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The multivariate option iPoD framework – assessing systemic financial risk

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Non-technical summary

Research Question

To measure the degree of systemic risk in a financial system it is necessary to capture beside the risk of the individual financial institutions (FIs) also the risk which is created through the linkages that exist between the different FIs in the economy. These linkages can cause financial distress in one or a small group of FIs to spill over to other institutions and to propagate within the financial sector to larger systemic distress, as seen, for instance, during the 2007/2008 sub-prime crisis. This paper aims to develop indicators that capture systemic financial distress in an adequate and timely manner.

Contribution

We suggest a novel framework to estimate systemic financial distress indicators from call option prices. In our framework we combine estimates for the individual institutions' asset distributions (RNDs) and probabilities of default (PoD) with estimates for the dependence structure between the different institutions in the system to obtain multivariate asset distributions (MADs) for a group of FIs. The RNDs and the dependence structure are estimated in a very flexible way such that we can capture the empirically usually observed deviations of asset distributions from normality as well as non-linear and dynamic dependence structures between the FIs. The MADs exhibit information about the conditional PoDs of the different FIs such that we can calculate distress measures based on the assumed default of one or more institutions in the system.

Results

The application of the framework to a sample of major US FIs during the period of the 2007/2008 financial crisis shows that the suggested framework identifies in a timely manner: i) the most distressed FIs in the system; ii) the systemically most important FIs; iii) implicit bailout guarantees given to some FIs; and iv) a "too connected to fail" problem in the US financial sector throughout the year 2008.

Nicht-technische Zusammenfassung

Fragestellung

Indikatoren für das systemische Risiko im Finanzsektor müssen sowohl das Einzelrisiko der Banken als auch das Risiko, das durch die Vernetzung der Institute im System entsteht, erfassen. Die Vernetzung der Institute kann dazu führen, dass Probleme in einem oder einer kleinen Gruppe von Finanzinstituten auf andere Institute übergreifen und sich so zu systemweiten Krisen ausweiten - wie z.B. während der Finanzkrise 2007/2008 beobachtbar. Dieses Forschungspapier versucht Indikatoren zu entwickeln, welche in geeigneter und frühzeitiger Weise das systemische Risiko im Finanzsektor anzeigen.

Beitrag

Wir präsentieren einen neuen Ansatz, der auf Basis von Optionspreisen Kennzahlen für das systemische Risiko im Finanzsektor schätzt. Unser Ansatz kombiniert die Vermögensverteilungen (RNDs) und Ausfallwahrscheinlichkeiten von Einzelbanken mit Schätzungen für die Abhängigkeitsstruktur zwischen den Banken im System um hierdurch gemeinsame (multivariate) Vermögensverteilungen für ein System von Banken zu erhalten. Die Schätzung der RNDs und der Abhängigkeitsstruktur erfolgt auf sehr flexible Art, so dass sowohl die empirisch beobachtbaren Abweichungen von Vermögensverteilungen von der Normalverteilung als auch nicht-lineare und dynamische Abhängigkeitsstrukturen modelliert werden können. Die multivariaten Verteilungen enthalten Informationen über die bedingten Ausfallwahrscheinlichkeiten der verschiedenen Institute, so dass Risikomaße berechnet werden können, die den Effekt eines Ausfalls eines oder mehrerer Institute auf das Restsystem evaluieren.

Ergebnisse

Die empirische Anwendung auf US-Finanzinstitute während der Zeit der US-Subprime-Krise zeigt, dass die vorgeschlagenen Risikomaße zeitnah: i) die am stärksten betroffenen Finanzinstitute; ii) die systemisch relevantesten Finanzinstitute; iii) die impliziten Staatsgarantien von einigen Finanzinstituten; und iv) ein „too connected too fail“ - Problem im US-Finanzsektor identifizieren.

BUNDESBANK DISCUSSION PAPER NO 20/2014

The Multivariate Option iPoD Framework - Assessing Systemic Financial Risk *

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Abstract

We derive multivariate risk-neutral asset distributions for major US financial institutions (FIs) using option implied marginal risk-neutral asset distributions (RNDs) and probabilities of default (PoDs). The multivariate densities are estimated by combining the entropy approach, dynamic copulas and rank correlations. Our density estimates yield information about the conditional distributions of the individual FIs, and we propose several financial distress measures based on default scenarios in the financial sector. Empirical results around the period of the US sub-prime crisis show that the proposed risk measures identify in a timely manner: i) the most distressed FIs in the system; ii) the systemically most important FIs; iii) the implicit bailout guarantees given to some FIs; and iv) a ”too connected to fail” problem in the US financial sector throughout the year 2008.

Keywords: Financial Distress, Conditional Probability of Default, Copulas, Option Prices, Entropy Principle.

JEL classification: C14, C32, G01, G21.

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1 Introduction

Systemic financial risk deals with the risk to the financial sector as a whole as opposed to the risk of just individual financial entities. In order to measure this type of risk, it is necessary to assess not only the risk of individual financial institutions (FIs), but also the risk that arises due to the interconnectedness among the FIs in the economy. Interconnectedness in the financial sector occurs due to direct links, as e.g. through the interbank market, and/or indirect links arising e.g. from similar exposures in the FIs' portfolios. These linkages can cause financial distress in one or a small group of FIs to spill over to other firms (contagion) and to propagate within the financial sector to large systemic crises, such as was seen during the 2007/2008 sub-prime crisis. Assessments of the sub-prime crisis at the G20 summits in Washington 2008 and subsequent summits resulted in an action plan to implement a so-called macro-prudential policy framework, of which the objective is to mitigate systemic risks and reduce the likelihood of future financial crises (Jenkins and Thiessen (2012); BIS (2011)). Unlike traditional, micro-prudential regulation, which focuses on the soundness of single FIs, macro-prudential policy focuses on the risk of the financial system as a whole. This is due to the recognition that the interconnectedness of FIs and markets increases financial risk to an extent that is not captured by focusing solely on individual institutions (Group of Thirty (2010), Bernanke (2010)). As part of the macro-prudential agenda, in recent years institutions such as the European Systemic Risk Board (ESRB) or the Financial Stability Oversight Council (FSOC) in the US were founded to assess systemic risk and to identify systemically important FIs (SIFIs). Further, new regulatory frameworks such as Basel III (global) and the Dodd-Frank Act (USA) implemented macro-prudential policy instruments such as counter-cyclical capital requirements and additional regulatory requirements for SIFIs (Murphy (2013), BoE (2009)). A prerequisite for successful macro-prudential policy is the timely measurement of systemic risk as well as the identification of the most vulnerable and the systemically most important FIs.

In this paper, we suggest a novel framework to derive informative measures for systemic financial risk. Our approach combines the (univariate) option iPoD framework, proposed in Capuano (2008) and Vilsmeier (2011), and the most entropic copula procedure, proposed in Chu (2011), such that we can estimate multivariate asset distributions (MADs) for a sample of financial institutions. The approach, which we refer to as the multivariate option iPoD procedure, derives the MADs in such a way that they exhibit information about the conditional Probabilities of Default (PoDs) of the FIs in the sample. Hence, we are able to implement distress scenarios that are based on the default of one or multiple institutions in the system and evaluate the impact on the residual FIs in the system. The distress levels measured by the proposed risk indicators incorporate the firm's individual risk level as well as risk induced by distress in other firms of the system and spilt over due to the interconnectedness among the FIs.

The cornerstone of our methodology is the option iPoD approach, in which option implied risk neutral densities (RNDs) are estimated in such a way that they reveal information about the PoD of the issuer of the underlying. Default in this framework corresponds to the event that the stock price of the firm falls to zero during the time to maturity of the options. Our framework to derive firms' PoDs is purely statistical and requires neither balance sheet nor recovery rate information, in contrast to competing methods such as Merton-type models[1] and Reduced Form (RF) models[2]. The RNDs are estimated using a semi-parametric estimation procedure based on the entropy concept (Shannon (1948)) and moment constraints that are given by risk neutral pricing theory. As illustrated in Vilsmeier (2011) the estimated distributions are, in general, very smooth and highly flexible with regard to their functional forms such that the empirically observed deviations of RNDs from normality can be easily modeled. Further, the framework provides highly plausible PoD estimates, which, as shown in Matros and Vilsmeier (2012), clearly outperform CDS spreads in identifying the most vulnerable FIs in the course of 2007/2008. Following the suggestions in Matros and Vilsmeier (2012) we derive daily time series of maturity-corrected RNDs, where each RND is estimated

[1]Merton-type models define a firm's default according to the structural approach of Merton (1974), which states that a firm defaults if its value of assets is lower than its value of debt. To calculate the PoD one calibrates asset distributions on basis of historical equity prices and defines a default point according to the firm's book value of debt. Examples of structural approaches to measure systemic risk are: Tudela and Young (2005), Chan-Lau and Gravelle (2005), Lehar (2005), Crosbie and Kocagil (2003), J.P.Morgan (1997).

[2]Reduced form models use debt-based market instruments and their market prices to calibrate the firms' default processes (usually modelled as Poisson processes). To calculate the implied PoDs, assumptions about unknown recovery rates in the case of the firm's default have to be made. Examples for reduced form models applied to measuring systemic risk are: Huang, Zhou, and Zhu (2012), Segoviano and Goodhart (2009), and Avesani, Pascual, and Li (2006).)

using only daily available option prices for different strikes and a given maturity.

To extend the option iPoD procedure to a multivariate framework, we apply the statistical concept of copulas. Copulas are a very flexible way to estimate multivariate probability distributions, as they allow us to model the dependence structure of a multivariate distribution (the copula) independently of its univariate marginals. Based on Sklar's theorem (Sklar (1959)), we can combine a given copula with arbitrary marginals and obtain different multivariate distributions. The most entropic copula (MEC) approach of Chu (2011) allows us to model the dependence structure between the different FIs on basis of Spearman rank correlations that we estimate on the basis of the time series of marginal RNDs. We estimate the rank correlations dynamically with exponentially decreasing influence of past observations and obtain time-varying measures of dependence that can capture linear as well as non-linear dependence and may change throughout the economic cycle. By applying Sklar's theorem, we combine the RNDs from the (univariate) option iPoD approach and the MEC to obtain time series of MADs whose marginals and dependence structure may change every day. As the framework is easily implemented in higher dimensions (i.e. $d > 2$), our methodology provides a high-dimensional, dynamic, non-Gaussian Copula framework that is mathematically tractable and straightforward to implement. This clearly puts our methodology in contrast to standard approaches of Merton-type and RF models that usually model the dependence structures between different firms in a static and linear way, based on historical equity return Pearson correlations.

We apply our framework to data of 13 of the largest US FIs during the period of January 2007 to September 2008 and calculate time series of five different distress indicators. The indicators are derived on the basis of conditional PoDs (CPoDs) and conditional lower quantiles (CQs) of the MAD. Compared to CPoDs, the CQ measures can be interpreted as broader definitions of distress that cover not only default but also events such as downgrades or large losses in market value. To our knowledge, these CQ risk measures are a new addition to the literature. The estimated distress indicators are able ot identify in a timely manner i) the most distressed FIs during 2007/2008, ii) the systemically most important FIs, iii) the implicit bailout guarantees given to some FIs and iv) the high degree of interconnectedness in the sector throughout the year 2008.

The contributions of our paper are as follows. We introduce a novel framework to measure systemic financial risk by estimating a financial sector's multivariate asset distribution (MAD). This framework has several attractive properties: i) the MADs incorporate information about the firm's PoDs and conditional PoDs ii) we do not need balance sheet or recovery rate information to derive the MADs, iii) the estimated MADs are high-dimensional, capture dynamic and non-linear dependence structures, and are very flexible with regard to their functional forms. Based on the MADs that we estimated for 13 major US FIs during the period of January 2007 to September 2008, we calculate five different distress indicators based on conditional PoDs and conditional lower quantiles. The analysis of the distress indicators shows the high informational content of the framework, which provides very plausible results given the historical events during the US sub-prime crisis.

The remainder of the paper is structured as follows. First, section 2 introduces the methodology of the multivariate option iPoD procedure. Section 3 presents the data used in our empirical application to the US financial sector. In section 4, we give a brief description of how to empirically implement the framework, and section 5 provides the estimation results. Section 6 concludes and gives some suggestions for future research.

2 Methodology

The multivariate option iPoD methodology uses copula theory to estimate MADs for a sample of FIs. Using this theory, one can model the univariate asset distributions (RNDs) of the FIs independently of their dependence structure (copula) and combine them to MADs using Sklar's theorem (Sklar (1959)). We apply the univariate option iPoD approach as suggested in Vilsmeier (2011)) to estimate the univariate asset distributions, and the most entropic copula (MEC) approach of Chu (2011) to model the dependence structure. Both frameworks apply a semi-parametric estimation procedure which maximizes the entropy function (Shannon (1948)) under moment constraints. In the case of the option iPoD procedure, the moment constraints are obtained from the theory of risk neutral-pricing and the observed option prices for different strikes. In the case of the MEC, the moment constraints are given by the dynamic Spearman rank correlations estimated on basis of the marginal RND time series. In the following we describe the individual building blocks of our framework in detail.

