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Do correlated defaults matter for CDS premia? An empirical analysis

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Non-technical summary

Research Question

Credit default swaps (CDSs) provide protection against the default of single-name borrowers. Their prices are therefore highly dependent on the future default probability of the respective single name. However, if CDSs on different single names are pooled together in a portfolio, prices of securities on that portfolio, e.g. collateralized debt obligation (CDO) tranches, reveal that not only single defaults are priced in the CDS pool, but correlated defaults are as well. Therefore, we analyze the question of whether correlated default risk factors matter for the pricing of CDSs and how their relevance has changed over time.

Contribution

We derive a cash flow based top-down model that translates CDO prices into CDS quotes and calibrate it to an extensive CDX data set. The data set comprises the most liquid daily CDS and CDO tranche quotes from September 2005 until September 2012 and can be divided into a pre-crisis, a crisis and a post-crisis period. More precisely, the prices of CDSs can be understood as prices of CDOs weighted with corresponding sensitivity parameters. The sizes of these sensitivities allow for meaningful insights into the pricing of CDSs.

Results

In the pre-crisis period, correlated default factors played a minor role in the pricing of CDSs. More than 80% of the observed default risk was caused by the single default factor. During the crisis, correlated default factors accounted for about 80% of the default risk, and even after the crisis, their fraction was still above 50%. Furthermore, our analysis of the sensitivity parameters provides us with the following relationship for the pricing of CDSs: high CDS premia are primarily driven by the single default factor. For low CDS quotes, the correlated default factors are a relevant issue.

Nicht-technische Zusammenfassung

Forschungsfrage

Kreditausfallversicherungen (Credit Default Swaps - CDSs) bieten eine Absicherung gegen den Ausfall von Einzeladressen. Die Preise solcher CDSs hängen somit stark von der künftigen Ausfallwahrscheinlichkeit der jeweiligen Einzeladresse ab. Werden CDSs auf verschiedene Einzeladressen allerdings in einem Portfolio zusammengefasst, so zeigen die Preise von Wertpapieren, die auf diesem Portfolio basieren (wie z. B. die Preise von Collateralised Debt Obligations (CDO)-Tranchen), dass nicht nur der Ausfall von Einzeladressen im CDS-Pool gepreist ist, sondern auch korrelierte Ausfälle. Aus diesem Grund wird im vorliegenden Forschungspapier untersucht, ob korrelierte Ausfallrisiken bei der Preisbildung von CDSs eine Rolle spielen und wie sich ihre Bedeutung im Zeitverlauf verändert hat.

Beitrag

Mit Hilfe eines cashflowbasierten Top-Down-Modells werden die Preise für CDO-Tranchen in CDS-Kurse umgerechnet. Die Kalibrierung des Modells erfolgt anhand eines umfangreichen CDX-Datensatzes, der die Tageskurse für die liquidesten CDSs und CDO-Tranchen von September 2005 bis September 2012 umfasst und sich der Zeit nach in Abschnitte vor, während und nach der Krise unterteilen lässt. Konkret können die Preise der CDSs als CDO-Preise verstanden werden, die mit entsprechenden Sensitivitätsparametern gewichtet wurden. Die Größenordnung dieser Sensitivitäten ermöglicht einen aussagekräftigen Einblick in die Preisbildung von CDSs.

Ergebnisse

Vor der Krise spielten Faktoren für korrelierte Ausfälle bei der Bepreisung von CDSs nur eine untergeordnete Rolle: ein Anteil von über 80 % des beobachteten Ausfallrisikos war auf den Faktor für den Ausfall von Einzeladressen zurückzuführen. Während der Krise hingegen waren Faktoren für korrelierte Ausfälle für etwa 80 % des Ausfallrisikos verantwortlich. Auch nach der Krise lag ihr Anteil noch über 50 %. Des Weiteren hat unsere Analyse der Sensitivitätsparameter in Bezug auf die Bepreisung von CDSs ergeben, dass hohe CDS-Prämien in erster Linie durch den Faktor für den Ausfall von Einzeladressen bedingt sind. Bei niedrigen CDS-Preisen kommen hingegen die Faktoren für korrelierte Ausfälle zum Tragen.

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Do Correlated Defaults Matter for CDS Premia? An Empirical Analysis*

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Abstract

Correlated defaults and systemic risk are clearly priced in credit portfolio securities such as CDOs or index CDSs. In this paper we study an extensive CDX data set for evidence whether correlated defaults are also present in the underlying CDS market. We develop a cash flow based top-down approach for modeling CDSs from which we can derive the following major contributions: (I) Correlated defaults did not matter for CDS prices prior to the financial crisis in 2008. During and after the crisis, however, their importance has increased strongly. (II) In line with a plausible default order, we observe that correlated defaults primarily impact the CDS prices of firms with an overall low CDS level. (III) Idiosyncratic risk factors for each single CDS play a major (minor) role when the CDS premia are high (low).

JEL Classification: G14, G21

Keywords: Correlated Defaults, Systemic Risk, Idiosyncratic Risk, Collateralized Debt Obligations, Credit Default Swaps, Credit Derivatives

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1 Introduction

Credit default swaps (CDSs) provide protection against the default of single name borrowers. Their prices are therefore highly dependent on the future default probability of the respective single name and they should only react to changes in their associated creditworthiness. However, if CDSs on different single names are pooled together in a portfolio, prices of securities on that portfolio, e.g. collateralized debt obligation (CDO) tranches, reveal that not only single defaults are priced in the CDS pool but correlated defaults are as well (see e.g. *Longstaff and Rajan*, 2008). As a consequence, we have a paradoxical situation for the pricing of CDSs on single entities. On the one hand, a default event and therefore a payoff from the CDS only depends on the solvency of the particular firm. On the other hand, the market price for the CDS is also impacted by other characteristics outside the firm such as correlation effects. This property suggests that not only the individual default risks of a single name are relevant for pricing CDS but also systemic factors that can lead to the default of many single names simultaneously due to default correlation.

The aim of this paper is to analyze risk factors for correlated defaults that drive CDS quotes. In particular, we strive for answers to the following research questions: (1) Has the financial crisis changed the relevance of correlated default factors? (2) Which CDSs are primarily impacted by correlated default factors? (3) Which CDSs require a further idiosyncratic factor beyond the common default factors of the portfolio to be reasonably explained?

To analyze these research questions, we use CDX data for CDSs and CDO tranches retrieved from Markit for the time period from September 2005 until September 2012. In the first step, we follow the approach proposed by *Longstaff and Rajan* (2008) to calibrating default risk factors that explain CDO prices. We also find that three factors — single defaults, industry defaults and systemic defaults — represent market prices reasonably well. In the second step, we derive a cash flow based top-down approach that translates CDO prices into CDS quotes. The notion behind this model is that an observed change of a CDS translates with different sensitivities to the various CDO tranches in which this entity is included. Furthermore, our top-down approach allows for idiosyncratic risk factors that can perfectly explain empirically observed CDS premia.

These estimations provide us with the following conclusions: before the crisis, correlated default factors, i.e. industry and systemic defaults, played a minor role in the pricing of CDSs. More than 80% of the observed default risk was caused by the single default factor. During and subsequent to the crisis, correlated default factors strongly enhanced their importance. During the crisis, correlated default factors accounted for about 80% of the default risk, and even after the crisis, their fraction is still above 50%.

Furthermore, we can observe that the CDO tranche sensitivities to the various CDSs contained therein exhibit a reasonable default order. In other words, the low CDS premia are primarily relevant for the senior tranche in a CDO, while the high CDS quotes drive the equity tranche of a CDO. As a consequence of the further observation that the equity tranche can be primarily characterized by the single default factor, whereas the correlated default factors explain the more senior tranches, we can confirm the following important relationship for the pricing of CDSs: high CDS premia are primarily driven by the single default factor. For low CDS quotes, the correlated default factors are a relevant issue.

The methods applied in this paper are related to other important studies. *Giesecke, Goldberg, and Ding* (2011) introduce a top-down approach based on a default matrix implied by CDO and CDS prices. We modify their approach by adapting the default matrix to observable cash flows. That way, we achieve a low parameterization and a high analytical and empirical tractability. To put our approach to work, we have to use a model for the CDO portfolio for which a wide variety of literature exists. First of all, models that belong to the category of top models can be employed within our framework. Top models are used to directly model the portfolio loss distribution without considering the single names of the underlying portfolio. Examples can be found in *Longstaff and Rajan* (2008), *Schönbucher* (2006), *Arnsdorf and Halperin* (2009), *Brigo, Pallavicini, and Torresetti* (2006), *Ding, Giesecke, and Tomecek* (2009) and many others. Since we do not impose certain restrictions on the CDO model but only assume the ability to model CDO cash flows, we could also use bottom-up models that capture the portfolio loss distribution from the underlying single-name portfolio. *Li* (2000), *Hull and White* (2004) and *Lopatin* (2011) belong to this category. *Ascheberg, Bick, and Kraft* (2013) investigate how these models perform empirically in hedging situations. *Junge and Trolle* (2013) construct a liquidity risk measure for CDS markets in comparison to index CDSs. Our paper contributes to their discussion through the introduction of idiosyncratic risk factors that explain the spread of index-to-theoretical bases.

The remainder of the paper is organized as follows: in Section 2, we introduce our cash flow based top-down approach. Section 3 presents a step-wise calibration procedure for the top-down model that we apply in Section 4 to the CDX North America Investment Grade index in order to analyze the question whether correlated defaults are priced in CDS markets. Section 5 concludes.

2 The Model

At first glance, single-name CDSs are only subject to individual default risk. However, if they are pooled in a portfolio such as for CDOs, the prices of tranches reveal that they contain not only individual, but also correlated default risks. In order to understand which portfolio risk factors are priced in single-name CDSs we propose a *top-down* approach that splits the cash flows of CDO tranches (*top* level) to the single name CDSs of the underlying portfolio (*down* level). The cash flow allocation is based on sensitivities that specify to what extent the cash flow of a specific CDS can be attributed to a given CDO tranche. This approach is based on the characteristic that a portfolio of CDSs provides equivalent cash flows to its corresponding CDO. Moreover, to capture potential deviations, we extend our top-down approach by including an idiosyncratic risk factor for each single-name CDS. This factor accounts for individual default risk that is not priced in the CDO market and should therefore facilitate the interpretation of any deviations between CDO-induced model premia and observed CDS premia.

In the following, we present how CDSs and CDO tranches are priced in our cash flow based top-down approach in general and how the idiosyncratic risk factor is embedded in our model for every single-name CDS. Furthermore, we outline the top model of *Longstaff and Rajan* (2008) that we use to price CDOs and a pointwise-homogeneous Poisson process for the idiosyncratic risk factors of single name CDS.

