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A model of mortgage losses and its applications for macroprudential instruments

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The Stability of the European Financial System and the Real Economy in the Shadow of the Crisis

held by the Deutsche Bundesbank, the Technische Universität Dresden and the Journal of Financial Stability in Dresden from 17 to 18 January 2013.

Thursday, 17 January 2013

8:45 – 9:00	Registration
9:00 – 9:10	**Opening Remarks** by **Alexander Karmann** (TU Dresden) and **Thilo Liebig** (Deutsche Bundesbank)
9:10 – 9:25	**Welcome Address** by **Hans Müller-Steinhagen** (Rector, TU Dresden)
Session 1	**Business Models and Risk-Taking of Banks I**
	Chair: Iftekhar Hasan (Fordham University)
9:25 – 10:15	**Which banks are more risky? The impact of loan growth and business model on bank risk-taking**
	Matthias Köhler (Deutsche Bundesbank)
	Discussant: Michel Dietsch (Université de Strasbourg and Banque de France)
10:15 – 10:25	Coffee Break
10:25 – 11:15	**Modelling and measuring business risk and the resiliency of retail banks**
	Mohamed Chaffai (Université de Sfax) and ***Michel Dietsch*** (Université de Strasbourg and Banque de France)
	Discussant: Matthias Köhler (Deutsche Bundesbank)
11:15 – 12:00	**Invited Talk**
	Capital Levels and Risk-Taking Propensity in Financial Institutions
	Giovanni Barone-Adesi (University of Lugano and Swiss Finance Institute)
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Session 2	**Business Models and Risk-Taking of Banks II**
	Chair: Thilo Liebig (Deutsche Bundesbank)
13:30 – 14:20	**The Common Drivers of Default Risk**
	Christoph Memmel (Deutsche Bundesbank), Yalin Gündüz (Deutsche Bundesbank) and Peter Raupach (Deutsche Bundesbank)
	Discussant: Wolfgang Bessler (Justus-Liebig University Giessen)
14:20 – 15:10	**Bank Risk Factors and Changing Risk Exposures of Banks: Capital Market Evidence Before and During the Financial Crisis**
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	Discussant: Christoph Memmel (Deutsche Bundesbank)
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Key Note Speech 15:25 – 16:10	**Why is the 'euro crisis' so difficult to deal with?** **Martin Hellwig** (Max Planck Institute for Research on Collective Goods)
Session 3	**The Financial Crisis and the Real Economy**
	Chair: Stefan Eichler (TU Dresden)
16:10 – 17:00	**Catharsis - Tracing the Real Effects of Bank Insolvency and Resolution**
	Josef Korte (Goethe University Frankfurt)
	Discussant: Camelia Minoiu (International Monetary Fund)
17:00 – 17:10	Coffee Break
17:10 – 18:00	**Liquidity Shocks, Bank Balance Sheets, and International Lending During the 2007-08 Crisis**
	Camelia Minoiu (International Monetary Fund) and Tümer Kapan (Fannie Mae)
	Discussant: Josef Korte (Goethe University Frankfurt)
19:00 – 22:00	**Reception and Conference Dinner** **Dinner Speech: Andreas Dombret** (Member of the Executive Board, Deutsche Bundesbank)

Friday, 18 January 2013

Session 4	**Regulatory Issues**
	Chair: Olivier De Bandt (L'Autorité de Contrôle Prudentiel France)
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	Christian Hott (Zurich Insurance Group)
	Discussant: Clemens Bonner (De Nederlandsche Bank)
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10:00 – 10:50	**Steering the LCR with the Interbank Money Market**
	Clemens Bonner (De Nederlandsche Bank) and Sylvester Eijffinger (CentER, Tilburg University)
	Discussant: Roman Horvath (Institute of Economic Studies, Charles University Prague and IOS Regensburg)
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11:00 – 11:50	**Central Bank Transparency and Financial Stability: Measurement, Determinants and Effects**
	Roman Horvath (Institute of Economic Studies, Charles University Prague and IOS Regensburg) and Dan Vaško (Institute of Economic Studies, Charles University Prague)
	Discussant: Christian Hott (Zurich Insurance Group)
11:50 – 12:35	**Invited Talk**
	Financial Contract: Bank versus Bank
	Iftekhar Hasan (Fordham University)
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Session 5	**Sovereign Risk**
	Chair: Alexander Karmann (TU Dresden)
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	Guglielmo Maria Caporale (Brunel University London, CESifo and DIW Berlin) and Alessandro Girardi (Istat, Rome)
	Discussant: Giovanni Calice (University of Southampton)
14:30 – 15:20	**Liquidity Spillovers in Sovereign Bond and CDS Markets: An Analysis of The Eurozone Sovereign Debt Crisis**
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	Discussant: Jorge Antonio Chan-Lau (International Monetary Fund)
15:20 – 15:35	Coffee Break
15:35 – 16:25	**Equity Returns in the Banking Sector in the Wake of the Great Recession and the European Sovereign Debt Crisis**
	Jorge Antonio Chan-Lau (International Monetary Fund), Estelle X. Liu (International Monetary Fund) and Jochen M. Schmittmann (International Monetary Fund)
	Discussant: Guglielmo Maria Caporale (Brunel University London, CESifo and DIW Berlin)
16:25 – 16:40	**Concluding remarks** by **Alexander Karmann** (TU Dresden) and **Thilo Liebig** (Deutsche Bundesbank)
	End of Conference

Non-technical summary

Loss rates on mortgages have increased substantially in the US and many other countries during the financial crisis. At the same time, financial institutions were heavily exposed to the mortgage market when the crisis started. As a result, many banks suffered high losses or even failed. One reason for this was that financial institutions underestimated the risk associated with mortgages. Another reason for the extent of the crisis was that also existing banking regulation and capital requirements did not adequately capture the build-up of risks in the mortgage market and the resulting systemic impact. As a reaction, we see increasing international efforts (especially by the Financial Stability Board, the BIS and the IMF) to enhance macroprudential instruments in order to identify and monitor systemic risks and to limit the build-up and/or the impact of these risks.

We develop a theoretical model of mortgage loss rates that evaluates their main underlying risk factors. Following the model, loss rates are positively influenced by the house price level, the loan-to-value of mortgages, interest rates, and the unemployment rate. They are negatively influenced by the growth of house prices and the income level. The calibration of the model for the US and Switzerland demonstrates that it is able to describe the overall development of actual mortgage loss rates in sample as well as out of sample. In addition, we show potential applications of the model for different macroprudential instruments: the calibration of loss rates under stress events, the calculation of the size and development of countercyclical buffers, and the setting of risk weights for different combinations of loan-to-value and loan-to-income.

Nicht-technische Zusammenfassung

Die Verluste auf Hypothekarkredite sind in den USA und vielen anderen Ländern während der Krise stark angestiegen. Gleichzeitig waren Finanzinstitute beim Ausbruch der Krise stark gegenüber dem Hypothekarmarkt exponiert. Als Resultat erlitten viele Banken hohe Verluste oder gerieten sogar in Insolvenz. Ein Grund hierfür war, dass Finanzinstitute das Risiko von Hypotheken unterschätzten. Ein anderer Grund für das Ausmass der Krise war, dass auch die Bankenregulierung und Kapitalanforderungen den Aufbau der Risiken auf dem Hypothekenmarkt und die resultierenden systemischen Auswirkungen nicht angemessen erfassten. Als Reaktion hierauf gibt es wachsende internationale Bemühungen (insbesondere durch das Financial Stability Board, die BIZ und den IWF), makroprudentielle Instrumente zu verbessern, um systemische Risiken zu identifizieren und zu monitoren und um den Aufbau und die Auswirkungen dieser Risiken einzuschränken.

