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A user cost approach to capital measurement in aggregate production functions

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Abstract

A method is proposed to measure capital services in production. This means that productive assets are weighted according to their user costs. The user costs of the individual asset classes are estimated based on data from the national accounts and other sources. The results show that, in the observation period between 1991 and 2010, enterprises' capital services expand faster than the officially published capital stock. For the economy as a whole, this applies only to phases of cyclical expansion. As the capital stock is aggregated using asset prices, the differences can be explained by the different weighting methods in conjunction with the varying speeds at which the individual asset types have accumulated over time. In growth accounting, different estimates of total factor productivity emerge. The methodological difference, however, does not significantly affect the estimates of parametric production function specifications.

Key words: Capital stock, aggregation, production function, TFP.

JEL classification: E01, O47, C43.

Non-technical Summary

Statistical data on capital services play an important role for empirical applications using a macroeconomic production function. An example is the estimation of potential output which is relevant for monetary, fiscal and structural policy issues. In Germany, the Federal Government, the Deutsche Bundesbank, the Council of Economic Experts and many economic research institutes usually provide estimates which are based on a production function approach, with capital measured by the aggregate capital stock data of the Federal Statistical Office.

For calculating aggregate capital services, the literature recommends to weight individual capital goods according to their shares in the user costs of all fixed assets. As regards aggregation, however, the fixed assets accounts of the Federal Statistical Office is based on a balance-sheet approach, meaning that individual assets are weighted by their stock values. The use of these data in production function applications is thus not intuitive from a conceptual point of view. By contrast, the detailed statistics can be used to construct aggregate indices of capital services. A prerequisite is the calculation of user costs for individual assets.

According to the breakdown of the fixed assets accounts, user costs were calculated for machinery and equipment, vehicles, intangible assets, commercial and residential property. The statistical data are mainly taken from the national accounts. Following the *ex ante* approach, expected values are taken from external sources and, if unavailable, they are modelled econometrically. The differences in the levels of asset-specific user costs are due primarily to differences in the depreciation rates for wear and tear as well as economic obsolescence, whereas it is of minor importance that expected capital gains/losses and effective marginal tax rates differ from one asset to another. In sum, the use of intangibles in the production process is most expensive, followed by machinery and equipment as well as vehicles. The services of commercial and residential property are cheapest.

In the observation period between 1991 and 2010, the index series of aggregate capital services increase somewhat more strongly and are visibly more procyclical than the aggregate capital stock series of the Federal Statistical Office. The difference can be explained by the fact that investment in movable and intangible assets, which are given a higher weight than structures in the user cost approach, has on average been slightly more dynamic than construction activity and is more strongly prone to cyclical fluctuations. This is relevant for the estimates of total factor productivity using Solow growth accounting. However, parametric estimates of approximate production functions in the Kmenta model remain unaffected by the form of capital measurement.

Nicht technische Zusammenfassung

Statistische Angaben zum Kapitaleinsatz spielen für empirische Anwendungen, die sich des theoretischen Konstrukts einer makroökonomischen Produktionsfunktion bedienen, eine wichtige Rolle. Hierzu gehört die Schätzung des gesamtwirtschaftlichen Produktionspotenzials, das für geld-, finanz- und strukturpolitische Fragestellungen relevant ist. In Deutschland führen unter anderem die Bundesregierung, die Deutsche Bundesbank, der Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Entwicklung und zahlreiche wirtschaftswissenschaftliche Forschungsinstitute regelmäßig Rechnungen durch, die allesamt auf einem produktionstheoretischen Zusammenhang fußen und hinsichtlich der Messung des Faktors Kapital die aggregierten Kapitalstockangaben des Statistischen Bundesamts verwenden.

Die wissenschaftliche Literatur empfiehlt, zur Bestimmung des aggregierten Kapitaleinsatzes Vermögensgüter entsprechend ihres Anteils an den Gesamtkosten für die Nutzung von Sachanlagen im Herstellungsprozess zu gewichten. Die amtliche Anlagevermögensrechnung verfolgt mit Blick auf die Aggregation hingegen einen bilanziellen Ansatz, d.h. die Vermögensarten gehen mit ihren Bestandswerten in das Gesamtergebnis ein. Für produktionstheoretische Analysen ist die Verwendung der aggregierten Angaben mithin aus konzeptioneller Sicht nicht naheliegend. Demgegenüber lassen sich auf Basis der Detailstatistiken Indexreihen für den aggregierten Kapitaleinsatz ermitteln. Voraussetzung ist dafür allerdings die Berechnung von Nutzungskosten nach Vermögensarten.

Entsprechend der Gliederung der Anlagevermögensrechnung werden Nutzungskosten für Maschinen und Geräte, Fahrzeuge, immaterielle Vermögensgegenstände, Gewerbe- und Wohnbauten berechnet. Die statistischen Daten stammen weitgehend aus den Volkswirtschaftlichen Gesamtrechnungen. Dem Grundprinzip des *Ex ante*-Ansatzes folgend werden aber Erwartungsgrößen externen Quellen entnommen; falls nicht vorhanden, wird die Erwartungsbildung modellhaft nachgezeichnet. Die Niveauunterschiede zwischen den anlagespezifischen Nutzungskosten sind primär auf Differenzen in den Abschreibungssätzen für technischen Verschleiß und wirtschaftliches Veralten zurückzuführen; von geringerer Bedeutung sind die güterspezifischen Abstufungen bei den erwarteten Kapitalgewinnen bzw. -verlusten und der effektiven Grenzsteuerbelastung. Insgesamt ist die Nutzung von immateriellen Vermögensgütern am teuersten, gefolgt von Maschinen und Geräten sowie Fahrzeugen. Am unteren Ende rangieren die Gewerbe- und Wohnbauten.

Die auf Basis der Nutzungskosten ermittelten Indexreihen für den aggregierten Kapitaleinsatz weisen im Betrachtungszeitraum von 1991 bis 2010 eine etwas größere Steigung auf und sind sichtbar stärker prozyklisch als die Kapitalstockreihen des Statistischen Bundesamts. Die Unterschiede lassen sich dadurch erklären, dass die Investitionen in bewegliche und immaterielle Anlagegüter, die im Vergleich zu den Bauten im Nutzungskostenansatz höher gewichtet werden, im Mittel etwas dynamischer waren als die Bauaktivitäten und vor allem ausgeprägteren zyklischen Schwankungen unterworfen sind. Dies hat Folgen für die Schätzung der totalen Faktorproduktivität im Rahmen von Solow-Wachstumszerlegungen. Auf parametrische Schätzungen approximativer Produktionsfunktionen im Kmenta-Modell ist die Verwendung verschiedener Reihen für den Faktor Kapital hingegen ohne Bedeutung.

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1 Introduction

In order to model macroeconomic production processes, empirical economic research frequently uses the theoretical construct of an aggregate production function, which links the goods produced in a specified period to the input factors used. An important area of application is the production function approach to potential output estimation. In Germany, this method is now widespread. The Deutsche Bundesbank (Bbk, 2003, 2007b) and the German Council of Economic Experts (*Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Entwicklung*, SVR, 2003, 2007) have many years of practical experience in this field. Since the debt brake was anchored in the German constitution, the government must apply the European Commission's estimation method (D'Auria et al., 2010) to determine the aggregate output gap when calculating the cyclical influence on net borrowing (BMF, 2011; BMWi, 2011). This, too, has for several years been based on the assumption of an aggregate production function. In their biannual joint economic projection exercises, the leading economic research institutions are also obliged to use this procedure when drawing up their medium-term growth projection. Among international institutions, the OECD, like the European Commission, has also committed itself to the use of the production function approach (Cotis et al., 2004, Box 1).

The aggregate production function models gross value added—or GDP when looking at the economy as a whole—as a function of hours worked and the use of the existing capital stock, taking into account a productivity component that is generally measured as total factor productivity (TFP). Nowadays, parametric specifications of the production function are usually not used when estimating potential growth;[1] calculations are instead based on the growth accounting framework initially suggested by Solow (1957). As the production function is a flows relationship, fixed assets can be used as capital input only under the assumption that an asset's productive output in a given period is proportionate to its (real) user cost in production.

The applications for Germany are based on the fixed assets accounts of the Federal Statistical Office (*Statistisches Bundesamt*). All mentioned institutions except the European Commission use the aggregate gross capital stock.[2] Schmalwasser and Schidlowski (2006, p 1110) point out that gross fixed assets are the suitable measure for analysing pro-

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[1]In the 1990s, parametric approaches were still being used (eg Bbk, 1995).

[2]The European Commission uses the net capital stock, ie the gross capital stock less cumulated depreciation.

duction processes. The aggregation, however, is done by making up the individual asset types on the basis of their stock values, whereas in this context the academic literature—in the tradition of Jorgenson and Griliches (1967)—and international experts (eg OECD, 2009) recommend weighting using the shares in the user costs of all capital goods used in production.