2.1 CDO Valuation

Let L_τ denote the accumulated loss of a CDO portfolio for any time τ with $t \leq \tau \leq T$, where t denotes the valuation date and T the maturity of the CDO with notional 1. Then, for the possible portfolio loss outcomes during the lifetime of the CDO $0 \leq L_\tau < 1$ holds. Furthermore, let a_p and d_p denote the attachment and the detachment point of the tranche p. The accumulated loss process L_τ^p of tranche p is then expressed by

$$L_\tau^p = \frac{1}{d_p - a_p} \left(\max[0, L_\tau - a_p] - \max[0, L_\tau - d_p] \right). \tag{1}$$

The equation shows that the notional $1 - L_\tau^p$ of the tranche p is not affected by portfolio losses that occur below its attachment point, $L_\tau < a_p$. For higher losses $L_\tau \geq a_p$, the notional $1 - L_\tau^p$ of p linearly decreases for increasing L_τ until L_τ hits the detachment point of p leading to $L_\tau^p = 1$.

The payment obligations of a CDO tranche become effective at payment dates t_n for which $t < t_n \leq T$ holds. We denote the time period between t_{n-1} and t_n as Δ_{t_n} where usually Δ_{t_n} takes values that are close to the quarter of a year depending on the day count convention of the CDO.

The protection leg of a CDO tranche compensates for losses in the underlying portfolio interval $[a_p, d_p)$ that occur between two payment dates t_{n-1} and t_n. Thus, the value of the protection leg at t under the risk-neutral measure $\mathbb{Q}$ is equal to

$$PF_t^{p,\text{prot}} = \sum_{n=1}^{N} b_{t,t_n} \cdot E_t^{\mathbb{Q}} \left(L_{t_n}^p - L_{t_{n-1}}^p \right) \tag{2}$$

where b_{t,t_n} denotes the discount factor at t with time horizion t_n. In exchange for the loss compensation, a CDO investor has to pay a premium c_t^p at every date t_n on the intact capital of the portfolio interval. The value of the premium leg referring to a premium amount of 1 is then expressed by

$$PF_t^{p,\text{prem}} = \sum_{n=1}^{N} b_{t,t_n} \cdot \Delta_{t_n} \cdot E_t^{\mathbb{Q}} \left(1 - L_{t_{n-1}}^p \right). \tag{3}$$

There are two conventions governing how CDO tranche premia are quoted in the markets. The approach that was mainly used before the financial crisis of 2008 is known as the running spread convention. At the trading date t, the two counterparties of a CDO tranche trade agree that the protection buyer will pay a premium $c_t^p \Delta_{t_n}$ on the remaining intact capital of the tranche to the protection seller at each payment date t_n. The premium c_t^p may change for every other CDO tranche trade and consequently, it is subject to market risk. Thus, as the market risky premium c_t^p is paid on a recurring basis its quoting convention is known as the running spread convention. Its value is derived from the assumption that under the risk-neutral measure $\mathbb{Q}$ the value of the protection leg has to equal the value of the premium leg leading to:

$$c_t^p = \frac{PF_t^{p,\text{prot}}}{PF_t^{p,\text{prem}}}. \tag{4}$$

The other approach to quoting CDO tranches is known as the upfront payment convention. It was already used for junior tranches before the financial crisis but it has since become the market standard. Its major benefit lies in the simplification of trade processing and the higher flexibility and efficiency in trade settlements. Unlike for c_t^p, the running spread $c^{p,\text{fix}}$ is set for a standard amount and is not subject to market risk. In order to account for the value of the CDO portfolio, the protection buyer pays an upfront payment $c_t^{p,\mathrm{up}}$ to the protection seller at t that is subject to market risk. When the value of the protection leg is equal to that of the premium leg, $c_t^{p,\mathrm{up}}$ is equal to zero. Otherwise,

$$c_t^{p,\mathrm{up}} = PF_t^{p,\mathrm{prot}} - c^{p,\mathrm{fix}} PF_t^{p,\mathrm{prem}} \tag{5}$$

holds. Equation (5) implies that $c_t^{p,\mathrm{up}}$ can take negative values which might seem unrealistic at first glance. But there are certain cases in which the value of the fixed premium leg might be too high with respect to the quality of an underlying portfolio interval and therefore the protection seller compensates the buyer for overpayments that occur during the course of the CDO tranche.

Typically, the intervals $[a_p, d_p)$ are parameterized in such a way that their union yields the interval $[0, 1)$. Furthermore, they are disjoint sets, making them adjacent intervals. In the following, we will assume that $p = 1$ marks the most junior tranche of a CDO which is commonly known as the equity tranche. As long as no default happens in the CDO portfolio, the capital of the equity tranche remains unaffected. But as soon as losses occur, the capital of the equity tranche will be reduced first until it is completely exhausted. Further portfolio losses will then affect the next most junior tranche after the equity tranche, $p = 2$, also known as the junior mezzanine tranche. Increasing portfolio losses will consume the capital of $p = 2$ until it is completely exhausted, too. In this way, at least in theory, the capital of the CDO is consumed tranche after tranche until the capital of the entire portfolio is consumed.

2.2 CDS Valuation

In our top-down approach, the value of a CDS k is represented by its sensitivities q_p^k towards the CDO tranches $p = 1, \ldots, P$. In other words, the sensitivities q_p^k split the cash flows of all CDO tranches to all the single-name CDSs of the underlying portfolio. Therefore, in a perfect world, the cash flow of a CDO tranche p is completely allocated to the underlying CDS portfolio leading to equation (6). Additionally, as we only split a cash flow into positive amounts, inequation (7) has to hold:

$$\sum_{k=1}^{K} q_k^p = 1, \forall p, \tag{6}$$

$$q_k^p \geq 0, \forall p, \forall k. \tag{7}$$

(6) and (7) represent restrictions that CDS model premia have to adhere to and that are especially important during calibration.

The cash flows of a CDO tranche are allocated to a single-name CDS as follows: as can be seen from equation (1) the cash flows of a CDO tranche refer to its notional with amount 1. However, the notional of the CDO portfolio adds up to 1 as well. Consequently, CDO tranche cash flows need to be rescaled to the original portfolio notional which can be achieved by multiplying equation (2) by $(d_p - a_p)$. In the next step, the rescaled CDO tranche cash flows are split to the single-name CDSs of the portfolio by multiplying them by the tranche sensitivities q_k^p. For the protection leg of a CDS k, the value of allocated tranche cash flows is expressed by:

$$PF_t^{k,\text{prot}} = \sum_{n=1}^{N} b_{t,t_n} \sum_{p=1}^{P} q_k^p \cdot (d_p - a_p) \cdot E_t^{\mathbb{Q}} \left(L_{t_n}^p - L_{t_{n-1}}^p \right) \tag{8}$$

If both markets, the CDO as well as the CDS market, valued risks equivalently, equation (8) would be sufficient for valuing the protection leg cash flow of any CDS. However, this may not always be the case, and in order to account for CDS premia that are not in line with CDO cash flows, we extend the CDS valuation by including idiosyncratic risk factors. For this purpose, we introduce a stochastic default time η_k for each CDS k where the distribution of η_k is driven by a k-specific idiosyncratic risk factor. Then the value of the protection leg induced by the idiosyncratic risk factor is

$$I_t^{k,\text{prot}} = (1 - \varphi) \cdot \sum_{n=1}^{N} b_{t,t_n} \cdot P\left[t_{n-1} < \eta_k \leq t_n \right], \tag{9}$$

where φ denotes the recovery rate. As in the protection leg, the portfolio-related part of the premium leg of k is computed as

$$PF_t^{k,\text{prem}} = \sum_{n=1}^{N} b_{t,t_n} \cdot \Delta_{t_n} \cdot \sum_{p=1}^{P} q_k^p \cdot (d_p - a_p) \cdot E_t^{\mathbb{Q}} \left(1 - L_{t_{n-1}}^p \right) \tag{10}$$

and the part of the idiosyncratic risk factor as

$$I_t^{k,\text{prem}} = \sum_{n=1}^{N} b_{t,t_n} \cdot \Delta_{t_n} \cdot P\left[t_n < \eta_k \right]. \tag{11}$$

Finally, the model premium of a CDS k in our top-down approach is expressed by

$$f_t^k = \frac{PF_t^{k,\text{prot}} + I_t^{k,\text{prot}}}{PF_t^{k,\text{prem}} + I_t^{k,\text{prem}}}. \tag{12}$$

If $q_k^p = 0$ for all p of a given k, then equation (12) reduces to the valuation formula that is commonly used in the literature, e.g. as in *Longstaff, Mithal, and Neis* (2005). It would also mean that a CDS is not correlated with a given CDO portfolio at all.

Default Order Equations (8) to (12) show that in our approach mainly four types of input are necessary for pricing CDS premia: the discount factors b_{t,t_n}, the expected tranche loss $E_t^{\mathbb{Q}}\left(L_{t_n}^p\right)$, the sensitivities q_k^p that determine the impact of the expected tranche loss on the CDS premium, and finally the survival probabilities $P\left[t_n < \eta_k\right]$ deduced from the idiosyncratic risk factor. While the expected tranche loss is computed from CDO data, the sensitivities q_k^p and the idiosyncratic risk factor are retrieved from information priced in the CDS market. The levels of their values have a strong economic impact as they reveal which kind of portfolio risk is priced in a given CDS. For example, let us consider the sensitivity q_k^1 that is associated with the equity tranche of a CDO. If q_1^1 is noticeably higher than all other q_k^1 then the bulk of the expected losses of the equity tranche are priced in CDS $k = 1$. Another example deals with the important question of whether systemic risk is priced in only a few single names or in all names of a portfolio. Given the nature of systemic risk, one might expect that the effects of a catastrophic event, e.g. a severe economic crisis, would lead to a substantial number of defaults in the portfolio. Therefore, if CDSs and CDO markets are priced consistently it is plausible to suggest that most of the single names are exposed to systemic risk, implying equally high q_k^p for all k with respect to the senior tranche $p = 4$. In other words, the risk of junior tranches should be mapped to only a few single names that have a high probability of default. By contrast, the risk inherent in senior tranches should be priced in single names with small CDS premia because they will most likely only be affected by a catastrophic event. Consequently, the values q_k^p shed light on the implicit default order that can be deduced from CDS and CDO premia. If the default order prevails, then a high (low) CDS quote is supposed to have a high sensitivity to the equity tranche $p = 1$ (senior tranche $p = 4$). A major question for our empirical study in Section 4 will be, whether the default order can be verified empirically.

2.3 Portfolio Model

There are several possibilities for modeling the loss distribution of a portfolio. One common way is to model the default of every single name first and then to aggregate the resulting single-name loss distributions to a portfolio loss distribution. This approach is known as the bottom-up approach and is used e.g. in the base correlation model (*O'Kane* *and Livesey*, 2004). Although we are free to employ such a model in our valuation approach, it seems to be more purposeful to model the portfolio loss distribution *directly* without considering the risks inherent in single names. In this way, one does not need to account for the dependence structure between all single names which results in a much lower parameterization and simpler calibration of the model. The model of *Longstaff and Rajan* (2008) combines these desirable properties and is the portfolio model that we use throughout this paper.