Copulas

A copula C can be isolated from any multivariate random vector $(X_1,\ldots,X_n)$ with multivariate distribution function $H(x_1,\ldots,x_n) = P(X_1 \leq x_1,\ldots,X_n \leq x_n)$ and marginal distributions $F_i(x_i) = P(X_i \leq x_i)$, $i = 1,\ldots,n$, by transforming the univariate random variables X_i to standard-uniform random variables using the probability integral transform, i.e. by applying the marginal cdfs F_i onto each random variable X_i leading to $U_i = F_i(X_i) \sim U(0,1)$. The distribution function of the resulting random vector $(U_1,\ldots,U_n)$ is given by the copula function, i.e.:

$$C(u_1,\ldots,u_n) = P(U_1 \leq u_1,\ldots,U_n \leq u_n) = H(X_i \leq F_{x_i}^{-1}(u_i),\ldots,X_n \leq F_{x_n}^{-1}(u_n)). \tag{1}$$

This means that a copula is a multivariate distribution function $C : [0,1]^d \rightarrow [0,1]$ with standard uniform (univariate) marginals.[3]

According to Sklar's theorem (Sklar (1959)), there exists a copula for all H and $F_i,\ldots,F_n$ such that for all $(x_1,\ldots,x_n) \in \bar{\mathbb{R}}^n$:

$$H(x_1,...,x_n) = C(F_1(x_1),...,F_n(x_n)), \tag{3}$$

where C is unique if all F_i are continuous. Using Sklar's theorem, we can create, for a given copula, different H, which all exhibit the same dependence structure but differ by their marginals F_i. Conveniently, the different marginals may come from different families

[3]Formally a copula function satisfies the following three properties:

$$C(1,...,1,u_i,1,...,1) = u_i \quad \text{for every } i \leq n \text{ and all } u_i \text{ in } [0,1]. \tag{2.1}$$

$$C(u_1,...,u_n) = 0 \quad \text{if } u_i = 0 \text{ for any } i \leq n. \tag{2.2}$$

C is n-increasing, i.e. for each n-box $R := \prod_{i=1}^{n}[x_i,y_i] \subseteq [0,1]^n$, $x_i \leq y_i$, the C-volume $V_C(R)$ is non-decreasing:

$$V_C(R) := \sum sgn(\mathbf{z})C(\mathbf{z}) \geq 0, \text{where the sum is over all vertices } \mathbf{z} \text{ of R and } sgn(\mathbf{z}) = \begin{cases} 1 & z_k = a_k \text{ for even } k\text{s} \\ -1 & z_k = a_k \text{ for odd } k\text{s} \end{cases} \tag{2.3}$$

of probability distributions. As copulas allow us to model the dependence structure and the marginals independently of one another and subsequently combine them arbitrarily, they provide a much more flexible framework to model multivariate densities than fitting parametrized multivariate distributions to the data.[4]

Another important property of copulas is that they are a "non-parametric" measure of dependence since they are invariant under strictly increasing and continuous transformations of their marginals. In contrast to traditional linear correlation coefficients, rank correlation measures share this property, since applying the probability integral transform to a random variable corresponds to a rank transformation of that random variable. This implies a direct representation of rank measures of dependence as a function of the copula. The Spearman rank coefficient that will be used in the context of the MEC estimation has the following copula representation (see e.g. Nelsen (2006), chapter 5):

$$\rho(X_i,X_j) = 12 \int_{[0,1]^2} (C(u_i,u_j) - u_i u_j) du_i du_j - 3 \tag{4}$$

Most Entropic Copula (MEC) Approach

In order to estimate the MAD, we need to find a copula that adequately captures the dependence structure among the different RNDs. Standard approaches assume specific parametric copulas, such as the Gaussian- or t-copula, and estimate the parameters such that they fit the data at hand. Since different parametric assumptions impose different dependence structures onto the data, the results in empirical applications can be quite sensitive to the specific choice of the copula, as shown in Frey, McNeil, and Nyfeler (2001). The MEC approach of Chu (2011) circumvents the copula choice problem to a large degree by estimating a semi-parametric copula that, from an information theoretic point of view, imposes as little information as possible on the shape of the copula beyond what is actually known from the data. This is achieved by applying the entropy principle to identify the optimal copula. The entropy principle, formulated by Jaynes (1957), states that, given the information from the data (expressed as moment constraints), the distribution which best describes the current

[4]For an in-depth description of the copula concept see e.g. Nelsen (2006).

state of knowledge is the one that maximizes the entropy function.

The entropy function for a multivariate density function $f(x_1,\dots,x_n)$ is defined as:

$$H[f(x_1,\dots,x_n)] = -\int_{-\infty}^{\infty} f(x_1,\dots,x_n)\log f(x_1,\dots,x_n)dx_1\dots x_n. \quad (5)$$

As shown by Shannon (1948), the entropy function can be be interpreted as a metric measure for the average uncertainty in a random variable, where the uncertainty refers to the predictability of an (average) outcome of the random variable. Jaynes (1957) suggested using the entropy function for density estimation purposes when there is information about the density in the form of expected values available (moment constraints).

In the MEC approach, a copula density function $c(u_1,\dots,u_n)$ is estimated by maximizing the entropy function subject to moment constraints that are given by i) constraints that guarantee that the n marginals of the density are uniform and ii) rank correlation functions expressed in terms of $c(u_1,\dots,u_n)$ that have to satisfy their empirical counterparts estimated from the data.

The MEC framework is very flexible with regard to modeling different types of dependence. As suggested in Chu (2011), different kinds of rank measures can be jointly used in the estimation. On the one hand, these can be traditional rank measures such as Spearman's ρ or Kendall's τ, which are non-parametric measures and capture linear and non-linear dependence structures. On the other hand, one can use measures such as Blest measures of correlation (Blest (2000)) or conditional Spearman coefficients (Schmid and Schmidt (2007)), which allow us to model asymmetric types of dependence or specific types of tail-dependence.[5] On the downside, as for the estimation of an n-dimensional copula all possible pairwise rank correlations among the different RNDs have to be included, each type of rank measure (e.g. Spearman's ρ) implies $n(n+1)/2$ additional constraints and hence additional parameters to estimate. We will restrict ourselves to a set-up which contains only ordinary Spearman rank correlations but estimated in a dynamic way such that possible asymmetric dependence structures should be captured and displayed to a large extent in the time-varying correlations.

[5] Another possible and interesting approach is to include multivariate versions of Spearman's ρ, or of other rank measures. Examples of multivariate rank measures are e.g. given in Schmid and Schmidt (2007).

The MEC Estimation Setup

The above described estimation problem is formalized in an n-dimensional set-up as follows:

$$\textit{Maximizing} \quad H\left[c(u_1,\ldots,u_n)\right] = -\int\limits_{[0,1]^n} c(u_1,\ldots,u_n)\,log\; c(u_1,\ldots,u_n)du_1...du_n \tag{6}$$

subject to:

$$\int\limits_{[0,1]^n} c(u_1,\ldots,u_n)du_1\ldots du_n = 1, \tag{7}$$

$$\int\limits_{[0,u_i]}\int\limits_{[0,1]^{n-1}} u_i^j c(u_1,\ldots,u_i,\ldots,u_n)du_1\ldots du_i\ldots du_n = \frac{1}{1+j}, \tag{8}$$

$\forall\; i = 1,2,\ldots,n$ and $j = 1,2,\ldots,m$,

$$\int\limits_{[0,1]^n} u_k u_l c(u_1,\ldots,u_n)du_1\ldots du_n = \frac{\widehat{\rho}^{\;k,l}+3}{12}, \tag{9}$$

$\forall\; k = 2,3,\ldots,n, l = 1,2,\ldots,n-1, l < k.$

Equation (6) is the entropy function with the multivariate copula density $c(u_1,...,u_n)$ as its argument. Equation (7) represents the additivity constraint which guarantees that the copula density integrates up to one. The set of equations (8) guarantees that the n marginals of the copula are standard-uniform by imposing that the first m moments (with m as finite integer) of the copula density satisfy the theoretical first m moments of a standard-uniform, where the j-th moment is given by $\mu_j = 1/(1+j)$. This is, of course, only an approximation but as shown in Chu (2011), a relatively small set of constraints is already enough to guarantee approximate standard-uniform marginals.

The system of equations (9) represents the dependence constraints given by the estimated Spearman's ρ in terms of the copula density and the pairwise Spearman rank correlations among all n FIs estimated from the respective RNDs. Given the RND time series for the different FIs, we obtain the empirical correlations by drawing for each firm randomly from its RNDs at all points in time such that we get a time series for the respective firm's asset value. On the basis of each pair of time series of different firms, we can estimate the rank correlation. This process is carried out repeatedly such that we obtain a distribution of Spearman's ρ for each pair of firms, and we take the median from this distribution as our estimate $\widehat{\rho}^{\,k,l}$. To model a time-varying dependence structure between the FIs, we estimate the correlation coefficients dynamically with exponentially decreasing influence of past observations. Following the suggestions in RiskMetricsTM (1996) our empirical correlation estimates are obtained according to:

$$\rho_t^{k,l} = \frac{\sum_{s=1}^{t-1} \alpha^{t-s-1}(R_{k,s} - \bar{R}_k)(R_{l,s} - \bar{R}_l)}{\sqrt{\sum_{s=1}^{t-1} \alpha^{t-s-1}(R_{k,s} - \bar{R}_k)^2 (R_{l,s} - \bar{R}_l)^2}}, \tag{10}$$

where α^{t-s-1} denotes the weighting coefficient, with $\alpha \in \,]0,1[$ and t as a moving endpoint that is equal to the period for which we calculate the respective correlation coefficient ("stretching window" approach).[6] $R_{k,s}$ and $R_{l,s}$ are the rank transformed asset values in time period s for two different firms, and $\bar{R}_k = \frac{1}{t-1}\sum_{s=1}^{t-1} R_k, s$ and $\bar{R}_l = \frac{1}{t-1}\sum_{s=1}^{t-1} R_l, s$ are the respective means of the rank transformed data. Equation (10) provides us with daily updated rank correlations and we obtain a framework in which the dependence structure among the FIs may change every day (dynamic copula approach).

The estimation of the copula function represented by the maximization set-up (6)-(9) can be carried out using ordinary Lagrange multiplier methods. Deriving the Lagrangian with respect to $c(u_1, \ldots, u_n)$ yields for a given time period t the following form for the optimal density:

$$c_t^*(u_1 \ldots u_n) = \exp\left\{ -\sum_{i=1}^{n}\sum_{j=1}^{m} \lambda_{i,j} u_i^j - \sum_{k=2}^{n}\sum_{\substack{l=1 \\ l<k}}^{n-1} \lambda_{n+k-1,l} u_k u_l \right\} \tag{11}$$

where the set of λs (the Lagrange multipliers) is obtained from the minimum of the following function (for a derivation see e.g. Chu (2011) or Alhassid, Agmon, and Levine (1978)):

$$Q(\lambda) = \int_{[0,1]^n} \exp\left\{ -\sum_{i=1}^{n}\sum_{j=1}^{m} \lambda_{i,j}\left(u_i^j - \frac{1}{1+j}\right) - \sum_{k=2}^{n}\sum_{\substack{l=1 \\ l<k}}^{n-1} \lambda_{n+k-1,l}\left(u_k u_l - \frac{\widehat{\rho}_t^{\,k,l} + 3}{12}\right) \right\} du_1 \ldots du_n. \tag{12}$$

[6]Of course, there are theoretically more appealing, but also more complex, methodologies for deriving rank correlations in a dynamic way. Especially the use of realized correlations theory (e.g. Barndorff-Nielsen and Shephard (2004)) might be a promising way to make the correlations less dependent on historical data.

The option iPoD framework

The cornerstone of our MAD estimation approach is the (univariate) option iPoD framework, which allows us to estimate the required time series of univariate asset distributions (RNDs) for all FIs considered in our sample. While the theoretical foundations of the option iPoD methodology were introduced by Capuano (2008) and Vilsmeier (2011), Matros and Vilsmeier (2012) show how the framework can be empirically implemented in such a way that time series of maturity-corrected RNDs/PoDs can be derived. We closely follow the suggestions made in Matros and Vilsmeier (2012).

The option iPoD procedure allows us to estimate risk-neutral densities in such a way that they exhibit information about the default probability of the issuing company of the option's underlying. The default of a firm is triggered when the stock price S of a firm falls to zero during time to maturity of the option. Importantly, the PoDs and RNDs are derived in a purely statistical manner without the use of balance-sheet information or recovery rate assumptions. All that is needed for the estimation is up-to-date information given by the set of daily observable equity (call) option prices to different strike prices K_i (and by a risk-free interest rate).

The idea of the option iPoD framework is to modify traditional RND estimation approaches such that it is possible to estimate a "mass point" in the RND which indicates the probability that the underlying of a stock option will have a value of zero at time of maturity T. The mass point is given as the integral over the density in a specific sub-domain of the RND. This sub-domain is obtained by shifting the domain of possible realizations for S_T upwards by some constant D and defining a new variable $V_T = S_T + D$. Then the RND $f(V_T)$ is estimated for this new variable. Using the domain of the new variable, the pay-off for a call option in T with strike K_i is defined by: $C_T^{K_i} = \max(V_T - D - K_i; 0)$. Hence, there will be no pay-off for the option in the interval of values $V_T \in [0, D]$. The applied estimation procedure ensures that there is an interaction between the (entire) density of the RND for $V_T \in [D, V_{max}]$, with V_{max} as the upper domain bound for V_T, and the level of the PoD (i.e. the size of the density assigned to $V_T \in [0, D]$) such that the combination of the RND shape and the PoD level that best fits the observed prices can be identified (see Vilsmeier (2011)). The choice of D is crucial. It determines the length of the sub-domain $[0,D]$ and, jointly with it, the PoD. Using numerical experiments, Vilsmeier (2011) found that for arbitrary reasonable RND forms and PoD levels, the procedure can approximate the respective probability mass point for $S_T = 0$ quite well if the constant D is chosen within the interval $[1;20]$. Since an exact rule for the determination of the optimal D has not yet been detected, the PoDs are obtained by averaging over RND estimates with different Ds ranging from 1 to 20. The optimal RND is then identified as the one that provides the PoD closest to the average PoD ("averaging approach"). Matros and Vilsmeier (2012) found that the "averaging approach" provides highly plausible and informative estimates in various empirical applications.

