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How useful is the carry-over effect for short-term economic forecasting?

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Abstract:

The carry-over effect is the advance contribution of the old year to growth in the new year. Among practitioners the informative content of the carry-over effect for short-term forecasting is undisputed and is used routinely in economic forecasting. In this paper, the carry-over effect is analysed 'statistically' and it is shown how it reduces the uncertainty of short-term economic forecasts. This is followed by an empirical analysis of the carry-over effect using simple forecast models as well as Bundesbank and Consensus projections.

Keywords: Forecast uncertainty, growth rates, carry-over effect, variance contribution, Chebyshev density

JEL-Classification: C53, E37, C16

Non technical summary

The carry-over effect is, put simply, the advance contribution of the old year to growth in the new year. This concept is generally applied to data with a frequency of less than one year, such as monthly and quarterly data, and has a firm place in the cyclical analysts' toolbox. It is used routinely, albeit heuristically, for short-term economic forecasts.

This discussion paper provides a statistical analysis of the carry-over effect. The annual growth rate of real gross domestic product (GDP) is represented as a weighted sum of quarterly growth rates and broken down into two components: the (already known) carry-over effect and the (still unknown) forecast component. The correlations of the carry-over effect and of the forecast component with the annual growth rate are derived for various forecast horizons.

During the recent economic and financial crisis, real GDP in Germany declined by almost 5% in 2009. This episode is taken as an example to demonstrate how the carry-over effect affected forecasts of annual GDP growth. Estimates of forecast intervals taking account of the carry-over effect based on the normal distribution for the quarterly growth rates seemed thoroughly appropriate before the crisis, but have become obsolete as a result of the dramatic worldwide slumps in output. The contribution of extreme observations to the variance of quarterly GDP growth rates is considerably larger than what would be compatible with the normal distribution. By contrast, forecast intervals based on the Chebyshev density are resistant to the volatility of any distribution with finite variance.

In order to examine empirically how the carry-over effect impacts on forecast uncertainty, two simple forecast models for the annual growth rates of real GDP are considered. The information of the carry-over effect is combined with mean value and random walk forecasts for the forecast component. It is found that

including of the carry-over effect reduces forecast uncertainty broadly in line with the profile derived analytically.

If the uncertainty of short-term forecasts decreases in line with the theoretically determined pattern, this is an indication that the forecasters have used the information of the progressively available carry-over effect. To investigate this issue, Bundesbank and Consensus Economics forecasts of annual real GDP growth in the period from 1991 Q2 to 2007 Q4 (without crisis) and to 2009 Q4 (with crisis), respectively, are analysed. It is found that forecast uncertainty has tended to decrease more slowly than would be expected solely on the basis of the carry-over effect.

The overall conclusion is that the carry-over effect provides extremely useful information for short-term forecasts of real GDP in Germany and it is easy to incorporate when calculating point and interval forecasts. Knowing the carry-over effect for the fourth quarter of the previous year reduces uncertainty about the annual growth rate in the current year to 68% of the unconditional variance. If information for the first quarter of the current year becomes available, uncertainty falls to 32%. Moreover, forecast ranges based on the Chebyshev density yield estimates of forecast uncertainty that are robust to distributions with ‘fat tails’.

Nicht technische Zusammenfassung

Der statistische Überhang ist, vereinfacht ausgedrückt, die Vorleistung des alten Jahres für das Wachstum im neuen Jahr. Das Konzept wird gewöhnlich auf Daten mit unterjähriger Frequenz wie Quartals- und Monatsdaten angewandt und hat einen festen Platz im Instrumentenkasten der Konjunkturanalytiker. Es wird routinemäßig, wenngleich auf heuristische Weise, bei kurzfristigen Wachstumsprognosen verwendet.

Dieses Diskussionspapier liefert eine statistische Analyse des statistischen Überhangs. Die Jahreswachstumsrate des realen Bruttoinlandsprodukts (BIP) wird als gewogene Summe von Quartalswachstumsraten dargestellt und in zwei Komponenten zerlegt: den (bereits bekannten) statistischen Überhang und die (noch unbekannte) Prognosekomponente. Die Korrelationen des statistischen Überhangs und der Prognosekomponente mit der Jahreswachstumsrate werden für unterschiedliche Prognosehorizonte hergeleitet.

Im Verlauf der jüngsten Finanz – und Wirtschaftskrise sank das reale BIP in Deutschland im Jahr 2009 um nahezu 5%. Am Beispiel dieser Episode wird demonstriert, wie sich der statistische Überhang auf Prognosen der Jahreswachstumsrate des BIP ausgewirkt hat. Schätzungen von Prognoseintervallen unter Berücksichtigung des statistischen Überhangs auf Basis der Normalverteilung für die Quartalswachstumsraten, die vor der Krise durchaus angemessen erschienen, sind durch die weltweit dramatischen Produktionseinbrüche obsolet geworden. Der Beitrag extremer Beobachtungen zur Varianz der Quartalswachstumsraten ist wesentlich größer als mit der Normalverteilung vereinbar. Prognoseintervalle auf Basis der Tschebyschow - Dichte sind dagegen resistent gegenüber der Volatilität beliebiger Verteilungen mit endlicher Varianz.

Um den Informationsgehalt des statistischen Überhangs auf die Prognoseunsicherheit empirisch zu untersuchen, werden zwei einfache Prognosemodelle für die Jahreswachstumsraten des realen BIP betrachtet. Die Informationen des statistischen Überhangs werden kombiniert mit

Mittelwertprognosen und Random – Walk – Prognosen für die Prognosekomponente. Wie sich zeigt, geht die Prognoseunsicherheit durch die Berücksichtigung des statistischen Überhangs etwa gemäß dem analytisch bestimmten Umfang zurück.

Wenn die Unsicherheit von kurzfristigen Prognosen gemäß dem theoretisch ermittelten Muster abnimmt, so ist dies ein Hinweis darauf, dass die Prognostiker die Informationen des sukzessive eintreffenden statischen Überhangs verwertet haben. Um dies zu untersuchen, werden die Prognosen der Deutschen Bundesbank und der Consensus Economics Inc. für das Jahreswachstum des realen BIP in Deutschland vom 2. Quartal 1991 bis zum 4. Quartal 2007 (ohne Krise) bzw. bis zum 4. Quartal 2009 (mit Krise) analysiert. Es zeigt sich, dass die Prognoseunsicherheit tendenziell langsamer gesunken ist, als allein aufgrund des statistischen Überhang zu erwarten ist.

Insgesamt ergibt sich, dass der statistische Überhang außerordentlich nützliche Informationen für die kurzfristigen Prognosen des realen BIP in Deutschland liefert und bei der Berechnung von Punkt- und Intervallprognosen einfach zu berücksichtigen ist. Die Kenntnis des statistischen Überhangs für das vierte Quartal des Vorjahres reduziert die Unsicherheit über die Jahreswachstumsrate im laufenden Jahr auf 68% Prozent der unbedingten Varianz. Liegt die Information für das erste Quartal des laufenden Jahres vor, geht die Prognoseunsicherheit auf 32% zurück. Prognoseintervalle auf Basis der Tschebyschow – Dichte liefern Schätzungen der Prognoseunsicherheit, die robust sind gegenüber Verteilungen mit ’dicken Enden’.