First, we assume that the loss dynamic of a portfolio is driven by three independent Cox processes $i = 1, 2, 3$. The intensity dynamic of each process is given in the form of a *Cox, Ingersoll, and Ross* (1985) process without drift as in

$$d\lambda_\tau^i = \sigma_i \sqrt{\lambda_\tau^i} dY_\tau^i, \tag{13}$$

where dY_τ^i marks the independent increment of a Wiener process related to process i and σ_i its volatility. λ_τ^i denotes the jump intensity. Let $P_t[j = N_T^i]$ denote the probability that conditional on time t process i has jumped j times at time T. It can then be shown that the following equation holds for the jump probabilities of each process:

$$P_t[j = N_T^i] = \exp(-A^i(T-t) \cdot \lambda_t^i) \cdot \sum_{k=0}^{j} B_{j,k}^i(T-t) \cdot (\lambda_t^i)^k, \tag{14}$$

$$A^i(T-t) = \frac{4\sigma_i^2}{\sqrt{2}\sigma_i^3 \cdot \left[1 + \exp(-\sqrt{2}\sigma_i \cdot (T-t))\right]} - \frac{\sqrt{2}}{\sigma_i} \tag{15}$$

where $B_{0,0}^i(T-t) = 1$, $B_{j,0}^i(T-t) = 0$ for $j > 0$, $B_{j,k}^i(0) = 0$ for $j > 0$, $k > 0$. The remaining functions $B_{j,k}^i(T-t)$, $1 \leq k \leq j-1$ are computed numerically from the following system of ODEs:

$$dB_{j,j}^i(\tau) = j \cdot (B_{j-1,j-1}^i(\tau) - \sigma_i^2 \cdot A^i(\tau) \cdot B_{j,j}^i(\tau))d\tau, \tag{16}$$

$$dB_{j,k}^i(\tau) = \left(jB_{j-1,k-1}^i(\tau) - k\sigma_i^2 A^i(\tau) B_{j,k}^i(\tau) + \frac{(k+1)k\sigma_i^2}{2} B_{j,k+1}^i(\tau) \right) d\tau. \tag{17}$$

As a result of the distributions for the number of jumps N_τ^i and the jump size γ_i, we obtain the following possible outcomes for the portfolio losses:

$$L_\tau = 1 - \exp\left(-\sum_{i=1}^{3} \gamma_i N_\tau^i \right). \tag{18}$$

Obviously, $0 \leq L_\tau < 1$ holds for $N_\tau^i \in \mathbb{N}_0$. If $N_\tau^i = 0, \forall i$ then the exponential function is equal to one and the portfolio loss takes the value $L_\tau = 0$. For increasing N_τ^i the portfolio loss will increase as well until it takes values close to one.

2.4 Single-Name Model

In general, there are two model classes that are suitable for computing the default probabilities of the idiosyncratic risk factors in equations (11) and (9): structural and reduced form models. While structural models are particularly useful for an economic explanation of the sources leading to the default of a company, reduced form models allow for a higher flexibility and do not need any assumptions with regard to the liability structure. For these two reasons, we employ a pointwise-homogeneous reduced form model in the context of *Lando* (1998) in order to compute default probabilities from the idiosyncratic risk factor. Let θ_τ^k mark the default intensity of the idiosyncratic risk factor of single name k at time τ. The associated solution for the survival probabilities $P_t[T < \eta_k]$ can be derived as

$$P_t[T < \eta_k] = \exp(-(\theta_t^k + \omega_t^k) \cdot (T - t)) \tag{19}$$

where ω_τ^k marks a technical intensity that does not exhibit a specific economic meaning. The need for ω_τ^k arises because of the structure of equation (12) and the calibration pattern introduced in Section 3. The first step in the pattern calibrates the ratio $PF_t^{k,\text{prot}}/PF_t^{k,\text{prem}}$ to observed CDS premia. Afterwards, for calibrating the idiosyncratic risk factor, it first needs to be adapted to the protection and the default leg induced by the tranche cash flows and the calibrated q_k^p as in (12). That means that

$$\frac{PF_t^{k,\text{prot}} + I_t^{k,\text{prot}}}{PF_t^{k,\text{prem}} + I_t^{k,\text{prem}}} = \frac{PF_t^{k,\text{prot}}}{PF_t^{k,\text{prem}}} \tag{20}$$

for calibrated ω_τ^k and $\theta_\tau^k = 0$. To match observed CDS premia, we allow θ_τ^k to take values that are greater than $-\omega_\tau^k$. This way, the calibrated θ_τ^k reflect the true level of idiosyncratic risk.

3 Model Calibration

For an accurate calibration, we propose a stepwise calibration pattern that successively calibrates the models involved from the *top* level of the CDO portfolio to the *down* level of each single-name CDS.

3.1 Portfolio Level

Let c_t^{*p} denote the observable market quote of the CDO tranche p at time t. For a given data set, we formulate the calibration problem as follows:

$$\begin{aligned} \min_{\lambda_t,\sigma,\gamma} \quad & \sum_t \sum_p \left| \frac{c_t^p - c_t^{*p}}{c_t^{*p}} \right| \\ \text{s.t.} \quad & \lambda_t^i \geq 0, \forall i, \forall t, \\ & \sigma_i \geq 0, \forall i, \\ & \gamma_i \geq 0, \forall i. \end{aligned} \tag{21}$$

Calibration errors that are composed of absolute values of relative differences offer several advantages in CDO calibration. First, the information of all tranches is incorporated into the calibration problem to the same extent. This effect avoids an overemphasis of junior tranches with relatively high premia and ensures that the information contained in senior tranches is taken into account in a balanced way. The approach is therefore superior for calibrating correlated default factors that drive prices of senior tranches. Moreover, in comparison with a quadratic calibration error, the absolute error forces the optimization algorithm to further minimize errors that are below one, whereas quadratic errors tend to overemphasize very large deviations and to neglect very small ones. The problem is solved with a gradient-based method.

3.2 Single-Name Level

In our approach, there are two types of risks that are priced in CDS premia: CDO-induced risk and idiosyncratic risk. While the latter is only related to the single-name k and therefore does not require any information about the other single names of a portfolio, the former is split from the portfolio to all single names. This is why restrictions (6) and

(7) have to be adhered to during the calibration of q_k^p. We formulate the general calibration problem as

$$\begin{aligned}\min_{q,\theta} \quad & \sum_t \sum_{k=1}^K (f_t^k - f_t^{*k})^2 \\ \text{s.t.} \quad & \sum_{k=1}^K q_k^p = 1, \forall p, \\ & q_k^p \geq 0, \forall p, \forall k, \\ & \omega_t^k \geq 0, \forall k, \\ & \theta_t^k \geq -\omega_t^k, \forall k,\end{aligned} \tag{22}$$

where f_t^{*k} denotes the values of observed CDS market premia. We do not choose an absolute error as in (21) because we want to make use of a quadratic optimization algorithm that facilitates very fast and accurate calibration[1].

Since θ_t^k is not restricted to positive values the idiosyncratic risk factor can lead to higher *or* lower CDS model premia compared to the case in which only portfolio risks are priced. Let us assume that the model premium which is only induced by portfolio risk is lower than the observed market premium. Then $\theta_t^k > 0$ has to hold as additional default mass needs to be induced by the idiosyncratic risk factor. For the other case that the portfolio model premium is too high, $\theta_t^k < 0$ leads to a reduction of the priced default mass in the model premium. This is why one can directly infer from the value of the intensities θ_t^k whether a single name is subject to high or low idiosyncratic risk. The restriction $\theta_t^k \geq -\omega_t^k$ needs to be imposed because otherwise negative intensities $\theta_t^k + \omega_t^k$ would be possible in (19), and the related default probabilities $P_t[T < \eta_k]$ would not meet the usual requirements for a probability measure.

Although the idiosyncratic risk factors provide easy and useful explanations they complicate the calibration of CDS premia. One possibility for addressing this problem would be to calibrate all the parameters related to CDSs at once: $q_k^p, \omega_t^k, \theta_t^k$. However, it is non-trivial to solve such a high-dimensional problem. For this reason, we propose a step-wise calibration approach that we outline in the following.

We split the problem (22) into two problems that are easier to solve. First, we calibrate the sensitivities q_k^p only and afterwards the idiosyncratic risk factor where we take the calibrated q_k^p from the first step as fixed values. This approach is motivated by the assumption that all single names — and only them — are part of the CDO portfolio and that all losses that are priced in the CDO should map, overall , to the single names. Any

[1]As problem (21) would still be highly non-linear with a quadratic error, the absolute error there is the better choice for an accurate calibration.

further deviations that cannot be explained by the portfolio dynamics are then captured by the idiosyncratic risk factor.

In accordance with the approach of *Giesecke et al.* (2011), let $q \in \mathbb{R}^{(k \cdot p) \times 1}$ denote the stapled vectors q_k each of which contains the sensitivities q_k^p of a given k. With e denoting a vector of ones for which $e \in \mathbb{R}^{p \times 1}$ holds, we solve the following quadratic problem for q:

$$\begin{aligned} \min_q \quad & \frac{1}{2} q^T \cdot Q \cdot q \\ \text{s.t.} \quad & A \cdot q = e \\ & q \geq 0. \end{aligned} \tag{23}$$

Q is a diagonal matrix with matrices Q_k on its diagonal and zeros otherwise with

$$Q_k = \sum_\tau \text{diag}(z_\tau^k) \cdot e \cdot e' \cdot \text{diag}(z_\tau^k), \tag{24}$$

where the elements of the vector $z_t^k \in \mathbb{R}^{p \times 1}$ are defined as

$$z_t^{k,p} = \sum_{n=1}^{N} b_{t,t_n} \cdot (d_p - a_p) \cdot \left(\Delta_{t_n} \cdot E_t^{\mathbb{Q}} \left(1 - L_{t_n}^p\right) - \frac{E_\tau^{\mathbb{Q}} \left(L_{t_n}^p - L_{t_{n-1}}^p\right)}{f_t^{*k}} \right). \tag{25}$$

Equation (25) is obtained by subtracting (10) from (8) and rearranging. The error is squared in equation (24) for a given k and stapled in Q for all single names. With the help of a quadratic optimizer, the calibration of q turns out to be very fast, accurate and unambiguous.

In the second step, we solve problem (22) for every k independently from the rest of the portfolio but with fixed q_k^p from the previous calibration. This way, the intensities θ_t^k are obtained easily by applying a gradient-based method.

4 Empirical Analysis

After describing the formal foundations of our analysis, we can turn to our overall question of which default factors are priced in CDS markets empirically. We outline the specifics of the CDX data set, provide corresponding results of the calibration routine presented in Section (3) and finally show the test results which reveal the role of correlated defaults in CDS markets in the years 2005 - 2012 which include the subprime credit crisis.

4.1 Data Set

Our data set comprises daily CDO, index CDS and single-name CDS quotes of the CDX North America Investment Grade Index from September 2005 until September 2012 with a maturity of five years which to our knowledge, is the most extensive CDX data set used so far in the literature. The data set was completely retrieved from Markit, which is the provider of the CDX index. The CDX comprises the cross-industry single names that exhibit the highest liquidity in the credit derivatives market. Therefore, our analysis focuses on the overall credit risk perception among representative single names during that period.

