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Investor fears and risk premia for rare events

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Non-technical summary

A risk premium is the compensation demanded by investors for holding a financial asset with risky payoffs that exceeds the risk-free rate. A recent strand of literature believes that the fear of large negative shocks is a component that drives asset prices because investors expect compensation for the risk that such a rare event occurs. This paper aims at estimating equity risk premia that are due to the compensation for rare events. Rare events, such as the collapse of Lehman in September 2008, trigger large price jumps and are seldomly observed in the data which makes it hard to estimate the distribution of such events. I use a newly developed method to extract price jumps from option data and high frequency futures price data of the S&P 500 for estimating the distribution of rare events and the equity risk premia for the US stock market. In addition, the method allows for constructing an investor fear index. I replicate the method expanding the data sample to include more recent years. Furthermore, I apply the method to German data using the DAX as the proxy for the German stock market.

The compensation for rare events accounts for a considerable part of the equity risk premia in both stock markets. The results are much higher than the results of similar analyses. The investor fear index works very well, as it spikes at all significant events that moved the stock markets. But the correlation of the fear index with commonly used volatility indices such as the VIX for the US market and the VDAX for the German market is about 90%. Moreover, in the financial crisis the fear index spikes only after the Lehman default, whereas indicators based on credit spreads increase sharply much earlier. Therefore, the fear index appears to be inappropriate as an early-warning tool but describes the prevalent situation in the stock markets well.

Nicht-technische Zusammenfassung

Eine Risikoprämie ist der Aufschlag gegenüber dem risikofreien Zinssatz, der von den Anlegern für das Halten eines finanziellen Vermögenswerts mit risikobehaftetem Kapitalrückfluss verlangt wird. In Teilen der aktuellen Fachliteratur wird die Auffassung vertreten, dass die Furcht vor ausgeprägt negativen Schocks eine Triebkraft der Vermögenspreisentwicklung ist, weil die Anleger einen finanziellen Ausgleich für das Risiko erwarten, dass ein derartig seltenes Ereignis tatsächlich eintritt. Im vorliegenden Diskussionspapier soll geschätzt werden, inwieweit die Aktienrisikoprämien auf diese Kompensation für seltene Ereignisse zurückgehen. Außergewöhnliche Ereignisse, wie etwa der Zusammenbruch von Lehman Brothers im September 2008, verursachen starke Preissprünge. Es gibt dazu nur wenige Beobachtungen in den Daten, was eine Schätzung der Verteilung solcher Ereignisse schwierig macht. Im vorliegenden Beitrag wird eine neu entwickelte Methode angewandt. Diese ermittelt Preissprünge aus Optionsdaten und Hochfrequenzdaten zu Preisen von Terminkontrakten auf den S&P 500, um die Verteilung seltener Ereignisse und die Aktienrisikoprämien für den US-Aktienmarkt zu schätzen. Außerdem läßt sich daraus ein Index der Anlegerangst konstruieren. Diese Vorgehensweise wird hier repliziert und der Untersuchungszeitraum um die letzten Jahre erweitert. Darüber hinaus wird die Methode auf deutsche Daten angewandt, wobei der DAX stellvertretend als Indikator für den deutschen Aktienmarkt herangezogen wird.

An beiden Aktienmärkten hat der Renditeaufschlag für seltene Ereignisse einen beträchtlichen Anteil an den Aktienrisikoprämien. Dieser Anteil fällt wesentlich höher aus als in den Ergebnissen ähnlicher Untersuchungen. Der Index der Anlegerangst erweist sich als sehr aussagekräftig, da er immer dann, wenn bedeutsame Ereignisse die Aktienmärkte bewegten, Spitzen aufweist. Allerdings liegt die Korrelation des Angstindex mit den gängigen Volatilitätsmessgrößen wie dem VIX für den US-amerikanischen oder dem VDAX für den deutschen Markt bei rund 90%. Zudem schlägt er in der Finanzkrise erst nach der Insolvenz von Lehman nach oben aus, während auf Kreditspreads basierende Indikatoren bereits deutlich früher einen kräftigen Anstieg verzeichnen. Folglich erscheint der Angstindex als Frühwarninstrument untauglich, zur Beschreibung des Geschehens an den Aktienmärkten eignet er sich indes gut.

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Abstract

This paper uses the method developed by Bollerslev and Todorov (2011b) to estimate risk premia for extreme events for the US and the German stock markets. The method extracts jump tail measures from high-frequency futures price data and from options data. In a second step, jump tail distributions are approximated using the extreme value theory. Applying the method to German data yields very similar results to the ones shown for the US data. The risk premia for rare events constitute a considerable part of the total equity and variance risk premia for both markets. When using the results to build an investor fear index for the US and Germany, I find that the correlation of the fear index for the US with the VIX is 89.5% and that of the fear index for Germany with the VDAX is 90.6%.

Keywords: Crisis indicator, extreme value theory, implied moments

JEL Classification: C13, G10, G12

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1 Introduction

Since the financial crisis evolved in August 2007 researchers and policymakers have concentrated on analyzing the sources of so-called systemic risk. Although no unique definition exists, most of the widely used definitions specify that systemic risk is the risk of a disruption in the financial system with the potential for serious negative consequences for the financial market and the real economy.[1] The aim of this strand of research is to identify the build-up of imbalances or vulnerabilities and estimate the probability of materialization as well as the potential impact on the financial system and the real economy. The build-up of imbalances is monitored employing various indicators, balance sheet data and macroeconomic variables. Stress tests or scenario analyses help to estimate the impact given a materialization of a systemic risk, but to date to estimate the probability of materialization no commonly accepted methodology has been found. Systemic events are triggered by large shocks that are seldomly observed. Thus, it is typically hard to estimate the distribution of these rare events. Nevertheless, it is important to understand the distribution of rare events, because it is crucial in forecasting or in finding adequate measures to mitigate systemic risks. It is not only the occurrence of rare events but the very fear of them that influences investors' behavior and market prices. The analyses of the perception of rare events are compounded by peso-type problems. This means that rare events may be perceived and priced by investors but may not materialize, which makes it even harder to extract them from the data.

The work of Bollerslev and Todorov (2011b) aims to estimate jump tail distributions as a proxy for the distribution of rare events that affect stock markets. More importantly, the authors try to shed light on the perception of rare events and the risk premia demanded. The risk premia considered are both the equity and the variance risk premia due to the compensation for rare events. They develop a new methodology for estimating model-free implied measures for expected jump tail distributions. In addition, building on their earlier work (Bollerslev and Todorov (2011a)) they apply a new extreme value theory method to circumvent peso type problems. The extreme value theory "seeks to assess the probability of events that are more extreme than any observed prior".[2] Therefore, medium-sized jumps from high-frequency data can be used to infer the distribution of tail events. The methods are applied to options and futures on the S&P 500. The risk premia for rare events reported are volatile and high. In addition, a fear index for investor fears is constructed, which spikes in times of crises but not prior to rare events such as the Lehman default.

In this study I replicate the method of Bollerslev and Todorov (2011b) extending the sample period to include the financial crisis and additionally applying it to German stock market data. As a proxy for the German stock market I use the DAX. I shed light on some properties of the estimation results not shown by Bollerslev and Todorov (2011b) and compare the results with other research conducted in the area of estimating risk

[1] The definition is taken from ESRB Regulation (2010) and FSB, IMF, and BIS (2009).
[2] Compare Rao (2013).

premia or indicators for crises, such as the Gilchrist-Zakrajsek spread.[3]

Since the seminal work of Mehra and Prescott (1985) the equity premium has been the focus of a great quantity of research papers. While some have tried to find theoretical models to better explain the quantities observed e.g. by adding stochastic volatility (Hull and White (1987) and Heston (1993)) or jumps (Merton (1976)) or both (Bakshi, Cao, and Chen (1997)), a vast strand of literature has concentrated on the estimation of risk premia.[4] Even before the recent financial crisis, several studies tried to explain the high observed risk premia by taking into account the premia for rare events. This idea was originated by Rietz (1988) and refined by Barro (2006) and Liu, Pan, and Wang (2005) among others.

The idea of estimating jump risk premia from S&P 500 index data or options has also been taken up by e.g. Pan (2002), Broadie, Chernov, and Johannes (2007), Todorov (2010) and Santa-Clara and Yan (2010). Broadie, Chernov, and Johannes (2009) analyze S&P 500 option portfolio returns and find that jump risk premia can explain parts of the returns. Extreme value theory, especially its application in finance, is extensively discussed in Embrechts, Klüppelberg, and Mikosch (2010). The new extreme value theory used and developed in Bollerslev and Todorov (2011a) is also applied by Bollerslev, Todorov, and Li (2013) who build on this method to estimate systematic and idiosyncratic jump tail risks in financial asset prices.