This paper proposes a procedure to estimate index series of capital services in production for the German economy as a whole, the enterprise sector (excluding housing) and corporations. The key sources of information are, first, the official fixed assets accounts—not in the aggregate, but broken down by the following published asset types: machinery and equipment, vehicles, intangible assets, non-residential property and residential property. Second, further national accounts data (such as depreciation and capital goods deflators) and other sources (eg inflation expectations and effective marginal capital income tax rates) are used to calculate the user costs of individual asset classes according to the neoclassical investment theory (Hall and Jorgenson, 1962).[3]

The OECD and EU KLEMS regularly publish time series of capital services for Germany as part of international data collections (Schreyer and Webb, 2006; O'Mahony and Timmer, 2009). Although these have the same methodological foundations as the approach put forward here, there are several differences in the concrete calculations. The OECD and EU KLEMS base their calculations on more than the asset classes reported in the official fixed assets accounts as they analyse information and communication technology (ICT) equipment, for instance, separately. Moreover, equal assumptions are made across countries (eg EU KLEMS measures the required rates of return *ex post*) and uniform parameters are set (eg depreciation rates, capital goods prices). This makes an international comparison of the data easier, which is in line with the intention of the OECD and EU KLEMS. The present paper, meanwhile, aims to achieve as much conformity with the fixed assets accounts of the Federal Statistical Office as possible.

In terms of measuring capital as a factor of production, the main outcome of the study is that, on average, capital services expand slightly faster than the capital stock in the period under review. The main reason is that the accumulation of buildings, which, because of their comparatively large asset value, have a lot of weight in the Federal Statistical Office's aggregate capital stock, failed to keep up with the formation of movable fixed assets. The difference is larger in the enterprise sector than in the total economy.

In non-parametric modelling of the macroeconomic production function, the measurement of capital automatically influences the TFP component, which is the residual in the growth accounting exercise. This may have implications for the calculated rate of technical

[3]In connection with the major revision of the national accounts in the summer of 2011, the price-adjusted calculation of fixed assets is currently being switched from a fixed-price basis to previous year's price basis, which has been common practice in GDP measurement for several years, for instance. The most striking difference is that price-adjusted information on the capital stock is no longer represented as a volume (ie in billions of euros of a base year) but as a chain index. This methodological difference is irrelevant for the calculations carried out in the analysis. However, to date, data are only available for the economy as a whole. The results for the enterprise sector and corporations are based on data that were estimated based on the assumption that the breakdown has not changed as a result of the switch.

progress. According to the empirical findings presented here, a small part of the increase in hourly productivity that is attributed to the TFP increase in the decomposition based on official capital stock data can, on average, be interpreted as capital deepening. In periods of cyclical expansion, the effect is more perceptible. The estimates of parametric production function specifications, however, are not significantly affected by the methods used to measure capital.

This paper is structured as follows. The next section outlines some basic principles for capital services measurement in the context of heterogeneous assets which are drawn from production and index theory. On that basis, we argue that, in the fixed assets accounts of the Federal Statistical Office, the aggregation requirements of the theoretical literature can be regarded as fulfilled in terms of time heterogeneity (ie as regards the different times at which capital goods are procured), but not in relation to the different asset types. In Section 3 the user costs of five asset types are calculated. In Section 4, the index series of aggregate capital services are presented and compared to aggregate capital stock data from the fixed assets accounts. User costs and capital services are determined for the economy as a whole, the enterprise sector (excluding housing)[4] and corporations. Section 5 looks at the implications for Solow growth accounting and for estimates of the parametric production function specifications. The paper concludes with a summary.

2 The capital aggregation problem from a theoretical and empirical perspective

Some basic principles for the measurement of capital services which are derived from production and index theory are outlined in the first part of this section. The second part explains why the fixed assets accounts of the Federal Statistical Office meet these requirements only in part.

2.1 Production and index theory foundations

Macroeconomic studies often assume an aggregate production function in the form $Y(t) = A(t)F(K(t), L(t))$, where Y is output, K and L are capital and labour as factors of production, A is the TFP index and t the time index. The function $F(\cdot)$ meets the condition of rising and diminishing marginal returns in both arguments and has constant returns to scale.

In general, there are aggregation problems with regard to the functional form and for all input and output variables.[5] This can be attributed to firm-specific production

[4]For simplicity, we will merely refer to the "enterprise sector" in the following.

[5]Felipe and Fisher (2003) provide a comprehensive review of the literature including very sceptical conclusions about the theoretical foundations of an aggregate production function.

technologies that can be summarised as $F(\cdot)$ only under very restrictive assumptions,[6] and to the fact that each measure comprises heterogeneous units. The issue, widely debated in the literature, of whether presenting an aggregate production function even makes sense from an aggregation theory perspective, is as irrelevant here as the question of an adequate interpretation of this central macroeconomic model component. Given the objective of the study, it is enough to point to the empirical relevance of aggregate production functions, which is considered to have been proven for standard functional forms such as Cobb-Douglas (CD) or a technology with constant (but not unit) elasticity of substitution (CES) between labour and capital (Fisher, 1971; Fisher et al., 1977).

In the following, capital alone is allowed to be heterogeneous. The production technology is assumed to possess the functional form

$$Y(t) = A(t)F(\{K_i(t)\}, L(t)) \tag{1}$$

This presupposes the existence of $F(\cdot)$. Y and L are assumed to be homogeneous variables, and K_i measures the services of homogenous asset type i with $i = 1, 2, ..., I$. Moreover, it is assumed that an aggregate measure of capital can be calculated using observable goods and factor prices and the functional distribution of income.[7]

Two results of aggregation theory are of immediate interest when measuring, and constructing indices for, capital services. First, it follows from Leontief's (1947) theorem that capital aggregation is possible if the marginal rate of substitution between each individual pair of capital goods is independent of labour. For time heterogeneity, this means that goods in the same asset class procured at different times may be aggregated, provided quality differences can be completely represented as capital-augmenting technological progress and investment vintages are expressed in efficiency units (Fisher, 1965). Second, it is advisable to aggregate assets of various classes using the user costs in the production process (Jorgenson and Griliches, 1967). Pursuant to the Euler theorem, this is implied by the equation

$$p(t)\bar{Y}(t) = w(t)\bar{L}(t) + \sum_{i=1}^{I} r_i(t)\bar{K}_i(t), \tag{2}$$

valid for the cost-minimising input combination $(\{\bar{K}_i\}, \bar{L})$ to produce output quantity $\bar{Y}$ where p is the output price, w the nominal wage and r_i the user cost of asset i.

While wages and goods prices can usually be observed, no market information is available for the user cost of assets—with just a few exceptions such as housing rents and leasing rates for some capital goods. In general, the user cost of capital is therefore calculated using an equation that is derived from neoclassical investment theory. Taking into account the statutory tax rate on income τ and future tax rebates and subsidies specific to each

[6] Nataf (1948) found that firm-specific production functions in labour and capital can—without assuming efficient production—be expressed as a functional form in aggregate arguments for labour and capital if each of them is additively separable in the factors of production.

[7] This led to an extensive scholarly debate years ago, which was triggered by Robinson (1953–54). However, what is known as the Cambridge-Cambridge capital controversy will not be discussed in any detail here.

capital good i at the present value u_i, the user cost measured according to Auerbach (1983) totals

$$r_i(t) = \frac{1-u_i(t)}{1-\tau(t)}\Big[\big(z(t)-\pi^e(t)\big)+\delta_i(t)-\big(\pi_i^e(t)-\pi^e(t)\big)\Big]q_i(t), \qquad (3)$$

where z is the return on an alternative investment (opportunity costs), δ_i is the rate of capital consumption and q_i is the procurement price of capital good i. The anticipated price increases in asset class i are given by π_i^e and the expected general inflation by π^e.

The measure for the services of all capital goods in production is obtained by aggregating the individual asset classes using their contributions to capital income. In terms of index theory, it is advantageous to use a Divisia index.[8] Applying the Törnqvist approximation in discrete time, the construction principle of the index is

$$\Delta \ln K(t) = \sum_{i=1}^{I} \frac{1}{2}\big[s_i(t)+s_i(t-1)\big]\Delta \ln K_i(t), \qquad (4)$$

where $s_i = r_i K_i / \sum_{j=1}^{I} r_j K_j$ is defined as the share of capital good i in the user cost of all capital goods. The difference operator is denoted by Δ.

2.2 The fixed assets accounts of the Federal Statistical Office

The fixed assets accounts of the Federal Statistical Office are designed such that the problem of time heterogeneity can be considered to have been solved, provided differences in the quality of capital goods within the same asset class acquired in different years are fully reflected in prices. This is the case due to improvements in quality adjustment methods when measuring the prices of capital goods, for instance through the widespread use of hedonic procedures (eg Bbk, 2005).

In formal terms, this result can be demonstrated as follows. The price of capital good i with the quality features $\Omega_i(t)$ can be written as $q_i(t) = (1+\pi_i(t))Q_i(\Omega_i(t))$, where $(1+\pi_i(t))$ is the pure inflation component and $Q_i(\Omega_i(t))$ the quality component in the price. Real investment $I_i(t)$ can be calculated from (nominal) investment spending $I_i^n(t)$ deflated by the pure inflation component. From $I_i(t) = I_i^n(t)/(1+\pi_i(t)) = Q_i(\Omega_i(t)) \times (I_i^n(t)/q_i(t))$ it follows that the vintages of real investment are measured in efficiency units because they can be expressed as the product of the capital good in "natural" units $(I_i^n(t)/q_i(t))$ and the degree of technical efficiency $\Phi_i(t) = Q_i(\Omega_i(t))$.

The capital stock is determined by aggregating the vintages of real investment taking into account the disposal of assets:

$$K_i(t) = \Phi_i(t)\frac{I_i^n(t)}{q_i(t)} + \big[1-d_i(t-1)\big]\Phi_i(t-1)\frac{I_i^n(t-1)}{q_i(t-1)} + ... + \big[1-d_i(0)\big]^t\Phi_i(0)\frac{I_i^n(0)}{q_i(0)},$$

[8] This is an "exact" index, ie it is consistent with a second-order approximation for a logarithmic general production function (Diewert, 1976).

where d_i is the separation rate of asset class i.[9] In line with Hulten (1992), this can be used to determine an index of average embodied technical efficiency $\Psi_i(t)$. Therefore, $K_i(t) = \Psi_i(t)K_i^*(t)$, where K_i^* is defined as the stock of assets i that are homogenous from a technical point of view.