There are some important characteristics of the CDX data set that we outline in the following. Before the beginning of the financial crisis in 2007, the CDX index was reconstituted every six months in March and September, a process which is commonly known as index roll. Immediately before an index roll occurs, Markit conducts a poll in which licensed CDS dealers agree on the most liquid 125 single names that will constitute the next CDX index. The first index of that kind was the CDX 1 that began trading in September 2003 and was labeled as an on-the-run index, which means that it was constituted of the most liquid single names at that time. Afterwards, the CDX 2 began trading in March 2004 as an on-the-run index and so forth. From this point, trades in the CDX 1 were still possible as long as the maturity of the underlying products was not reached. However, the CDX 1 was entitled to continue as an off-the-run index as it did not represent the most liquid single names at that time. However, no CDO data are available for the first four CDX indices and for this reason, we exclude them from our analysis.

We include the four subsequent indices CDX 5 through 8 that exhibit workable time series on daily CDO as well as CDS quotes and the same CDO tranche borders: $0 - 0.03$, $0.03 - 0.07$, $0.07 - 0.10$, $0.10 - 0.15$ and $0.15 - 0.30$. For these indices, we also include index CDS[2] data to supplement the missing tranche that would cover the last CDO interval ending with detachment point 1. The only tranche of these indices that was quoted according to the upfront payment convention (5) was the equity tranche with $c^{1,\text{fix}} = 500$ bp, $a_1 = 0$ and $d_1 = 0.03$. The quotes of all other tranches correspond to the running spread convention (4).

For the following three years, the CDX data exhibit a peculiar feature: although index rolls were conducted every half-year, CDX 9 is considered to be the most liquid reference index during the crisis. Because of this and the fact that CDX 9 has the most workable

[2]The index CDS can be considered as a tranche on the whole portfolio interval $0 - 1$.

time series in these three years, we exclude the indices CDX 10 through 13. Furthermore, we do not supplement CDX 9 data with index CDS because the time series of the $0.3 - 1$ tranche is available.

After CDX 9, three major changes occured for liquidity reasons. First, index rolls were only conducted every full year. Thus, even index numbers are not available any more because the odd numbers started to trade in the September of each year and traded for the entire following year. Second, the tranche borders of the CDO were restructured to four tranches $0 - 0.03$, $0.03 - 0.07$, $0.07 - 0.15$ and $0.15 - 1$. Third, the quotation convention was changed to the upfront convention for all tranches which — in that order — exhibit the fixed running premia 500 bp, 100 bp, 100 bp and 50 bp. The last two indices that we include in our data sample are CDX 15 and CDX 17, of which the former was on-the-run from September 2010 until September 2011 and the latter the subsequent year.

In the whole observation period, four credit events occured that led to payouts of CDO tranches and CDSs. The first two are related to Fannie Mae and Freddie Mac, which were placed into conservatorship on September 7, 2008. Washington Mutual filed for Chapter 11 bankruptcy on September 27, 2008, followed by CIT Group about one year later on November 1, 2009. Until the respective credit event, we include the CDS time series of all four entities. The absolute CDO tranche premia were reduced after each credit event, but unlike the CDSs they continued trading on the markets as the capital of the underlying portfolio intervals was not exhausted. After each credit event, the version of the corresponding index was increased by one. So before September 7, 2008, the CDX 9 data referred to its first version. Afterwards, the second version of CDX 9 began trading and so forth.

In the CDS space, major changes occured on April 8, 2009 and they are known as the CDS Big Bang. The thrust of these changes was to improve the efficiency of central clearing and trade processing in CDS markets. Of all the changes in the context of the CDS Big Bang, the only one that may be important to us is the change of the quotation convention. Before that date, CDS premia were quoted according to the running spread convention. Afterwards, CDS dealers quoted them in terms of the upfront payment convention. Another characteristic of the CDX data set is that all CDS quotes after the CDS Big Bang were still quoted in terms of the running spread convention. The conversion from upfront payments to running spreads is facilitated by a model converter that Markit offers on its website. It is common market practice to use models because otherwise — as can be seen from equations (4) and (5)[3] — conversion would not be possible.

[3]The facts also hold for CDS model premia.

The characteristics of the data set show that a uniform presentation of all indices is not possible because of changing tranche borders and quotation conventions. Therefore, we have two possible ways to describe the data set: first, we could present the descriptive statistics for each single CDX index with its fixed characteristics in terms of quotation convention and tranche borders. This would truly reflect all observed data and provide full transparency but would involve costs regarding the associated scope and readability. For this reason, we choose the second option which entails the use of model premia computed from the calibrated top model. The advantage of this approach lies in the fact that it facilitates the presentation of uniform time series for CDO tranche premia which can be compared among indices. Clearly, this implies the drawback of imposing model risk on the descriptive statistics. Because of the good model fit, which we will present later, we consider this handicap to be negligible.

All CDX indices that we include in our study exhibit different tranche borders and quotation conventions. As we seek to unify tranches across all indices we need to fix the tranche borders and conventions. This leads us to the problem that we need to reduce and convert observed data to premia that can be computed from any index. For example, from the premia of the $0.07 - 0.10$ and the $0.10 - 0.15$ tranche we can compute a model-implied premium for a non-existent $0.07 - 0.15$ tranche, but not vice versa because of missing information. Consequently, we have to fix our uniform tranche borders to the borders of the CDX15 and CDX17 indices: $0.00 - 0.03$, $0.03 - 0.07$, $0.07 - 0.15$ and $0.15 - 1.00$. The model gives us the flexibility to use the quotation convention for tranches that best fits our purposes. As outlined above, CDS premia are quoted in accordance with the running spread convention throughout our data set. Furthermore, according to the running spread convention, quotations can never take negative values, which attributes a higher expressiveness to statistics. For these two reasons, we use the running spread convention to represent tranche data that are, provided that no workable observed time series exists, computed from the top model.

4.2 Descriptive Statistics

In the course of this paper, we investigate how the market perception of default factors has changed over time. In order to draw meaningful conclusions from our data set, we divide it into three parts that are in line with *Kahle and Stulz* (2010): the pre-crisis period ranging from September 2005 until September 2007, which comprises the CDX 5 through 8 indices. The crisis period lasting from September 2007 until September 2010, which consists solely of CDX 9 data. And finally, the post-crisis period from September 2010 until September 2012, which contains the CDX 15 and 17 indices. Thus, we are capable of comparing the role of default factors in the CDS market before, during and after the financial crisis.

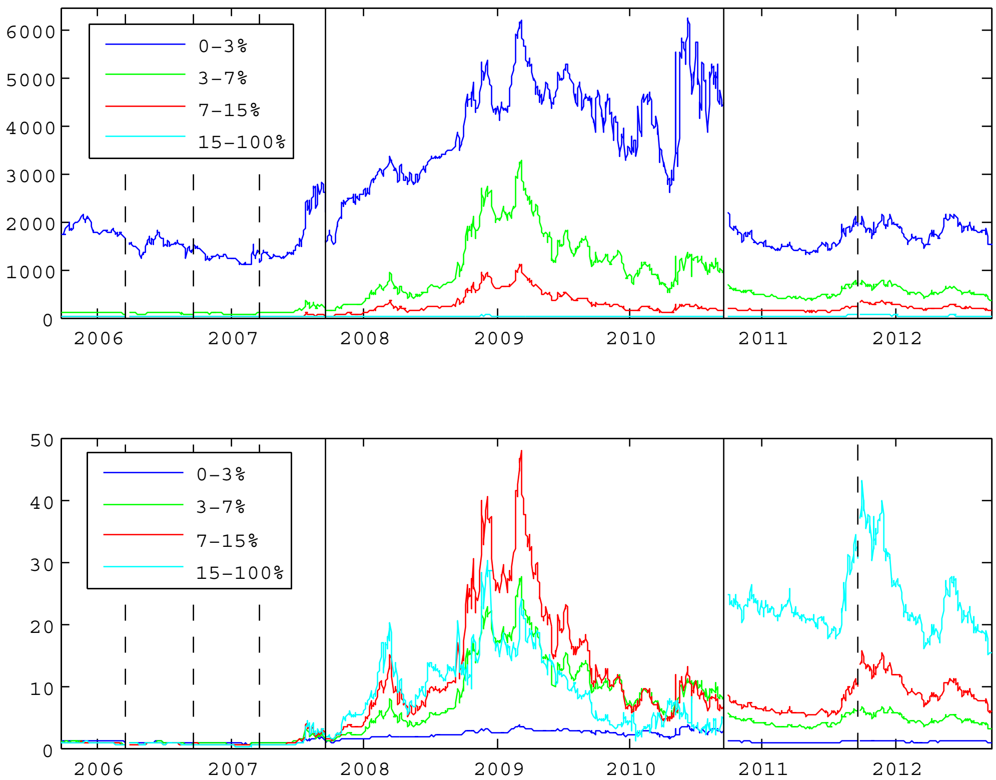
Figure 1 plots CDO premia that were retrieved from the data set and that were supplemented by model premia where necessary. The upper graph shows that until spring 2007, the CDO market was in a comparably smooth state with equity tranche premia just short of 2000 basis points. The premia of the $0.03-0.07$, $0.07-0.15$ and $0.15-1.00$ tranches were negligibly small during that period. The outbreak of the crisis in summer 2007 is reflected in increased premia for all tranches. The roll to the CDX 9 index saw a sharp drop in the equity tranche premia followed by very high market uncertainty in the overall credit derivatives market. The credit events of Fannie Mae, Freddie Mac and Washington Mutual in September 2008 caused the CDX 9 tranches to peak at the beginning of 2009. Afterwards, the situation relaxed until summer 2010 when premia widened again, although no credit event occured in the index. This rise may have been driven by the Greek sovereign debt crisis which started at that time and may have channelled through to North American credit derivatives markets. Another explanation might be the drop in CDX 9 liqudity in anticipation of the roll to the CDX 15 index which started trading at considerably lower premia. The low premium levels of CDX 15 persisted throughout CDX 17 and were — at least for the equity tranche — at levels similar to those immediately before the crisis. However, for the more senior tranches the premium levels after the crisis are considerably higher than before the crisis. As can be seen from the lower graph of Figure 1, the more senior tranches in particular gained in relative premium levels during the crisis. This observation indicates that correlated defaults may have played an increasingly important role during the crisis because they mainly affect the premium levels of tranches with high seniority.

Table 1 presents the descriptive statistics for the four tranche premia throughout the complete data set[4]. The correlation between the time series decreases with the seniority of the tranche. For example, the equity tranche is highly correlated with the junior mezzanine tranche but exhibits almost no correlation with the senior tranche with borders $0.15 - 1.00$. This property suggests that different risk factors are driving the premia of different tranches and therefore justify the use of the three-factor model. Furthermore, premium levels decrease with the seniority of a tranche because the capital of junior tranches is consumed first when defaults occur. Since all time series exhibit a very high serial correlation, the descriptive statistics for their first differences are also reported in Table 1. The values show that the main findings of the original time series hold for

[4]Descriptive statistics for the pre-crisis, crisis and post-crisis periods are reported in the appendix.

the differentiated time series as well and that the former are not due to the high serial correlation.