Related literature for the model-free tail measure estimates is Breeden and Litzenberger (1978), who derive prices of state-contingent claims from option prices in a model-free manner, and Britten-Jones and Neuberger (2000), Bakshi, Kapadia, and Madan (2003) as well as Carr and Wu (2009), who use model-free approaches to derive prices for higher-moment contracts from option prices.

This paper is organized as follows. Section 2 briefly summarizes the main thrust of the methods used to estimate the risk premia for rare events. In Section 3 the data and preliminary estimates are described and depicted. The risk premia due to compensation for large jumps are shown in Section 4, where I also compare the results with the results of other analyses. Section 5 describes the construction of the fear index. Finally, Section 6 concludes.

2 Estimation of risk premia for rare events

The method for estimating the risk premia for rare events is taken from Bollerslev and Todorov (2011b). It is based on the idea that risk premia are the difference between the expected return under the objective probability measure and the expected return under the risk neutral measure. Adopting the notation used in Bollerslev and Todorov (2011b) we can write the equity risk premium and the variance risk premium as:

[3]This spread is developed in Gilchrist and Zakrajsek (2012).

[4]A summary of different methods and estimates of equity risk premia can be found in Damodaran (2012).

$$ERP_t \equiv \frac{1}{T-t}\left(E_t^P\left(\frac{F_T - F_t}{F_t}\right) - E_t^Q\left(\frac{F_T - F_t}{F_t}\right)\right), \tag{1}$$

and

$$VRP_t \equiv \frac{1}{T-t}\left(E_t^P\left(QV_{[t,T]}\right) - E_t^Q\left(QV_{[t,T]}\right)\right), \tag{2}$$

with F_t being the futures price for the aggregate market portfolio with an unspecified maturity date and $QV_{[t,T]}$ being the quadratic variation from t to T of the log price process. The future price follows a jump diffusion process with time-varying drift, α_t, and volatility, σ_t, parameters and a compensated jump measure, $\tilde{\mu}(dt, dx) \equiv \mu(dt, dx) - \nu_t^P(dx)dt$, with $\mu(dt, dx)$ being a counting measure for jumps and $\nu_t^P(dx)dt$ the stochastic jump density, denoted by:

$$\frac{dF_t}{F_t} = \alpha_t dt + \sigma_t dW_t + \int_R (e^x - 1)\tilde{\mu}(dt, dx). \tag{3}$$

Now, the risk premia can be decomposed into a part deriving from the diffusion process and a second part resulting from the jump process. Let c indicate the superscript for the premium from diffusion risk and d the superscript for jump risk. Then the risk premia are:

$$ERP_t = ERP_t^c + ERP_t^d, \tag{4}$$

and

$$VRP_t = VRP_t^c + VRP_t^d. \tag{5}$$

As we want to estimate the risk premia for extreme events, which are approximated by "large" jumps in the data, we will only look at the discontinuous premium parts and we will estimate these parts for jumps larger than $k > 0$. This leaves us with the challenge of estimating:[5]

$$\begin{aligned} ERP_t(k) = \frac{1}{T-t}\Bigg(& E_t^P\left(\int_t^T \int_{|x|>k} (e^x - 1)\nu_s^P(dx)ds\right) \\ & - E_t^Q\left(\int_t^T \int_{|x|>k} (e^x - 1)\nu_s^Q(dx)ds\right)\Bigg) \end{aligned} \tag{6}$$

As large jumps are seldomly observed in the data, medium-sized jumps are used to approximate the jump tail measures and, in a second step, estimate tail distributions using the extreme value theory. The estimation method builds on the ideas of Bollerslev and Todorov (2011a).

The risk-neutral and objective measures are estimated separately.

[5]For the derivation of the ERP and the corresponding expression for VRP please refer to Bollerslev and Todorov (2011b).

2.1 Estimating expected jumps under the risk-neutral measure

According to Bollerslev and Todorov (2011b), options that are deep out-of-the-money and have few days to maturity should only have a strictly positive value if the market believes in the existence of big jumps in the price process. Otherwise these options should be worthless. Therefore, these options are used to extract the compensation demanded for jump risk. Of course this assumption neglects the possibility of the diffusion process generating large price movements. But the authors claim that the diffusion part of the price process only triggers small price movements such that it can be ignored when analyzing short maturity options. Building on the ideas of Carr and Wu (2003) and adopting this assumption, we can express the price of a European short maturity deep out-of-the-money call, $C_t(K)$, as:

$$e^{r(t,T]}C_t(K) \approx \int_t^T E_t^Q \left(\int_R I_{\{F_{s-}<K\}}(F_{s-}e^x - K)^+ \nu_s^Q(dx) \right) ds, \tag{7}$$

and the corresponding put:

$$e^{r(t,T]}P_t(K) \approx \int_t^T E_t^Q \left(\int_R I_{\{F_{s-}>K\}}(K - F_{s-}e^x)^+ \nu_s^Q(dx) \right) ds, \tag{8}$$

where I denotes the indicator function and F_{s-} denotes the price of the future directly before a jump occurs. It is further assumed that for short maturity options only one large jump can occur before expiration. So both the compensation for price movements from the diffusion process and the compensation for two or more medium-sized or large jumps is ignored, attributing the option price fully to the compensation for the event that exactly one large jump might happen. Let $k \equiv \frac{K}{F_{t-}}$ and $\kappa \equiv ln(k)$, then the model-free[6] risk-neutral tail measures can be written as:

$$RT_t^Q(k) \equiv \frac{1}{T-t}\int_t^T \int_R (e^x - e^\kappa)^+ E_t^Q(\nu_s^Q(dx))ds \approx \frac{e^{r(t,T]}C_t(K)}{(T-t)F_{t-}}, \tag{9}$$

and

$$LT_t^Q(k) \equiv \frac{1}{T-t}\int_t^T \int_R (e^\kappa - e^x)^+ E_t^Q(\nu_s^Q(dx))ds \approx \frac{e^{r(t,T]}P_t(K)}{(T-t)F_{t-}}. \tag{10}$$

These jump tail measures can then be used to estimate parameters of the jump tail distribution under the risk-neutral measure. The relevant equations can be found in the appendix to Bollerslev and Todorov (2011b). The number of parameters for each tail to be estimated is 3. Therefore, the authors use 3 different levels of moneyness such that the equations are exactly identified.

[6] The measure is model-free as it constitutes scaled option prices with observable scaling measures. But the interpretation as a jump tail measure and its use for the estimation of tail density parameters are only valid given the model for the futures price in equation (3) and the assumptions mentioned above.

2.2 Realized and expected jumps under the objective measure

In order to produce similar expected values to the ones estimated for the Q measure, high-frequency data is used to estimate realized variations, as a first step. In a second step, these realized variation measures are fed into a VAR to estimate the expected integrated volatility. In addition, based on the scores of the log-likelihood function of the generalized Pareto distribution and on the results from Bollerslev and Todorov (2011a), the parameters for the jump tail distribution are estimated using the extreme value theory.

The realized variation measures that we seek to estimate constitute total realized variation, RV_t, which equals the quadratic variation, a variation measure just for the continuous part of the price movements, CV_t, as well as measures for the jump tail variation, JT_t, which can be further separated into positive or right and negative or left jump variations, RJT_t and LJT_t respectively. These measures are extracted from 5-minute interval price data from a broad stock index.[7] The separation between continuous and jump variation is performed using a threshold. The variations for the continuous part and the jump parts add up to the total realized variation. Therefore, it is assumed that price movements below the threshold are solely attributable to the continuous part of the price process, the Brownian motion, and the movements above the threshold are solely attributable to jumps. Now, the threshold used for price movements is not a fixed value but varies depending on the prevalent variation level. The continuous variation from the preceding day as well as the variation at the specific time of the day are used to adapt the threshold for each day and each time of the day. The latter accounts for the fact that at the beginning and the end of the trading period the variation is generally larger than in the middle of the trading day and is further defined in Appendix A.

