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Evaluation of minimum capital requirements for bank loans to SMEs

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Non-technical summary

The purpose of this paper is to examine firm size as a driver of systematic credit risk in lending to small and medium enterprises (SMEs) by means of an empirical analysis based on historical default rates of German SMEs. The dependence of systematic risk on firm size is compared with the size-dependent capital relief granted to SME lending in the regulatory minimum capital requirements of Basel II and Basel III. Key contributions of this paper are the use of a unique data set of SME lending by over 400 German banks covering both small and large banks and an evaluation of the asset correlations as a measure for systematic risk in the context of current regulatory capital requirements. Since regulatory capital requirements can affect the interest margins required by the lender, only their appropriate risk sensitivity will ensure an optimal credit supply.

We find that the relative differences between the capital requirements for large corporates and those for SMEs (in other words, the capital relief for SMEs in Basel II) are in two cases lower in the current regulatory framework than implied by the empirically estimated asset correlations: (1) In the Internal Ratings-Based (IRB) Approach this difference amounts to up to 24 percentage points on average across rating categories. This concerns only SME loans in the corporate portfolio. (2) For all loans assigned to the SME portfolio in the Revised Standardized Approach (RSA), this effect is considerably stronger. Before interpreting these results as an indication to lower international regulatory capital requirements, one should also consider the deliberately more conservative calibration of the less risk-sensitive RSA (compared with the IRB approach) and the Germany-specific robustness of the credit quality of SMEs within the sample period.

Nicht-technische Zusammenfassung

In dieser Studie soll mithilfe einer empirischen Analyse auf Basis historischer Ausfallraten deutscher kleiner und mittlerer Unternehmen (KMU) untersucht werden, inwiefern sich die Unternehmensgröße auf das systematische Kreditrisiko bei der Kreditvergabe an KMUs auswirkt. Die Abhängigkeit des systematischen Risikos von der Unternehmensgröße wird der in den regulatorischen Eigenkapitalanforderungen von Basel II und III gewährten Eigenkapitalentlastung bei der Kreditvergabe an KMUs gegenübergestellt. Die Hauptbeiträge dieses Papiers sind die Verwendung eines einzigartigen Datensatzes zur Kreditvergabe an KMUs von mehr als 400 kleinen und großen deutschen Banken und eine Beurteilung der Assetkorrelationen als Maß für systematisches Risiko im Kontext der aktuellen regulatorischen Eigenkapitalanforderungen. Da sich die regulatorischen Eigenmittelanforderungen potentiell auf die Zinsmarge des Kreditgebers auswirken, kann eine optimale Kreditvergabe nur durch deren adäquate Risikosensitivität sichergestellt werden.

Die Studie zeigt, dass die relativen Unterschiede zwischen den Kapitalanforderungen für Großunternehmen und jenen für KMUs (in anderen Worten die Eigenkapitalentlastung für KMUs in Basel II) im derzeitig geltenden regulatorischen Rahmen in zwei Fällen geringer ausfallen als die empirisch geschätzten Assetkorrelationen vermuten lassen: (1) Im IRB-Ansatz beläuft sich die Differenz auf bis zu 24 Prozentpunkte auf Basis von gewichteten Durchschnitten über die Ratingklassen hinweg. Dies betrifft ausschließlich KMU-Kredite des Unternehmensportfolios. (2) Bei sämtlichen KMU-Krediten im Kreditrisikostandardansatz ist dieser Effekt deutlich größer. Bevor man diese Ergebnisse im Sinne einer Senkung der internationalen regulatorischen Eigenkapitalanforderungen interpretiert, sollte man auch die bewusst konservativere Kalibrierung des weniger risikosensitiven Kreditrisikostandardansatzes (im Vergleich zum IRB-Ansatz) und die für Deutschland typische robuste Bonität der KMUs im Untersuchungszeitraum berücksichtigen.

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Abstract

Our paper addresses firm size as a driver of systematic credit risk in loans to small and medium enterprises (SMEs). Key contributions are the use of a unique data set of SME lending by over 400 German banks and relating systematic risk to the size dependence of regulatory capital requirements. What sets our sample apart is its comprehensive coverage of the particularly rich and well developed credit market for SMEs in Germany. We estimate asset correlations as the key measure of systematic risk from historical default rates. Our results suggest that systematic risk tends to increase with firm size, conditional on the respective rating category. We also compare the size of this effect with the capital relief that has been granted in Basel II for SMEs relative to large firms. For SME loans in the corporate portfolio of the Internal Ratings-Based Approach and also for SME loans treated under the revised standardized approach of Basel II, our asset correlation estimates suggest a significantly larger relative difference from large firms than reflected in the regulatory capital requirements.

Keywords: Asset Correlation, Basel II, Minimum Capital Requirements, Single Risk Factor Model.

JEL classification: G 21, G 33, C 13.

*Phone: +49 69 9566 8404, +49 69 9566 3343. E-mail: klaus.duellmann@bundesbank.de, philipp.koziol@bundesbank.de. We have benefited from comments by Michel Dietsch, Signe Krogstrup, Massimo Libertucci, Caroline Liesegang, Jens Mehrhoff, Joël Petey, Peter Raupach and participants of the Deutsche Bundesbank Research Seminar, the LaRGE Workshop on SME Finance 2013, and the 16th Annual Conference of the Swiss Society for Financial Market Research (SGF). We thank Katharina Erhardt, Arne Kramer, Irina Sareika, and Gregor Singer for excellent research assistance. Discussion Papers represent the authors' personal opinions and do not necessarily reflect the views of the Deutsche Bundesbank or its staff.

1 Motivation and Overview

Our paper belongs to a well-established strand of empirical work on the systematic risk in loans to small and medium enterprises (SMEs). We explore in particular the dependence of systematic risk on firm size and compare the size of this effect with the capital relief granted to SME lending in the regulatory minimum capital requirements of Basel II.[1] Another key contribution is the use of a unique data sample of SME lending by over 400 German banks. What sets this sample apart is its comprehensive coverage of the particularly rich and well-developed credit market for SMEs in Germany, the availability of banks' internal ratings, and the capture of the recent financial crisis in the time series. The asset correlation is used as the key measure of systematic risk. It also drives the systematic risk in the Asymptotic Single Risk Factor (ASRF) model of Gordy (2003) that is the basis of the regulatory minimum capital requirements in the Internal Ratings-Based Approach (IRBA) of Basel II.

The asset correlation is estimated in the first step from historical default rates by the Maximum-Likelihood (ML) estimator of Gordy and Heitfield (2002). In the second step and based on the asset correlation estimates, the dependence of capital requirements on firm size is compared both with the dependence implicit in the current IRBA risk weight functions and with risk weights in the Revised Standardized Approach (RSA).[2] For this purpose we consider the relative reduction in systematic risk which is measured as a capital requirement in the ASRF model, with respect to large firms. Thereby, our study also contributes to the empirical question of an appropriate (relative) calibration of regulatory capital requirements for SME lending.

For our analysis it is important to separate a potentially higher *firm-specific* (idiosyncratic) risk of SMEs – that is typically reflected in higher default probabilities – from a potentially lower *systematic* risk of SMEs. Since capital requirements in the ASRF model refer by construction only to systematic risk, lower asset correlations (and therefore lower systematic risk) compared with large firms would *ceteris paribus* also suggest lower calibrated capital requirements for SMEs. Capital requirements of an SME loan in the IRBA depend on both, the default probability and the risk weight function that in turn depends on the asset correlation value. As a consequence lower systematic risk for SMEs can well be in line with higher capital requirements for SMEs if SMEs have higher default probabilities, i.e., higher firm-specific risk, than large firms.

[1] This treatment has been continued without change in the Basel III framework.

[2] In the IRBA the capital requirements are computed by multiplying the credit exposure by a risk weight that is a function of the default probability, the recovery rate, the maturity and the asset type of the loan. In the RSA the risk weight is tabulated and depends both on the borrower type and an external rating, i.e., a rating given by an acknowledged rating agency. Very often in this paper the terms "capital requirement" and "risk weight" can be used interchangeably.

Our empirical results can also be useful for an evaluation of current regulatory capital requirements – subject to taking into account various conceptual and empirical caveats that are described in more detail in Section 5. Since regulatory capital requirements can affect the interest margins required by the lender, only their appropriate calculation in the sense that they reflect the actual risk posed by the borrower will ensure an optimal credit supply for the economy. Since SMEs are the backbone of the economy in many countries, such as Germany, appropriate capital requirements are crucial for economic growth.

In principle, an evaluation of regulatory capital requirements should distinguish between the *level* of capital and the *relative* difference to other asset classes. In the development of Basel II the second aspect, often referred to as *relative calibration*, was addressed first. The *level calibration* instead was guided by the requirement to keep the overall level of capital in the international banking system broadly constant when transitioning from Basel I to Basel II. This was achieved in an iterative top-down calibration, guided by several *quantitative impact studies* coordinated by the Basel Committee on Banking Supervision.