Unlike time heterogeneity, the official fixed assets accounts do not deal with the problem of aggregating different asset classes in a way that is in line with the theoretical principles presented in the previous section. The weighting scheme for the aggregate capital stock published by the Federal Statistical Office is based on the asset classes' shares in the overall value of all capital goods, which mimics a balance sheet approach based on gross data.[10] From a conceptual perspective, the use of this statistic for estimating potential output is therefore not intuitive.

The detailed raw data provided by the official fixed assets accounts can, however, be used to approximate aggregate capital services. According to (4), the calculation is based on gross capital stock data reported for individual asset types and the adequate weighting regime, determined by the user costs of the respective asset types in production.

The breakdown of fixed assets into machinery and equipment, vehicles, intangibles, commercial property and residential property is rather broad, implying a considerable degree of heterogeneity within these classes. Hence, the aggregation problem cannot be solved fully based on publicly accessible data, because the balance sheet approach of the Federal Statistical Office still determines the way in which the asset types are aggregated at lower levels. However, the measurement error this causes is likely to be low compared to the conceptual progress as a result of user cost-based aggregation at the higher level.

Finally, it should be noted that the Federal Statistical Office reports fixed assets in relation to a point in time at the turn of the year.[11] In the present study it is useful, however, as in numerous macroeconomic applications, to look at data averaged over the year. In fixed prices of a base year, the holdings at the end of a year is identical to the holdings at the beginning of the subsequent year. The average annual holdings can therefore be approximated using the arithmetic mean of the holdings in neighbouring years (Schmalwasser and Schidlowski, 2006, p 1108). We stick to this simple procedure to form the average in the regime of previous year's prices, although the construction means that the two figures are no longer identical.[12]

[9] For simplification, the separation rate is assumed to be constant here. In fact, the Federal Statistical Office calculates the disposal of assets by linking investment years with a separation function, in which the effective service life is randomly drawn from a probability distribution (Schmalwasser and Schidlowski, 2006).

[10] In the fixed-price basis, that means that the volume data for the asset types (in billions of euros of a base year) are added up. This is equivalent to weighting in line with the value shares of the asset types in the base year. In the previous year's price basis, by contrast, the value shares of the previous year are used as the weighting regime. For more information, see eg Tödter (2007).

[11] Before the revision of the national accounts in the summer of 2011, holdings were reported as at the beginning of the year; since then, the holdings at the end of the year have been reported in line with international conventions (Räth and Braakmann, 2011).

[12] According to Räth and Braakmann (2011), the index would change by „not much" (p 853) if year-end data were used instead of annual averages.

3 Calculating the user cost of capital

No representative market information is available on the user cost of most assets. Residential property is an important exception. The use of market information on this asset class is an option in principle, because the residential rental market in Germany is largely organised in a competitive way, and the entire range of supply is covered—albeit not in an entirely representative manner. However, in order to avoid methodological differences when calculating the user costs of the various asset types, residential property will be treated like other asset classes in the following.

User cost of capital is determined using (3). In the literature, a distinction is made between *ex post* and *ex ante* calculations, with hybrid forms also discussed (Oulton, 2007). The *ex post* approach is based on the objective of calculating the user cost of capital using only data from national accounts which is a consistent framework, making the results transparent and simply reproducible. To determine the return on fixed assets, however, the national accounts figures on operating surpluses and entrepreneurial income must be used, which are measured with a high degree of inaccuracy, at least, in real time. Even more weighty is the theoretical objection that the key factor in investments is not the actually realised rate of return, but the rate of return anticipated when the decision is taken. In the *ex ante* approach, the investor's required return is approximated by the rate of return on alternative investments and expectations on future relative capital gains/losses have to be formed.

The calculation of the user cost of capital proposed here is, in principle, based on the *ex ante* method. However, where theoretically appropriate, national accounts data are used. Expectation formation is modelled econometrically—unless survey data (eg inflation expectations) or other sources (eg effective marginal rates of taxation on income from capital) provide adequate information. From an empirical perspective, this means that the components of the user cost of capital fluctuate less over time than when using *ex post* data from the national accounts. Leaving aside the question of economic justification, which is virtually impossible to answer objectively, the advantage of avoiding volatility in the user cost of capital is that the resulting index series for aggregate capital services are not affected by random noise that is mainly attributable to the user cost of capital.[13]

3.1 The components of the user cost of capital

The user cost of an asset employed in the production process is, according to (3), calculated as the sum of the real return required by the investor, the regular depreciation for technical wear and tear as well as economic obsolescence and the anticipated capital gains/losses multiplied by the replacement price, with due account taken of the effective marginal rates

[13] Diewert (2005) suggests, for instance, ensuring that the resultant index series is as smooth as possible and that the results are easy to reproduce when choosing the methods of calculating capital services.

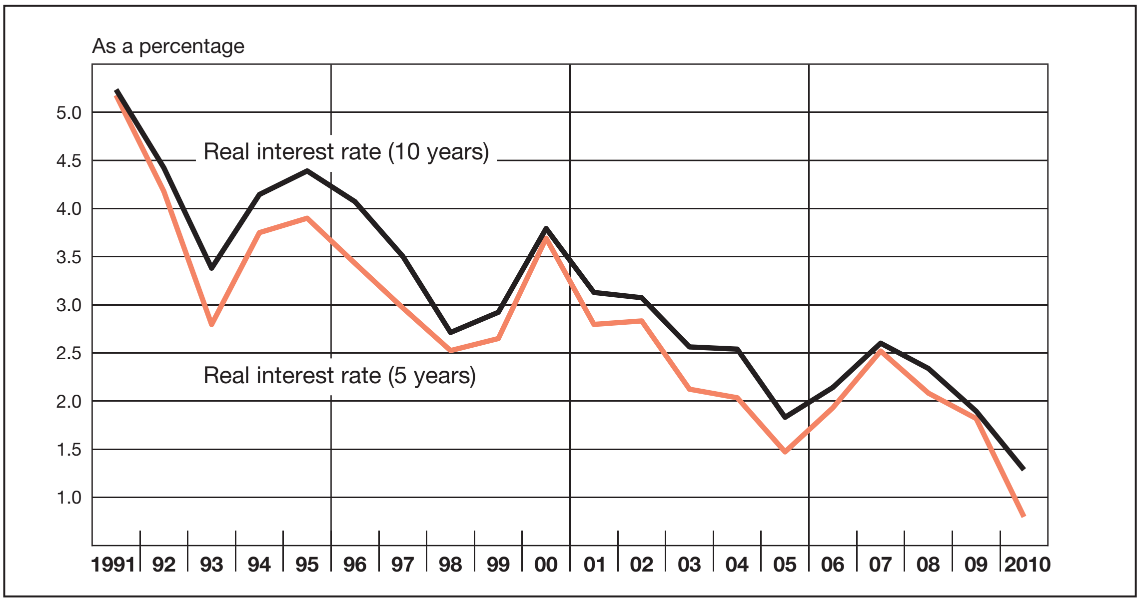
of taxation on income from capital specific to the asset class, which reflects the fiscal component.[14]

Required real return

Ex ante real interest rates are used as a measure of the investor's expected real return. Specifically, the *Consensus Forecasts* inflation expectations are deducted from the yield on corporate bonds (Bbk, 2001). The medium-term (5 years) and long-term (10 years) real interest rates are averaged. The yield requirement applies to any investment and is therefore independent of the asset type under examination. This means that no account is taken of possible premiums attributable to the length of time for which capital is tied up and which may differ from one capital good to another in line with their service lives.

To take into account the risk of default associated with entrepreneurial activity, the nominal return on corporate bonds is used which incorporate an appropriate spread over safe government bonds. Business activity as a whole naturally has to fulfil the market test and thus individual investments cannot be regarded in isolation (OECD, 2009, page 67). Assets do differ somewhat in terms of how easy they are to recover in the event of insolvency. However, such differences cannot be accounted for here as there is insufficient information.

Figure 1: Real interest rates



[14]„Effective“ means in this context that the expected effects of tax rebates and subsidies specific to the asset are included in the marginal tax rate. This includes, in particular, the difference between the actual time profile of depreciation and the depreciation scheme relevant for tax purposes.

