X_0,Y_0\right)=0$. Below, the expression $D\left(X,Y\right)$ will also be called an approximation error, although it is not an exclusive property of the mapping V but a random variable.

Definition 1. Following BJRS, *malign risk interaction* (MRI) takes place at points where $D\left(x,y\right)<0$; the opposite case is denoted as *benign risk interaction.* On the level of risk measures, cases in which the inequality $\kappa\left(V\left(X,Y\right)\right)>\kappa_s\left(V\left(X,Y\right)\right)$ holds are defined as *underestimation by separation* (UbS) of risks. Otherwise, separate risk measurement for X and Y is referred to as *conservative.*

MRI is useful in finding conditions under which κ_s is in fact conservative:

Proposition 1 (Breuer et al., 2010)**.** *Let the law-invariant risk measure κ be coherent. If there is no malign risk interaction, meaning that $D\left(X,Y\right)$ is $\mathbf{P}$-almost surely non-negative, separate risk measurement for X and Y is conservative.*

Proof. Supposing monotonicity[12] and subadditivity, I observe

$$\begin{aligned}\kappa\left(V\left(X,Y\right)\right) &= \kappa\left(V\left(X,Y_0\right)+V\left(X_0,Y\right)+D\left(X,Y\right)\right)\leq\kappa\left(V\left(X,Y_0\right)+V\left(X_0,Y\right)\right)\\ &\leq \kappa\left(V\left(X,Y_0\right)\right)+\kappa\left(V\left(X_0,Y\right)\right)=\kappa_s\left(V\left(X,Y\right)\right),\end{aligned}$$

which was to be shown. □

For coherent risk measures that exclusively build on a loss tail, such as the ES, a global non-appearance of MRI is overly restrictive. Below, I show that MRI can become a problem only if it appears in the loss tail.

"Building on a loss tail" requires definition. The concept is based on the observation that a law-invariant risk measure could equivalently be defined on the cdf of a random variable or on its quantile function. The following definition filters out those quantile levels in $[0,1]$ which a risk measure is not sensitive to.

Definition 2. Given a number $\alpha\in[0,1]$, the random variables U, V are *equivalent on the α-tail* if $Q_h\left(U\right)=Q_h\left(V\right)$ holds for all $h\leq\alpha$. The risk measure κ is *concentrated on the α-tail* if any α-tail-equivalent variables U, V must have the same risk measure $\kappa\left(U\right)=\kappa\left(V\right)$.

VaR_α and ES_α are obviously concentrated on the α-tail. [13]

It is important to note that a random variable can be modified in the following way, without having any effect on the risk measure. If κ is concentrated on the α-tail, then

$$\kappa\left(Z\right)=\kappa\left(I_A Z+C\,I_A\right)\ \text{for every}\ A\supseteq\left\{Z\leq Q_\alpha\left(Z\right)\right\},$$

[12]Coherence includes monotonicity. If κ is law-invariant, monotonicity means that $X\leq Y$, $\mathbf{P}$-a.s., implies $\kappa\left(Y\right)\leq\kappa\left(X\right)$.

[13]Including the *whole* tail in the definition of tail equivalence appears unnecessarily special as, for instance in the case of the VaR_α, the set of relevant quantile levels $[0,\alpha]$ could be replaced by the singleton $\{\alpha\}$. However, I am not aware of a related case if the measure is coherent. In the case of spectral risk measures, such as the ES, the set of relevant quantile levels *must* have the form $[0,\alpha]$ (Acerbi, 2002, Theorem 4.1).

where C is an arbitrary constant larger than $Q_\alpha(Z)$. If the quantile is negative, C can be chosen to be zero so that

$$\kappa(Z) = \kappa(I_A Z) \text{ for every } A \supseteq \{Z \leq Q_\alpha(Z)\}. \tag{4}$$

I return now to the comparison of separate and joint risk measurement on the particular space $\mathfrak{X} \times \mathfrak{Y}$ to show the main theoretic result of the paper:

Proposition 2. *Let the law-invariant, coherent risk measure κ be concentrated on the α-tail. Furthermore, assume that each of the quantiles*

$$Q \equiv Q_\alpha(V(X,Y)), \ Q^X \equiv Q_\alpha(V(X,Y_0)), \text{ and } Q^Y \equiv Q_\alpha(V(X_0,Y))$$

is negative.[14] *If there is no malign risk interaction in the α-tail of V, meaning that $D(X,Y)$ is $\mathbf{P}$-almost surely non-negative on the inverse image of the tail, separate risk measurement for X and Y is conservative.*

Proof. To utilize (4), "tail dummies" are defined as follows:

$$\mathscr{T} \equiv I_{\{V(X,Y)\leq Q\}}, \ \mathscr{T}_X \equiv I_{\{V(X,Y_0)\leq Q^X\}}, \ \mathscr{T}_Y \equiv I_{\{V(X_0,Y)\leq Q^Y\}}.$$

As the quantiles Q, Q_X, and Q_Y are negative, (4) applies so that I obtain

$$\kappa(V(X,Y)) = \kappa(\mathscr{T} V(X,Y)) \text{ and } \kappa(V(X,Y_0)) = \kappa(\mathscr{T}_X V(X,Y_0)) \tag{5}$$

(analogously for Y). Accordingly, decomposing $V(X,Y)$ and the absence of MRI in the tail (meaning that $\mathscr{T} D(X,Y) \geq 0$, $\mathbf{P}$-a.s.) yield

$$\begin{aligned}\kappa(V(X,Y)) &= \kappa(\mathscr{T} V(X,Y)) = \kappa(\mathscr{T} V(X,Y_0) + \mathscr{T} V(X_0,Y) + \mathscr{T} D(X,Y)) \\ &\leq \kappa(\mathscr{T} V(X,Y_0) + \mathscr{T} V(X_0,Y)) \\ &\leq \kappa(\mathscr{T} V(X,Y_0)) + \kappa(\mathscr{T} V(X_0,Y)),\end{aligned} \tag{6}$$

where monotonicity is utilized in the second line and subadditivity in the third. By construction of $\mathscr{T}_X$, the inequality $\mathscr{T}_X V(X,Y_0) \leq 0$ holds and, a fortiori, $\mathscr{T}_X(1-\mathscr{T}) V(X,Y_0) \leq 0$ so that monotonicity again yields

$$\kappa(\mathscr{T} V(X,Y_0)) \leq \kappa(\mathscr{T} V(X,Y_0) + \mathscr{T}_X(1-\mathscr{T}) V(X,Y_0)) = \kappa((\mathscr{T} \vee \mathscr{T}_X) V(X,Y_0)),$$

where $\vee$ is the maximum operator. As $\{\mathscr{T} \vee \mathscr{T}_X = 1\}$ is a superset of $\{V(X,Y_0) \leq Q^X\}$, I can apply (4) to obtain

$$\kappa(\mathscr{T} V(X,Y_0)) \leq \kappa((\mathscr{T} \vee \mathscr{T}_X) V(X,Y_0)) = \kappa(V(X,Y_0)).$$

This estimate and a related expression for $V(X_0,Y)$ can be used to continue (6) by

$$\kappa(V(X,Y)) \ \leq \ \kappa(V(X,Y_0)) + \kappa(V(X_0,Y)) = \kappa_s(V(X,Y)).$$

This means that separate risk measurement is conservative. □

Judging the usefulness of Proposition 2, two kinds of potential application should be distinguished:

[14] As $V(X_0,Y_0)$ was assumed to be zero, negative quantiles just mean quantiles below the current value. This assumption is unlikely to be hurt over short risk horizons. Longer horizons, however, may require that $V(X_0,Y_0)$ is replaced by a more suitable reference value; see Section A.1 for a problematic case.

Its use is probably limited in attempts to prove that some separate risk measurement is conservative by showing that D does not become negative on the loss tail. Such an attempt has good prospects only if the factor spaces $\mathfrak{X}$ and $\mathfrak{Y}$ both have low dimension, which is most easily seen using the analogy to derivatives trading drawn in the introduction: if $\mathfrak{X} \times \mathfrak{Y}$ is 1×1-dimensional only, a well-behaved *cross gamma* is often sufficient for the conditions of Proposition 2. If, in contrast, $\mathfrak{X} \times \mathfrak{Y}$ is $M \times N$-dimensional, then $M \times N$ different cross gammas may become relevant.

However, Proposition 2 turns out to be useful also in cases where UbS *can* occur, as it helps to disentangle the drivers of UbS: whatsoever effect will turn $D(X, Y)$ into a negative value, it must show up in the loss tail.

In the next subsection I present a – fairly simple – type of interaction between risk factors where no general statement about the sign of the approximation error D seems possible: in the valuation of corporate bonds, a suitable credit spread must be selected, based on the rating of the issuer. The selection is of key importance: if valuation requires that a risk factor is selected from a range of factors, and if the selection criterion is a risk factor, too, then risk separation can easily fail.

2.2 The case of bond and CDS portfolios

In this part of the paper I introduce the main example of risk interaction analyzed. I focus on two risk factors, the rating of a bond and the credit spread, which depends on the bond's rating. Ratings are defined in two alternative ways, first by abstract labels on a discrete scale, as external ratings, and second by a probability of default over a given risk horizon. In this section it is not necessary to interpret this probability directly, nor is the risk horizon important. Rather, it is sufficient to suppose the existence of some $R \in [0, 1]$, to be known as the "rating", such that low values mean good credit quality. The credit spread is given by a time dependent function s of the rating. Here, I consider two points of time $t \in \{0, 1\}$, the present and the end of the risk horizon. Keeping t fixed, $s_t(\cdot)$ is a mapping from $[0, 1]$ to $\mathbb{R}_+$.[15]

Definition 3. *Spread changes* are understood as differences $s_1(R) - s_0(R)$ for a *fixed* rating R or, if the rating is indeterminate, as changes in the whole spread function.

Spread changes under this definition must be distinguished from changes in the spread of a certain bond. The former is due to a change in the premium required for bearing a certain – unchanged – credit risk, whereas the latter combines changes in the premium and in underlying fundamental credit risk.

To get an idea under which conditions risks of a defaultable bond may interact in a malign way, let me assume for simplicity that the bank holds a position worth $-s_t(R_t)$ at time t. This neglects default-free interest rates and the non-linearity of bond valuation. The value is obviously linear in the spread but in no way linear if considered as a function of the spread function and the rating.[16] Setting $X \equiv s_1(\cdot)$, $X_0 \equiv s_0(\cdot)$, $Y \equiv R_1$, $Y_0 \equiv R_0$, and $\Delta X = X - X_0$ and $\Delta Y = Y - Y_0$, the approximation error can be written

$$D = -[s_1(R_1) - s_0(R_1)] + [s_1(R_0) - s_0(R_0)] = -[\Delta s(R_1) - \Delta s(R_0)]. \tag{7}$$

If the risk measure in use is concentrated on the α-tail, Proposition 2 indicates that the sign of

[15] Such a function is not observable in practice, but financial companies provide indices for corporate bond spreads. In the empirical part, I use rating specific Merill-Lynch corporate spreads, which are reported for 7 rating classes, and interpret them as $s(t, \cdot)$ being observed at 7 grid points on $[0, 1]$.

[16] This simple mapping actually equals the famous Dirac delta and is therefore a linear functional in its first argument. There is no linearity in the rating, however.

D is relevant for risk underestimation only at points where the position's value $-s_1(R_1)$ is in the α-tail. This tail event is given by $\{s_1(R_1) > Q\}$ where Q stands for the quantile $Q_{1-\alpha}(s_1(R_1))$.

I split the tail event into disjoint parts, depending on the sign of $R_1 - R_0$:

$$\{s_1(R_1) > Q\} = \{R_1 > R_0, s_1(R_1) > Q\} \cup \{R_1 \leq R_0, s_1(R_1) > Q\} \tag{8}$$

and obtain a disaggregated form for the approximation error restricted to the tail event:

$$\begin{aligned} D \times I_{\{s_1(R_1)>Q\}} &= -[\Delta s(R_1) - \Delta s(R_0)]\, I_{\{R_1>R_0, s_1(R_1)>Q\}} && (9)\\ &\quad -[\Delta s(R_1) - \Delta s(R_0)]\, I_{\{R_1\leq R_0, s_1(R_1)>Q\}} && (10)\end{aligned}$$

If the probability is high enough that this expression has significant negative values, UbS appears. Note that only one of the two terms can deviate from zero at the same time. The term in (10) is rather unlikely to do so in general because the indicator function requires the new rating R_1 to be *better* than the old one, which entails a lower spread in the sense of $s_1(R_1) < s_1(R_0)$, whereas the second condition requires the ultimate spread $s_1(R_1)$ to be particularly high. In a sense, the conditions counteract each other.

The term on the r.h.s. of (9) is more relevant for the identification of conditions under which UbS is "likely" to appear. Turning to the indicator function, the probability that the conditions $R_1 > R_0$ and $s_1(R_1) > Q$ are both fulfilled is comparably high if the correlation between rating deterioration and the spread $s_1(R_1)$ is positive. While such a correlation would exist even if the spread function never changed (simply because spread functions are generally rising), the correlation becomes stronger if there is an additional link between deteriorating credit and increases in the whole spread function, which would show up in a correlation between $R_1 - R_0$ and $\Delta s(R_1)$. The effect is summarized as follows.

Conjecture 1. *Suppose that the value of a position negatively depends on a rating-sensitive spread. The stronger the link between credit deterioration and spread increases (in the sense of Definition 3), the larger the propensity to UbS.*

Turning now to the bracketed term in (9), the way in which the variance of spread changes depends on the rating also matters. If $\Delta s(R_1)$, where R_1 is worse than R_0, has higher variance than $\Delta s(R_0)$, the bracketed difference will be larger in size than in a situation where the variances are the same.

To illustrate, assume for the moment that $R_1 > R_0$ is deterministic and $\Delta s(R_1)$ and $\Delta s(R_0)$ are perfectly correlated.[17] They differ only by a constant factor, meaning that the bracketed term takes the form

$$[\Delta s(R_1) - \Delta s(R_0)] = \Delta s(R_1)\left(1 - \frac{\mathrm{var}(\Delta s(R_0))}{\mathrm{var}(\Delta s(R_1))}\right),$$

which gains in size if the variance of $\Delta s(R_0)$ decreases. Hence, if $D \times I_{\{s_1(R_1)>Q\}}$ has a negative realization stemming from the first term, the malign approximation error will be pushed up in size if the spread volatilities of bad ratings are higher than those of good ones. Similar but less obvious effects are at work if the spread changes are not perfectly correlated, even in the case of independence. Note that negative outcomes of D would be "compensated" by positive outcomes only if they were in the same domain of relevance $\{R_1 > R_0, s_1(R_1) > Q\}$. However, the condition that $s_1(R_1)$ must exceed a high quantile excludes most of these. Summing up, referring to the same exposure as in Conjecture 1 I state :

[17] The empirical correlation is actually very high; cf. panel A of Table 2.

Conjecture 2. *The more steeply* $\operatorname{var}(\Delta s(R))$ *rises as a function of* R*, the larger the propensity to UbS.*

Returning to bonds, let me consider the simplest case of a defaultable one-dollar zero bond maturing one year after the end of the risk horizon. Abstracting again from the default-free interest rate, its price is assumed to be $e^{-s(R)}$. The approximation error of separate risk measurement is

$$D_{\text{bond}} = \left[e^{-s_1(R_1)} - e^{-s_0(R_1)}\right] + \left[e^{-s_1(R_0)} - e^{-s_0(R_0)}\right] = \Delta e^{-s(R_1)} - \Delta e^{-s(R_0)}. \tag{11}$$

As the tail event is the same as that of the negative-linear position, the expression for the approximation error is very similar to (9) and (10):

$$\begin{aligned} D_{\text{bond}} \times I_{\{s_1(R_1)>Q\}} &= \left[\Delta e^{-s(R_1)} - \Delta e^{-s(R_0)}\right] I_{\{R_1>R_0, s_1(R_1)>Q\}} \\ &\quad + \left[\Delta e^{-s(R_1)} - \Delta e^{-s(R_0)}\right] I_{\{R_1\leq R_0, s_1(R_1)>Q\}}. \end{aligned}$$

While all former arguments on a possible tendency to UbS hold to an equal extent, I expect the effects to be slightly weaker than for the linear position as e^{-x} is convex. Hence, deep losses are dampened, as are the most extreme realizations of the approximation error.

