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**The term structure of interest rates and
the macroeconomy: learning about economic
dynamics from a FAVAR**

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Non-technical summary

Research Question

In this paper, I analyze how agents form their expectations about the future development of the term structure of interest rates based on macroeconomic information.

Contribution

I adjust an established approach in two ways in order to better understand the expectation formation of the agents: First, I restrict the information that the agents can use for pricing bonds to what is actually available to them at each point in time. Second, I analyze whether agents act rationally and (equally) exploit all the information that is observable, or whether the expectation formation of agents is more influenced by recent developments (learning approach). In the latter case, agents behave with only bounded rationality.

Results

My results suggest that agents behave with only bounded rationality: Instead of taking equal account of all available information, they tend to focus on the latest economic and financial developments when forming their expectations about future yields. Yet the results also indicate that the agents' focus on the latest developments may lead them to make higher forecasting errors than if they dedicate equal attention to older information. The evaluation of the model also sheds light on the time variation of both the economic dynamics and the macro-financial linkage. Particularly in times of crisis, the estimation results for the agents acting with bounded rationality do indeed show a faster adjustment to the latest economic situation. The macro-financial linkage is likewise stronger in turbulent economic periods.

Nicht-technische Zusammenfassung

Fragestellung

In diesem Papier untersuche ich, wie Marktteilnehmer ihre Erwartungen über die zukünftige Entwicklung der Zinsstrukturkurve auf Basis von makroökonomischen Informationen bilden.

Beitrag

In zwei Aspekten passe ich einen etablierten Modellansatz an, um die Erwartungsbildung der Marktteilnehmer genauer nachzuvollziehen: Erstens beschränke ich zu jedem Zeitpunkt des Untersuchungszeitraums den Datenkranz, den die Marktteilnehmer zur Bewertung von Anleihen nutzen, auf die zu diesem Zeitpunkt tatsächlich bekannten Informationen. Zweitens untersuche ich, ob die Marktteilnehmer vollkommen rational sind und alle ihnen zur Verfügung stehenden Informationen (mit gleichem Gewicht) nutzen, oder aber Informationen aus der jüngeren Vergangenheit bei ihrer Erwartungsbildung stärker gewichten (Learning-Ansatz). Diese Marktteilnehmer handeln nicht vollkommen rational.

Ergebnisse

Meine Ergebnisse deuten darauf hin, dass die Marktteilnehmer eher nicht vollkommen rational handeln: Anstatt alle ihnen zur Verfügung stehenden Informationen gleichmäßig gewichtet zu nutzen, neigen sie vielmehr dazu, sich in ihrer Erwartungsbildung eher auf jüngere Entwicklungen von Wirtschaft und Finanzmärkten zu konzentrieren. Die Ergebnisse deuten jedoch darauf hin, dass die Marktteilnehmer mit dieser Fokussierung auf die jüngeren Entwicklungen größere Vorhersagefehler machen, als wenn sie älteren Informationen gleichermaßen Aufmerksamkeit widmeten. Mit der Schätzung kann die zeitliche Varianz sowohl der ökonomischen Dynamik als auch deren Verhältnis zu finanzwirtschaftlichen Entwicklungen näher beleuchtet werden. Insbesondere für Krisenzeiten zeigt sich, dass sich die Schätzergebnisse für die begrenzt rationalen Marktteilnehmer schneller an das geänderte wirtschaftliche Umfeld anpassen. Auch die Verbindung zwischen der makroökonomischen und finanzwirtschaftlichen Entwicklung ist in ökonomisch unruhigen Phasen stärker.

The Term Structure of Interest Rates and the Macroeconomy: Learning about Economic Dynamics from a FAVAR*

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Abstract

Expectations about macroeconomic developments are important determinants of long term interest rates. In this paper, I compare two different assumptions on how agents may form their expectations about the economy and yields in a pseudo real time exercise. Based on the no-arbitrage factor-augmented vector autoregression model developed by Moench (2008), I apply a purely econometric learning scheme as proposed by Laubach, Tetlow, and Williams (2007) in the estimation and compare the results to those of an estimation without discounting. In- and out-of-sample performance indicates that the agents are more inclined to form their expectations according to the learning approach.

Keywords: Affine Term Structure Models, Factor Models, Learning.

JEL classification: C38, E43, E44.

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1 Introduction

In this paper, I analyze how agents form their expectations about the future development of the term structure of interest rates based on macroeconomic information. I analyze whether agents act rationally and equally exploit all the information that is available, or whether they behave with only bounded rationality and form their expectations by putting a higher weight on recent developments (learning approach).

Macroeconomic information can help to explain the dynamics of the term structure of interest rates at least in part. For instance, [Ang and Piazzesi \(2003\)](#) use inflation and output related data and prove the usefulness of these variables both for in-sample analysis and out-of-sample forecasting. While [Ang and Piazzesi \(2003\)](#) rely additionally on latent factors, there are merely factors extracted from a huge panel of macroeconomic time series used by [Moench \(2008\)](#) to perform a forecasting analysis for the term structure of interest rates. Generally, I follow the setup of [Moench \(2008\)](#) to estimate the U.S. term structure of interest rates with information extracted from macroeconomic data. The main focus of this paper, however, is the implementation of a different approach to estimating the expectations of the agents. First of all, I will consider which information was actually available for the agents to price the bonds at each point in time. The information set will be restricted to that, and in each period, as new information arrives, the model will be reestimated. Current yields and expectations about future yields that are derived from these estimates are hence derived in a time-consistent manner as in an out-of-sample forecasting analysis. This real time estimation procedure fits well with the motivation of [Moench \(2008\)](#) for extracting information from an extensive data set:¹ Central banks base their decisions on the evaluation of multitudinous time series. Whether central bankers or bond investors, all agents update their evaluations when new data becomes available. New observations may influence their decision making more than data which is already decades old. Therefore, the interpretation of the modeling approach can be amplified to steer the agents' attention to the parts of such huge data sets which are intuitively most relevant, namely the most recent observations. Hence, second, I will apply a parsimonious and purely econometric mechanism that puts an emphasis on more recent information in the coefficient estimation. [Laubach et al. \(2007\)](#) present such an approach to discount older in favor of newer macroeconomic information for a term structure estimation and already suggest its combination with a factor model. They also relate it to the literature on learning; the rational expectations assumption thus does not need to be rejected completely, but expectations are rather assumed to have bounded rationality ([Evans and Honkapohja, 1994](#)).² The learning approach works in complement with the factor extraction to obtain the most relevant information from a huge data set - the latter along the cross section, the former along the time dimension.

The estimation results reflect the considerable time variation in the relation of the yield data to the macroeconomic factors. Their correlation reached their highest levels

¹The notion of 'real time estimation' refers to the estimation approach. The data that are used for the estimation, however, are revised. See Section 5 for details.

²For ease of exposition, I refer to the estimation approach with discounting as learning approach and to the approach without discounting as benchmark approach. Since the benchmark approach implies also a reestimation of the model coefficients in every period, however, agents in the benchmark approach can also be considered as learners who do not discount ([Laubach et al., 2007](#)).

shortly after the burst of the dotcom bubble at the beginning of the 2000s as well as during the recent crisis. The overall volatility of the state variables of the affine term structure estimation likewise supports the application of an estimation approach that incorporates time variation. In line with the literature, I also find the volatility of both types of state variables, the macroeconomic factors and the short-term interest rate, to be higher, for instance, at the end of the 1980s than between 2002 and 2007. A comparison of both in-sample fit and out-of-sample forecasting performance of the real time approach without discounting (henceforth, benchmark approach) and of the learning scheme is undertaken in order to understand which approach better approximates the expectation formation of agents: If investors have rational expectations and take all available information equally into account, then the estimated coefficients of the benchmark approach should provide a better in-sample fit. If they update their expectations by focussing on the most recent economic developments, the estimated coefficients of the learning scheme should fit the yield data better. I find that the learning scheme generally delivers better forecasts for longer maturities over shorter horizons than the benchmark approach. I interpret this pattern as an indication that the learning approach is indeed better able to reflect the agents' actual expectations for future yields. This strategy, however, is not necessarily recommendable in the long run: The benchmark approach produces better out-of-sample forecasts over longer horizons for shorter maturities. Given that realized yields are better forecasted by the benchmark approach, investors would in the end be better off if they had formed their expectations by dedicating equal attention to older information.

The remainder of this paper is organized as follows. First, I motivate the learning approach in a parsimonious regression analysis (Section 2). Then, I present the underlying model (Moench, 2008) and the estimation methodology in Section 3. In Section 4, I discuss the incorporation of a learning approach into such a model. Section 5 provides a description of the data that is used for the implementation of the model. The estimation results of the affine term structure model are shown in Section 6. These contain both in-sample yield estimates and out-of-sample yield forecasts, as well as a brief analysis of the relation to the underlying economy's dynamics. Section 7 provides a robustness check of the main results.

2 Real Time Estimation and DLS: A Brief Motivation

Discounting information from past observations is the basic mechanism that I aim to exploit in the analyses of this paper. Therefore, I will start by illustrating the effects of this concept in a parsimonious least square exercise. I regress the ten-year bond yield on a state vector containing the short rate r_t and four contemporaneous macroeconomic factors F_t which summarize the common dynamics of a macroeconomic data set (see Section 3 and 5 for details). This unrestricted regression can be described by

$$Y_t^{120m} = \alpha + \beta \cdot (F_t', r_t)' + u_t. \quad (1)$$

It is straightforward to estimate this regression using ordinary least squares (OLS) for the entire sample. If one wishes to allow the coefficients to vary over time, one can repeatedly reestimate the regression for expanding samples: Starting from an initializing

subsample $t = 1, \dots, t_{init.}$, one continuously expands and reestimates the equation and strings each subsample's last period yield estimate together to assemble the real time estimates \hat{y}_t^{120m} for the entire sample $t = t_{init.}, \dots, T$. [Laubach et al. \(2007\)](#) discuss the advantages of such repeated real time estimations, namely that they may improve the estimation accuracy in comparison to a full sample estimation, and that it restricts the information set of the econometrician at each point in time to what was then actually observable. Therefore, they call their recursively repeated estimation of a VAR a quasi-real-time learning approach, as it simulates the situation of a bond investor who evaluates the yields based on information that is currently available and updates its estimation when new information arrives. The learning effect may be intensified when one allows the bond investor or the econometrician to focus particularly on the most recent information (see also [Piazzesi and Schneider, 2006](#)). Downweighting information from observations further in the past places a greater weight on more recent information for the coefficient estimation. Downweighting is attained using discounted least squares (DLS), which I will discuss in more detail in Section 4. It implies that data receive less and less weight in the estimation as they become older. For instance, the discounting rate that I choose gives a four year old observation only about half of the weight of a recent observation.