Assuming that the jump intensity under the objective probability measure is a linear function of the stochastic volatility i.e.:

$$\nu_t^P(dx) = \left(\alpha_0^- I_{x<0} + \alpha_0^+ I_{x>0} + (\alpha_1^- I_{x<0} + \alpha_1^+ I_{x>0})\sigma_t^2\right)\nu^P(x)dx, \tag{11}$$

where $\nu^P(x)$ is a Lévy density and $\alpha_i^\pm > 0$, $i = 0, 1$ are free parameters, the jump tail measures can be written as:

$$RT_t^P(k) = \left(\alpha_0^+ + \frac{\alpha_1^+}{T-t}E_t^P\left(\int_t^T \sigma_s^2 ds\right)\right)\int_R (e^x - e^\kappa)^+ \nu^P(x)ds, \tag{12}$$

and

$$LT_t^P(k) = \left(\alpha_0^- + \frac{\alpha_1^-}{T-t}E_t^P\left(\int_t^T \sigma_s^2 ds\right)\right)\int_R (e^\kappa - e^x)^+ \nu^P(x)ds. \tag{13}$$

The parameter estimation for the jump intensity relies again on medium-sized jumps and the extreme value theory to infer the jump tail distribution. The estimation equa-

[7] In Bollerslev and Todorov (2011b) the S&P 500 futures contract is used. For the US market I use the same data but for the German market I use DAX futures.

tions can be found in the appendix to Bollerslev and Todorov (2011b). A further unknown component of $RT_t^P(k)$ and $LT_t^P(k)$ is the expected integrated volatility denoted by $E_t^P\left(\int_t^T \sigma_s^2 ds\right)$. As mentioned above it is estimated using a restricted VAR(22) model of the following vector $(CV_t, RJV_t, LJV_t, onret_{t-1;t}^2)$, where $onret_{t-1;t}$ is the overnight return on the future from day $t-1$ to day t. The VAR is restricted in the sense that only the parameters for lags 1, 5 and 22 are estimated and all other parameters are set to zero. These lags represent the lags for one day, one week and one month, discounting non-trading days. Using the parameters of the VAR, forecasts are generated for each day for the relevant time horizons. The relevant time horizon is given by the days to maturity of the options used for the Q measure.

3 Description of the data and estimates of the tail measures

Unless explicitly stated, I follow the methods used by Bollerslev and Todorov (2011b). The analysis for the US market uses the exact same data but the time period is expanded by two and a half years to include more data from the recent financial crisis. The data for the US stock market uses the S&P 500 as a proxy for the market portfolio. The options as well as the proxy for the risk-free rate are taken from OptionMetrics and run from January 1996 until May 2011. After cleaning the data following Carr and Wu (2009) and filtering out the shortest-to-maturity options for each day with a minimum maturity of 8 calendar days, I use all options available on each day and calculate the implied volatility of the target levels of moneyness needed for the tail measures using linear interpolation. The target levels of moneyness used are $(1.075, 1.0875, 1.1)$ for the right tail and $(0.925, 0.9125, 0.9)$ for the left tail. If there is no option that is deeper out-of-the-money, such that I cannot interpolate the implied volatility, I take the implied volatility of the deepest out-of-the-money option. From this implied volatility the Black-Scholes option price is calculated for moneyness k. One might of course question the interpretation of e.g. call options with moneyness 1.075 as deep-out-of-the money options. But it is convenient as the data availability is very good for the levels of moneyness used. For deeper out-of-the-money options there are rarely prices available. The days to maturity vary between 8 and 40 (50) for the US (German) options. So the maturity horizon can exceed one month. Now, taking these option prices and interpreting the calculated measures as ones solely attributable to one potential large price jump seems a doubtful operation. First, the options are not very deep out-of-the-money and second, if one believes in the existence of large jumps, than more than one large jump may surely occur within 40 or 50 days, especially in times of crises.

In the next step a time series of tail measures is calculated using equations (9) and (10) to calculate $RT_t^Q(k)$ and $LT_t^Q(k)$. The tail measures with moneyness $k = 1.1$ for the right tail and $k = 0.9$ for the left tail are illustrated in Figure 12 in Appendix B. The correlation between $RT_t^Q(k)$ and $LT_t^Q(k)$ is 86.64%, which is very high, meaning that changes in market perception and demand for compensation for positive as well as negative jumps go in the same direction and happen at the same time. It is counterintuitive that the market demands similar compensation for large positive shocks as for large negative shocks. This rather symmetric perception of jumps could also be interpreted as an indication of the lack of jumps, whereby it is a volatility process that is responsible for this type of option pricing.

The quadratic variation, QV_t^Q, is calculated following Bakshi, Kapadia, and Madan (2003). It is similar to the method that is used to calculate the CBOE VIX index. Comparing the quadratic variation with the CBOE VIX index I find a correlation of 91.99%, which is not suprising. Comparing the VIX with the Q tail measures I find that the correlation with $RT_t^Q(k)$ and $LT_t^Q(k)$ is 70.99% and 90.21% respectively.

The statistical probability measures are calculated using trade data for S&P 500 futures. The data is from Tick Data Inc. and starts in January 1990 and ends in May 2011. I omit days where the price at 8:35 (opening price) is missing, because the analysis depends on the overnight returns. I also delete days with less than 40 price observations. From the trade data I extract the last price for every 5-minute interval starting at 8:35 (CST) and ending at 15:15. This leaves us with 81 intraday prices and 80 intraday returns to calculate the realized variation measures which are depicted in Figure 1. The realized variation measures are positively correlated and seem to have their largest outcomes in times of crisis. RJT_t and LJT_t have a correlation of 65.54%; RJT_t and CV_t of 31.81%; and CV_t and LJT_t of 24.99%.

It is worth mentioning that the method for the P measure strongly depends on the data used. More precisely, it is crucial to use the last trade price for each time interval. When using 1-minute averages of the prices from all trades that were conducted in the respective minute, the method fails to generate estimates of the jump tail distribution. This is because there are not enough large price jumps in the 1- minute average data such that the jump variation measure comes close to zero and the parameter estimations do not converge. The critical reader should reflect on whether large differences in prices of trades that vanish when taking 1-minute averages can really be interpreted as jumps in the price process and thus as reactions to rare events. One could also interpret the identified jumps as price differences due to differences in bargaining power or trades between well-informed and uninformed investors. They could also just be random outliers from misquoted prices or result from other data issues. It is open to question, too, whether such a large time interval as 5 minutes should be used to identify jumps. A few hundred thousand trades are processed each trading day for the S&P 500 as well as the DAX futures. Therefore one can observe more than ten thousand trades within a 5-minute interval. Taking just the last trade price for every 5 minutes seems arbitrary and neglects the potential minor price movements in-between these 5-minute interval, which could easily be interpreted as movements from continuous parts of the price process.

These realized variation measures are used to estimate the parameters of the jump

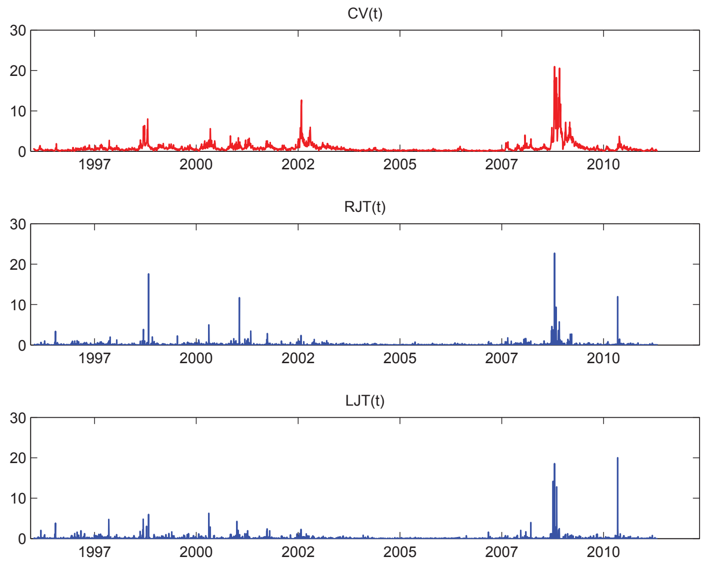

Figure 1: US realized variation measures from trade data for S&P 500 futures.

intensity and estimate the expected integrated volatility using a VAR. Then we can compute the corresponding P measure tails. These will be presented in Section 4.

For the German stock market I have chosen the DAX as the market index. In order to have comparable results I use the exact same methods as for the US data. The options are taken from Datastream and span a time period from March 2006 to December 2012. The Q measure results are shown in Figure 2. The correlation of the measures with the VDAX for the German data are very high. VDAX and QV_t^Q have a correlation of 94.94%; for VDAX and $RT_t^Q(k)$ it is 82.95%, and for VADX and $LT_t^Q(k)$ it is 91.35%, where again $k = 1.1$ and $k = 0.9$ respectively. For the proxy of the risk-free rate I have downloaded the Svensson parameters from the website of the Deutsche Bundesbank and calculated the risk-free rate for the relevant time horizons - the respective days to maturity.[8] The parameters are daily values estimated from listed German sovereign bonds.

The trade data for the DAX future is from Eurex. Although I have a longer time period available, trading seems to be quite infrequent in the initial years. Therefore, I start the estimation period in January 2000 and end it in December 2012. I also delete days where the price for the first 5 trading minutes is missing and if I have less than 40 price observations for that day. The difficulty with this data is that the trading time on Eurex changed several times during the sample period. This complicates the identification of jumps from the data, because the threshold for returns to be classified as jumps depends on the day and on the time-of-day factor. This is described in more

[8]Compare Svensson (1994).