3 Risk model

My base case analysis relates to a portfolio of simple corporate bonds. As credit spreads are assumed to be the only type of market risk, while changes in default-free interest rates are ignored, the case does not directly apply to fixed-rate bonds. It does apply, however, to floating-rate bonds, which are not to be neglected in the corporate sector.[18] The case also holds for a fixed-rate bond portfolio, the interest rate risk of which is being hedged.

Furthermore, selling credit protection through a range of single-name CDS has a similar risk profile. Abstracting from the differences between CDS and bond portfolios is in line with regulatory practice. Indeed, the way market and credit risk of bonds and CDS protection-selling are to be modeled for regulatory purposes is pretty much the same, the main difference being that the spread curves may be different, as may the treatment of double-default risk. Reiterating the notion of market spread risk in the context of CDS, spread changes (as defined in this paper and by many regulators) are changes in a credit risk premium for *fixed* credit risk; this is different from a change in the spread of a particular CDS, which combines spread changes with fundamental credit risk.

I investigate UbS in two model versions. The *threshold model* takes a traditional approach to credit risk, very much in line with CreditMetrics, one of the most popular credit portfolio models in the banking industry. It generates rating transitions on a scale of 7 grades and the default state. The *continuous model* borrows concepts from Moody's KMV and builds on a continuous scale of ratings representing default probabilities. Both model versions are driven by the same latent background factors for spread changes and the same single systematic credit risk factor.

Many of the modeling assumptions in this paper could certainly be relaxed and replaced by others that would, for instance, allow for a better compatibility between the spread dynamics and the data. However, the role of the risk model is to provide a *sufficiently realistic* framework for the investigation of UbS; an ideal data fit is not part of the focus of this paper.

[18] About 30 percent of outstanding corporate bond debt has floating rates (BIS Quarterly Review, Sept. 2014).

3.1 Threshold model

The set-up is similar to Kiesel et al. (2003). In contrast to capital requirements, I consider a joint risk horizon of 3 months both for credit and market risk, which is a compromise between the ten-day horizon of the regulatory horizon for market risk and the one-year horizon of the IRC.

The first building block of the model determines the joint movements of spreads and the systematic component of the credit risk model. They are commonly driven by an 8-dimensional normally distributed random vector $W \sim N(\mu, \Sigma)$.

The first 7 components of W generate movements of spread indices, each of which belongs to a rating class AAA, AA, A, BBB, BB, B, or CCC–C. These are the rating classes for which empirical spread indices exist. In the formulas, the rating labels are replaced by numbers, where 1 stands for AAA and 7 for CCC–C. The normally distributed variables define quarterly changes of log spreads, called *spread returns*; the mean vector μ is set such that spread changes have expectation zero:

$$log \begin{bmatrix} s_1(1)/s_0(1) \\ \cdots \\ s_1(7)/s_0(7) \end{bmatrix} \sim N\left(\mu_{1,\ldots,7}, \Sigma_{1,\ldots,7}\right), \text{ where } \mu_{1,\ldots,7} = -\frac{1}{2} diag\left(\Sigma_{1,\ldots,7}\right). \qquad (12)$$

A fully fledged model would include idiosyncratic spread factors responsible for bond-individual deviations from the spread index. They are the subject of a robustness check in Section 6.

The eighth component of W is the single systematic credit risk factor (SCRF). It is normalized to $Y \sim N(0,1)$ (but correlated with the spread returns via Σ) and enters the credit risk model in the same way as in various “structural” credit risk models: for each obligor i, there is an independent idiosyncratic shock $Z_i \sim N(0,1)$ which linearly combines with the systematic factor Y to form a latent “asset return” X_i:

$$X_i = \sqrt{\rho} Y + \sqrt{1-\rho} Z_i. \qquad (13)$$

The parameter ρ, called *asset correlation*, is set to 20% and is therefore close to a representative value of 19.98% as reported by Zeng and Zhang (2001).[19]

The latent X_i triggers rating migrations by a threshold mechanism. Let R_0 denote the initial rating of a single bond and R_1 the rating 3 months ahead. The default state D is now introduced as a further rating state and given the number 8. A bond migrates from R_0 to rating state R_1 whenever

$$c(R_0, R_1+1) < X_i \le c(R_0, R_1)$$

where the $c(\ldots)$ are thresholds calibrated as follows. For given probabilities $p(R_0, R_1)$ of migrations from R_0 to R_1, $1 \le R_0, R_1 \le 8$, the thresholds are chosen in such a way that

$$\mathbf{P}\left(c(R_0, R_1+1) < X_i \le c(R_0, R_1)\right) = p(R_0, R_1),$$

which is achieved by setting $c(R_0, 8) = -\infty$ and[20]

$$c(R_0, R_1) = \Phi^{-1}\left(\sum\nolimits_{R \ge R_1} p(R_0, R)\right), \ 1 \le R_0, R_1 \le 7. \qquad (14)$$

The matrix of 3-month transition probabilities is calculated from a one-year transition matrix under the Markov assumption, using a matrix based on S&P rating events 1982–2012 (Standard

[19] This value is the average over Zeng and Zhang’s sub-sample of firms with the fewest missing observations.

[20] As I consider only one period, the default state no. 8 is no relevant start rating.

and Poor's, 2013, Table 9); for further details, see Table 9 in the appendix.

The next step is valuation. In the base case analysis I assume the bank holds a portfolio of coupon bonds. To avoid aging effects, bonds are assumed to pay a continuous coupon stream δ until maturity T. Principal is due at T. All payments are discounted at a continuous rate $r_f + s$, where r_f is the default-free interest rate and s is the spread; the two are insensitive to the bond's time to maturity. Unlike in Section 2.2, the spread function $s(\cdot)$ is defined on the discrete domain $\{1, \ldots, 7\}$ corresponding to the labels AAA,...,CCC–C. Later, I will distinguish the current spread function $s_0(\cdot)$ from the future function $s_1(\cdot)$. Default-free interest r_f is fixed at 4 percent in the base case analysis; interest rate risk is left to a robustness test. Given a spread function s, the value of an R-rated bond at time t is

$$V_{\text{bond}}(t, s, R) = \begin{cases} \left[\frac{\delta}{r_f + s(R)} + \left(1 - \frac{\delta}{r_f + s(R)}\right) e^{-\left(r_f + s(R)\right)(T-t)}\right] \times N & \text{if } R < 8 \\ (1 - \lambda) \times N & \text{if } R = 8, \end{cases}$$

where N is the notional amount of the bond and λ the random loss given default (LGD), which is independent of all other random variables and beta distributed with a mean of 52.3% and a standard deviation of 26.7 percent. These are moments found by Altman and Kishore (1996) for senior unsecured bonds across all industries.

I set the maturity equal to 5 years and assume that δ equals $r_f + s_0(R_0)$ at issuance so that the bond is priced at par throughout its lifetime, provided there is no change in its spread.

As idiosyncratic spread risk is neglected in the base case, two bonds with the same rating R and notional N have exactly the same value $V_{\text{bond}}(t, s, R)$. The value function is convex in the spread, the same as the value of the zero bond introduced at the end of Section 2.2. A convex value function appears to aggregate spread and credit risk in a harmless manner but the approximation error under separate risk measurement is similar to that given in (11) so that there is a possibility of UbS; Conjecture 1 and Conjecture 2 hold accordingly.

The P/L of a bond results from revaluation at the end of the risk horizon. The total P/L, integrating credit and spread risk, amounts to

$$\Delta V_{\text{total}} \equiv V_{\text{bond}}(1, s_1, R_1) - V_{\text{bond}}(0, s_0, R_0). \tag{15}$$

Under separate risk measurement, the purely *migration induced* P/L (which includes defaults) is defined as

$$\Delta V_{\text{mig}} \equiv V_{\text{bond}}(1, s_0, R_1) - V_{\text{bond}}(0, s_0, R_0), \tag{16}$$

i.e., the time-0 spread function is evaluated at the new rating, or the bond defaults. The purely *spread induced* P/L is defined as

$$\Delta V_{\text{spread}} \equiv V_{\text{bond}}(1, s_1, R_0) - V_{\text{bond}}(0, s_0, R_0). \tag{17}$$

Given a risk measure κ, which either stands for the VaR or the ES in this paper, integrated risk measurement means calculating $\kappa(\Delta V_{total})$, which is the *total risk*, whereas the final outcome of separate risk measurement is given by $\kappa(\Delta V_{mig}) + \kappa(\Delta V_{spread})$, according to (1). I follow BJRS in calculating the *risk interaction index*

$$RI \equiv \frac{\kappa(\Delta V_{\text{total}})}{\kappa(\Delta V_{\text{mig}}) + \kappa(\Delta V_{\text{spread}})}. \tag{18}$$

If RI exceeds 1, UbS is observed, meaning that separate measurement underestimates the total risk.

3.2 Continuous model

This model version works on a continuous rating scale. It is motivated by three arguments:

First, the rating scale in the threshold model is discrete. Over a rather short risk horizon of 3 months, migrations between only 7 ratings are rare events so that much of the latent credit risk factors' interaction with spread changes may be filtered out. In contrast, *expected default frequencies* (EDFs) – the key variable of Moody's KMV – act on a continuous scale and change frequently. EDFs are derived from a structural credit model, combined with an empirical fitting to default rates, and can directly be interpreted as default probabilities.

Second, changes in external ratings tend to lag behind market implied measures of credit risk, which EDFs belong to. As I also observe that external ratings lag behind spread changes, the EDFs may easily have strong links to spread changes while rating changes do not or do so to a lesser extent. Hence, an EDF-based portfolio model could assign a much higher probability to domains of MRI than a model relying on historical rating migrations. Indeed, this is what I find.

Third, the general level of credit risk likewise matters. The continuous model's level may be different from the threshold model, particularly over a relatively short risk horizon. Presumably, it will be larger inasmuch as external ratings are often designed to "look through the cycle". Rating agencies try to filter out changes in credit quality that they perceive to be transient, thus lowering migration rates. This may give a biased picture of "actual" credit risk in the plain sense of what a bond might be worth at the end of the risk horizon.

EDFs are not subject to filtering or smoothing in the time dimension. If credit risk is now larger in an EDF-based continuous model, it may foster UbS because separate risk measurement relies on spread changes for the *current* rating while correct measurement is based on spread changes for the *future* rating. The more distant the future rating tends to be from the current one, the worse the current rating's spread changes will function as a proxy for the actually relevant ones.

The continuous model relies on the same vector W of latent systematic factors and the "asset returns" X_i responsible for credit risk as in the threshold model. Apart from the spread function being extended from 7 ratings to the continuum of possible EDFs, the model chiefly differs from the threshold model in the way in which the latent factors X_i from (13) transform into credit changes. Below I list the main steps involved; more details are presented in the calibration section.

$EDF_{i,0}$ denotes the *expected default frequency* of obligor i at time 0. EDFs can directly be interpreted as a one-year default probability.[21] Each EDF is traced back to a *distance to default* (DtD) denoted $D_{i,0}$, which is conceptually linked to a firm's capital structure and the "asset returns" X_i. The DtD can be thought of as a capital buffer held by the obligor against default, however it is not measured in dollars but in standard deviations of the obligor's random value of assets. The relationship between DtD and EDF is established by a decreasing deterministic function $M : DtD \mapsto EDF$. In data work, the inverse of M is applied to the given $EDF_{i,0}$ in order to obtain $D_{i,0}$, the DtD at time zero.

In simulations, random DtDs at the end of the risk horizon are generated by $D_{i,1} = \Delta_i + D_{i,0}$. As the variable Δ_i primarily has the role of an asset return, it is set to be generated by the standard normal factor X_i that otherwise drives credit migrations in the threshold model. Each Δ_i is therefore obtained from a transformation $H : X_i \to \Delta_i$ that is simply linear in the base case analysis and matched to moments of a sample of Δ_i (see Section 4.1); in a robustness test, H becomes nonlinear to generate heavy tails, as found in the sample.

Each DtD at the end of the risk horizon defines a new $EDF_{i,1} \equiv M(D_{i,1})$, supposing that

[21] KMV also reports EDFs over other horizons, but data coverage is best for one year.

Table 1: Assigning default probabilities to rating classes. Source: Standard and Poor's (2013), Table 9 "Global Average One-Year Transition Rates, 1981–2012". *The value for AAA (originally 0) is raised to 0.01%, as a result of exponential fitting (cf. Bluhm et al., 2003).

Rating	AAA*	AA	A	BBB	BB	B	CCC–C
EDF	0.01%	0.02%	0.07%	0.22%	0.86%	4.29%	26.88%

$D_{i,1}$ is large enough; if not, a default is triggered. Interpreting the phrase "distance to default" literally, a default should occur if $D_{i,1}$ is negative. As this does not lead to the exact default rates implied by the EDF (which is a default probability, albeit on a one-year horizon), I choose a default threshold different from zero. It is calibrated such that the default probability over the horizon of 3 months corresponds to the one-year EDF. Assuming a constant hazard rate[22], the default threshold DT_i is implicitly defined by

$$\mathbf{P}\left(D_{i,1} < DT_i\right) = 1 - \left(1 - EDF_{i,1}\right)^{0.25}. \tag{19}$$

A default is triggered if $D_{i,1} < DT_i$, in the event of which the same independent default loss λN is drawn as in the threshold model.

In parallel to the factors X_i, new time-1 spreads are drawn as in Section 3.1. However, the spread function must now be defined on the whole unit interval as EDFs can take any value in [0.1]. To this end, the ratings that spread indices refer to are mapped to fixed EDFs (Table 1), which gives random future spreads on a grid of 7 points. I connect them using suitably calibrated Brownian bridges and extend the spread function beyond the worst EDF's gridpoint (corresponding to rating CCC-C) by an extrapolation that also involves a Brownian bridge. The exact details are outlined in Section A.2.

The initial spread function $s_0(\cdot)$, now defined on $[0, .1]$, is set once and kept fixed.[23] The value of each bond i at time 0 is given by $V_{\text{bond}}(0, s_0, EDF_{i,0})$. Each simulation round involves a drawing of s_1 and new EDFs, followed by the revaluation of bonds at $V_{\text{bond}}(1, s_1, EDF_{i,1})$ for total risk measurement, $V_{\text{bond}}(1, s_0, EDF_{i,1})$ for migration induced risk measurement, and $V_{\text{bond}}(1, s_1, EDF_{i,0})$ for spread risk measurement. VaR, ES, and RI are calculated as before.

4 Data and calibration

All the time series used span the period from July 1999 to October 2012.

Spread indices are given by quarterly Merrill Lynch Euro corporate spread indices for the following rating classes: AAA, AA, A, BBB, BB, B, and CCC–C as an aggregate class. Further types of differentiation, such as between countries, industries or maturities, are not available in combination with rating classes.[24] The upper graph in Figure 2 shows the time series of the spread indices. The first panel of Table 2 contains covariances of spread returns, which make up

[22] A constant hazard rate is not fully consistent with the structural model that EDFs stem from. I neglect this imperfection for simplicity.