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How useful is the carry-over effect for short-term economic forecasting?*

1. Introduction

The carry-over effect (or statistical overhang) has a firm place in the cyclical analysis toolbox. It is used routinely, albeit heuristically, for short-term economic forecasting.[1] For instance, Deutsche Bundesbank (2009, p. 21) and the German Institute for Economic Research (DIW, 2010, p. 10, author's translation) recently pointed out with regard to Germany and China, respectively: *„Owing to the statistical carry-over, the increase in average annual growth will be higher in 2010 than in 2011." "Given a carry-over effect of almost four percent, growth of real gross domestic product in the area of ten percent in 2010 is very probable."* The carry-over effect is sometimes misinterpreted as the lower limit for annual growth.[2] As yet, there does not appear to be a statistical foundation for the carry-over effect.

The paper is organised as follows. In section 2 the carry-over effect is defined in terms of levels and quarterly growth rates. Section 3 shows how much forecast uncertainty in the annual growth rates is explained by the (already known) carry-over effect and how much is due to its counterpart, the (still unknown) 'forecast component'. In section 4, the most recent financial and economic crisis is taken as an example to determine confidence intervals for short-term forecasts of the annual growth rates of real GDP in Germany. As an alternative to the normal distribution, distribution-free confidence bands are calculated on

* I wish to thank Malte Knüppel, Hermann-Josef Hansen, Thomas McClymont, Guido Schultefrankenfeld and Hans-Eggert Reimers for their helpful comments and suggestions. Of course, they are not responsible for any remaining errors and omissions. Moreover, the views expressed in this paper are not necessarily those of the Deutsche Bundesbank.

[1] See, for example, Nierhaus (1999, 2007), Sachverständigenrat (2005/06), European Central Bank (2010), Deutsche Bundesbank (2010).

[2] See, for example, Economy and Trade, Online Dictionary German-English, definition of 'Statistischer Überhang' (carry-over effect): *„Since GDP growth is mostly in positive territory, the carry-over effect may also be expressed as the average annual growth rate that is* ***at***

the basis of Chebyshev's density. In the next two sections, the contribution of the carry-over effect to the reduction in forecast uncertainty is examined empirically. In section 5, forecasts of real growth are used with two simple forecast models and, in section 6, the historical forecast errors of Bundesbank and Consensus projections are analysed in the light of the carry-over effect. Section 7 concludes.

2. Defining the carry-over effect

The carry-over effect is defined for variables with a frequency of less than one year, such as quarterly or monthly data. Below, quarterly data for real gross domestic product (GDP) are analysed.[3] Where GDP in quarter i, i=1,2,3,4, of year t is denoted by $Q_{t:i}$,

(1) $$w_t = \frac{Q_{t:1} + Q_{t:2} + Q_{t:3} + Q_{t:4}}{Q_{t-1:1} + Q_{t-1:2} + Q_{t-1:3} + Q_{t-1:4}} - 1$$

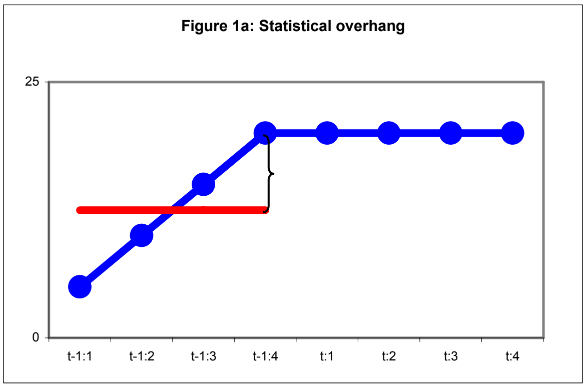
is the GDP growth rate in year t compared with the previous year. The **carry-over effect** is the GDP growth rate produced if the level attained in the fourth quarter of the previous year were to remain unchanged throughout the current year:

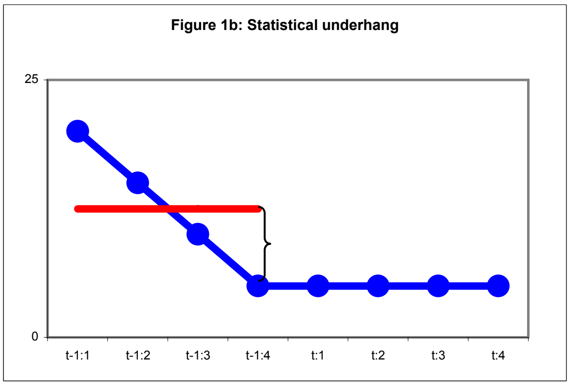
(2) $$u_t = \frac{Q_{t-1:4} + Q_{t-1:4} + Q_{t-1:4} + Q_{t-1:4}}{Q_{t-1:1} + Q_{t-1:2} + Q_{t-1:3} + Q_{t-1:4}} - 1 = \frac{Q_{t-1:4}}{\bar{Q}_{t-1}} - 1.$$

In this fictitious case, the annual growth rate would be equal to the statistical overhang. Thus, a positive carry-over effect is always given if the level of GDP attained in the fourth quarter of the previous year $Q_{t-1:4}$ is greater than the quarterly average in the previous year $\bar{Q}_{t-1}$. In the opposite case ($u_t < 0$) one speaks of a statistical underhang (see **Figure 1a and 1b**).

***least** to be recorded in year 2 owing to the development in year 1" (author's translation and emphasis)* http://www.mijnwoordenboek.nl/EN/theme/FI/DE/EN/S/18

3 References below to quarterly values of real GDP are to seasonally and calendar-adjusted data in all cases. The sum of the adjusted quarterly figures of a given year may deviate marginally from the sum of the unadjusted values.





The carry-over effect u_t may also be interpreted as the advance contribution of the old year to GDP growth in the new year. Its counterpart, the contribution of the current year r_t – which is still unknown at the beginning of the year – is positive if the quarterly average is greater than the value achieved in the fourth quarter of the previous year:

(3) $$r_t = w_t - u_t = \frac{\bar{Q}_t - Q_{t-1:4}}{\bar{Q}_{t-1}}.$$

Rather than using quarterly level data, the growth rates in equations 1, 2 and 3 can also be expressed by quarterly growth rates $q_{t:i} = Q_{t:1} / Q_{t-1:4} - 1$ for i=1 and $q_{t:i} = Q_{t:i} / Q_{t:i-1} - 1$ for i=2,3,4. As shown in **Appendix 1**, the annual growth rates can be approximated using a weighted sum of quarterly growth rates:

(1') $$w_t = \frac{1}{4} q_{t-1:2} + \frac{2}{4} q_{t-1:3} + \frac{3}{4} q_{t-1:4} + \frac{4}{4} q_{t:1} + \frac{3}{4} q_{t:2} + \frac{2}{4} q_{t:3} + \frac{1}{4} q_{t:4}.$$

The weights (γ_τ) follow a triangular distribution:

(4) $$\gamma_\tau = 1 - \frac{|4-\tau|}{4}, \quad \tau = 8,7,...,2,1$$

The first quarter of the current year has the largest weight, followed by the two neighbouring quarters. These three central quarters contribute 10/4 to the total weight of 16/4, while the four peripheral quarters account for 6/4. If the value attained in the fourth quarter of the previous year were to remain unchanged, the last four terms on the right-hand side of (1') would be zero. The carry-over effect and the growth contribution of the current year, referred to below as forecast component, are therefore

(2') $$u_t = \frac{1}{4}q_{t-1:2} + \frac{2}{4}q_{t-1:3} + \frac{3}{4}q_{t-1:4}$$