Figure 1: Graphs for the Time Series of the CDX North America Investment Grade CDO Tranche Model Premia



Note: The upper graph shows the time series of unified CDO tranche premia computed from the portfolio model for the period from September 2005 until September 2012. The dashed vertical division lines indicate the roll of one CDX index to the next. The straight lines indicate the rolls for the CDX 9 index and simultaneously mark the borders of pre-crisis, crisis and post-crisis periods in our data sample. The lower panel shows the same premia but divided by the first values of their respective time series. Values of the upper panel are reported in basis points. Values of the lower panel are normalized to the first observation of the respective time series.

Table 1: Summary Statistics for the Levels and First Differences of the CDX North America Investment Grade CDO Tranche Premia

	Correlations											
	3–7	7–15	15–100	Mean	SD	Min.	Med.	Max.	Skew.	Kurt.	Serial corr.	N
0–3 Tranche	0.862	0.736	-0.006	2668.40	1361.06	1059.23	1960.04	6271.22	0.80	2.27	0.999	1706
3–7 Tranche		0.960	0.330	686.59	638.48	57.69	515.66	3302.73	1.51	5.05	0.999	1706
7–15 Tranche			0.497	199.15	198.74	7.65	162.75	1126.05	1.81	6.70	0.998	1706
15–100 Tranche				17.39	15.67	0.34	13.17	67.81	0.63	2.38	0.998	1706
Δ 0–3 Tranche	0.439	0.466	-0.035	-0.12	138.42	-2322.54	1.18	2102.15	-1.31	116.80	0.063	1705
Δ 3–7 Tranche		0.874	0.219	0.14	48.01	-453.48	-0.01	387.07	-0.07	21.51	0.074	1705
Δ 7–15 Tranche			0.567	0.07	15.80	-115.09	-0.02	144.42	0.62	20.11	0.136	1705
Δ 15–100 Tranche				0.01	1.53	-8.50	0.00	33.16	6.41	139.23	-0.020	1705

Note: This table reports summary statistics for unified CDO tranche premia computed from the top model. The model was calibrated to the original, observed data and the statistics are computed from the whole time series ranging from September 2005 until September 2012. Values are reported in basis points.

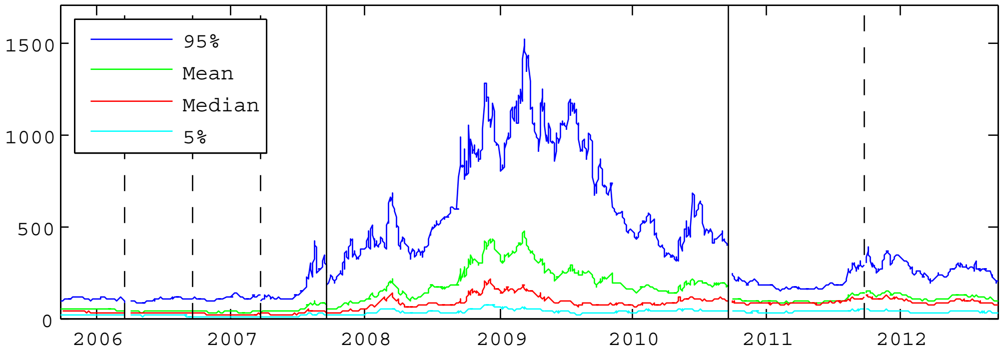
Each CDX index comprises the 125 most liquid single names during its on-the-run time period. Figure 2 plots the 5% and 95% quantiles, the mean and the median of the cross-section of CDS premia. Until the outbreak of the crisis, the overall CDS premium level was comparably low, with the 95% quantile moving below 150 basis points and the mean below 60 basis points. The outbreak of the financial crisis in summer 2007 led to a considerable increase in overall CDS premium levels which peaked at the beginning of 2009. The level of the 95% quantile reached more than 1500 basis points, indicating that during that time, the market priced high default probabilities for the single names with the lowest credit quality among the index constituents. The median CDS premium reached a maximum of more than 200 basis points. This level does not reflect a very high default probability but signals that in the overall market perception, protection sellers took high premia for single names with a good credit quality, which may suggest that correlated defaults were priced in CDS markets.

Table 2 shows the corresponding summary statistics[5]. In line with Table 1, the observed CDS time series exhibit high serial correlations which are considerably reduced by taking first differences. In addition, the time series of all considered cross-sectional statistics are highly correlated. The finding suggests that CDSs follow overall market movements, which again indicates that correlated defaults are likely to be priced in the CDS market.

Interest rate data are the last missing piece that we need for the calibration of the top-down approach. We compute discount factors from the Svensson parameters published on the website of the Federal Reserve. The parameters reflect the interest rate term structure of US Treasuries that are considered to be the most liquid interest rate products in the US market. Therefore, they are ideally suited to our purposes. An explanation of the related methodology can be found in *Gürkaynak, Sack, and Wright* (2006).

[5] Descriptive statistics for the pre-crisis, crisis and post-crisis periods are reported in the appendix.

Figure 2: CDS Time Series of the CDX North America Investment Grade Index Constituents



Note: The graph shows daily cross-sectional statistics for the CDX index constituents. The dashed vertical division lines indicate the roll of one CDX index to the next. The straight lines indicate the rolls for the CDX 9 index and simultaneously mark the borders of pre-crisis, crisis and post-crisis periods in our data sample. Values are reported in basis points.

Table 2: Summary Statistics for the Levels and First Differences of the Cross-Section of CDX Index Constituents

	Correlations														
	SD	Min.	5	Med.	95	Max.	Mean	SD	Min.	Med.	Max.	Skew.	Kurt.	Serial corr.	N
Mean	0.934	0.648	0.788	0.859	0.955	0.908	131.69	91.09	30.46	109.35	477.61	1.27	4.35	0.999	1706
SD		0.446	0.584	0.653	0.941	0.986	200.64	210.67	23.21	86.56	982.81	1.30	3.63	0.997	1706
Min.			0.822	0.801	0.548	0.414	18.23	9.31	0.00	19.10	45.35	0.20	2.26	0.994	1706
5				0.961	0.633	0.586	30.68	13.99	6.81	33.04	77.58	0.10	2.58	0.999	1706
Med.					0.710	0.642	73.82	37.74	16.85	78.66	214.73	0.47	3.12	0.999	1706
95						0.895	385.00	313.25	76.06	273.67	1515.47	1.33	3.88	0.999	1706
Max.							1633.29	1833.96	125.42	585.74	8861.85	1.34	3.90	0.993	1706
Δ Mean	0.791	0.100	0.557	0.691	0.715	0.648	0.03	5.87	-64.15	-0.02	49.93	-0.93	28.08	0.232	1705
Δ SD		-0.001	0.174	0.268	0.412	0.934	0.01	23.65	-404.46	0.03	289.86	-2.94	91.24	0.012	1705
Δ Min.			0.292	0.177	0.058	-0.014	0.00	2.27	-38.52	0.00	38.48	-0.26	169.08	-0.138	1705
Δ 5				0.706	0.420	0.097	0.01	1.02	-6.06	-0.01	9.13	0.75	14.39	0.229	1705
Δ Med.					0.555	0.173	0.02	2.82	-20.92	-0.03	25.91	0.48	16.17	0.242	1705
Δ 95						0.302	0.06	22.58	-227.62	-0.01	176.99	-0.35	21.56	0.213	1705
Δ Max.							-0.01	296.20	-5800.93	0.09	3615.09	-3.72	122.65	-0.070	1705

Note: This table reports summary statistics for the cross-sectional statistics time series of CDX index constituents. Values are reported in basis points.

4.3 Calibration Results

In the following we present the results of the calibration procedure introduced in Section 3. First, as a preparation for the CDS study, we discuss the calibrated parameters of the top model and the associated goodness of fit. Afterwards, we turn towards the question of which default factors are priced in CDS markets and investigate the calibrated CDS parameters.

4.3.1 Portfolio Level

Jump Size and Volatility Parameters We applied the proposed calibration procedure to each CDX index. Table 3 presents the resulting parameter values. The jump size parameters $\gamma_i, i = 1, 2, 3$ are comparably stable across the different CDX indices. The value of the first parameter γ_1 ranges from 0.0042 to 0.0083. Given the fact that each constituent is weighted with $1/125 = 0.008$ in a CDX index, the jump size allows the interpretation of the first factor to model the default of a single name while the recovery rates vary from 47.5% to 0%[6]. For this reason, we refer to the first factor as the single default factor of the portfolio. The jump size parameter of the second factor lies between 0.0592 and 0.0792 which are clearly above the jump sizes reported for the single default factor. Thus, a jump of the second factor induces a default that corresponds with the simultaneous default of 9.11 single names given a recovery rate of 0%, or 18 firms with a recovery rate of 49.38%. Since a CDX index is composed of single names from 12 different industries, the average number of single names per industry is equal to 10.42. Consequently, the second factor can be interpreted as an industry default factor. This interpretation implies that correlated defaults are priced in CDOs because more than 10 firms default at the same time in case the second factor jumps. In contrast, if there was no default correlation priced at all, more than one single name could not default at the same time in the model. The jump sizes of the third factor range from 0.3459 to 0.4044. A default event of the third factor may thus wipe out more than 40% of the portfolio capital. This means that more than 50 single names with recovery rate 0% would default at the same time if the third factor jumps. This seems to be a very unlikely event because it is tantamount to an extremely severe economic crisis with consequences beyond the scope of the last financial crisis in 2008. We interpret this factor as modeling a systemic default event and therefore call it the systemic default factor. The standard errors of the jump size parameters show that the calibration worked particularly well in all cases. Furthemore, they are in line with *Longstaff and Rajan* (2008), who report similar values.

[6] Or even less than 0%. In that rare case, two single names default if the first factor jumps.

The stability observed for jump size parameters across CDX indices does not apply to the volatility parameters $\sigma_i, i = 1, 2, 3$. In the pre-crisis and crisis periods, the σ_i are relatively stable with the exception of σ_1, which drops for the CDX 8 index. However, the picture changes for the post-crisis period, where the σ_i can take values that are more than three times larger than before. The finding suggests that despite decreased premium levels, and hence lower default risk, the market environment exhibited a high degree of uncertainty. In this context, it seems plausible that the market perception of default volatility has changed because of the financial crisis.