[23] At the gridpoints that correspond to ratings, the spread is set equal to the data mean. Between these points I apply log-log-linear interpolation. This procedure is used for time-0 spreads only.

[24] The indices cover "EUR denominated corporate debt publicly issued in the eurobond or Euro member domestic markets..." for the respective rating categories. (download from http://www.mlindex.ml.com/gispublic/bin/IndexRules.asp). I use the indices ER10, ER20, ER30, ER44 for investment grade and HE10, HE20, and HE30 for high-yield bonds. The eurobond market is not restricted to European corporates; however, the indices put, at least, more weight on Europe than the global indices. Indices restricted to European corporates exist but do not differentiate between rating classes, only between investment and non-investment grade. Consistent spread indices are available from April 1999 on.

the first 7 rows and columns in the covariance matrix Σ of the latent systematic factors W.

As Conjecture 1 suggests that the relationship between spread changes and the single systematic credit risk factor (SCRF) is important for UbS, I try a number of alternative empirical proxies for the SCRF, based on three different data sources. The first source is a sample of one-year EDFs (plus the variables *equity*, *asset volatility* and *default point*) for all Western European firms contained in the KMV database. This data is used to construct a time series that can be interpreted as a systematic component of firms' asset returns. S&P issuer rating events are the second data source, from which I calculate quarterly downgrade frequencies that are further transformed into a SCRF time series. The third alternative data source consists of returns taken from the MSCI-Europe stock index, chosen in accordance with the European scope of the KMV data.

Despite the conceptual link between the threshold model and a downgrade-based SCRF, just as between the continuous model and a KMV-implied SCRF, I will treat the empirical specification of the SCRF independently of the model into which they feed. This approach merely represents an attempt to cope with model risk. For instance, aside from the possibility that one model simply may do better than the other, it could well be true that downgrade frequencies capture large-scale changes in credit risk best, while a continuous model hits quantiles of losses in bond portfolios best – or vice versa.

The next two subsections explain how the KMV-based and the downgrade-implied SCRF are constructed. Section 4.3 introduces further alternatives for defining a SCRF and explores the empirical relationship between spread changes and each of the SCRFs.

4.1 A systematic credit risk factor based on KMV

The KMV dataset consists of $EDF_{i,t}$, the expected default frequency of an obligor i at time t, and, furthermore, $A_{i,t}$ (asset value), $DP_{i,t}$ (the so-called default point, a weighted sum of short- and long-term debt), and $\sigma^A_{i,t}$ (asset volatility). There are between 4,361 and 5,091 companies in the cross-section, depending on the quarter.

The EDF has a close, almost deterministic empirical relationship with the *distance to default* (DtD in short), which is calculated as[25]

$$D^{\text{data}}_{i,t} \equiv \left(\log\left(A_{i,t}/DP_{i,t}\right) - \frac{1}{2}\left(\sigma^A_{i,t}\right)^2 \right) \Big/ \sigma^A_{i,t} \tag{20}$$

The superscript "data" signalizes that this DtD is not the variable I ultimately work with.

It seems particularly suitable to construct a SCRF from DtDs for two reasons. First, as the DtD is related to a capital buffer, changes in the DtD are related to asset returns which, in turn, act as the role model for the latent credit risk factors X_i. Second, the asset volatility in the denominator of (20) normalizes DtD changes so that their distribution is much more homogeneous across firms, and especially so compared to EDF changes; the DtD distribution also exhibits greatly reduced asymmetry. For these reasons it makes sense to interact credit risk in the DtD space and to empirically filter a SCRF out of co-moving DtD changes.

However, as I am ultimately interested in EDF changes, and as the noise in the relationship between $EDF_{i,t}$ and $D^{\text{data}}_{i,t}$ is not a feature of the credit risk model but a consequence of insufficient information on the KMV model, I wish to eliminate this randomness. Consequently, $D^{\text{data}}_{i,t}$ is only used for the purpose of fitting a theoretic link between DtDs and EDFs to the data, and is subsequently neglected. This fitting procedure is the first of the next three steps towards a time series for the KMV-based SCRF:

[25] The EDF is actually a deterministic function of the variables mentioned, and further input factors unobservable to the researcher.

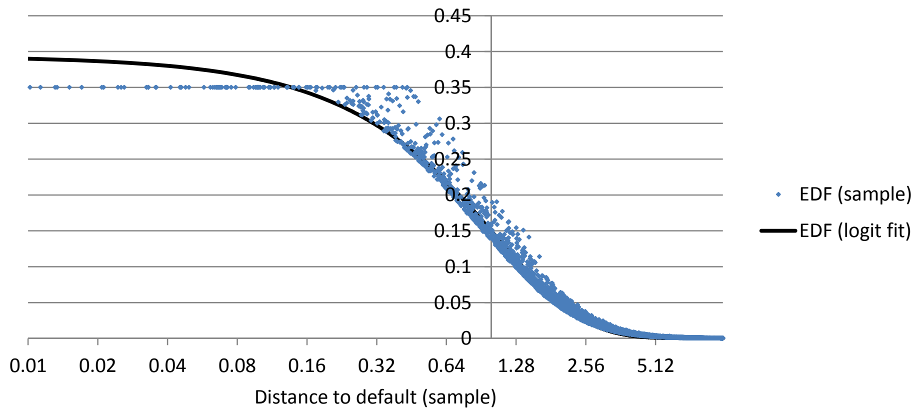

Figure 1: Fitted function M, mapping DtDs to EDFs. DtD on the axis is on a logarithmic scale. This is $D_{i,t}^{\text{data}}$, as defined in (20).

Step 1 concerns the decreasing deterministic function $M : DtD \mapsto EDF$. Plotting $EDF_{i,t}$ against $D_{i,t}^{\text{data}}$ reveals a close relationship to which M is fitted by minimizing mean squared errors. Various tests have shown that a logit transform of a power function of DtD performs well. Figure 1 shows a scatter plot of DtD/EDF pairs and the fitted function

$$M(D) \quad = \quad \left(1 + \exp\left\{\frac{D^p - c_1}{c_2}\right\}\right)^{-1} = \left(1 + \exp\left\{\frac{D^{0.9791} + 0.3321}{0.7667}\right\}\right)^{-1}.$$

Having fitted M, I replace $D_{i,t}^{\text{data}}$ with the EDF-implied DtD

$$D_{i,t} \equiv M^{-1}\left(EDF_{i,t}\right),$$

where M^{-1} is the inverse of M.[26]

In **Step 2**, the distribution of DtD changes is fitted to the data. The pooled empirical distribution of quarterly DtD changes

$$\Delta_{i,t} = D_{i,t+1} - D_{i,t}$$

is fairly symmetric, but it has heavy tails which give rise to a robustness check performed in Section 6. In base-case simulations, DtD changes are simply drawn from a normal distribution, with moments being matched to the pooled sample of $\Delta_{i,t}$.

In **Step 3**, DtD changes are aggregated to a SCRF. The unweighted average

$$Y_t^{\text{KMV}} \equiv N_t^{-1} \sum_{i=1}^{N_t} \Delta_{i,t}$$

is taken over all firms in the sample for each quarter t.[27]

The construction of Y_t^{KMV} allows me to check the choice for the asset correlation $\rho = 20\%$,

[26]The inverse is undefined for EDFs above 0.393. However, it is needed for *initial* EDFs only, which are always below this value for the portfolios analyzed.

[27]Simple averaging may appear strange for an index. However, DtD changes are already fairly homogeneous by construction. Furthermore, as a DtD change with a low initial EDF behind it may be equally informative about systematic changes as one with a high EDF behind, I see no good reason for weighting. Similar arguments apply to firm size. To draw an analogy to credit default indices, the CDX and the iTraxx are both also unweighted averages in a sense because, if one of its constituents defaults, the payoff is the same, whoever has defaulted.

which is close to typical values used in the calculation of IRB[28] capital requirements for corporate loans. The average correlation between Y_t^{KMV} and $\Delta_{i,t}$ is 0.465, which corresponds to an estimated $\rho = 21.6\%$. This value confirms my choice for the base case; other correlations are subject to a robustness test.

4.2 A systematic credit risk factor based on rating downgrades

As the threshold model is calibrated to frequencies of credit migrations, it is perfectly natural to distill a SCRF from the same data source. For simplicity, I forgo an elaborate ML estimate that would distinguish migrations both by source and destination rating. The estimation is instead restricted to downgrade and default events.

The "asset returns" X_i as defined in (13) remain the same as in Section 3.1, but there is now just one uniform threshold: all downgrades (including defaults) from all ratings are triggered by X_i falling below that threshold.[29] Let p_t be the aggregate rate of downgrades in quarter t and $\hat{p}$ its average over all periods. Assuming that ρ is known and $\Phi^{-1}(\hat{p})$ is the exact default threshold, ideally I would like to determine the value Y_t for the SCRF that maximizes the likelihood of the observed rate p_t, conditional on Y_t. The precise ML value is influenced by discreteness intricacies which, however, do not contribute much to the estimation if the number of rated firms in a certain quarter is large enough. If the number goes to infinity, literature shows that the ML estimate converges to

$$Y_t^{\text{dg}} \equiv \frac{1}{\sqrt{\rho}} \left(\Phi^{-1}(\hat{p}) - \sqrt{1-\rho}\,\Phi^{-1}(p_t) \right),$$

(Bluhm et al., 2003, ch. 2.5). Y_t^{dg} defines the downgrades-implied SCRF used in this paper. The particular choice of ρ is actually irrelevant as Y_t^{dg} is subsequently normalized, meaning that ρ cancels out.

The sample of S&P rating events used for estimation spans the period 1999:Q1–2010:Q4. It conveys 36,743 rating events relating to 13,630 firms.

4.3 Further alternatives and comparison

I observe considerable autocorrelation in the estimated time series of the two SCRFs introduced thus far: Y^{KMV} has an AR(1) coefficient of 0.50 and Y^{dg} of as much as 0.73. Such a high autocorrelation raises the question of whether it is perhaps better to focus on the unpredictable component of the risk factors. In fact, it could be argued that banks can – and in fact should – use loss forecasts to build suitable provisions for expected future losses from credit events. Where they do so, it is the unpredictable component of losses that is to be covered by capital. This idea prompts me to include the AR(1) residuals of the time series estimates, denoted by $Y^{\text{KMV,res}}$ and $Y^{\text{dg,res}}$, as further SCRF specifications. Their autocorrelations around -0.05 are insignificant.

As a further alternative, an equity index can also contain relevant information on the credit risk of its constituents. Indeed it should do because, in line with the idea behind structural credit risk models, changes in a firm's asset value simultaneously carry over to equity and debt. I calculate quarterly log returns of the MSCI Europe, denoted by Y_t^{MSCIE}.

[28]IRB refers to the Internal-Ratings Based Approach to capital requirements under Basel III.

[29]Strictly speaking, a uniform threshold could only be consistent with the migration model if all quarterly downgrade probabilities were the same across source ratings. The variation of probabilities is moderate though, with one exception: 2.80% for downgrades from AAA, 2.17% (AA), 1.59% (A), 1.46% (BBB), 2.81% (BB), 3.03% (B), but 12.50% for downgrades from class CCC–C; these are actually defaults. As the sample of such bonds is small anyway, I accept the inconsistency. For more advanced methods, see e.g. Meyer (2009).

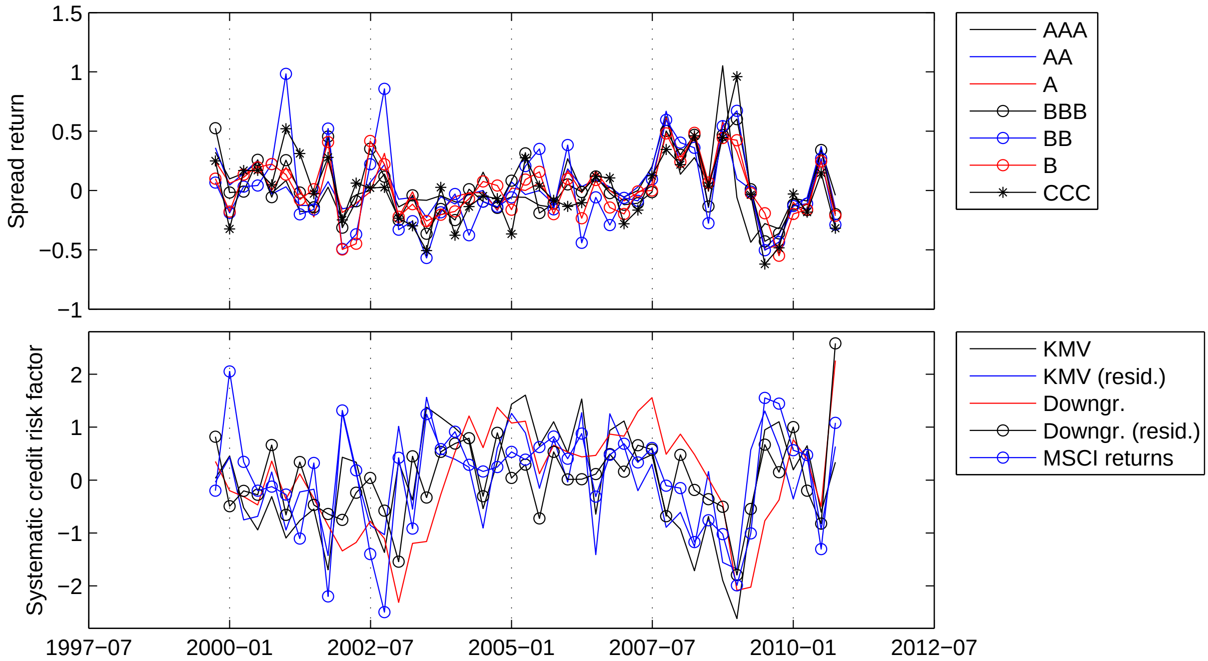

Figure 2: Spread indices and specifications of the systematic credit risk factor. Quarterly data. Rating-specific spread indices are taken from the Merrill Lynch Euro corporate indices. *Spread returns* (upper plot) are changes in log spreads. Credit risk factors (lower plot) are standardized. *KMV* is the change in the average KMV distance-to-default across European corporates. *KMV (resid.)* consists of the AR(1) time series residuals of *KMV*. *Downgr.* consists of ML estimates using an asymptotic single-risk-factor threshold model for quarterly frequencies of S&P corporate rating downgrades. *Downgr. (resid.)* consists of its AR(1) residuals. *MSCI* is defined by log returns of the MSCI Europe stock index.

Figure 2 shows how spread returns and SCRFs evolve through time. Spread returns in the upper graph are obviously highly correlated, which also shows up in the covariances presented in panel A of Table 2. Inspecting standard deviations in panel B, they are rather homogeneous with one exception. The markedly higher standard deviation for BB spread returns will affect the occurrence of UbS.

The lower panel in Figure 2 gives less of a clear pattern for SCRFs. While some of the factors co-move in certain periods such as the financial crisis, there are periods of diverging movements as well.

Panel C of Table 2 shows correlations of the SCRFs with the spread returns, which Conjecture 1 predicts to be important. Three of the factors, Y^{KMV}, $Y^{\mathrm{KMV,res}}$ and Y^{MSCI}, are strongly negatively related to spread widening. Recalling that low SCRF values mean high credit risk, a negative correlation indicates that bond investors tend to require a high premium for credit risk – which is fixed to a certain rating – in times when the general level of credit risk rises. Observing strong negative correlations exactly for the factors that depend much on equity prices, matters look different in the case of the the downgrade-based factor Y^{dg}. No clear relationship to spreads is identified as its correlations range from slightly positive to slightly negative. By contrast, the residuals of Y^{dg} are more similar to the equity-based factors, with moderately negative correlations.