(3') $$r_t = w_t - u_t = \frac{4}{4}q_{t:1} + \frac{3}{4}q_{t:2} + \frac{2}{4}q_{t:3} + \frac{1}{4}q_{t:4}$$

A carry-over effect $u_{t:\tau}$ can be defined not only at the end of the year but also for each of the eight quarters within two consecutive years, which are indexed with τ = 8,7,…,2,1 in accordance with the decreasing forecast horizon. After the first quarter of the previous year (τ=8), the carry-over effect is $u_{t:8} = 0$, i.e. there is no information on annual growth in t. After two quarters (τ=7), the carry-over effect – the growth rate that would result in t if GDP were to remain at the level attained in the second quarter of t-1 – namely, $u_{t:7} = \frac{4Q_{t-1:2}}{Q_{t-1:1} + 3Q_{t-1:2}} - 1 \approx \frac{4(1+q_{t-1:2})}{1+3(1+q_{t-1:2})} - 1 \approx \frac{1}{4}q_{t-1:2}$, already carries information for annual growth in t. For the third quarter of the previous year (τ=6), $u_{t:6} = (1/4)q_{t-1:2} + (2/4)q_{t-1:3}$, etc. After the fourth quarter of the current year (τ=1), the information on annual growth is complete, the actual forecast horizon is zero and the carry-over effect corresponds to annual growth, such that $u_{t:1}=w_t$.[4]

In 2009, real GDP in Germany declined by almost 5% in the wake of the financial and economic crisis – a slump unparalleled in the history of the Federal Republic of Germany. **Table 1** shows how the carry-over effect

[4] Actually, quarterly data of the national accounts are not available until roughly six weeks after the end of the quarter, these being provisional values.

developed during the eight quarters of 2008-09. Errors due to the use of quarterly growth rates in (2') compared with the level data in (2) are small.

Table 1: Quarterly growth rates of real GDP in Germany

Quarter	$Q_{t:i}$	$q_{t:i}$	Carry-over effect ($u_{t:\tau}$), based on levels	growth rates
2008 Q1	574.52		0	0
2008 Q2	571.27	-0.57	-0.14	-0.14
2008 Q3	569.47	-0.32	-0.30	-0.30
2008 Q4	555.55	-2.44	-2.14	-2.13
2009 Q1	536.00	-3.52	-5.58	-5.65
2009 Q2	538.38	0.44	-5.27	-5.32
2009 Q3	542.30	0.73	-4.92	-4.95
2009 Q4	543.28	0.18	-4.88	-4.91

3. Information content of the carry-over effect

Forecasts of annual growth made during a given year or in the course of the previous year are not true annual forecasts as they can already draw on information regarding the carry-over effect (Stekler and Sakamoto, 2008). In the following it is assumed that the quarterly growth rates $q_{t:i}$ are independently and identically distributed random variables with expectation ω and finite variance σ^2: $q_{t:i} \sim \text{iid}(\omega, \sigma^2)$. Defining the coefficients

(5) $\alpha_\tau = \sum_{\kappa=\tau}^{8} \gamma_\kappa$, $\beta_\tau = \sum_{\kappa=\tau}^{8} \gamma_\kappa^2$,

the expected values E(*) and variances V(*) for the carry-over effect, the forecast component and the annual growth rate are as follows:

(6) $E(u_{t:\tau}) = \alpha_\tau \omega$, $E(r_{t:\tau}) = (4-\alpha_\tau)\omega$, $E(w_t) = 4\omega$

(7) $V(u_{t:\tau}) = \beta_\tau \sigma^2$, $V(r_{t:\tau}) = (44/16 - \beta_\tau)\sigma^2$, $V(w_t) = (44/16)\sigma^2$

The weights and coefficients are shown in **Table 2**.[5]

[5] **Appendix 2** provides some charts of density functions and illustrative probability calculations for different values of ω, assuming normality of the quarterly growth rates.

Table 2: Quarterly weights

Quarter	t-1:1	t-1:2	t-1:3	t-1:4	t:1	t:2	t:3	t:4
Horizon (τ)	8	7	6	5	4	3	2	1
γ_τ	0	1/4	2/4	3/4	4/4	3/4	2/4	1/4
α_τ	0	1/4	3/4	6/4	10/4	13/4	15/4	16/4
ß$_\tau$	0	1/16	5/16	14/16	30/16	39/16	43/16	44/16

Owing to the assumed independence of the quarterly growth rates, the covariance between the carry-over effect and the annual growth rate is equal to the variance of the overhang, and the variance of the annual growth rate is equal to the sum of the variances of the two components. For the correlation between the carry-over effect and the annual growth rate and for the correlation between the forecast component and the annual growth rate, this implies:

$$(8) \qquad corr_{u_{t:\tau}w_t} = \frac{Cov(u_{t:\tau}, w_t)}{\sqrt{V(u_{t:\tau})V(w_t)}} = \sqrt{\frac{V(u_{t:\tau})}{V(w_t)}} = \sqrt{\frac{\beta_\tau}{44/16}}$$

$$(9) \qquad corr_{r_{t:\tau}w_t} = \frac{Cov(r_{t:\tau}, w_t)}{\sqrt{V(u_{t:\tau})V(w_t)}} = \sqrt{\frac{V(r_{t:\tau})}{V(w_t)}} = \sqrt{1-\frac{\beta_\tau}{44/16}} = \sqrt{1-corr^2_{u_{t:\tau}w_t}}$$

It follows from equation 8 that the carry-over effect in the second quarter of the previous year shows a correlation with the annual growth rate of 0.15. In the fourth quarter of the previous year, this correlation amounts to 0.56 and jumps to 0.83 one quarter later (see **Table 3**). The correlations between the (known) carry-over effect and the (as yet unknown) annual growth rate reflect the increasing informative value of the carry-over effect for annual growth, while the importance of the forecast component decreases.

Table 3: Correlations with the annual growth rate

Quarter	t-1:1	t-1:2	t-1:3	t-1:4	t:1	t:2	t:3	t:4
Horizon (τ)	8	7	6	5	4	3	2	1
Corr. $(u_{t:\tau}, w_t)$	0	0.151	0.337	0.564	0.826	0.941	0.989	1
Corr. $(r_{t:\tau}, w_t)$	1	0.989	0.941	0.826	0.564	0.337	0.151	0
Squared corr. $(r_{t:\tau}, w_t)$	*1*	*0.977*	*0.886*	*0.682*	*0.318*	*0.114*	*0.023*	*0*

If $u_{t:\tau} = \bar{u}_{t:\tau}$ becomes available, the conditional expectation and the conditional variance of the annual growth rate are:

(10) $$E(w_t | \bar{u}_{t:\tau}) = \bar{u}_{t:\tau} + E(r_{t:\tau}) = \bar{u}_{t:\tau} + (4 - \alpha_\tau)\omega,$$

(11) $$V(w_t | \bar{u}_{t:\tau}) = V(r_{t:\tau}) = (44/16 - \beta_\tau)\sigma^2$$

The carry-over effect reduces the variance of the annual growth rate to the uncertainty on the forecast component. The part of the variance of the annual growth rate that is still unexplained, even knowing the carry-over effect, is

(12) $$\frac{V(w_t | \bar{u}_{t:\tau})}{V(w_t)} = \frac{(44/16 - \beta_\tau)\sigma^2}{(44/16)\sigma^2} = corr^2_{r_{t:\tau} w_t}$$

In a regression context, viewing $w_t = \bar{u}_{t:\tau} + r_{t:\tau}$ as a regression of w_t on $\bar{u}_{t:\tau}$ with residuals $r_{t:\tau}$, equation 12 can be interpreted as the ratio of unexplained to total sum of squares, i.e. one minus the coefficient of determination ($1-R^2$).