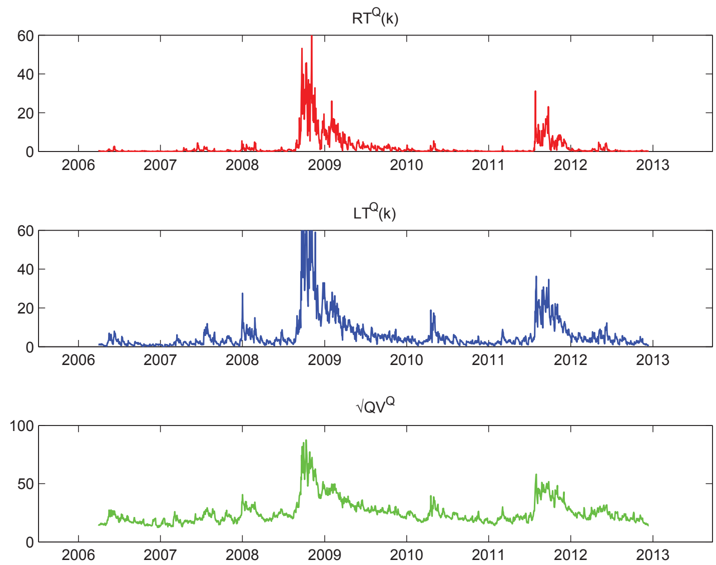

Figure 2: Measures for the German stock market: tail measures and quadratic variation implied from DAX options with moneyness $k = 1.1$ for the right tail and $k = 0.9$ for the left tail.

detail in Appendix A.

The German tail measures seem to reveal very similar patterns to the US data, just with a slightly higher realization. The German data includes the time period of the European sovereign debt crisis and the Greek default. What is interesting to see is that the perceived tail measures extracted from the options are much less pronounced during the sovereign debt crisis compared to the period after the default of Lehman.

4 Estimated equity and variance risk premia

Using target moneyness levels $(1.075, 1.0875, 1.1)$ for the right tail and $(0.925, 0.9125, 0.9)$ for the left tail to get daily tail measure estimates I obtain the parameter estimates for the Q measure jump tail distribution shown in Table 1.

For the P measure the parameter estimates obtained using the realized variation measures are listed in Table 2. The estimates are obtained by solving a system of equations that can be found in the appendix to Bollerslev and Todorov (2011b). π denotes the fraction of the total variation that is due to the variation in the overnight period. Note that there is only one unique value of π which does not correspond to the right tail or the left tail measure alone.

Table 1: Parameter estimates of Q measure tails.

		ξ	σ	$\alpha\bar{\nu}_\psi(tr)$
USA	RT^Q	0.2943	0.0263	0.7391
	LT^Q	0.4315	0.0476	1.3120
Germany	RT^Q	0.1179	0.0342	1.0750
	LT^Q	0.2409	0.0667	1.4765

Table 2: Parameter estimates of P measure tails.

		ξ	σ	$\alpha_0\bar{\nu}_\psi(tr)$	$\alpha_1\bar{\nu}_\psi(tr)$	π
USA	RT^P	0.2442	0.0022	0.000005	0.2014	0.3032
	LT^P	0.3180	0.0019	0.000004	0.1901	0.3032
Germany	RT^P	0.3079	0.0018	0.000006	0.1253	0.1812
	LT^P	0.2072	0.0023	0.0000003	0.1277	0.1812

Having estimated the P measure parameters and the forecasts of the expected integrated volatility using a VAR, we can calculate the P tail measures. These are depicted in Figures 13 and 3. It is very interesting to see that the P tail measures are of a much lower order of magnitude than the corresponding Q measures. As the desired risk premia are calculated scaling the difference of P and Q measure expectations, the results for the risk premia which will be shown below are almost solely driven by the Q measures.

Both tail measures are positive and reveal their peak after the default of Lehman. The measures for the right tail seem to be very similar for the US and for Germany. Values of LT^P are much smaller for Germany than for the US.

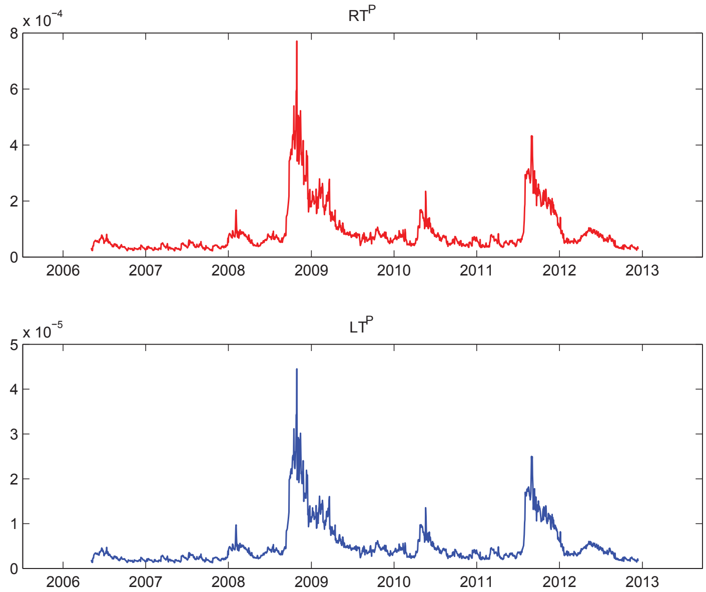

Figure 3: Tail measures for the German stock market under the objective measure extracted from DAX futures trade data.

The parameter estimates for the jump tail distribution can now be used to calculate the relevant integrals of the expectations in the risk premia for rare events. Therefore, we can finally calculate the values for the equity risk premium as in equation (6) and the variance risk premium. The risk premia are scaled such that they can be interpreted as the risk premia demanded for a time horizon of one year. In the original paper the authors claim "Also, for the ease of interpretation, both premia are reported at a monthly frequency based on 22-day moving averages of the corresponding daily estimates" (compare Bollerslev and Todorov (2011b) p. 2184). By doing this the authors conceal very important features of their risk premia. First, the risk premia are very volatile, and second, the risk premia can reach negative values for the equity premium and positive values for the variance premium. The unsmoothed risk premia for the US stock market are illustrated in Figure 14 in Appendix B and for the German stock market in Figure 4.

The unsmoothed equity risk premia can reach values as small as -102.22% for the US and -105.72% for Germany. It is of course hard to interpret that the market demands a negative risk premium for rare events and the magnitude is extremely high. The extreme magnitude suggest an instability in the estimation methods used.

In addition, I show the smoothed daily values for the risk premia. Smoothing the time series almost conceals the above-mentioned features of the risk premia - only the equity risk premia for the US market still have counterintuitive values as they become negative several times at the beginning of the sample. The smoothed risk premia for the

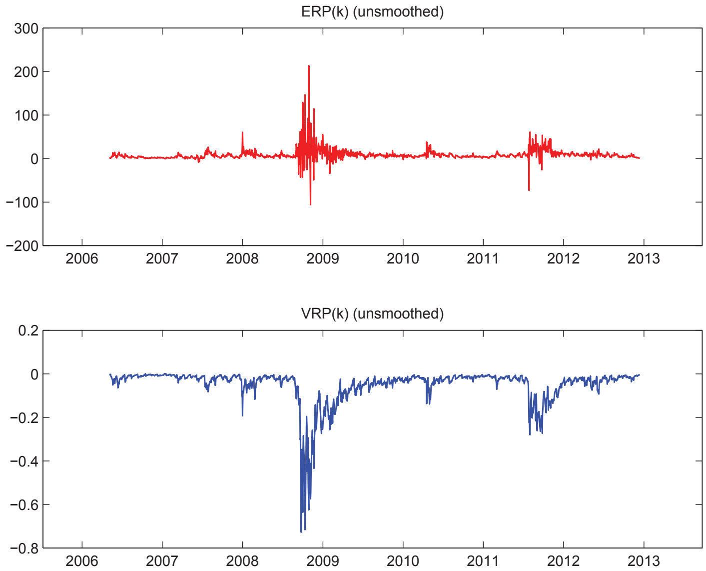

Figure 4: Equity and variance risk premia for the German stock market (unsmoothed values).

US stock market are illustrated in Figure 15 and for the German stock market in Figure 16, both in Appendix B.