Our analysis is very much in the spirit of previous analyses that were carried out for the relative calibration of Basel II. The asset correlations are estimated based on the ASRF model underlying the IRBA capital requirements. We use large corporates as a *benchmark* which means that they are assumed to be correctly calibrated in level. Then we compare the relative difference of both, capital requirements based on estimated asset correlations and the current IRBA capital requirements from the capital requirements for this benchmark. Comparing these two relative differences can provide useful information for an evaluation of the capital relief for SMEs granted in Basel II.

We have to confine ourselves to this *relative* calibration since the appropriate *level* of regulatory capital cannot be satisfactorily assessed for the following two reasons:

(i) The overall level of capital requirements was determined in the top-down calibration of the whole Basel II framework, involving also for example the 99.9% confidence level of the value-at-risk, the scaling factor of 1.06 of credit risk weighted assets, and the benchmark maturity of 2.5 years. There is no reason to believe that this very different calibration goal will provide asset correlations similar to the estimates from time series of default rates.

(ii) The asset correlation parameter in the regulatory risk weight function was calibrated in order to reflect a certain degree of sectoral concentration in portfolios of representative banks.[3] For this reason, any direct estimation based on the ASRF model would be, technically speaking, an estimation within a wrongly specified model.

A well-established strand of empirical work now exists on the systematic risk in SME loans. Although the findings on the level of asset correlations in the ASRF model vary substantially, they tend to indicate lower rather than higher asset correlations overall compared to the values used in the IRBA capital requirements. Table A.1 in the Appendix provides a comprehensive overview of the existing empirical studies on asset correlations. Further studies are summarized in the survey of Berg, Gehra, and Kunisch (2011). We distinguish between two strands of empirical literature which lead to quite different results in terms of the level of asset correlations. The first strand uses historical default rates to determine default correlations or asset correlations. These studies include Rösch (2003), Dietsch and Petey (2004), Düllmann and Scheule (2006), and Bams, Pisan, and Wolff (2012). These authors generally estimate lower values than the ones used in the IRBA. In the second strand Düllmann, Kunisch, and Küll (2010) have shown that asset correlation estimates based on equity prices tend to be somewhat higher than those based on default rates. Their results are reflected, for instance, in studies by Hahnenstein (2004) or Lopez (2004).

Several studies which originated from Hahnenstein (2004) also assess the dependence of asset correlations on size, creditor quality (i.e., rating), and sector. For a sample of German firms Düllmann and Scheule (2006) find that asset correlations increase with size, but they do not find an unambiguous relation to the creditor rating. In contrast, Dietsch and Petey (2004) find that for French and German SMEs "asset correlations decrease significantly on average with the SME size". However, their sample is considerably smaller than the one used by Düllmann and Scheule (2006).

Castro (2012), who distinguishes between sectors, world regions, and rating finds an "inverse relation between correlation and the quality of the issuer" using Bayesian estimation techniques. Also, Hansen, van Vuuren, and Ramadurai (2008) come to the conclusion that asset correlations vary geographically and that the relation between asset correlations and probability of default (PD) is ambiguous. For certain asset classes, as opposed to the Basel II predictions, they find a higher PD being associated with higher asset correlations.

In summary, previous empirical work is by and large inconclusive on the size of the

[3] Since the ASRF model cannot capture sectoral concentration because of the assumption of the single risk factor, the asset correlation for the risk weight function was "calibrated" as follows: It was set so that the value-at-risk of the ASRF model was on average the same as the value-at-risk of representative portfolios of real banks in a multi-factor asset value model. In this way, the risk weight function implicitly also reflects the sectoral concentration of those bank portfolios on average.

appropriate asset correlations, although a tendency towards lower values than the ones chosen for the current regulatory capital requirements is observable.
Our empirical results confirm previous findings that asset correlations increase with firm size conditional on the rating category. Furthermore, they suggest that the relative differences between the capital requirements for large corporates and those for SMEs (in other words, the capital relief for SMEs) are in two cases lower in the current regulatory framework than implied by our empirically estimated asset correlations:

1. In the IRB approach the empirically observed potential for increasing the difference in capital requirements between SME loans in the corporate portfolio and large corporates might amount up to 24 percentage points dependent on firm size. This could be achieved by adjusting the asset correlation parameters of the IRBA formula. For SMEs in the IRBA retail portfolio, however, there is no empirical indication supporting a change of the current minimum capital requirements.

2. For all loans assigned to the SME portfolio in the RSA, the empirical results suggest a significantly higher relative reduction compared to large firms than reflected in the current capital requirements. The capital relief potential amounts to values between 15 and 35 percentage points.

Before the capital relief reflected in these figures is translated into a policy message to adjust the current regulatory capital requirements, several caveats also need to be considered, which are described in Section 5.

2 Data Sample

2.1 Desirable Properties of the Dataset

The sample should ideally cover as large a part of the entire SMEs and large corporates universe as possible. Since defaults are rare events and since the sample needs to be split into several buckets the small-sample noise in the estimation results should be reduced to the extent possible. Systematic risk is driven by the evolution of the credit cycle over time; therefore, it can only be measured from time series. Thus, it is important to have a long time series that covers at least one full credit cycle.

It is also crucial that the definition of default is the same throughout the observation period since any change in the definition may have a significant impact on the level of observed default rates and ultimately on the asset correlation estimates. This definition should ideally be the Basel II definition of default: "A default is considered to have occurred with regard to a particular obligor when either or both of the two following events have taken place. (1) The bank considers that the obligor is unlikely to pay its credit obligations to the banking group in full, without recourse by the bank to actions such as realising security (if held). (2) The obligor is past due more than 90 days on any material credit obligation to the banking group. Overdrafts will be considered as being past due once the customer has breached an advised limit or been advised of a limit smaller than current outstandings." (See Basel Committee on Banking Supervision (2006)) Since this definition is wider than the insolvency criterion[4] that has often been used in previous studies (e.g., Düllmann and Scheule (2006)), we expect the number of defaults to be on average higher and consequently the results to be more robust to small-sample noise in buckets of low default rates. A recent study by Dietsch (2013) investigates the impact of different definitions of default on measures of credit risk.

There should be no selection bias of SMEs included in the sample, e.g., by larger SMEs entering at an earlier stage and smaller SMEs at a later stage. Pooled data of a large number of banks can also help to reduce any bank-specific bias in the data.

Based on empirical findings regulators chose to make the asset correlation in the IRBA risk weight functions for corporate SMEs dependent both on the PD and the firm size. In order to disentangle both dependencies, the sample should be broken down not only into size buckets but also into rating buckets. This two-dimensional break-down further underlines the need for the total sample to be of a sufficient size.

There should be no structural breaks in the sample caused, for example, by a significant change of the rating methodology.

Lending policies as well as the cyclicality of the credit risk in SME loans may differ across

[4]This is specified in the German Insolvency Code.

jurisdictions, for example, because of economic differences between the SME segments, structural differences in the capital structure of SMEs, different lending policies (e.g. relationship oriented banking vs. transaction oriented banking), or legal differences, e.g. in the insolvency code. Since our results pertain only to the German SME sector, an analysis on a country-by-country basis is required before generalizing the outcome to SME sectors of other countries.

2.2 Data Description

The data that have been provided by both small and large German banks feature the desirable properties listed in the previous subsection. They comprise submissions by more than 400 banks and cover the time period from 1 January 2005 to 31 December 2011. Defaults are counted for 14 semi-annual periods starting at the end of June and the end of December, respectively. In each considered time period our sample on average includes approximately 250,000 rated borrowers. Although the vast majority of banks have adopted the RSA, their rating systems have been designed along the requirements of an IRBA rating system. The considered creditors include all domestic firms (except credit institutions) for which an IRBA PD has been available. Retail and specialized lending are not considered.

Default rates for certain rating-size buckets exhibit semi-annual seasonality due to banks' provisioning policy. Thus, the underlying time series have to be seasonally adjusted. Since the data set contains only 14 half-yearly observations for every rating-size bucket, it is challenging to identify the seasonal pattern. In this regard, we apply the difference-from-moving-average method to seasonally adjust the time series. In order to avoid overadjustment during the financial crisis (second half year of 2008, both half years of 2009), these outliers are excluded from the estimation of the moving average. It should be noted, however, that all data points are seasonally adjusted and visual inspection of the new time series confirms the high quality of this approach.