Knowing the carry-over effect at the end of the previous year $(\bar{u}_{t:5})$ leads to a theoretical reduction of forecast uncertainty to 68% of the unconditional variance. If information on the first quarter of the current year is available $(\bar{u}_{t:4})$, forecast uncertainty declines to 32% of the unconditional variance (see **Table 3**).[6] Measuring the forecast errors as squared deviations between forecasts and observations, the mean squared error (MSE) may be considered as an empirical estimate of the conditional variance given by equation 11. If the

[6] Interestingly, in a comprehensive international study on Consensus forecasts, Isiklar and Lahiri (2007, p. 186) found that *"the largest improvement in forecasting performance comes when the forecast horizon is around 14 months", i.e. 4 to 5 quarters..*

empirically observed reduction in forecast uncertainty with a decreasing forecast horizon is roughly equivalent to the pattern in equation 12, this indicates that the information on the carry-over effect that was progressively made available was effectively used. Equation 12 can thus serve as a benchmark for comparisons with actual forecasts.

If the quarterly growth rates are not independent as assumed above, but (either positively or negatively) autocorrelated random variables, the relative conditional variance declines faster with forecast horizon τ than given in equation (12).[7] Thus, the informative value of the carry-over effect is even bigger than shown in Table 3.

4. Forecast uncertainty in the recent economic crisis

GDP fell heavily worldwide in the wake of the financial crisis. In Germany, the slump in GDP was most severe in 2008 Q4 and 2009 Q1. **Table 4** summarises some data on real GDP before the recent economic and financial crisis (1991 Q2 to 2007 Q4) and including the crisis (up to 2009 Q4).

Before the crisis, growth of real GDP in Germany averaged $\overline{q} = 0.37\%$ per quarter, the standard deviation was s=0.63%, and the quarterly growth rates were only very weakly autocorrelated. According to the Jarque-Bera (JB) test, the null hypothesis of normally distributed quarterly growth rates cannot, at all events, be rejected at the 5% level. The contribution of realisations outside the range of two standard deviations around the mean (2s range) came to 81.4%, which is very close to the share of 80.1% in the case of a normally distributed random variable. The contribution of realisations outside the 4s range, at 17.1%, was even smaller than the normal distribution suggests (26.1%). The correlation between the carry-over effect at the end of the year and the annual growth rates came to 0.62, which is close to the theoretical value of 0.56 shown in Table 3.

[7] See Patton and Timmerman (2010, p. 5)

Table 4: Quarterly growth rates of real GDP

	without crisis 1991 Q2 – 2007 Q4	with crisis 1991 Q2 – 2009 Q4
Number of observations	67	75
Mean value	0.367	0.276
Standard deviation	*0.631*	*0.833*
First-order autocorrelation	0.085	0.261
JB statistic[1]	0.386	142.19
Variance contribution 2s range[2]	0.814	0.902
Variance contribution 4s range[2]	0.171	0.524
Correlation $(corr_{u_{t:5}w_t})$	0.616	0.595

1) Jarque-Bera statistic for testing normality with 5% critical level of 5.99.
2) Variance contribution of realisations outside the interval $\overline{q} \pm s$ $(\overline{q} \pm 2s)$, where $\overline{q}$ is the mean value and s is the standard deviation in the sample; see eq. (13) below.

Including the crisis years 2008-09 leads to a fall of the average growth rate by one fourth, to $\overline{q} = 0.28\%$ per quarter, with a standard deviation that is one-third higher of s=0.84%. According to the JB statistic, which has risen to 142, the normal distribution for the quarterly growth rates must be clearly rejected.[8] This is reflected, above all, in the variance contribution of observations outside the 4s range, which has tripled from 17% to 52%. By contrast, the correlation between the carry-over effect and the annual growth rate in the larger sample has hardly changed.

The declines in GDP that were observed in two quarters of the years 2008-09 are extremely unlikely under the normal distribution. A year-on-year decline in GDP of more than 2% has a probability of less than 1/10,000, and a decline of more than 3% a probability of less than 1/10,000,000. Under the normal distribution, such events should not occur more than once in 3,000 years and once in 5,000,000 years respectively.[9]

[8] Thus, we refrain from viewing extreme observations in times of crisis as mere outliers, with the normal distribution still valid in 'normal' times.

[9] It is worth recalling the words of the head of quantitative equity strategies for Lehman Brothers: "*Wednesday is the type of day people will remember in quant-land for a very long time," said Mr. Rothman, a University of Chicago Ph.D. who ran a quantitative fund before joining Lehman Brothers. "Events that models only predicted would happen once in 10,000 years happened every day for three days."* http://seekingalpha.com/article/44338-quant-fund-pain-is-the-worst-over. For another perspective, see Haas et al. (2010).

It is worthwhile to have a closer look at the variance contributions mentioned above. If $J_{t:i}(k)$ is defined as an indicator function which assumes the value 1 when the observation $q_{t:i}$ is greater in absolute terms than $\bar{q} + ks$ (and null otherwise), then the contribution of 'extreme' observations *outside* this range to the variance of the quarterly growth rates is

$$(13) \qquad Y(k) = \frac{\sum J_{t:i}(k)(q_{t:i} - \bar{q})^2}{\sum (q_{t:i} - \bar{q})^2} \ , \ k \geq 0 \ , 0 \leq Y \leq 1 .$$

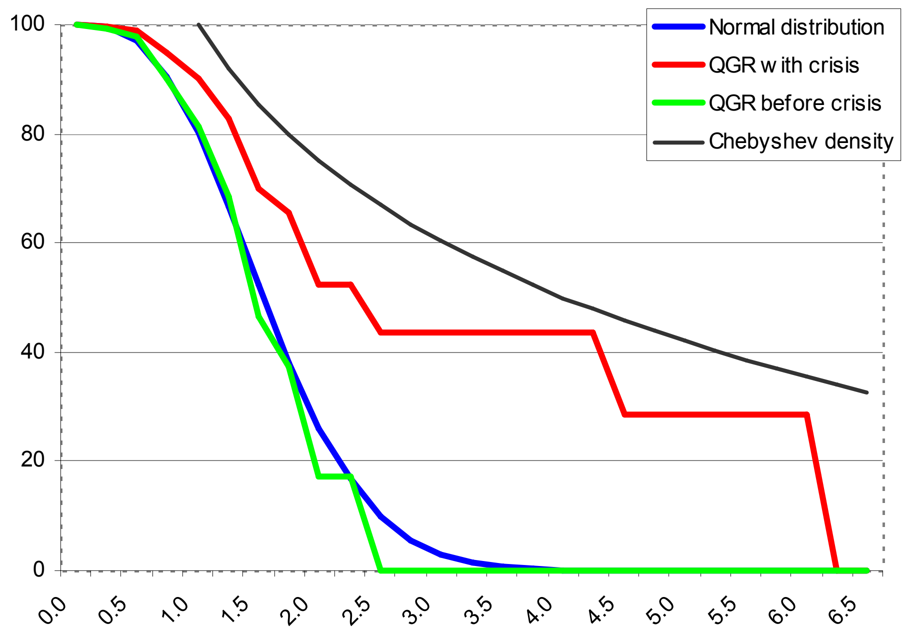
The function 1-Y(k) may be interpreted as distribution of relative volatility. As shown in **Appendix 3**, if a random variable has a $N(\mu, \sigma^2)$ - distribution, then the corresponding volatility density is independent of the parameters of the underlying normal distribution:

$$(13') \qquad \upsilon_N(k) = \frac{\partial[1 - Y(k)]}{\partial k} = \sqrt{2/\pi}\, k^2 e^{-k^2/2} .$$

Figure 2 shows the variance contribution for the quarterly growth rates (QGR) of real GDP (before and with crisis) as a function of k in comparison with the theoretical contribution under normality of growth rates. As may be seen, the profile prior to the crisis (lower line) corresponds very closely with the theoretical profile in the case of normally distributed growth rates (second line from below). This is no longer the case if both years of the crisis 2008-09 are included (third line). The contribution of extreme observations to the overall variance is far greater than is compatible with normally distributed growth rates. The upper line shows the variance contribution for the Chebyshev density (truncated at Ω=16).[10]

[10] For more details on the Chebyshev density see eq. 15 below and **Appendix 4**.