Despite the purely statistical nature of the option iPoD approach, the variables can be given a theoretical meaning by applying a Merton (1974)-type interpretation. In this case, V_T denotes the value of assets, S_T represents the value of equity and D denotes the value of debt. Using this interpretation, a firm defaults if the value of its assets does not cover the value of debt.

In order to obtain the risk-neutral density $f(V_T)$ for a given D, we use moment constraints given by the continuous risk-neutral option pricing formula (Cox and Ross (1976)) and the observed option prices for different strikes K_i at time $t = 0$:[7]

$$C_0^{K_i} = e^{-rT} \int_{V_T=D+K_i}^{V_{max}} (V_T - D - K_i) f(V_T) dV_T, \quad i = 1, \ldots, B. \tag{13}$$

Equation (13) depends on the unknown RND $f(V_T)$ for observable option prices $C_0^{K_i}$ at different strikes. The formula states that today's observed option prices must be equal to the discounted expectation over all possible pay-offs under the risk-neutral probability

[7]While the described pricing formula holds for European-style call options, in our empirical application we will use American-style call options. European-style and American-style options theoretically trade at the same price in the absence of dividend payments (see e.g. Hull (2009)). As we do not correct the option prices for dividend payments we face some slight inaccuracies in our applications. In future research this inaccuracies can be avoided by correcting the observed option prices for dividend payments via binomial tree approaches.

density, where B denotes the number of observable option prices for different strikes K[8] and r represents the annualized risk free rate. The current stock price S_0 is included as an option with strike $K_1 = 0$. One face an under-determined estimation problem, as there is an infinite number of densities compatible with a finite number of moment conditions. In order to obtain a unique solution for $f(V_T)$, the principle of maximum entropy is used. In contrast to the MEC estimation set-up from above, the option iPoD approach uses the related cross-entropy (CE) function to derive the optimal density, and not the entropy function. The CE function, introduced by Kullback and Leibler (1951)), minimizes the so-called entropic distance (see e.g. Cover and Thomas (2006), chapter 2), $CE(x) = \int_0^{V_{max}} f(x) \log \frac{f(x))}{f^0(x)}$, between the density of interest $f(x)$ to be determined and some prior function $f^0(x)$. The CE function allows for a more general density estimation set-up than the entropy function by providing the possibility to consider a priori information in the estimation process. Importantly, in the case of a finite support and uniform prior, the minimization of the CE function and the maximization of the entropy function lead to the same optimal density for any given set of moment constraints. [9]

Given the moment constraints in (13) and an additivity constraint $\int_0^{V_{max}} f(V_T) dV_T = 1$, which ensures that the density integrates up to one, the RND estimation problem of finding $f(V_T)$ can be formalized by the following Lagrangian function:

$$
\begin{aligned}
L \;=\; & \int_{V_T=0}^{V_{max}} f(V_T) \left[\log \frac{f(V_T)}{f^0(V_T)} dV_T \right] + \lambda_0 \left[1 - \int_{V_T=0}^{V_{max}} f(V_T) dV_T \right] \\
& + \sum_{i=1}^{B} \lambda_i \left[C_0^{K_i} - e^{-rT} \int_{V_T=D+K_i}^{V_{max}} (V_T - D - K_i) f(V_T) dV_T \right], \qquad (14)
\end{aligned}
$$

where $f^0(V_T)$ denotes a uniform prior function on the interval $[0, V_{max}]$.

[8]Note that the strike prices in equation (13) are denoted in ascending order, i.e. i=1 denotes the smallest and i=B the largest strike price.

[9]More precisely, the prior function has to be of maximal entropy on the defined domain for x. On a closed interval this will be the uniform distribution, on an unbounded positive real valued domain (for a given mean) the exponential distribution, and on a unbounded real valued interval (given a mean and a variance) the normal distribution.

Optimizing (14) with respect to $f(V_T)$ yields (see e.g. Cover and Thomas (2006), chapter 12):

$$f^*(V_T) = \frac{1}{\mu(\lambda)} f^0(V_T) \exp\left[\sum_{i=1}^{B} \lambda_i e^{-rT} \mathbf{1}_{V_T > D + K_i}(V_T - D - K_i)\right], \tag{15}$$

with

$$\mu(\lambda) = \exp(1-\lambda_0) = \exp(-\lambda_0') = \int_{V_T=0}^{V_{max}} f^0(V_T) \exp\left[\sum_{i=1}^{B} \lambda_i e^{-rT} \mathbf{1}_{V_T > D + K_i}(V_T - D - K_i)\right] dV_T. \tag{16}$$

Under the assumption of a finite domain for $V_T \in [0, V_{max}]$, the optimal set of Lagrange multipliers λ_i, $i = 1, 2 \ldots B$, is obtained as the minimum of the following function (see Vilsmeier (2011) for a derivation):

$$\begin{aligned} F = {} & \log\left(\frac{1}{V_{max}}\right) + \log\Bigg\{\exp\left(-\sum_{i=1}^{B} w_i \lambda_i C_0^{K_i}\right) D \\ & - \sum_{i=1}^{B-1} \Bigg[\frac{\exp\left(\sum_{j=1}^{i} w_j \lambda_j (e^{-rT}(K_i - K_j) - C_0^{K_j}) - \sum_{k=i+1}^{B} w_k \lambda_k C_0^{K_k}\right)}{e^{-rT}(\sum_{j=1}^{i} w_j \lambda_j)} \\ & \qquad - \frac{\exp\left(\sum_{j=1}^{i} w_j \lambda_j (e^{-rT}(K_{i+1} - K_j) - C_0^{K_j}) - \sum_{k=i+1}^{B} w_k \lambda_k C_0^{K_k}\right)}{e^{-rT}(\sum_{j=1}^{i} w_j \lambda_j)} \Bigg] \\ & - \left[\frac{\exp\left(\sum_{j=1}^{B} w_j \lambda_j (e^{-rT}(K_B - K_j) - C_0^{K_j}\right) - \exp\left(\sum_{j=1}^{B} w_j \lambda_j (e^{-rT}(V_{max} - D - K_j) - C_0^{K_j}\right)}{e^{-rT}(\sum_{j=1}^{B} w_j \lambda_j)}\right]\Bigg\}, \end{aligned} \tag{17}$$

where w_i denotes liquidity weights that are pre-multiplied to the Lagrange multipliers λ_i. The weights ensure that more liquid option contracts (measured in our approach in terms of open interest) have to be approximated more closely by the estimated RND. As stressed in Matros and Vilsmeier (2012), the assignment of the liquidity weights is very important in order to obtain timely consistent and smooth PoD estimates.

The option iPoD framework has several appealing properties that are of interest when assessing systemic financial risk. First, since the methodology provides PoD estimates, it allows us to implement default scenarios in the financial sector and to assess (in combination with the MEC approach) the impact on the resilience of the financial system. Second, the PoDs are derived without the use of balance-sheet data or recovery rate assumptions. As pointed out e.g. in Vilsmeier (2011), the assumptions made in Merton-type models and RF approaches are especially severe when applied to FIs. This is due to the fact that these types of firms exhibit complex, opaque and very volatile asset and liability structures. This makes the exact definition of default points, as required by Merton-type models, or the estimation of recovery rates, as necessary for RF frameworks, very difficult. Third, compared to recently suggested frameworks such as Segoviano and Goodhart (2009), which exclusively use PoD estimates to model a financial sector's MAD, our approach enables us to derive not only CPoD estimates but also broader measures of distress based on the CQs of the MADs. This is because the PoDs are estimated jointly with the corresponding RNDs. As we will see in section 5, the CQ measures make it possible to identify government bailout guarantees given to some FIs.

3 Data

To calculate the MAD for a set of firms on a specific day, we need a risk-free interest rate and, for each firm, the prices of the call options written on this firm's stock to different strike prices (including the stock price itself). As the risk-free interest rate, we use the 3-month treasury bill secondary market rate obtained from the FRED database. The option data are daily option closing prices for contracts with maturities ranging from five to seven months obtained from the New York Stock Exchange (NYSE) via the data provider Stricknet. As an example shows Table 8 (Appendix) the stock option dataset of JPMorgan Chase (JPM) on January 1, 2007.

Our option data set comprises call prices for 13 different FIs over a period from January 1, 2003 to September 10, 2008. While we use the entire sample to calculate the RNDs and the dynamic Spearman correlations, we calculate the MADs from January 1, 2007 to September

10, 2008. The considered institutions in our sample are: Goldman Sachs (GS), Wells Fargo (WFC), Citigroup (C), Bank of America (BAC), JPMorgan Chase (JPM), the American International Group (AIG), Morgan Stanley (MS), Lehman Brothers (LEH), Bear Stearns (BSC), Wachovia Bank (WB), Merrill Lynch (MER), Countrywide Financial (CFC) and Washington Mutual (WM). All of these institutions were among the largest US FIs during the period of 2007/2008.

The time span for which we calculate the MADs covers the beginning of the sub-prime crisis period, with growing losses in the financial sector and initial (smaller) collapses in 2007, until shortly before the crisis brokeout in earnest around September 15, 2008, nearly bringing the US financial system to its knees. Our sample contains several FIs involved in incisive events of the sub-prime crisis. These are: the rescue/takeover of BSC at March 15, 2008 by JPM (orchestrated and financially backed by the US Federal Reserve Bank (FED)); the takeover of CFC on July 1, 2007 by BAC; the collapse of LEH on September 15, 2008; the takeover of MER by BAC on September 14, 2008; the rescue of AIG by the FED at September 16, 2008; the collapse of WM on September 26, 2008; the announcement of the takeover of WB by WFC on October 3, 2008 (the purchase was finalized on December 31, 2008); and the rescue of Citigroup by the FED on November 23, 2008 (see e.g. Wheelock (2010)). Given this historical timeline, we make the following classifications for the FIs in our sample:

1. "Surviving" FIs: JPM, GS, WFC, BAC, MS
2. Acquired FIs: BSC, CFC, WB, MER
3. "Rescued" FIs: BSC, AIG, C, WB
4. "Bankrupt" FIs: LEH, WM

The historical classification gives us a rough guideline on how to assess the informational content of our risk measures. In general, we will assume that the "surviving" FIs were less distressed during the crisis than the other institutions. By contrast, one can expect the "bankrupt" banks to be highly distressed and they should therefore be indicated as very risky. For the 'acquired/rescued' FIs there might be effects of implicit government or bailout guarantees in our estimated MADs as such guarantees, if anticipated by the market, would influence the risk perception of the investors for these institutions.

4 Empirical Implementation

As described in section 2, the first step in implementing the multivariate option iPoD framework is to estimate univariate RNDs for all FIs in our sample. This is done using the univariate option iPoD approach as described in Matros and Vilsmeier (2012). Following their suggestions, we first calculate the RNDs on the basis of an option maturity cycle of 5, 6, and 7 months, and use open interest[10] for the contracts as liquidity weights in the optimization of equation (17). The described maturity cycle means that we use a newly initiated 7-month contract for a specific firm for two months, until a new 7-months contract is initiated. For the RND domain, we define a finite interval $[0, V_{max}]$ for V_T, where V_{max} is set equal to five times the current stock value. To remove the maturity dependence inherent to the estimates, we apply the regression-based maturity dependence correction scheme to the estimates described in Matros and Vilsmeier (2012). However, instead of using it solely for the PoDs, we also apply it to the first ten moments of the RND estimates. In the correction scheme, the maturity effect on the RNDs is calculated by carrying out a (pooled) non-linear quantile regression of the RND estimates onto the respective time to maturity of the option contracts used to estimate the RNDs. We obtain correction factors and apply these to the moments and PoDs. To obtain a maturity-corrected RND, we use the cross-entropy procedure with a uniform prior and estimate a density that satisfies the moment constraints given by the maturity-corrected moments and PoDs.[11] Carrying out this procedure for every day, we eventually obtain a daily time series of maturity-corrected RNDs for each of the 13 considered FIs for the period from January 1, 2003 to September 10, 2008. After the maturity correction, the RNDs imply a theoretical time to maturity and, hence, evaluation/forecast

[10]The use of open interest (contracts traded in the past and not exercised or evened up yet) results in less volatile RND/PoD estimates than the use of trading volume. The weights are calculated by dividing open interest for a specific strike by the sum of open interest over all available strikes for a firm's stock option.

[11]The Lagrangian of the estimation set-up for a specific firm and time period t is equal to equation (14) where the third term is replaced by: $\sum_{i=1}^{10} \lambda_i (M_i - \int_0^{V_{max}} (V_T - \overline{V_T})^i f(V_T) dV_T)$, with M_i as the i-th central moment, and $\lambda_{11}(PoD - \int_0^D f(V_T))dV_T$.

horizon of 7 months.[12]

In the second step, we use the time series of the RNDs to estimate all possible dynamic Spearman rank correlations among the 13 RND time series (78 in total). To do so, we sample for each FI for a given period t randomly from its RND and obtain time series of their asset values V_T. Then we calculate the Spearman correlation coefficients dynamically and with exponentially decreasing influence of past observations between the time series. As the persistence parameter in the exponential weighting scheme (see equation (9)), we use $\alpha = 0.99$, as we assume that the dependence among the FIs is subject to some inertia.[13] We repeat this procedure 5000 times in order to obtain a distribution of Spearman coefficients for each pair of FIs. Finally, we use the median from these distributions as our empirical estimate of the Spearman correlations between the different FIs/RNDs. In Figure 4 (Appendix) four examples of the dynamically estimated Spearman correlations are shown. It can be clearly seen that, except for the correlation between JPM and LEH, all time series sharply increase during the year 2007, after moving in quite moderate cycles in previous years.