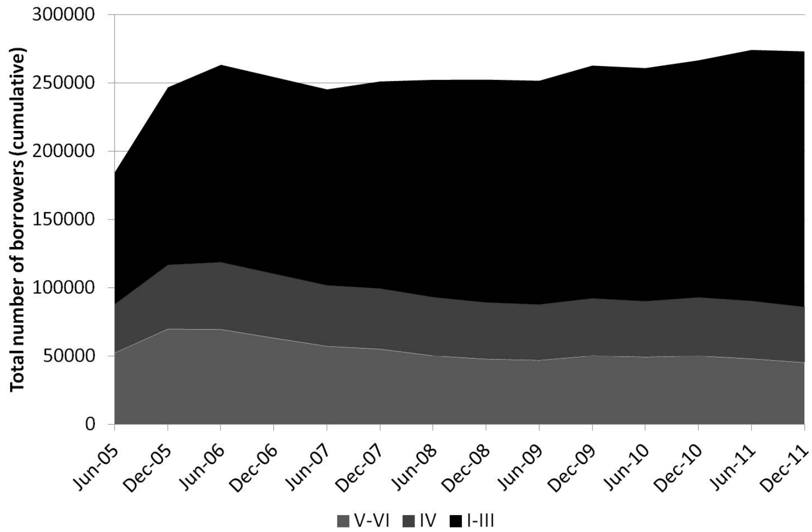
In our sample every observation includes two figures: the number of borrowers at the beginning of the respective period in the respective bucket, and the number of defaults up to the end of that period. Buckets are built in three dimensions: yearly turnover, rating, and time.[5] If a borrower is included in the credit portfolio at the beginning of the semi-annual horizon and its credit is redeemed in the following half year, the credit is counted as 0.5 for the number of credits. Nevertheless this effect is minor. The upper panel of Figure 1 shows the number of borrowers and the lower panel the number of defaults with respect to the rating category. The increase in numbers in the first two semi-annual

[5]The availability of "number of borrowers" and "number of defaults" per bucket allows us to merge buckets quite flexibly if the number of observations becomes too low for robust estimation results.

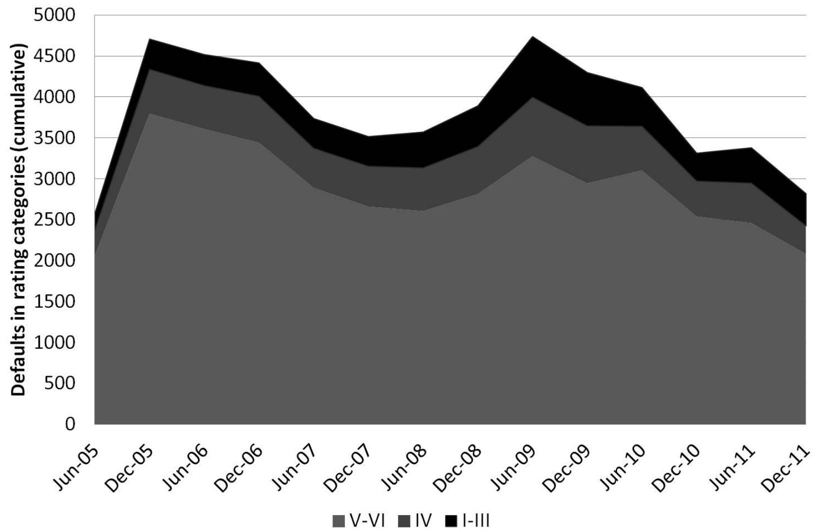
periods is clearly due to the step-by-step adoption of the rating methodology by the banks in 2004–2005. The fluctuation of the number of defaults is indicative for point-in-time or "hybrid" PDs, i.e., PDs that follow a mixture between a pure point-in-time and a pure through-the-cycle concept.

Figure 1: **Semi-annual number of borrowers and defaults with respect to rating category**

(a) Number of borrowers



(b) Number of defaults



The rating buckets are defined according to the size and the rating of the borrower. Rating categories are determined by a master scale invented by the Joint Banking Initiative for the Financial Location of Germany (2010) (IFD). Its original purpose was to improve the transparency of IRBA ratings and to enable borrowers to compare their own rating across banks. This master scale comprises six rating classes I to VI with I being the best rating. The different rating systems of each bank in Germany can be converted into the IFD system. The sample period of seven years covers the recent global financial crisis and roughly meets the requirement to include one full credit cycle. However, due to the robustness of foreign demand from outside Europe and due to special national arrangements to support the German economy (for example, extensive use of flexible time arrangements in order to avoid lay-offs, or the temporary introduction of the car scrapping premium), the sample period does not capture a period of severe recession in the SME sector.

Since observations are available for half years, the sample in total contains 14 semi-annual periods in the time dimension. As the data is pooled across many banks it is important to ensure that double counting of defaults due to two or more banks granting loans to the same customer is avoided. In our sample, the majority of banks adhere to the "regional principle" which states that banks are only allowed to serve customers within a specified region. Therefore, there should be no significant distortion from the double counting of defaults and the estimations should not be significantly biased. This has been assured via various robustness checks.

The size of the borrowers is proxied by the turnover which is published in the balance sheet of each borrower. The following six size categories measured in € million are chosen: $[0; 0.3]$, $(0.3; 1]$, $(1; 2.5]$, $(2.5; 5]$, $(5; 50]$, and $(50; +\infty)$. Figure 2 illustrates the number of borrowers for each rating and size bucket. The majority of buckets is sufficiently filled by borrowers; however, for the higher size categories, the number of borrowers is comparatively lower.

Figure 2: **Number of average borrowers with respect to rating and size category**

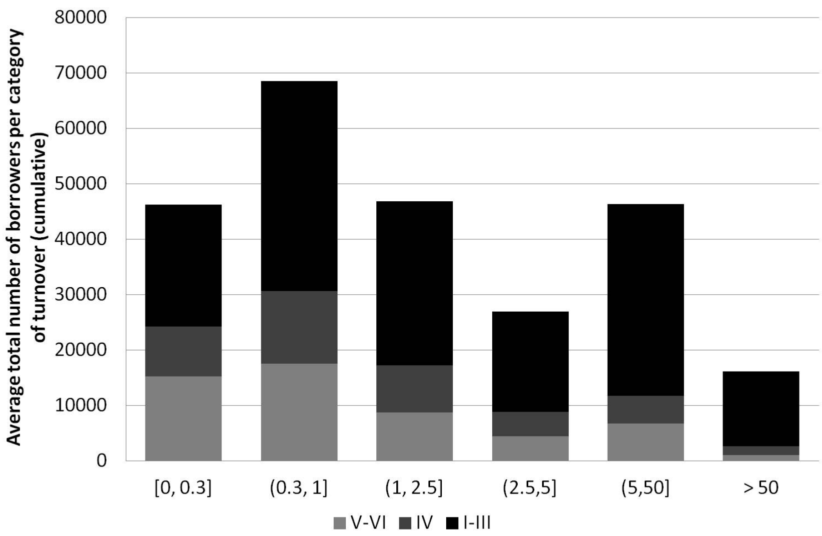
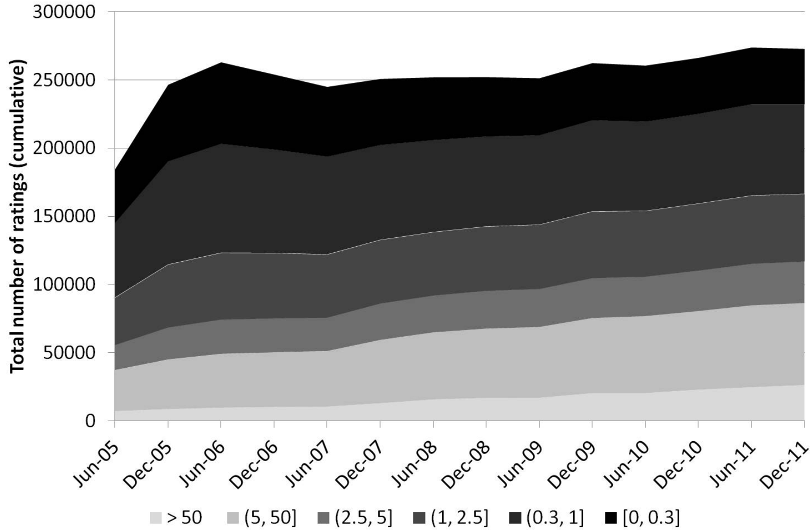


Figure 3 shows the evolution of the number of borrowers and defaults, respectively, over time in each turnover class. Disregarding the large increase in the number of rated borrowers in all turnover classes from the first to the second half of 2005, the number of borrowers has grown moderately for smaller borrowers and increased considerably for larger enterprises. The number of defaults, as also evident from Figure 1, behaves in a rather volatile fashion, which can be attributed to the fact that banks usually report more write-offs at the end of the year.