Figure 2: Variance contribution of quarterly GDP growth rates (%)



Before the crisis, with quarterly data up to and including 2008 Q1, on the basis of equation 10 one would have predicted the annual growth rate for 2009 at 4*0.367 = 1.47%, with a standard deviation of $\sqrt{44/16*0.631^2} = 1{,}05\%$. Three quarters later, with data up to 2008 Q4, one would have had to revise the forecast sharply downwards to $\bar{u}_{t-1:4} + \beta_4\hat{\omega} = -2.13 + (10/4)*0.367 =$ -1.23%, with a more precise estimate of 0.89% for the standard deviation. The information on the first quarter of 2009 would have led to a forecast close to the later realised annual value.

Table 5 shows the profile of the growth forecasts for 2009 and the forecast uncertainty, measured by the estimated standard deviation, with the carry-over effect progressively being taken into account.

Table 5: Carry-over effect and growth forecasts in the crisis

Quarter	2008 Q1	2008 Q2	2008 Q3	2008 Q4	2009 Q1	2009 Q2	2009 Q3	2009 Q4
Quarterly growth rates		-0.57	-0.32	-2.44	-3.52	0.44	0.73	0.18
Carry-over effect	0	-0.14	-0.30	-2.13	-5.65	-5.32	-4.95	-4.91
Forecast component	1.47	1.44	1.21	0.90	0.48	0.20	0.07	0
Growth forecst. for 2009	1.47	1.30	0.91	-1.23	-5.17	-5.12	-4.89	-4.91
Est. standard deviation	*1.05*	*1.05*	*1.01*	*0.89*	*0.68*	*0.47*	*0.21*	*0*
ND confidence band[1]	4.10	4.13	3.97	3.48	2.66	1.86	0.83	0
CH confidence band[2]	9.35	9.43	9.06	7.95	6.06	4.24	1.88	0

Figures are based on the equations (10) and (11). Mean values and standard deviations $(\overline{q}, s)$ were recalculated to the current end in each case.

1) Width of a 95% confidence interval based on the normal distribution (see eq. 14).

2) Width of a 95% confidence interval based on the Chebyshev density (see eq. 15/16).

Before the crisis it could be assumed (though ‘unsafely’ with hindsight) that the quarterly growth rates ($q_{t:i}$) were normally distributed. Under this hypothesis, it is possible to determine approximate 1-α confidence intervals for the forecast annual growth rates $\hat{w}_t$ using equation (7) and the estimated moments $\hat{\omega} = \overline{q}$, $\hat{\sigma}^2 = s^2$,

$$(14) \qquad P\left(\left| \frac{w_t - \hat{w}_t}{\sqrt{\hat{V}(w_t)}} \right| \le k \right) \approx \Phi(k) - \Phi(-k) = 1 - \alpha ,$$

where $\hat{w}_t$ is the forecast for the expected value, $\hat{V}(w_t)$ is the estimated variance and $\Phi(*)$ is the distribution function of the standard normal distribution.[11]

As explained above, however, the recent crisis gave rise to serious doubts concerning the validity of the normal distribution for the quarterly growth rates of real GDP in Germany (Sornette, 2009, p. 1):

[11] Refined confidence intervals based on a t-distribution with 30 or more degrees of freedom are broader by a factor of not more than 1.04 than those based on the normal distribution.

"One of the most remarkable emergent properties of natural and social sciences is that they are punctuated by rare large events, which often dominate their organization and lead to huge losses. This statement is usually quantified by heavy-tailed distributions of event sizes."

Thus, the prudence principle suggests to use a distribution with thicker tails than those of the normal distribution for calculating forecast intervals. It is not clear, however, how thick those tails should be. Nevertheless, a density function can be determined such that its tails are *at least as thick* (but not thicker) than those including all random variables with an unknown distribution but existing variance σ^2.[12] Such an 'enveloping' density function for the tails of a pseudo-random variable X, which is defined outside the interval $(\mu-\sigma, \mu+\sigma)$, can be given as follows:

(15) $$f(x) = \frac{\sigma^2}{|x-\mu|^3}, \quad x \notin (\mu-\sigma, \mu+\sigma).$$

For reasons about to be explained, this function is denoted in the following as Chebyshev's density. The Chebyshev density has two branches with poles at μ-σ and μ+σ.[13]

Figures 3a and 3b show the Chebyshev density and the corresponding Chebyshev distribution function for μ=0 and σ^2=1 compared with standard normal distribution and a t - distribution with 3 degrees of freedom.[14]

[12] By implication, the expected value μ is finite as well; see Hogg, McKean and Craig (2005, p. 69).

[13] For a centred variable (μ=0), the right branch is proportional to a **Pareto** density $[f(x) = ca^c / x^{c+1}, a > 0, c > 0]$ with c=2 and a=σ.

[14] The t - distribution has fatter tails than the normal, its variance exists if the number of degrees of freedom exceeds 2.