The third step consists of estimating the copula density function $c(u_1, \ldots, u_{u_n})$ on the basis of the calculated Spearman correlation coefficients. To guarantee that the n marginals of c are uniform, we impose eight moment constraints according to $m_j = \frac{1}{1+j}$ onto the marginal of each of the variables $u_1, \ldots, u_n$. In addition to the constraints on the marginals, $n(n+1)/2$ Spearman moment conditions are imposed on the estimation of the copula. As the Spearman correlations change every day, we estimate the copula density for each day anew.

In order to estimate the copula density, the n-dimensional integral in the objective function (12) has to be solved numerically. The optimization with regard to the Lagrange multipliers requires a very precise computation of the integrals, which cannot be achieved using ordinary Monte Carlo methods. Therefore, we use a deterministic quadrature rule suggested in Berntsen and Espelid (1991). The method employs a globally adaptive subdivision scheme

[12] In fact, the informational content in the used option prices implies an average time to maturity of 6 months as this is the average time to maturity of the option contracts used.

[13] Note, that the weight of a one year ($= 250$ days) old observation is $\approx 8\%$ with $\alpha = 0.99$, $\approx 0.5\%$ with $\alpha = 0.98$ and close to zero for smaller α.

and uses a cubature rule for the subregion estimation.[14] As pointed out in Hahn (2005), the method is very reliable in moderate dimensions but quite slow if applied to more than nine dimensions. For this reason, MADs for the entire sample of FIs were only calculated for March 10, 2008 and September 10, 2008, at which we carry out static risk analyses. For the dynamic evaluation, we restrict our analysis to the eight banks in the ”rescued/acquired” and ”bankrupt” group plus JPMorgan as a representative of the ”surviving” group.

In the fourth and final step we obtain the MADs by combining information about the dependence structure and the marginal RNDs using Sklar's theorem. Each copula density is integrated to the copula cdf function $\hat{C}$, such that Sklar's theorem can be applied. Then we integrate the FI's RNDs to the cumulative RND $\hat{F}(V_T)$ and use all of them as arguments in $\hat{C}$. We then obtain the MAD $\hat{H}$ as:

$$\hat{H}(x_1,\dots,x_n) = \hat{C}(\hat{F}_1,\dots,\hat{F}_n). \tag{18}$$

$\hat{H}$ exhibits marginals and a dependence structure that change dynamically every day.

Figures (5)-(7) (Appendix) illustrate graphically the procedure of the multivariate option iPoD framework by showing exemplarily the RNDs, the time series of Spearman correlations and the resulting MAD for the firm's BSC and LEH on March 10, 2008.

5 Results

In this section, we present the results from the empirical application of our framework to the sample of 13 US major FIs. The informational content of our framework is captured by five different risk measures (”systemic distress indicators”) obtained from the time series of MADs. We analyse the risk measures statically on March 10, 2008 (five days before the collapse of BSC) and on September 10, 2008 (five days before the failure/default of LEH) and dynamically from January 1, 2007 to September 10, 2008.

[14]The method is available in the statistical software **R** using the ”cuhre” algorithm from the package R2Cuba.

5.1 Systemic Distress Indicators

Based on our estimation framework described in section 2, we suggest five macro-prudential stability measures in order to evaluate the financial system's resilience. The first indicator consists in bivariate conditional PoDs (CPoDs) which measure the PoD of a particular FI A given that another FI B in the system defaults. The CPoDs of all respective institutions under consideration are summarized in a default distress dependence matrix (D-DDM). The CPoDs are formalized as follows:

$$CPoD(A|B) = P(V_A \leq D_A | V_B \leq D_B) = \frac{P(V_A \leq D_A \cap V_B \leq D_B)}{P(V_B \leq D_B)}. \tag{19}$$

On the same rationale, we calculate the bivariate conditional quantile risk (CQR), which gives us the probability of an FI's (A) asset value falling below the unconditional 25% quantile ($Q_{25\%;A}$) given that another FI B defaults.[15] Compared to the event of actual default by FI A as measured by CPoDs, this broader distress measure indicates more sensitively an increase in banks' conditional distress and also captures events such as downgrades or large losses in market value. Again, the CQRs for a given sample of FIs are summarized in a quantile distress dependence matrix (Q-DDM) and are defined by:

$$CQR(A|B) = \frac{P(V_A \leq Q_{25\%;A} | V_B \leq D_B) - 0.25}{0.75}. \tag{20}$$

We subtract 0.25 to adjust the conditional measure for the 25% unconditional probability of the asset value falling below its 25% quantile, which would result in the case of independence. We further normalize the CQR indicator such that its values are for positive dependence structures $\in [0, 1]$.

Third, we estimate the probability that at least one other firm in the sample will default given a specific firm A defaults (PAO). This risk indicator measures the impact of a specific FI's default on the resilience of the other institutions in the system. It can be regarded as a

[15] Note that in our estimation approach we can specify any continuous quantile.

measure of the systemic relevance of a particular FI.[16]

$$PAO = P(\text{at least one other FI defaults}|V_A \leq D_A). \quad (21)$$

We refer to the average over the PAOs of all FIs in the sample, as the Financial Interconnectedness Index (FII), which can be regarded as a proxy for the degree of system-wide interlinkage.

Fourth, we evaluate the vulnerability of a FI A to systemic default events (D-VSE), i.e. the sensitivity of a certain FI (measured in terms of CPoD) given that at least one FI in the system defaults. This measure is given by the following equation:[17]

$$D-VSE = P(V_A \leq D_A|\text{at least one other FI defaults}). \quad (22)$$

Our fifth risk measure considers the vulnerability of a specific firm A to systemic default events in terms of conditional quantile risk (Q-VSE). The quantile-based definition of the VSE provides a broader definition of distress than its CPoD-based counterpart. Its normalized version is given by:

$$Q-VSE = \frac{P(V_A \leq Q_{25\%;A}|\text{at least one other FI defaults}) - 0.25}{0.75}. \quad (23)$$

As a proxy for the system-wide level of distress, we define the Financial Vulnerability Index (FVI), more precisely the D-FVI and Q-FVI, which are calculated as the averages of the D-VSEs and Q-VSEs over all FIs in the sample, respectively.

While the D-DDM and the PAO are risk measures that were suggested in Segoviano and

[16]Example of PAO calculation in the case of three FIs A,B, C:

$$PAO(A) = P(V_B \leq D_B \cup V_C \leq D_C|V_A \leq D_A).$$

[17]Example of D-VSE calculation in the case of three FIs A, B, C:

$$D\text{-}VSE(A) = P(V_A \leq D_A|V_B \leq D_B \cup V_C \leq D_C).$$

Goodhart (2009), the remaining distress measures are, to the best of our knowledge, a novel addition to the literature.

5.2 Static Analysis

Table 1 shows the bivariate CPoDs for all FIs in our sample five days before the bankruptcy of Lehman Brothers (September 15, 2008), summarized by the default distress dependence matrix (D-DDM). Given the default of an FI in the column, the D-DDM provides the probability that an FI in the row will default between now and the time to maturity of the underlying options (theoretically seven months). Looking at the table from a row perspective, we learn how sensitive a specific institution is to defaults of the other FIs. From the column perspective, we learn about the impact of a specific FI's default on the remaining institutions. Hence, the row averages presented in the last column of the D-DDM can be regarded as a proxy for the (average) vulnerability of a particular FI to distress in one of the other FIs, and the column averages, presented in the last row of the D-DDM, as a proxy for the (average) systemic impact/importance of a specific FI's default on the other institutions.

Starting with the row averages, we see that the group of bankrupt FIs (see section 3), LEH and WM, exhibits by far the highest sensitivities to distress in other institutions (65.10% and 81.27%), while the institutions which weathered the financial crisis comparably well (surviving FIs: GS, WFC, BAC, JPM, MS) have by far the lowest sensitivities, with values ranging from 0.66% for GS to 4.28% for MS. The group of acquired/rescued institutions exhibits vulnerabilities that are much lower than for the bankrupt FIs, but much higher than for the surviving FIs (from 8.13% for C to 28.97% for WB). From the column averages, one can see that the systemic impact of the bankrupt FIs is the lowest among all institutions in the sample, while the acquired/rescued FIs are identified as the institutions with the highest systemic importance, with C as the most important. Taking into account only the "distressed" FIs (bankrupt/acquired/rescued) we find an inverse relationship between systemic importance and the level of vulnerability of the institutions (see highlights in the table). This might be explained by implicit government guarantees given to the systemically important FIs and anticipated by the investors in the option market. Finally, in line with the results above, one finds in the D-DDM that the "distressed" FIs were the most sensitive to LEH's default.

	GS	WFC	C	BAC	JPM	AIG	MS	LEH	WB	MER	WM	Row Ø
GS	100	0.63	0.70	0.74	0.60	0.66	0.71	0.64	0.68	0.70	0.53	0.66
WFC	2.59	100	2.83	2.73	2.18	2.62	2.75	2.30	2.75	2.80	2.06	2.56
C	7.73	5.47	100	8.60	5.35	10.03	9.98	6.43	10.79	10.86	6.08	8.13
BAC	2.70	2.00	3.26	100	1.92	3.11	3.01	2.37	3.09	3.24	1.89	2.66
JPM	1.16	1.23	1.56	1.47	100	1.51	1.58	1.36	1.52	1.58	1.15	1.41
AIG	33.66	17.11	33.65	27.60	17.55	100	28.57	22.30	30.61	31.71	18.35	26.11
MS	4.29	3.07	5.77	4.59	3.14	4.91	100	3.51	5.08	5.31	3.10	4.28
LEH	54.37	51.24	72.62	70.84	53.89	74.50	68.93	100	73.71	78.24	52.62	65.10
WB	22.82	20.58	41.01	31.37	20.28	35.03	33.73	25.22	100	37.94	21.67	28.97
MER	23.59	11.51	23.01	18.17	11.54	20.03	19.50	14.89	21.02	100	11.66	17.49
WM	56.43	67.66	96.04	83.79	66.82	90.09	89.00	77.48	92.99	92.43	100	81.27
Col. Ø	20.93	18.05	28.05	24.99	18.33	24.25	25.78	15.65	24.22	26.48	11.91	21.69

Table 1: Default distress dependence matrix (D-DDM) on September 10, 2008. CPoDs of banks in rows given default of banks in columns (in %).

In the appendix, we find in Table 9 the quantile distress dependence matrix (Q-DDM), which depicts the bivariate CQRs for the banks in our sample on September 10, 2008; in Tables 10 and 11 we find the D-DDM and Q-DDM five days prior to BSC's acquisition by JPM on March 15, 2008. The interpretation of the values of the Q-DDMs are as follows: Given the default of the bank in the column, the Q-DDM provides the probability of the row bank's asset value falling below its unconditional 25% quantile between now and the time to maturity of the underlying option. We will not discuss the results of Tables 9-11 in detail as the upcoming results for the PAO and VSE measures provide qualitatively the same insights that can be obtained from these tables. Compared to the column and row averages of the DDMs, the PAOs and VSEs are more precise measures of the systemic importance and financial vulnerability of a FI as they take into account all possible systemic events including cascade effects.[18].

We start by looking at the PAO estimates five days prior to the BSC (Table 2) and the LEH "event" (Table 3).

	GS	WFC	C	BAC	JPM	AIG	MS	LEH	BSC	WB	MER	CFC	WM
PAO:	43.12	50.31	78.47	59.78	50.90	65.45	75.43	56.82	75.29	75.09	74.64	54.12	66.02

Table 2: Probability that at least one other firm in the sample defaults given a specific firm defaults (PAO) at March 10, 2008 in percent (average 63.50).

	GS	WFC	C	BAC	JPM	AIG	MS	LEH	WB	MER	WM
PAO:	81.12	77.19	98.60	92.16	77.73	96.78	94.55	81.53	97.82	98.12	65.69

Table 3: Probability that at least one other firm in the sample defaults given a specific firm defaults (PAO) at September 10, 2008 in percent (average 87.39).

Most strikingly, the PAO levels of all FIs except for WM rose sharply from March 10, 2008 to September 10, 2008 (average ≈ 24 percentage points). This indicates the growing tension and interconnectedness in the US financial sector closer to LEH's bankruptcy. Whereas BSC had an above-average systemic importance five days prior to its acquisition, LEH's systemic importance five days before its collapse is below average. However, the fact that the PAO of LEH on September 10, 2008 is higher than for any FI on March 10, 2008, is a first indication of the extreme interconnectedness prevailing in the sector prior to the failure of LEH. In line with the results of the D-DDM (Table 1), the PAOs identify the rescued/acquired FIs on September 10, 2008 as systemically most important institutions while the bankrupt FIs exhibit PAOs below average. Interestingly, the group of acquired/rescued institutions, including BSC, already show above average PAO values on March 10, 2008.

Tables 4 to 7 depict the D-VSEs and Q-VSEs for March 10, 2008 and September 10, 2008 respectively.

[18]The PAO corresponds to the sums of the respective columns of the DDM corrected for "overlapping" events (e.g. two FIs default at the same time). The VSE corresponds to the sum over the respective rows of the DDM corrected for "overlapping" events in the conditioning set.