Table 2: Variance measures of the systematic factors. All values are in percent. Quarterly data from July 1999 to October 2010. Spreads are Merrill Lynch Euro corporate spread indices. Y^{KMV} is derived from changes in KMV's average distance-to-default, calculated for a large panel of European corporates. $Y^{\text{KMV,res}}$ consists of the AR(1) time series residuals of Y^{KMV}. Downgrades of S&P corporate ratings are the basis for Y^{dg}. It is the ML estimate of the systematic factor in an asymptotic single-risk-factor threshold model for downgrade frequencies. $Y^{\text{dg,res}}$ consists of its AR(1) residuals. Y^{MSCI} is defined by log returns of the MSCI Europe stock index.

Panel A – Covariance matrix of quarterly log returns of spreads							
	AAA	AA	A	BBB	BB	B	CCC–C
AAA	6.07						
AA	4.64	4.88					
A	4.57	4.81	5.47				
BBB	4.11	4.71	5.35	6.89			
BB	4.80	5.42	6.77	7.17	13.65		
B	3.88	4.14	4.84	5.44	7.58	6.52	
CCC–C	3.55	4.13	5.27	6.39	7.86	5.55	9.04
Panel B – Standard deviations							
	24.6	22.1	23.4	26.2	36.9	25.5	30.1
Panel C – Correlations with systematic credit risk factors							
Y^{KMV}	–42.9	–52.1	–67.1	–65.2	–64.9	–74.3	–71.6
$Y^{\text{KMV,res}}$	–55.8	–60.9	–74.0	–69.7	–72.0	–77.3	–70.3
Y^{dg}	15.9	6.7	1.9	–4.3	–6.1	–1.3	–9.5
$Y^{\text{dg,res}}$	–13.9	–27.8	–32.3	–28.7	–35.3	–28.9	–36.3
Y^{MSCIE}	–30.6	–48.3	–63.1	–68.3	–64.1	–71.6	–66.3

5 Results

5.1 Threshold model

The base-case portfolio contains 1000 homogeneous, equally rated bonds. The rating is worsened in steps from 1 (AAA) to 7 (CCC–C), while the spread-related part of the covariance matrix Σ has constant values as presented in panel A of Table 2; the covariances of the SCRF with the spread returns are set according to the correlations given in panel C, depending on the choice of the SCRF. The asset correlation ρ is set to 20%, as explained below (13).

In each step, represented by the selection of a uniform time-0 rating and a SCRF (which shapes Σ), I simulate one million portfolio returns and calculate the risk measures VaR and ES at security levels of 5%, 1%, 0.5% and 0.1%, for ΔV_{total}, ΔV_{mig}, and ΔV_{spread}. Formula (18) yields the risk interaction index RI, which indicates UbS if value 1 is exceeded.

Table 3 presents results for the value-at-risk (VaR), with all values shown in percent. Columns [1] and [2] list the separate spread and credit VaRs, depending on the uniform rating and α. Of course, the separate VaRs are independent of the way credit and spread risks interact; so only the total VaR is dependent on the empirical specification of the SCRF. The total VaR is displayed only for Y^{KMV}, as an example, since it contains the same information as the corresponding RI, as total VaR divided by the sum of separated VaRs. RI is displayed for all five versions of the SCRF.

My general finding from the threshold model is that total risk is seldom, and never strongly, underestimated by separate risk measurement. Whether UbS is in place depends in large part on the initial rating and, more importantly, on the SCRF chosen: RI is highest for those SCRFs that are most strongly negatively correlated with spread returns (cf. Table 2). This result confirms Conjecture 1.

UbS proves to be strongest with BBB rated bonds. As described in Conjecture 2, this finding is due to an exceptional rise in spread volatility from BBB (26.2%) to the next-worse rating BB (36.9%). Here, the spread part of separate risk measurement simply ignores the fact that the valuation of a bond that has been downgraded to BB is subject to a much higher volatility than that of a bond with a stable rating. Generally speaking, RI as a function of the rating appears to have a hump shape. There is a multitude of possible explanations (such as independent drawings of LGDs), which are disentangled in Section 6.

Finally, I also observe a dependency of RI on the severity level of the VaR: the smaller α is, i.e., the more the measure zooms into the loss tail, the larger RI is. This finding is somewhat surprising as the systematic factors interact under a normal copula, which is known to "lose dependency" if increasingly conditioned on tails. Copulae with a higher tail dependency might make RI more sensitive to the severity level.

The results presented thus far refer to the VaR, while Table 10 in the appendix shows values for the expected shortfall. ES- and VaR-based RI values are very similar, with a maximum relative deviation of 4.7%. There seems to be a strong proportionality between the exceedances of RI_{ES} and RI_{VaR} above the critical 100%. Indeed, interpreting all RI values reported in Table 3 and Table 10 as random "observations" (depending on the SCRF, the initial rating, and α) and regressing their exceedances of 100% on each other, the OLS estimate

$$RI_{ES} - 1 = -0.00414 + 1.074\,(RI_{VaR} - 1) + \varepsilon$$

has an R^2 of 99.2 percent.

Finding the propensity to UbS to be more or less the same for ES and VaR emphasizes yet again that the known flaws of the VaR are not to blame when it comes to UbS. Rather, the problem is the very risk modeling itself that occurs before risk measures comes into play.

Although all analyses in this paper are performed for both VaR and ES, I have chosen not to report any further ES related figures as all the results are qualitatively the same.

All in all, while the threshold model indeed includes cases where total risk is underestimated by separation, the extent to which this happen is not alarming. UbS totally vanishes for Y^{dg} and $Y^{\text{dg,res}}$, which are most consistent with the threshold model in that they both rely on rating migration data. Nevertheless, as explained at the beginning of Section 3.2, some skepticism is warranted as to whether any actual malign risk interaction is potentially filtered out by the particular design of the threshold model.

5.2 Continuous model

Although ratings are replaced by EDFs in the continuous model, the base case portfolio is actually the same as before, except for the singular difference that ratings are renamed as EDFs according to Table 1. In simulations, the uniform EDF progresses in steps through these values, from one basis point up to nearly 27%. In each step I calculate VaRs (unreported and ESs) at varying security levels, again for total risk, credit/migration risk, and spread risk.

Table 4 shows VaRs and risk interaction indices as Table 3 does for the threshold model. Unlike in the latter, I identify significant UbS, with cases of total risk almost 50 percent larger than the sum of spread and credit risk. UbS is evident for rather low EDFs, corresponding to good ratings: fairly exactly, the investment grade is affected. There is a stronger relationship between the security level α and RI than in the threshold model. Again, the choice of the SCRF has a large impact on the strength of UbS, and the order is the same as before.

Table 3: Separate and total value-at-risk (VaR) in the threshold model. All numbers are shown in percent. The table presents the VaR (at varying security level α) of a portfolio of 1,000 homogeneous bonds with a uniform rating. Risks are either exclusively induced by *spreads*, by migrations and defaults (*credit*), or by both risk types (*total*). *RI* is the total risk over the sum of the separate risks; values above 1 (bold) indicate an underestimate of total risk by separate measurement. The results depend on estimated correlations with the empirical proxy for the systematic credit risk factor (SCRF): Y^{KMV} is derived from changes in KMV's distances-to-default of European corporates. $Y^{\mathrm{KMV,res}}$ consists of its AR(1) residuals. Y^{dg} is based on downgrades of S&P corporate ratings. $Y^{\mathrm{dg,res}}$ consists of its AR(1) residuals. Y^{MSCI} is defined by log returns of the MSCI Europe stock index.

Rating	α	Spread	Credit	Total	Risk interaction index *RI*				
		(all Y)	(all Y)	Y^{KMV}	Y^{KMV}	$Y^{\mathrm{KMV,res}}$	Y^{dg}	$Y^{\mathrm{dg,res}}$	Y^{MSCIE}
		[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
AAA	0.1	1.95	0.90	2.57	90.1	95.4	71.8	78.4	85.7
	0.5	1.50	0.52	1.85	91.4	94.5	77.7	83.3	88.5
	1.0	1.31	0.39	1.57	92.5	95.4	80.6	85.5	90.2
	5.0	0.83	0.18	0.97	96.3	97.9	87.5	91.9	95.0
AA	0.1	2.89	1.03	3.83	98.0	**102.0**	77.5	85.6	96.5
	0.5	2.25	0.62	2.80	97.6	**100.4**	82.3	89.0	96.3
	1.0	1.96	0.49	2.39	97.7	**100.2**	84.5	90.6	96.7
	5.0	1.25	0.24	1.48	99.3	**100.4**	90.2	94.8	98.8
A	0.1	5.09	1.29	6.32	99.1	**100.6**	81.9	87.4	98.2
	0.5	3.95	0.75	4.64	98.7	**100.2**	86.1	90.9	98.0
	1.0	3.45	0.57	3.97	98.7	**100.0**	88.2	92.3	98.3
	5.0	2.20	0.25	2.45	99.8	**100.5**	92.9	96.1	99.9
BBB	0.1	8.25	3.44	11.93	**102.1**	**105.1**	76.2	85.1	**102.3**
	0.5	6.40	2.14	8.55	**100.1**	**102.4**	81.0	88.0	**100.4**
	1.0	5.58	1.68	7.24	99.6	**101.7**	83.3	89.4	**100.4**
	5.0	3.53	0.81	4.35	**100.3**	**101.2**	89.0	93.7	**100.6**
BB	0.1	30.11	5.19	31.03	87.9	87.9	84.4	85.7	87.9
	0.5	23.39	3.35	24.35	91.1	91.6	86.7	88.6	91.0
	1.0	20.32	2.67	21.27	92.5	92.9	87.9	89.9	92.5
	5.0	12.72	1.33	13.47	95.8	96.2	90.6	93.0	95.7
B	0.1	25.49	11.52	31.47	85.0	85.7	71.4	76.1	84.3
	0.5	20.20	8.03	25.12	89.0	89.1	75.6	80.1	88.1
	1.0	17.91	6.63	22.16	90.3	90.9	77.4	82.0	89.9
	5.0	11.72	3.63	14.49	94.4	94.6	83.3	87.5	93.7
CCC–C	0.1	47.03	28.52	49.16	65.1	65.0	61.4	63.2	64.9
	0.5	39.06	23.05	43.50	70.0	70.0	63.7	66.9	69.6
	1.0	35.12	20.44	40.28	72.5	72.4	65.1	68.8	72.0
	5.0	23.95	13.85	30.00	79.4	79.2	69.8	74.3	78.6

However, even the downgrade-implied SCRF Y^{dg}, which is largely uncorrelated to spread returns, includes an example of UbS for $EDF = 0.22\%$ at $\alpha = 0.1\%$. This observation is in line with Conjecture 2, which predicts UbS at points where the EDF-dependent spread volatility grows the most steeply.

The remarkable drop in RI at $EDF = 0.86\%$ tells the same story, with an opposite sign: spread volatility peaks at that point (Table 2, panel B), implying a decreasing volatility if EDF continues to grow. That decrease leads to an overestimate in separate risk measurement because bonds receiving a worse EDF in fact leave a domain of very uncertain spreads, while separate spread risk measurement presumes the volatility to remain the same.

It comes as no surprise that no UbS is evident for the worst EDF tested as the loss tail in the integrated model is dominated by defaults, which make up more than one-fourth of the individual loans' returns. As the LGD distribution is independent of any spread factor, separate credit risk measurement is simply perfect as far as default events are concerned; no relevant risk factor is missing, as is true if downgrades to non-default states are relevant for large portfolio losses. By contrast, separate spread risk measurement detects virtual losses from rising spreads of bonds that have already defaulted.

Different effects mix when the EDF is 4.29 percent. On the one hand, defaults also play an important role here, working against UbS, as explained above. On the other hand, the spread volatility rises for $EDF > 4.29\%$[30], thus making Conjecture 2 relevant, and the correlation between spread changes and the SCRF is closest to -1 among all spread factors (with the exception of $Y^{\text{dg,res}}$), meaning that Conjecture 1 applies. The counteracting forces result in RI values around 100 percent.

While the effects of SCRF, rating and α on RI_{VaR} are similar in both models, the similarity is much weaker than that discernible between RI_{VaR} and RI_{ES} within each model. To gain an impression of the relationship, Figure 3 plots all RI_{VaR} values of Table 3 (threshold model, x-axis) against Table 4 (continuous model, y-axis). Clearly, a link exists, but it is moderate in nature; an OLS regression delivers an R^2 of 55 percent. Naturally, the fact that UbS is substantial in the continuous model but not in the threshold model is the most important difference here. This observation is investigated in the next subsection.

5.3 What makes the difference?

This section investigates two hypotheses regarding why UbS is evident in the continuous model but negligible in the threshold model. There is no simple, general answer to this question as the models differ in several ways. It is also difficult to determine which model might prove more realistic: neither of them is "right" or "wrong" in their approach to the market valuation of credit, as the dynamics of external ratings (relevant for CreditMetrics) and equity prices (for KMV) each have their own merits in explaining bond and CDS price changes.

However, if we make a comparison of columns [2] and [3] in Table 5, which show credit VaRs at $\alpha = 0.1\%$ for both models, using Y^{KMV} as SCRF, it becomes clear that the models generate very different amounts of credit risk. While the credit VaRs of the continuous model are at the same order of magnitude as spread risk (which is identical for both models), the threshold model generates much smaller credit VaRs.

It is possible that virtually no UbS is present in the threshold model, solely on account of the fact that there are so few migrations: recalling Section 2.2, migration is critical as it shifts assets to classes where spread volatility and correlation are different from those in the initial class. It is this shift that is neglected by risk separation. I perform two exercises in order to gain

[30] Spread volatility increases from 25.5% for $EDF = 4.29\%$ to 30.1% for $EDF = 26.88\%$; see Table 2.

Table 4: Separate and total value-at-risk (VaR) in the continuous model. All numbers are shown in percent. The table presents the VaR at varying security levels α of a portfolio of 1000 homogeneous bonds with a uniform EDF, which correspond to the ratings (in brackets) of the threshold model. Risks are either exclusively induced by *spreads*, by migrations and defaults (*credit*), or by both risk types (*total*). *RI* is the total risk over the sum of the separate risks; values above 1 (bold) indicate an underestimate of total risk by separate measurement. Results depend on estimated correlations with the empirical proxy for the systematic credit risk factor (SCRF): Y^{KMV} is derived from changes in KMV's distances-to-default of European corporates. $Y^{\text{KMV,res}}$ consists of its AR(1) residuals. Y^{dg} is based on downgrades of S&P corporate ratings. $Y^{\text{dg,res}}$ consists of its AR(1) residuals. Y^{MSCI} is defined by log returns of the MSCI Europe stock index.