Figure 3a: Chebyshev density

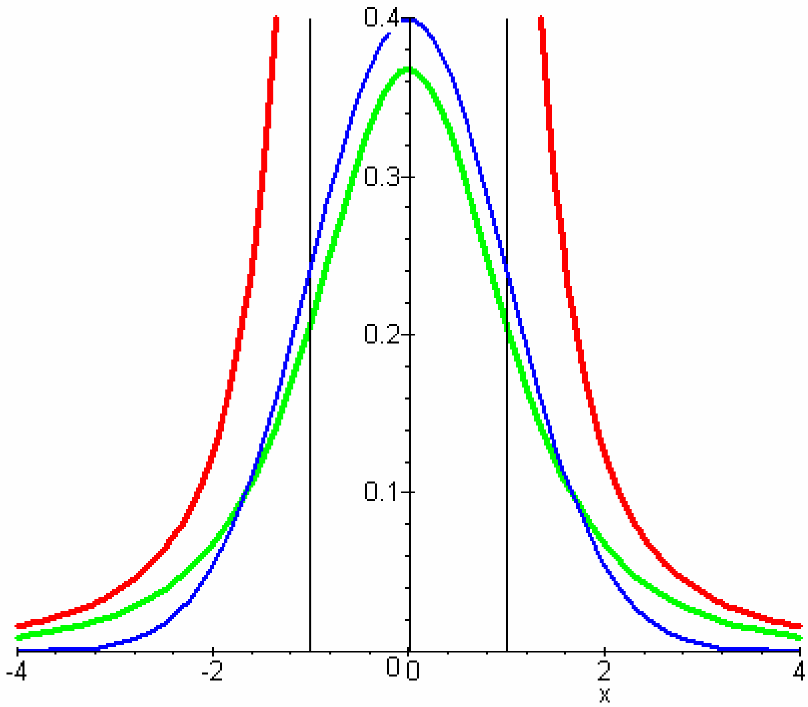
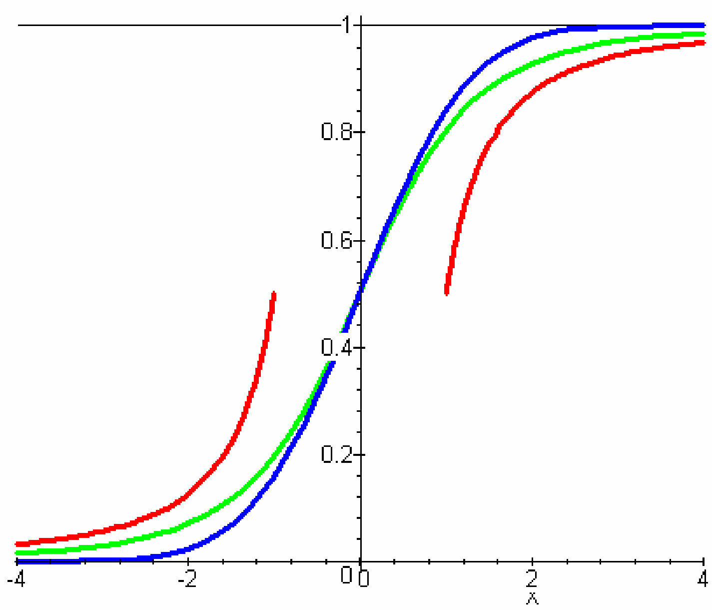


Figure 3b: Chebyshev distribution



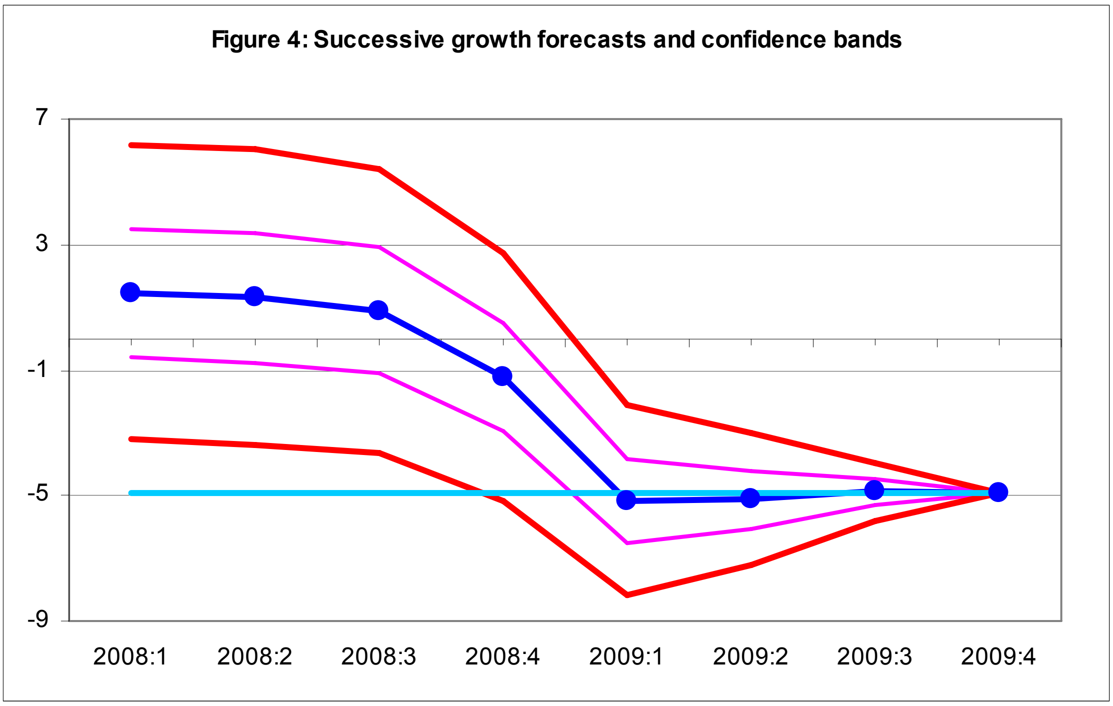
The density function in equation 15 describes a random variable with expectation μ, however, with a non-existent variance: $\int_{-\infty}^{\mu-\sigma}(x-\mu)^2 f(x)dx$ $+\int_{\mu+\sigma}^{\infty}(x-\mu)^2 f(x)dx \to \infty$. The higher even moments do not exist either, while the higher uneven moments are null. Because equation 15 is the envelope density of random variables with finite variance of arbitrary size, its variance cannot be finite.

Realisations of X outside the interval $(\mu-k\sigma, \mu+k\sigma)$ have the probability:

(16) $$P(|X-\mu| \geq k\sigma) = \int_{-\infty}^{\mu-k\sigma} f(x)dx + \int_{\mu+k\sigma}^{\infty} f(x)dx = \frac{1}{k^2}, \quad k \geq 1.$$

This is exactly the lower bound of the Chebyshev's (1867) inequality. For $k = 1/\sqrt{\alpha}$ a 1-α confidence interval is obtained which includes the uncertainty about the shape of the distribution.

Figure 4 (and Table 5) shows conditional forecasts (middle line) and the corresponding 95% confidence intervals based on the normal distribution (inner band) and the Chebyshev density (outer band). The Chebyshev intervals are more than twice as wide as those that result from a normal distribution. Both confidence bands become increasingly narrow as more and more information arrives regarding the carry-over effect.



5. An empirical application with simple forecast models

To examine empirically how the carry-over effect impacts on forecast uncertainty, an ex-post analysis is conducted for real GDP in Germany applying two simple forecast models. Annual growth rates are forecast not just for calendar years but after each quarter. To simplify the notation, below, the quarterly values are indexed throughout with j = 1,2,…, i.e. the annual growth rate with data up to and including quarter j is:

$$w_j = \frac{Q_{j-3} + Q_{j-2} + Q_{j-1} + Q_j}{Q_{j-7} + Q_{j-6} + Q_{j-5} + Q_{j-4}} - 1. \qquad (1'')$$

The forecasts ($\hat{w}_{j|n}$) are made in the period $n \leq j$ and combine the information on the carry-over effect available up to then with short-term forecasts for the forecast component: $\hat{w}_{j|n} = \bar{u}_{j|n} + \hat{r}_{j|n}$. Note, that the previously defined subscript is τ=j–n and the forecast horizon is τ-1 quarters.