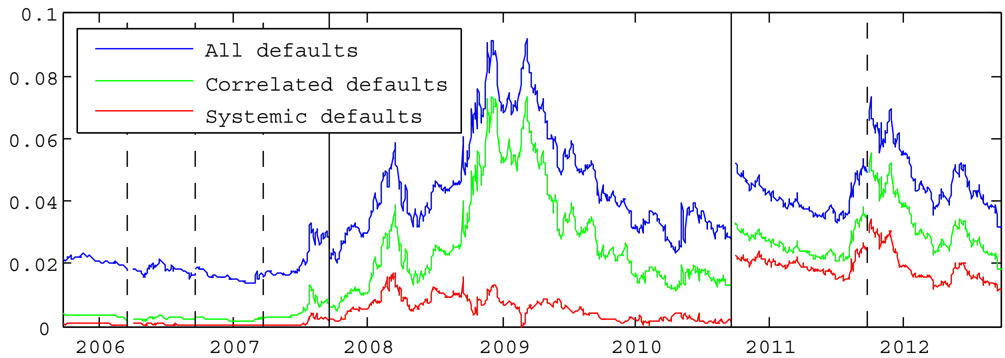
Priced Defaults by Portfolio Risk Factors Since we are interested in the question of how the importance of the factors has changed relative to each other on the one side, and how this translates to the portofolio loss distribution on the other side, we have plotted the priced defaults resulting from the top model in Figure 3. Before the crisis, more than 80% of priced defaults can be attributed to the single default factor. The industry and the systemic factor played a negligible role during that period. Furthermore, only around 2% of the portfolio capital were expected to default under the risk-neutral measure. Afterwards, the financial crisis changed the situation dramatically. At the beginning of 2009, the market expected almost 10% of the portfolio capital to default. Roughly 80% of these defaults were priced by the industry and the systemic risk factor, with the former clearly the more dominant of the two. The market situation eased in 2010 and the expected default mass declined to 4%, half of which can be attributed to the single default factor. The index roll from CDX 9 to 15 saw a sharp increase in the time series of systemic defaults. This increase might be explained by the new index composition or the new tranche borders, which may have stimulated a greater awareness of systemic risk in the senior tranche. After the crisis, the systemic risk factor accounts for more than 40% of priced defaults while the proportion between the first and the second factor remains relatively stable. Summing up, the single default factor played the major role during the pre-crisis period whilst the correlated default factors dominated afterwards. Thus, the market perception of correlated defaults and particularly systemic risk has changed during and after the financial crisis.

Table 3: Top Model Parameter Estimates for the CDX North America Investment Grade Indices

	Volatility parameters			Jump size parameters		
	First	Second	Third	First	Second	Third
CDX5	0.1872	0.2115	0.1573	0.0045	0.0592	0.3459
	(0.0006)	(0.0001)	(0.0007)	(0.0000)	(0.0000)	(0.0006)
CDX6	0.2901	0.1928	0.1573	0.0048	0.0619	0.3459
	(0.0032)	(0.0004)	(0.0007)	(0.0000)	(0.0001)	(0.0002)
CDX7	0.3522	0.1960	0.1573	0.0048	0.0611	0.3459
	(0.0008)	(0.0002)	(0.0009)	(0.0000)	(0.0000)	(0.0004)
CDX8	0.0006	0.2003	0.1573	0.0043	0.0602	0.3459
	(0.5597)	(0.0001)	(0.0005)	(0.0000)	(0.0000)	(0.0003)
CDX9	0.1343	0.2003	0.1573	0.0042	0.0602	0.3459
	(0.0371)	(0.0085)	(0.0115)	(0.0001)	(0.0005)	(0.0150)
CDX15	0.7130	0.0000	0.4044	0.0079	0.0729	0.4044
	(0.0022)	(0.8375)	(0.0013)	(0.0000)	(0.0001)	(0.0002)
CDX17	0.4740	0.4550	0.3834	0.0083	0.0656	0.3834
	(0.0101)	(0.0054)	(0.0037)	(0.0001)	(0.0003)	(0.0013)

Note: This table reports parameter estimates for the top model. The volatility parameters are $\sigma_i, i = 1, 2, 3$ and the jump size parameters are $\gamma_i, i = 1, 2, 3$. Standard errors are in parantheses and are computed according to *Gallant* (1975).

Figure 3: Time Series of Priced Defaults in CDX CDO Tranches



Note: The graph shows the priced defaults in CDX CDO tranches computed from the top model. All defaults refer to the factors $i = 1, 2, 3$, and correlated defaults refer to the factors $i = 2, 3$ combined. Systemic defaults refer to factor $i = 3$ only. The maximum possible default loss is equal to the portfolio notional with amount 1.

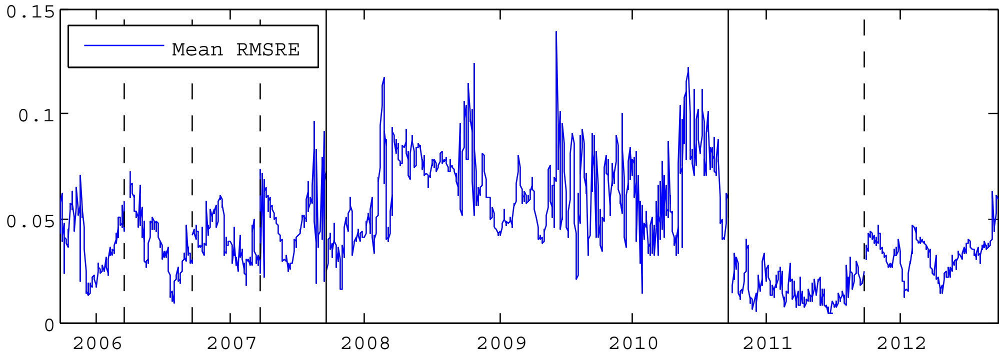
Goodness of Fit To assess the overall model fit, we examine the fit of model premia with regard to observed premia of the data set. A typical measure that is applied in this context is the so-called root mean square error (RMSE). The value of the RMSE reports the absolute deviation between observed and model premia. In our study the RMSE would measure the difference between observed and model premia in basis points. Although this would provide a high degree of comparability across tranches, one major drawback lies in the explanatory power, since a deviation of e.g. 10 basis points for the equity tranche is relatively lower than a deviation of 10 basis points for the senior tranche. In order to satisfy this circumstance for CDO tranches, we introduce the root mean square relative error (RMSRE) that is defined as follows:

$$RMSRE^{p}_{t,T} = \sqrt{\sum_{\tau=t}^{T} \left(\frac{c^{p}_{\tau} - c^{*p}_{\tau}}{c^{*p}_{\tau}} \right)^{2}} \tag{26}$$

The advantage of the RMSRE lies in its comparability across tranches: if the model premia of an equity tranche and a senior tranche are both calibrated with a deviation of 10% then they can be considered to fit the data equally well. Another favorable property for our study is the robustness of the RMSRE towards a change in quotation conventions. As mentioned earlier, the quotation convention for some on-the-run tranches in the data set suddenly changes from running to upfront, which would directly lead to a sharp increase or drop in RMSEs. Since this does not hold for RMSREs they are clearly better suited for our purposes. Furthermore, the RMSRE is closely related to the error in problem (21) and thus coincides with the top model calibration. However, there is a slight drawback associated with the RMSRE: if the observed premium c^{*p}_{t} is close to zero it takes huge values and distorts a further analysis. Therefore we have to erase outliers.

Figure 4 plots the mean RMSRE across tranches. During the pre- and post-crisis periods, the mean RMSRE ranges from roughly 1% to 10% with high fluctuations. For a senior tranche with a premium of 50 basis points this translates to a maximum deviation of 5 basis points which can be considered a good model fit. However, during the crisis the RMSRE takes higher values and has partially higher fluctuations than in the other periods. This might be considered a weakness of the model but given the fact that, during the crisis, observed premia are usually subject to greater distortion owing to market uncertainty, the model fit in the crisis is still good.

Figure 4: Time Series of Mean RMSRE across CDO Tranches



Note: The graph shows the time series of the mean RMSRE across CDO tranches adjusted for outliers. The RMSRE refers to observed CDO tranche premia.

4.3.2 Single-Name Level

As an outcome of top model calibration, we know that the CDO market prices correlated defaults, especially during and after the crisis. But are these risks also perceived in CDS markets? With the help of our top-down model, we are able to answer this question empirically by analyzing the calibrated values of the q_k^p and θ_t^k.

Test Setup for Portfolio Sensitivities In the first step, we hypothesize how correlated defaults are reflected in CDS prices within our model. For this reason, let us assume the following two cases: first, a single name with a very high CDS premium in the cross-section of the portfolio. Second, a single name with a low CDS premium. For the first single name, a high value of the associated CDS suggests a high default probability and favors an early default time. For the single name with the low CDS premium, the opposite relation holds: since a default is unlikely it is expected that other single names will default beforehand, if at all. Therefore, the low CDSs should be priced in the senior tranche $p = 4$ whose capital is only affected by defaults that occur last of in a portfolio. One could also argue that single names with a very low CDS premium may only default in catastrophic scenarios such as natural disasters or very severe economic crises. In this case, they should also exhibit a high sensitivity towards the senior tranche. Accordingly, single names with high CDS premia should be priced in the equity tranche since they are expected to default as one of the first portfolio constituents.

To test whether these considerations hold empirically, we formulate the regression equation

$$q^p = a_p + b_p \cdot f^* + \varepsilon^p, \tag{27}$$

$$q^p = \begin{pmatrix} q_1^p \\ q_2^p \\ \vdots \\ q_K^p \end{pmatrix}, f^* = \begin{pmatrix} E(f_t^{*1}) \\ E(f_t^{*2}) \\ \vdots \\ E(f_t^{*K}) \end{pmatrix},$$

where K denotes the total number of single names in the portfolio and ε^p a zero mean error term. The notion behind this regression is to determine the relationship between the size of CDS f^* and the sensitivities q^p toward a tranche p. If b_p is positive, we can conclude that high CDSs are priced in p. If it is negative, this holds for low CDSs.

We compute the q^p and f^* for each single CDX index and regress them according to equation (27) per period. That means that the first regression refers to the parameters q_k^p and f^* of CDX 5 to 8 combined, the second regression to CDX 9 and the third regression to CDX 15 und 17 combined. The packages assure that there is an equal amount of data involved in each regression and thus they have comparable explanatory power.

From our notion that only single names with high CDS premia are priced in the equity tranche, we retrieve the following hypothesis:

Hypothesis 1 *The tranche sensitivity q_k^1 is higher for high CDS premia than for low CDS premia.*

Empirically, we can consider hypothesis 1 to hold if the regression parameter b^1 is positive and significant.

Accordingly, since single names with high CDS premia are supposed to default first, they should have no influence on the pricing of the senior tranche, and the respective tranche sensitivity should equal zero. Therefore we retrieve:

Hypothesis 2 *The tranche sensitivity q_k^4 is higher for low CDS premia than for high CDS premia.*

The validity of hypothesis 2 can be verified by a negative parameter b^4.

Test Setup for Idiosyncratic Risk Factors To capture potential pricing deviations between observed and CDO-induced CDS premia, we introduced idiosyncratic risk factors.

Since we are interested in finding out which CDSs they particularly apply to, we conduct the following regression with respect to the calibrated idiosyncratic intensities θ_t^k:

$$\theta = a + b \cdot f^* + \varepsilon \tag{28}$$

$$\theta = \begin{pmatrix} E(\theta_t^{*1}) \\ E(\theta_t^{*2}) \\ \vdots \\ E(\theta_t^{*K}) \end{pmatrix}. \tag{29}$$

where ε indicates a zero mean error term. If idiosyncratic risk factors are especially present in high CDS premia, b should be positive. In the opposite case that they are present in low CDS premia, b should be negative. If the CDS level is in no way related to the amount of idiosyncratic risk, b should be close to zero.