Figure 3: **Semi-annual number of borrowers and defaults with respect to turnover class**

(a) Number of borrowers



(b) Number of defaults

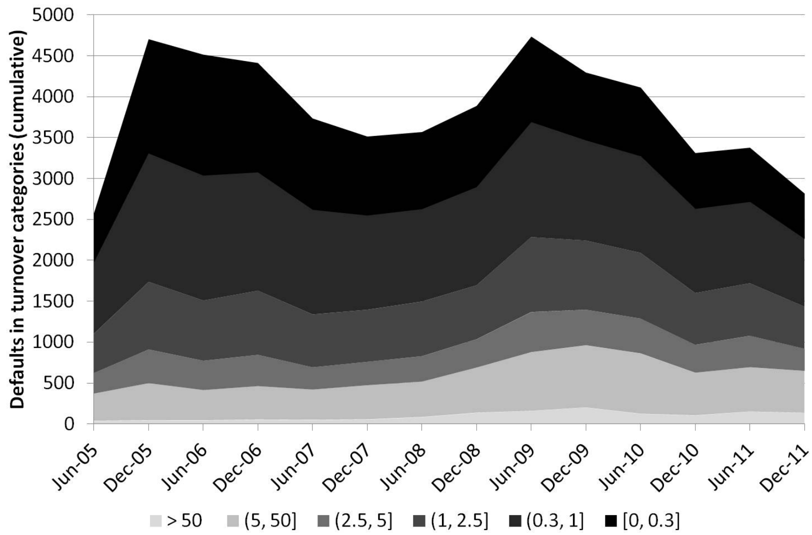
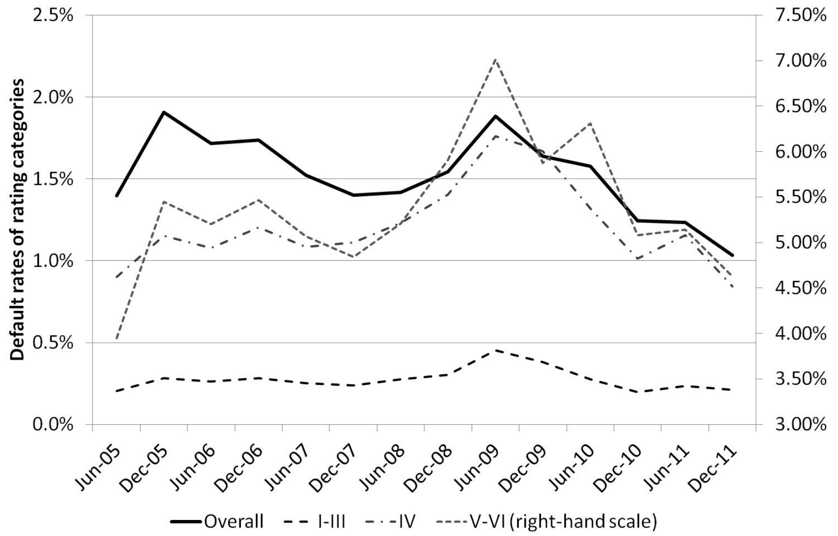


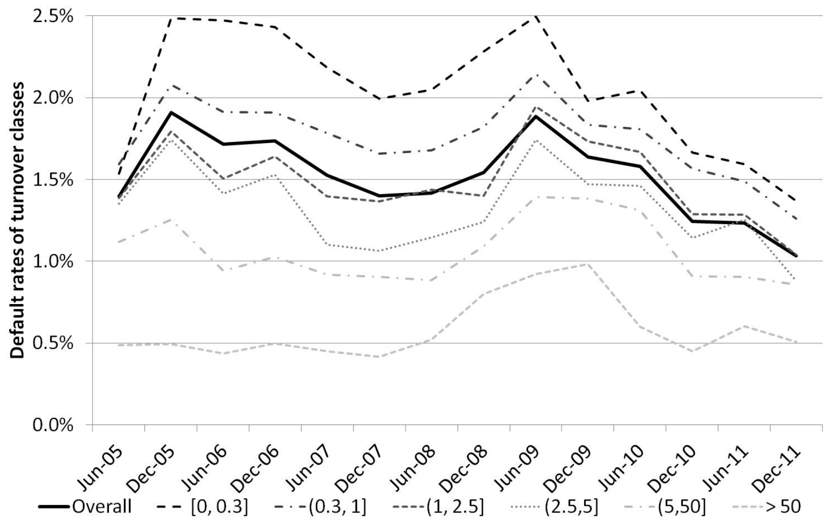
Figure 4 illustrates the evolution of the default rates for each rating class (upper panel) and each turnover class (lower panel) over the considered time horizon. The highest default rates for the total sample are observed in 2005, well before the financial crisis started. This observation supports the view that German SMEs mastered the financial crisis quite well. In general, the majority of empirical studies dealing with the estimation of asset correlations from default rates use yearly data.

Figure 4: **Semi-annual default rates in percent with respect to rating category and turnover class**

(a) Default rates by rating category



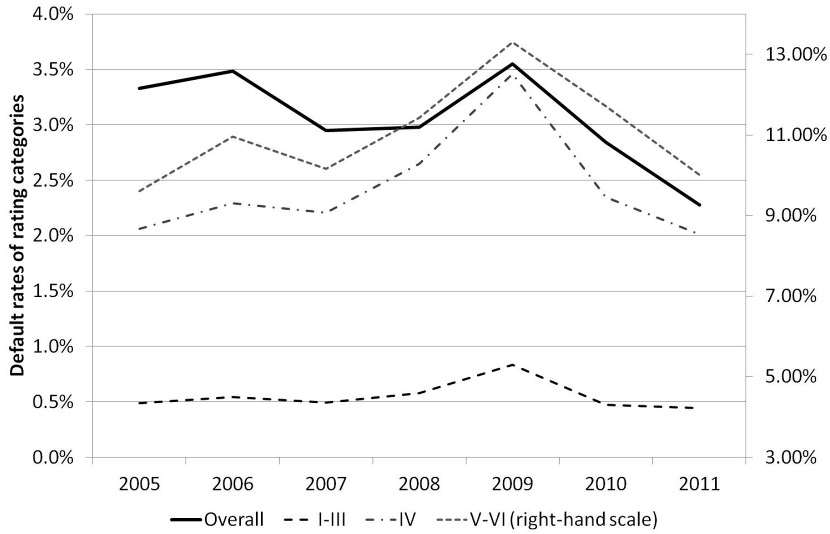
(b) Default rates by turnover class



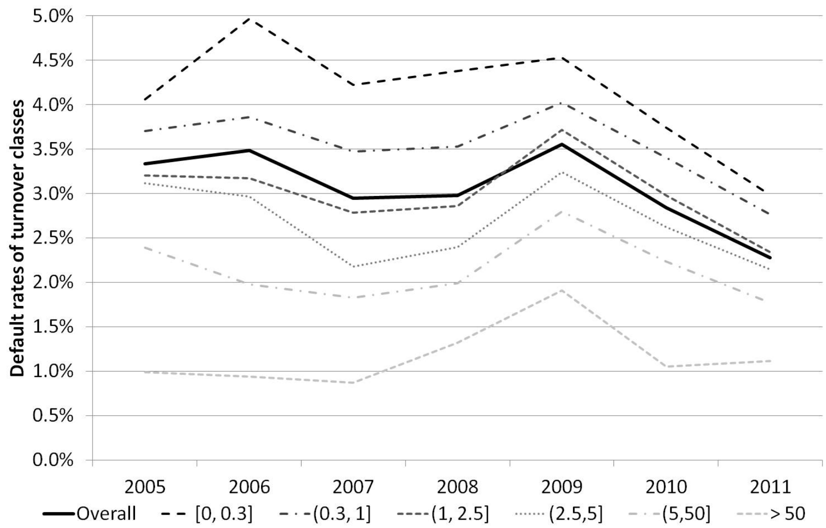
In order to validate the robustness of the estimation results based on semi-annual data, we also aggregate the observation periods to an annual time span so as to conduct all the estimations with annual default rates. The main difference in yearly data is that they reduce the variation in the number of defaults over time. This smoothing effect is revealed in the time series of default rates in Figure 5 which shows the yearly default rates in each rating category (upper panel) as well as in each turnover class (lower panel).

Figure 5: **Annual default rates in percent with respect to rating category and turnover class**

(a) Default rates by rating category



(b) Default rates by turnover class



3 Model and Estimation Methodology

The analysis is based on the widely known ASRF model of Gordy (2003) that is also the foundation of the IRBA risk weight functions for credit exposures in the banking book. Default is triggered in this model if the ability-to-pay process Y_i of firm i falls below an exogenous default threshold γ_i. Y_i follows a standard normal distribution. It can be decomposed into the return of a systematic and unobservable factor X and an idiosyncratic firm-specific part ε_i:

$$Y_i = \sqrt{\rho_i} \cdot X + \sqrt{1 - \rho_i} \cdot \varepsilon_i.$$

X and ε_i are independent for every obligor i and follow a Gaussian distribution. The factor loading $\sqrt{\rho_i}$ of the systematic risk factor can be interpreted either as the sensitivity against systematic risk or as the square root of the asset correlation ρ_i. For this analysis the common assumption of a constant ρ_i is applied which is typical for such empirical studies as it allows this parameter to be estimated from a cross section. The Bernoulli variable L_i describes if a credit event has occurred during the considered horizon ($L_i = 1$) or not ($L_i = 0$). It is important to differentiate between the unconditional and the conditional default probability. The unconditional default probability of obligor i for the time period t is defined as follows:

$$P(L_i = 1) = P(Y_i < \gamma_i) = \Phi(\gamma_i)$$

where Φ denotes the cumulative distribution function of a standard normal distribution. Since homogeneity in the obligor buckets is assumed, the index i for the distance to default γ_i of a specific firm is dropped.