I hence estimate the regression in Equation (1) three times, once with OLS over the full sample, once repeatedly for each period with OLS (OLS real time estimation), and once repeatedly for each period with discounted least squares (DLS real time estimation). For these three regressions, Figure 1 compares the resulting yield estimates \hat{y}_t^{120} with the data. The time span from January 1983 to January 1994 serves as the initialization period for both of the real time estimations, thus I compare the results of the approaches for the period from January 1994 to April 2010. Contrary to the results of [Laubach et al. \(2007\)](#), my OLS real time estimates are less precise than the OLS full sample estimates (the root mean squared errors are 1.01 compared to 0.90). In this application, the advantages of reestimations in the real time estimation are apparently overcompensated by the disadvantageous limitation of the information set. However, focusing on the information provided by the most recent data improves the precision remarkably, as can be seen from the lowest panel of Figure 1. The root mean squared error (RMSE) of the DLS real time estimation is 0.70.

Obviously, discounting old information requires the real time procedure of repeated estimations. Evaluating Equation (1) only once using DLS for the full sample induces a y_t^{120m} -estimate that clearly misses both the level and the variation of the data at the beginning of the sample. This can be seen from Figure 2. It also highlights how the DLS real time estimates become successively more precise than the OLS real time estimates towards the end of the sample.

In the following sections, I will apply more elaborate mechanisms for estimating bond yields based on macroeconomic factors. Specifically, I will estimate the macroeconomic dynamics in a factor-augmented VAR and use the coefficients from it for the yield estimation with an affine term structure model. At both of the two stages, however, I apply discounting in order to give more weight to the most recent observations. To assess the impact of discounting, I compare the results to a real time estimation of the model without discounting (henceforth, the benchmark model).

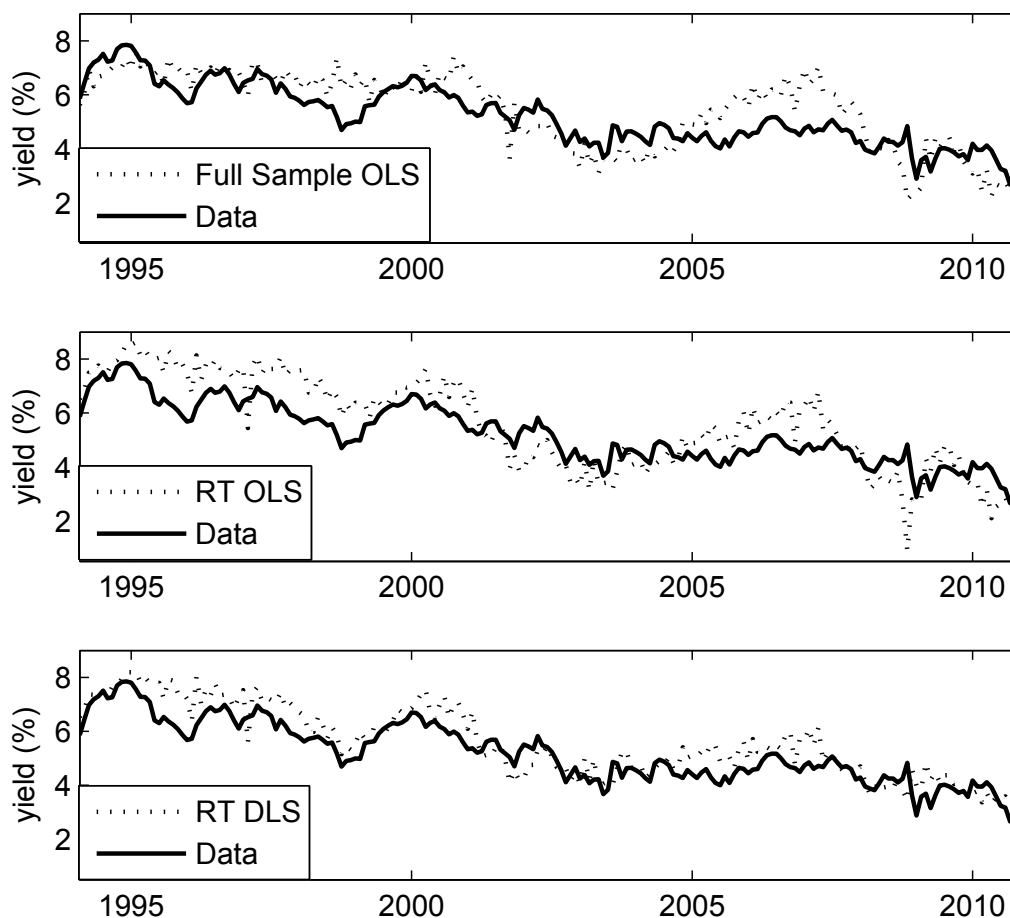


Figure 1: Full sample estimation and real time estimation of the ten-year bond yield: The upper figure plots the OLS full sample estimate against the data, the figure in the middle the OLS real time estimate, and the lower one the DLS real time estimate.

3 The Benchmark Model

I estimate a no-arbitrage affine term structure model in which the underlying state variables follow a factor-augmented vector autoregression (FAVAR). The approach is taken from [Moench \(2008\)](#), and I refer the reader to his paper for a thorough description of the procedure, since I present here only the elements that are most relevant for the implementation.

FAVAR models were introduced to the monetary policy literature by [Bernanke, Boivin, and Eliasziw \(2005\)](#) and have proven to be an appropriate approach for taking information from extensive data sets into account. Since the number of state variables that can be incorporated into affine term structure (ATS) models is very limited, the reduction of dimensionality that such factor methods allow is particularly interesting for the estimation of ATS models. ATS models have become workhorse models for researchers who want

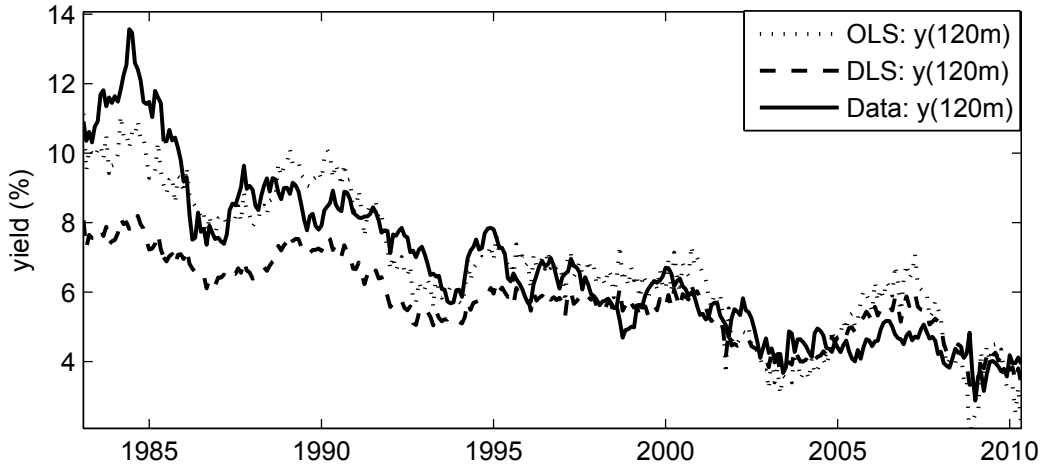


Figure 2: OLS and DLS regression estimates of the ten-year yield on four contemporaneous macro-factors and the three-month yield.

to analyze the term structure of interest rates in tractable no-arbitrage models. Their development dates back at least to [Duffie and Kan \(1996\)](#). For a review of the model class see, for instance, [Piazzesi \(2010\)](#).

As I will explain below in detail, I use some 150 transformed time series of various economic activities as macroeconomic data. Using principal components, the common dynamics of all the time series can be condensed into a few (k) factors. That keeps the number of parameters that have to be estimated throughout the analysis low without restricting the information set to only a few selected observable time series. The factors F_t that summarize the macroeconomic data X_t and their loadings C_t^F are hence estimated by

$$X_t = C_t^F \cdot F_t + \epsilon_t, \quad (2)$$

where the error term ϵ_t is a vector whose length is equal to the number of time series in X_t . It is assumed to be normal and to have mean zero (see [Bernanke et al., 2005](#)). The state equation of the model is

$$Z_t = \mu_t + \phi_t Z_{t-1} + \omega_t, \quad (3)$$

where Z_t contains both the macroeconomic factors and the short-term interest rate r_t and their lags, $Z_t = (F_t', r_t, F_{t-1}', r_{t-1}, \dots, F_{t-p+1}', r_{t-p+1})'$. The vector μ_t is the companion form of the $(k+1) \times 1$ vector μ_t . ϕ_t is the companion form of $\phi_t(L)$, a $(k+1) \times (k+1)$ matrix of lag polynomials of order p , and ω_t is the companion form of an error term of size $(k+1) \times 1$.

The application of model selection criteria supports the choice of the lag length $p = 4$ as in [Moench \(2008\)](#). I choose to extract $k = 4$ factors from the macroeconomic data, as an ATS estimation with a higher number of factors would be barely feasible.³

³Among others, [Ang and Piazzesi \(2003\)](#) and [Hamilton and Wu \(2012\)](#) discuss the problem of overfitting in ATS models with a large number of parameters that need to be estimated.

The model parameters are estimated in the same two-step procedure as in [Moench \(2008\)](#). First, the FAVAR parameters $\boldsymbol{\mu}_t$, $\boldsymbol{\phi}_t$, and $\boldsymbol{\omega}_t$ are estimated using ordinary least squares. Then these parameters are taken as given for the estimation of the term structure model parameters $\lambda_{0,t}$ and $\lambda_{1,t}$, which form the market price of risk according to $\lambda_t = \lambda_{0,t} + \lambda_{1,t}Z_t$. Then the model implied yield for a bond maturing in n -quarters, $\hat{y}_t^{(n)}$, can be calculated:⁴

$$\hat{y}_t^{(n)} = -(A_{t,n}/n) - (B'_{t,n}/n) \cdot Z_t, \quad (4)$$

where

$$A_{t,n} = A_{t,n-1} + B'_{t,n-1}(\boldsymbol{\mu}_t - \boldsymbol{\Omega}_t\lambda_{0,t}) + (1/2) \cdot B'_{t,n-1}\boldsymbol{\Omega}_tB_{t,n-1}, \quad (5)$$

$$B'_{t,n} = B'_{t,n-1}(\boldsymbol{\Phi}_t - \boldsymbol{\Omega}_t\lambda_{1,t}) - \delta'. \quad (6)$$

The coefficient δ is a $(k+1) \cdot p \times 1$ vector of zeros with a one on its $(k+1)$'s entry. $\boldsymbol{\Omega}_t$ is the variance-covariance matrix of the residuals of regression Equation (3). $A_{t,0}$ is zero, as $B_{t,0}$ is a vector of zeros. In this way, the recursive formulas above deliver for the three-month ($n = 1$ quarter) yield the coefficients $A_{t,1} = 0$ and $B_{t,1} = \delta'$, which leads by Equation (4) to the identity $\hat{y}_t^{(1)} = r_t$. From that point on, yields of higher maturities are obtained by adding information contained in the FAVAR estimates for $\boldsymbol{\mu}_t$, $\boldsymbol{\Phi}_t$, and $\boldsymbol{\Omega}_t$.