The mean and median of the risk premia for rare events are shown in Table 3. Comparison of mean and median reveals a skewed distribution of the risk premia. In view of the fact that the risk premia for the US market are based on a different sample period from the risk premia for the German market, it is not very useful to compare the values with each other. But comparing the equity risk premia for the US with the estimates of Mehra and Prescott (2003), with a value of 4.1% to 8.4% depending on the data set and time period or comparing them with Cochrane (2005), who finds 8% for the total equity risk premium for the US postwar period, the median value of 5.78% seems rather high.[9] Taking the 8%, this means that almost three-quarters of the average equity risk premium is the compensation for rare events. These results are higher than the results from Broadie, Chernov, and Johannes (2007) who find that the contribution of jumps to the total risk premium is 3% of the total 8%. For the German market I calculated the realized market excess returns by taking the annual DAX returns and substracting the German risk-free rate calculated using the Svensson method. The mean realized excess return of the DAX is 0.0189, which is much lower than the 6.6% stated by Mehra and Prescott (2003) for the period 1978-1997, and the median is 0.1217. Taking the median

[9] Damodaran (2012) reports various historical risk premia varying from -1.92% to 7.2% depending on the time horizon of the data sample and the risk-free rate chosen.

as a proxy for the total equity risk premium, more than 50% of the total risk premium can be attributed to compensation for rare events, which is similar to the values found by Santa-Clara and Yan (2010). The annual realized risk premium and the model equity risk premium due to rare events series are depicted in Figure 5. Of course the comparison is flawed. The rare events risk premia are based on perceived risks for rare events and the empirical risk premia are realized measures. Therefore the comparison is prone to peso-type problems. But what is interesting to see is that the series are negatively correlated with a correlation of -29.19%. That means when the market demands a high level of compensation for rare events the actual excess return is low (or negative).[10]

Table 3: Risk premia for rare events: annualized ERP(k) and VRP(k) for the US and German stock markets.

		mean	median
USA	ERP(k)	0.0773	0.0578
	VRP(k)	-0.0443	-0.0262
Germany	ERP(k)	0.0843	0.0626
	VRP(k)	-0.0520	-0.0240

In addition, the variance risk premia due to rare events seem large in magnitude. When calculating the mean total variance risk premium, $\overline{VRP}$, with a very simple method using $\sum_{t=1}^{T}(RV_t - QV_t^Q)$, I obtain a value of -0.0212 for the US.[11] Comparing this result with the mean variance risk premium due to rare events, $\overline{VRP}(k)$, the latter is larger in absolute terms than the total variance risk premium (which should include the rare events premium). The results seem implausible and also stand in contrast to the findings of Broadie, Chernov, and Johannes (2007) who find that the variance risk premia for jumps account for only 24.4% of the total variance risk premia. Hence, it seems that in this analysis either the estimator for the mean total variance risk premium is too low or the variance risk premia due to rare events are too high.[12]

In order to check, whether the mean total variance risk premium is too low, I compare it with the results from Bollerslev, Tauchen, and Zhou (2009) and Drechsler (2013). Both analyses use almost the same method to estimate total variance risk premia, defined in a very similar way to those in this paper. Both take the squared value of the VIX as the risk-neutral expectation of return variance. For the objective measure Bollerslev,

[10] According to Mehra and Prescott (2003) the realized US equity premium is countercyclical in the sense that in times of high stock market valuation the equity premium is low. In contrast, the annual DAX returns shown in Figure 5 seem to be very cyclical. Comparing the annual realized excess returns with the market capitalization of listed companies as a percentage of GDP downloaded from the World Bank website, the correlation coefficient is 57.9%. But, the equity risk premium demanded for rare events is countercyclical.

[11] Bollerslev and Todorov (2011b) find a value of -0.02 in their sample period.

[12] Alternatively it could be that the variance risk premia for small jumps and for diffusion are positive and considerable in magnitude.

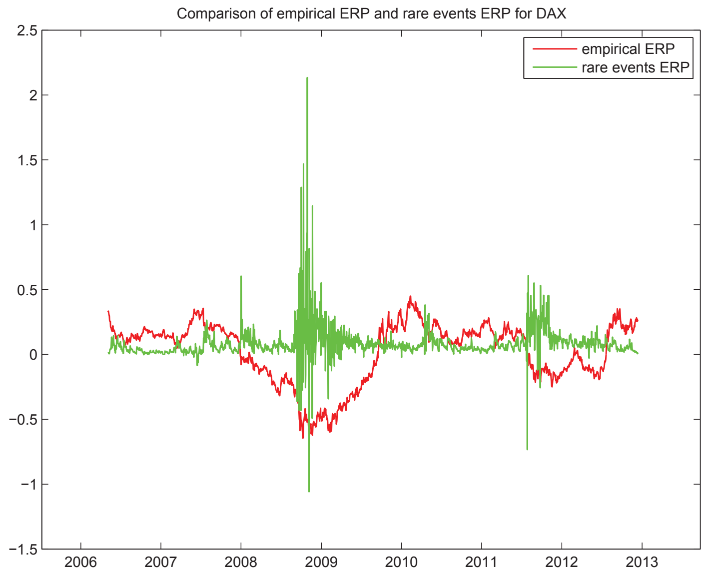

Figure 5: Comparison of realized market excess returns and equity risk premia for rare events.

Tauchen, and Zhou (2009) use the squared 5-minute log returns for S&P 500 index and Drechsler (2013) uses a forecast of the realized variance based on the squared 5-minute log returns of S&P 500 futures. They find a mean monthly variance risk premium of 18.30 and 10.55 (in percentages squared) respectively. In annualized terms this means 0.02196 and 0.01266 respectively, which indicates that the mean total variance risk premium estimate seems not too low compared to estimates of other studies.[13] One may conclude that the values calculated by Bollerslev and Todorov (2011b) for the variance risk premia due to rare events seem very large.

4.1 Decomposition of risk premia for Germany

Now I will take a closer look at the risk premia by decomposing them into parts due to negative and positive jumps. I will only show the graphs for the German market as they reveal similar patterns to the US market graphs and these can be seen in Bollerslev and Todorov (2011b). The decomposition of the equity risk premia is shown in Figure 6 and the decomposition of the variance risk premia is shown in Figure 7. The equity risk premia for large positive jumps are negative and for large negative jumps positive. But

[13]Bollerslev, Tauchen, and Zhou (2009) use a sample period from January 1990 to December 2007 and Drechsler (2013) extends this sample to December 2009. Note that Bollerslev, Tauchen, and Zhou (2009) and Drechsler (2013) define the risk premium as the risk-neutral expectation minus statistical expectation, therefore we have positive variance risk premia here.

the magnitude for the negative jumps is larger than for positive jumps. The variance risk premia are both negative with more pronounced values for the negative jumps.

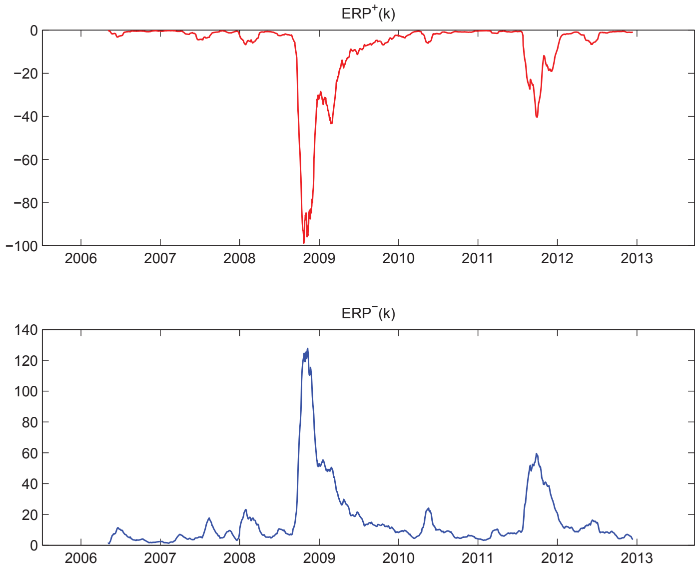

Figure 6: Decomposed equity risk premia for the DAX due to large positive and negative jumps.

Looking at the contribution of the P Measures and the Q measures to the equity risk premia I find that the P expectation of equation (6) indeed yields very small positive values with a median of 0.000002. The Q expectation has a median of −0.0626. Therefore the risk premia results are hardly influenced by the results from the high-frequency data.

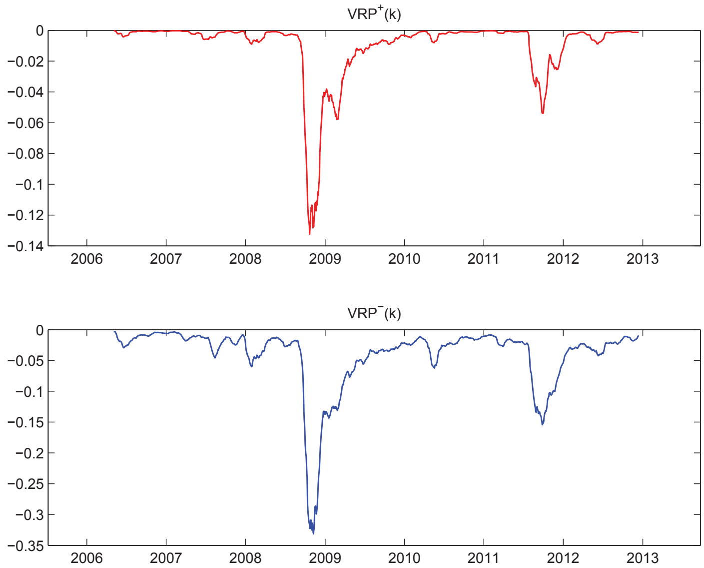

Figure 7: Decomposed variance risk premia for the DAX due to large positive and negative jumps.