	GS	WFC	C	BAC	JPM	AIG	MS	LEH	BSC	WB	MER	CFC	WM
D-VSE:	0.40	2.61	10.04	1.24	2.82	2.95	3.50	5.77	7.27	5.06	4.86	53.55	37.05

Table 4: Vulnerability of a firm to systemic default events (D-VSE) at March 10, 2008 in percent (average 10.55).

	GS	WFC	C	BAC	JPM	AIG	MS	LEH	WB	MER	WM
D-VSE:	0.51	1.97	5.27	1.74	1.09	17.15	2.63	52.47	19.81	10.81	78.99

Table 5: Vulnerability of a firm to systemic default events (D-VSE) at September 10, 2008 in percentages (17.50).

We first take a look at the D-VSE tables. Equivalently to the results in Table 1, on September 10, 2008 the bankrupt FIs have by far the highest values, the "safe" institutions have by far the lowest, and the rescued/acquired FIs have values of medium size. Comparing D-VSEs and PAOs for the "distressed" FIs, we find again an inverse relationship between systemic importance and (assessed) financial soundness/vulnerability. The D-VSEs increase in average from March 10, 2008 to September 10, 2008, which shows the decreasing financial soundness in the financial sector in the course of 2008. Already in March most of the "distressed" institutions, including BSC and especially CFC, exhibit larger values than the "safe" FIs. However, except for CFC, WM and C, the classification is less clear than in September. The relatively small D-VSE value for BSC five days prior to its acquisition might be explained by the relatively high systemic importance implied by its PAO.

	GS	WFC	C	BAC	JPM	AIG	MS	LEH	BSC	WB	MER	CFC	WM
Q-VSE:	3.08	7.24	26.54	11.72	7.72	17.59	24.67	18.58	23.77	27.69	27.71	30.47	27.12

Table 6: Vulnerability of a specific firm to systemic default events in terms of conditional quantile risk (Q-VSE) at March 10, 2008 in percent (average 19.53).

The picture becomes clearer if we look at the Q-VSEs in Tables 6 and 7. From a theoretical point of view we expect that the Q-VSEs do not discriminate between systemically important and less important FIs, as the likelihood of large losses in market value and downgrades

	GS	WFC	C	BAC	JPM	AIG	MS	LEH	WB	MER	WM
Q-VSE:	12.54	16.12	38.16	20.53	17.20	34.06	21.96	30.01	37.64	36.01	35.80

Table 7: Vulnerability of a specific firm to systemic default events in terms of conditional quantile risk (Q-VSE) at September 10, 2008 in percentages (average 27.28).

should not be affected by implicit government bailout guarantees. The obtained results confirm these expectations, as the Q-VSE for the "distressed" FIs are very similar to one another and hence converge compared to the D-VSEs, where the values between the bankrupt and acquired/rescued FIs differed largely.[19] Consequently, FIs with comparably low D-VSEs but high Q-VSEs are a clear indication of implicit bailout guarantees for these institutions. Following this reasoning, we find on September 10, 2008 bailout guarantees for C, AIG, WB and MER as they exhibit Q-VSEs similar to those of the "bankrupt" FIs, WM and LEH, but much lower D-VSEs. On March 10, 2008 we find bailout guarantees for C, MS, WB, MER and BSC[20] since their D-VSEs are much lower than those of the high-risk firms CFC and WM (according to D-VSE) but their Q-VSEs are similar.

Besides helping to identify bailout guarantees, the Q-VSEs further provide sharper and more timely discrimination between "safe" and "distressed" FIs than the D-VSEs. On September 10, 2008 all "distressed" FIs exhibit above-average Q-VSE values, while all 'safe' FIs have below average values. On March 10, 2008 all "distressed" institutions, including BSC, show above- or close-to-average values, while most "safe" FIs (except MS) show clear below-average Q-VSEs.

In summary, the main findings of the static systemic risk analysis are:

- The D-VSEs clearly discriminate between the three groups: "bankrupt", "acquired/rescued" and "surviving" FIs .
- The "acquired/rescued" FIs exhibit lower D-VSEs than the "bankrupt" FIs but higher D-VSEs than the "surviving" FIs.

[19]This convergence can also be found for the row averages of the Q-DDM in Table 9 (Appendix).

[20]From Table 11 (Appendix) one may infer that all of the "distressed" FIs were highly sensitive to the default of BSC, which indicates that these FIs had similar exposures as BSC and/or were strongly linked to this firm.

- The ”acquired/rescued” FIs are found to be more systemically important (as measured by the PAOs) than the ”bankrupt” FIs.

- A comparison of D-VSEs and Q-VSEs gives clear indications for implicit bailout guarantees conceded to the ”acquired/rescued” FIs, since the indicated risk levels for the ”bankrupt” and for the ”rescued/acquired” firms converge in the case of Q-VSEs.

- The Q-VSEs are able to identify the ”distressed” FIs more clearly at an early stage.

5.3 Dynamic Systemic Risk Analysis

For the dynamic analysis of our risk indicators we estimate time series for the PAOs, D-VSEs and Q-VSEs from January 1, 2007 to September 10, 2008. Due to the problems with high-dimensional integration (see section 4) we restrict our sample to the ”distressed” FIs plus JPMm representing the ”surviving” FIs.

Figure 1.(a) shows, as examples the time series of PAOs (in percent) for C, LEH and WM. All PAOs start to increase sharply on July 30, 2007 and rise relatively continuously until September 10, 2008. There are two periods of comparative recovery, the first from January 11, 2008, when BAC announced the acquisition of CFC, to the end of February and the second from March 15, 2008, when JPM announced the acquisition of BSC, until the end of April. Interestingly, WM has relatively high systemic importance until January 2008. This is due to its high connectedness with CFC (see also Table 10 (Appendix)), but after the announced acquisition of CFC by BAC on January 11, 2008, WM‘s impact decreases and in June 2008 with the looming completion of the takeover of CFC on July 1, 2008 even becomes the least important bank in the sample. In contrast, C is regarded as the most important FI throughout the whole of 2008, with LEH and WM not even coming close.

However, as already suspected in the static analysis, the historically extreme PAO levels, observable since at least November 2007, even for the less important FIs (WM and LEH) give clear indications that the collapse of any bank in our sample will have an immense impact on the stability of the financial sector. This impression is strongly backed by Figure 1.(b), which shows the evolution of the Financial Interconnectedness Index (FII) over time.

The FII is calculated as the average over the PAOs of all FIs in the sample (see Figure 8 (Appendix) for the PAO time series of all considered FIs) and can be seen as a proxy for the degree of interconnectedness prevailing in the financial sector. We see that the degree of interconnectedness in the sector reaches historically unprecedented levels starting in November 2007, and especially since June 2008. This strongly suggests a "too connected too fail" problem prevailing in the sector also around the time of LEH's collapse in September 2008, which implies that the FED's decision to let LEH go bankrupt might have been correct from a cross-dimensional but not from a time-dimensional perspective.

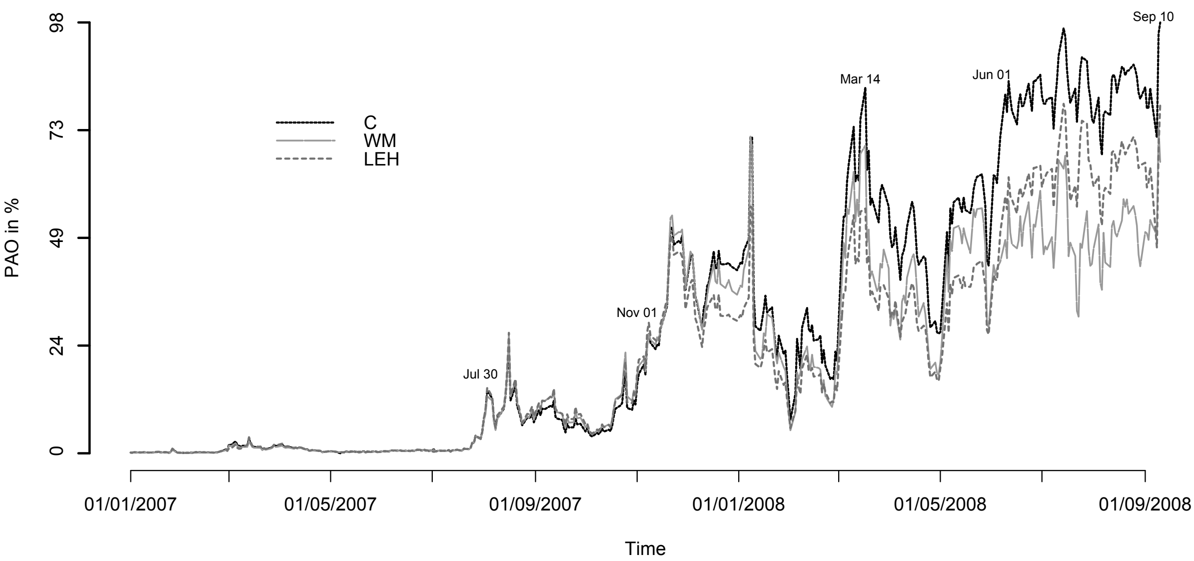

Figure 1.(a): Probability that at least one other firm in the sample defaults given a specific firm defaults (PAO) of C, LEH and WM from January 2007 to September 10, 2008 (in %).

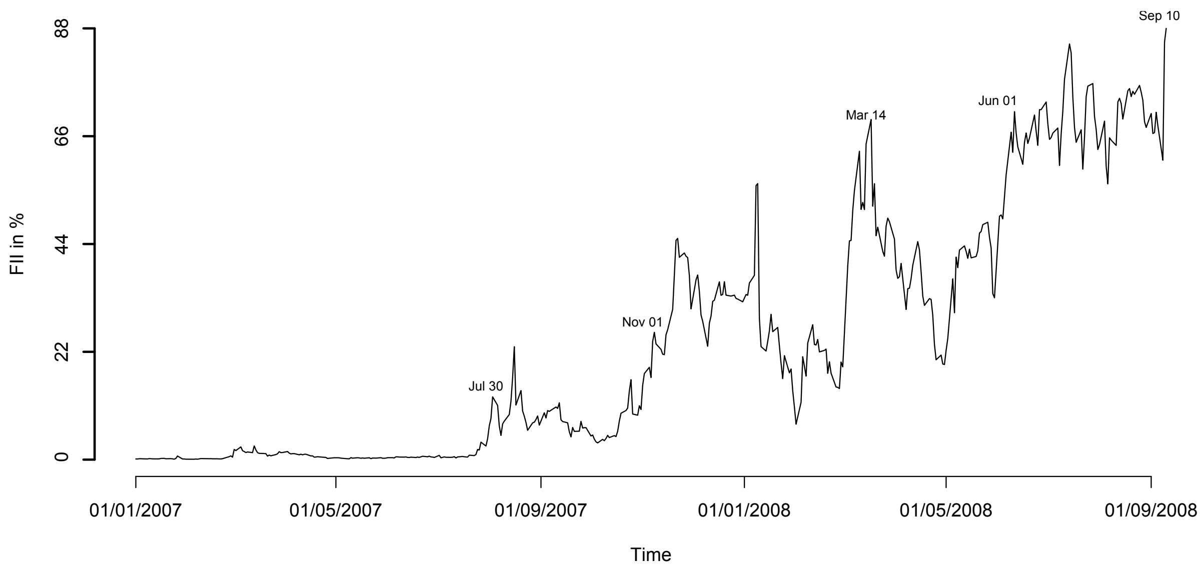

Figure 1.(b): Financial Interconnectedness Index (FII) from January 2007 to September 10, 2008 (in %).

Figure 2.(a) shows the time series of D-VSEs for C, BSC, LEH and JPM from January 2007 to September 10, 2008. The D-VSEs start to increase at the end of July 2007, and in November 2007 we see the beginning divergence between the time series of the "distressed" FIs, C, BSC and LEH, and the time series of JPM (as representative of the "safe" FIs). This divergence reaches extreme levels in the weeks prior to BSC's collapse in mid-March 2008 and remains high in the post-BSC period as well. In May 2008 the time series of LEH and C also begin to diverge strongly, a clear indication that a default is much more likely for LEH than for C, which might be explained by a possible bailout guarantee given to C.

In Figure 2.(b) we see the timely evolution of the average value of the D-VSEs for all FIs in our sample (see Figure 9 (Appendix) for the VSE time series of all considered FIs), which we refer to as the Default Financial Vulnerability Index (D-FVI). The increasing financial distress in the sector shows up as early as July 30, 2008 and rises similarly to the PAOs above continuously until September10, 2008.

Figure 3.(a) depicts the Q-VSE time series of C, WM, JPM, WB and BSC. Here we see a more clear divergence between the "safe" bank JPM and the "distressed" FIs than for the

D-VSEs. The divergence begins in October 2007 and gets more and more extreme over the remainder of the period. As already suspected in the static analysis, we see that the values of the "distressed" FIs move very closely to each other over time. Hence, already in the last months of 2007 the group of FIs which faced the most severe problems during the sub-prime crisis can be identified clearly on basis of the Q-VSE measure. This is also true for BSC, whose values rise sharply from November 2007 until its collapse in March 2008. When comparing the time series of Q-VSEs and D-VSEs for the individual banks, we find strong indications that the investors at the option markets anticipated the implicit bailout guarantees for C and BSC since their D-VSE levels did not rise to such extreme levels as seen for LEH, CFC and WM[21] (see Figure 9 (Appendix) for the D-VSE time series of the latter two FIs), but their distress levels as measured by the Q-VSEs were very similar (see Figure 10 in the Appendix for Q-VSE time series of CFC and WM). Using the time series in the Appendix (Figure 9 and Figure 10), we further find bailout guarantees for AIG, WB and MER.