EDF	α	Spread	Credit	Total	Risk interaction index *RI*				
		(all Y)	(all Y)	Y^{KMV}	Y^{KMV}	$Y^{\text{KMV,res}}$	Y^{dg}	$Y^{\text{dg,res}}$	Y^{MSCIE}
		[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
0.01	0.1	1.96	3.05	6.40	**128.0**	**133.8**	83.1	**104.2**	**127.5**
(AAA)	0.5	1.51	2.42	4.67	**119.1**	**123.7**	78.9	98.6	**117.9**
	1.0	1.31	2.14	3.94	**114.3**	**118.7**	77.0	95.9	**113.2**
	5.0	0.83	1.41	2.31	**103.5**	**107.3**	71.9	88.4	**102.1**
0.02	0.1	2.85	3.62	8.77	**135.5**	**140.0**	84.7	**107.8**	**134.6**
(AA)	0.5	2.23	2.81	6.27	**124.5**	**128.8**	81.4	**101.4**	**124.0**
	1.0	1.95	2.46	5.25	**119.3**	**124.4**	79.3	98.5	**119.1**
	5.0	1.25	1.58	3.06	**108.3**	**111.6**	73.3	90.5	**107.4**
0.07	0.1	5.10	7.24	17.88	**144.9**	**149.1**	95.2	**118.6**	**144.9**
(A)	0.5	3.95	5.73	12.97	**134.0**	**139.1**	90.1	**110.2**	**134.5**
	1.0	3.45	5.05	11.00	**129.4**	**133.3**	88.3	**107.0**	**128.8**
	5.0	2.21	3.37	6.48	**116.2**	**119.4**	83.5	98.4	**116.3**
0.22	0.1	8.17	12.81	29.76	**141.9**	**144.1**	**101.4**	**120.3**	**141.2**
(BBB)	0.5	6.38	10.78	23.17	**135.1**	**138.2**	98.5	**115.6**	**134.9**
	1.0	5.55	9.86	20.31	**131.7**	**134.4**	96.6	**113.1**	**131.2**
	5.0	3.52	7.46	13.25	**120.7**	**123.5**	91.8	**105.4**	**120.7**
0.86	0.1	30.09	12.19	33.82	80.0	80.8	59.1	68.2	79.0
(BB)	0.5	23.50	9.46	26.80	81.3	82.2	59.6	69.3	80.7
	1.0	20.49	8.22	23.39	81.5	82.5	59.3	69.2	80.8
	5.0	12.89	5.10	14.59	81.1	82.3	57.8	68.2	80.4
4.29	0.1	25.30	23.56	47.84	97.9	98.3	77.8	86.6	96.7
(B)	0.5	20.28	19.63	40.01	**100.2**	**100.3**	78.6	88.3	99.2
	1.0	17.89	17.74	36.04	**101.1**	**101.0**	79.0	89.0	**100.1**
	5.0	11.75	13.04	25.16	**101.5**	**101.6**	80.0	89.5	**100.3**
26.88	0.1	46.94	26.75	50.27	68.2	68.2	60.1	64.7	67.6
(CCC–C)	0.5	39.06	21.38	43.99	72.8	72.5	61.8	67.4	72.0
	1.0	35.00	18.73	40.20	74.8	74.6	62.4	68.5	73.8
	5.0	23.96	11.83	28.40	79.3	79.1	63.9	71.3	78.2

Table 5: Risk interaction in modifications of the threshold model. All values are shown in percent. VaRs, all for $\alpha = 0.1\%$, refer to the base case portfolio of 1,000 bonds with a uniform rating (corresponding EDFs in brackets). Correlation assumptions rely on Y^{KMV} as SCRF. The pure spread risk is identical in all set-ups. RI is total VaR (unreported), divided by the sum of credit and spread VaR; bold figures indicate UbS. Transition probabilities (away from the current rating to any other state) are reported on a 1-year basis for ease of comparison. Columns [1], [3] and [11] are from Table 3; [2] and [10] are from Table 4. In columns [4], [8] and [12], the threshold model's SCRF Y is scaled up (by rating dependent constants) to harmonise the credit VaR [4] with that featured in the continuous model [2] (neglected for CCC–C, as this would require down-scaling). Columns [5], [9] and [13] show results from the threshold model using a transition matrix from the literature (Gupton et al., 1997, Table 6.3) which was calibrated to changes in KMV's EDFs.

Variable:	VaR_{spread}	VaR_{credit}				Transition probability (1 year)				Risk interaction index RI			
Model:	all	cont.	threshold			cont.	threshold			cont.	threshold		
Trans. matrix:	–	–	orig.	scaled	KMV	–	orig.	scaled	KMV	–	orig.	scaled	KMV
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
AAA (0.01)	1.96	3.05	0.90	3.05	2.77	100	9.8	14.2	33.7	**128.0**	90.1	**108.0**	**113.9**
AA (0.02)	2.85	3.62	1.03	3.62	3.45	100	10.1	15.7	57.0	**135.5**	98.0	**110.2**	**122.3**
A (0.07)	5.10	7.24	1.29	7.24	4.33	100	8.4	17.0	55.8	**144.9**	99.1	**105.4**	**121.8**
BBB (0.22)	8.2	12.8	3.44	12.8	10.1	100	9.1	17.4	57.5	**141.9**	**102.1**	**108.0**	**133.2**
BB (0.86)	30.1	12.2	5.19	12.2	6.73	100	15.7	22.2	55.6	80.0	87.9	78.6	81.6
B (4.29)	25.3	23.6	11.5	23.6	15.5	100	16.4	23.7	47.0	97.9	85.0	78.5	96.1
CCC–C (26.8)	46.9	26.7	28.5	–	14.7	100	48.8	–	30.1	68.2	65.1	–	77.4

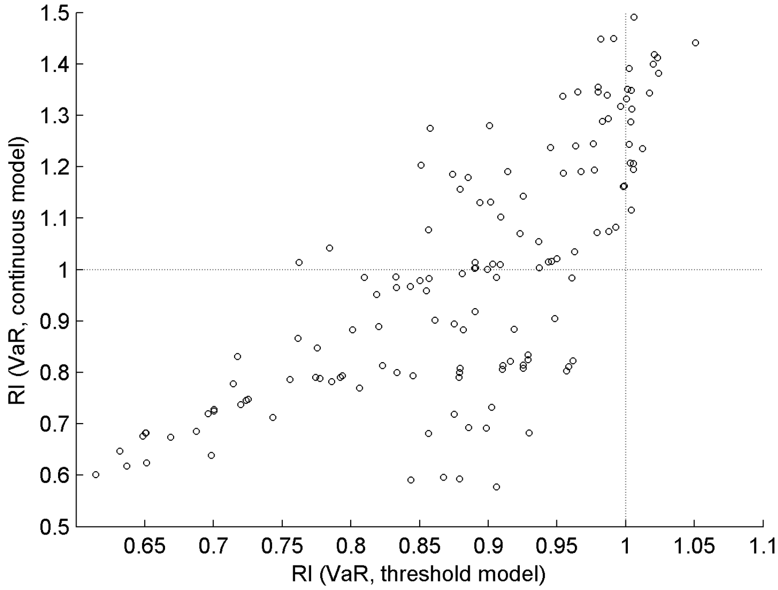

Figure 3: Risk interaction indices (VaR) for both models. Values on both axes are risk interaction indices, indicating an underestimation of total risk if above 1. Values on the x-axis are calculated under the threshold model, conveying all RI values from Table 3. They vary depending on SCRF, rating and α. Values on the y-axis are from Table 4, calculated under the continuous model.

an impression of the extent to which RI in the threshold model depends on the level of portfolio credit risk.

In the first exercise, I scale the systematic credit risk factor artificially up by a factor h such that the latent credit risk factor of bond i is

$$X_{i,h} = h\sqrt{\rho}Y + \sqrt{1-\rho}Z_i. \tag{21}$$

Raising h increases transition probabilities (cf. column [8] in Table 5) and, consequently, the portfolio credit VaR. In doing so, I force the threshold model to generate higher credit VaRs than in the base case. Depending on the initial rating, the scaling factor h is calibrated such that the threshold model generates the same credit VaR as in the continuous model (so that column [4] matches [2]).[31]

The RI values in column [12] indicate that increased credit risk actually leads to UbS, however at a moderate level, compared to the continuous model in column [10]. There is again no UbS outside the investment grade. Another specification where the idiosyncratic factor Z_i is scaled along with the systematic factor (such that $X_{i,h} = hX_i$) gives similar results.

In the second exercise, I run the model using a transition matrix from the literature: alongside matrices based on agency ratings, Gupton et al. (1997, Table 6.3) present a transition matrix for 7 ratings plus default that had been calibrated to a sample of EDF changes. While its default probabilities are comparable to those of agency ratings (probably because it was the matching criterion), this matrix exhibits much higher total annual transition probabilities that are around one half, as shown in column [9]. Such a high level of migration is typical for transition matrices of bank-internal *point-in-time* ratings (see below). If transition frequencies are of this order, RI values in column [13] reach levels comparable to those seen in the continuous model, again

[31]Matching factors are 1.42 (AAA), 1.48 (AA), 1.65 (A), 1.59 (BBB), 1.39 (BB), and 1.43 (B). As the threshold model's credit VaR for the class CCC–C is already larger than the one under the continuous model, h would have to scale credit risk *down*, which is neglected.

exclusively for investment grade ratings.

I conclude from both exercises that the general migration risk is an important factor in terms of risk underestimation. A symmetric effect on the part of spread risk is not to be expected, however, at least not if the volatility of spread changes is only proportionally up- or downsized: while the credit rating determines which spread variation (plus correlations with the credit factor) is chosen from a vector of distributions (which is ignored in the isolated calculation of spread risk), there is no reciprocal influence in the sense that different levels of spread risk would heavily change credit migrations, as long as the copula of the systematic factors remains untouched. The ultimate reason for the asymmetry in the effects of credit vs. spread risk on RI is that spread risk is multidimensional, while credit risk is not. Simulations confirm that a proportional rescaling of the spread risk factors has virtually no influence on RI.[32]

To what extent should the two exercises be taken seriously? I believe they include some degree of realism as the threshold model is calibrated to external ratings, which are intended to "look through the cycle", in contrast to internal ratings which are often labeled "point-in-time". Literature documents that internal ratings have much higher transition rates, similar to the KMV-implied rates of Gupton et al. (1997) used in the second exercise.[33] As the trading book is defined to convey investments over a short period (and not over a credit cycle), point-in-time ratings – and therefore the corresponding transition matrices – are in fact more suitable for the trading book than external ratings. Hence, the second exercise in particular matters, as does UbS in this set-up. If transition probability is as high as is typically the case with internal ratings, UbS can also appear in a threshold model even though the rating scale is quite coarse. The latter aspect points to another hypothesis regarding the differences in UbS:

As there are only seven rating classes, credit migrations – and therefore events of migration-induced bond repricing – become rare events if the risk horizon is 3 months only.[34] It is possible that the latent credit risk factors X_i and the spread factors interact in a malign way but the interaction does not materialize in credit migrations as it is largely filtered out by the threshold mechanism.

To test this hypothesis, I refine the rating scale by the notches "+" and "–" so that the ratings become less persistent. As empirical spread indices are not specific to notches, the modeled spread changes are extended to notch classes in the same manner as in the continuous model.

The impact of using a more granular rating scale on UbS is weak. While some of the RI values rise slightly, there are very few, and still only small exceedances of the critical 100 percent figure.

6 Robustness and extensions

Ultimately, robustness considerations are not essential to this paper. Rather, the whole analysis constitutes a robustness test of the manner in which risks are split up when calculating trading book capital, the main message being that a separation of spread and credit risk is not robust. While the primary task of this paper has already been achieved, it is nonetheless interesting to examine which parameters strengthen or weaken the effects.

In **Test 1**, I check for the potential sensitivity to a systematic relationship between default rates and LGDs. This analysis is motivated by the fact that UbS is weak or non-existent for bad

[32]There is a weak impact through the non-linearity of the bonds' pricing function.

[33]Transition matrices of internal ratings or accounting based proxies are found in Segoviano and Lowe (2002), Mählmann (2006), Carey and Hrycay (2001), or Krüger, Stötzel, and Trück (2005). Araten, Jacobs, Varshney, and Pellegrino (2004) constitute an exception as they report lower transition rates similar to those of external ratings.

[34]See Table 9 for 3-months transition probabilities.

ratings / high EDFs. When credit risk is high, losses from defaults are relatively large compared to those arising from downgrades. This means that when defaults become more frequent they are increasingly represented in the portfolio loss tail. If the LGD is independent of systematic factors, the "tail conditional" correlation between credit losses and spreads could become less pronounced when default probabilities rise, with the result that the domain of MRI might lose weight. Reciprocally, if the LGD tends to be high in times of high default rates, UbS might remain strong also for higher initial PDs where it is not to be found in the base case.

In fact, there is empirical evidence of a relationship between default frequency and LGD; cf. Düllmann and Trapp (2004). I establish a relationship between LGDs and the systematic credit risk factor Y with a normal bivariate copula, while the marginal beta distribution of the LGDs is left unchanged.[35] Testing correlations in the bivariate copula between 0 and -0.9[36], there is no measurable effect on RI at all, in both model variants (whereas the risk measures do indeed change, as expected). The rather high volume of 1,000 bonds, which limits the influence of idiosyncratic LGDs, might explain this finding.

In **Test 2**, I consider a portfolio of corporate bonds with fixed bond rates. Holding them in the trading book (i.e. not to maturity) includes interest rate risk, i.e. the risk that the default-free interest rate r_f changes. The natural data source for pricing would be rating-dependent indices of corporate bond rates, which are not at my disposal. I approximate them by simply adding a time series of a default-free interest rate to the time series of spread indices.[37] From these indices, I estimate the same correlation matrices as before; the only difference to the base case appears in the valuation of bonds, where the discount rate $r_f + s$ now has two random components.

The main outcome of the continuous model with interest rate risk, using Y^{KMV} as SCRF, is presented in Figure 4. I observe markedly lower levels of RI when EDFs are low, while UbS is still substantial for medium credit risk. The drop in RI is caused by higher correlations between corporate interest rates and Y^{KMV} (correlation values are found on the x-axis of the graph). They are now positive for low EDFs; as spreads for high-quality bonds are generally low, corresponding interest rates are mainly determined by the default-free rate, which is *positively* correlated with Y^{KMV}, in contrast to spread indices.

In fact, there is a hedging effect between default-free interest rates and credit risk: when default risk goes up on average, which would lead to larger discount rates even if credit risk were fixed, default-free interest rates counteract this loss by a tendency to go down. The trough of RI at $EDF = 0.0086$ is again caused by a volatility peak, as expressed in Conjecture 2 and explained in Section 5.2. In the threshold model, again using Y^{KMV} as SCRF, I observe no UbS whatsoever.

Test 3 concerns idiosyncratic spread risk. In reality, the spread risk of an individual bond also has an idiosyncratic component in the sense that the spread of an individual bond could change while neither its credit risk does so nor the spread index representative for this bond. This may be due to a singular liquidity shock in the market for a particular bond, for example.

[35] To do so, I have to account for the fact that, conditional on a bond defaulting, Y has a downward-biased non-normal distribution. The latter is numerically available, however. I apply this default-conditional CDF to Y (such that the transformed number is uniformly distributed) and transform the outcome using the standard normal inverse CDF. Having created a standard normal default-conditional systematic credit risk factor, I then linearly couple it with another independent normal r.v. to obtain a standard normal variable again. Multiple variables of this kind are correlated. A further transformation using the normal CDF and the beta inverse results in LGD variables that have the same univariate beta distribution as in the base case set-up. However, the LGDs of two defaulting bonds are correlated with each other and Y.

[36] Note that, following empirical evidence, large LGDs are associated with high default rates, i.e., with *low* outcomes of Y. Only negative correlations are therefore of interest.

[37] I use internal return rates of exchange-listed German 3-to-5-years treasuries; Bundesbank time series BBK01.WT9552. Applying the US CBOE 5 years treasury yield index gives very similar results.