Two alternative simple models are examined to predict the forecast component: mean value forecasts (MV) and random walk (RW) forecasts. In the case of the MV forecasts, the expected value of the forecast component is estimated using the historical mean of the quarterly growth rates: $\hat{\omega}_n = (1/n)\sum_{s=1}^{n} q_s$. The estimation period begins in 1991 Q2 (j=1). The first estimation period ends in 2001 Q1 (n=35) and is extended successively until 2007 Q4 (n=67) and until 2009 Q4 (n=75) respectively. The forecast component is then predicted according to $\hat{r}_{j|n} = \beta_\tau \hat{\omega}_n$. This results in T=32 (without crisis) and T=40 (with crisis) forecasts. With the RW forecasts, the last observed quarterly growth rate (q_n) in each case is extrapolated: $\hat{r}_{j|n} = \beta_\tau q_n$.[15]

Four forecast variants are calculated. They differ according to the scale of the carry-over effect considered. The benchmark forecasts A4 with a horizon of four quarters ignore the information already known about the carry-over effect from three previous quarterly growth rates. In the case of the variant B4, the horizon is also four quarters, but the information about the carry-over effect from the three previous quarters is included. For example, in quarter n=70 (2008 Q3) the MV forecast for the annual growth rate in the quarter j=74 (2009 Q3) is calculated according to $\hat{w}_{74|70} = \left(\frac{1}{4}q_{68} + \frac{2}{4}q_{69} + \frac{3}{4}q_{70}\right) + \frac{10}{4}\hat{\omega}_{70}$. With the RW forecasts, $\hat{\omega}_{70}$ is replaced by q_{70}. The variants B3 and B2 are structured analogously, but only have a horizon of three and two quarters respectively, i.e. they use information about the carry-over effect from four and five quarters respectively.

The forecast errors that occur ex post are measured using the mean absolute error (MAE) or, as an alternative, by the root mean squared error (RMSE):

$$\text{(17)} \qquad \begin{aligned} MAE &= (1/T)\sum \left| w_{j|n} - \hat{w}_{j|n} \right| \\ RMSE &= \sqrt{(1/T)\sum (w_{j|n} - \hat{w}_{j|n})^2} \end{aligned} .$$

[15] A first-order autoregressive forecast model led to consistently poorer forecasts.

Moreover, relative MSE statistics that refer to forecasts of the A4 variant are shown in **Table 6**.

Table 6: Carry-over effect and forecast error measures

	without crisis				with crisis			
Forecast variant	A4	B4	B3	B2	A4	B4	B3	B2
Number of forecasts	32				40			
Forecst. horizon (τ-1)	4	4	3	2	4	4	3	2
	Mean value forecasts							
MAE	1.22	0.84	0.52	0.27	1.55	1.16	0.73	0.39
RMSE	1.37	1.00	0.64	0.33	2.08	1.69	1.10	0.60
Relative MSE	*1*	*0.53*	*0.21*	*0.06*	*1*	*0.66*	*0.28*	*0.08*
	Random walk forecasts							
MAE	1.27	1.06	0.62	0.30	1.55	1.32	0.92	0.43
RMSE	1.55	1.28	0.76	0.36	2.11	1.83	1.43	0.70
Relative MSE	*1*	*0.69*	*0.24*	*0.05*	*1*	*0.75*	*0.46*	*0.11*

If no information about the carry-over effect is used, the mean error of the MV forecasts for the pre-crisis annual growth rates amounts to 1.22 (MAE) or 1.37% (RMSE). Information about the carry-over effect of three, four or five quarter-on-quarter growth rates reduces the forecast uncertainty to 53%, 21% and 6%, respectively, of the MSE in the benchmark forecast. This is a somewhat sharper decline than was theoretically to be expected on the basis of independent random variables (Table 3). Both the MAE and the RMSE of the RW forecasts are greater than those of the MV forecasts.[16] This is probably a reflection of the greater amount of information used in the MV forecasts. Again, the reduction of forecast uncertainty is roughly in line with the theoretical results.

[16] Clements and Hendry (1998, p. 84) define forecasts as informative if the variance of the forecast errors is greater than the variance of the variable being forecast. Thus, the RW forecasts would not qualify as informative since the MSE of the MV forecasts is essentially the variance of the growth rates.

If the financial crisis is included, the MAE of the benchmark forecasts increase markedly from 1.22 to 1.55%, and the RMSE even more sharply from 1.37 to 2.08%. By including the carry-over effect, forecast uncertainty declines roughly to the theoretically expected extent. Again, the RW forecasts perform worse than the MV forecasts.

For the MV forecasts (variant A4), the point predictions $\hat{w}_t$ as well as their variance $\hat{V}(w_t)$ were estimated in order to calculate forecast intervals. Like the expected value of the quarterly growth rates $\hat{\omega}_n$, the variance was calculated with a fixed starting point (1991 Q2) and variable endpoint up to 2007 Q4 (without crisis) or up to 2009 Q4 (with crisis) as $\hat{\sigma}_n^2 = (1/n)\sum(q_n - \hat{\omega}_n)^2$. Then, 50% and 95% confidence intervals were calculated according to $\hat{w}_{j|n} \pm h\sqrt{\hat{V}(w_{j|n})}$, where $\hat{V}(w_{j|n}) = (44/16)\hat{\sigma}_n^2$, with h = (0.67; 1.96) for the confidence intervals on the basis of the normal distribution and h = (1.41; 4.47) for the confidence intervals based on the Chebyshev density.

With 32 and 40 forecasts respectively, it was to be expected that 16 and 20 (1.6 and 2) realisations would drop out of the 50% (95%) confidence band based on normal distribution. In the case of the Chebyshev bands, these values are to be viewed as upper limits by construction. **Table 7** shows that the intervals based on a normal distribution were violated too often before – and even more so after – the crisis, i.e. the forecast uncertainty was underestimated by these bands. This was not the case with the Chebyshev bands: the number of violations, even including the crisis period, remained within the expected range.

Table 7: Violations of confidence intervals

Forecast period…	without crisis		with crisis	
Coverage of confidence bands (CB)	50%	95%	50%	95%
CB according to normal distribution				
expected violations	16	1.6	20	2
observed violations[1]	24	2	30	6
CB according to Chebyshev density				
expected violations	≤16	≤1.6	≤20	≤2
observed violations[1]	9	0	13	2

1) Number of realisations of the annual growth rates of GDP in Germany outside the confidence bands.

Feldstein (1971) already suggested to quantify forecast uncertainty with the aid of Chebyshev's inequality. In the light of the recent crisis, the practice of a number of central banks, including the Deutsche Bundesbank, the European Central Bank and the Federal Reserve, of publishing confidence bands with only 50 to 70% coverage ought to be reviewed. Such narrow bands, even more so when based on the normal distribution, do not reflect the inherent risks entailed of growth forecasts adequately.

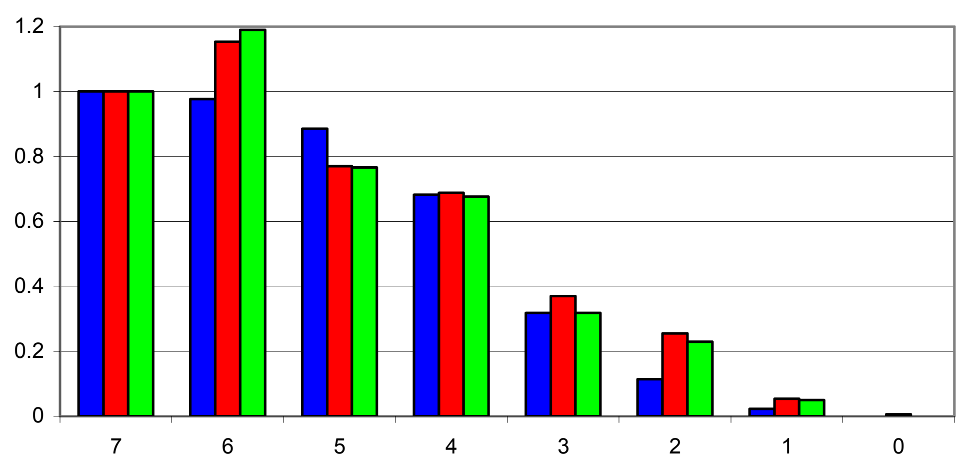
6. Bundesbank and Consensus forecasts

With a shorter forecast horizon, the relative variance of the forecast errors should decrease owing to the progressively available information of the carry-over effect. If the relative variance of actual forecasts decreases more quickly than is to be expected according to equation 12, the reason may be that the forecasters have used more information than only the carry-over effect. Conversely, if the relative variance of the forecast errors decreases more slowly, this is an indication that the available information was not processed efficiently.