The model implied yields are estimated by minimizing the sum of squared errors, \hat{S}_t^{bm} , with respect to the risk parameters $\lambda_{0,t}$ and $\lambda_{1,t}$:

$$\hat{S}_t^{bm} = \sum_{n=1}^N \sum_{t=1}^T (\hat{y}_t^{(n)} - y_t^{(n)})^2. \quad (7)$$

Hence, the estimation is performed with a general method of moments (GMM) estimator in which the weighting matrix is an identity matrix.⁵

The subscript t indicates the time variation of the coefficients at all stages of the estimation. The time variation enters via the repeated reestimation of the model. Starting from an initialization sample, the model is always reestimated after expanding the subsample by one period until the end of the sample, T , is reached. As already discussed above, the actual yield estimates that are subsequently evaluated are the chained last period's estimates from each of the considered subsamples. This expanding window approach both restricts the data set to what is actually observed and enables all parameters to vary over time. Alternatively, a time-varying parameter VAR could be applied (see, for example, [Canova, 1993](#), and [Sims, 1993](#), or, particularly for the case of a FAVAR, [Koop and Korobilis, 2014](#)). However, this would induce further structural assumptions that are avoided in the parsimonious approach applied here. Moreover, expanding window estimation intuitively fits well with the view of a real time investor who updates his expectations when new data becomes available.

⁴As explained in Section 5, I use the three-month yield as the short rate. It is therefore more convenient to state the ATS model in terms of a quarterly frequency.

⁵[Hamilton \(1994\)](#), for example, describes the GMM estimator and discusses the choice of the weighting matrix.

4 The Learning Approach

4.1 The Exponential Weighting Scheme

The application of a FAVAR model allows the information of a data set to be captured efficiently with a large cross sectional dimension in a few common factors. The methodology which is described below aims additionally to focus on the data set's information along the time dimension that is most important for nowcasting and forecasting. This is achieved by downweighting past information compared to information from more recent observations. Old observations are not completely discarded from the sample, but they are assumed to be of minor relevance. The methodology can be related to the literature on learning, and it was shown that the discounting of older information is, at least asymptotically, not a deviation from the rational expectations assumption.

The standard formula for a weighted least squares estimator can be found, for example, in [Pollock \(1999\)](#) as $\hat{\beta}_t = (X_t' \Gamma_t X_t)^{-1} X_t' \Gamma_t Y_t$, where $\Gamma_t = \text{diag}(\gamma^{t-1}, \dots, \gamma^1, \gamma^0)$ is the discount matrix with the discount rate γ on its diagonal.⁶ [Montgomery and Johnson \(1976\)](#) relate the asymptotic properties of the DLS estimator to standard OLS estimators and also show that the DLS estimator is unbiased. Applying the estimator to the FAVAR equation (3) yields the coefficient estimates $[\hat{\mu}_t, \hat{\phi}_t] = (Z_{t-1}' \Gamma_t Z_{t-1})^{-1} Z_{t-1}' \Gamma_t Z_t$.

There are various applications of learning approaches in term structure models, including, for example, [Kozicki and Tinsley \(2001\)](#), [Piazzesi and Schneider \(2006\)](#) and [Laubach et al. \(2007\)](#). [Kozicki and Tinsley \(2001\)](#) show that the assumption on the limiting conditional forecast of the short rate process is a crucial determinant of long-horizon forecasts. As the “endpoint” of the short rate process depends on the information set it is conditioned on, this finding is directly related to the pseudo real time estimation applied in this paper. Given that the benchmark approach and the exponential weighting scheme differ in the degree of persistence of the state variables, this paper also builds on the finding of [Kozicki and Tinsley \(2001\)](#) that the assumed persistence of the short rate process influences its endpoint and thus the forecasts. However, while agents learn about the evolution of the short rate and the macroeconomy in this paper, [Kozicki and Tinsley \(2001\)](#) consider agents that learn about inflation expectations. The short rate endpoint is then directly calculated from the implied expected inflation value. Technically, I will instead follow [Laubach et al. \(2007\)](#) in their approach to downweighting old information. I differ from their methodology only slightly to account for having factors as state variables and not observed macroeconomic time series. Recursively updating the coefficients for each period as [Laubach et al. \(2007\)](#) do is unsuitable in a factor model due to the factors' limited identification. However, both extracting principal components and estimating the FAVAR coefficients using linear regressions is fast, hence the costs of estimating them from scratch at each point in time are negligible.

As discussed by [Laubach et al. \(2007\)](#), discounted least squares converge to constant gain learning. Specifically, the gain term $\frac{1-\gamma}{1-\gamma^t}$ converges to $1-\gamma$ in t . [Evans and Honkapohja \(2001\)](#), for example, consider agents in such constant gain models to be of bounded

⁶Note that this estimator is equivalent to GLS if Γ_t was the variance-covariance matrix of the regression's residuals ([Pollock, 1999](#)). Since I impose Γ_t exogenously, and γ is a discount factor, I use, like [Pollock \(1999\)](#), the notion discounted least squares to distinguish it from standard generalized least squares.

rationality to emphasize that the rationality assumption is not entirely abandoned.

In addition, the ATS estimation needs to be adjusted to the learning approach. The FAVAR-DLS parameters are estimated to fit the latest data best. Consequently, past errors in Equation (7) have to be downweighted accordingly:

$$\hat{S}_t^{ew} = \sum_{t=1}^T \sum_{n=1}^N (\hat{y}_t^{(n)} - y_t^{(n)}) \cdot \gamma^{-(t-T)} \cdot (\hat{y}_t^{(n)} - y_t^{(n)}). \quad (8)$$

As outlined above, if the discounting matrix was replaced by the inverse variance-covariance matrix, this estimator would coincide with the optimal weighted GMM estimator. The choice of the discount factor remains with the applicant. See Section 4.2 for a rationalization of my choice.

Such an exponential weighting scheme alleviates the adjustment to structural changes in the economy, because the meaning of old information smoothly decreases over time. Particularly, the exponential weighting scheme suits structural changes that do not occur abruptly, but rather evolve over months or even years. Typical events discussed in the literature as pivotal points of fundamental relations fit well with this characteristic. Even changes that appear to happen overnight, such as the appointment of Paul Volker as new central bank chairman, affect structural relations in the economy rather gradually. However, a model that incorporates regime switching may be a better choice if the structural changes do occur from one period to the next, or if one is interested in analyzing the difference itself between two regimes.⁷

An alternative but similar way of dealing with structural breaks is simply to discard older data completely from the estimation sample (Pollock, 1999). However, an exponential weighting scheme has at least two advantages compared to such a rolling sample estimation: First, one has to justify the choice of the memory horizon. For example, why should a 10-year old observation still be of relevance for the model, but an observation one month older than that completely discarded? Second, one easily runs into numerical problems by estimating short sub-samples. This point is of particular importance for an affine term structure application, as such models require a relatively large time dimension. These two drawbacks make the exponential weighting scheme appear the better choice when one wishes to apply a parsimonious learning scheme to an ATS yield estimation.

4.2 The Choice of the Discount Rate

As discussed in Section 4.1, the discount rate γ is exogenous in my model and can be freely chosen. Piazzesi and Schneider (2006) set their discount rate to 0.99, which leaves observations that are 17 years old half of the weight of the most recent observation in their estimation of quarterly data. Laubach et al. (2007) set the discount rate to the value that minimizes the one-month forecast error of the VAR of the macroeconomic time series. In my model, with principal components as state variables and a focus rather on the ATS model, such an approach is not very suitable. Due to the limited identification of the factors of the FAVAR, which may represent different aspects of the data in different

⁷For a term structure of interest estimation that incorporates regime switching, see, for example, Dai, Singleton, and Yang (2007).

periods, factor forecast precision matters in my model only as much as it leads to yield forecast precision. Optimizing the forecast precision of the ATS estimation over different discounting rates, on the other hand, would require massive computational efforts. In order to understand the influence of the discount rate value on bond yield estimates, I run a parsimonious least squares yield estimation, described in Section 7, for different values of γ . It turns out that the question of which value of γ is optimal depends crucially on both the forecasting horizon and the maturity under consideration. Figure 3 shows the RMSEs for the in-sample estimation and the 12-months ahead forecast. The longer the forecasting horizon, the more disadvantageous strong discounting is. Stronger discounting, nevertheless, improves the in-sample fit of long maturity yields (this higher precision of long term yield estimates is also reflected in the results of [Laubach et al., 2007](#)). To avoid pushing the results of my analyses too much in one of these directions, I choose $\gamma = 0.985$ for this paper, a value lying in between the values of [Laubach et al. \(2007\)](#) and [Piazzesi and Schneider \(2006\)](#). 0.985 is the value that optimizes the in-sample fit and out-of-sample forecasting performance of the 12-month bond yield. Figure 4 shows the average RMSE for the 12-month yield for all considered forecasting horizons, $h = \{0, 1, 6, 12\}$. The minimal RMSE of 0.55 is obtained with $\gamma = 0.985$. It may be interesting to explore whether

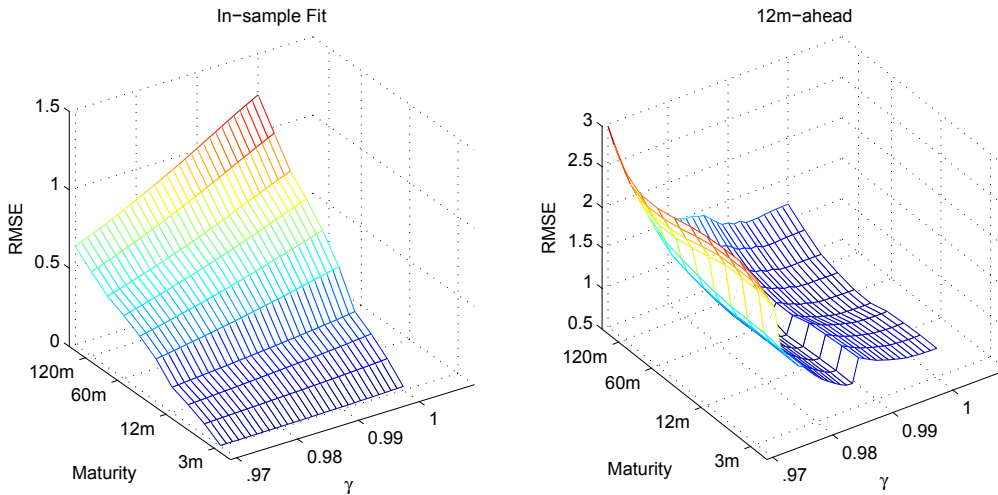


Figure 3: Root mean squared errors of the discounted least squares regression in dependence of the discount rate γ for different maturities, describing the in-sample fit in the left panel, and the 12 months forecasts in the right one.

the relations of the RMSEs to the maturity and forecasting horizons in the search for the optimal γ -choice are only specific to the considered sample. That could be checked through similar analyses over different samples or for different countries. For data other than yields, a forecasting exercise for different discount rates could be undertaken, for example, with the Survey of Professional Forecasters on expected inflation, because this survey also reports the forecasters' expectations of the variables over different horizons.