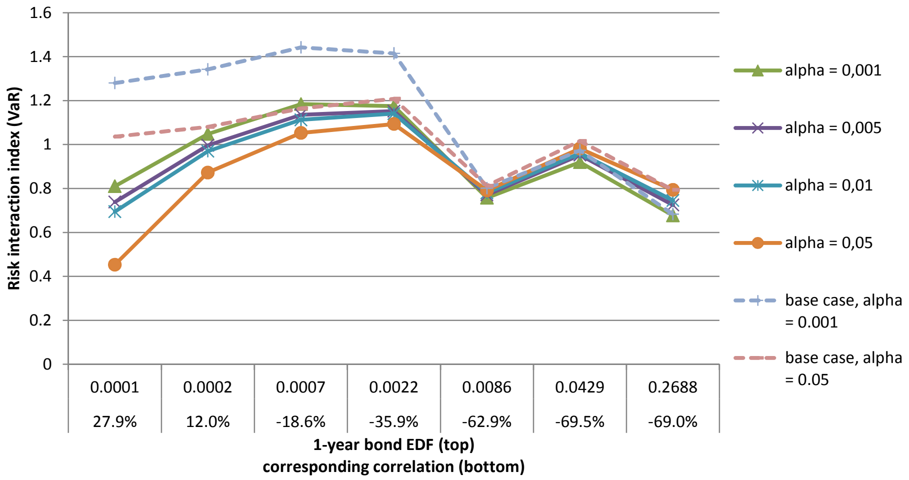

Figure 4: Risk interaction index of the VaR in the continuous model, including interest rate risk. The x-axis features uniform bond EDFs (top line); the bottom line reports the correlation of the corresponding corporate interest rate and the systematic credit risk factor Y^{KMV}. Vertical axis: risk interaction index; values above 1 indicate UbS. Labels identify the security level α of the VaR. Dotted lines show the base case results excluding interest rate risk.

If idiosyncratic shocks add to a bond's spread, the correlation between its credit risk and the discount rate at which it is priced should decrease, which could dampen UbS overall.

I check for this effect by allowing the individual spread of bond i to vary around the systematic spread. The latter is multiplied by a bond specific (approximately) log-normal random factor U_i, which adds idiosyncratic noise. The U_i must not simply be independent, as this would add heteroskedasticity to the individual spreads and lead to strange effects on the risk measures. I solve this problem by making the variance of U_i dependent on the spread it is multiplied with. However, conditional on all spreads being given, the U_i are independent lognormals. Details are given in Section A.3.

For lack of reliable information on typical amounts of idiosyncratic spread risk, the standard deviation σ_U is varied over a wide range from zero to 15%. The highest value is approximately one half of the average standard deviation in the spread returns. Such a degree of fluctuation would already generate substantial overlaps of individual spread levels between neighboring rating classes, which should not be a particularly frequent phenomenon in practice. The test is performed in the continuous model only because RI, if affected, should fall in σ_U so that the virtual absence of UbS in the threshold model would merely be confirmed.

I can find no effect of idiosyncratic spread risk across all EDF levels. This is because not even the risk measures that underlie RI change. Diversification between the bonds is strong enough to eliminate any measurable effect, even if the portfolio consists of no more than 20 bonds; in this case, risk measures change by less than 2%, as does RI.

In **Test 4**, I modify the distribution of the changes Δ_i of the distance-to-default in the continuous model. The base case treats them as normally distributed with moments matched to the sample. But a sample kurtosis of 9.8 indicates that Δ_i may actually have fat tails, which is of course important in the context of risk measures. Even if the sample is censored on both sides at 0.5% quantiles, the kurtosis is still 1.76.

To capture the tails correctly, Δ_i is simulated using the MATLAB class `paretotails`. Tails are fitted by Pareto distributions, while the center part is covered by a kernel density estimation. A QQ-plot in the left-hand panel of Figure 8 in the appendix indicates a satisfactory fit.

Table 6 shows RI in the base case and with Pareto tails, depending on the initial EDF. While

Table 6: Risk interaction using different distributions for changes in the distance to default.

EDF	0.01	0.02	0.07	0.22	0.86	4.29	26.88
RI, base case	1.28	1.34	1.45	1.41	0.80	0.98	0.68
RI, Pareto tails	1.34	1.35	1.34	1.31	0.81	0.96	0.68

fattening the tail of the credit factor Δ_i clearly raises credit and total risk, it has a mixed impact on UbS. *RI* goes up for the best EDF, confirming intuition, but there are also two cases in the middle range where *RI* drops by 9 to 11 percent.

This outcome may surprise and appear to conflict with the result of Section 5.3 where rising credit risk increases *RI* in the threshold model. It is, however, less surprising if account is taken of the fact that the alternative distributions of Δ_i are reconciled by moment matching. Given the presence of a fat tail in one of the distributions, which pushes variance up, the matching thin-tailed normal distribution must compensate for this by having more variance around the center. To illustrate, I compare two sorted samples of Δ_i, each from one of the alternative distributions. Only 7.8% of the most negative outcomes from the fat-tailed sample are below the most negative normal ones,[38] meaning that most of the negative half of normally distributed Δ_i push the DtD further down than a fat-tailed Δ_i, and thus further away from the point where separate spread risk measurement would be correct.

Precisely under which distribution UbS is stronger depends on the speed at which spread risk is changing as DtD decreases or, equivalently, EDF rises; this change is ignored by separate risk measurement. Either moderate changes in spread risk dominate, as they combine with *many* Δ_i realizations near the origin, or there is a preponderance of large spread risk changes applying to few realizations of Δ_i in the lower tail. Table 6 suggests that the former scenario is valid for my data.

A similar observation can be made for the two exercises performed in Table 5, where transition probabilities in the threshold model are larger than in the base case. When the SCRF is scaled up to the extent that the credit VaR is as large as in the continuous model, the transition probabilities in column [8] grow by some 5–7 percent only, combined with only moderate *RI* values of around 109 percent. *RI* is much higher in the second exercise (column [13]) where the credit VaR is still smaller (!) than in the continuous model but annual transition probabilities exhibit very high levels around 1/2. Consistent with Table 6, the big mass of migrations near the origin seems to matter, but not so much the ones in the tail of credit factors.

In **Test 5**, I investigate the impact of the asset correlation ρ, which is the (squared) loading of the SCRF in (13). It is crucial for the risk of large portfolios, as it determines the non-diversifiable part of portfolio risk. Testing values $\rho = 15\%$, 20% and 25%, the risk measures change significantly as expected, whereas *RI* is virtually unaffected; see also Figure 9 in the appendix. The effect is equally small in the threshold model.

Test 6 concerns the initial spread function in the continuous model. While it is given by average spread indices over the full data period in the base case, the spread function is now set to sample index values at different time points, manually selected for their "interesting" shape. Running the base case simulation with the only modification being the choice of the spreads at time 0, Figure 5 shows that this choice has a surprisingly strong impact. While the resulting *RI*s as functions of the bonds' EDF have a similar shape, their general level is quite diverse.

Motivated by the strong effect evident in the continuous model, I repeat the exercise for the threshold model, using Y^{KMV} as the systematic credit risk factor. Depending on the spread at

[38] For illustration, the left-hand panel of Figure 8 plots cdfs of both distributions in the lower half. They cross at $(-1.69; 0.039)$; the probability of a negative realization is roughly one half in both distributions.

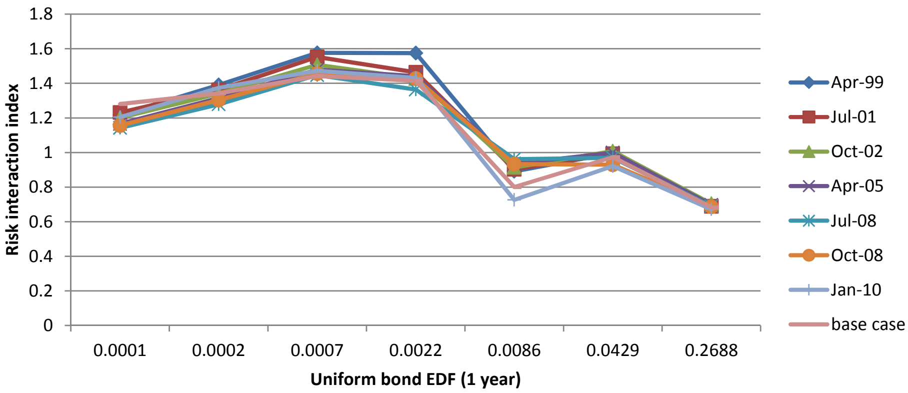

Figure 5: Risk interaction index of the VaR in the continuous model, assuming different historic spread indices as the initial spread function. The x-axis features uniform obligor EDFs while the y-axis has risk interaction indices RI for the VaR at security level $\alpha = 0.001$. Values above 1 indicate UbS. Labels indicate points of time from which the historic spread indices were taken and assumed to be the spread index at time 0.

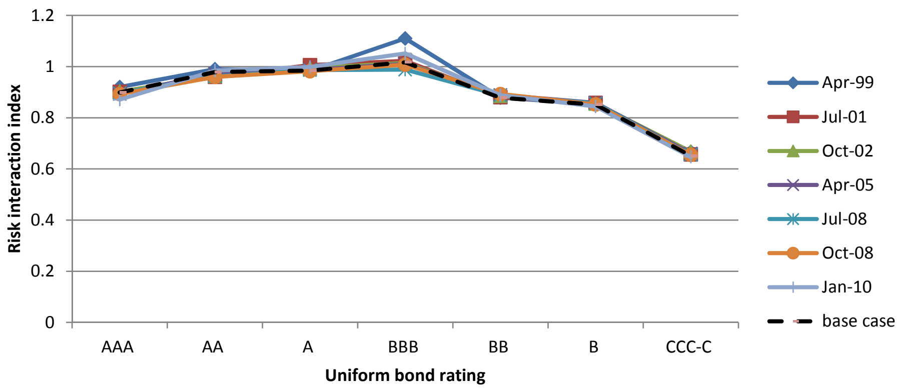

Figure 6: Risk interaction index of the VaR in the threshold model, assuming different historic spread indices as the initial spread function. The x-axis features uniform obligor ratings while the y-axis has risk interaction indices RI for the VaR at security level $\alpha = 0.001$. Values above 1 indicate UbS. Labels indicate points of time from which the historic spread indices were taken and assumed to be the spread index at time 0.

time 0, Figure 6 shows that there are now also cases of substantial UbS. RI at rating B is much more sensitive to the start spread than the others, for which I have no explanation up to now. The effect is all the more surprising if one takes into account that the volatility structure of the factors is the same throughout.

In **Test 7**, I lower the number of bonds in the portfolio from 1,000 to 50. The effect is weak: risk measures for the smaller portfolio are larger than benchmark measures by 0.5% to 3%, while RI falls by around 1.5%.

Test 8 goes a step beyond plain bonds, considering a portfolio of 1,000 derivatives each of which is a linear short position of a certain obligor's spread; its dollar value is w.l.g. simply $-s_1\left(EDF_{i,1}\right)$. Figure 7 displays RI for the VaR at the two most extreme security levels and their counterparts from the bond portfolio.

RI values up to 1.71 are found to be markedly stronger than in the base case (RI is even more extreme for the ES, where it reaches a maximum of 1.78). This finding is consistent with the comparison made at the end of Section 2.2, stating that the tendency towards UbS for bonds should be weaker than for linear positions.

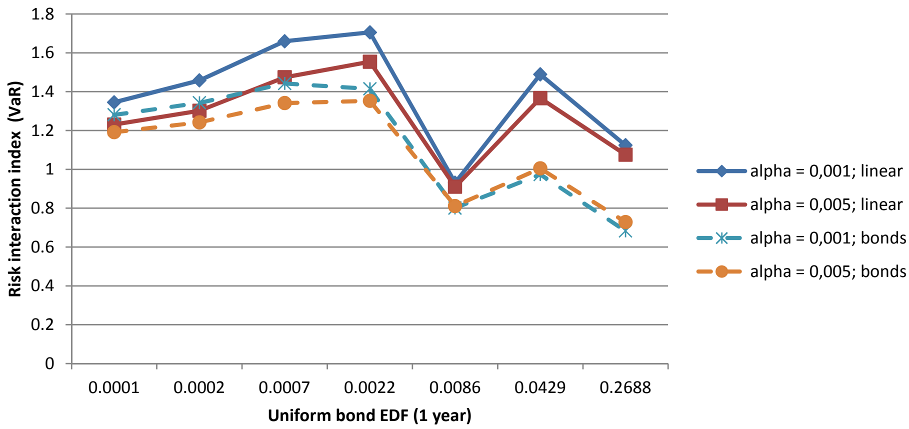

Figure 7: Risk interaction index for VaR and ES of linear positions in the continuous model. The portfolio consists of linear positions, the value of which is minus the spread of an obligor. The x-axis features uniform EDFs of the obligors; risk interaction indices RI for selected security levels α are found on the y-axis; values above 1 indicate UbS. For comparison, the graph includes corresponding values of RI for the bond portfolio (dotted lines).

Finally, **Test 9** checks for UbS when the base-case bond portfolio is held short. While a direct shorting of bonds is difficult to implement in practice, a bond's short value in the absence of an interest rate risk is similar to buying CDS protection. It transpires that UbS vanishes completely; RI values are fairly similar, lying at around 0.9. The result might be surprising at first glance, as a short bond is a concave function of the spread, which should foster MRI. Consistent with Conjecture 1, the positive association between changes of the spread function and credit risk should also have the same effect as before, albeit now combining upgrades and a lowering spread function to portfolio losses. However, the forces described in Conjecture 2 act against it. The portfolio loses value if a bond is *up*graded, which now entails a shift from a regime of higher spread volatility to one of lower volatility. This effect outweighs the others.

Summarizing the robustness tests, risk underestimation persists under many conditions, with the important exception that the introduction of interest rate risk alleviates the problem substantially. In other words, risk underestimation is more a problem of portfolios containing floating rate corporate bonds and CDS protection-selling than of fixed-rate corporate bonds.

Two possible extensions of the analysis would be particularly relevant in terms of validating the current capital requirements for the trading book.

First, the bond portfolio is treated as if the bank did not hold anything else in the trading book. Sensitivities and UbS could differ from the current results in the presence of a larger portfolio with its own factor sensitivities. The bond portfolio's risk *contribution* to the rest of the trading portfolio would be interesting. However, as the latter can feature such very different risk profiles, the results would probably boil down to a big "it depends on...".

Second, the actual IRC is calculated in a multi-period set-up over one year, which calls for a corresponding approach in my framework. Such an analysis would, however, require a sound theory of how risks over different horizons should be aggregated. As yet, this seems to be outstanding.

7 Evaluating the Revised Market Risk Framework

The Basel Committee is in the process of undertaking a fundamental review of the trading book. Two of the changes proposed in the consultative document (BCBS, 2013) are particularly relevant for corporate bonds or related positions. First, the VaR is replaced by the ES in the calculation of market risks. This change is clearly important in a general sense but, as has been shown, less so regarding UbS in the context of bond portfolios. Second, and much more importantly in the context of this paper, migration risks are now to be considered part of the market risk model, with the important exception that defaults are not included. Capital for default risk is still calculated using a separate credit risk model. The resulting capital part has accordingly been relabeled from an incremental credit risk charge (IRC) to the incremental default risk (IDR) charge.

The logical question is whether the new approach also includes risk underestimation and, if so, how large this is. While other changes[39] to the regulatory framework may also matter, I restrict the analysis to the shifting of migration risk from the separate credit risk measure to the market risk measure. Contrary to the risk separation defined in (15)–(17), risks are now simply split into

$$\Delta V_{\text{no default},i} \equiv I_{\{D_{i,1} \geq DT_i\}} \Delta V_{\text{total},i} \ \text{ and } \ \Delta V_{\text{default},i} \equiv I_{\{D_{i,1} < DT_i\}} \Delta V_{\text{total},i} \ ,$$

where DT_i is the default threshold of bond i as defined in (14) or (19). The way in which risks are split up is the only difference from the base case; risk interaction (now at the portfolio level) is defined as before:

$$RI_{\text{reform}} \equiv \frac{\kappa\left(\Delta V_{\text{total}}\right)}{\kappa\left(\Delta V_{\text{no default}}\right) + \kappa\left(\Delta V_{\text{default}}\right)}. \tag{22}$$

Table 7 presents results of the continuous model for VaR and ES. Total risk is no longer underestimated under virtually all conditions, which may also be due to the fact that default risk is only tiny compared to market risk, especially for the lower EDFs where UbS prevailed in the base case.