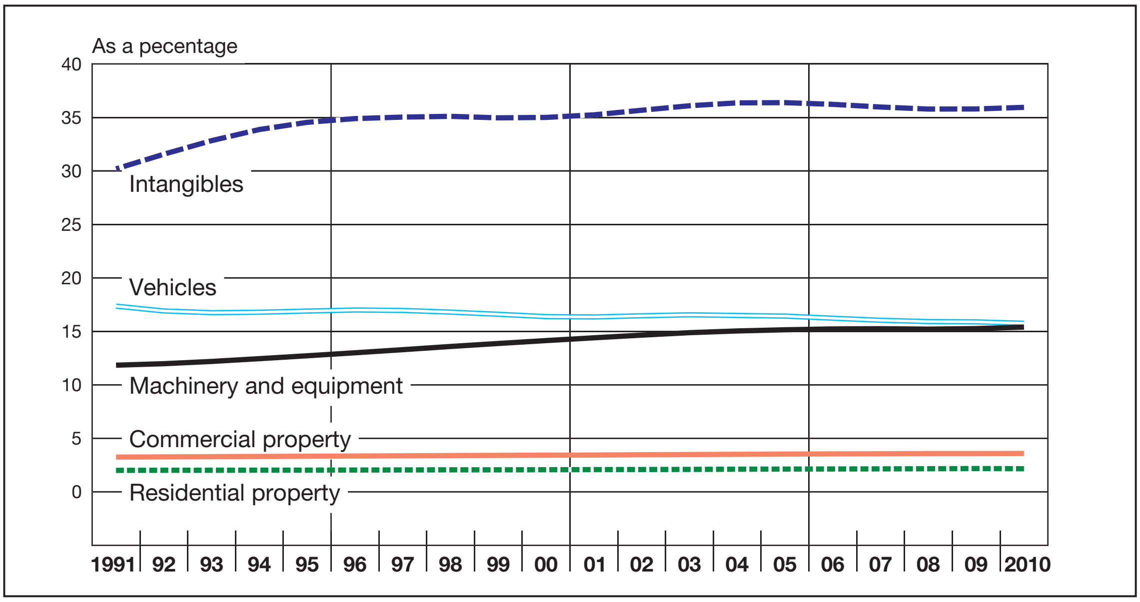
Figure 1 shows that real interest rates have, since 1991, not only been subject to cyclical volatility, but that they have also moderated considerably in trend terms. The assumption of constant real interest rates—as proposed by Diewert (2005), for instance—does not therefore appear appropriate for Germany in the observation period.

Depreciation for technical wear and tear and economic obsolescence

The annual rate of depreciation for assets subject to technical wear and tear and economic obsolescence is calculated from national accounts data.[15] The approach used by the Federal Statistical Office generally assumes straight-line depreciation based on randomly distributed services lives at a highly disaggregated level (Schmalwasser and Schidlowski, 2006). Capital consumption is reported on an annual basis, broken down by asset types.

The ratio of price-adjusted depreciation to the corresponding net fixed assets yields a time series for the average rate of depreciation (OECD, 2009, p 97).[16] This calculation is in line with the theoretical model and uses all the information contained in the fixed assets accounts on the composition of the asset classes and their depreciation methods, which influence the effective rate of depreciation over time.

Figure 2: Depreciation rates



[15]The Federal Statistical Office also considers that firms may sometimes be insured against the risk of capital losses.

[16]The rate of depreciation is calculated on the basis of constructed volume series, ie multiplying the index series for the price-adjusted fixed assets and the price-adjusted depreciation by the corresponding figures in the reference year 2005.

The asset-specific depreciation rates are, first, different in level. Figure 2 shows that in the years 1991 to 2010 residential and commercial property had the lowest annual depreciation rates, at 2% and $3\frac{1}{2}$% respectively, followed by machinery and equipment, and vehicles with an average depreciation rate of around 15% in the last two decades. At annual rates of between 30% and 35%, intangible assets were written down most rapidly in this period.

Second, the asset types differ in terms of whether their depreciation rate has increased over time or has essentially remained stable. While the depreciation rates for property and vehicles have remained largely unchanged since 1991, those of machinery and equipment and intangibles have seen a trend increase, at least until the middle of the last decade. This is probably partly due to the fact that the average service life of these asset types has shortened as obsolescence effects have increased due to the rapid technical progress in the ICT sector (eg Bbk, 2007a).

Anticipated relative capital gains/losses

Moreover, the ratio of anticipated asset price changes to general inflation is a component of the user cost of capital. This component assumes a considerable magnitude for some capital goods, because their purchase prices have decoupled from the general price trend as a result of technological progress embodied in capital, which mainly acts to depress prices. This applies to machinery and equipment as well as intangibles, which have lost $1\frac{1}{2}$% and 2% of their value a year on average over the last two decades. By contrast, prices for vehicles and property rose by an average of $1\frac{1}{4}$% and $1\frac{1}{2}$% respectively per year, just a little less than general consumer price inflation ($1\frac{3}{4}$%).

Unlike for consumer price inflation, data on expected capital goods prices are not observable directly, for instance in terms of survey information. Expectations formation must therefore be modelled. The expected price increase in an asset class is calculated here using simple exponential smoothing. This method mimics the concept of adaptive expectations formation

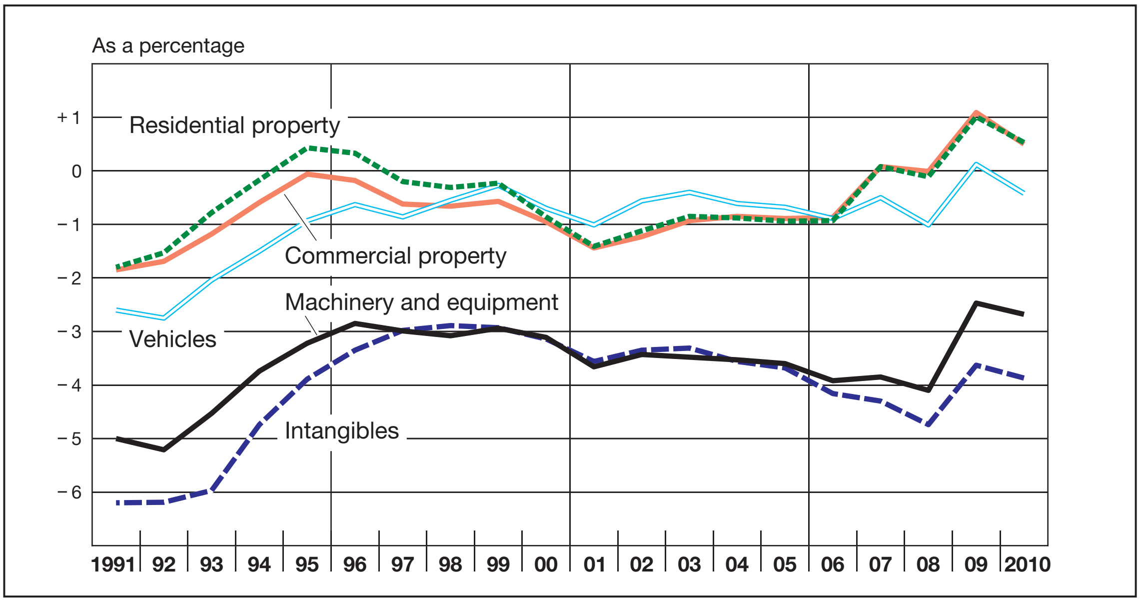
$$\pi_i^e(t) = \beta\pi_i^e(t-1) + (1-\beta)\pi_i(t), \tag{5}$$

where $0 \leq \beta \leq 1$ is the smoothing parameter. The method meets the condition that only time series information available at the relevant point in time is used. However, this does not guarantee efficient information processing, which means that this does not generally fulfil the postulate of rational expectations. By contrast, the modelling used is robust, transparent and reproducible at any time.[17]

By setting β to 0.8, the year-on-year volatility in the expected figures is relatively moderate. However, Figure 3 also shows that there are periods of several years' duration in which value changed appreciably more or less. Machinery and equipment as well as intangibles lost the most value on average as compared to general consumer inflation, at

[17] The initial value for the year 1991 is chosen such that the average difference between the mean of the original series and that of the smoothed series is just zero for a given smoothing parameter.

Figure 3: Anticipated relative capital gains/losses



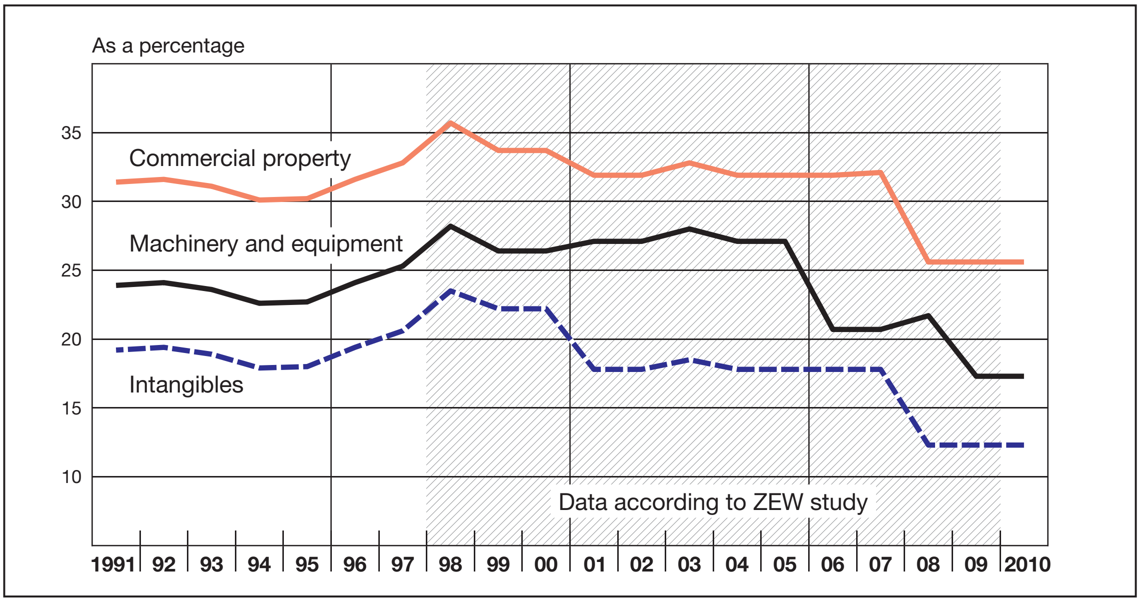
$3\frac{1}{2}\%$ and 4% per annum respectively. However, property and vehicles also fell moderately in value over the past two decades.