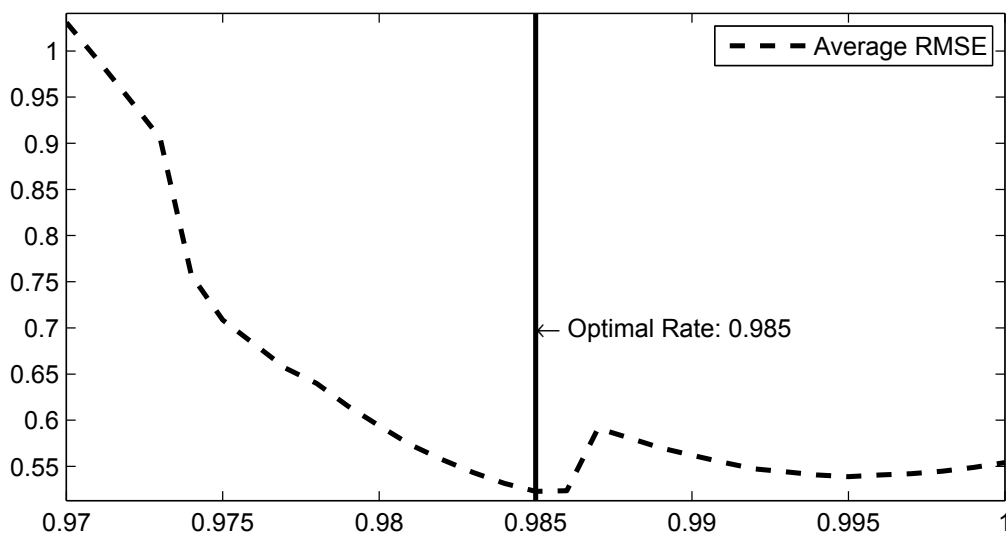


Figure 4: Root mean squared errors for the 12-month yield estimates from the discounted least squares regression, averaged over all considered forecasting horizons, $h = \{0, 1, 6, 12\}$, in dependence of the discount rate γ : The minimum is reached at $\gamma = 0.985$.

5 Data

5.1 Macroeconomic Data Set and Yield Data

I estimate the U.S. term structure of interest rates using macroeconomic data and yield data on a monthly frequency, ranging from January 1982 to April 2011. I use some 150 time series that provide measures for various economic activities such as industrial production, employment, wages, inflation, new orders, surveys, and outstanding loans.

There are two aspects of the data that are not in line with the real time approach that I take in the estimation: I do not consider publication lags in the data, but treat an observation for a certain month as already observable at the end of that month. Additionally, the time series may have been revised after their first publication. Instead of considering the original vintages of the data set, I use the data in their revised version. Although pure real time data are available for some of the series,⁸ it is not possible to compile a data set comparable to the one I use over almost 30 years for a monthly frequency from such sources. [Bernanke and Boivin \(2003\)](#) found that an increased number of time series is more important for forecasting performance in factor analyses than the use of real time data instead of revised data. [Giannone, Reichlin, and Sala \(2005\)](#) also argue that revision errors are typically idiosyncratic to specific series and thus vanish when common factors are extracted. However, due to the use of revised data in a real time estimation exercise, they call their approach “pseudo real time”. A detailed list of the time series I use is included in [Appendix A.2](#). The original data are transformed before factors are extracted as described in the next section.

⁸See, in particular, the ALFRED data base of the St. Louis Fed.

For the term structure of interest rates, I use freely available continuously compounded (zero coupon) yield data from Gurkaynak, Sack and Wright,⁹ see [Gurkaynak, Sack, and Wright \(2007\)](#) for a thorough description of the data. As is pointed out there, the Svensson parameters¹⁰ that the authors provide do not allow yields of very short maturities to be backed out, because they exclude all bonds that have maturities of fewer than three months from their parameter estimation. Therefore, the shortest maturity yield that I incorporate into my analysis is the three-month yield. The three-month yield receives thus a distinctive role in my ATS model estimation, as the shortest yield serves as a state variable. The other yields that are used for estimation are those of bonds of 6, 9, 12, 24, 36, 48, 60, 84, and 120 months to maturity.

5.2 Preparing the Data: Transformations and Time Series Selection

The factor analysis requires preparative transformations of the data. In order to obtain stationary factors, some of the macroeconomic time series need to be considered in first differences. Note, however, that this widely used pre-adjustment does not guarantee consistent factor estimates: As discussed by [Bai and Ng \(2004\)](#), the presence of a nonstationary idiosyncratic component in a time series can cause the extracted principal components to be inconsistent factor estimates.

Particularly the real time estimation procedure and the application of DLS make the imposition of stationarity a challenging task: Time series are not only required to be stationary over the entire sample from 1983 to 2010. They also need to be stationary for the sample that is actually available at each point in time, from the starting time of the real time estimation in January 1994 until April 2010. Moreover, the estimation using discounted least squares makes an autoregression of a time series even more prone to nonstationarity due to its focus on the latest evolution of the series.

The severity of the stationarity issue becomes obvious in a simple regression analysis. I run AR(1)-regressions for each time series of the data set separately, starting in January 1994 and then expanding the sample continuously by one period until reaching the last of the 328 observations in the data set. With some transformation effort, all of the considered time series are stationary in 202 OLS-AR(1) evaluations. However, in 428 of the 21426 regressions that are undertaken, estimates are nonstationary if the AR(1) coefficients are estimated using DLS. Thus, more time series need to be differentiated for the application of DLS. This removes information from the data and therefore has negative effects on the precision of the following estimations.

If I wanted to transform all time series to expose stationary dynamics in all DLS regressions, I would have to differentiate almost all series, especially since for certain groups of time series it is desirable that they enter the analysis in the same conceptual manner. For example, if total capacity utilization is considered in logarithms, capacity utilization of the durable goods production should not be considered in first differences but also in logarithms. In any case, imposing a certain transformation scheme for the

⁹See <http://www.federalreserve.gov/econresdata/researchdata.htm>.

¹⁰For a description of the parameters and the corresponding functional form describing the yield curve, see also [Svensson \(1994\)](#).

entire sample may not only lead to over-differentiating; it actually also does not follow the real time orientation of my approach.

Given these prerequisites, I apply a transformation procedure that reduces the need for differentiation. In line with the real time approach, I choose the transformation scheme for all points in time separately: To achieve this, I first define a preferred transformation setup, namely a setup that leads the OLS-AR(1) coefficients of all time series at each point in time to be stationary. This scheme incorporates only limited differencing. I also specify logical groups of time series that should be transformed in the same way, for example a group containing all series related to capacity utilization. Then, I check separately for all periods whether all series are stationary given the preferred transformation scheme. If one series is not, I deviate from the preferred scheme and apply first differences not only on that series but also on all others in its group. The outcome was that factors resulting from data treated using such a real time transformation approach do indeed preserve more valuable information for the term structure estimation. The list in Appendix A.2 provides details on the preferred transformation scheme and the grouping of the time series.

An additional source of nonstationarity of the ATS-model's state variables is, of course, the short rate. I cannot react to a nonstationary short rate process by taking first differences, as the ATS equations (4) to (6) imply its inclusion in levels. Fortunately, the short rate exposes nonstationary dynamics only three times under the assumption that it is estimated with DLS as a fourth order autoregressive process as in the FAVAR described in Equation (3). Therefore, after assembling the state variables, I follow several contributions to the literature on learning and project the last period's estimates for the FAVAR for the current period if it exposes nonstationary dynamics.¹¹

Another preparative task is selecting those series from the extensive data set that are of relevance for estimating interest rates. I choose a parsimonious procedure, namely discarding those series from the analysis that do not exceed a correlation of 0.04 to the short-term interest rate. To remain consistent, I repeat the selection at each point in time. Therefore, the number of series from which the principal components are extracted varies between 93 and 123 in the learning approach and between 97 and 123 in the benchmark approach. The difference is caused by divergent data transformations in both approaches. More elaborate techniques for data selection are available,¹² but their application would go beyond the scope of this paper.

Before the factors are finally extracted through principal component analysis, the time series are also standardized to have zero mean and unit variance. This guarantees that each series' movements, if the series is originally of low or high variance, has the same relevance in the principal component analysis. Appendix A.1 provides plots of the macroeconomic factors.

6 Affine Term Structure Model Estimation

I start the evaluation of the benchmark model and of the exponential weighting scheme by comparing their in-sample fit of the yield data. The real time estimation is deduced for the subsample from January 1994 to April 2010. There is no specific reason for the

¹¹See, for example, [Marcet and Sargent \(1989\)](#), [Evans and Honkapohja \(1994\)](#), or [Laubach et al. \(2007\)](#).

¹²See, for example, the least angle regression of [Efron, Hastie, Johnstone, and Tibshirani \(2004\)](#).

choice of the starting date, except that especially the real time estimation of the ATS model requires a fairly long initialization period.