All in all, my results suggest that the Revised Market Risk Framework represents an important step in the right direction.

8 Conclusion

This paper contributes to the discussion of whether different types of risk applying to the same business can be measured separately. While there are good reasons for such a block-based approach to total risk, especially as a way of keeping results robust and the model risk low, it is clear that an aggregate of separate risk measures must not indicate less risk than the risk measure resulting from an integrated approach. Otherwise, the error made by risk separation would not be made on the safe side.

My first contribution is a theoretic one. Generalizing a proposition of Breuer et al. (2010), I have weakened a sufficient condition for risk separation to be conservative. While this more general condition will continue to apply only rarely in practice, Proposition 2 eases the analysis of opposite cases where separate risk measurement leads to an *underestimation by separation* (UbS).

The second contribution is applied and directly linked to current regulatory practice. I find plausible set-ups in which certain market and credit risks, if measured separately and added up, fall short of the total risk measured under an integrated model. Importantly, they can do so for very plain exposures such as corporate bond portfolios or CDS protection selling, which the literature has not yet identified as being subject to UbS.

These bonds are exposed to fundamental credit risk on the part of the issuer and to changes in the spread (understood as changes unrelated to underlying credit risk). When credit and

[39] For instance, the current positions are assumed to remain constant over the full default-risk horizon; the default risk model must include two systematic factors.

Table 7: Effect of the reform: Separate risk measures, induced by (1) total risk without defaults and (2) pure default risk vs. total risk in the continuous model (base case). All numbers are shown in percent. The table presents VaR and ES at varying security levels α of a portfolio of 1000 homogeneous bonds with a uniform rating. Risks are either induced by spread changes and migrations, excluding default losses, purely by defaults ("Only defaults"), or by both ("Total"). *RI* is the total risk over the sum of the separate risks; values above 1 (bold) indicate an underestimate of the total risk by separate measurement. The results are calculated in the continuous model, based on the systematic credit risk factor Y^{KMV}, derived from changes in KMV's distances-to-default of European corporates.

EDF	α	Value-at-Risk				Expected Shortfall			
		Spread & migration	Only defaults	Total	*RI*	Spread & migration	Only defaults	Total	*RI*
0.01	0.1	6.34	0.14	6.40	98.9	7.02	0.18	7.12	98.8
	0.5	4.64	0.08	4.67	98.8	4.00	0.08	4.03	98.9
	1.0	3.92	0.06	3.94	98.9	2.70	0.05	2.72	98.9
	5.0	2.31	0.00	2.31	**100.3**	0.76	0.00	0.76	99.9
0.02	0.1	8.66	0.19	8.77	99.1	9.70	0.26	9.85	98.9
	0.5	6.22	0.10	6.27	99.2	5.40	0.11	5.45	99.1
	1.0	5.22	0.08	5.25	99.0	3.63	0.06	3.66	99.1
	5.0	3.05	0.00	3.06	**100.4**	1.00	0.01	1.01	99.7
0.07	0.1	17.57	0.43	17.88	99.3	19.35	0.57	19.73	99.1
	0.5	12.82	0.23	12.97	99.4	11.05	0.24	11.21	99.3
	1.0	10.89	0.17	11.00	99.5	7.49	0.14	7.58	99.4
	5.0	6.45	0.07	6.48	99.5	2.10	0.03	2.12	99.5
0.22	0.1	29.02	1.01	29.76	99.1	30.82	1.34	31.72	98.6
	0.5	22.78	0.54	23.17	99.4	18.92	0.55	19.32	99.3
	1.0	20.03	0.39	20.31	99.5	13.28	0.32	13.52	99.4
	5.0	13.13	0.15	13.25	99.7	4.07	0.06	4.12	99.7
0.86	0.1	31.94	2.68	33.82	97.7	33.51	3.32	35.83	97.3
	0.5	25.59	1.56	26.80	98.7	21.06	1.53	22.21	98.3
	1.0	22.47	1.17	23.39	98.9	14.88	0.92	15.59	98.7
	5.0	14.15	0.50	14.59	99.6	4.49	0.19	4.65	99.3
4.29	0.1	41.49	8.29	47.84	96.1	42.28	9.51	49.18	95.0
	0.5	35.57	5.41	40.01	97.6	28.27	4.95	32.19	96.9
	1.0	32.37	4.29	36.04	98.3	20.67	3.16	23.28	97.7
	5.0	23.17	2.18	25.16	99.2	6.91	0.77	7.60	98.9
26.88	0.1	34.58	24.91	50.27	84.5	35.32	26.48	50.57	81.8
	0.5	29.36	19.71	43.99	89.7	23.43	16.32	34.64	87.2
	1.0	26.64	17.25	40.20	91.6	17.08	11.46	25.54	89.5
	5.0	18.31	11.37	28.40	95.7	5.59	3.51	8.55	93.9

spread risk are separately measured, total risk is larger than their sum by up to 50 percent. The occurrence and degree of UbS depend on a number of parameters. Total risk is most strongly underestimated if the following conditions apply: (1) low to medium credit risk; (2) a high correlation between credit downgrades and spread widening (NB widening in excess of the simple sensitivity of spreads to credit risk); (3) spread volatility is a rising function of credit risk; (4) large (non-default) migration risk; and (5) extreme quantile levels of the risk measures.

The effect is slightly stronger for the expected shortfall compared to the value-at-risk. When fluctuations of the default-free interest rate are included, the underestimation problem vanishes for particularly low PDs while it is preserved for medium ones. Fixed-rate corporate bonds are therefore less affected than floating rate bonds.

While this paper includes a number of robustness checks, it should be clear that robustness is not central to the identification of UbS as an underestimation of risk is evidence of non-robustness. Acknowledging the many ways in which I deviate from banks' risk modeling practice and regulatory capital rules (for instance in the risk horizons), I do not claim that UbS *must* appear in the context of bonds. Rather, I caution that it *can* appear, as UbS shows up under plausible conditions.

In which respect do the results apply to bank regulation?

As regards Pillar 1 of Basel III, the current capital rules actually instruct banks to use an internal model for trading book capital to treat spread risk as a market risk factor feeding into market risk capital, while default and migration risk is to be treated by a separate, internal credit risk model resulting in the *incremental risk charge* (IRC). This split is the same as in my paper, which is why I consider it quite possible that total risk is sometimes underestimated under Pillar 1.

Two main caveats limit a direct transfer of my results to the case of Pillar 1. First, there is no strict concept of a unified framework into which regulatory market risk capital and the IRC would fit, if only for the different risk horizons and VaR security levels. Hence, there is no clear total-risk benchmark. Second, a bank will typically hold a large variety of trading book assets, whereas I perform an isolated analysis of ultimately eight risk factors applying to a limited range of exposures. Other risk factors and exposures are left to future research.

As mentioned in Section 7, trading book capital is currently being reformed by the Basel Committee (BCBS, 2013, 2014). Its Trading Book Group proposes to integrate migration risk – without defaults – into the market risk model, while still using a separate model to calculate the default risk capital, known as the *incremental default risk* (IDR) charge. My results suggest that this step is a move in the right direction, as UbS almost completely vanishes for the positions analyzed in this paper. However, in the cases where UbS appeared in the first place, the default risk is low, so that the disappearance of UbS under the proposed reform cannot be taken as a general "all-clear". At the current stage of knowledge it seems possible that exposures other than corporate bonds and CDS might be subject to UbS under the new risk split while they are harmless under the old splitting approach.

Another important proposal of the Trading Book Group, i.e. replacing the VaR with the expected shortfall (ES), does not seem to interact with the appearance of UbS. Throughout the various models and parametrizations, UbS appears with the VaR almost exactly as and when it appears with the ES, with the problem being slightly more serious in the second case.

Besides relating to Pillar 1 of Basel III, my results directly relate to banks' internal calculation of economic capital, as regulated under Pillar 2. Internal economic capital is frequently determined in a piecemeal manner for separate risk types and aggregated afterward. While current European regulation (EBA, 2014) prescribes that risk measures must be summed up in SREP capital assessments, other aggregation methods (such as the square-root approach) are allowed for bank-internal purposes. If an aggregation method includes a discount for diversification effects between market and credit risk, my findings of UbS hold a fortiori, and there may be additional cases where adding risk measures up would be conservative while the square-root approach is not.

This work has three obvious policy implications. First, it raises questions concerning the reliability of the risk split in current capital requirements for the trading book. Second, it shows that the proposed reform of capital, in which migration risk is assigned to market risk, helps to close the regulatory gap, as regards the positions analyzed in this paper. Third, with respect to Pillar 2, the supervisory skepticism about diversification effects between different risk types is confirmed. In general, regulators should consider establishing incentives for banks to develop reliable integrated risk models, be it as primary models for economic capital or as backstops for types of risk interaction that do not show up in separated risk measurement.

A Appendix

A.1 Risk aggregation in Kupiec (2007)

Kupiec calculates capital for a bond portfolio. He chooses a 6-month risk horizon and includes accrued interest in the expected loss (EL) such that this EL is usually negative; in his examples it is around −2.6%. Doing so is not wrong per se but must be accounted for when risk measures are compared. To illustrate, assume separate risk measurement is exact as regards unexpected losses $UL \equiv VaR - EL$. In other words, if the equation

$$UL_{\text{interest}} + UL_{\text{credit}} = UL_{\text{integrated}} \tag{23}$$

holds, there is nothing wrong with separate risk measurement. When Kupiec compares separate and joint risk measures (Table 9), he first aggregates the separate VaRs and subsequently subtracts the (approximate) EL from the result to obtain "piecemeal capital" equal to

$$K_{\text{piecemeal}} \approx VaR_{\text{interest}} + VaR_{\text{credit}} - EL = UL_{\text{interest}} + UL_{\text{credit}} + EL.$$

On the contrary, integrated capital is essentially

$$K_{\text{integrated}} \approx VaR_{\text{integrated}} - EL = UL_{\text{integrated}}.$$

Even in the presence of (23), the "error" of piecemeal capital (if calculated in this way) is $\approx EL$ (notably, a negative number). It therefore comes as no surprise that the capital shortfall from separate risk measurement in Table 9 is roughly the same as the accrued interest for 1/2 year. An underestimation of risk is unlikely to be the main driver.

The example shows that particular care must be taken in the aggregation of risks when expected returns are substantially different from zero. Adding unexpected losses makes more sense in this instance.

A.2 Extending the spread function

While the threshold model has just seven ratings, EDFs in the continuous model can take any value in $[0, 1]$, meaning that the spread function must act on the full unit interval. To make its modeling as consistent with the threshold model as possible, each of the seven ratings is assigned a fixed 1-year EDF. One-year PDs and EDFs are treated as synonyms, for the match of which I use a study of S&P rating transitions 1981–2012. Table 1 gives further details. The spread function is defined on the seven EDF gridpoints by the corresponding spread indices as before.

Accordingly, simulation on the gridpoints is effected by drawing spread returns as specified in (12).

To simulate spread returns between the gridpoints (and beyond, for EDFs larger than 26.88%), linear interpolation between the gridpoints would be inappropriate. If the grid values were simply combined by straight lines (or splines), the variance curve would resemble a suspension bridge with seven pillars.[40] I correct for this artifact by adding independent noise of a suitable size to ensure that the variance curve becomes a linear interpolation of the variances at the nodes. It is straightforward to show that Brownian bridges between the nodes represent the right amount of noise to be added.

In the following calibration, I switch to the logarithm of EDFs, so that the gridpoints now lie between $-\infty$ and 0. Assume X_j and X_{j+1} are log spreads at two nodes. Assume further ω to be a standard Wiener process on $[0,1]$ and B its Brownian bridge on the same time interval, i.e. $B_\alpha \equiv \omega_\alpha - \alpha\omega_1$. If B is independent of X_j and X_{j+1}, define the process Y on $[0,1]$ by

$$Y_\alpha \equiv (1-\alpha) X_j + \alpha X_{j+1} + \text{stdev}(X_j - X_{j+1}) B_\alpha,$$

where the standard deviation results from Σ, the systematic factors' covariance matrix. Y_α has the desired variance structure:

$$\text{var}(Y_\alpha) = (1-\alpha)\,\text{var}(X_j) + \alpha\text{var}(X_{j+1}),$$

i.e., variances at neighboring gridpoints are connected by a straight line. Generating six independent Brownian bridges, one for each interval between the gridpoints, the total variance curve is a polygon.

While interpolation is possible between ln(0.01%) and ln(19.2%), which constitute the best and the worst EDF for which spread indices exist, spreads are also needed outside this interval. On the low-risk side, I add an artificial gridpoint with a de-facto zero spread and zero variation. On the high-risk side, spreads for EDFs larger than 26.88% are simulated by linear extrapolation from X_6 via X_7 plus a Brownian bridge. However, the slope of the extrapolation line is dampened to ensure that the variance curve $\text{var}(X_6)$ and $\text{var}(X_7)$ is a straight line. In particular, I make the following choices. Denote by $q_1, \ldots, q_7$ the nodes of the spread curve, i.e. the log of PDs from Table 1. I choose another node $q_* > q_7$ up to which the variance curve is to be linear. Given some log EDF q between q_7 and q_*, I define $\alpha \equiv (q - q_7)/(q_* - q_7)$ and

$$Y_\alpha \equiv X_7 + \alpha\beta(X_7 - X_6) + \gamma B_\alpha.$$

Requiring that the variance curve of Y_α is a straight line from $\text{var}(X_6)$ via $\text{var}(X_7)$ up to the variance at q_* implies

$$\beta = \frac{\text{var}(X_7) - \text{cov}(X_7, X_6)}{\text{var}(X_7 - X_6)}\left(\sqrt{1+Q} - 1\right)$$

with

$$Q \equiv \frac{q_* - q_7}{q_7 - q_6} \times \frac{[\text{var}(X_7) - \text{var}(X_6)]\,\text{var}(X_7 - X_6)}{[\text{var}(X_7) - \text{cov}(X_7, X_6)]^2}.$$

As with interpolation, the Brownian bridge B_α scaled by $\gamma = \beta\text{stdev}(X_7 - X_6)$ gives $\text{var}(Y_\alpha)$ a linear shape on $[q_6, q_*]$. On $(q_*, 1]$, the variance grows more than linearly.

[40] This is because a straight line between two gridpoint spreads consists of weighted averages. The part of randomness that is idiosyncratic to each gridpoint spread partly averages out in the inner points of the line.

A.3 Modeling idiosyncratic spread risk

In Section 6 I introduce additional bond-specific spread risk. In the base case, the uniform spread function s_1 is individually evaluated, resulting in $s_{i,1} \equiv s_1(EDF_{i,1})$. This spread is now multiplied by a bond-specific random factor U_i to add idiosyncratic noise. If undesired heteroskedasticity is to be avoided, $(U_i)_{i=1,\dots,N}$ must not be completely independent from $(s_{i,1})_{i=1,\dots,N}$. As the U_i are still to be independent, lognormal variables, conditional on $(s_{i,1})_{i=1,\dots,N}$, I define the function

$$\tau^2(s) \equiv \log\left(\sigma_U^2 s_0^2(R_0)\, s^{-2} + 1\right),$$

where σ_U is a given parameter representing the amount of idiosyncratic spread risk. Introducing a bond-specific independent standard normal variable ζ_i, the idiosyncratic spread factor is defined as

$$U_i \equiv \exp\left(-\frac{1}{2}\tau^2(s_{i,1}) + \tau(s_{i,1}) \times \zeta_i\right).$$

This approach avoids heteroskedasticity, as it fulfills

$$\mathbf{E}\left(U_i | \, s_{i,1}\right) = 1 \text{ and } \operatorname{var}\left(s_{i,1}U_i | \, s_{i,1}\right) = \sigma_U^2.$$

A.4 Supplementary tables and figures

Table 8: Descriptive statistics. All numbers are shown in percent. Quarterly data from April 1999 to October 2010. Statistics for EDFs are calculated from a pooled sample of European corporates.