Finally, Figure 3.(b) shows the Quantile Financial Vulnerability Index (Q-FVI), calculated as the average of the Q-VSE of all considered FIs. At an early stage, the Q-FVI shows more distinctly the high degree of distress in the financial sector than the D-FVI, because it rises in a more continuous manner and in a concave rather than in a convex way.

[21]While CFC finally was acquired by BAC on July 1, 2008 and did not go bankrupt like LEH and WM, its high D-VSE levels indicate that CFC did not have an implicit bailout guarantee.

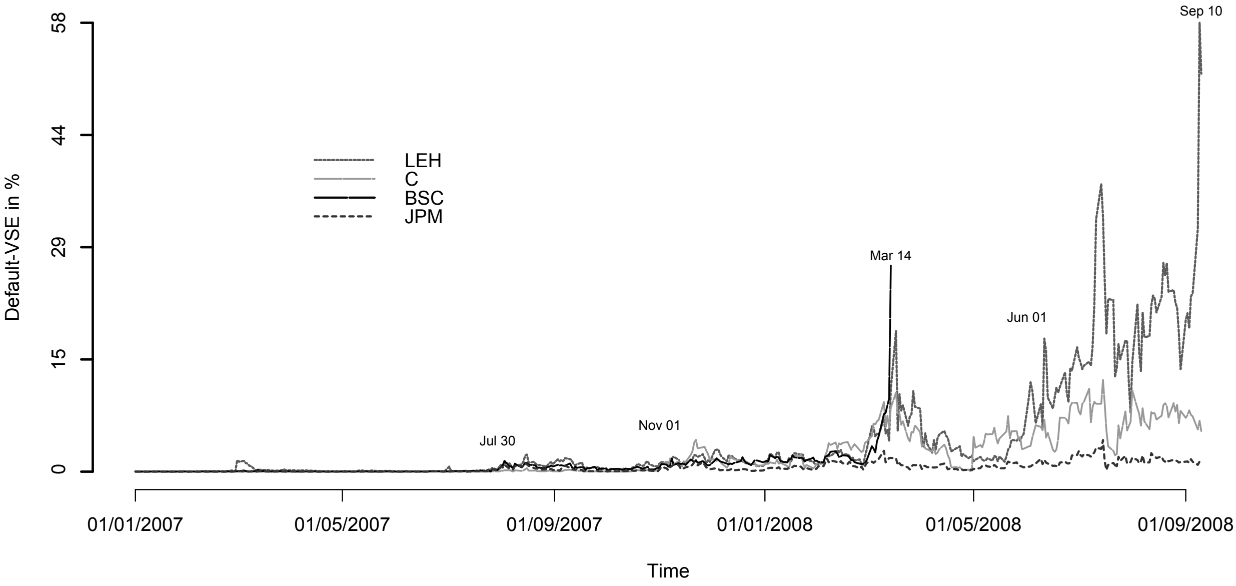

Figure 2.(a): Vulnerability of a firm to systemic default events (D-VSE) of C, JPM, LEH and BSC from January 2007 to September 10, 2008 (in %).

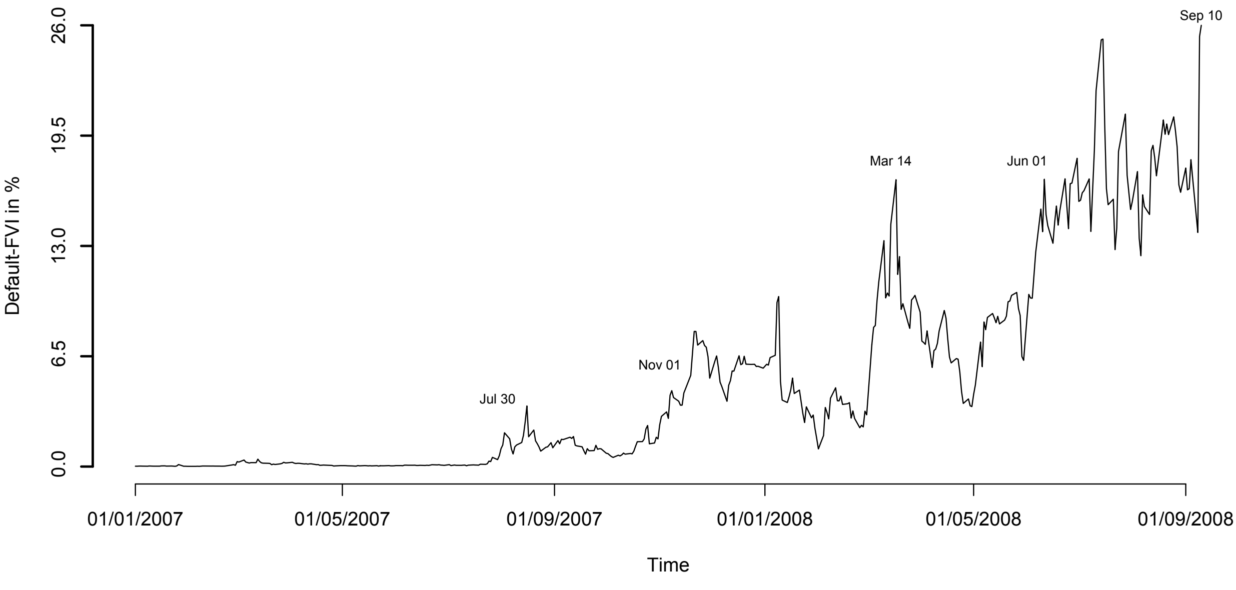

Figure 2.(b): Default Financial Vulnerability Index (D-FVI) of C, AIG, LEH and JPM from January 2007 to September 10, 2008 (in %).

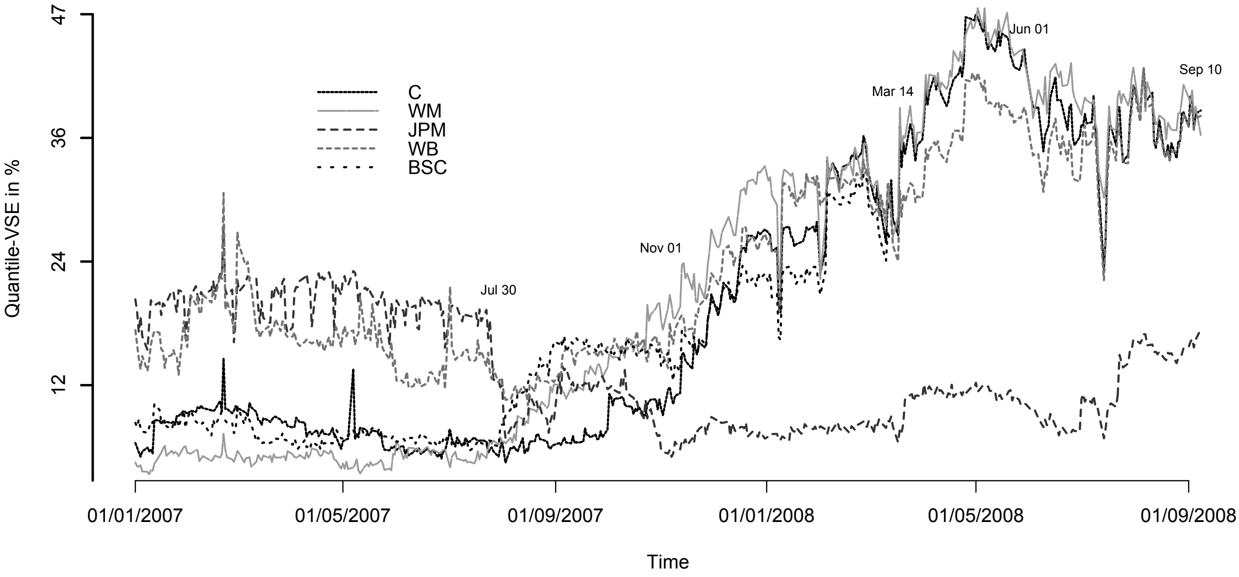

Figure 3.(a): Vulnerability of a specific firm to systemic default events in terms of conditional quantile risk (Q-VSE) of C, AIG, LEH and JPM from January 2007 to September 10, 2008 (in %).

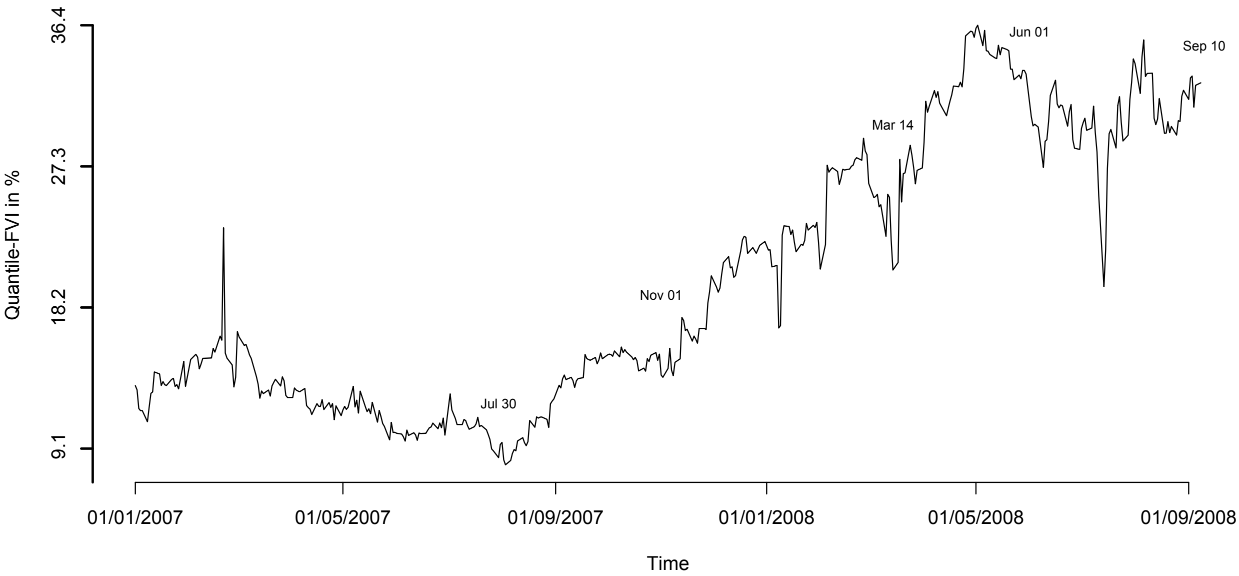

Figure 3.(b): Quantile Financial Vulnerability Index (Q-FVI) of C, AIG, LEH and JPM from January 2007 to September 10, 2008 (in %).

The main insights from the dynamic risk analysis can be summarized as follows:

- The PAO levels indicate a ”too connected too fail” dilemma since November 2007, since even the least systemically important FIs exhibited historically extreme values.
- All indicators (PAOs, D-VSEs, Q-VSEs) begin to sharply increase in July 2007.
- The discrimination/identification between ”distressed” and ”safe” FIs happens as early as November 2007 (for CFC even in July 2007) and is highly stable over time.
- At an early stage of distress the Q-VSEs identify more clearly the ”distressed” FIs than the D-VSEs.
- The Q-VSEs of all ”distressed” FIs exhibit in a stable manner similar distress levels and hence, by comparing the displayed riskiness of D-VSE and Q-VSE for the individual FIs, we obtain timely indications of implicit bailout guarantees.
- The derived indices FII, D-FVI and Q-FVI are able to display the increasing degree of distress and interconnectedness in the financial sector over the period of July 2007 to September 2008. Here, the Q-FVI indicates more distinctly at an early stage the extreme degree of distress as it increases more continuously than the other indices and in a concave rather than in a convex way.

6 Conlusion

In this paper we have proposed a novel framework for assessing systemic risk in financial sectors. Our framework uses daily option prices to estimate time series of multivariate (risk-neutral) asset distributions (MADs) for a sample of financial institutions (FIs). The MADs are obtained by combining the (univariate) option iPoD approach of Capuano (2008) and Vilsmeier (2011) with the most entropic copula methodology of Chu (2011). While the option iPoD procedure provides us with time series of the individual FIs' asset distributions (RNDs) and their probabilities of default (PoDs), the MEC methodology allows us to estimate a copula for the multivariate asset distribution on the basis of Spearman rank correlations that we calculate from the RND time series. Using Sklar's theorem, we combine the RND time series and the copula to obtain the MADs. For the estimation of the RNDs neither balance sheet data nor recovery rate assumptions are required, as the option iPoD framework derives the individual bank's asset distributions in a purely statistical way using only daily sets of equity option prices. The Spearman rank correlations are estimated dynamically with exponentially decreasing influence of past observations such that the dependence structure of the MAD described by the copula may change every day. The dynamic rank correlations measure linear as well as non-linear dependence structures and capture the increasing correlations during times of economic downturns that are usually detected.

Both the RNDs and the copula are estimated using a semi-parametric estimation procedure based on the entropy function. The framework provides smooth density estimates that are very flexible with regard to their functional forms and MADs whose marginals (RNDs) and dependence structure are updated on a daily basis.