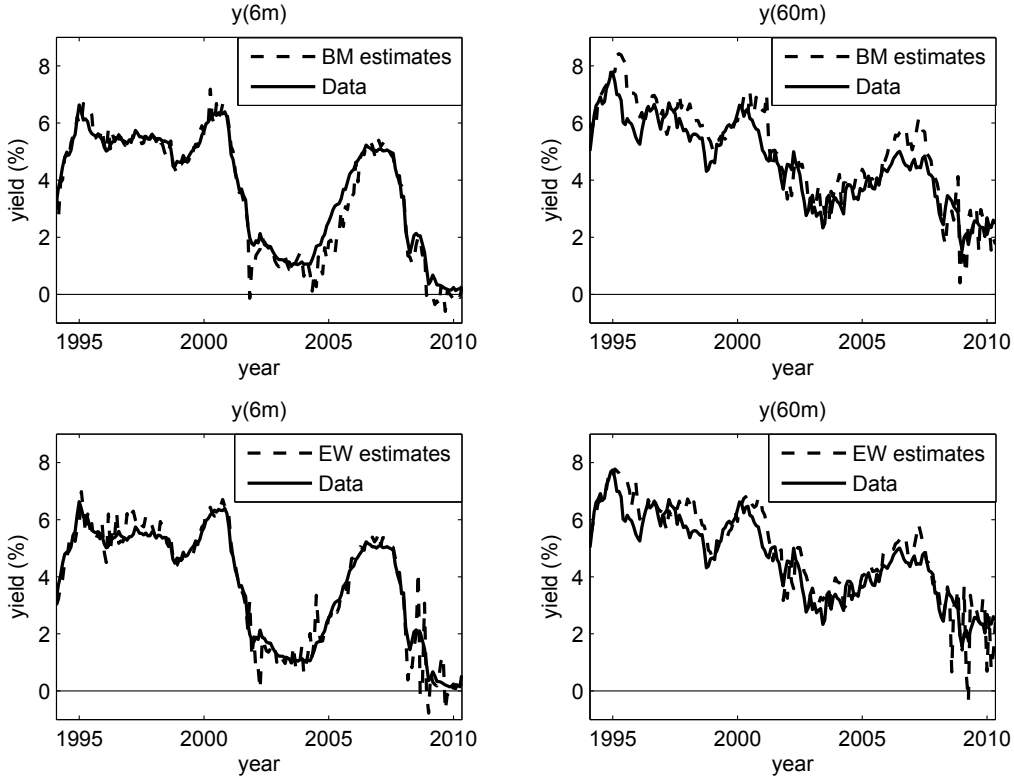


Figure 5: Estimates for the six and sixty-month yields in comparison to the yield data: In the upper (lower) panel, the estimates from the benchmark approach (exponential weighting scheme) are shown for the subsample from 01:1994 to 04:2010.

Because of the point specific estimation, the in-sample estimation coincides with a zero step ahead forecast. Therefore, the RMSEs for the estimations can be found separately for each maturity in column $h = 0$ of Table 1. The RMSEs are relatively high given that the term structure of interest rates can already be fitted very well with three latent factors (Litterman and Scheinkman, 1991). However, no latent factors are used in this estimation. Moench (2008), who does not use latent factors either, reports RMSEs for short-horizon forecasts that indicate just a slightly better fitting ability for a forecasting period from 1994 to 2003. As in the least squares analysis in Section 2, the average RMSE of the benchmark estimation over the full sample is slightly lower than for the benchmark real time estimation (0.63 compared to 0.67). Again, if one does not restrict the information set to what was actually known at each point in time, one overstates the agents' ability to price bond yields. The exponential weighting scheme delivers slightly more precise estimates (RMSE=0.66). The table reveals differences over the maturity spectrum, namely that long yields are fitted better with the exponential weighting scheme, while the benchmark model is advantageous for shorter maturity yields. This pattern will be discussed in Section 6.2. The fitting abilities of the different approaches also differ over

the course of time. Figure 5 depicts the yield estimates of both approaches, displayed in the upper two graphs for the benchmark model, and in the lower two graphs for the exponential weighting scheme. It appears that the focus on more recent developments in the exponential weighting scheme makes its estimates react faster to the latest trends. The two-year increase in the six-month yield starting in mid-2004, for example, is followed more precisely by \hat{y}_t^{ew} than by \hat{y}_t^{bm} . Also, in the case of the 60-month yield, one sees that \hat{y}_t^{ew} reacts faster to the end of the upward trends in the summers of both 2000 and 2006. However, the higher meaning of newer information disturbs \hat{y}_t^{ew} of any maturity when the financial market turmoil escalated at the beginning of 2008. Apparently, the focus on the most recent developments helps little when current perspectives are very uncertain.

Additionally, it is worth noting that the means and variances of the yield data of all maturities are better fitted with the exponential weighting scheme, with the sole exception of the variance in the ten-year yield, which is fitted more precisely with the benchmark model. Nevertheless, the comparison of the fitting ability makes it hard to claim that one of the approaches does far better overall, but rather reveals different strengths and weaknesses that can be linked to their underlying characteristics.

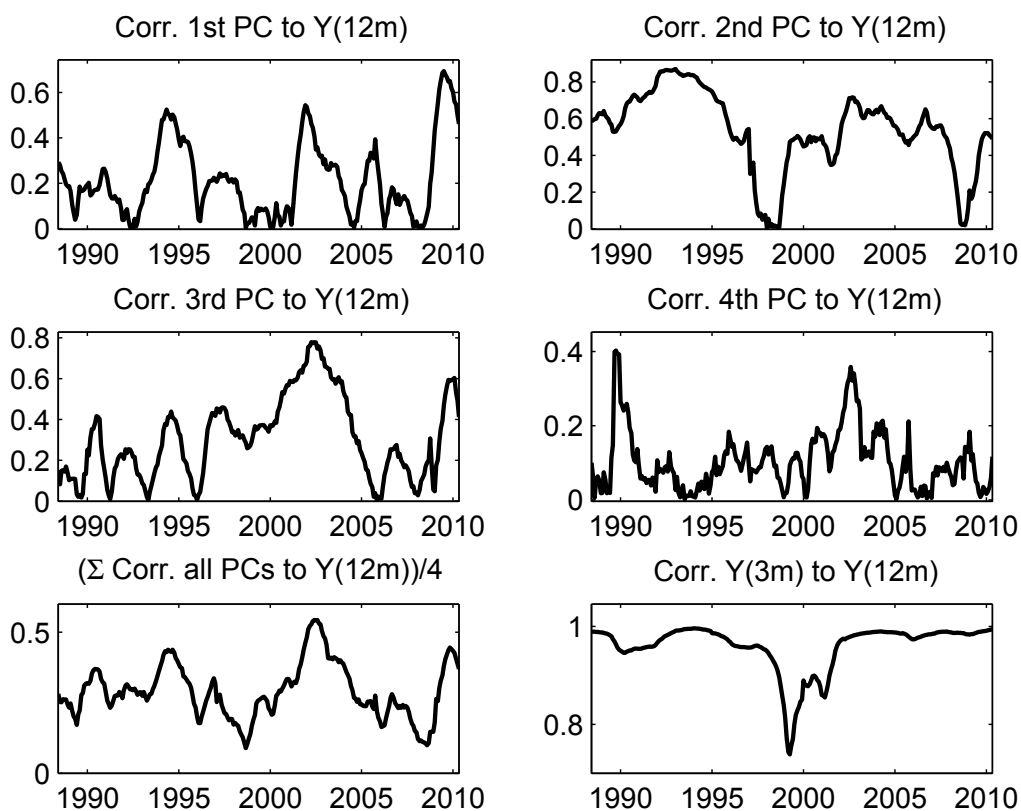


Figure 6: Absolute values of correlations of state variables with the one-year bond yield for the period of 06:1988 to 04:2010. For the macroeconomic factors, the simple mean of correlations of the four factors to the one-year bond is shown.

6.1 Economic Relations and Dynamics

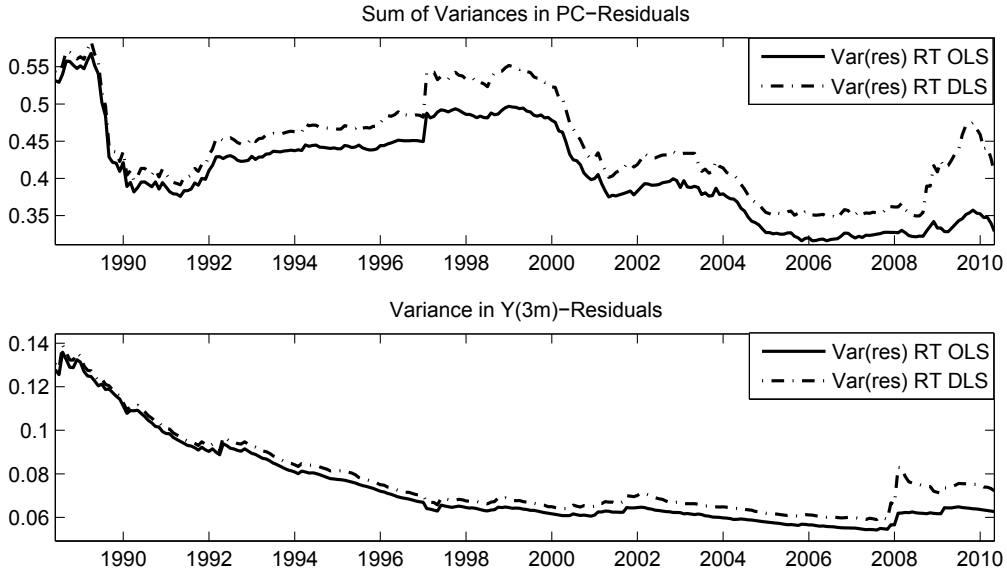


Figure 7: Time variation of macroeconomic factor and interest rate volatility for the subsample from 06:1988 to 04:2010. The graphs plot the diagonal elements of the degrees of freedom-adjusted variance-covariance matrix of the FAVAR, $\Sigma_t = \Omega_t \cdot \Omega_t'$. For the macroeconomic factors, the variances are accumulated as the arithmetic mean of the four variances.