Wir entwickeln ein theoretisches Modell eines Hypothekarmarktes und untersuchen die Hauptrisikofaktoren für die Banken. Aus dem Modell folgt, dass die Verlustraten bei Hypotheken positiv vom Niveau der Hauspreise, dem Verhältnis von Hypotheken zum Wert der Immobilien, dem Zinssatz und der Arbeitslosigkeit abhängen. Sie hängen dagegen negativ vom Wachstum der Hauspreise und vom Einkommensniveau ab. Die Kalibrierung des Modells für die USA und die Schweiz zeigt, dass unser Modell die tatsächliche Entwicklung der Verlustraten auf Hypothekarkrediten sehr gut wiedergibt, sowohl in-sample als auch out-of sample. Darüber hinaus zeigen wir mögliche Anwendungen des Modells für makroprudentielle Instrumente auf: Die Kalibrierung von Verlustraten unter Stress-Szenarien, die Entwicklung von antizyklischen Puffern und die Berechnung deren Höhe und sowie die Bestimmung von Risikogewichten für verschiedene Kombinationen der Verhältnisse von Kredit und Immobilienwert bzw. von Kredit und Einkommen.

Bundesbank Discussion Paper No 34/2013

A Model of Mortgage Losses and its Applications for Macroprudential Instruments*

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Abstract

We develop a theoretical model of mortgage loss rates that evaluates their main underlying risk factors. Following the model, loss rates are positively influenced by the house price level, the loan-to-value of mortgages, interest rates, and the unemployment rate. They are negatively influenced by the growth of house prices and the income level. The calibration of the model for the US and Switzerland demonstrates that it is able to describe the overall development of actual mortgage loss rates. In addition, we show potential applications of the model for different macroprudential instruments: stress tests, countercyclical buffer, and setting risk weights for mortgages with different loan-to-value and loan-to-income ratios.

Keywords: Mortgage Market, Credit Risk, Macroprudential Instruments

JEL classification: E5, G21

*christian.hott@zurich.com. The author was a senior economist at the Swiss National Bank until the end of 2011. The paper was substantially written during his employment with the Swiss National Bank. The opinions expressed in the paper are those of the author and do not necessarily reflect the views of his present or former employer. I would like to thank Terhi Jokipii, Pierre Monnin, Pius Matter, Peter Westerheide, as well as the participants of the conference "The Stability of the European Financial System and the Real Economy in the Shadow of the Crisis" and the Marie Curie ITN Conference on Financial Risk Management & Risk Reporting for their helpful comments.

1 Introduction

Loss rates on mortgages have increased substantially in the US and many other countries during the financial crisis. At the same time, financial institutions were heavily exposed to the mortgage market when the crisis started. Figure 1 displays the relationship between mortgages and charge-off rates in the US. As we can see, mortgages increased strongly as long as charge-off rates were low. Therefore, the banks' exposure was at its peak when the bubble burst and charge-off rates suddenly increased strongly. As a result, many banks suffered high losses or even failed. In 2009 the FDIC recorded 140 bank failures for the US. As a comparison, in 2008 there were 25 bank failures and from 2002 to 2007 only 21.

One reason for this was that financial institutions underestimated the risk associated with mortgages. The development of the lending standards of banks and their the mortgage supply[1] indicates that financial institutions base their risk models on the past performance of mortgages rather than on the main economic risk drivers. Among others, these risk drivers are the income of borrowers and the development of house prices.

Another reason for the extent of the crisis was that also existing banking regulation and capital requirements did not adequately capture the build-up of risks in the mortgage market and the resulting systemic impact. As a reaction, we see increasing international efforts (especially by the Financial Stability Board, the BIS and the IMF) to enhance macroprudential instruments in order to identify and monitor systemic risks and to limit the build-up and/or the impact of these risks.[2] The process is still ongoing; however, key methodologies to identify systemic risks are constructing aggregated indicators for systemic imbalances like the credit-to-GDP ratio and conducting macro stress tests. The most prominent macroprudential instrument is the countercyclical capital buffer, under which banks have to hold more capital in boom phases and can use the buffer to cover losses in a downturn phase. A further instrument is the limitation of loan-to-value and loan-to-income ratios, which are important parameters for banks to influence the risk profile of their mortgage loan portfolio.

Various papers evaluate the driving forces behind mortgage defaults and the resulting losses. Campbell and Cocco (2011), for example, examine default risks and develop

[1] Lown and Morgan (2006) show that lending standards (as measured by the Loan Officer Opinion Survey of the Federal Reserve) are a key driver of loans.

[2] See e.g. Financial Stability Board (2011).

a model where households maximize their discounted future utility from consumption and housing. They finance their house by a mortgage and decide in each period whether or not to default on the mortgage. The authors assume that the mortgage lender has no recourse to the defaulter's income or savings. Following their model, households decide to default when their home equity turns negative, meaning the value of the house becomes smaller then the outstanding mortgage loan. However, the authors show that if borrowing constrains are less binding (due to a higher income), households might decide not to default even when home equity is negative.

Deng, Yongheng, Quigley, and Van Order (2000) argue that a mortgage borrower has two separate options: a prepayment option and a default option. The authors develop a unified model of these two options and show that the simultaneity of the two options can help to explain borrower behavior. However, since in many countries there are prepayment penalties and lenders have recourse to defaulters' income, the results are mainly relevant for some US states like California.

Haughwout, Peach, and Tracy (2008) evaluate possible reasons for the strong increase in early mortgage defaults in the US in 2006 and 2007. In their empirical estimation they use credit risk variables like loan-to-value ratios and debt service-to-income ratios as well as variables that capture the economic conditions like regional unemployment rates or house prices. Their results indicate that both bad credit standards and bad economic conditions contributed to the increase in defaults and that the economic conditions had the largest impact. However, the empirical model only predicts less than half of the strong increase in early defaults after 2006.

While most studies focus on default rates of mortgages, Qi and Yang (2009) evaluate different influence factors of the loss given default. Their empirical study is based on a large pre-crisis loan level data set and indicates that the current loan-to-value ratio is the single most important determinant of the loss given default.

The aim of this paper is to develop a theoretical model of mortgage loss rates, meaning the product of default probabilities and loss given defaults. Furthermore, the model serves as a basis for macroprudential instruments like countercyclical buffers. The development of a foundation for these macroprudential instruments is the main contribution of this paper.

In the theoretical model, banks provide mortgages to *a priori* identical households. A heterogenous development of the households' income and house prices leads to the default of some households in the following period. In order to demonstrate the empirical relevance of the model, we calibrate it for two countries that experienced pronounced real estate crises within the past 25 years but had a different development: the US and Switzerland. The results of the calibration are used to demonstrate the ability of the model to estimate the impact of stress scenarios on mortgage losses, to calculate the size and development of countercyclical buffers, and to set standards for risk weights on mortgages with different loan-to-values and loan-to-incomes.

The paper is organized as follows: in the next section we develop the theoretical model, in section 3 we calibrate the model, section 4 shows potential applications of the model for macroprudential instruments, and section 5 offers some concluding remarks.

2 The Model

In this section we develop a theoretical two-period model that enables us to calculate loss rates for mortgage loans. The basic setting of the model is very similar to Hott (2011): in period $t = 1$ banks provide mortgages to *a priori* identical households. In period $t = 2$ each household receives a random labor income and defaults if this labor income plus the value of the house is too low to fulfill the mortgage duties. In contrast to Hott (2011), however, we assume that the constant loan-to-value of mortgages is less then 100%. In addition, we consider unemployment, maintenance costs for houses, foreclosure costs, and a heterogenous development of house prices. By introducing these additional features, the model becomes more realistic and, hence, more suitable for describing actual loss rates on mortgages.