In this study the ML estimator advocated in Gordy and Heitfield (2002) is applied for retrieving the main results. The ML estimator is a very general estimator and allows for the possibility that obligors in different rating and size buckets may be sensitive to different risk factors. For robustness tests we employ a Method-of-Moments (MM) estimator[6] and also use annual time periods in addition to semi-annual ones for computing the default rates (see Appendix A.3–A.5). The estimation methodology is described in Appendix A.2.

[6]In addition we also employ the Asymptotic Maximum Likelihood Estimator that has been analysed by Düllmann et al. (2010) in small samples. The results for this estimator are available from the authors on request.

4 Results

4.1 Asset Correlation Estimates

Table 1: **ML estimates for asset correlations and PDs (in percent)**

Rating Category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Asset correlation estimates						
I–III	0.51	0.59	0.62	0.66	0.81	1.71
	(0.26)	(0.26)	(0.27)	(0.32)	(0.34)	(0.80)
IV	0.50	0.43	0.62	0.74	0.70	1.72
	(0.26)	(0.20)	(0.28)	(0.37)	(0.32)	(0.93)
V–VI	0.56	0.31	0.49	0.64	0.80	1.54
	(0.22)	(0.13)	(0.20)	(0.28)	(0.32)	(0.81)
PD estimates (one-year horizon)						
Rating Category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	0.66	0.56	0.56	0.56	0.49	0.42
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)
IV	2.11	2.33	2.54	2.70	2.48	2.56
	(0.12)	(0.12)	(0.15)	(0.18)	(0.16)	(0.28)
V–VI	10.08	10.52	11.31	10.69	9.72	8.97
	(0.41)	(0.32)	(0.43)	(0.48)	(0.49)	(0.71)

Standard errors determined analytically from asymptotic Fisher information matrices are given below in brackets.

In order to evaluate the relative calibration we need to consider that besides firm size measured by yearly turnover, credit quality, i.e., the rating, has also been found to be a potential driver of the estimation of asset correlations (e.g., Hahnenstein (2004) and Düllmann and Scheule (2006)). This two-dimensional dependency is also reflected in the current IRBA risk weight functions. Therefore, we estimate the asset correlation for a matrix of rating and turnover buckets. This procedure enables us to compare capital requirements for different size buckets conditional on the rating with the respective IRBA capital requirements. The estimation results are presented in Table 1. Since the time periods in the sample cover six months we have transformed the estimates of a half-year PD_h by the formula $PD = 1 - (1 - PD_h)^2$ into PDs for a one-year horizon. This transformation is necessary for the analysis of the capital requirements since PDs in Basel II always refer to a one-year horizon.

The asset correlation estimates in Table 1 tend to increase with firm size when holding the rating constant. This increase, however, is not perfectly monotonic and more pronounced in some rating categories than in others. The level of asset correlations never exceeds two percent and is on average considerably below the asset correlations in the IRBA capital requirements. A possible underestimation of the asset correlations could result from the

fact that for each size and rating bucket the correlations were estimated for well-diversified portfolios with respect to business sectors. Since the applied time series is still relatively short at seven years, as already discussed in Section 2, it is questionable whether one full business cycle is captured in the estimations. If this is not the case, negative biases can arise in the estimation of the asset correlations. (See for instance Gordy and Heitfield (2002), Dietsch and Petey (2004), and Düllmann and Scheule (2006).)
In the next subsection we compare the capital requirements in Basel II dependent on turnover with the estimated capital requirements based on the asset correlation and PD estimates. Afterwards in Subsection 4.3 we evaluate the estimated capital requirements with respect to the RSA. The asset correlation estimates depend not only on the turnover but also on the rating. In the following we account for this two-dimensional dependence by weighting the IRBA or RSA risk weights with respect to the number of borrowers in each rating category. The advantage of this aggregation is that we can condense the assessment of the asset correlation estimates in a single figure.

4.2 Evaluation of IRBA Capital Requirements

We consider the "empirical risk weight function", i.e., the risk weight function based on the empirically estimated asset correlations, rather than the asset correlation estimates themselves in order to assess the calibration of the IRBA capital requirements. The current Basel II capital requirements are calculated according to the IRBA formulas. Turnovers above €50 million are lumped together in a single bucket since the risk weight curve would remain flat above this turnover threshold (for a constant PD). For a turnover between €2.5 and €50 million we have applied the corporate risk weight function including the capital relief due to the turnover dependence of the asset correlation. The retail risk weight curve (Other Retail) has been applied for a turnover below €2.5 million.[7]
By comparing the size dependence of "estimated capital requirements" (i.e., based on empirical asset correlation estimates) with the size dependence "hard-wired" into the corresponding IRBA capital requirements we want to answer the question whether the size dependence of IRBA capital requirements is appropriate in light of the new empirical results. For this purpose, and for different size buckets, we consider the relative difference of the (estimated and Basel II) capital requirements from the corresponding capital requirements of "large" corporates (i.e., firms with a yearly turnover higher than €50

[7]Analyses of the Bank for the Accounts of Companies Harmonised (BACH) database from the European Committee of Central Balance Sheet Data Offices support the consideration of the first three turnover classes as Other Retail since the average ratio of turnover to liabilities of credit institutions amounts to 3.1 in 2009 and €1 million is the exposure threshold for the retail portfolio. 2009 is the most recent final data point.

million) which serve as a benchmark[8]; by assumption they are correctly calibrated. If both (relative) differences are negative (indicating a capital relief) and if the absolute value of the difference for the empirical estimates is higher than that of the difference for the regulatory numbers, this may be interpreted as an indication that our empirical results *ceteris paribus* would support lower Basel II capital requirements for SMEs. Table 2 shows the estimated capital requirements and the Basel II ones in terms of risk weights.[9]

Table 2: **Capital requirements in terms of risk weights per rating class (in percent)**

Estimates						
	Other Retail			Corporate		
Rating Category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	4.0	3.9	4.0	4.2	4.3	6.4
IV	9.6	9.4	12.6	14.6	13.2	23.9
V–VI	30.3	22.6	30.2	33.9	36.3	50.8
Basel II						
	Other Retail			Corporate		
Rating Category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	39.8	36.6	36.6	61.2	62.4	67.8
IV	62.3	63.6	64.8	100.9	107.7	130.3
V–VI	80.3	81.4	83.6	159.7	167.1	196.5

In order to determine the differences in capital requirements we calculate the relative difference between each risk weight in all turnover classes up to €50 million and the corresponding risk weight for the largest turnover class (benchmark). As an example, consider the relative difference between the estimated risk weight of turnover class (5, 50] and turnover class >50 in rating category I-III:

$$\Delta_{5-50}^{Est,I-III} = \frac{RW_{5-50}^{Est,I-III} - RW_{>50}^{Est,I-III}}{RW_{>50}^{Est,I-III}} = \frac{4.3\% - 6.4\%}{6.4\%} = -32.8\%.$$

The same is done for the Basel II risk weights:

$$\Delta_{5-50}^{BII,I-III} = \frac{RW_{5-50}^{BII,I-III} - RW_{>50}^{BII,I-III}}{RW_{>50}^{BII,I-III}} = \frac{62.4\% - 67.8\%}{67.8\%} = -8.0\%.$$

[8] This segment comprises firms with a yearly turnover of at least €50 million; the size adjustment in the IRBA risk weight function is zero for this segment.

[9] Appendix A.2.3 shows the empirical risk weight function based on estimated PDs and asset correlations.

Doing this for each risk weight gives us the following reductions for the estimated and the Basel II capital requirements:

Table 3: **Relative differences in capital requirements by rating and turnover class (in percent)**

Estimates						
	Other Retail			Corporate		
Turnover (€ m) / Rating Category	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	-37.3	-39.1	-37.3	-34.6	-32.8	0.0
IV	-59.9	-60.6	-47.5	-38.9	-45.0	0.0
V–VI	-40.4	-55.5	-40.5	-33.3	-28.5	0.0
Basel II						
	Other Retail			Corporate		
Turnover (€ m) / Rating Category	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	-41.3	-46.0	-46.0	-9.8	-8.0	0.0
IV	-52.2	-51.2	-50.3	-22.6	-17.4	0.0
V–VI	-59.1	-58.6	-57.5	-18.7	-15.0	0.0

Since the dependence of capital requirements on size and rating based on the asset correlation estimates is compared with the dependence implicit in the current IRBA risk, total differences are considered and presented in Table 4. Total differences of capital requirements with respect to the benchmark "large corporates" are simply the absolute differences of the relative differences of the estimated capital requirements and the relative differences within Basel II. Again using the example of turnover class (5, 50] and rating category I-III the total difference of capital requirements amounts to

$$\Delta_{5-50}^{I-III} = \Delta_{5-50}^{Est,I-III} - \Delta_{5-50}^{BII,I-III} = -32.8\% - (-8.0\%) = -24.8\%.$$

Table 4: **Total differences of capital requirements in the Basel II IRBA for all rating and size buckets (in percent)**

Rating Category \ Turnover (€ m)	Other Retail			Corporate		
	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	3.9	6.9	8.7	-24.8	-24.8	0.0
IV	-7.7	-9.4	2.8	-16.4	-27.5	0.0
V–VI	18.8	3.0	17.0	-14.5	-13.5	0.0

The total differences in Table 4 vary to some extent, which means that it is difficult to draw essential conclusions from this representation. Thus, for an overall assessment of these results, average total differences of the capital requirements are applied in terms of a weighted average of all total differences for each turnover class using weights with respect to the number of loans per rating class. Table 5 shows all weights for each turnover class.