Time series of MADs were estimated for 13 major US FIs during the period of the US sub-prime crisis from January 2007 to September 2008. On the basis of these MADs, we derived five different systemic risk indicators which are based on conditional PoDs and conditional lower quantiles. The indicators were analyzed statically on March 10, 2008 and September 10, 2008 as well as dynamically over the whole sample period and provide strong evidence for the high informational content resulting from our estimation approach. The static analysis showed that the derived indicators are able: i) to clearly discriminate between the institutes that had the most severe problems during the financial crisis ("distressed" FIs) and those who weathered the turmoil comparably well ("safe" FIs), ii) to distinctly classify the "distressed" FIs into the group of institutes that were acquired or rescued during the crisis and those banks that went bankrupt, iii) to show that the acquired/rescued FIs were assessed as more systemically important than the bankrupt FIs and iv) to provide strong evidence that implicit bailout guarantees were granted to the systemically most important institutions. From the dynamic analysis we obtain the following insights: i) all indicators begin to increase sharply in July 2007, ii) the discrimination between "distressed" and "safe" institutions happens as early as November 2007 (in case of Countrywide Financial (CFC) even in July 2007) and is highly stable over time, iii) the implicit bailout guarantees given to some FIs can be identified at an very early stage (differing among institutes) and iv) the degree of dependence among the FIs indicates a severe "too connected too fail" dilemma in the US financial sector since (at least) November 2007.

Future research should intend to minimize the dependence of the estimated correlation coefficients on outdated historical data. A major improvement might be obtained by using intra-day data and realized correlations theory as, for instance, in Huang, Zhou, and Zhu (2009). In addition, more efficient ways to solve the high-dimensional integrals in the MEC estimation should be found. A valuable step in this direction might be to use the connection between the MEC and the FRAME model (a type of Markov Random Field model) of Zhu, Wu, and Mumford (1998) as suggested in Huang and Freedman (2010).

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Appendix

Option Price	Strike Price	Open Interest	Liquidity Weight
48.30	0.00	1	1.00
16.05	32.50	353	0.01
13.60	35.00	27	0.00
11.20	37.50	375	0.01
8.90	40.00	265	0.01
6.80	42.50	248	0.01
4.80	45.00	2076	0.05
3.10	47.50	16430	0.38
1.80	50.00	7525	0.17
0.90	52.50	3781	0.04
0.40	55.00	10758	0.09
0.10	60.00	40	0.25

Table 8: Dataset of JPM stock options on January 1, 2007 (Estimated PoD: 2.7×10^{-6}).

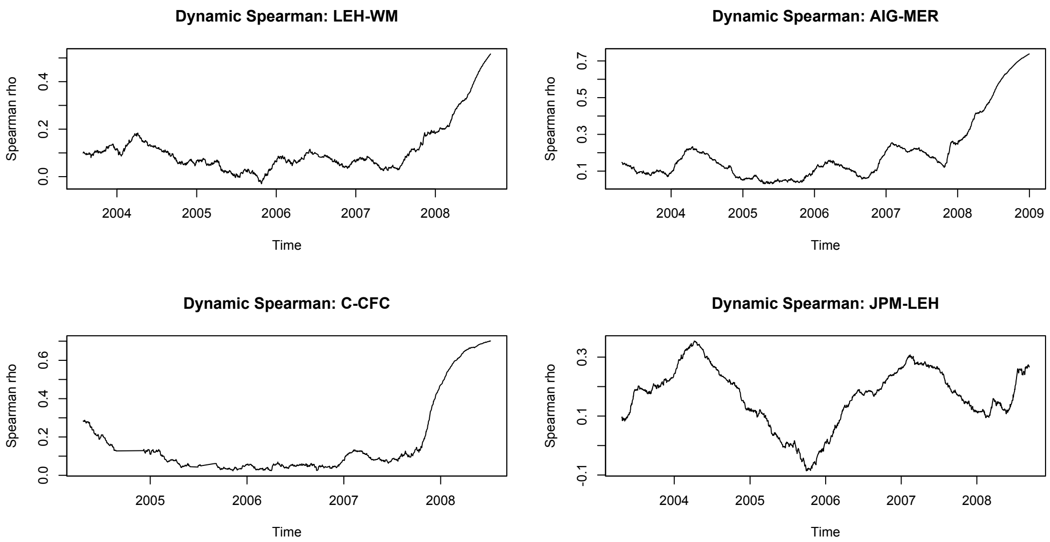

Figure 4: Examples of dynamic Spearman correlations.

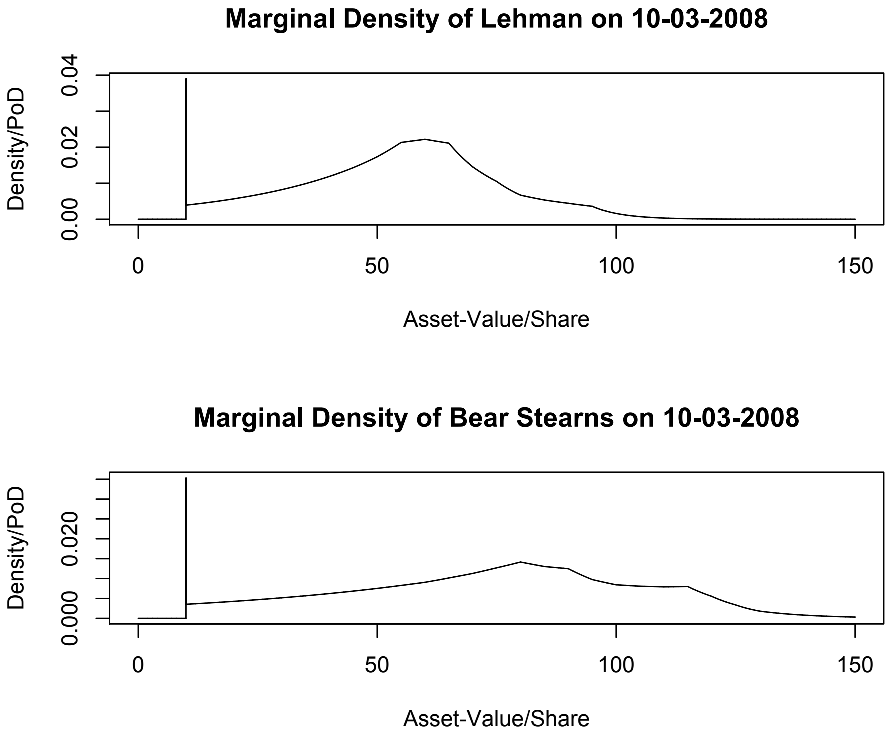

Figure 5: Marginal density functions of Lehman Brothers and Bear Stearns on March 10, 2008.

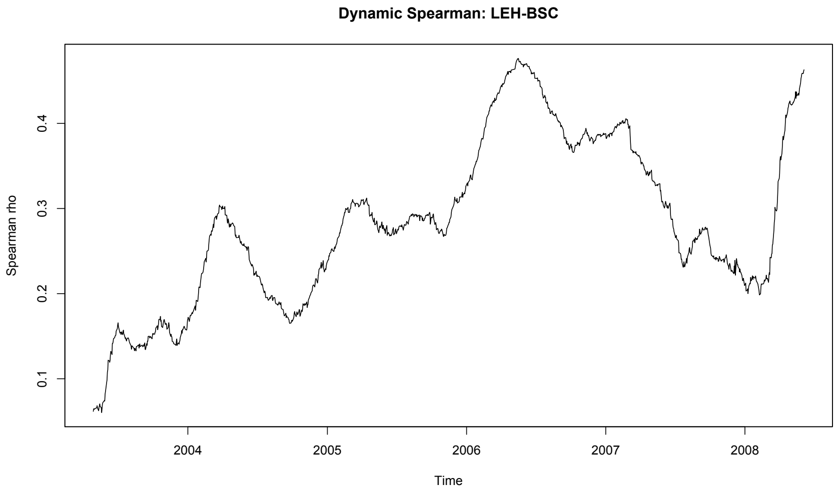

Figure 6: Dynamic Spearman rank correlations ($\alpha = 0.99$) between Lehman Brothers and Bear Stearns from January 1, 2003 to September 10, 2008.

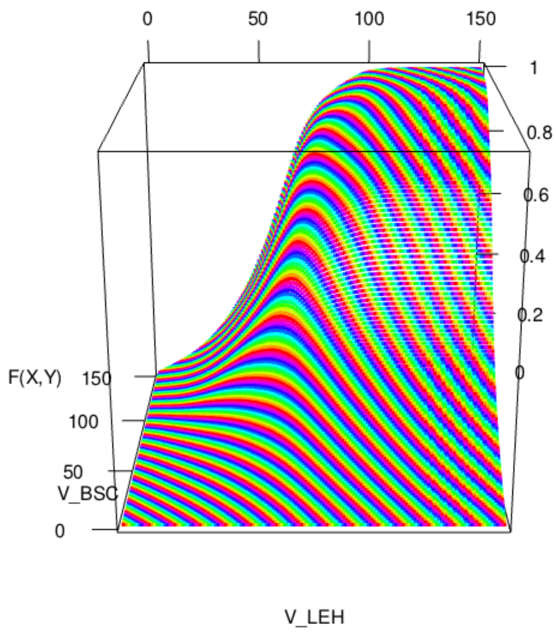

Figure 7: Bivariate cumulative density function of Lehman Brothers and Bear Stearns on March 10, 2008.

	GS	WFC	C	BAC	JPM	AIG	MS	LEH	WB	MER	WM	Row Ø
GS	100	8.83	12.00	13.49	7.96	10.28	12.18	8.36	11.01	11.84	5.02	10.10
WFC	9.01	100	14.36	13.27	7.17	12.14	13.49	8.82	13.60	14.03	6.10	11.20
C	12.58	14.58	100	30.78	13.85	38.49	37.07	21.33	41.59	41.49	20.24	27.20
BAC	12.97	13.36	30.37	100	12.17	29.01	27.17	20.05	28.85	30.42	12.94	21.73
JPM	8.31	6.98	13.39	11.85	100	12.63	13.88	10.05	12.91	13.84	5.61	10.95
AIG	13.21	13.41	41.66	31.93	14.23	100	33.53	24.20	37.67	39.06	17.04	26.59
MS	12.32	13.64	36.76	27.31	14.29	30.65	100	18.98	32.16	33.58	15.56	23.52
LEH	11.97	13.53	33.13	31.44	15.86	34.72	29.59	100	33.88	38.67	14.07	25.69
WB	12.72	15.23	45.56	32.23	14.73	38.26	35.70	23.96	100	41.93	18.58	27.89
MER	14.08	14.51	42.60	31.65	14.59	37.01	34.77	25.40	39.19	100	16.95	27.07
WM	9.15	11.93	45.80	27.54	11.22	34.50	33.83	19.21	38.43	37.92	100	26.95
Col. Ø	11.63	12.60	31.56	25.15	12.61	27.77	27.12	18.04	28.93	30.28	13.21	21.72

Table 9: Quantile distress dependence matrix (Q-DDM) on September 10, 2008. CQRs of banks in rows given default of banks in columns (in %).

	GS	WFC	C	BAC	JPM	AIG	MS	LEH	BSC	WB	MER	CFC	WM	Row Ø
GS	100	0.46	0.45	0.45	0.46	0.47	0.49	0.46	0.46	0.44	0.48	0.39	0.39	0.45
WFC	2.56	100	3.15	2.97	2.45	2.67	3.17	2.34	3.03	2.94	3.07	2.63	2.69	2.81
C	5.94	7.47	100	9.96	7.19	11.13	13.16	7.76	12.87	12.77	13.02	11.99	12.22	10.46
BAC	1.07	1.26	1.78	100	1.04	1.48	1.65	1.16	1.63	1.61	1.63	1.45	1.50	1.44
JPM	2.79	2.63	3.26	2.65	100	2.95	3.42	2.90	3.14	3.04	3.34	2.94	2.89	3.00
AIG	2.41	2.44	4.28	3.20	2.50	100	4.02	2.82	3.88	3.85	3.89	3.50	3.64	3.37
MS	2.47	2.86	5.01	3.51	2.87	3.98	100	3.09	4.61	4.68	4.67	4.26	4.18	3.85
LEH	5.12	4.65	6.51	5.44	5.36	6.16	6.81	100	6.83	6.40	6.54	6.43	6.16	6.03
BSC	4.62	5.45	9.78	6.93	5.26	7.66	9.21	6.18	100	9.04	8.91	8.62	8.49	7.51
WB	3.16	3.73	6.85	4.85	3.59	5.37	6.60	4.09	6.38	100	6.28	5.88	6.02	5.23
MER	3.40	3.85	6.88	4.82	3.89	5.34	6.48	4.12	6.20	6.19	100	5.83	5.86	5.24
CFC	28.15	33.81	63.43	43.87	35.10	49.08	59.66	41.51	60.33	58.45	58.79	100	60.07	49.35
WM	19.38	23.85	45.08	31.33	23.82	35.26	40.69	27.44	41.38	41.59	41.10	41.59	100	34.38
Col. Ø	6.76	7.70	13.04	10.00	7.79	10.96	12.95	8.66	12.56	12.58	12.64	7.96	9.51	10.24

Table 10: Default distress dependence matrix (D-DDM) on March 10, 2008. CPoDs of banks in rows given default of banks in columns (in %).