2.1 Basic Assumptions

2.1.1 Houses

There are S *ex ante* identical houses and in period $t = 1$ the price of house i is $P_1^i = P_1$, where i=1,...,S. Therefore, the value of the entire housing stock is SP_1. Further, we assume that houses are subject to depreciation and that owning a house leads to maintenance costs. The sum of the depreciation and the maintenance costs (as a fraction of the house price) is assumed to be $1 > \rho' \geq 0$.

In period $t = 2$ the price of house i is assumed to be uniformly distributed between $(1 - \delta)P_2$ and $(1 + \delta)P_2$, where P_2 is the average house price in $t = 2$ and $\delta > 0$ is the maximum relative deviation from this average price. Therefore, in $t = 2$ the value of the entire housing stock is SP_2.

2.1.2 Mortgages

In period $t = 1$ banks provide mortgages to households at the interest rate $m_1 > 0$. Since banks cannot *ex ante* differentiate between different households and different houses, this mortgage rate is identical for each household. The loan-to-value (LTV) is the constant l, where $0 < l < 1$.[3] Therefore, in period $t = 1$ the amount of all mortgages is lSP_1. The maturity of each mortgage loan is assumed to be one year. This implies that households have to refinance their mortgage every year.

When a mortgage borrower defaults, the house goes into the ownership of the bank. We assume that the foreclosure and change in ownership causes costs. These foreclosure costs are assumed to be a fixed fraction ϕ of the amount of the mortgage. We further assume that the bank has recourse to the defaulter's income.[4]

2.1.3 Households

There are N *ex ante* identical households. For simplicity, we assume that $N = S$. Each of them buys the same fraction of the housing stock: $S/N = 1$. It uses its entire wealth to finance the fraction $(1 - l)$ of its housing investment. To finance the rest of the investment, each household takes out a mortgage in the amount of lP_1.

On the one hand, in period $t = 2$ each household bears depreciation and maintenance costs of $\rho l P_1$, where $\rho = \rho'/l$. In addition, each household has to pay the mortgage rate and has to pay back the mortgage: $(1 + m_1)lP_1$. Hence, its total expenses are: $(1 + \rho + m_1)lP_1$. Further we assume that depreciation and maintenance costs are senior to the mortgage payments.

On the other hand, household i owns a house which can be sold at the price P_2^i, where P_2^i is uniformly distributed between $(1-\delta)P_2$ and $(1+\delta)P_2$. In addition, we assume that in $t = 2$ household i receives a labor income Y_2^i that is uniformly distributed between zero and Y_2. With probability u_2 a household becomes unemployed and receives an income of zero. In reality, in most countries the minimum (transfer) income

[3]Iacoviello (2005) and Kiyotaki and Moore (1997) also assume fixed LTV ratios when evaluating the link between real estate prices and output. Campbell and Cocco (2011) show that in the US, LTV ratios were relatively stable between 1984 and 2008.

[4]This might not be the case in some US states. However, following the results of Campbell and Cocco (2011), income also matters if there is no recourse. In addition, as we will see, our model provides relatively good results also for the US (see section 3).

is greater then zero. However, since we do not consider consumption expenditures, we assume that the minimum income is used for an autonomous consumption and can, therefore, not be used for mortgage payments.

Household i becomes insolvent in $t = 2$ if:

$$P_2^i + Y_2^i < (1 + \rho + m_1)lP_1 \qquad \text{or}$$
$$Y_2^i < \left(\rho + m_1 - \tfrac{P_2^i - lP_1}{lP_1}\right) lP_1. \tag{1}$$

The right-hand side of equation (1) is also known as the imputed rent for housing. This imputed rent is in line with the assumptions of many other studies on real estate and mortgage markets. Poterba (1984 and 1992), McCarthy and Peach (2004), Himmelberg, Mayer, and Sinai (2005), and Hott and Monnin (2008), for example, use very similar factors to define imputed rents.

2.2 Calculation of Loss Rates

To calculate loss rates on mortgage loans, we first calculate the default probability (PD) and the loss given default (LGD) of mortgages independently. The expected loss rate is given by the multiplication of the PD and the LGD.

The calculation of the PD and the LGD depends strongly on the development of house prices in $t = 2$. There are two extreme cases. In the first case, house prices increase so strongly that no household defaults ($PD = 0$). This is the case if $(1-\delta)P_2 \geq (1 + \rho + m_1)lP_1$. In the other extreme case, house prices fall so strongly that all households default ($PD = 1$). This is the case if $(1+\delta)P_2+Y_2 \leq (1+\rho+m_1)lP_1$. Both cases are not very realistic and will, therefore, not be considered. The intermediate case can be divided into several subcases as well. However, we will only consider the following case as the relevant case:[5]

$$\frac{(1 + \rho + m_1)lP_1}{1 + \delta} \leq P_2 \leq \frac{(1 + \rho + m_1)lP_1}{1 - \delta}$$
$$\text{and } \frac{(1 + \rho + m_1)lP_1 - Y_2}{1 - \delta} \leq P_2. \tag{2}$$

[5]The calibration of the model confirms that this is the relevant case. See also Hott (2011).

In this range, neither households with the best performing houses ($P_2^i = (1+\delta)P_2$) nor households with the highest labor income ($Y_2^i = Y_2$) default. However, there are always at least some households that default.

2.2.1 Probability of Default

Figure 2 illustrates which households default and which stay solvent. As we can see, for a household with a labor income greater than $[(1+\rho+m_1)lP_1 - (1-\delta)P_2]$, the probability of a default is zero. Therefore, the probability that an employed household has an income that leads to a positive default probability $\Pr(PD > 0|employed)$ is:

$$\Pr(PD > 0|employed) = \frac{(1+\rho+m_1)lP_1 - (1-\delta)P_2}{Y_2}. \tag{3}$$

If a household is unemployed, this probability is one.

For households that can default ($PD > 0$), the default probability decreases with labor income. For an unemployed household with zero labor income ($Y_2^i = 0$), the probability of default is:

$$PD(Y_2^i = 0) = \frac{(1+\rho+m_1)lP_1 - (1-\delta)P_2}{2\delta P_2}. \tag{4}$$

Under the condition that the labor income of an employed household is in a range where there is a positive default probability, the default probability is 50% of $PD(Y_2^i = 0)$.[6] Multiplied by the probability $\Pr(PD > 0|employed)$ from equation (3) we obtain the default probability of an employed household:

$$PD(employed) = \frac{[(1+\rho+m_1)lP_1 - (1-\delta)P_2]^2}{4\delta P_2 Y_2}. \tag{5}$$

The unconditional PD is given by:

$$\begin{aligned} PD = \quad & u_2 \tfrac{(1+\rho+m_1)lP_1 - (1-\delta)P_2}{2\delta P_2} \\ & + (1-u_2)\tfrac{[(1+\rho+m_1)lP_1 - (1-\delta)P_2]^2}{4\delta P_2 Y_2}. \end{aligned} \tag{6}$$

[6] Since each income in this range is equally likely.

As we can see, the PD depends positively on the unemployment rate (u_2), the depreciation and maintenance costs (ρ), the LTV, the mortgage rate (m_1), the house price level in $t = 1$ (P_1), and the maximum deviation from the average house price in $t = 2$ (δ).[7] It depends negatively on the house price increase ($P_2 - P_1$) and on income (Y_2). Note that condition (2) assures that (3) to (6) lie between 0 and 1.