Once the model is estimated for each point in time, the results also allow light to be shed on the dynamic relations of the factors to the yield data. I derive correlations of the state variables with the yield of a bond maturing in 12 months. Correlations are calculated for a rolling window of five years for the sample from June 1988 to April 2010. Figure 6 shows these correlations in absolute values. Their time variation appears to be high. Generally, and not surprisingly, the correlation is highest between the three-month and 12-month yield.¹³ Among the macroeconomic factors, the second factor appears to be the most relevant state variable according to this correlation criterion. I also provide the arithmetic mean of the separate correlations of the macroeconomic factors as an accumulated measure. It allows the overall meaning of the macroeconomic factors to be assessed, as a drop in the correlation of one factor may be compensated by a peak in another factor's correlation. The measure of the accumulated correlations peaks in times of major financial market stress. The reaction is just slightly delayed, as the financial market stress first has to be reflected in the macroeconomic data. Hence, the highest level is reached in early 2002 in the aftermath of the burst of the dotcom bubble and the recession following the terror attacks of September 2001. The indicator declines afterwards in the tranquil economic period around the middle of the decade, before starting to rise again due to the recent financial crisis. The economic interpretation of this pattern

¹³The drop in this correlation down to 0.74 in April 1999 originates from a five-year estimation window that covers an eightfold inversion of the relation of the 12-month yield to the three-month yield.

depends on the actual drivers of these increases in correlation, and the model setup in this paper is not the most appropriate to analyze it further. Studies like [Backus and Wright \(2007\)](#), [Cochrane and Piazzesi \(2005\)](#) and [Ludvigson and Ng \(2009\)](#) decompose long term yields and relate their components to business cycle developments. They find that term premia (or unexpected yield components) behave countercyclically. Nevertheless, on the aggregated level that is considered here, it is interesting to see that macroeconomic information appears to be particularly helpful for forecasting yields in times of crisis. I also find that most of the correlations of the factors to the yields are higher when the factors are considered in lags. The stronger correlation in lags may be caused in part by the fact that macroeconomic data are usually published with some delay and/or that macroeconomic data may indeed contain predictive information for forecasting yields ([Moench, 2008](#)). There is no clear trend regarding the meaning of factors for yields of different maturities. Overall, the correlations do not indicate a lower or higher meaning of macroeconomic factors for yields of shorter or longer maturities. However, the third and fourth factors provide more information related to the longer maturities, while the second factor is more correlated with the shorter maturities. The first factor's correlation, by contrast, is highest for intermediate maturities. Again, it is important to mention that these results are heavily driven by the transformations that were imposed on the data.¹⁴ A lack of interpretability remains one of the obvious disadvantages of using principal components to summarize macroeconomic data.

The derivation of the real time estimates also allows us to consider the time variation of the volatility of the state variables. To illustrate the time variation of the factors' variances, I sum them up again for all four principal components. Looking at [Figure 7](#) makes it clear, first of all, that there is considerable time variation in the subsample from June 1988 to April 2010 that would go unnoticed in a full sample estimation. The OLS estimated volatility decreases over most of the sample, interrupted once by a rise at the end of the 1990s. The short rate likewise exhibits a decline in volatility until late-2007. That is in line with the findings of [Laubach et al. \(2007\)](#), who interpret this gradual reduction as an outcome of the great moderation of interest rates and other key economic indicators after the turbulent economic times of the 1970s.

DLS volatilities are generally higher, and the ups and downs as observed for the OLS estimates are stronger. For the macroeconomic factors, this overlays a clear downward trend seen in the OLS volatilities. This is no surprise, as the idea of DLS is to focus on the latest developments. The short rate volatilities originating from the OLS and DLS estimations are far more similar to each other. They diverge just at the end of the sample. The start of the financial crisis is thus reflected less clearly in the OLS evaluation. The DLS estimator, however, appears once more to react faster to the change in the economic environment.

¹⁴Note that the imposed transformations necessary for stationarity of the data do not vary from period to period as in the other parts of the paper, because that would add another source of variance volatility. Hence, for the calculation of the correlation figures and the state variable volatilities, I apply a constant transformation regime and accept the projection facility to be triggered more often.

Table 1: Root mean squared errors of ATS estimation

Benchmark:				
Maturity	h=0	h=1	h=6	h=12
y(3m)	0.00	0.25	0.87	1.44
y(6m)	0.48	0.50	1.09	1.66
y(9m)	0.60	0.62	1.12	1.64
y(12m)	0.56	0.58	1.06	1.60
y(24m)	0.59	0.64	1.07	1.57
y(36m)	0.65	0.70	1.09	1.56
y(48m)	0.71	0.77	1.11	1.54
y(60m)	0.76	0.81	1.14	1.53
y(84m)	0.82	0.88	1.16	1.49
y(120m)	0.88	0.94	1.14	1.42
Exponential Weighting:				
Maturity	h=0	h=1	h=6	h=12
y(3m)	0.00	0.23	0.87	1.49
y(6m)	0.51	0.55	1.06	1.65
y(9m)	0.65	0.64	1.10	1.63
y(12m)	0.66	0.72	1.22	1.78
y(24m)	0.53	0.65	1.08	1.57
y(36m)	0.62	0.74	1.14	1.60
y(48m)	0.68	0.78	1.16	1.57
y(60m)	0.69	0.79	1.13	1.53
y(84m)	0.77	0.86	1.14	1.50
y(120m)	0.84	0.92	1.14	1.46
$RMSE^{BM} - RMSE^{EW}$:				
Maturity	h=0	h=1	h=6	h=12
y(3m)	0.00	0.02	-0.01	-0.05
y(6m)	-0.04	-0.05	0.02	0.01
y(9m)	-0.06	-0.02	0.02	0.00
y(12m)	-0.11	-0.14	-0.16	-0.18
y(24m)	0.06	-0.02	-0.01	0.00
y(36m)	0.03	-0.04	-0.05	-0.04
y(48m)	0.03	-0.01	-0.04	-0.03
y(60m)	0.07	0.02	0.00	-0.01
y(84m)	0.05	0.02	0.02	0.00
y(120m)	0.04	0.01	0.00	-0.04

RMSEs for in-sample estimates and out-of-sample forecasts for the benchmark model (upper panel) and the exponential weighting scheme (middle panel), separately for all maturities and for forecasting horizons of $h=\{1,6,12\}$ months. The $h=0$ values are the values for the real time estimation and thus provide information on the in-sample fit. The lower panel depicts the difference between the upper and the middle panel. I use data from the period from 01:1983 to 04:2010 to estimate and forecast for the time span from 01:1994 to 04:2011.

6.2 Out-of-sample Forecasting

After the in-sample analyses in the previous sections, I will assess the out-of-sample forecasting performance of both the benchmark model and the exponential weighting scheme. Out-of-sample forecasts can be easily calculated once real time estimates have been obtained. The FAVAR coefficients, both from the benchmark model and the exponential weighting scheme, are used to obtain state vector forecasts according to

$$\hat{Z}_{t+h|t} = \hat{\phi}_t^h Z_t + \sum_{i=0}^{h-1} \hat{\phi}_t^i \hat{\mu}_t, \quad (9)$$

as it is described, for example, in [Moench \(2008\)](#). Given the state vector forecast, the FAVAR parameter estimates, and the ATS risk parameters $\hat{\lambda}_0$ and $\hat{\lambda}_1$, the yield forecasts are calculated according to Equations (4) to (6). I evaluate the forecasting performance using root mean squared errors that are reported in the upper two panels of [Table 1](#). The RMSEs are higher overall than in comparable studies, especially [Moench \(2008\)](#). This is, first of all, due to the forecasting sample. I use data from January 1983 to April 2010 to forecast for the time span from January 1994 to April 2011, hence the sample includes about two and a half turbulent years of the recent financial crisis. Moreover, as discussed in [Section 5.2](#), my modeling approach with real time estimations and discounting requires more first differencing to induce stationarity, which also removes information from the data that may be useful for forecasting.

The lower forecasting accuracy of the exponential weighting scheme over longer horizons reflects the findings of [Clark and McCracken \(2007\)](#). Among the techniques which they compare in order to deal with structural instabilities in VAR frameworks, DLS is one of the least successful methods in terms of forecasting performance. In my context, however, DLS is applied because of its relation to learning. Hence, before comparing the forecasting performance of the benchmark model and the exponential weighting scheme, it is important to remember the original motivation of applying a learning scheme: Agents are assumed to learn about the evolution of the considered state variables, they down-weight old information in favor of new information in order to form their expectations about the future values of the state variables. However, whether the exponential weighting scheme is indeed better in that respect than the standard approach is difficult to prove. Particularly, a better ability to capture the agents' expectations only coincides with a better out-of-sample forecasting performance if the agents' expectations were indeed perfect forecasts. If agents, on the other hand, expected the state variables to evolve very differently from what actually occurred, the overall forecasting ability of a model does not say anything about the agents' expectations. Still, particularly term structure data allows expectations about the future to be detected in the cross section of yields. Yields of bonds with long maturities are equal to the sum of the average of the investors' expectations about future short rates and term premia.¹⁵ Hence, a good in-sample fit of the cross section of yields reveals that a model is able to capture the expectations of investors about future short-term interest rates and/or term premia, while the forecasting ability rather reveals whether the investors' model implied expectations turned out to be right. The lower panel of [Table 1](#) provides a convenient overview of this, namely the

¹⁵See for example [Wright \(2011\)](#) for a decomposition of long term yields into these two elements.

Table 2: Root mean squared errors of alternative forecasting approaches

Random Walk			
Maturity	h=1	h=6	h=12
y(3m)	0.23	0.89	1.56
y(6m)	0.22	0.90	1.57
y(9m)	0.23	0.91	1.56
y(12m)	0.25	0.91	1.53
y(24m)	0.28	0.90	1.41
y(36m)	0.29	0.87	1.28
y(48m)	0.30	0.82	1.18
y(60m)	0.29	0.78	1.09
y(84m)	0.29	0.71	0.96
y(120m)	0.28	0.64	0.84
Diebold Li (AR)			
Maturity	h=1	h=6	h=12
y(3m)	0.24	0.86	1.45
y(6m)	0.21	0.83	1.41
y(9m)	0.22	0.82	1.38
y(12m)	0.24	0.82	1.35
y(24m)	0.27	0.82	1.23
y(36m)	0.29	0.78	1.10
y(48m)	0.30	0.74	1.00
y(60m)	0.30	0.71	0.92
y(84m)	0.29	0.66	0.83
y(120m)	0.29	0.63	0.78
Diebold Li (VAR)			
Maturity	h=1	h=6	h=12
y(3m)	0.28	0.91	1.51
y(6m)	0.25	0.88	1.46
y(9m)	0.26	0.87	1.43
y(12m)	0.27	0.86	1.40
y(24m)	0.28	0.84	1.26
y(36m)	0.28	0.79	1.12
y(48m)	0.29	0.74	1.01
y(60m)	0.29	0.70	0.93
y(84m)	0.28	0.65	0.83
y(120m)	0.29	0.63	0.78

RMSEs of forecasts from a random walk evaluation and from Diebold-Li's VAR and AR models. The considered sample coincides with the forecasting sample of the benchmark model.

difference between the RMSEs of benchmark and exponential weighting scheme as they are presented in the panels above. A tradeoff is revealed between fitting expectations and good forecasting performance, as the benchmark model provides smaller RMSEs for longer-horizon forecasts, and the exponential weighting scheme performs better in the cross section. In other words, agents appear to form their expectations by learning, but in the long run they actually should not do so. This pattern becomes more pronounced for values of the discount rate γ that induce stronger discounting.¹⁶

Decomposing model implied yields into term premia and expected short rates could provide details on which of these two components is better estimated by the learning scheme. However, the yield curve decomposition is avoided for at least two reasons. First, identifying significant differences in term premia and short rate expectations between the two approaches may be hampered by the assumption of an exogenous short rate process (see Section 3 for details). Second, the pricing accuracy of the model setup is also limited since no latent factors are used. Yield decompositions would thus be imprecise in principle. Therefore, I limit the analysis to a comparison of implied expectations of overall yields from both approaches. The question of whether the better fitting ability of long term rates is caused by a better fit of term premia or short rate expectations may better be analyzed in a latent factor model.