	GS	WFC	C	BAC	JPM	AIG	MS	LEH	BSC	WB	MER	CFC	WM	Row Ø
GS	100	7.56	6.90	7.17	7.90	8.51	9.72	7.69	7.60	6.60	9.39	3.03	2.96	7.09
WFC	7.84	100	15.00	12.85	6.44	9.23	15.21	5.00	13.53	12.44	14.07	8.84	9.55	10.83
C	7.16	15.25	100	27.63	13.81	33.11	42.13	16.80	41.04	40.52	41.59	39.16	39.35	29.80
BAC	7.19	12.51	26.48	100	6.36	18.77	23.04	9.67	22.59	22.20	22.56	18.29	19.48	17.43
JPM	7.93	6.16	13.31	6.33	100	9.77	15.08	9.23	11.96	10.84	14.20	9.81	9.21	10.32
AIG	9.16	9.59	32.79	19.60	10.41	100	29.64	14.77	27.99	27.64	28.00	24.27	25.68	21.64
MS	9.80	14.93	40.79	23.21	15.11	28.88	100	17.97	36.31	37.02	36.81	34.11	32.20	27.27
LEH	7.53	4.52	16.16	9.55	9.08	14.03	17.96	100	18.05	15.52	16.35	16.05	14.21	13.25
BSC	7.75	13.48	40.43	23.16	12.19	27.73	36.96	18.40	100	36.05	35.29	35.39	33.76	26.72
WB	6.99	12.60	39.95	22.92	11.28	27.50	37.73	16.05	36.11	100	35.23	33.56	34.11	26.17
MER	9.51	13.88	40.53	22.89	14.32	27.56	37.08	16.51	34.91	34.77	100	33.45	33.00	26.53
CFC	4.00	11.52	50.92	24.88	13.23	31.81	45.89	21.75	46.79	44.28	44.73	100	46.43	32.19
WM	3.43	11.09	45.65	23.60	11.05	30.04	38.72	17.17	39.85	40.15	39.39	41.20	100	28.44
Col. Ø	7.36	11.09	30.75	18.65	10.93	22.24	29.09	14.25	28.05	27.33	28.15	24.76	25.00	21.36

Table 11: Quantile distress dependence matrix (Q-DDM) on March 10, 2008. CQRs of banks in rows given default of banks in columns (in %).

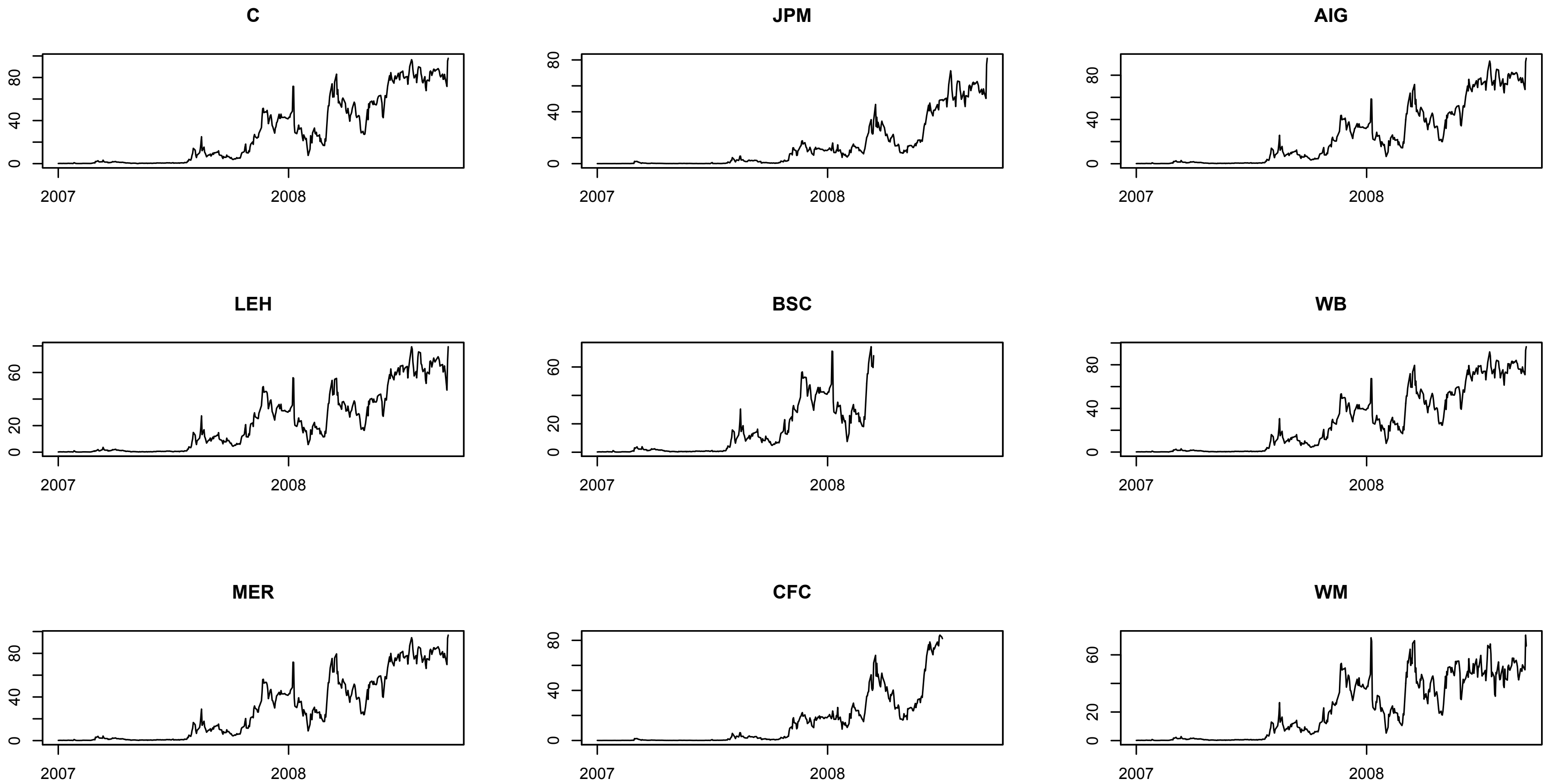

Figure 8: Probability that at least one other firm in the sample defaults given a specific firm defaults (PAO) of all FIs considered in the dynamic analysis from January 1, 2007 to September 10, 2008.

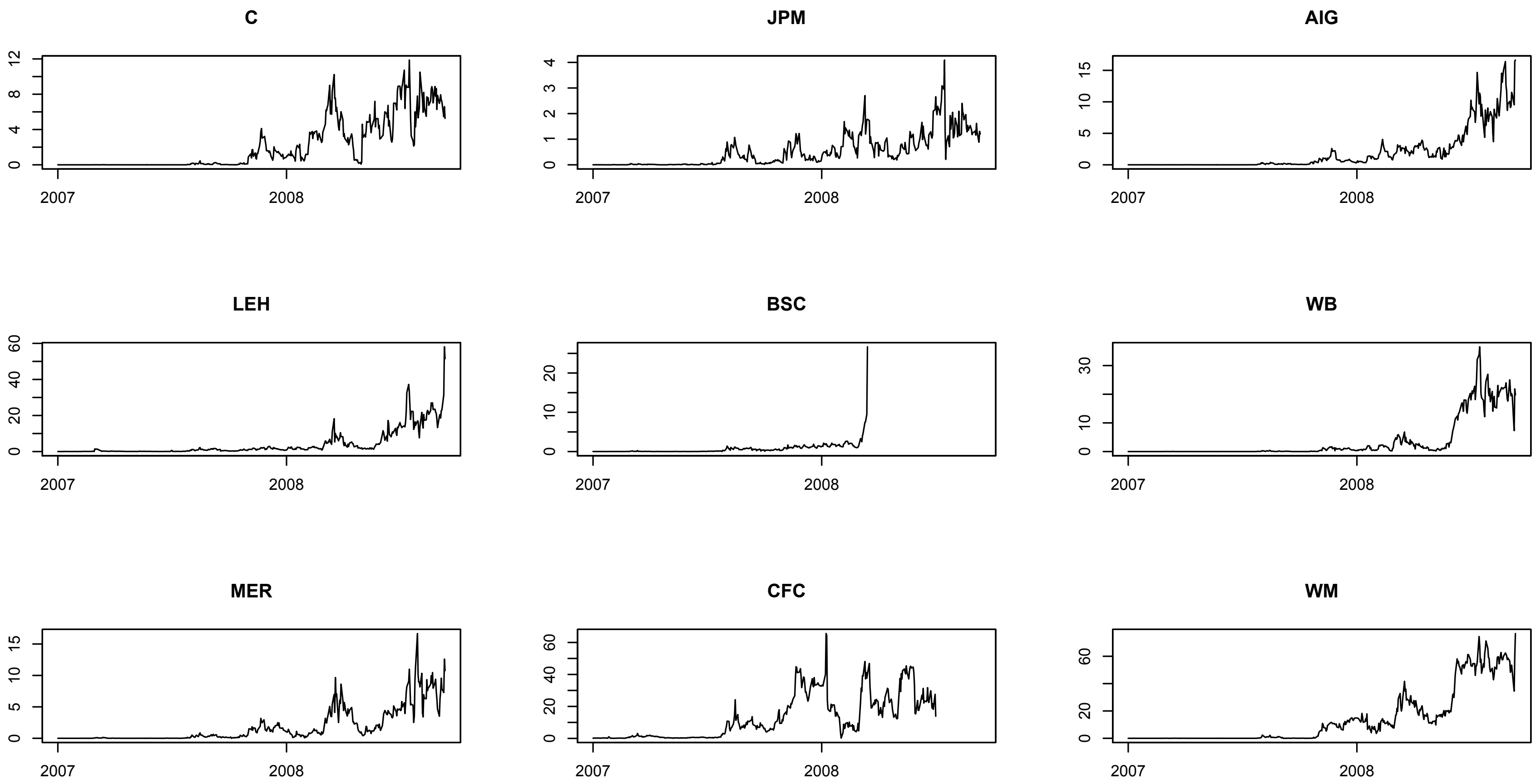

Figure 9: Vulnerability of a firm to systemic default events (D-VSE) of all FIs covered in the dynamic analysis from January 1, 2007 to September 10, 2008.

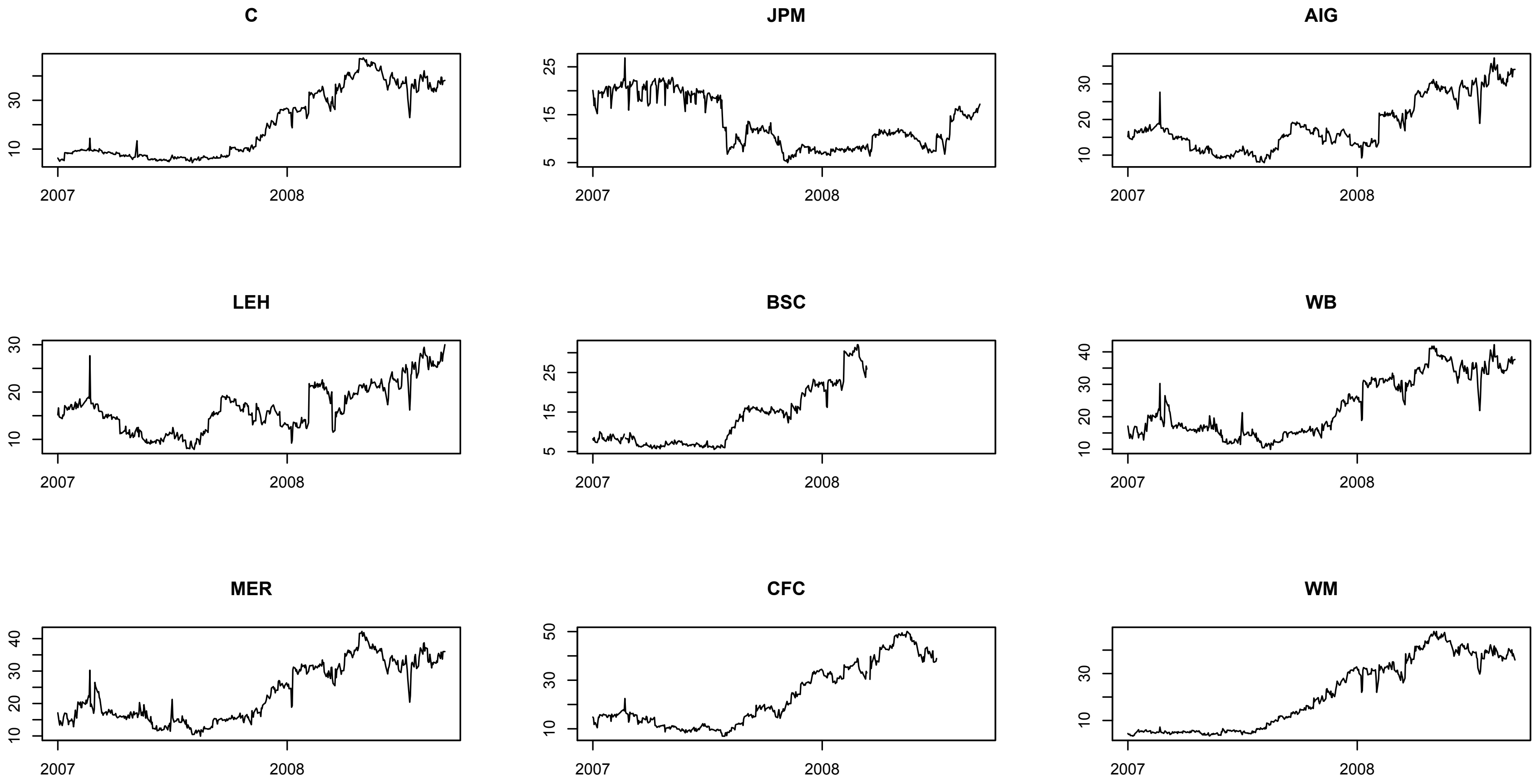

Figure 10: Vulnerability of a specific firm to systemic default events in terms of conditional quantile risk (Q-VSE) of all FIs covered in the dynamic analysis from January 1, 2007 to September 10, 2008.