2.2.2 Loss Given Default

There are two factors that have an influence on losses for a bank when a mortgage borrower defaults. Firstly, the difference between the outstanding mortgage (lP_1) plus interest, depreciation and maintenance costs[8] ($(\rho+m_1)lP_1$), and the value of the house in period 2 (P_2^i) plus the income of the house (Y_2^i). The second factor are the foreclosure costs (ϕlP_1). As a result, the loss given default (LGD) of a mortgage to household i is:

$$LGD(P_2^i, Y_2^i) = \frac{(1 + \rho + m_1)lP_1 - P_2^i - Y_2^i}{lP_1} + \phi \tag{7}$$

For an unemployed household with zero income, the LGD can vary between ϕ and $[(1+\rho+m_1)lP_1 - (1-\delta)P_2]/[lP_1] + \phi$. Since each of the LGD in this range is equally likely, the expected LGD of an unemployed household is:

$$LGD(Y_2^i = 0) = \frac{(1 + \rho + m_1)lP_1 - (1 - \delta)P_2}{2lP_1} + \phi \tag{8}$$

The LGD of a mortgage to an employed household decreases linearly with the labor income of the borrower. It varies between ϕ for $Y_2^i = (1 + \rho + m_1)lP_1 - (1 - \delta)P_2$ (the maximum income for a PD greater than zero) and $LGD(Y_2^i = 0)$ for an employed household with zero income. As we have seen in section 2.2.1, the different LGDs are not equally likely, however. The probability that a household defaults varies between zero and the result of equation (4). The expected LGD of a mortgage to an employed household is therefore:

$$LGD(employed) = \frac{(1 + \rho + m_1)lP_1 - (1 - \delta)P_2}{3lP_1} + \phi. \tag{9}$$

[7] The relationship between δ and PD depends on the development of the house price. In the relevant range, however, $\partial PD/\partial\delta$ is positive.

[8] Depreciation and maintenance costs have to be borne by the household or the bank. However, in both cases the loss for the bank would be the same.

This LGD depends positively on ϕ, ρ, m_1, l, δ, and P_1, it depends negatively on P_2 and it is independent of the maximum income Y_2.

2.2.3 Expected Loss Rate

The expected loss rate in period $t = 2$ (EL_2) is given by the weighted average of the product of the PD and LGD for unemployed and employed households:

$$EL_2 = u_2 \frac{(1+\rho+m_1)lP_1-(1-\delta)P_2}{2\delta P_2}\left[\frac{(1+\rho+m_1)lP_1-(1-\delta)P_2}{2lP_1}+\phi\right] + (1-u_2)\frac{[(1+\rho+m_1)lP_1-(1-\delta)P_2]^2}{4\delta P_2 Y_2}\left[\frac{(1+\rho+m_1)lP_1-(1-\delta)P_2}{3lP_1}+\phi\right]. \quad (10)$$

As we can see, the expected loss rate depends negatively on income (Y_2) and on the house price increase ($P_2 - P_1$) and it depends positively on the unemployment rate (u_2), the foreclosure costs (ϕ), the house price level (P_1), the loan-to-value (l), and the mortgage rate (m_1). Furthermore, in the relevant range[9] the expected loss rate also depends positively on the heterogeneity of the house price development (δ).

These results are in line with the usual empirical findings. Deng et al. (2000), for example, find empirical evidence that increasing interest rates and decreasing house prices increase the probability of default. Furthermore, the authors find evidence that a higher LTV leads to higher defaults. Qi and Yang (2009) examine defaults in the US mortgage market and find the current LTV is the most important determinant of the loss given default of a mortgage. Lambrecht, Perraudi, and Satchell (1997) examine the UK mortgage market and find a positive relationship between interest rates and default probabilities and a negative relationship between income and default probabilities. Surprisingly, the authors also find a negative relationship between the LTV and default probabilities. One explanation for this finding could be that high LTV mortgages are only provided to households with low credit risk.

3 Model Calibration

To examine the empirical relevance of the model, we calibrate it for two different countries: the US and Switzerland. Both countries have experienced a major real estate crisis within the past 25 years. However, the overall development of the two mortgage

[9] See equation (2).

markets was rather different. Besides an in sample calibration we also calibrate the crisis development out of sample, i.e. using only data up to 2005 Q4.

3.1 Data

According to equation (10) we need data on mortgage rates (m_t), house prices (P_t), unemployment rates (u_t), and the maximum household income (Y_t) to calibrate loss rates. The maximum household income is calculated as $(2GDP_t)/[(1-u_t)N_t]$ and by using nominal GDP and population data (N_t). To compare the resulting theoretical expected loss rates with reality, we also need data on actual loss rates (L_t). The frequency of most data series is quarterly. Annual population data and the annual loss rates for Switzerland are transformed into quarterly data by linear interpolation. Table 1 provides a brief description of the data, the sources, and transformations.

3.2 Parameter Values

According to equation (10) we need parameter values for the LTV (l), the heterogeneity of the house price development (δ), the depreciation and maintenance rate (ρ), and the foreclosure costs (ϕ) for both countries. In addition, we need a conversion factor α to adjust the level of GDP to the house price level ($Y_t = \alpha 2GDP_t/[(1-u_t)N_t]$). Firstly, house prices (index values) and GDP are not expressed in the same unit and, secondly, Y reflects only the (constant) fraction of the income that is available for housing expenditures.

The first parameter is the loan-to-value (l). According to Green and Wachter (2005), in the US the average LTV is 75% but it can go up to 97%. For the calibration we use $l = 0.75$ for the average LTV in the US. In Switzerland a mortgage usually has a LTV of 67% and it can go up to 100%. We use $l = 0.67$ for the calibration. To capture the variation of the loan-to-value in both countries, we consider the difference between the maximum and the average LTV in the heterogeneity of the house price development (δ).

The second parameter is the heterogeneity of the house price development (δ). When calibrating a mortgage model for the UK, Miles (2005) uses 12.5% as the standard deviation of house price growth. Campbell and Cocco (2003) consider 11.5% when evaluating the US mortgage market. This would imply around 22% for our parameter

δ.[10] As already mentioned, we will also consider the difference between the maximum and the average LTV in the parameter δ. We assume that the difference between the maximum and the average LTV is equal to three standard deviations of the LTV. Therefore, the add-on for δ (maximum of a uniform distribution) is $\sqrt{3}/3 = 1/\sqrt{3}$ times the difference between the maximum and the average LTV. For the US the resulting δ is 35%, and for Switzerland it is 41%.

The literature offers different estimates of the depreciation and maintenance rate. Harding, Rosenthal, and Sirmans (2006) estimate that the annual rate is between 2.4 and 2.9 percent. McCarthy and Peach (2004) assume that the depreciation rate plus repairs is 2.5 percent per year and Poterba (1992) assumes that the sum of depreciation and maintenance rate is four percent. For simplicity, we use $\rho' = 2.5\%$ for each of the two countries. Therefore, we obtain $\rho = \rho'/l = 3.3\%$ for the US and $\rho = \rho'/l = 3.7\%$ for Switzerland.

Foreclosure costs (ϕ) involve legal fees, taxes, insurance, commissions etc. Although these costs might vary between countries, for simplicity, we assume that they are 15% in both countries. This figure is based on anecdotal evidence from contacts with banks.

The fifth parameter is the conversion factor α. For this parameter we do not have an *a priori* assumption (except that it is positive). Therefore, we chose a parameter value for α that solves the following minimization problem:

$$\min_{\alpha} \sum_{t=0}^{T} \left[EL_t - L_{t+k}\right]^2 \tag{11}$$

subject to $\alpha > 0$. While $T - k$ is equal to the end of the data sample (2010 Q4 for Switzerland and 2011 Q2 for the US) in our in sample calibration, it is 2005 Q4 in our out of sample calibration. The parameter k reflects the lead of the theoretical series compared to the actual series expressed as the number of quarters. The lead is derived by looking at the maximum cross correlation of the two series. The choice of the different parameter values is summarized in Table 2.

[10] The maximum deviation of the uniformly distributed heterogeneity of the house price development is $\sqrt{3}$ times the standard deviation.