Table 5: **Mean weights for ratings per turnover class (in percent)**

Rating Category \ Turnover (€ m)	Other Retail			Corporate		
	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	47.7	55.3	63.1	67.3	72.1	83.9
IV	19.4	19.0	18.2	16.2	13.8	9.9
V–VI	32.9	25.7	18.6	16.5	14.1	6.2

By weighting the total differences with the number of obligors for each rating category, a representative number of possible differences between Basel II and the empirical estimations for each turnover class is obtained in Table 6. For example, the average total difference in capital requirements for the size category $(5, 50]$ is obtained as

$$\Delta^{T}_{5-50} = \Delta^{Est}_{5-50} - \Delta^{BII}_{5-50} = -33.9\% - (-10.0\%) = -23.9\%.$$

Table 6: **Average total differences of capital requirements in the Basel II IRBA (in percent)**

Differences \ Turnover (€m)	Other Retail [0, 0.3]	Other Retail (0.3, 1]	Other Retail (1, 2.5]	Corporate (2.5, 5]	Corporate (5, 50]	Corporate > 50
Basel II IRBA	-49.3	-50.2	-48.9	-13.3	-10.3	0.0
Estimated	-42.7	-47.4	-39.7	-35.1	-33.9	0.0
Average total difference	**6.6**	**2.8**	**9.2**	**-21.8**	**-23.6**	0.0

Table 6 shows that the average total differences for the corporate portfolio and the retail portfolio move in different directions. For the retail portfolio, the gap between both relative differences from the benchmark is positive but below ten percent. We define average total differences below this threshold as economically insignificant. In contrast, for all SME loans assigned to the corporate portfolio, the capital requirements of the estimated curve show significantly higher negative differences than Basel II does with a gap of about 22 percentage points.

These results for the IRBA indicate that there is a potential for increasing the relative distance between the capital requirements for SME loans in the corporate portfolio and the capital requirements for loans to larger corporates. This could be achieved, for example, by providing a capital relief for lowering the risk weights of SMEs relative to their current treatment only for a certain turnover class or by adjusting the asset correlation parameters of the IRBA formula. In general, the results of Table 6 are not surprising and confirm a perception already discussed when the Basel II framework was designed, namely that splitting the SMEs between the corporate portfolio and the retail portfolio gave rise to a cliff effect between both portfolios.

4.3 Evaluation of Capital Requirements in the Standardized Approach

In this subsection we compare the relative level of capital requirements implied by the asset correlation estimates with the RSA capital requirements. Table 7 calculates the average total differences in the RSA. The RSA risk weight function is simply a step function with a risk weight of 100% if the firm is treated as a corporate exposure and 75% if it is assigned to the retail portfolio, i.e., if the exposure to the borrower does not exceed €1 million which is comparable with a turnover of up to €2.5 million (see footnote 7).

The results for the RSA are considerably stronger and economically more significant than those for the IRB approach. The estimated capital requirements differ to a much greater

Table 7: **Average total differences of capital requirements in the RSA (in percent)**

Differences \ Turnover (€ m)	Other Retail [0, 0.3]	Other Retail (0.3, 1]	Other Retail (1, 2.5]	Corporate (2.5, 5]	Corporate (5, 50]	Corporate > 50
Basel II RSA	-25.0	-25.0	-25.0	0.0	0.0	0.0
Estimated	-42.7	-47.4	-39.7	-35.1	-33.9	0.0
Average total difference	**-17.7**	**-22.4**	**-14.7**	**-35.1**	**-33.9**	0.0

extent from the benchmark "large corporates" (-34% up to -47%) than the regulatory figures (0% up to -25%). For SMEs in the corporate portfolio, the results are directionally in line with those for the IRBA, but the average total differences are higher up to a level of 35 percentage points. In comparison to the corporate portfolio, the empirical results for the SME loans in the retail portfolio indicate a lower but economically significant capital relief potential between 15 and 22 percentage points. To sum up, for all loans assigned to the SME portfolio, the empirical results suggest that the relative reduction compared to large firms is significantly higher than reflected in the current capital requirements. Before these results are interpreted in terms of a policy message, it needs to be considered that the RSA is less risk-sensitive than the IRB approach in general which justifies a more conservative calibration. The robustness checks conducted support our results for possible capital reliefs for SMEs and show to some extent higher effects. (Appendix A.3–A.5)

5 Policy Message

In our paper we have identified two cases in which our empirical results suggest that the relative differences between the capital requirements for large corporates and those for SMEs (in other words, the capital relief for SMEs) are lower in the current regulatory framework than implied by our empirically estimated asset correlations. Since these average total differences reflect the capital relief granted for SMEs by the regulators they may indicate – in certain cases and if taken at face value – a potential for increasing this capital relief. This would be equivalent to lowering the regulatory capital requirements for SMEs, for instance by lowering the asset correlation values in the IRBA formula or by lowering the RSA risk weights directly. Before drawing this inference as the policy message of this paper, the following important caveats need to be carefully considered:

The RSA was deliberately calibrated more conservatively than the IRB approaches. This can be explained by the significantly lower risk sensitivity of the RSA and the regulatory intention to retain incentives in terms of a *ceteris paribus* capital relief when banks switch to the more risk sensitive IRB approaches. The more conservative calibration is one reason why the capital requirements in the RSA are currently independent of firm size, which is one important driver for the empirically observed lower potential for reductions of the capital requirements. It also suggests that at least a substantial part of the 15%–35% difference between the current capital relief in the RSA and the capital relief implied by our new empirical results can be explained by this original calibration target.

The time series is still relatively short and the use of semi-annual rather than annual time intervals for measuring the default rates does not remove the limitation that the development of the German economy is only captured over seven years. The substantial noise in asset correlation estimates from such short time series has been well-documented, for example in Düllmann et al. (2010) by Monte Carlo simulations. Furthermore, although the sample period comprises the recent global financial crisis, the German SME sector appears to have been surprisingly unaffected.

Since the regulatory minimum capital requirements are harmonized internationally nowadays, their modification appears reasonable only if the results of this study are also broadly representative for other countries. This applies all the more since the development of the German economy during the financial crisis differs positively from that in other European countries. Therefore, further analyses – possibly based on the same methodology used in this paper – appear to be useful, especially for countries which also have a strong SME business sector. This is left to further research.

The indicated potential for increasing the relative difference between risk weights for certain size buckets for SMEs and risk weights for larger corporates is derived from the current regulatory minimum capital requirements for large corporates which are assumed to be correctly calibrated. The assumption to use large corporates as a benchmark is well motivated since, for reasons stated above (see Section 1), the study design does not allow us to quantify the absolute level of the minimum capital requirements. One could argue that a compensation in the overall calibration of minimum capital requirements would be necessary if the requirements are lowered for specific SMEs, in order to retain the overall level of capital in the banking system.