Fiscal component

The national accounts contain no statistical raw material that could be used to calculate the fiscal component of the user cost of capital. This is based on the idea of the *ex ante* approach that investors take future tax conditions (including investment incentives) into account when making investment decisions. These cannot be adequately approximated, for instance, using average *ex post* tax rates that are calculated from actual tax revenues and the appropriate tax base. Moreover, this information alone does not allow the tax burden to be broken down according to the individual asset types.

Effective marginal rates of taxation on income from capital, which reflect the tax conditions at the time that the respective investment decision is made, are conceptually more suitable. These generally differentiate not only by asset type, but also by funding form. In the literature, the effective tax paid on income from capital used for investments is widely determined using the procedure developed by Devereux and Griffith (1999). In a broad study ordered by the European Commission, the Centre for European Economic Research (ZEW) presented relevant data for all European Union countries and for some other countries (eg United States, Switzerland) in the period from 1998 to 2009 (Devereux et al., 2009). From this, the effective marginal tax rates of corporate income calculated for Germany are extracted, broken down by industrial property, machinery and equipment as well as intangible assets. In general, the tax burden differs in the form of funding selected. This, however, is not taken into account; instead, the different rates are averaged.

Figure 4: Effective marginal tax rates of corporate income



One of the problems with using this data is that the ZEW study models the effective marginal tax burden of an investment based on a series of assumptions that is not identical in terms of economic parameters such as real interest rates, inflation and depreciation rates to the corresponding assumptions in the user cost of capital calculated here. In addition, problems of coverage and definitions cause inaccuracies. First, the asset classes observed in the ZEW study are not fully consistent with the breakdown by asset types in the national accounts definition used here.[18] Second, the effective tax rates of corporate income can only be regarded as a rough guide to the representative marginal tax burden of total capital income, as many companies are managed by sole proprietors or unincorporated partnerships, and their revenues are therefore subject to the personal income tax. This problem is particularly striking for residential property because most rented property in Germany is owned by households.[19]

However, these indisputable shortcomings appear acceptable compared to the considerable theoretical and statistical faults with alternative calculation methods. The systematic distortion is probably fairly small as compared to the *ex post* analysis pursuant to Mendoza et al. (1994), which ultimately reflects an average burden and thus likely significantly underestimates effective marginal tax rates. This is evident from a comparison of the cost of capital, which is also reported in the ZEW study, if it is adjusted to the concept of the user cost of capital by also taking into consideration economic depreciation. Disregarding

[18]In the ZEW calculations, the rate for machinery and equipment is also used for vehicles, and the rate for industrial properties is also applied to residential property units.

[19]According to surveys, households own three-quarters of residential property in Germany, while only just over 40% is owner-occupied (ECB, 2009).

the fiscal component is not an option either, as that would mean that user cost of capital would be understated by $15 - 55\%$ depending on asset type and time period. Irrespective of the impact on absolute levels, the fact that the effective marginal tax rates of corporate income according to the ZEW study differ substantially not only from one capital good to another, but are also subject to appreciable change over time is also significant (see Figure 4).

The results of the ZEW study do not cover the entire period of the capital stock accounts. At the current end, a calculation of effective marginal tax rates is missing for 2010. It makes sense to use the results for 2009 given that the tax conditions for investments did not change in these two years. Extrapolating for the period from 1991 to 1997 is more difficult. It is clear that there were significant changes in the tax charged on income from capital in this period, not least from the *ex post* analysis pursuant to Mendoza et al. (1994). With no alternative data available in the literature, these calculations are used, correcting for their systematic distortion to the downside. As this procedure does not permit differentiation by capital good, the effective marginal tax rates for the individual asset types roughly approximated in this manner are parallel.

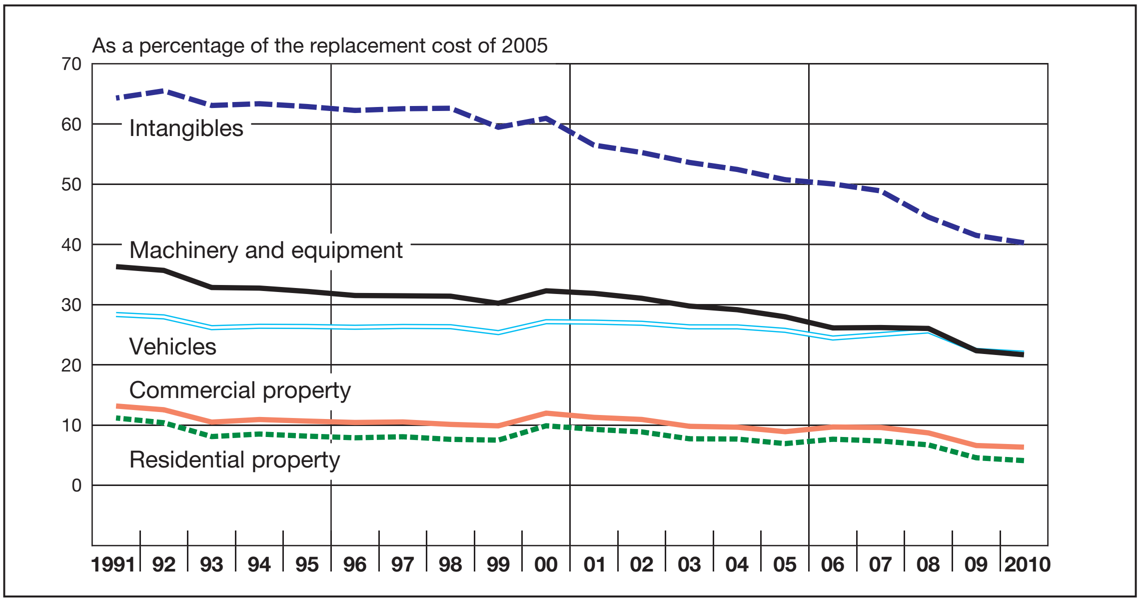
3.2 Asset-specific and aggregate user cost of capital

Linking the individual components according to (3) yields estimates of the user cost of the asset types under consideration, which differ, at times substantially, in terms of amount and trend (see Figure 5). Relative to the respective replacement cost, the use of intangible assets in the production process is most expensive, followed by machinery and equipment, and then by property. The depreciation rates are the main reason for the different levels of user costs. Gradations in the expected relative capital gains/losses are also a considerable factor.

The ranking has not changed in the last two decades. However, the differences between tangible and intangible assets have narrowed significantly. With the exception of vehicles, the user costs of all asset types fell visibly between 1991 and 2010. For instance, it is estimated that the annual cost of using intangibles was just two-fifths of the replacement cost of 2005 at the current end, compared with almost two-thirds in the first few years following reunification. The costs calculated for machinery and equipment fell from more than a third to just over one-fifth. The user cost of vehicles remained comparatively stable in a relatively tight range around 25%. In the past two decades, the user cost of property halved to around $6\frac{3}{4}\%$ in 2010 for commercial use and $4\frac{1}{2}\%$ for residential property.

The trend decline in real interest rates as a proxy for the return required by investors is equally relevant for all asset classes. For intangibles as well as machinery and equipment a further factor reducing costs is that replacement prices for goods with unchanged quality have fallen, not only relative to general inflation but also in absolute terms. The associated effect of expected capital losses implies a considerable markup. By contrast, the trend towards shorter service lives observed in these asset clases until the middle of the last decade had no noteworthy impact on the user cost of capital through heightened depreciation rates.

Figure 5: User cost of capital by asset types

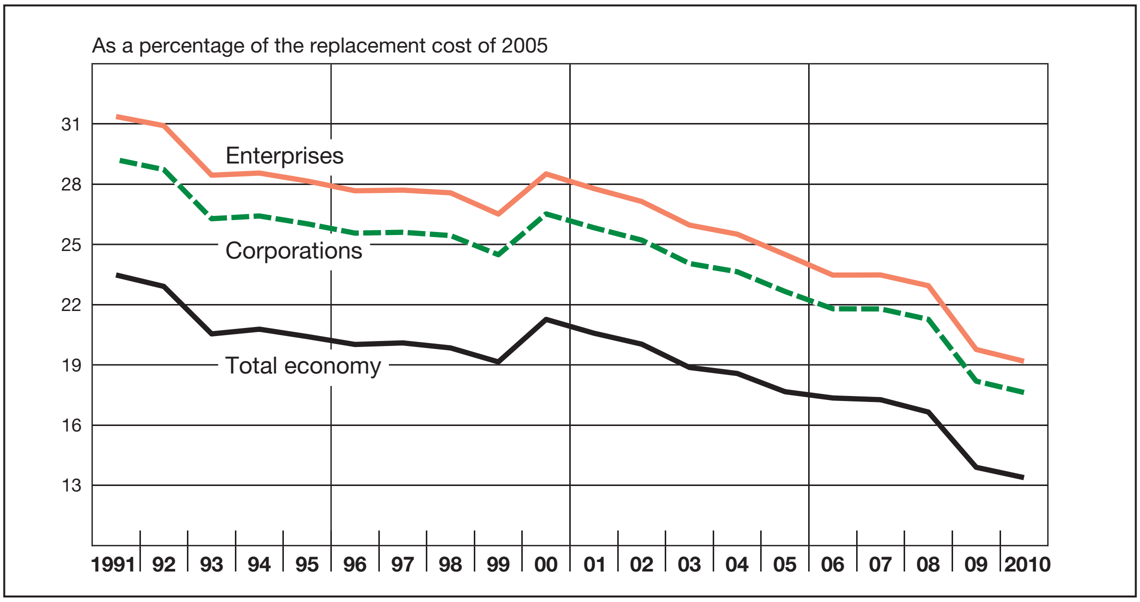


Since 1998, but no later than the second half of the last decade, effective marginal tax rates of corporate income have fallen appreciably for all asset types.