3.3 Calibration Results

Figure 3 displays the development of actual (L_t) and theoretical expected (EL_t) in and out of sample loss rates for the US and Switzerland. As we can see, the fit between actual and theoretical in sample loss rates is quite good. In particular the model is able to mirror the high loss rates during the crisis episodes: The current real estate crisis in the US and the crisis in the early 1990s in Switzerland. According to Table 2, the theoretical expected losses have a lead of $k = 4$ quarters. This reflects that in reality it can take several periods until a bank knows how high the losses from a defaulted mortgage really are.

For Switzerland, the difference between the in and out of sample expected loss rates is negligible. For the US, however, we can observe a difference. This is no surprise, since charge-off rates in the US experienced an unprecedented strong increase after the end of 2005 and, therefore, the end of the considered data for the out of sample calibration. Nevertheless, even the out of sample calibration of the crisis leads to a relatively good fit with the actual development.

4 Macroprudential Applications of the Model

Our model and its calibration give us information about the impact of macroeconomic developments on mortgage losses. This information can be used as an input for macroprudential instruments. Important examples of macruprudential instruments are conducting stress tests, introducing countercyclical buffers, and setting risk weights for mortgages with different loan-to-value and loan-to-income ratios.

4.1 Stress Tests

As a reaction to the recent crisis and in order to identify systemic risks, banking supervisors and central banks are conducting more and more stress tests. In theses stress tests banks are asked to calculate the impact of an adverse combination of macroeconomic shocks on their profits and losses and, therefore, on their capital needs.

Our model can provide us with an alternative view on the impact of an adverse scenario on a macro level. To demonstrate this, we calculate the aggregated impact of the adverse scenarios from the 2009 Supervisory Capital Assessment Program (SCAP) of the Federal Reserve Board for the US and the adverse scenario from the 2011 EU-wide stress test of the European Banking Authority (EBA) for the UK.[11] The scenarios are summarized in Table 3.

The impact of the scenarios on expected losses (EL_t) is presented in Figure 4. The solid (SCAP) and the broken grey (EBA) lines indicate that both scenarios lead to an increase in expected losses. As we can see, the impact of the SCAP scenario is much stronger then the impact of the EBA scenario. This is not a surprise since the assumed GDP and house price declines are much higher. Another noteworthy outcome is that both scenarios have a stronger impact on expected losses in the US than on expected losses in Switzerland. The main reason for this is that we assume a higher LTV for the US.

Compared to the estimates of the Fed with regard to the impact of the SCAP scenario on US banks, our expected loss rates are relatively low. While the Fed estimates indicate an aggregated two year loss rate of 7 to 8.5%,[12] our model indicates only 5% with an annual maximum of 3.3%. One reason for this might be that we are calculating the expected losses under an adverse scenario and not adverse losses under this scenario. According to Table 2, the difference between our estimate and the Fed estimate is about three times the average difference between the expected and actual loss rates.

4.2 Countercyclical Buffers

With a countercyclical buffer, banks have to hold more capital when imbalances in the credit market are building up. The aim of this additional requirement is that banks have a higher capital buffer which they can use when the bubble bursts. Therefore, the countercyclical buffer should compensate for the additional risk that arises from the increasing imbalances. A further potential effect of the countercyclical buffer is that it can help to lean against emerging bubbles.

The Basel Committee on Banking Supervision (BCBS, 2010) proposes monitoring the ratio of credit to GDP and the deviation from its trend to assess the phase of the cycle and the build-up of the imbalances. The size of the buffer should then increase with this indicator between zero and a (more or less arbitrary) maximum buffer size.

In our model the credit (or mortgage) volume is defined as a constant fraction l of the house price P_t multiplied by the housing stock $S = N$. GDP, on the other hand,

[11] The EBA stress test included scenarios for all EU countries. However, we only consider the scenario for the UK.

[12] See Federal Reserve Board (2009b).

is given by $(1-u_t)NY_t/2$. Hence, the ratio of credit to GDP (i_t) is given by:

$$i_t = \frac{2SlP_t}{(1-u_t)NY_t} = \frac{2lP_t}{(1-u_t)Y_t}. \quad (12)$$

As an indicator for imbalances we take the difference between i_t and its average $i^{average}$. To determine the size of the countercyclical buffer, we look at the impact of an above-average i_t on expected losses (EL_t). We rewrite equation (10) to:

$$\begin{aligned} EL_t = \quad & u_t \tfrac{lR_{t-1}-(1-\delta)(1+w_t)}{2\delta(1+w_t)} \left[\tfrac{lR_{t-1}-(1-\delta)(1+w_t)}{2l} + \phi\right] \\ & + i_t(1-u_t)^2 \tfrac{[lR_{t-1}-(1-\delta)(1+w_t)]^2}{8\delta l(1+w_t)} \left[\tfrac{lR_{t-1}-(1-\delta)(1+w_t)}{3l} + \phi\right], \end{aligned} \quad (13)$$

where $R_{t-1} = 1 + \rho + m_{t-1}$ and $1 + w_t = P_t/P_{t-1}$. The impact of an above-average i_t on expected losses is then given by:

$$\begin{aligned} & buffersize = \Delta EL_t = \\ & (i_t - i^{average})(1-u_t)^2 \tfrac{[lR_{t-1}-(1-\delta)(1+w_t)]^2}{8\delta l(1+w_t)} \left[\tfrac{lR_{t-1}-(1-\delta)(1+w_t)}{3l} + \phi\right]. \end{aligned} \quad (14)$$

Capital requirements should make sure that banks hold enough capital to cover losses in stress events. As a stress scenario for the capital requirements we take the strongest historical drop in real estate prices in combination with the average mortgage rate or R_t, respectively. According to equation (14), the unemployment rate has a negative effect on the buffer size. One reason for this is that the loan-to-income (LTI) is only important for households that are employed. With a lower unemployment rate the LTI is relevant for a higher fraction of the population. Another reason is that, given a level of GDP, the income of an employed household has to be higher the higher the unemployment rate is. Hence, for the stress scenario we take the average unemployment rate. The scenarios for the two countries are summarized in Table 4.

Figure 5 shows the evolution of the resulting buffer size. It is assumed (as in the BCBS proposal) that the buffer cannot be negative. As we can see, the countercyclical buffer increases before the crises begin and loss rates increase. Hence, banks can use the capital buffer to cover losses and the buffer requirements are gradually reduced to zero.

The maximum buffer size is about 0.47% of the mortgage volume in the US and about 0.25% in Switzerland. Reasons for the higher buffer size in the US are the more adverse scenario and a higher LTV but also a greater imbalance during the stress episodes. In contrast to the BCBS proposal, our buffer is not based on risk-weighted assets but on loan volumes. If we assume average risk weights of about 35%, the buffer size would reach a maximum value of about 1.34% in the US and about 0.71% in Switzerland.

4.3 Risk Weights

The loan-to-value and the loan-to-income are the most important parameters for banks to influence the risk profile of their mortgage loan book. According to equation (13), both parameters have a positive effect on expected losses. Hence, risk weights for capital requirements should increase with LTV and LTI.

Our model can help to estimate to which extent risk weights should increase with both parameters and how strong the tradeoff between both parameters is. Figure 6 shows the impact of a change in LTV on expected losses for different LTI. These expected losses are calculated according to equation (13). As in section 4.2 we consider a stress scenario with the strongest historical drop in real estate prices and an average interest rate level. Since the unemployment rate has a negative effect on expected losses, we take the highest historical unemployment rate (US: 10.7%, Switzerland: 5.4%).

According to Figure 6, expected losses are higher in the US than in Switzerland. The main reason for this is that the US scenario assumes a much higher drop in real estate prices. In general, however, the relationship between LTV, LTI, and expected losses is very similar: Expected losses increase with increasing LTV and with increasing LTI. In addition, we can see that the impact of an increasing LTV is stronger when the LTI is higher. These effects should be reflected when risk weights for capital requirements for mortgages loans are set.