A Appendix

A.1 Literature Review on Asset Correlation Studies

Table A.1: **Asset correlation estimates of related studies**

Study	**Data**	**Time Period**	**Estimation Method**	**Estimated AC (in percent)**
Akhavein, Kocagil, and Neugebauer (2005)	Rating agency data (Fitch Rating) on 7,886 US corporate issuers, factor loadings underlying the Fitch Default Vector Model Version 2.0	1970–2004	Asset correlations are derived in three ways: 1) estimating joint default probabilities and transforming them into asset correlations, 2) based on the direction of rating changes, and 3) using an equity-based factor model	18
Bams et al. (2012)	Dun & Bradstreet data set	2005–2011	multi-factor model	0–2.58
Bluhm and Overbeck (2003)	Moody's	1970–2001	Estimation of joint default probabilities and their transformation into asset correlations	11.77–42.51
Cassart, Castro, Langendries, and Alderweireld (2007)	Moody's database on issuer/obligator senior rating (corporates and financial institutions)	1970–2005	Asset correlations are estimated based on the direction of rating changes. They use two approaches, 1) based on default events only (STRM), and 2) considering all possible rating transformations (DRTM)	STRM approach: 21 DRTM approach: 2
Carlos Cespedes (2002)	Moody's	1970–2000	Simple estimations within a one-factor and two-factor model	About 10
Castro (2012)	Moody's Corporate Default database on issuer/obligator senior rating (corporates and financial institutions) and Moody's database on structured products	1970 (1981)–2007	Asset correlations are estimated for aggregated data as well as for different sectors, world regions, and structured products. Estimation is carried out using Bayesian techniques. For aggregated data, a one-factor model of default risk is estimated, for the sub-groups, a two-factor model is estimated, where the second factor can be viewed as "local" systemic factor	Aggregated US corporates: 10.2–24.1
Chang, Yu, and Liu (2011)	Default data published in Standard & Poor's default report on global corporates	1981–2009	Estimation of a serially dependent factor model using Bayesian techniques	8.3 (average of rating classes A to CCC)
Chernih, Henrard, and Vanduffel (2010)	Monthly asset returns from Moody's KMV Credit Monitor database	1998–2007	Monthly asset returns are calculated as the ratio of the ending (market) asset value minus the value of liabilities issued during that month to the starting (market) asset value. From these time series, the default and asset correlation is calculated according to the Merton model	11.1

Curcio, Gianfrancesco, and Malinconico (2010)	Quarterly statistics on Italian banks' SME loan portfolio quality	1990–2010	Adapted from Hansen et al. (2008)	1.06–2.3
Dietsch and Petey (2002)	Internal rating transition data of 224,000 French SMEs	1995–1999	MM, based on Basel II one-factor model	EU: 7.9
Dietsch and Petey (2004)	Internal rating transition data on 440,000 French and 280,000 German SMEs, from Coface and Creditreform	1995–2001	MM, based on Basel II one-factor model	France: 1.28 [0; 17.2] Germany: 0.93 [0; 6.52]
Düllmann and Scheule (2006)	Time series of default histories of 53,280 German firms	1991–2000	MM and ML, based on Basel II one-factor model	3.3
Fu (2005)	Moody's rating data on about 10,000 corporates and financial institutions; Moody's KMV's Global Correlation Model estimation database for about 7,000 U.S. firms	1970–2002	Asset returns correlations are derived in two ways: 1.) based on the direction of joint rating changes 2.) factor model approach based on asset returns which are generated from equity returns and liability structure information by using an option pricing approach	21
Gordy (2000)	S&P rating-specific default frequencies of corporate bonds	1981–1997	Basel II one-factor model, MM estimator	U.S.: 23.75
Gordy and Heitfield (2002)	Moody's / S&P	1970–1998 / 1981–1997	ML estimators and MM estimator	5.51–11.14/4.94–8.86
Hahnenstein (2004)	Equity returns of 241 German corporates (55 weekly observations)	2001	One-index model (industry-specific stock indices as factor), equity return correlations derived from their sensitivity to stock indices and from stock index return correlations	10.7
Hamerle, Liebig, and Rösch (2003a)	Default rate time series of G7 countries (industry and country-specific)	varying; Germany, UK: 20 years; Canada, Japan: 1991–1999	ML, based on Basel II one-factor model	EU: 0.2–2.1 US: 0.3–2.3
Hamerle, Liebig, and Rösch (2003b)	S&P	1982–1999	Generalized factor model for credit risk with observable factors	3.91–6.95
Hansen et al. (2008)	Federal Reserve loss rate data and UK loss rate data on corporates	US: 1985–2007 UK: 1998–2007	They fit a beta distribution to empirical loss rates and calculate the empirically implied UL. The asset correlation is derived by calculating the implied correlation that equates the empirical UL to the theoretical Basel II UL.	US: 5.15 UK: 2.24

Hashimoto (2009)	Time-series data on 1 million active and defaulted companies in Japan by industry, size, credit rating and region	1985–2005	MM and ML based on Basel II one-factor model	1.5–4.5
Hrvatin and Neugebauer (2004)	Equity data of 6,100 firms in the Dow Jones Global Universe, grouped into 25 Fitch industries and 34 countries		Factor model based on Fitch's Default Vector Model (Version 2.0); asset return correlations approximated by factor model equity return correlations	U.S.: 23.2
Jakubik (2006)	Monthly default rate in Finland	1988/2–2004/1	ML estimation within a one-factor and multi-factor model	1.52, 1.66
Jobst and De Servigny (2005)	S&P's CreditPro ratings and default database; S&P's monthly equity time series of approximately 2,200 firms that are also contained in the CreditPro and Compustat data	1981–2003 ratings, 1962–2003 Compustat (North America) data	Using rating data as well as equity default swap data, the De Servigny and Renault (2003) approach is applied to estimate joint default probabilities, then default correlations are estimated and transformed into asset return correlations based on the Basel II one-factor model, and an ML approach based on the Basel II two-factor model is used.	U.S.: 15.5
Kitano (2007)	Monthly default rate in Japan, data of Tokyo Shoko Research Ltd. and National Tax Agency	1982/7–2002/7	ML, two-factor model	About 4–15
Lee, Lin, and Yang (2011)	36,957 firm-year observations on corporates in 9 sectors	1988–2007	First, they estimate the asset beta from the equity beta, then they substitute it into the ASRF asset correlation formula	10.5
Lopez (2004)	Credit portfolios of US, Japanese, and European obligors from Moody's KMV Credit Monitor database	year-end 2000	Asset correlations are determined by minimizing the absolute difference between credit losses indicated by the unconstrained Portfolio Manager model (of Moody's KMV) and by the ASRF-constrained version	EU av.: 14.4 US av.: 19.3
Pitts (2004)	Monthly asset values from Moody's KMV dataset supplied with Credit Monitor for 27 Airlines	1997/7 – 2001/8	Mixed RE and FE model incorporating industry and size effects, based on an extended Merton (1974) credit risk model	U.S., EU: 9.9
Rösch (2003)	Default rates of German corporates, from Federal Statistical Office	1980–2001	ML based on one-factor model	0.5–3.5
Van Landschoot (2007)	CreditPro Corporate rating transitions (U.S. 72%, EU 20%)	1990–2004	Asymptotic MLE, applied to default rates	U.S., EU: 13.5
Van Landschoot (2007)	CreditPro Corporate rating transitions	1990–2004	MLE approach used to estimate transition probabilities applying cohort and homogeneous duration methods	U.S., EU: 7

Zhang, Zhu, and Lee (2008)	5,040 publicly traded US non-financial firms from Moody's KMV historical default database	1981–2006	Joint default probabilities are estimated using MM, from which asset correlations are derived	16.4

This table is adapted from Berg et al. (2011).

A.2 Estimation Methodology

A.2.1 Method-of-Moments (MM) Estimator

The (asymptotic) MM estimator matches the first and second moments of the conditional default probability with the first and second moment of the observable default rates. The first moment is estimated by the average default rate, given by

$$E[g(x)] = \bar{p},$$

with $g(x)$ denoting the default probability conditional on $X = x$ and the second moment by the sample variance of the default rate, given by

$$Var[g(x)^2] = \Phi_2\left(\Phi^{-1}(\bar{p}), \Phi^{-1}(\bar{p}), \rho\right) - \bar{p}^2,$$

where $\Phi_2(\cdot)$ is the cumulative bivariate Gaussian distribution function.

A.2.2 ML Estimator

The ML estimator proposed by Gordy and Heitfield (2002) draws on the property that the number of defaults D in a (homogeneous) obligor bucket with n obligors follows a binomial distribution in each period, conditional on systematic factor X. The default probability conditional on $X = x$ is defined as

$$P(D = d|X = x) = \binom{n}{d} g(x;\rho,\gamma)^d \left(1 - g(x;\rho,\gamma)\right)^{n-d}.$$

The ML estimator of ρ is determined numerically by maximizing the log-likelihood function

$$LL(a, b; \rho, \gamma) = \sum_t \log\left(L_t(a_t, b_t; \rho, \gamma)\right),$$

where a_t denotes the $(T \times 1)$ vector of the total number of obligors for T time periods, b_t the $(T \times 1)$ vector for the number of defaulted obligors and

$$L_t(a_t, b_t; \rho, \gamma) = \int\limits_{\mathbb{R}} \binom{a_t}{b_t} g\left(\Phi^{-1}(x); \rho, \gamma\right)^{b_t} \left(1 - g(\Phi^{-1}(x); \rho, \gamma)\right)^{a_t - b_t} \varphi(x) dx$$

with φ representing the probability density function of the standard normal distribution.

A.2.3 Relation to the Corporate Risk Weight Function

The PD function conditional on the systematic factor in the ASRF model provides the link to the proposed corporate risk weight function of the IRB approaches in Basel II

$$RW(LGD, PD, M, \rho) = 1.06 \cdot 12.5 \cdot LGD \cdot \left[\Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho} x^*_{99.9\%}}{\sqrt{1-\rho}}\right) - PD\right] \cdot f(M, PD)$$

where LGD denotes the Loss Given Default, $x^*_{99.9\%}$ the 99.9% quantile of the standard normal distribution function and $f(M, PD)$ the maturity adjustment dependent on the effective maturity M and the PD with $f(M, PD) = (1 + (M - 2.5) \cdot b(PD))/(1 - 1.5 \cdot b(PD))$ and $b(PD) = (0.11852 - 0.05478 \cdot \log(PD))^2$.