A measure for the user cost of all capital goods is obtained by aggregating the results for the individual asset classes, with weighting regime given by their shares in overall expenditure for the use of capital in the reference year 2005. Various sector definitions can be looked at. In terms of interpreting the aggregate user cost of capital as an independent indicator, using the asset structure of corporations as a weighting regime is the appropriate choice, as the fiscal component is calculated exclusively based on effective marginal tax rates for corporations. The representativeness for the economy as a whole is limited, meanwhile, as corporations in Germany generate only two-thirds of gross value added. The total-economy aggregate has the advantage of employing all assets used to generate GDP as the basis for deriving the weighting regime. The user costs are, however, distorted in that enterprises subject to income tax are implicitly allocated the same effective marginal tax rates as corporations. This is likely to cause inaccuracies, particularly for residential property. When calculating enterprises' aggregate user cost of capital, the stock of residential property is, by definition, excluded.

Figure 6 shows the time series of the aggregate user cost of capital for the total economy, the enterprise sector and corporations. The differences in level can mainly be attributed to the weight of residential property in the respective categories. By contrast, the time series move more or less in parallel in the observation period. They are characterised by a trend decline, showing the sharpest decrease in the recession years of 1993 and 2009. Between 1999 and 2000, the user cost of capital picked up perceptibly, not least due to

Figure 6: Aggregate user cost of capital



temporarily increased real interest rates. On balance, corporations annually spent almost three-tenths of the replacement cost of the year 2005 for capital services in the early 1990s, while this figure was more than one-fifth in the years immediately preceding the financial and economic crisis. In 2009 and 2010, the rate fell visibly below this mark.

4 Index series for the aggregate capital services

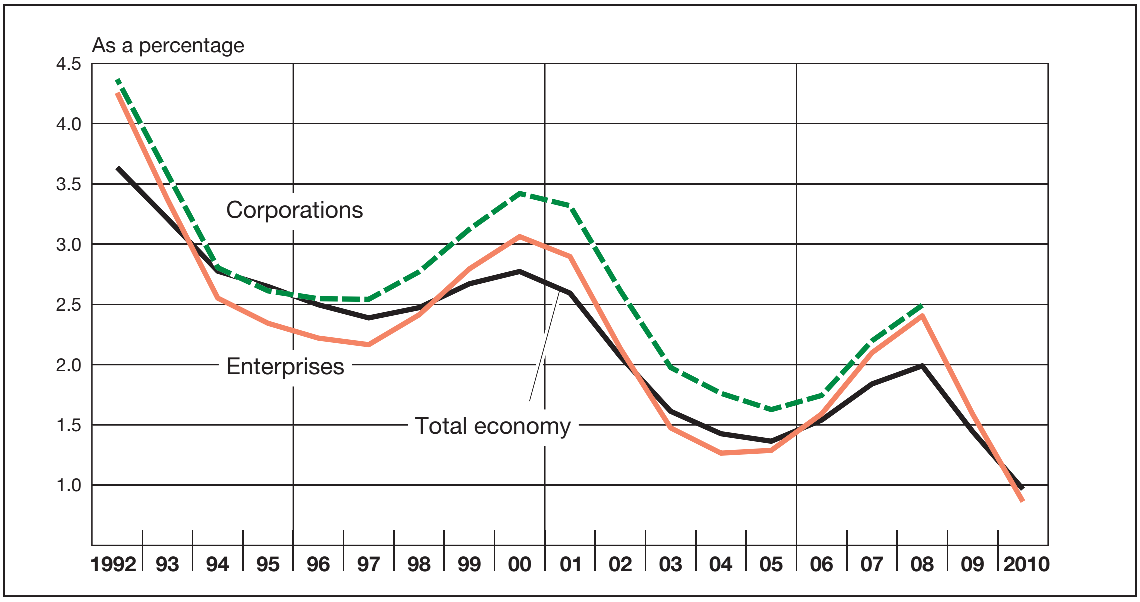
Divisia-type index series describing aggregate capital services are derived pursuant to (4) from the weighted mean of the rates of change of the gross stock of individual assets, with their shares in the total expenditures for capital services used as the weighting regime. The breakdown is possible based on estimates of user cost. The available dataset allows calculations for the total economy, the enterprise sector and corporations.[20]

It is equally true of all three categories that the use of capital has increased from year to year, but annual growth has tended downwards amid strong procyclicality (see Figure 7).[21] The cyclical pattern can be attributed to the fact that investment in movable and intangible assets, which has a comparatively high weighting using this method, is particularly sensitive to cyclical developments. It is, moreover, striking that the use of capital in corporations expanded more strongly than in the enterprise sector and in the

[20]In the breakdown by asset types, corporations' fixed assets (in prices of the base year 2000) are available only up until 2009 (as holdings at the beginning of the year.)

[21]In the period under observation, boom periods were experienced in the years 1991 and 1992, between 1998 and 2000 and from 2006 to 2008.

Figure 7: Annual change in aggregate capital services



economy as a whole. This is because the average asset accumulation by corporations has, in the last two decades, outpaced that in the broader sectoral categories in all asset classes except residential property. The increase in the capital services of the enterprise sector responded slightly more procyclically than the total-economy counterpart. This can largely be attributed to the weighting structure as the stock of residential property accounts for around three-tenths in the total economy, while it is, by definition, inexistent in the enterprise sector.

Compared with the aggregate capital stock series of the Federal Statistical Office, capital use is characterised by greater momentum (see Figure 8). In the enterprise sector, it expanded by an average of 2.4% per annum from 1991 to 2010, while the real replacement value for assets tied within this sector increased by just 2.0% a year. In the economy as a whole, the gap was smaller, at 2.3% versus 2.1%. Moreover, cyclical volatility is slightly smaller in capital accumulation than in capital services.

When aggregating pursuant to the user cost approach, intangible and movable assets are accorded more weight than in the Federal Statistical Office's capital stock accounts, in which property makes up a substantial share given that it represents a large percentage of holdings (see Table 1). For the economy as a whole, more than four-fifths of gross fixed assets constitute residential and commercial properties, while these asset types are estimated to account for only just under three-fifths of the expenditures for capital services. Although the enterprise sector includes no residential properties, structures nonetheless represent half of the commercial capital stock. In terms of user costs, commercial properties make up only one-quarter.

Figure 8: Annual change in capital services and the capital stock

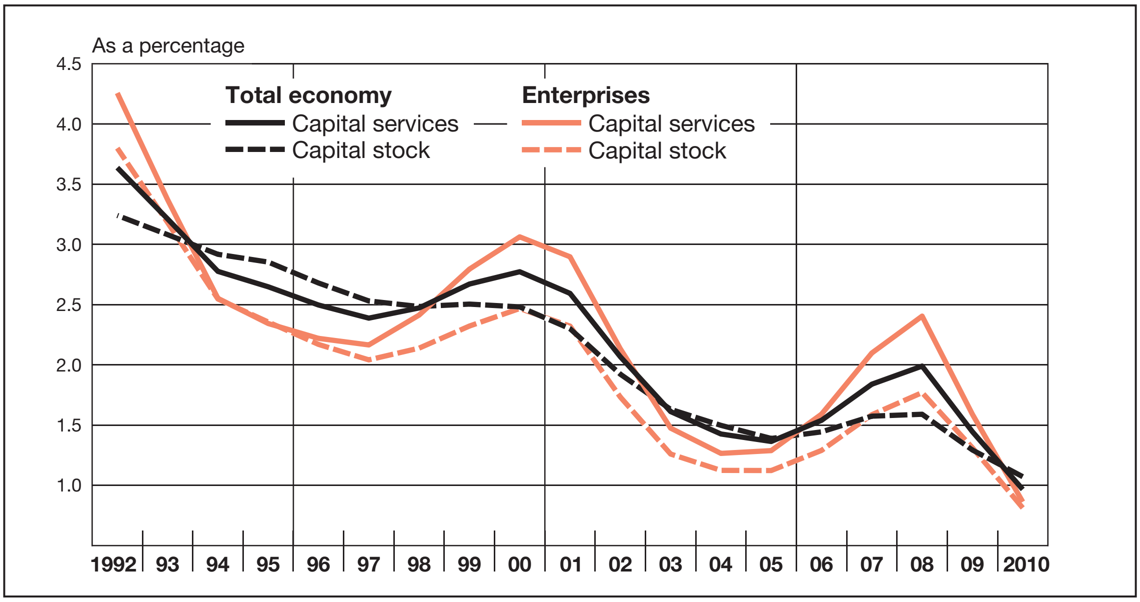
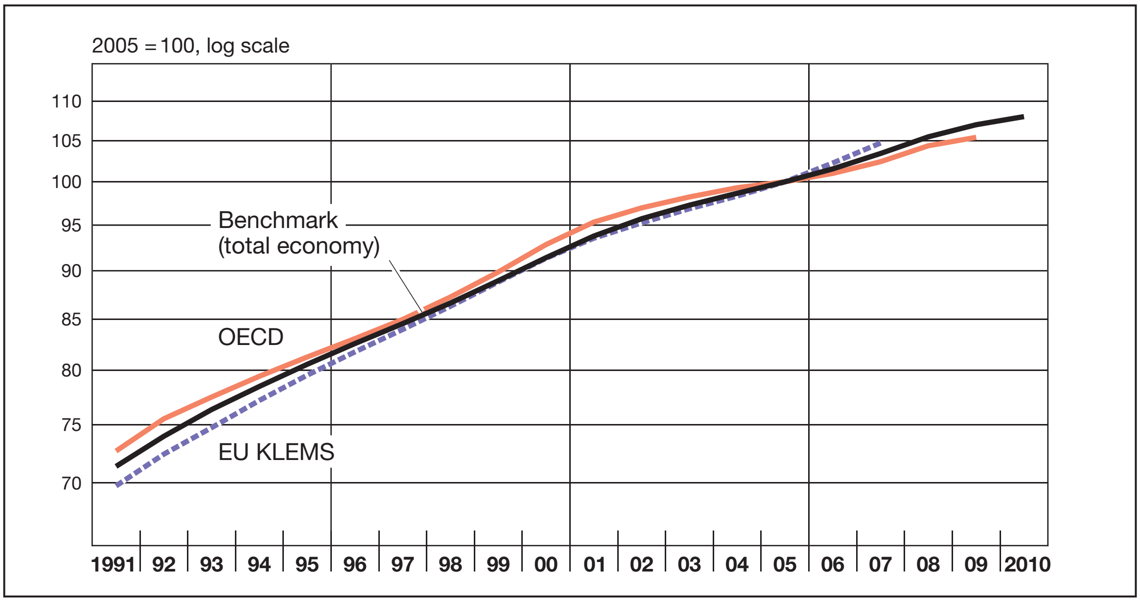