5 Conclusions

In this paper we develop a theoretical model of mortgage loss rates. Following the model, loss rates are positively influenced by the house price level, the unemployment rate, the loan-to-value, and interest rates. They are negatively influenced by the income level and the growth of house prices. The calibration of the model for the US and Switzerland has demonstrated that it is well able to describe the overall development of the actual loss rates.

Potential applications of the model are the calibration of loss rates under stress events, the calculation of the size and development of countercyclical buffer, and the setting of risk weights for different combinations of loan-to-value and loan-to-income.

The model has two main shortcomings: First, the maturity of mortgage loans is assumed to be one year. In reality, however, most mortgages have a much longer maturity. Therefore, actual reactions to house price declines are smaller than predicted by the model when they are only temporary, but they can become larger when house prices decline over a longer period of time. A second shortcoming of the model is that all the parameters are assumed to be constant. In reality, especially the loan-to-value can react to the market sentiment and the lending conditions of banks and can therefore vary over time. In spite of these shortcomings, however, the model is able to explain the main developments of mortgage loss rates in the US and Switzerland. In addition, it can serve as a basis for various macruprudential instruments. Efforts to make the model more realistic should be aligned to the specific needs of its application.

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Figure 1: **Mortgages and Charge-Off Rates in the US**

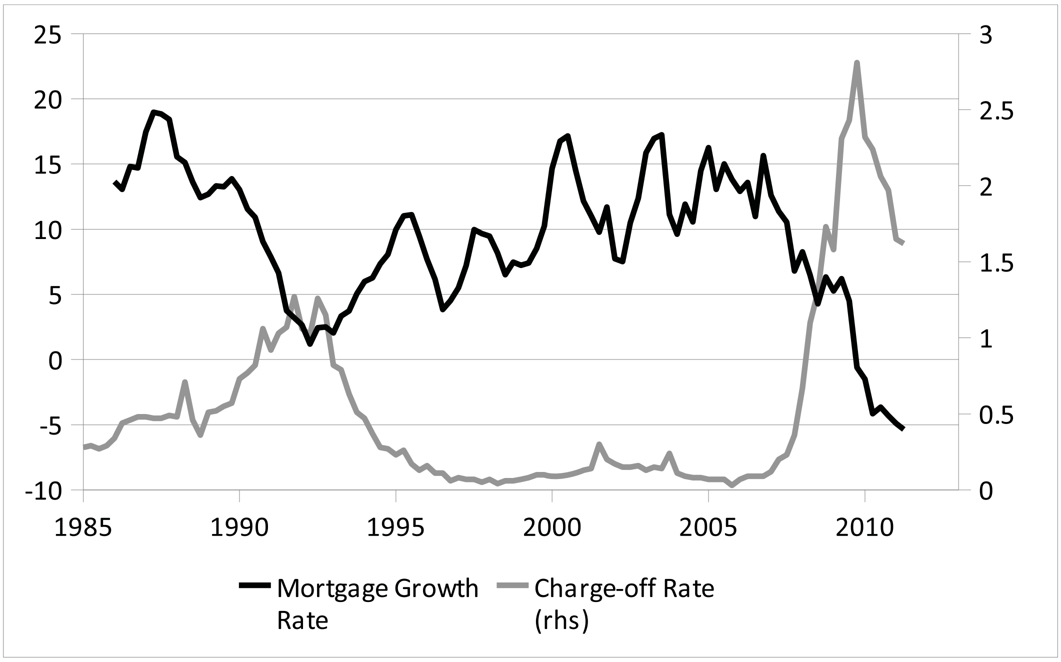


Figure 2: **Income, Mortgage Payments and Defaults**

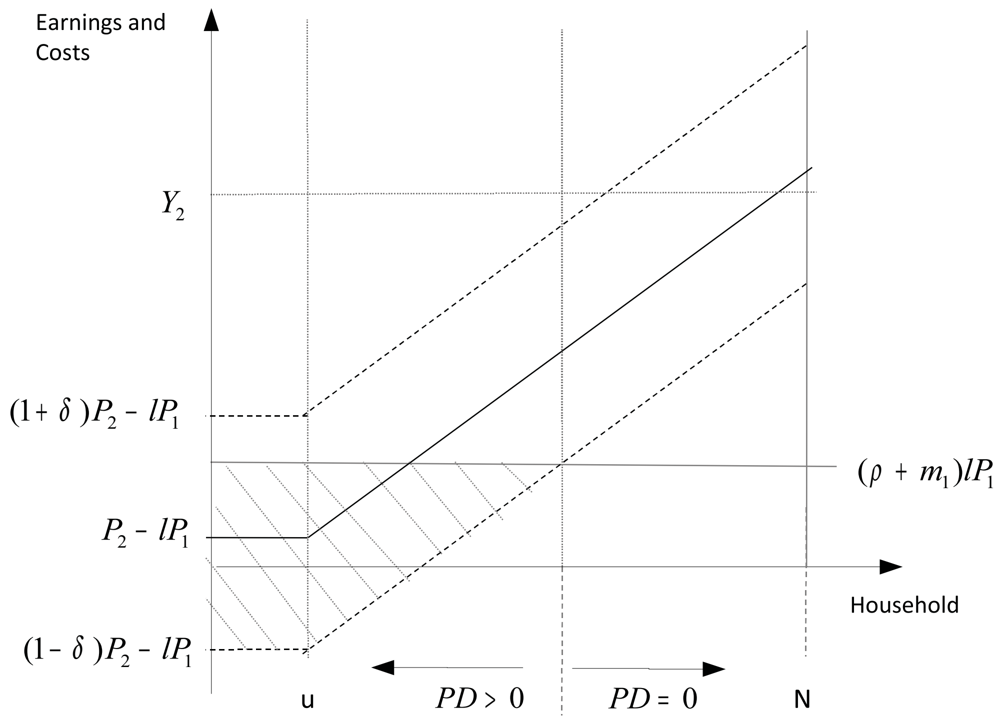
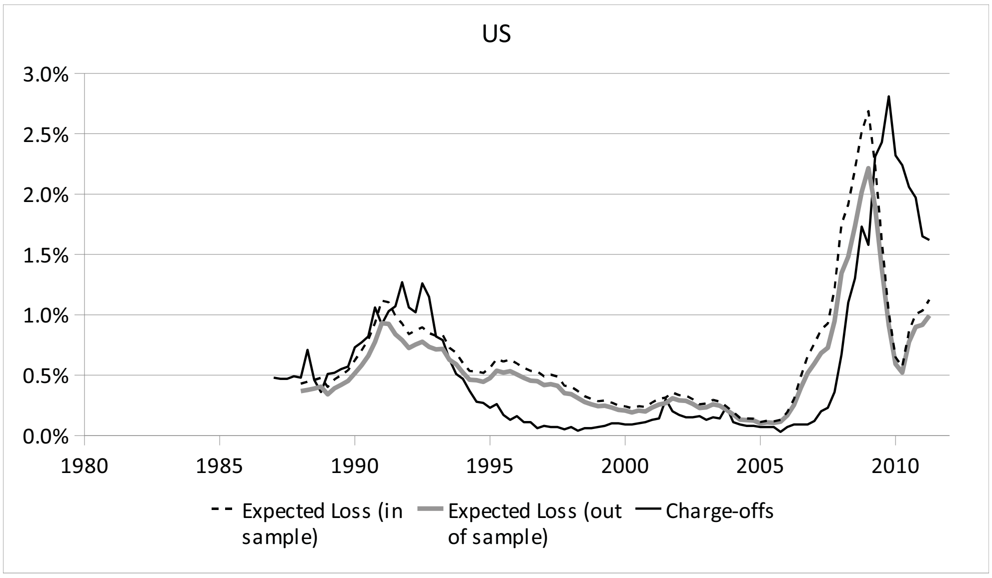