The forecasting performance was compared to the results of other models, namely to a simple random walk and the Diebold and Li (2006) AR and VAR setups.¹⁷ The results indicate that these models forecast much better over short horizons. Nevertheless, for a horizon of 12 months, the RMSEs of these models' forecasts also approach levels of 1.50 over this sample (see Table 2).

7 Robustness Checks

The ATS estimation, especially the forecasting performance, reveals the cost of inducing stationarity. The results require a check for robustness. ATS models appear to be particularly prone to nonstationarity. Since the idiosyncratic components of the principal component analysis can be nonstationary (see Section 5.2), the variance-covariance matrix Ω_t constitutes a second possible way for nonstationarity of the data to enter the ATS analysis. Hence, I use simple linear regressions instead to see whether they deliver similar results. The real time transformation approach described in Section 5.2 may also cause particular outcomes. By applying the same transformation scheme at all points in time, I therefore also check whether the real time transformation scheme drives the main characteristics of my results.

To do so, I use either OLS or DLS to regress yields of all maturities on the same state variables which are used for the ATS estimation. The approach is hence the same as the real time estimation described in Section 2, but with $L = 4$ lags of the state variables as in the ATS estimation:

$$Y_t^n = \alpha_t + \beta_t(L)Z_t + u_t. \quad (10)$$

RMSEs of these regressions are reported in Table 3. When comparing those results with

¹⁶In addition to $\gamma = 0.985$, the ATS model was estimated for γ -values of 0.973, 0.988 and 0.995.

¹⁷The estimation of the Diebold-Li model was facilitated by the codes of Carlo Favero of Bocconi university that are available on his website.

Table 3: Root mean squared errors of least squares analyses

Benchmark:				
Maturity	h=0	h=1	h=6	h=12
y(6m)	0.14	0.21	0.61	0.85
y(9m)	0.24	0.26	0.62	0.85
y(12m)	0.32	0.32	0.62	0.84
y(24m)	0.58	0.51	0.66	0.83
y(36m)	0.79	0.67	0.74	0.87
y(48m)	0.95	0.80	0.84	0.95
y(60m)	1.08	0.90	0.93	1.03
y(84m)	1.29	1.07	1.09	1.19
y(120m)	1.49	1.23	1.25	1.36
Exponential Weighting:				
Maturity	h=0	h=1	h=6	h=12
y(6m)	0.10	0.18	0.63	1.01
y(9m)	0.17	0.23	0.64	1.01
y(12m)	0.24	0.28	0.64	0.99
y(24m)	0.43	0.46	0.66	0.93
y(36m)	0.57	0.61	0.72	0.93
y(48m)	0.68	0.72	0.80	0.97
y(60m)	0.77	0.81	0.88	1.02
y(84m)	0.91	0.96	1.01	1.13
y(120m)	1.05	1.10	1.15	1.27
$RMSE^{BM} - RMSE^{EW}$:				
Maturity	h=0	h=1	h=6	h=12
y(6m)	0.04	0.03	-0.02	-0.16
y(9m)	0.06	0.03	-0.02	-0.16
y(12m)	0.08	0.03	-0.02	-0.15
y(24m)	0.16	0.04	0.00	-0.10
y(36m)	0.22	0.06	0.02	-0.06
y(48m)	0.27	0.08	0.04	-0.02
y(60m)	0.31	0.09	0.05	0.01
y(84m)	0.38	0.11	0.08	0.06
y(120m)	0.44	0.13	0.10	0.09

RMSEs for in-sample estimates and out-of-sample forecasts from OLS (upper panel) and DLS (middle panel), separately for all maturities and for forecasting horizons of $h=\{1,6,12\}$ months. The $h=0$ values are the values of the real time estimation and thus provide information on the in-sample fit. The lower panel depicts the difference between the upper and the middle panel. The forecasting sample is from 01:1983 to 04:2011, with the real time estimation (and forecasting) starting in 01:1994.

the ATS results, one should keep in mind that differences may be caused either by the yield estimation approach or by the applied transformation scheme. Yields are fitted very well using least squares in-sample, and also the 12-months ahead forecasts are mostly better than those from the ATS model. Nevertheless, least squares methods cannot keep up with the ATS forecasting performance over intermediate horizons. More interestingly, the evaluation of least squares results delivers a similar pattern as it is found in the ATS estimation: Like the ATS benchmark model, the OLS regression performs better for long horizon forecasts. On the other hand, the DLS regression allows a better fit of the cross section on shorter forecasting horizons. As in the ATS estimation, the better performance of DLS on long maturities overcompensates its weak long horizon forecasting ability for maturities at the long end of the yield curve. From five years to maturity onwards, DLS provides better forecasts than OLS over all horizons. Hence, the findings from a least squares analysis are very much in line with the results of the ATS estimation.

8 Conclusion

In this paper, I have compared two pseudo real time approaches in their ability to forecast the term structure of interest rates based on macroeconomic information. To achieve this, I have combined the ideas of [Laubach et al. \(2007\)](#) on applying a learning scheme to an ATS model and of [Moench \(2008\)](#) on how to estimate an ATS model with macroeconomic factors from a FAVAR.

The pseudo real time approach with repeated coefficient estimations that I have applied is supported by both the high volatilities of the macroeconomic factors and the time variation of the correlations of these factors to the bond yields. The learning approach in which past information is downweighted in favor of a higher meaning of more recent information turns out to be more successful for forecasting long maturity yields over short horizons. Long horizon forecasts, particularly for shorter maturity yields, are on the other hand, more precisely obtained through the benchmark model without discounting. Since long term yields are compounded from expectations about short term yields and risk premia, the successful fitting of the cross section of yields with the exponential weighting scheme can be interpreted as a better ability to capture the agents' expectations for future yields. It would be interesting to see whether this pattern also emerges in other ATS setups including, for instance, latent factors that guarantee a higher estimation precision overall. As a byproduct of the analysis, I found that the choice of the discount rate that delivers the best forecasting performance depends heavily on the forecasting horizon and the time to maturity under consideration.

A Appendix

A.1 Macroeconomic Factors

The macroeconomic factors are plotted in Figure 8 for the benchmark approach and in Figure 9 for the exponential weighting scheme. 5% confidence bands are obtained through a Monte Carlo procedure with the stationary block bootstrap of Politis and Romano (1994). See also Moench (2008) for details on the application.

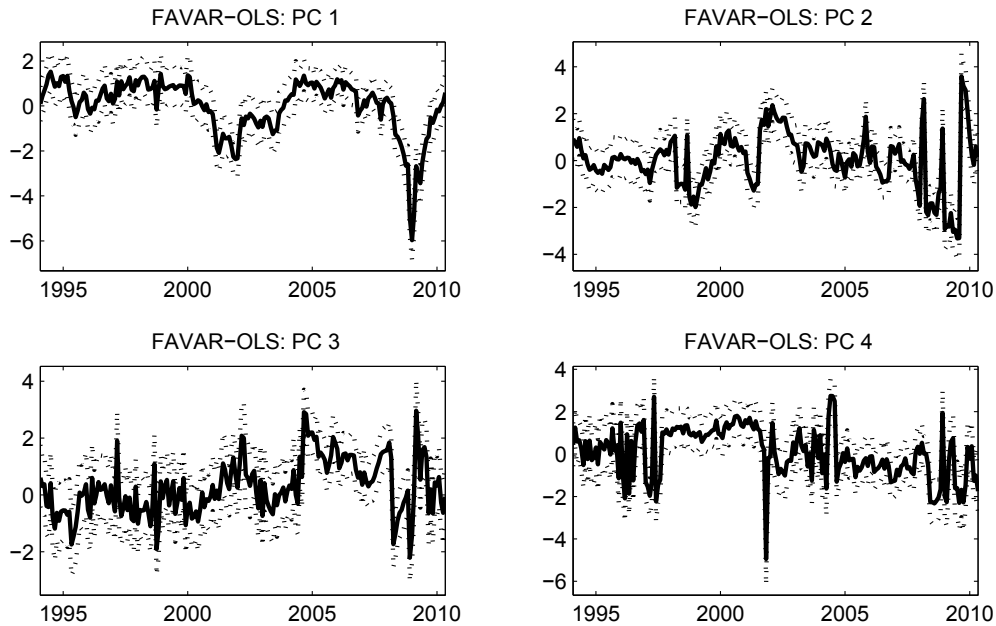


Figure 8: Macroeconomic factors with 5% confidence bands for the benchmark approach.

A.2 Macroeconomic Data

The macroeconomic data set was kindly provided by Domenico Giannone. All time series that are actually used in the analysis are listed below. The third column exhibits the transformation key for the preferred transformation scheme, the legend below gives the corresponding transformation. The preferred transformation scheme complies in general with transformation schemes reported in the literature, for example Giannone et al. (2005), but is adjusted to the sample and model specification. By including the series related to prices in terms of annual growth rates, I follow Ang and Piazzesi (2003) and Moench (2008). The last column shows groups of time series which are imposed to be transformed in the same way in the real time transformation approach.