Table 1: Weighting systems (in percentages)

Aggregate	(Weighting) variable	Maschinery and equipment	Vehicles	Intangible assets	Commercial property	Residential property
Total economy	stocks	13.4	3.6	0.8	37.2	44.9
	user cost	31.3	7.5	3.6	29.6	27.9
	memo item:					
	asset growth*	1.8	3.1	5.9	1.6	2.5
Enter-prises	stocks	35.3	9.6	2.2	52.9	–
	user cost	53.6	12.8	6.2	27.4	–
	memo item:					
	asset growth*	1.9	3.2	5.9	1.6	–
Corpo-rations	stocks	28.1	7.5	1.5	45.4	17.6
	user cost	48.7	11.2	4.7	27.2	8.4
	memo item:					
	asset growth*	2.2	3.5	6.2	2.2	1.2

* Average annual percentage change.

Figure 9: Capital services according to OECD and EU KLEMS



As the velocity at which the individual asset types are accumulated differs considerably, weighting differences have a substantial impact on trends in the observed aggregates. Since 1991, the stock of commercial property has risen at a fairly sluggish rate of, on average, 1.6% a year. Machinery and equipment also saw fairly weak annual asset growth in the overall period (1.8%)—particularly when substantial increases in quality are taken into consideration, which drive up stocks as measured in constant efficiency units. Even the stock of residential property witnessed greater average growth, at 2.5% a year. Commercial vehicles expanded at 3.1% a year, intangibles at 5.9%.

Finally, it is worth comparing the index of capital services that is calculated here for the total economy and the corresponding user-cost-based measures that are published regularly by the OECD and EU KLEMS. The current OECD data go up to 2009, while the EU KLEMS index currently already ends in 2007. These alternatives are thus only available with a considerable lag. Moreover, the approaches differ substantially in terms of implementation. For instance, both the OECD and EU KLEMS differentiate further between ICT and non-ICT equipment because these asset types differ considerably in terms of the speed of capital-embodied technical progress. This affects the user cost calculation through deviating depreciation rates and investment-specific price trends. The breakdown means that ICT equipment is allocated a greater weight in aggregation, which *ceteris paribus* causes the aggregate to have a steeper trend path given the more dynamic accumulation. Figure 9 demonstrates that, for EU KLEMS, this effect appears to be evident over the entire period. By contrast, the OECD index outpaces the benchmark series only in the 1990s.

5 Capital measurement and production function

The aggregate measure of capital services is important in the context of the production function approach to potential output estimation. The first part of this section deals with the implications of the alternative measurement procedures for the nowadays predominant non-parametric estimation approach, which is based on growth accounting pursuant to Solow (1957). The regression results of an approximated CES production function (Kmenta, 1967) for the various measures of capital are presented as another application example.

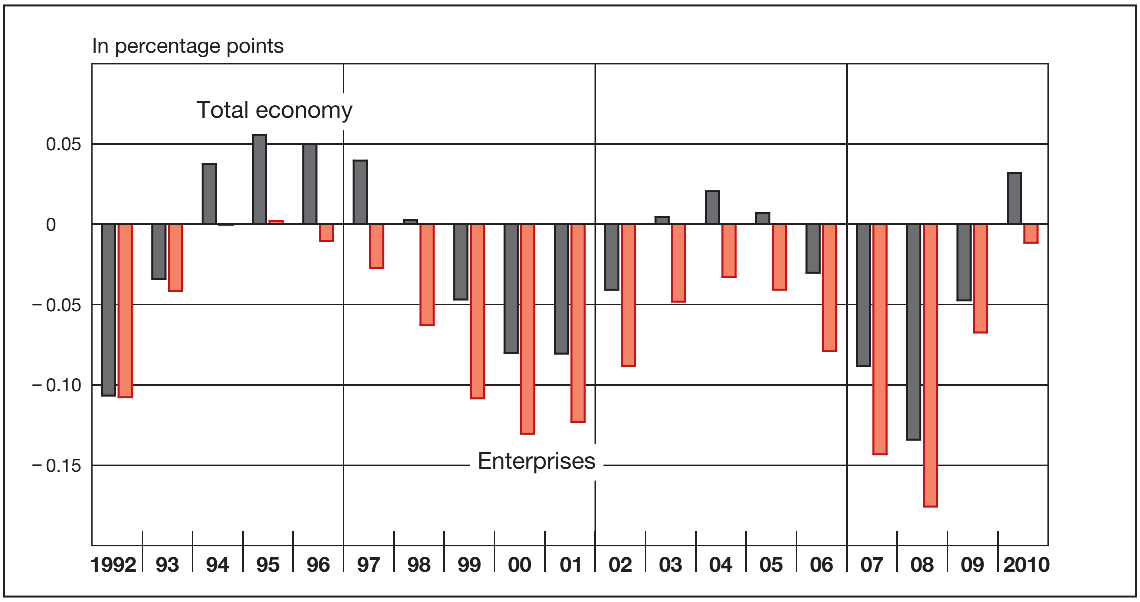
5.1 The influence of capital measurement in growth accounting

According to Solow growth accounting, the rate of change in output is decomposed into the contributions made by the factors of production labour and capital, and the TFP rate. In analogy with the model frameworks used to measure capital, the specification based on Jorgenson and Griliches (1967) is applied:

$$\Delta \ln Y(t) = \bar{\alpha}(t)\Delta \ln L(t) + \big(1 - \bar{\alpha}(t)\big)\Delta \ln K(t) + \Delta \ln A(t), \tag{6}$$

where $\alpha = (wL)/(pY)$ is the labour income ratio and $\bar{\alpha}(t) = \frac{1}{2}\big(\alpha(t) + \alpha(t-1)\big)$ is the corresponding Törnqvist weight. Labour is measured as effective hours worked.

Figure 10: Difference between the TFP rates resulting from growth accounting with capital services or capital stock



From a conceptual perspective, the TFP component is borne by disembodied technical progress. In the empirical implementation, however, it is a residual containing all inaccuracies associated with the measurement of the observed series (see Hulten, 2001, for a detailed overview). A change in the measurement of capital *ceteris paribus* necessarily changes the rate of TFP growth. When using the user cost approach, the TFP component becomes slightly flatter on average than under the conventional calculation based on aggregate capital stock data provided by the Federal Statistical Office. This can be attributed mainly to the higher contributions that capital formation makes to output growth.

The average gradation is slightly larger in the enterprise sector than in the total economy, but in both cases stands at less than 0.1 percentage point in relation to the annual rate of change. For a mean TFP rate of around 0.9% a year, this yields a distortion of $2\frac{1}{2}$% for the total economy and still $7\frac{1}{2}$% in the enterprise sector, if the aggregation according to the official fixed asset accounts is used. Figure 10 reveals that distortion of the TFP rate has a counter-cyclical pattern. This is because fluctuations in investment in movable and intangible assets are much stronger than in construction activity, which means that the capital services are more procyclical than the capital stock.

The breakdown of the rate of change of hourly productivity exemplifies that the choice of measurement framework can have an impact on the relative strength of the supply-side growth factors in a magnitude that is relevant to economic interpretation. A transformation of (6) yields

$$\Delta \ln \frac{Y(t)}{L(t)} = \big(1 - \bar{\alpha}(t)\big)\Delta \ln \frac{K(t)}{L(t)} + \Delta \ln A(t). \tag{7}$$

Studying the rate of productivity has the advantage over output growth that all components of the decomposition are affected by the way in which capital is measured.[22] Alongside the TFP rate, this is the contribution made by capital deepening, which can be determined by multiplying the increase in capital intensity K/L by the capital income ratio.