Figure 3: **Actual (L) and Theoretical (EL) in and out of Sample Mortgage Loss Rates**



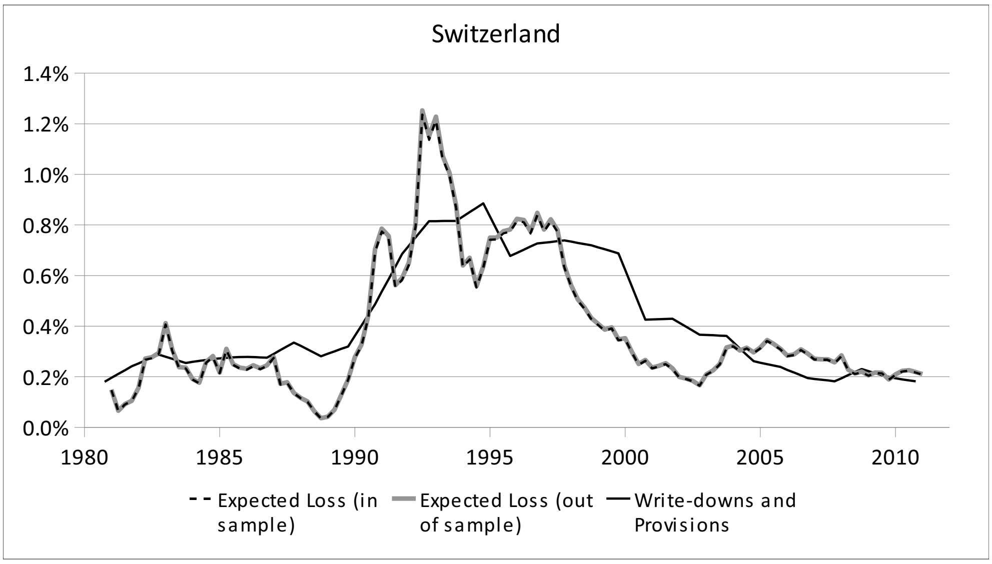


Figure 4: **The Impact of Adverse Scenarios on Expected Losses**

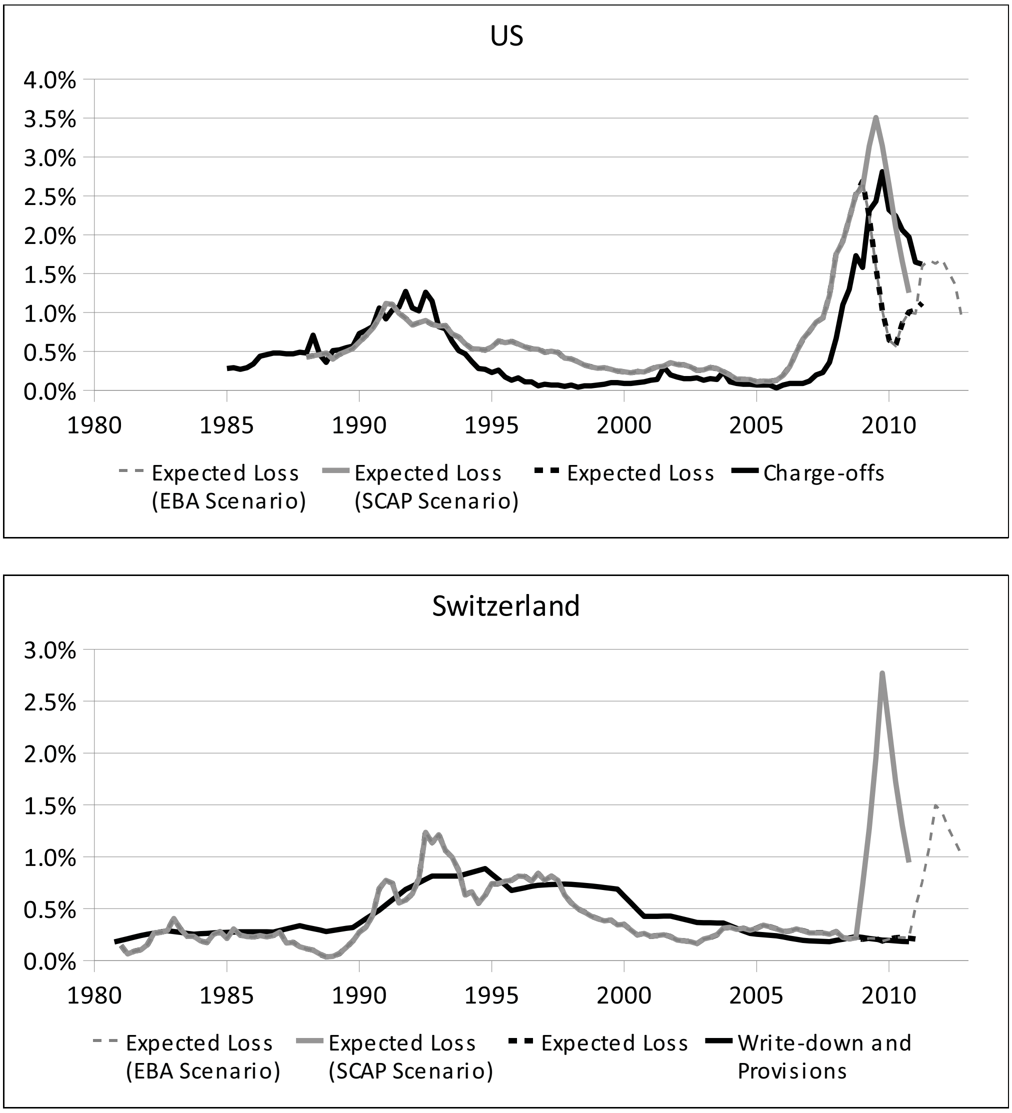


Figure 5: **Development of the Countercycal Buffer**

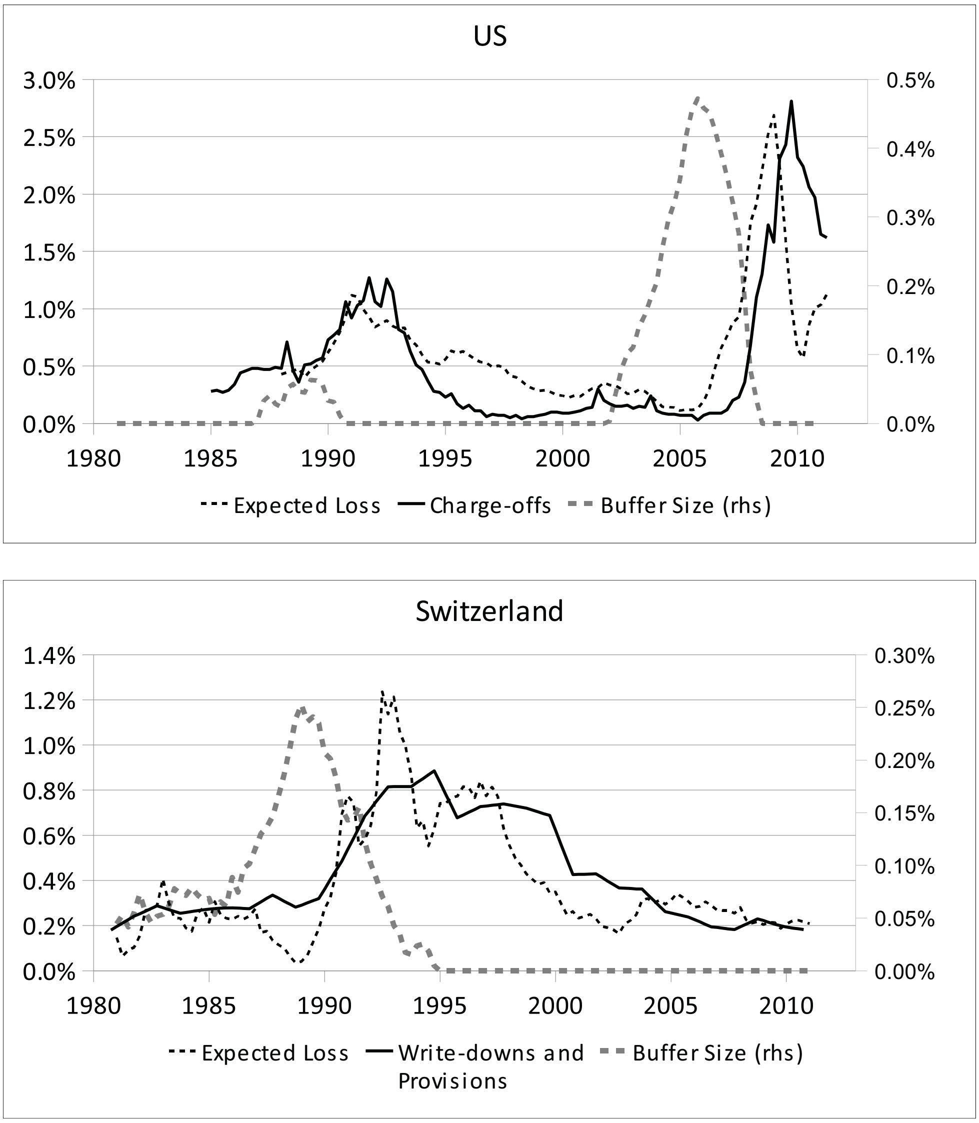


Figure 6: **Impact of LTV on Expected Losses for Different LTI**

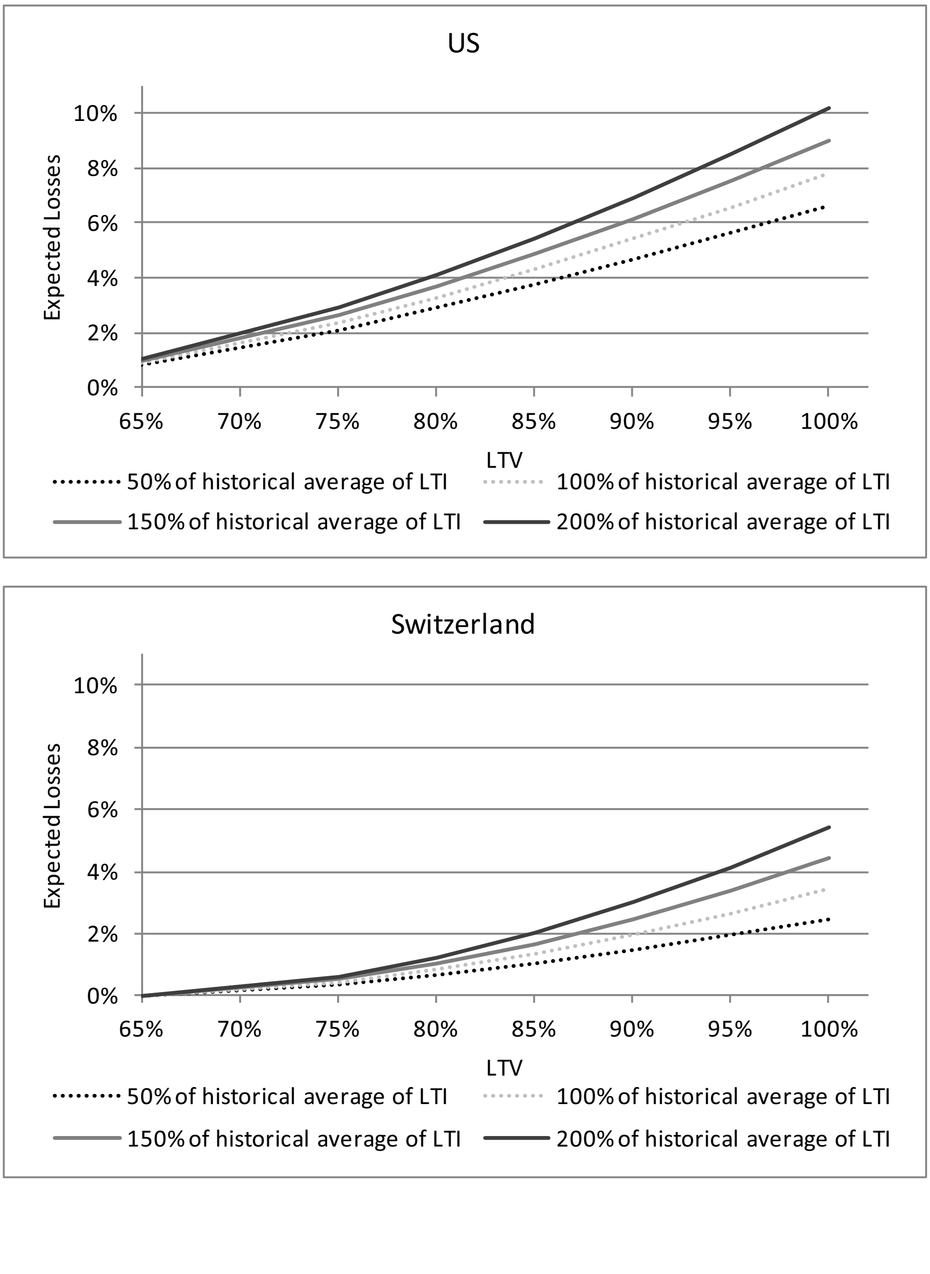


Table 1: **Data description**

		US	CH
m_t	*name*:	Interest rate on conventional mortgages, 30 years	Average mortgage rate
	source:	BIS	Swiss National Bank
P_t	*name*:	Case-Shiller National Seasonally Adjusted Home Price Values	Residential Prop. Pr., All One Family Houses
	source:	Standard & Poors	Wuest & Partner
GDP_t	*name*:	Gross Domestic Product SA	Gross Domestic Product SA
	source:	IMF (IFS)	IMF (IFS)
N_t	*name*:	Population	Population
	source:	IMF (IFS)	IMF (IFS)
	transf.:	Annual data is transformed into quarterly data by linear interpolation.	Annual data is transformed into quarterly data by linear interpolation.
u_t	*name*:	Unemployment Rate SA	Unemployment Rate SA