	Merrill Lynch Euro corporate spread indices							EDFs
	AAA	AA	A	BBB	BB	B	CCC–C	
Mean	0.43	0.74	1.25	1.80	4.99	7.30	16.91	3.28
Stdev.	0.38	0.63	1.12	1.28	3.52	4.43	12.88	6.72
Stdev. of differences	0.22	0.28	0.53	0.70	2.58	2.46	7.40	
Quantiles								
10	0.17	0.29	0.49	0.77	1.67	3.23	4.90	0.05
25	0.19	0.34	0.55	0.88	2.42	3.97	6.65	0.12
50	0.29	0.49	0.92	1.45	4.03	6.09	12.90	0.47
75	0.51	0.97	1.41	2.29	6.22	8.87	24.65	2.58
90	0.76	1.57	2.15	3.04	9.29	13.88	35.51	10.41

Table 9: Rating transition probabilities for 3 months. The 3-month matrix is calculated under the Markov assumption from S&P's one-year transition matrix based on rating events in the years 1982–2012 (Standard and Poor's, 2013, Table 9). The original matrix has proportionally been rescaled to eliminate transitions to the "not rated" state. The original default probability for AAA is zero. Using exponential fitting, I raised the value to 0.01% before calculating the 3-month matrix. Bluhm et al. (2003) report that using this type of fitting is common practice.

Rating	AAA	AA	A	BBB	BB	B	CCC–C	D
AAA	9.75E-1	2.43E-2	6.26E-4	7.07E-5	2.20E-4	5.69E-5	1.66E-4	8.07E-7
AA	1.52E-3	9.74E-1	2.34E-2	1.04E-3	1.16E-4	2.06E-4	5.91E-5	3.51E-5
A	7.17E-5	5.24E-3	9.78E-1	1.55E-2	7.54E-4	3.66E-4	4.31E-5	1.62E-4
BBB	2.59E-5	2.61E-4	1.01E-2	9.76E-1	1.14E-2	1.37E-3	4.07E-4	4.91E-4
BB	6.05E-5	1.09E-4	1.91E-4	1.59E-2	9.57E-1	2.26E-2	2.04E-3	1.85E-3
B	0	8.93E-5	3.15E-4	2.65E-4	1.75E-2	9.54E-1	1.70E-2	1.06E-2
CCC–C	0	0	5.70E-4	8.32E-4	1.23E-3	5.47E-2	8.44E-1	9.84E-2

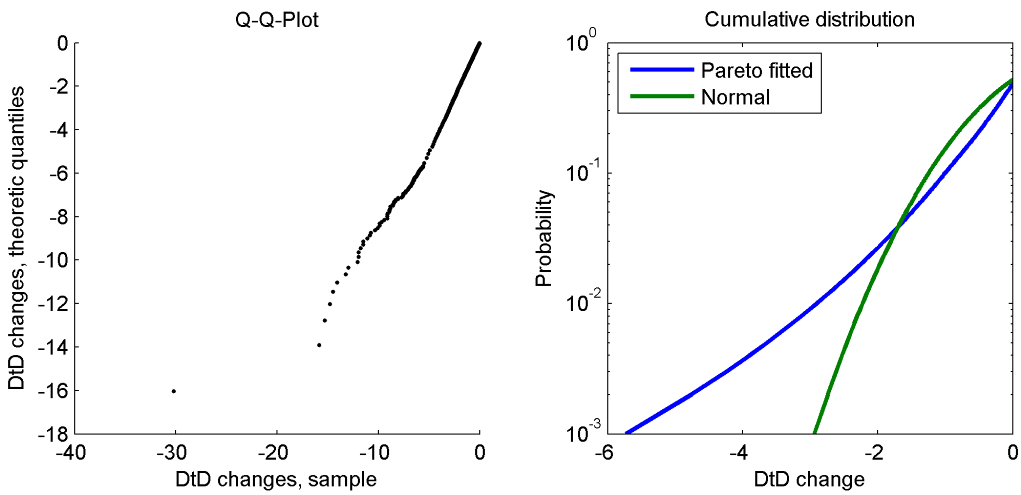

Figure 8: Fitting the distribution of changes in the distance-to-default. The left-hand panel compares the empirical distribution of DtD changes Δ_i with a numerical fit in a Q-Q-Plot of the lower half of observations. The numerical distribution has a Pareto distribution in its upper and lower 10%-tail and a smoothed empirical distribution in between. The right-hand panel shows the lower part of the fitted cdf (log scale) and a normal cdf with matching first and second moment. The lines cross at $(-1.69; 0.039)$.

Table 10: Separate and total expected shortfall (ES) in the threshold model. All numbers are shown in percent. The table presents the ES (at a varying security level α) of a portfolio of 1,000 homogeneous bonds with a uniform rating. Risks are either exclusively induced by *spreads*, by migrations and defaults (*Credit*), or by both risk types (*Total*). *RI* is the total risk over the sum of the separate risks; values above 1 (bold) indicate an underestimate of total risk by separate measurement. The results depend on estimated correlations with the empirical proxy for the systematic credit risk factor (SCRF): Y^{KMV} is derived from changes in KMV's distances-to-default of European corporates. $Y^{\text{KMV,res}}$ consists of its AR(1) residuals. Y^{dg} is based on downgrades of S&P corporate ratings. $Y^{\text{dg,res}}$ consists of its AR(1) residuals. Y^{MSCI} is defined by log returns of the MSCI Europe stock index.

Rating	α	Spread	Credit	Total	Risk interaction index *RI*				
		(all Y)	(all Y)	Y^{KMV}	Y^{KMV}	$Y^{\text{KMV,res}}$	Y^{dg}	$Y^{\text{dg,res}}$	Y^{MSCIE}
AAA	0.1	2.11	1.13	2.98	92.0	97.6	68.5	77.3	86.5
	0.5	1.26	0.51	1.61	90.7	94.8	74.6	80.8	87.1
	1.0	0.88	0.31	1.08	91.5	94.9	77.6	83.1	88.5
	5.0	0.26	0.07	0.31	94.4	96.6	84.1	88.8	92.6
AA	0.1	3.09	1.26	4.30	98.7	**103.4**	75.1	84.2	97.6
	0.5	1.87	0.59	2.41	97.9	**100.9**	79.8	87.2	96.4
	1.0	1.31	0.37	1.64	97.8	**100.5**	82.2	88.8	96.5
	5.0	0.39	0.09	0.47	98.5	**100.2**	87.4	92.8	97.7
A	0.1	5.45	1.61	7.02	99.4	**101.7**	79.1	85.3	98.5
	0.5	3.30	0.73	3.98	98.7	**100.6**	83.8	88.9	98.1
	1.0	2.30	0.44	2.71	98.7	**100.3**	86.0	90.7	98.2
	5.0	0.69	0.10	0.78	99.4	**100.3**	90.6	94.3	99.2
BBB	0.1	8.90	4.11	13.39	**102.9**	**106.6**	74.1	84.1	**103.7**
	0.5	5.36	2.01	7.45	**101.1**	**103.6**	78.6	86.4	**101.5**
	1.0	3.73	1.26	5.01	**100.3**	**102.7**	80.9	87.8	**100.9**
	5.0	1.12	0.29	1.41	99.9	**101.4**	86.2	91.5	**100.4**
BB	0.1	32.17	6.08	32.93	86.1	86.4	83.1	84.0	85.9
	0.5	19.53	3.08	20.23	89.5	89.9	85.5	87.2	89.3
	1.0	13.61	1.96	14.17	91.0	91.4	86.7	88.5	90.9
	5.0	4.05	0.47	4.26	94.3	94.7	89.2	91.5	94.1
B	0.1	26.81	12.91	33.16	83.5	84.0	69.9	74.1	82.7
	0.5	16.67	7.09	20.68	87.1	87.3	73.4	78.0	86.4
	1.0	11.79	4.69	14.62	88.7	89.1	75.5	80.0	88.1
	5.0	3.65	1.23	4.51	92.3	92.8	80.5	84.8	91.8
CCC–C	0.1	48.56	29.82	49.23	62.8	62.7	60.2	61.8	62.7
	0.5	31.52	18.86	34.06	67.6	67.5	62.5	65.1	67.2
	1.0	22.73	13.41	25.33	70.1	70.0	63.8	66.9	69.6
	5.0	7.31	4.23	8.80	76.3	76.1	67.7	71.7	75.6

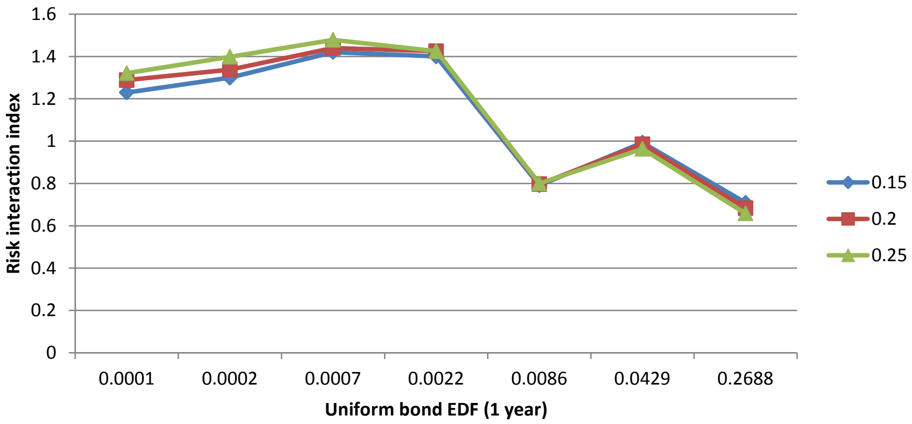

Figure 9: Risk interaction index of the VaR in the continuous model, for different asset correlations. x-axis: uniform bond EDF, y-axis: risk interaction index for the VaR at $\alpha = 0.001$; values above 1 indicate UbS. Labels identify the uniform asset correlation ρ in the credit risk model.

References

Acerbi, C. (2002). Spectral measures of risk: a coherent representation of subjective risk aversion. *Journal of Banking & Finance 26*(7), 1505–1518.

Acerbi, C. and D. Tasche (2002). On the coherence of expected shortfall. *Journal of Banking & Finance 26*(7), 1487–1503.

Alessandri, P. and M. Drehmann (2010). An economic capital model integrating credit and interest rate risk in the banking book. *Journal of Banking & Finance 34*, 730–42.

Altman, E. and V. Kishore (1996). Almost everything you wanted to know about recoveries on defaulted bonds. *Financial Analysts Journal*, 57–64.

Araten, M., M. Jacobs, P. Varshney, and C. R. Pellegrino (2004). An internal ratings migration study. *RMA Journal 86*(7), 90–97.

Artzner, P., F. Delbaen, J. Eber, and D. Heath (1999). Coherent measures of risk. *Mathematical finance 9*(3), 203–228.

Barnhill Jr. T. M. and W. F. Maxwell (2002). Modeling correlated market and credit risk in fixed income portfolios. *Journal of Banking & finance 26*(2), 347–374.

BCBS (2013). *Fundamental review of the trading book: A revised market risk framework.* Basel Committee on Banking Supervision. Consultative Document, Oct 2013.

BCBS (2014). *Fundamental review of the trading book: outstanding issues.* Basel Committee on Banking Supervision. Consultative Document, 19 Dec 2014.

Bluhm, C., L. Overbeck, and C. Wagner (2003). *An introduction to credit risk modeling.* Chapman & Hall.

Böcker, K. and M. Hillebrand (2008). Interaction of market and credit risk: an analysis of inter-risk correlation and risk aggregation. Bundesbank Discussion Paper, Series 2, No 11/2008.

Breuer, T., M. Jandačka, K. Rheinberger, and M. Summer (2010). Does adding up of economic capital for market-and credit risk amount to conservative risk assessment? *Journal of Banking & Finance 34*(4), 703–712.

Brockmann, M. and M. Kalkbrener (2010). On the aggregation of risk. *Journal of Risk 12*(3), 45–68.

Carey, M. and M. Hrycay (2001). Parameterizing credit risk models with rating data. *Journal of Banking & Finance 25*(1), 197–270.

Düllmann, K. and M. Trapp (2004). Systematic risk in recovery rates: An empirical analysis of US corporate credit exposures. Bundesbank Discussion Paper, Series 2, No 02/2004.

EBA (2014). *Guidelines on common procedures and methodologies for the supervisory review and evaluation process (SREP).* European Banking Authority. 19 Dec 2014.

Grundke, P. (2005). Risk measurement with integrated market and credit portfolio models. *Journal of Risk 7*(3), 63–94.

Grundke, P. (2013). On the reliability of integrated risk measurement in practice. *Journal of Risk 15*(3), 87–110.

Gupton, G. M., C. Finger, and M. Bhatia (1997). Creditmetrics: technical document. Technical report, JP Morgan & Co.

Jarrow, R. A. and S. M. Turnbull (2000). The intersection of market and credit risk. *Journal of Banking & Finance 24*(1), 271–299.

Kiesel, R., W. Perraudin, and A. Taylor (2003). The structure of credit risk: spread volatility and ratings transitions. *Journal of Risk 6*(1), 1–27.

Krüger, U., M. Stötzel, and S. Trück (2005). Time series properties of a rating system based on financial ratios. Deutsche Bundesbank Discussion Paper No 14/2005, Series 2.

Kupiec, P. (2007). An integrated structural model for portfolio market an credit risk. Federal Deposit Insurance Corporation.

Mählmann, T. (2006). Estimation of rating class transition probabilities with incomplete data. *Journal of Banking & Finance 30*(11), 3235–3256.

Marsala, C., M. Pallotta, and R. Zenti (2004). Integrated risk management with a filtered bootstrap approach. *Economic Notes 33*(3), 375–398.

Martin, M. R. W., H. Lutz, and C. S. Wehn (2011). A practical anatomy of incremental risk charge modeling. *Journal of Risk Model Validation 5*(2), 45–60.

Medova, E. A. and R. G. Smith (2005). A framework to measure integrated risk. *Quantitative Finance 5*(1), 105–121.

Meyer, C. (2009). Estimation of intra-sector asset correlations. *The Journal of Risk Model Validation 3*(3), 47–79.

Segoviano, M. A. and P. W. Lowe (2002). Internal ratings, the business cycle and capital requirements: some evidence from an emerging market economy. BIS Working paper no. 117.

Skoglund, J. and W. Chen (2011). On the choice of liquidity horizon for incremental risk charges: are the incentives of banks and regulators aligned? *Journal of Risk Model Validation 5*(3), 37–57.

Standard and Poor's (2013, May). Default, transition, and recovery: Global corporate quarterly default update and rating transitions.

Wilkens, S., J.-B. Brunac, and V. Chorniy (2013). IRC and CRM: Modelling framework for the Basel 2.5 risk measures. *European Financial Management 19*(4), 801–829.

Zeng, B. and J. Zhang (2001). An empirical assessment of asset correlation models. Moody's Investors Service.