The way in which capital is measured has no visible impact on the decomposition of the rate of change in hourly productivity in the economy as a whole. In the enterprise sector, capital deepening "explains" the increase in productivity to a slightly higher percentage than the TFP component if capital is aggregated using the user cost approach. At 0.1 percentage point, the difference is greatest in the period from 2006 to 2010 (see Table 2).

The fact that the TFP component is a residual implies that, when decomposing actual economic growth, all that happens arithmetically is a shift between the contributions made by capital and TFP, which are fairly small according to the results outlined. As part of non-parametric estimates of potential output, these shifts may, however, have an impact on potential growth, as capital accumulation generally enters into the estimates in unfiltered form, while TFP changes, which are very susceptible to fluctuation, are smoothed over time.

[22]Solow growth accounting contains, in the contribution of labour, a component that is not influenced by the form of capital measurement.

Table 2: Decomposition of changes in hourly productivity

Period	Measurement approach	Total economy: Hourly productivity*	Total economy: Capital deepening†	Total economy: TFP†	Enterprises: Hourly productivity*	Enterprises: Capital deepening†	Enterprises: TFP†
1992 – 1995	services	2.19	1.11	1.09	1.67	1.02	0.65
	stocks		1.10	1.10		0.98	0.69
1996 – 2000	services	1.96	0.71	1.25	2.03	0.60	1.43
	stocks		0.71	1.25		0.53	1.49
2001 – 2005	services	1.62	0.75	0.87	1.71	0.67	1.05
	stocks		0.73	0.89		0.60	1.11
2006 – 2010	services	0.93	0.35	0.58	0.73	0.33	0.40
	stocks		0.30	0.63		0.24	0.49

* Average annual percentage change.
† Contribution to change in hourly productivity in percentage points.

5.2 Estimates of approximated CES production functions

Kmenta (1967) proposes a linear regression model, which specifies a CES production function as a second-order Taylor approximation. Based on this approach, the CES production function $Y(t) = A(0)e^{\lambda t}\left[\eta K(t)^{-\rho} + (1-\eta)L(t)^{-\rho}\right]^{-1/\rho}$ with $0 < \eta < 1$ and $\rho > -1$ can be approximately estimated from the following regression:

$$\ln \frac{Y(t)}{L(t)} = \gamma_0 + \gamma_1 t + \gamma_2 \ln \frac{K(t)}{L(t)} + \gamma_3 \left[\ln \frac{K(t)}{L(t)}\right]^2 + \varepsilon(t). \tag{8}$$

The mean TFP rate is given by $\lambda = \gamma_1$. The parameters of the CES function can be calculated using $\rho = -2\gamma_3/[\gamma_2(1-\gamma_2)]$ and $\eta = \gamma_2$. The special case of the CD production function ($\rho = 0$) is reflected by the restriction $\gamma_3 = 0$.

The least squares estimates are carried out using unadjusted quarterly data in the period from the first quarter of 1991 to the fourth quarter of 2010 (80 observations), with the annual index series for capital quartalised with the aid of the univariate interpolation method suggested by Boot et al. (1967) using the *minimising squared second differences* option. The series for output and hours worked, which are, like the capital series from the year 2005, normed to the index level of 100, display a distinct seasonal pattern. Therefore, seasonal dummy variables are included in the regression.

Unit root tests indicate that the logarithmic time series for hourly productivity and capital intensity are not (trend-)stationary. In econometric terms, the regression could therefore be understood as a cointegration model if the error term ε displays covariance stationarity. To check whether this is the case, cointegration tests are conducted based on the procedure developed by Engle and Granger (1987). As reported in Table 3(b), for the total economy the Kmenta estimate can be interpreted as a static cointegrating regression. By contrast, when using the data for the enterprise sector, the null hypothesis of a unit root in the error term of the CES specification cannot be rejected. The estimates of the CD function, meanwhile, meet this condition.

Table 3: Approximate production function estimates

	Total economy				Enterprises			
	Services		Stocks		Services		Stocks	
	CES	CD	CES	CD	CES	CD	CES	CD
(a) Least squares estimates (standard errors in parentheses), regressand: $\ln(Y/L)$								
const	-0.13 (0.04)	-0.10 (0.02)	-0.13 (0.03)	-0.11 (0.02)	-0.23 (0.05)	-0.18 (0.04)	-0.22 (0.05)	-0.19 (0.03)
S(Q1)	-0.027 (0.005)	-0.028 (0.005)	-0.028 (0.005)	-0.028 (0.005)	-0.043 (0.008)	-0.045 (0.008)	-0.044 (0.008)	-0.045 (0.008)
S(Q2)	0.036 (0.009)	0.030 (0.007)	0.034 (0.008)	0.031 (0.006)	0.059 (0.013)	0.048 (0.011)	0.057 (0.013)	0.049 (0.010)
S(Q3)	0.012 (0.005)	0.012 (0.005)	0.012 (0.005)	0.012 (0.005)	0.008 (0.008)	0.008 (0.008)	0.008 (0.008)	0.008 (0.008)
trend	0.0023 (0.0006)	0.0019 (0.0004)	0.0023 (0.0005)	0.0020 (0.0004)	0.0038 (0.0009)	0.0030 (0.0006)	0.0038 (0.0007)	0.0033 (0.0005)
$\ln(K/L)$	0.31 (0.12)	0.43 (0.06)	0.35 (0.12)	0.41 (0.06)	0.03 (0.16)	0.22 (0.09)	0.06 (0.16)	0.21 (0.09)
$[\ln(K/L)]^2$	-0.13 (0.12)		-0.08 (0.13)		-0.21 (0.15)		-0.19 (0.18)	
R^2	0.98	0.98	0.98	0.98	0.95	0.95	0.95	0.95
DW	0.44	0.45	0.45	0.46	0.37	0.37	0.37	0.37
LLH	221.46	220.82	222.07	221.85	185.63	184.52	185.04	184.44
(b) Engle/Granger cointegration test								
ADF(4)	$4.52^{\star}$	$4.50^{\star}$	$4.53^{\star}$	$4.51^{\star}$	3.63	$3.94^{\star}$	3.68	$3.89^{(\star)}$

S(Q1), S(Q2) and S(Q3) are seasonal dummy variables for the first, second and third quarter. R^2 is the adjusted coefficient of determination, DW the Durbin/Watson statistic and LLH the value of the log-likelihood function. The Engle/Granger cointegration test is based on an augmented Dickey/Fuller test (ADF) where the SBC information criterion suggests including four lagged differences. The critical values are obtained from MacKinnon (1991). Taking into account the sample length, they are 4.92 for rejection at the 1% significance level [$\star\star$], 4.28 at the 5% level [$\star$] and 3.96 at the 10% level [$(\star)$] in the case of the CES specification. In the case of the CD specification, the corresponding critical values are 4.54, 3.91 and 3.59.

The parameter estimates in Table 3(a) are all in line with the theoretical requirements. For instance, the distribution parameter η is always between 0 and 1, and for the mean TFP growth, plausible rates are estimated, at $\frac{3}{4} - 1\%$ per year for the total economy and $1\frac{1}{4} - 1\frac{1}{2}\%$ in the enterprise sector. Evidence for the existence of an aggregate production function whose elasticity of substitution deviates from unity is weak on the whole. In terms of estimation accuracy, the coefficients γ_2 and γ_3 can be used to calculate a reasonably reliable figure for the elasticity of substitution $\sigma = (1+\rho)^{-1}$ at best for the total economy. The regression coefficients yield a point forecast for the elasticity of substitution between labour and capital of 0.45 (services) and 0.59 (stock). This confirms the findings of other empirical studies (eg Bbk, 2010) of an elasticity of substitution of less than unity.

Finally, it can be observed that the quantitative difference between the two ways of measuring capital is too small to make a perceptible difference in parametric approaches to production function estimation. Statistically significant differences can be identified neither in the regression results nor in diagnostics. From the value of the log-likelihood function, it can at best be derived that using the capital stock provided by the Federal Statistical Office is marginally better for analysing the economy as a whole, while the evidence for the enterprise sector is the opposite.

6 Summary

The use of the aggregate gross capital stock as published by the Federal Statistical Office in empirical analyses that assume a macroeconomic production function is not theoretically intuitive. This also relates to the estimation of potential output based on the widely used production function approach. Appropriate aggregation requires that different types of assets should be weighted by their share in the total user cost of capital. In this study, the user costs necessary for deriving such a weighting structure are estimated, and index series for aggregate capital services are determined. The model calculations and applications display differences to the official capital stock data which are sometimes quantitatively significant.

The proposed measurement method meets the aggregation requirements deriving from production, investment and index theory and at the same time allows the official fixed assets accounts to be used as much as possible. There is doubtless scope for improvement in the concrete implementation of this approach. This depends, not least, on whether existing statistical information gaps can be filled. This relates, first, to the estimate of individual components of user cost. Second, the precision of the calculation would benefit from the publication of fixed assets data with a detailed breakdown by asset type.

The use of the fixed assets data reported by the Federal Statistical Office to measure capital as a factor in production is based on the assumption that real estate as well as inventories and non-produced assets have no productive value. Although this view is not without its critics (eg OECD, 2009), this definition for capital services is in line with standard practice, which represents the framework for the present paper. The calculation of user cost of capital offers options in terms of estimation methods and parameter settings. A systematic examination of the sensitivity of the results to variations in method and parameters is an important issue, yet beyond the scope of this paper. To obtain a comprehensive understanding of the quantitative effects of the proposed model framework and measurement procedure, sensitivity analyses are therefore recommendable.

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