	source:	IMF (IFS)	IMF (IFS)
L_t	*name*:	Charge-off rate on real estate loans	Write-downs and Provisions Rate
	source:	Federal Reserve Board	Swiss National Bank
	transf.:		Write-downs and provisions to total credits of Regional Banks and Raiffeisen banks (more than 90% mortgages in their loan portfolio); annual into quarterly data by linear interpolation.

Table 2: **Parameter values for conversion factor α, heterogony of house price development δ, LTV l, foreclosure costs ϕ, maintenance costs ρ and number of lags k in quarters.**

	US	CH
δ	35%	41%
l	75%	67%
ϕ	15%	15%
ρ	3.3%	3.7%
α in sample	6.5830	0.0022
k in sample	4	4
α out of sample	10.688	0.0022
k out of sample	-1	4

Table 3: **Adverse scenarios for the SCAP and the EBA exercise. Source: Federal Reserve Board (2009a) and European Central Bank (2011).**

		GDP (Y on Y)	Unemployment Rate (Level)	House Prices (Y on Y)
SCAP	2009	-3.30%	8.9%	-22.00%
	2010	0.50%	10.3%	-7.00%
EBA UK adverse scenario	2011	-0.70%	9.0%	-12.50%
	2012	0.90%	10.6%	-7.00%

Table 4: **Scenario for the calculation of the countercyclical buffer.**

	US	CH
w	-18.9%	-7.4%
m	8.9%	4.7%
u	5.9%	2.5%