5 Index of investor fears

From the decomposition of the variance risk premia, Bollerslev and Todorov (2011b) construct an investor fear index, which is a measure that can be interpreted as showing the risk premia for the variation in jump intensities and is defined as:[14]

$$FI_t(k) \equiv |VRP_t^-(k) - VRP_t^+(k)|. \tag{14}$$

For this index the smoothed values of the risk premia are used. As mentioned above the time period spanned by the data for the US market starts in January 1996 and ends in May 2011. The fear index shows sharp increases for the following events, which I list in chronological order: Asian crisis stock market crash in October 1997, Russian default and collapse of Long-Term Capital Management in August and September 1998, terror attacks of September 2001, the market turmoils of March 2002, Lehman collapse in September 2008 and the Greek request for a bail-out package from the EU and IMF in April 2010. The German fear index peaks sharply after the Lehman default, the Greek request for a bail-out package and after the Greek default in June 2011.

I compare the fear index for the US with the VIX. The correlation is very high with a value of 89.46%. The fear index is at a low value - even below its mean value - before the

[14]In Bollerslev and Todorov (2011b) the fear index is defined without the absolute value. But in crisis times their fear index peaks downwards. Because in crisis times investor fears rise, I find it more intuitive to take the absolute value such that the index rises in crisis times.

Lehman collapse, rises modestly on the Friday before the collapse and reveals a sharp increase starting from the day after the collapse. The options used for the Q measure tails expire on the Saturday following the collapse. These are the shortest-to-maturity options starting from 11 August 2008. So this is the date when the fear index could theoretically already show signs of stress, as the time to maturity of the options already spans the time of the collapse. The VIX uses options with approximately a one-month maturity. Interestingly the VIX is above its mean value already from 4 August 2008 and rises apruptly on the day of the Lehman default.

For the German market I obtain similar results when I compare the fear index with the VDAX. The correlation is 90.58% and reduces to 83.45% when using the unsmoothed fear index. Even more important is the fact that the smoothed fear index lags the VDAX, which is not surprising. In Figure 8 I depict the comparison using standardized values for the indices. The vertical blue line indicates the day of the Lehman default. One can see that the fear index as defined by Bollerslev and Todorov (2011b) reacts more slowly to rare events than the VDAX. Using unsmoothed risk premia values to calculate the fear index helps to eliminate this unwanted property. Looking at the unsmoothed fear index in comparison to the VDAX since the financial crisis in Figure 9, where I choose August 2008 as the start date, reveals that the fear index neither leads nor lags the VDAX.[15] In a similar way to what I find for the US variables, although both indicators use forward-looking measures they react only after rare events happen. Therefore, neither indicator can be used as an early-warning indicator.

[15] A VAR(2) using the first-differenced variables does not give a clear picture either as VDAX seems to be dependent on past values of the fear index and vice versa.

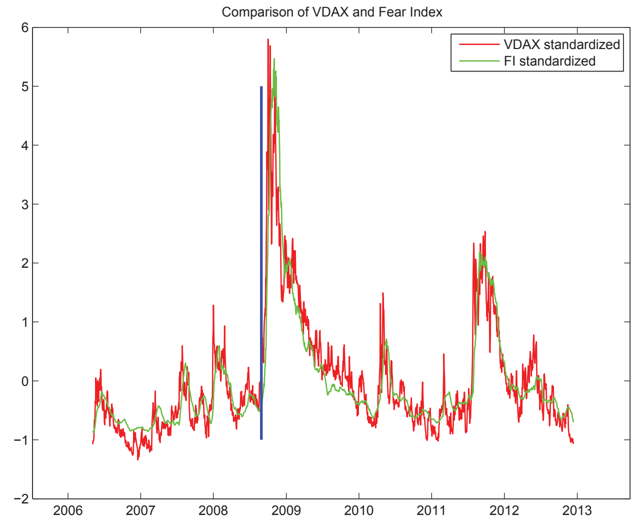

Figure 8: Comparison of the fear index for Germany with the VDAX. The vertical blue line indicates the day of the Lehman default. Both indicators are standardized to make them comparable.

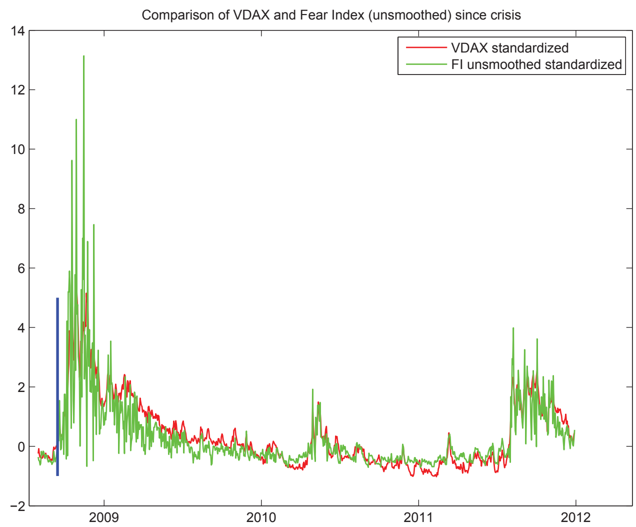

Figure 9: Comparison of the unsmoothed fear index for Germany with the VDAX showing only the values since August 2008. The vertical blue line indicates the day of the Lehman default. Both indicators are standardized to make them comparable.

A recent strand of literature analyzes the predictive power of bond market prices. Philippon (2009) argues that the equity market is more susceptible to mispricing and bubbles. He finds that bond market prices can predict future corporate investments better than equity market prices. Therefore, I also compare the fear index for the US to the Gilchrist-Zakrajsek spread developed by Gilchrist and Zakrajsek (2012). The spread is a newly developed credit spread index based on month-end secondary bond market prices of US non-financial firms. For ease of comparison I choose a time period when both the US fear index and the spread are available starting in January 1996 and ending in September 2010. I take the mean value of the fear index for each month[16] and standardize both variables. I show both graphs in Figure 10. The Gilchrist-Zakrajsek spread shows a small peak in August 1998, it rises starting from the beginning of 2000 probably due to the bursting of the dot-com bubble, shows only a very small peak in September 2001, peaks with the March 2002 market turmoils and rises sharply in the financial crisis. Thus, the Asian crisis as well as the September 2001 attacks seem to have had little impact on corporate lending rates, whereas the bursting of dot-com bubble starting in 2000 seems to have influenced bond prices considerably. Most importantly, the Gilchrist-Zakrajsek spread increases in the financial crisis starting from July 2007. The start of the financial crisis is dated 9 August 2007 when the overnight index swap spread over LIBOR rose dramatically.[17] Early signs were already prevalent in February 2007 with several events, such as Bear Stearns' capital injections into its own hedge funds in June and July 2007 as documented by Brunnermeier (2008). Therefore, the Gilchrist-Zakrajsek spread seems to have picked up the early signs of the crisis. In contrast, the fear index increases just slightly at the beginning of the financial crisis in August 2007 and even seems to indicate market recovery just before the Lehman collapse, when the values of the standardized monthly fear index become negative meaning that before Lehman the fear index is below its average value.

Alternative measures for tail risks or investor fears can be constructed from higher moments of the return distribution, namely skewness and kurtosis. Therefore, I take measures of realized and implied skewness and kurtosis and compare it with the fear index for the US. I use the implied skewness and kurtosis measures of Chang, Christoffersen, and Jacobs (2013) and the CBOE SKEW as an additional measures for implied skewness. I also calculate monthly realized skewness and kurtosis using daily returns of the S&P 500 futures. In linear regressions of the fear index on the higher moment measures and the VIX it shows that realized skewness and kurtosis seem to have little or no influence on the fear index. In contrast, the corresponding implied measures and SKEW[18] show significant positive parameter values and increased adjusted R^2 although not surprisingly the VIX exhibits the largest impact on the fear index. Orthogonalizing the fear index with respect to the VIX via a linear regression and regressing the respective residuals on the implied measures confirms that implied higher moments carry additional information

[16] Taking the month-end values only does not change the results.

[17] Compare Taylor (2009).

[18] Implied skewness and SKEW are used interchangeably in regressions.