Test Results for Pre-Crisis Period The regression results for the three periods are presented in Table 4. It can be seen that the only tranche sensitivities for which the regression coefficient b_p is significant during the pre-crisis periods, are the equity and the junior mezzanine tranche with $p = 1$ and $p = 2$. The coefficient for the junior mezzanine tranche is higher than for the equity tranche, which is fairly surprising because from hypotheses 1 and 2 the coefficient should decrease with the seniority of the CDO tranche. The scatter plots of Figure 5 reveal that this is due to outliers with a comparably low CDS premium that lead to a high b_2. The plots also show that the sensitivities q^2 are higher than q^1 for the highest CDS premia of the portfolio. The finding suggests that hypothesis 1 holds and that the highest CDS premia are priced in the most junior tranches. Regarding the senior mezzanine $p = 3$ and the senior tranche $p = 4$, there is no evidence that the single names follow a particular default order since the coefficients b_p are not significant. This is not surprising because correlated defaults were less important in the pricing of CDOs during the pre-crisis period and hence they were not perceived by the single-name CDS market.

Test Results for Crisis Period Later, though the CDO market apparently attached greater importance to correlated defaults owing to the outbreak of the financial crisis. Table 4 shows that both hypotheses 1 and 2 hold in the CDS market because the values of the coefficients b_1 and b_4 are both significant and they exhibit the expected sign. This translates to the single default factor being priced in single names with high CDS premia and systemic risks being priced in low CDSs. Thus, the CDS market follows the suggested default order, especially if both single and correlated default risks play an important and visible role in the CDO market. Only the coefficients b_2 and b_3 are not in line with the results for the equity and the senior tranche. $b_2 > b_1$ holds because of an outlier and b_3 is not significant. A possible explanation lies in the high volatility of the industry factor which accounts for most of the risk inherent in the junior and senior mezzanine tranches. Since the time series of those tranches and the CDSs that should — from a theoretical viewpoint — be in line with them are very volatile, the calibrated tranche sensitivities do not take the suggested values. Therefore, we cannot conclude that the default order holds for the mezzanine tranches.

Test Results for Post-Crisis Period Almost the same picture applies to the post-crisis period with the difference that all b_p are strongly significant. Hypotheses 1 and 2 hold but again b_2 and b_3 take the highest values. Our findings show that correlated defaults were not priced in CDS markets during the pre-crisis period; only single defaults were. The higher relevance of industry and systemic risk for CDO portfolios during the crisis was anticipated by participants in the CDS market who considered possible correlated defaults in their CDS trades.

Test Results for Idiosyncratic Risk Factors The relevance of the idiosyncratic price effect for single-name CDSs is revealed in Table 4 and Figure 5. Idiosyncratic risk factors prevailed during all three time periods of our analysis but were especially dominant during the crisis period. Furthermore, idiosyncratic risk increases with the level of CDS premia. Single names which are under high financial distress are very volatile on top of the single default factor that prices them. Similarly, not all dynamics observed on the CDS market automatically influence the time series of the equity tranche. A possible explanation could include illiquidity or diversification effects that act as a buffer between high CDSs and the equity tranche premium. However, these effects have a much weaker influence on low CDSs and the senior tranche premium. Thus, low CDS premia mainly follow overall market movements which can be characterized in terms of the systemic risk factor.

Goodness of Fit The presence of idiosyncratic risk factors implies that CDS premia cannot be fully explained by the top model and the tranche sensitivities. A perfect match is only possible with the idiosyncratic extension of CDS model premia that we introduced in equations (9) and (11). Nevertheless, it is still interesting to assess the model fit with tranche sensitivities, if only because it reveals how well they can explain observed CDS premia. We measure the deviations between observed and model premia with the

RMSRE. Figure 6 plots the RMSRE time series for the cross-section of CDS premia. It shows that more than one half of CDS premia are mispriced by less than 20%, which is tolerable for various applications. However, the top 5% of deviations take very large values that are even above 100%. In these cases, the idiosyncratic risk factors are required. The graph also shows that the mispricings are comparably higher in the crisis period than outside of it because the top-down model has its difficulties capturing the overall high market volatility at that time. But the high deviations could also be attributed to high levels of idiosyncratic risk caused by increased illiquidity in the markets. Despite some outliers, the overall model fit can be regarded as good enough to conclude that the tranche sensitivities have high explanatory power to back the findings of our analysis.

Table 4: Regression Results for Single-Name CDSs during the Pre-Crisis, Crisis and Post-Crisis Periods

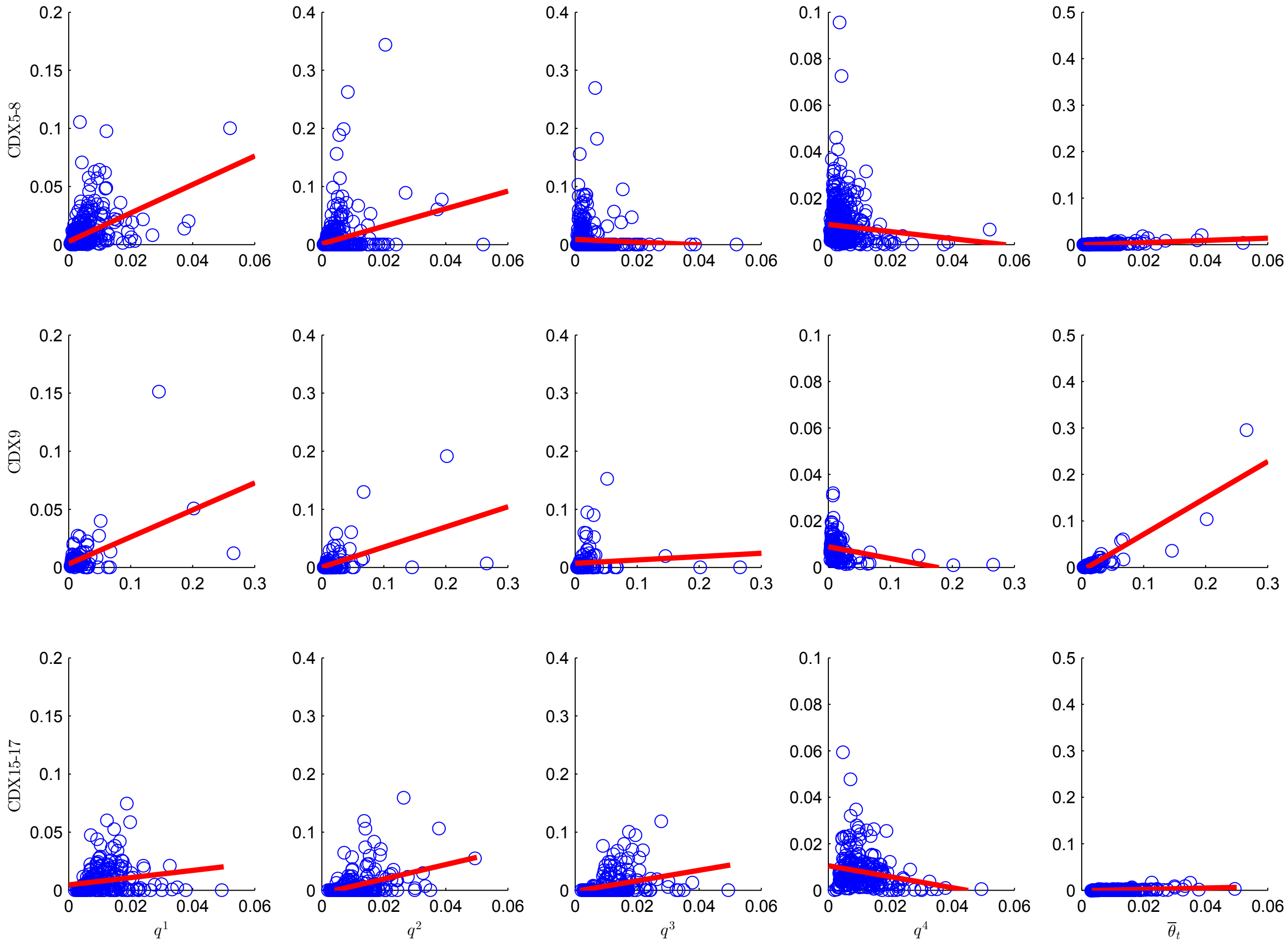
		Tranche sensitivities				
		q^1	q^2	q^3	q^4	θ_t
Pre-Crisis	a_p, a	0.0027	0.0015	0.0090	0.0087	-0.0005
	p-Value(a_p, a)	0.0001	0.3727	0.0000	0.0000	0.0000
	b_p, b	1.2261	1.5074	-0.2282	-0.1524	0.2369
	p-Value(b_p, b)	0.0000	0.0000	0.2883	0.0636	0.0000
	R^2	0.2107	0.0652	0.0023	0.0069	0.4552
Crisis	a_p, a	0.0030	0.0004	0.0071	0.0089	-0.0070
	p-Value(a_p, a)	0.0320	0.8553	0.0017	0.0000	0.0000
	b_p, b	0.2323	0.3469	0.0577	-0.0502	0.7823
	p-Value(b_p, b)	0.0000	0.0000	0.3176	0.0007	0.0000
	R^2	0.2589	0.2522	0.0084	0.0933	0.7922
Post-Crisis	a_p, a	0.0047	-0.0054	-0.0017	0.0105	-0.0008
	p-Value(a_p, a)	0.0006	0.0165	0.4260	0.0000	0.0000
	b_p, b	0.3107	1.2431	0.9000	-0.2351	0.1415
	p-Value(b_p, b)	0.0036	0.0000	0.0000	0.0009	0.0000
	R^2	0.0336	0.1683	0.1050	0.0433	0.2862

Note: This table reports results for the regressions of the single-name parameters q_k^p and θ_t^k against the respective absolute CDS mean levels. Regression data comprise pre-crisis, crisis and post-crisis period CDX data.

5 Conclusion

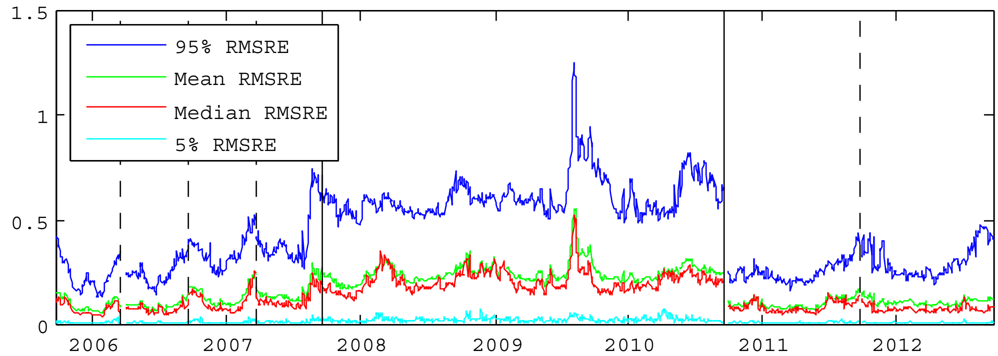
In this paper, we studied the impact of correlated default factors on CDS premia. Therefore, we first recapitulated the top model of *Longstaff and Rajan* (2008) to model CDO

Figure 5: Regression Results for Single-Name CDSs during the Pre-Crisis, Crisis and Post-Crisis Period



Note: The graphs show scatter plots for the regressions of the single-name parameters q_k^p and θ_t^k against the respective absolute CDS mean levels. Regression data comprise pre-crisis, crisis and post-crisis period CDX data.