Transformations:

- 0: No transformation
- 1: Logarithms
- 2: First differences
- 3: Monthly growth rates

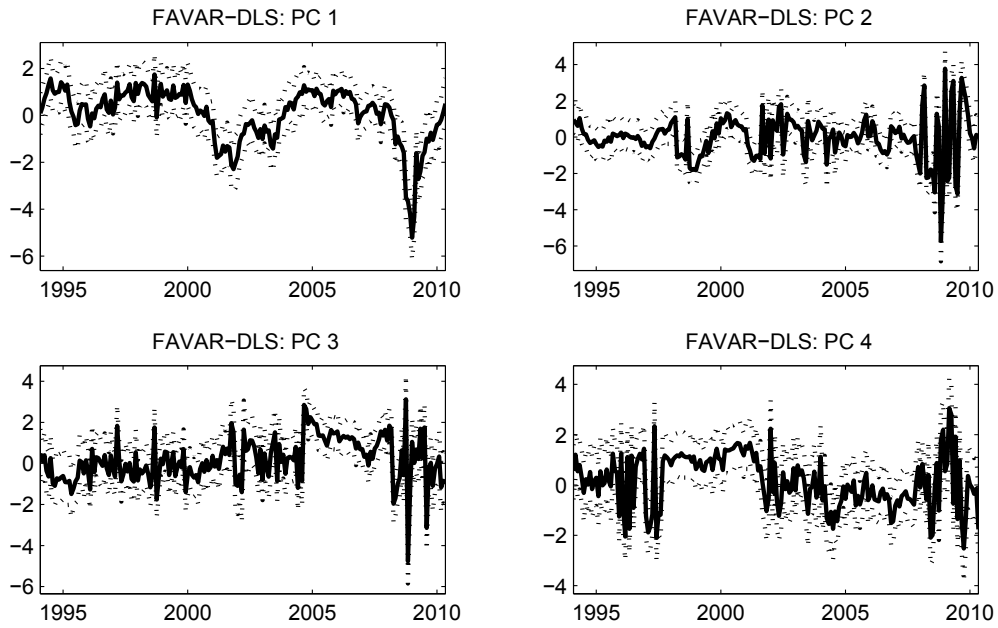


Figure 9: Macroeconomic factors with 5% confidence bands for the exponential weighting scheme.

- 4: Annual growth rates

Notes:

*: In millions of chained 1996-USD.

** : In millions of USD.

#	Series	Trf.	T.-Gr.
1	IP: Total	2	1
2	IP: Final products and non-industrial supplies	2	1
3	IP: Final products	2	1
4	IP: Consumer goods	2	1
5	IP: Durable consumer goods	2	1
6	IP: Nondurable consumer goods	2	1
7	IP: Business equipment	2	1
8	IP: Materials	2	1
9	IP: Materials, nonenergy, durables	2	1
10	IP: Materials, nonenergy, nondurables	2	1
11	IP: Mfg (NAICS)	2	1
12	IP: Mfg, durables (NAICS)	2	1
13	IP: Mfg, nondurables (NAICS)	2	1
14	IP: Mining (NAICS)	2	1
15	IP: Utilities (NAICS)	2	1
16	IP: Energy, total (NAICS)	2	1
17	IP: Non-energy, total (NAICS)	2	1
18	IP: Motor vehicles and parts (MVP) (NAICS)	2	1

#	Series	Trf.	T.-Gr.
19	IP: Non-energy excl. CCS (NAICS)	2	1
20	IP: Non-energy excl. CCS and MVP (NAICS)	2	1
21	Capacity utilization: Total (NAICS)	1	2
22	Capacity utilization: Mfg, durables (NAICS)	1	2
23	Capacity ut.: Computers, comm. equip., semiconductors	1	2
24	Purchasing Managers Index (PMI)	1	3
25	ISM Mfg index: Production	1	3
26	Real disposable personal income	2	2
27	Mean duration of unemployment	2	4
28	Persons unemployed fewer than 5 weeks	2	4
29	Persons unemployed 5 to 14 weeks	2	4
30	Persons unemployed 15 to 26 weeks	2	4
31	Persons unemployed 15+ weeks	2	4
32	Avg. weekly initial claims	2	16
33	Employment on nonag payrolls: Mining	2	5
34	Emp. on nonag payrolls: Construction	2	5
35	Emp. on nonag payrolls: Manufacturing, nondurables	2	5
36	Emp. on nonag payrolls: Service-producing	2	5
37	Emp. on nonag payrolls: Transportation and warehousing	2	5
38	Emp. on nonag payrolls: Utilities	2	5
39	Emp. on nonag payrolls: Retail trade	2	5
40	Emp. on nonag payrolls: Wholesale trade	2	5
41	Emp. on nonag payrolls: Professional and business services	2	5
42	Emp. on nonag payrolls: Education and health services	2	5
43	Emp. on nonag payrolls: Leisure and hospitality	2	5
44	Emp. on nonag payrolls: Other services	2	5
45	Emp. on nonag payrolls: Government	2	5
46	Avg. wkly. hrs. of prod. of nonsupervisory workers: Tot. priv.	2	5
47	Average weekly hours of PNW: Mfg.	2	5
48	Avg. weekly overtime hrs. of PNW: Mfg.	2	5
49	ISM Mfg index: employment	1	3
50	Sales*: Mfg & trade: Total	2	6
51	Sales*: Mfg & trade: Mfg, total	2	6
52	Sales*: Mfg & trade: Mfg, durables	2	6
53	Sales*: Mfg & trade: Mfg, nondurables	2	6
54	Sales*: Mfg & trade: Merchant wholesale	2	6
55	Sales*: Mfg & trade: Merchant wholesale, durables	2	6
56	Sales*: Mfg & trade: Merchant wholesale, nondurables	2	6
57	Sales*: Mfg & trade: Retail trade (mil of chained 96\$)	2	6
58	PCE: Total (bil of chained 96\$)	2	6
59	PCE: Durables (bil of chained 96\$)	2	6
60	PCE: Nondurables (bil of chained 96\$)	2	6
61	PCE: Services (bil of chained 96\$)	2	6
62	PCE: Durables - MVP - new autos (bil of chained 96\$)	2	6
63	Privately-owned housing, started: Total (thous)	2	7
64	New privately-owned housing authorized: Total (thous)	2	7

#	Series	Trf.	T.-Gr.
65	New 1-family houses sold: Total (thous)	2	7
66	New 1-family houses - months supply @ current rate	2	7
67	New 1-family houses for sale at end of period (thous)	2	7
68	Mobile homes - Mfg shipments (thous) (SA)	2	7
69	Construction put in place: Total (mil of current \$)	2	7
70	Construction put in place: Private (mil of current \$)	2	7
71	Inventories*: Mfg & trade, total	2	8
72	Inventories*: Mfg & trade, Mfg	2	8
73	Inventories*: Mfg & trade, Mfg, durables	2	8
74	Inventories*: Mfg & trade, Mfg, nondurables	2	8
75	Inventories*: Mfg & trade, merchant wholesales	2	8
76	Inventories*: Mfg & trade, retail trade	2	8
77	ISM Mfg index: Inventories	1	3
78	ISM Mfg index: New orders	1	3
79	ISM Mfg index: Suppliers deliveries	1	3
80	New orders**: All manufacturing industries	2	16
81	New orders**: All manufacturing industries w/unfilled orders	2	16
82	New orders**: Durable goods industries	2	16
83	New orders**: Nondurable goods industries	2	16
84	New orders**: Nondefense capital goods	2	16
85	Unfilled orders**: All manufacturing industries	2	16
86	Nominal effective exchange rate	2	9
87	Spot Euro/US (2)	2	9
88	Spot SZ/US (2)	2	9
89	Spot Japan/US	2	9
90	Spot UK/US	2	9
91	Spot CA/US	2	9
92	Commercial paper month-end outstanding: Total (mil of \$)	2	10
93	M1 (mil of \$)	2	10
94	M2 (mil of \$)	2	10
95	Monetary base (mil of \$)	2	10
96	Depository institutions reserves: Total (mil of \$)	2	10
97	Depository institutions: Nonborrowed (mil of \$)	2	10
98	Loans and securities @ all commercial banks**: Total	2	10
99	Loans and sec. @ all comm. banks**: Securities, total	2	10
100	Loans and sec. @ all comm. banks**: Securities, U.S. govt.	2	10
101	Loans and sec. @ all comm. banks**: Real estate l.	2	10
102	Loans and sec. @ all comm. banks**: Comm. and indus. l.	2	10
103	Loans and sec. @ all comm. banks**: Consumer l.	2	10
104	New car loans at auto finance comp. (NSA): loan to value ratio	2	10
105	New car loans at auto finance comp. (NSA): Amount finance	2	10
106	PPI: Finished goods (1982=100 for all PPI data)	4	11
107	PPI: Finished consumer goods	4	11
108	PPI: Intermediate materials	4	11
109	PPI: Crude materials	4	11
110	PPI: Finished goods excl. food	4	11

#	Series	Trf.	T.-Gr.
111	PPI: Crude nonfood materials less energy	4	11
112	PPI: Crude materials less energy	4	11
113	CPI: All items (urban)	4	11
114	CPI: Food and beverages	4	11
115	CPI: Housing	4	11
116	CPI: Apparel	4	11
117	CPI: Transportation	4	11
118	CPI: Commodities	4	11
119	CPI: All items less food	4	11
120	CPI: All items less shelter	4	11
121	CPI: All items less medical care	4	11
122	PCE: Chain weight price index: Total	3	12
123	PCE prices: Total excl. food and energy	3	12
124	PCE prices: Durables	3	12
125	PCE prices: Nondurables	3	12
126	PCE prices: Services	3	12
127	Avg. hourly earnings: Total nonagricultural (\$)	3	13
128	Avg. hourly earnings: Construction (\$)	3	13
129	Avg. hourly earnings: Mfg (\$)	3	13
130	Avg. hourly earnings: Transportation (\$)	3	13
131	Avg. hourly earnings: Retail trade (\$)	3	13
132	Avg. hourly earnings: Wholesale trade (\$)	3	13
133	Avg. hourly earnings: Finance, insurance, real estate (\$)	3	13
134	Avg. hourly earnings: Professional, business services (\$)	3	13
135	Avg. hourly earnings: Education and health services (\$)	3	13
136	Avg. hourly earnings: Other services	3	13
137	Total merchandise exports, total census basis (mil of \$)	2	15
138	Total merchandise imports, total census basis (mil of \$)	2	15
139	Total merchandise imports (CIF value) (mil of \$) (NSA)	2	15
140	Michigan survey: Index of consumer sentiment	1	-
141	Outlook: General activity	0	14
142	Outlook: New orders	0	14
143	Outlook: Shipments	0	14
144	Outlook: Inventories	0	14
145	Outlook: Unfilled orders	0	14
146	Outlook: Prices paid	0	14
147	Outlook: Prices received	0	14
148	Outlook: Employment	0	14
149	Outlook: Work hours	0	14
150	Federal govt. deficit or surplus (bil of \$) (NSA)	2	-
151	Chicago Fed Midwest Mfg survey: General activity	2	-

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