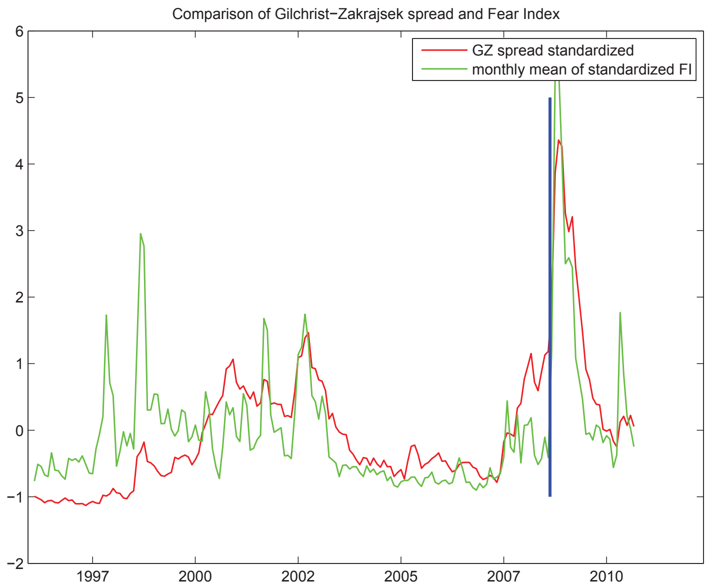

Figure 10: Comparison of the fear index for the US with the Gilchrist-Zakrajsek spread. The vertical blue line indicates the day of the Lehman default. Both indicators are standardized to make them comparable.

about the fear index.

Finally, I compare the fear index of the US with the fear index of Germany. Ideally this comparison should account for the fact that the fear indices are estimated based on prices of financial instruments with different trading times.[19] Due to the nature of the index one cannot easily adjust for the different trading times, because there are no intraday values - just one value per day is at hand. Therefore, one should expect the US fear index to be leading as the daily values of the index should include information that was not available when German markets closed.

My analyses seem to confirm this prior. Running a VAR on the first-differenced values of the indices using 3 lags I find that the past values of itself and of the US fear index seem to have explanatory power for the German fear index. For the US fear index only past values of itself seem to explain the current value. The parameters for the past values of the German index are not significant on the 5%-level. Therefore, one could conclude that the US index is leading.[20]

[19] The S&P 500 futures and options are both traded until 15:15 CST, so all events happening prior to this time should be incorporated into the prices at closing time. For Germany the calculation of the fear index starts in May 2006. Since then the DAX futures were traded until 22:00 CET, which is just 15 minutes before the closing time of the market for the S&P 500 instruments. However, the closing time of the market for the DAX options is 19:00 CET.

[20] A graph with the comparison of the indices is shown in Figure 17 and the results of the VAR are

6 Conclusion

I have replicated the paper by Bollerslev and Todorov (2011b) for the US stock market, expanding the sample to include more data on the recent financial crisis. I also use German data to compare the results. The paper aims to estimate equity and variance risk premia due to rare events. The method has several flaws in the identification of jumps. It is assumed that large price movements are solely attributable to jumps in the price process, more precisely to one large jump, ignoring that diffusive processes or several jumps may cause large price movements. Therefore, the basis for estimating the jump tail distributions seems unreliable. When estimating the jump tail distributions for the risk-neutral measure, prices of out-of-the-money options are used to proxy the price demanded for the potential occurrence of one large jump. As the days to maturity vary between one week and up to more than one month and the levels of moneyness are at only 1.1 for calls and 0.9 for puts, this approximation seems inadequate. A price rise or drop of 10% in equity markets within in some cases more than one month seems not unusual and not the outcome of a rare event. For the objective measure, the last trade price of each 5-minute interval is used to identify jumps. With more than ten thousand trades possible during such an interval, one can easily observe “jumps”, even if there are only small price movements.

The equity risk premium demanded due to large jumps is highly volatile and reaches large negative values after the Lehman default. This is hard to interpret and justify and stands in contrast to the findings of Santa-Clara and Yan (2010), who find an equity premium for jumps between 0 and 45.4%. The median equity risk premia for the US and Germany are 5.78% and 6.26% respectively, which is relatively high given that the reported values by Mehra and Prescott (2003) for the realized equity premia range from 4.1% to 8.4% in the US and were stated as 6.6% in Germany. The variance risk premia due to rare events are also very high compared to the values found in the literature.

The fear index constructed from variance risk premia shows spikes at times of crises, but shows a very low level of investor fears just before the Lehman collapse where other indicators such as the Gilchrist-Zakrajsek spread already show signs of stress. Therefore, it seems impractical as an early-warning indicator, but it works well as an indicator of prevalent stress. Furthermore, it has a very high correlation with corresponding volatility indices (the VIX for the US and the VDAX for Germany), which can easily be downloaded from several websites. Given the enormous effort needed to estimate the fear index and the very high correlation between the two indicators, it is more convenient to use the VDAX as a proxy for investor fears.

shown in Table 4 in Appendix C.

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A Time-of-day factor

The time-of-day factor is designed to account for the fact that volatility is higher at the beginning and the end of the trading day compared to the volatility around noon. It can be interpreted as the average volatility at a certain trading time over the sample period.

For S&P 500 future the time-of-day factor is depicted below.

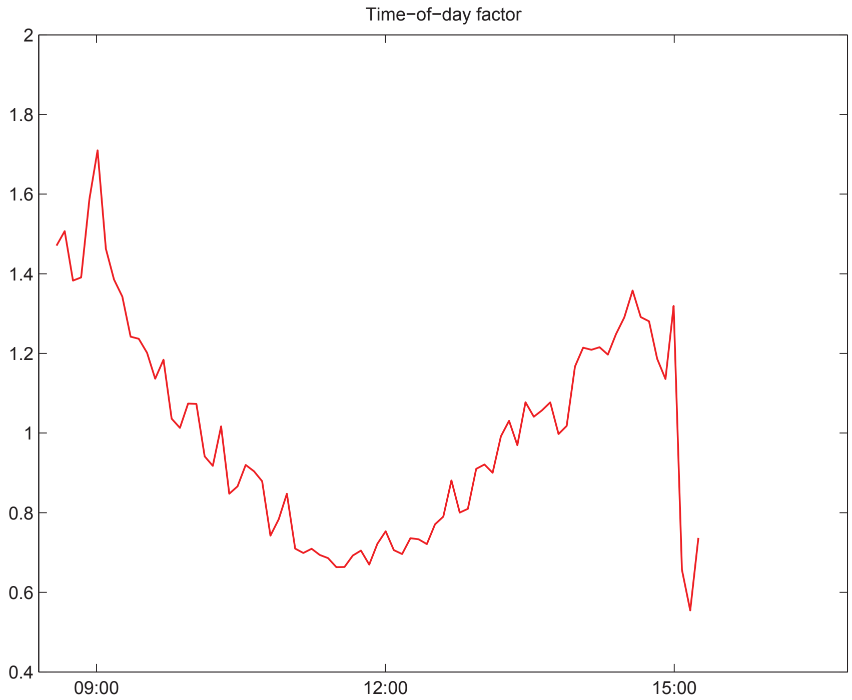

Figure 11: The time-of-day factor is estimated using 5-minute price data for S&P 500 futures.

For DAX futures it is not possible to set up a unique graph as the trading times of the futures on the Eurex changed 3 times during the estimation period: until 1 June 2000 the trading time on the FDAX was from 09:00 to 17:30; afterwards until 25 November 2005 the trading time was extended to 09:00 - 20:00; then until 31 May 2006 it was set at 09:00 - 22:00; and finally since then the trading times have been 08:00 - 22:00. For the variation measures estimated daily the changes in trading hours on Eurex do not pose a problem. But for the estimation of the threshold used to identify jumps from the high-frequency data the sample has to be split into 4 periods with different time-of-day factors.

B Selected figures of tail measures and risk premia

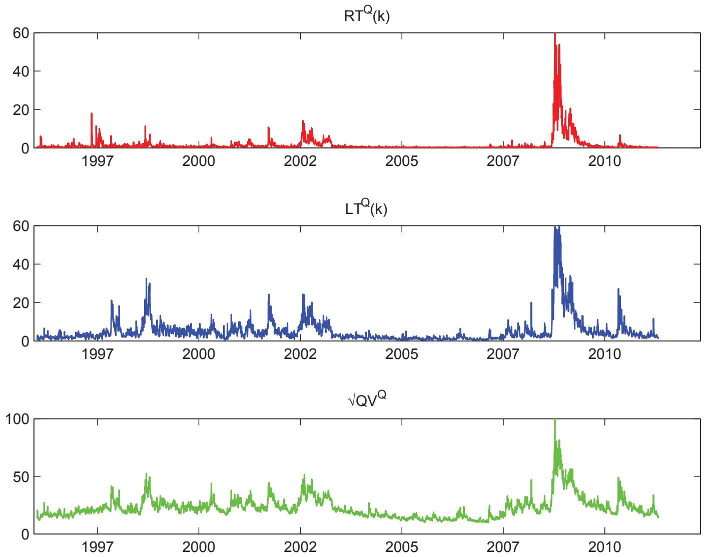

Figure 12: Measures for the US stock market: tail measures and quadratic variation implied from S&P 500 options with moneyness $k = 1.1$ for the right tail and $k = 0.9$ for the left tail.