Figure 6: Time Series of Mean RMSRE across CDS Premia



Note: The graph shows the time series of the mean RMSRE without idiosyncratc risk across CDS premia. The RMSRE refers to observed CDS premia.

tranche premia. Afterwards, we derived a cash flow based top-down approach that links theoretical CDS model premia to any kind of CDO model. A special feature of our framework lies in the ability to allow for idiosyncratic risk factors that are not priced in the CDO portfolio. With the help of these risk factors, empirically observed CDS premia can be perfectly matched by our model.

The top-down model was calibrated to an extensive CDX data set that covers daily tranche and CDS quotes from September 2005 until September 2012. We found that before the outbreak of the financial crisis in 2007, the influence of correlated default factors on CDS premia was only minor. However, the financial crisis led to a dramatic increase in those factors which accounted for most of the expected defaults in CDO and CDS markets at that time. After the crisis, the market situation eased, leading to an overall lower default risk level but still with high importance attached to the correlated default factors. Accordingly, we found correlated default factors to be priced in CDS markets when they were particularly high, that is, during and after the financial crisis. Our analysis revealed that the prices of single names with a low pricing level are mainly driven by correlated default factors. Furthermore, we found idiosyncratic risk to be priced in CDS markets in the whole data set but especially during the financial crisis. Single names with high CDS premia were in particular subject to high levels of idiosyncratic risk.

Appendix A Tables

Table 5: CDO Pre-Crisis

	Correlations											
	3–7	7–15	15–100	Mean	SD	Min.	Med.	Max.	Skew.	Kurt.	Serial corr.	N
0–3 Tranche	0.818	0.842	0.853	1593.06	376.29	1059.23	1503.54	2827.16	1.28	4.49	0.996	480
3–7 Tranche		0.986	0.939	110.58	50.81	57.69	97.89	365.22	2.59	9.92	0.994	480
7–15 Tranche			0.971	19.55	14.32	7.65	15.47	93.59	2.87	11.28	0.990	480
15–100 Tranche				1.25	1.04	0.34	0.97	6.63	3.02	12.04	0.985	480
Δ 0–3 Tranche	0.719	0.737	0.596	0.99	52.78	-295.43	-1.64	307.52	0.28	11.61	0.177	479
Δ 3–7 Tranche		0.912	0.739	0.07	9.81	-61.94	-0.35	87.17	0.76	25.27	0.162	479
Δ 7–15 Tranche			0.881	0.03	2.78	-16.08	-0.02	26.68	1.36	32.15	0.144	479
Δ 15–100 Tranche				0.00	0.23	-1.46	0.00	1.51	0.37	25.40	0.103	479

Table 6: CDO Crisis

	Correlations											
	3–7	7–15	15–100	Mean	SD	Min.	Med.	Max.	Skew.	Kurt.	Serial corr.	N
0–3 Tranche	0.819	0.690	0.328	3985.24	1030.54	1515.93	4120.93	6271.22	-0.15	2.31	0.998	742
3–7 Tranche		0.957	0.654	1158.30	690.56	116.71	1070.02	3302.73	0.78	3.02	0.998	742
7–15 Tranche			0.814	317.72	231.23	25.80	234.69	1126.05	1.27	3.91	0.998	742
15–100 Tranche				14.92	9.34	2.03	13.12	47.47	0.85	3.35	0.996	742
Δ 0–3 Tranche	0.396	0.537	0.164	3.89	182.21	-1944.84	6.69	2102.15	1.45	53.60	0.070	741
Δ 3–7 Tranche		0.905	0.333	1.11	70.07	-453.48	3.08	387.07	-0.01	10.56	0.078	741
Δ 7–15 Tranche			0.605	0.15	22.71	-115.09	0.18	144.42	0.38	10.47	0.150	741
Δ 15–100 Tranche				0.00	1.52	-8.50	0.05	12.40	0.83	16.93	0.000	741

Table 7: CDO Post-Crisis

	Correlations											
	3–7	7–15	15–100	Mean	SD	Min.	Med.	Max.	Skew.	Kurt.	Serial corr.	N
0–3 Tranche	0.868	0.817	0.580	1716.06	218.26	1330.79	1726.76	2186.54	0.15	1.99	0.997	484
3–7 Tranche		0.893	0.814	534.68	101.00	352.34	512.60	795.25	0.50	2.30	0.997	484
7–15 Tranche			0.795	195.50	59.40	119.23	182.72	367.29	0.79	2.71	0.998	484
15–100 Tranche				37.20	8.40	23.49	34.77	67.81	1.34	4.32	0.998	484
Δ 0–3 Tranche	0.918	0.840	0.792	-1.37	45.75	-161.71	-0.92	201.65	-0.01	5.31	0.022	483
Δ 3–7 Tranche		0.852	0.692	-0.67	18.74	-73.08	-0.99	80.91	0.07	5.01	0.006	483
Δ 7–15 Tranche			0.891	-0.13	8.78	-35.82	-0.25	86.28	1.86	23.83	0.007	483
Δ 15–100 Tranche				-0.03	1.55	-6.68	-0.06	10.21	0.42	9.43	-0.025	483

Table 8: CDSs Pre-Crisis

	Correlations														
	SD	Min.	5	Med.	95	Max.	Mean	SD	Min.	Med.	Max.	Skew.	Kurt.	Serial corr.	N
Mean	0.920	0.797	0.897	0.853	0.871	0.844	43.48	11.66	30.46	39.68	92.55	2.07	7.08	0.996	480
SD		0.685	0.713	0.621	0.907	0.950	47.64	24.06	23.21	40.70	166.19	2.35	8.92	0.994	480
Min.			0.844	0.772	0.612	0.662	7.14	1.85	4.04	6.97	14.88	1.17	4.88	0.995	480
5				0.941	0.637	0.687	12.90	4.20	6.81	13.39	25.16	0.81	3.57	0.996	480
Med.					0.538	0.618	28.04	6.40	16.85	27.00	47.27	0.55	2.60	0.997	480
95						0.751	123.06	52.24	76.06	108.81	417.74	3.19	13.22	0.994	480
Max.							365.00	215.47	125.42	297.83	1392.62	1.78	6.75	0.992	480
Δ Mean	0.789	0.219	0.581	0.722	0.829	0.523	0.04	1.35	-8.53	-0.04	8.88	0.22	17.17	0.441	479
Δ SD		0.039	0.278	0.336	0.728	0.882	0.13	3.09	-23.66	0.03	24.45	0.10	30.52	0.345	479
Δ Min.			0.383	0.261	0.141	0.023	0.00	0.39	-3.80	0.00	2.71	-1.58	34.06	-0.121	479
Δ 5				0.588	0.352	0.173	0.01	0.38	-2.10	-0.01	3.55	1.96	25.58	0.202	479
Δ Med.					0.487	0.110	0.00	0.82	-4.55	-0.04	6.36	1.16	16.74	0.113	479
Δ 95						0.475	0.32	8.55	-61.04	-0.06	71.63	0.93	28.12	0.331	479
Δ Max.							0.90	35.43	-380.18	0.17	350.26	-1.14	56.35	0.170	479

Table 9: CDSs Crisis

	Correlations														
	SD	Min.	5	Med.	95	Max.	Mean	SD	Min.	Med.	Max.	Skew.	Kurt.	Serial corr.	N
Mean	0.915	0.520	0.727	0.900	0.920	0.866	204.25	90.12	47.83	183.57	477.61	0.75	3.08	0.999	742
SD		0.304	0.516	0.714	0.869	0.968	385.41	202.21	57.67	386.07	982.81	0.36	2.46	0.996	742
Min.			0.569	0.603	0.447	0.249	22.54	7.73	0.00	21.21	45.35	0.29	3.90	0.990	742
5				0.921	0.489	0.563	38.32	11.69	16.62	37.69	77.58	0.53	3.20	0.999	742
Med.					0.710	0.720	92.79	35.47	27.92	83.30	214.73	0.82	3.46	0.999	742
95						0.767	653.58	300.38	177.54	544.66	1515.47	0.65	2.28	0.999	742
Max.							3225.57	1790.45	350.55	3423.20	8861.85	0.38	2.86	0.992	742
Δ Mean	0.784	0.134	0.583	0.705	0.696	0.637	0.17	8.33	-52.99	0.22	49.93	-0.18	11.19	0.239	741
Δ SD		0.041	0.189	0.273	0.381	0.931	0.39	34.28	-404.46	0.40	289.86	-1.57	42.23	0.005	741
Δ Min.			0.300	0.173	0.079	0.020	0.01	3.32	-38.52	0.00	38.48	-0.32	85.75	-0.145	741
Δ 5				0.719	0.432	0.105	0.02	1.39	-6.06	-0.01	9.13	0.61	9.20	0.262	741
Δ Med.					0.562	0.176	0.08	3.93	-20.92	0.00	25.91	0.33	9.65	0.264	741
Δ 95						0.273	0.30	32.41	-227.62	0.58	176.99	-0.13	10.74	0.209	741
Δ Max.							4.07	435.97	-5800.93	4.64	3615.09	-2.35	57.94	-0.078	741

Table 10: CDSs Post-Crisis

	Correlations														
	SD	Min.	5	Med.	95	Max.	Mean	SD	Min.	Med.	Max.	Skew.	Kurt.	Serial corr.	N
Mean	0.955	0.164	0.669	0.941	0.964	0.930	107.94	17.58	84.21	103.13	162.35	0.58	2.24	0.998	484
SD		-0.064	0.458	0.814	0.982	0.954	69.11	17.71	41.79	69.99	111.24	0.05	1.77	0.998	484
Min.			0.631	0.356	-0.026	0.083	22.61	6.38	9.59	23.93	35.59	-0.37	2.07	0.997	484
5				0.837	0.497	0.534	36.59	4.97	24.96	36.56	52.86	0.31	3.17	0.998	484
Med.					0.850	0.833	90.13	12.50	72.41	85.33	129.59	0.83	2.62	0.998	484
95						0.930	233.01	55.69	153.45	226.82	385.68	0.35	2.13	0.998	484
Max.							450.04	111.10	275.21	457.66	691.41	0.17	1.97	0.998	484
Δ Mean	0.924	0.367	0.744	0.858	0.865	0.688	-0.04	2.14	-9.39	-0.08	12.27	0.45	7.27	0.294	483
Δ SD		0.206	0.648	0.762	0.856	0.851	-0.02	1.89	-9.02	-0.09	10.83	0.39	7.92	0.293	483
Δ Min.			0.298	0.335	0.247	0.039	-0.04	0.93	-2.89	-0.06	8.59	1.82	18.73	-0.067	483
Δ 5				0.675	0.608	0.514	-0.01	0.76	-2.83	0.00	3.21	0.21	6.07	0.069	483
Δ Med.					0.701	0.571	-0.03	1.93	-6.46	-0.07	9.04	0.48	5.76	0.120	483
Δ 95						0.628	-0.09	7.38	-31.46	-0.24	36.82	0.47	6.77	0.284	483
Δ Max.							-0.43	13.91	-114.33	-0.78	85.28	-0.36	16.12	0.162	483

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