The capital charge is determined by multiplying the exposure at default with the risk weight and the solvability coefficient of 0.08.

A.3 Results for Semi-Annual Data, MM Estimator

Table A.2: **MM estimates for asset correlations and probabilities of default (in percent)**

Asset correlation estimates						
Turnover (€ m) / Rating Category	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	0.69	0.78	0.89	0.95	1.09	2.10
	(0.33)	(0.35)	(0.40)	(0.45)	(0.49)	(0.96)
IV	0.66	0.56	0.91	1.14	1.05	2.45
	(0.32)	(0.26)	(0.41)	(0.54)	(0.48)	(1.23)
V–VI	0.57	0.36	0.60	0.82	1.00	2.19
	(0.23)	(0.15)	(0.25)	(0.36)	(0.41)	(1.06)
PD estimates (one-year horizon)						
Turnover (€ m) / Rating Category	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	0.66	0.56	0.56	0.56	0.49	0.41
	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.03)
IV	2.10	2.32	2.54	2.70	2.48	2.53
	(0.07)	(0.07)	(0.09)	(0.11)	(0.10)	(0.16)
V–VI	10.07	10.52	11.31	10.70	9.72	8.84
	(0.22)	(0.18)	(0.25)	(0.28)	(0.28)	(0.43)

Standard errors determined by bootstrapping are given below in brackets.

Table A.3: **Capital requirements in terms of risk weights by rating and turnover class, based on MM estimates (in percent)**

Estimates						
Turnover (€ m) / Rating Category	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	4.9	4.7	5.1	5.3	5.3	7.4
IV	11.3	11.0	15.8	19.1	17.0	30.2
V–VI	30.7	24.5	34.0	39.1	41.4	62.4
Basel II						
Turnover (€ m) / Rating Category	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	39.8	36.6	36.6	61.2	62.2	67.3
IV	62.2	63.6	64.8	100.9	107.6	129.9
V–VI	80.3	81.5	83.6	159.7	167.0	195.4

Table A.4: **Average total differences of capital requirements in the Basel II IRBA based on MM estimates (in percent)**

Turnover (€ m) / Differences	Other Retail			Corporate		
	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II IRBA	-49.0	-49.9	-48.6	-12.7	-9.9	0.0
Estimated	-45.2	-48.1	-36.8	-30.9	-31.7	0.0
Average total difference	3.8	1.9	11.8	-18.1	-21.8	0.0

Table A.5: **Average total differences of capital requirements in the Basel II RSA based on MM estimates (in percent)**

Turnover (€ m) / Differences	Other Retail			Corporate		
	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II RSA	-25.0	-25.0	-25.0	0.0	0.0	0.0
Estimated	-45.2	-48.1	-36.8	-30.9	-31.7	0.0
Average total difference	-20.2	-23.1	-11.8	-30.9	-31.7	0.0

A.4 Results for Yearly Data, ML Estimator

Table A.6: **ML estimates for asset correlations and probabilities of default, yearly data (in percent)**

Asset correlation estimates						
Turnover (€ m) / Rating Category	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	0.21	0.64	0.71	0.67	0.96	1.79
	(0.17)	(0.40)	(0.44)	(0.44)	(0.58)	(1.13)
IV	0.28	0.45	0.76	0.84	0.79	1.72
	(0.22)	(0.29)	(0.48)	(0.56)	(0.51)	(1.18)
V–VI	1.72	1.72	1.72	0.96	1.25	3.33
	(1.18)	(1.18)	(1.18)	(0.59)	(0.73)	(2.08)
PD estimates (one-year horizon)						
Turnover (€ m) / Rating Category	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	1.32	1.17	1.13	1.15	0.10	0.92
	(0.08)	(0.11)	(0.12)	(0.12)	(0.12)	(0.16)
IV	4.26	4.66	5.16	5.60	5.13	5.41
	(0.25)	(0.32)	(0.45)	(0.52)	(0.46)	(0.75)
V–VI	22.98	20.88	19.84	21.52	19.36	18.16
	(1.22)	(0.82)	(0.78)	(1.42)	(1.49)	(2.42)

Standard errors determined analytically from asymptotic Fisher information matrices are given below in brackets.

Table A.7: **Capital requirements in terms of risk weights by rating and turnover class, yearly data (in percent)**

Rating Category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Estimates						
I–III	4.1	7.2	7.4	7.3	8.3	11.9
IV	11.4	15.9	23.4	26.2	23.8	39.6
V–VI	87.6	84.3	82.5	62.0	68.2	115.7
Basel II						
I–III	54.1	51.7	51.0	80.4	83.1	95.0
IV	69.3	70.0	70.6	123.6	131.9	163.1
V–VI	112.2	108.2	106.0	203.5	213.0	247.2

Table A.8: **Average total differences of capital requirements in the Basel II IRBA, yearly data (in percent)**

	Other Retail			Corporate		
Differences \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II IRBA	49.6	50.5	50.2	-17.2	-13.4	0.0
Estimated	53.2	40.1	36.5	-39.1	-33.1	0.0
Average total difference	3.5	10.4	13.8	-21.9	-19.8	0.0

Table A.9: **Average total differences of capital requirements in the Basel II RSA, yearly data (in percent)**

	Other Retail			Corporate		
Differences \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
Basel II RSA	-25.0	-25.0	-25.0	-0.0	-0.0	-0.0
Estimated	53.2	40.1	36.5	-39.1	-33.1	0.0
Average total difference	-28.2	-15.1	-11.5	-39.1	-33.4	0.0

A.5 Results for Yearly Data, MM Estimator

Table A.10: **MM estimates for asset correlations and probabilities of default, yearly data (in percent)**

Asset correlation estimates						
Rating Category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	0.35	0.86	0.95	0.89	1.21	2.34
	(0.24)	(0.51)	(0.57)	(0.56)	(0.70)	(1.33)
IV	0.45	0.54	1.11	1.19	1.16	2.46
	(0.30)	(0.34)	(0.67)	(0.74)	(0.73)	(1.16)
V–VI	0.53	0.33	0.60	0.53	1.20	2.66
	(0.31)	(0.20)	(0.36)	(0.34)	(0.72)	(1.65)
PD estimates (one-year horizon)						
Rating Category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I-III	0.66	0.56	0.56	0.56	0.49	0.41
	(0.04)	(0.06)	(0.06)	(0.06)	(0.06)	(0.08)
IV	2.12	2.35	2.58	2.74	2.51	2.57
	(0.14)	(0.16)	(0.25)	(0.27)	(0.25)	(0.39)
V–VI	10.61	11.11	12.00	10.61	10.22	9.26
	(0.52)	(0.42)	(0.61)	(0.53)	(0.75)	(1.09)

Standard errors determined by bootstrapping are given below in brackets.

Table A.11: **Capital requirements in terms of risk weights by rating and turnover class, based on MM estimates, yearly data (in percent)**

Estimates						
Rating Category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	3.2	5.0	5.3	5.1	5.7	8.0
IV	9.0	10.9	18.1	19.8	18.3	30.6
V–VI	30.4	24.1	35.2	30.4	47.3	72.3
Basel II						
Rating Category \ Turnover (€ m)	[0, 0.3]	(0.3, 1]	(1, 2.5]	(2.5, 5]	(5, 50]	> 50
I–III	39.8	36.6	36.7	61.3	62.3	67.4
IV	62.4	63.8	64.9	101.1	108.0	130.4
V–VI	81.7	83.0	85.5	159.2	170.5	198.9

Table A.12: **Average total differences of capital requirements in the Basel II IRBA based on MM estimates, yearly data (in percent)**

Differences \ Turnover (€ m)	Other Retail [0, 0.3]	Other Retail (0.3, 1]	Other Retail (1, 2.5]	Corporate (2.5, 5]	Corporate (5, 50]	Corporate > 50
Basel II IRBA	-49.0	-50.0	-48.5	-13.0	9.9	0.0
Estimated	-61.6	-50.3	-38.3	-40.0	-31.8	0.0
Average total difference	-12.6	-0.4	10.3	-27.0	-22.0	0.0

Table A.13: **Average total differences of capital requirements in the Basel II RSA based on MM estimates, yearly data (in percent)**

Differences \ Turnover (€ m)	Other Retail [0, 0.3]	Other Retail (0.3, 1]	Other Retail (1, 2.5]	Corporate (2.5, 5]	Corporate (5, 50]	Corporate > 50
Basel II RSA	-25.0	-25.0	-25.0	0.0	0.0	0.0
Estimated	-61.6	-50.3	-38.3	-40.0	-31.8	0.0
Average total difference	-36.6	-25.3	-13.3	-40.0	-31.8	0.0

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