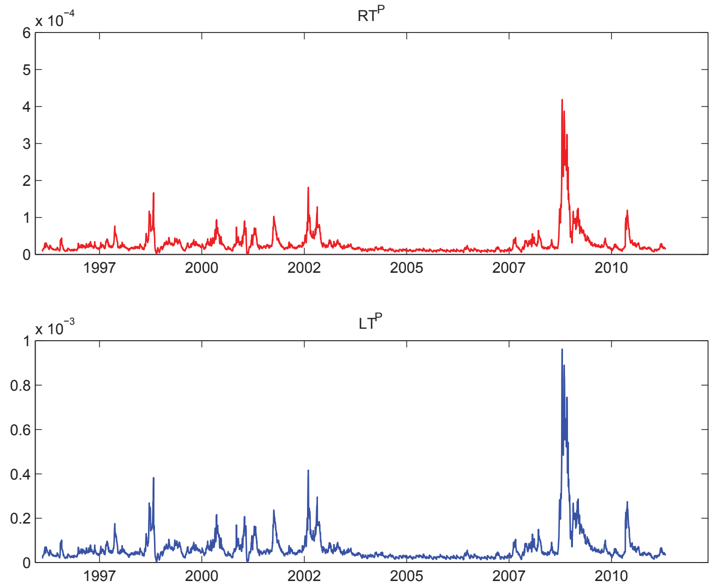

Figure 13: Tail measures for the US stock market under the objective measure extracted from S&P 500 futures trade data.

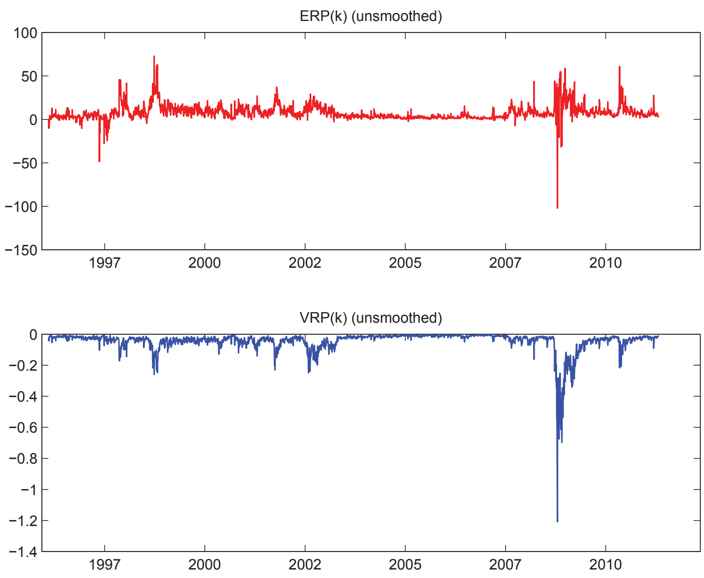

Figure 14: Equity and variance risk premia for the US stock market (unsmoothed values).

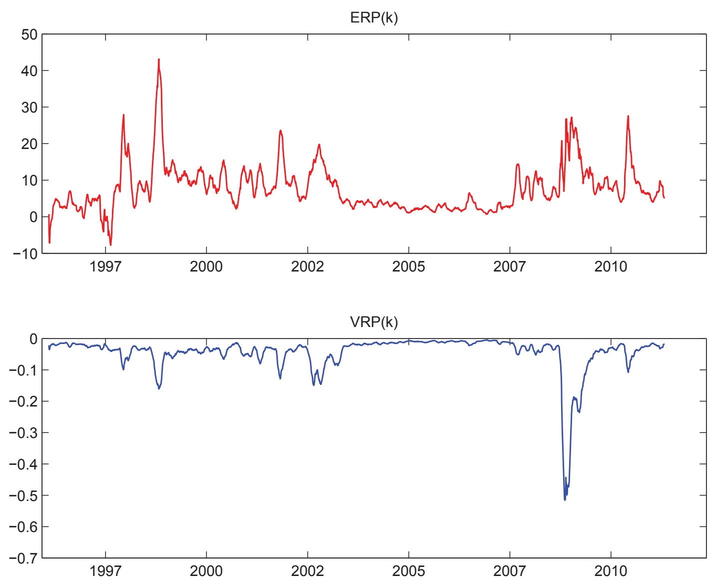

Figure 15: Equity and variance risk premia for the US stock market (smoothed using 22-day moving averages).

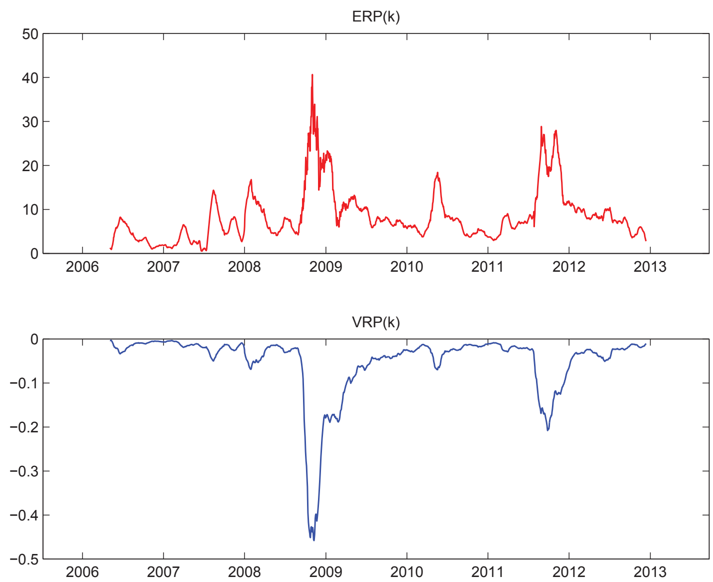

Figure 16: Equity and variance risk premia for the German stock market (smoothed using 22-day moving averages).

C Comparison of Fear Indices

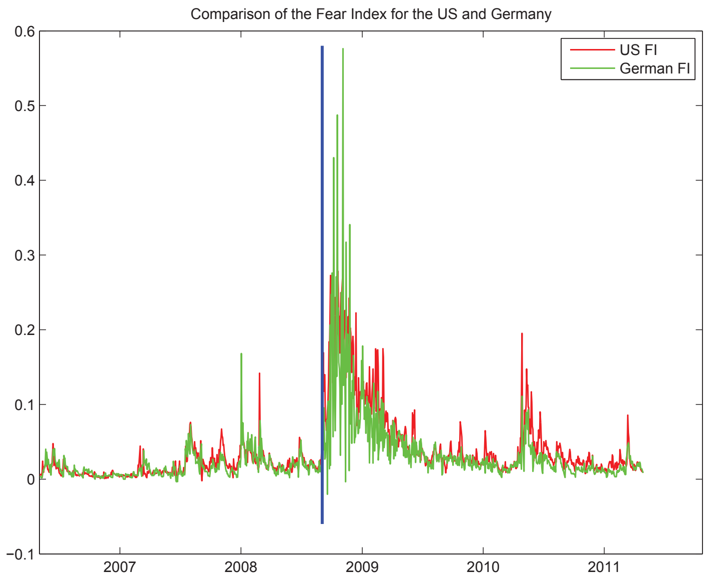

Figure 17: Comparison of the fear index for the US with the fear index for Germany. The vertical blue line indicates the day of the Lehman default.

Table 4: VARs of the German and US Fear Indices.

Δ German Fear Index		
$\Delta FI_{DE(t-1)}$	-0.715***	(-24.65)
$\Delta FI_{DE(t-2)}$	-0.460***	(-13.89)
$\Delta FI_{DE(t-3)}$	-0.151***	(-5.33)
$\Delta FI_{US(t-1)}$	0.422***	(7.77)
$\Delta FI_{US(t-2)}$	0.171**	(3.07)
$\Delta FI_{US(t-3)}$	0.134*	(2.42)
Δ US Fear Index		
$\Delta FI_{DE(t-1)}$	0.0223	(1.42)
$\Delta FI_{DE(t-2)}$	-0.0330	(-1.84)
$\Delta FI_{DE(t-3)}$	-0.0201	(-1.31)
$\Delta FI_{US(t-1)}$	-0.254***	(-8.64)
$\Delta FI_{US(t-2)}$	-0.218***	(-7.25)
$\Delta FI_{US(t-3)}$	-0.0633*	(-2.12)
N	1160	

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$