Below, the forecasts of real GDP in Germany published by the Deutsche Bundesbank and Consensus Economics Inc. are examined to determine the extent to which the forecast errors decline with a shorter forecast horizon. Quarterly forecasts compiled on a half-yearly basis in the period from 1999 Q4 to 2009 Q4 are analysed.[17] However, when comparing the actual and the theoretical development of the relative variance, there exists the problem of what the 'true' forecast horizon is. With real forecasts, the actual development is not yet known at the time the forecast is made and even the data from the previous quarters are provisional and subject to later revisions. For these reasons, even the 'forecasts' for the current quarter (nowcasts) are uncertain and show errors. It is therefore assumed that the true forecast horizon is one quarter longer than the nominal forecast horizon. The comparison is based on the relative RMSE of the above-mentioned two institutions.

Figure 5 shows the development of the relative forecast errors (eq. 12 and Table 3) [left bars] in comparison to the Bundesbank forecasts [middle bars] and the Consensus forecasts [right bars] depending on the horizon (τ-1). The differences between both institutions are very small. However, the decline of relative forecast uncertainty broadly follows the theoretical profile. Except for the horizon τ-1=5, the relative errors are greater than would be expected solely on the basis of the carry-over effect.

Figure 5: Relative RMSE of Bundesbank - and Consensus Forecasts



[17] For details, see Deutsche Bundesbank (June 2010, pp. 40-41).

Table 8 summarises the results. The first line shows the analytically determined decline in relative forecast uncertainty on account of information about the carry-over effect. The following block states the relative MSE of different forecasts: MV and RW forecasts with and without crisis, Bundesbank and Consensus forecasts. If these values are smaller than those in the first line, they indicate that the forecasts have processed more information than can be explained by the carry-over effect alone, while, in the opposite case, forecast uncertainty has not fallen in line with the gain in information.

Table 8: Utilisation of the carry-over effect in forecasts

Horizon (τ-1)	7	6	5	4	3	2	1	0
Relative forecast uncertainty ($corr^2_{r;w}$)	1	0.98	0.89	0.68	0.32	0.11	0.02	0
	Relative MSE							
MV without crisis	1	-	-	0.53	0.21	0.06	-	0
MV with crisis	1	-	-	0.66	0.28	0.08	-	0
RW without crisis	1	-	-	0.69	0.24	0.05	-	0
RW with crisis	1	-	-	0.75	0.46	0.11	-	0
Bundesbank	1	1.15	0.77	0.69	0.37	0.26	0.05	0
Consensus	1	1.19	0.77	0.68	0.32	0.23	0.05	0

In most cases, the MV forecasts and even the RW forecasts show a sharper decline in forecast uncertainty for both periods than is suggested by the carry-over effect. One reason for the deviations of the measured values from unity is probably that the quarterly growth rates are not strictly realisations of independently and identically distributed random variables, as was assumed. Another reason is likely to be the relatively small sample size of 32 and 40 observations respectively. In the case of the Bundesbank forecasts – much the same as in the Consensus forecasts – it is apparent that, in particular with the shorter horizons, the decline in forecast uncertainty is smaller than would be expected on the basis of information about the carry-over effect. However, the problem of later revisions of real-time data prevents a ‘pure’ comparison with the theoretical pattern.[18]

7. Conclusions

The carry-over effect contains valuable information for forecasting the annual growth rates of German GDP. Under relatively weak assumptions about the distribution of the quarterly growth rates, the information available at the beginning of the year on GDP at the end of the previous year can reduce forecast uncertainty, measured in terms of the mean squared forecast error, to roughly two-thirds of the value that would result if this information were absent. Data on the first or on the first and second quarter of the current year reduce the forecast uncertainty to one-third and one-tenth of this value, respectively.

The relative forecast errors of mean value (MV) and random walk (RW) forecasts for real GDP in Germany are largely in keeping with the theoretically expected profile. Regarding the Bundesbank and Consensus forecasts, the decline in forecast uncertainty is smaller than expected on the basis of information about the carry-over effect, in particular with the shorter horizons, for which the forecast errors are very small in any case.

One important lesson of the recent financial and economic crisis is that normal distribution is not to be trusted as a basis estimating forecast uncertainty with regard to real growth. The extreme slumps in real GDP in Germany in 2008-09, and elsewhere, cannot be explained with normally distributed growth rates. The quarterly growth rates evidently have a distribution with broader tails than is implied by the normal distribution. In order to take account of the possibility of extreme events such as those observed during the recent crisis, distribution-free forecast intervals based on Chebyshev density are suggested as an alternative method of quantifying short-term forecast uncertainty.

[18] Moreover, major conceptual revisions in the German national accounts took place in 2005; see Deutsche Bundesbank (2005, pp. 36).

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Appendix 1: Approximation of the annual growth rates

Approximation of the annual growth rates (AGR) using quarterly growth rates (QGR) is based on the fact that for $|x|, |y| << 1$ the following applies: $(1+x)(1+y) \approx 1+x+y$ and $1/(1+x) \approx 1-x$.

If numerator and denominator of the AGR from equation 1 are defined as $Z_t \equiv Q_{t:1} + Q_{t:2} + Q_{t:3} + Q_{t:4}$ and $Z_{t-1} \equiv Q_{t-1:1} + Q_{t-1:2} + Q_{t-1:3} + Q_{t-1:4}$ and if the quarterly values are written as $Q_{t:2} = Q_{t:1}(1+q_{t:2})$ etc., it follows that

$$\begin{aligned} Z_t &= Q_{t:1}\left[1+(1+q_{t:2})+(1+q_{t:2})(1+q_{t:3})+(1+q_{t:2})(1+q_{t:3})(1+q_{t:4})\right] \\ &\approx Q_{t:1}\left[4+3q_{t:2}+2q_{t:3}+q_{t:4}\right], \end{aligned}$$

$$Z_{t-1} \approx Q_{t-1:1}\left[4+3q_{t-1:2}+2q_{t-1:3}+q_{t-1:4}\right].$$

If $Q_{t:1}$ is expressed by $Q_{t-1:1}$,

$$\begin{aligned} Q_{t:1} &= Q_{t-1:1}(1+q_{t-1:2})(1+q_{t-1:3})(1+q_{t-1:4})(1+q_{t:1}) \\ &\approx Q_{t-1:1}\left[1+q_{t-1:2}+q_{t-1:3}+q_{t-1:4}+q_{t:1}\right], \end{aligned}$$

it is found that

$$\begin{aligned} Z_t &\approx Q_{t-1:1}\left[1+q_{t-1:2}+q_{t-1:3}+q_{t-1:4}+q_{t:1}\right]\left[4+3q_{t:2}+2q_{t:3}+q_{t:4}\right] \\ &\approx Q_{t-1:1}\left[4+4q_{t-1:2}+4q_{t-1:3}+4q_{t-1:4}+4q_{t:1}+3q_{t:2}+2q_{t:3}+q_{t:4}\right]. \end{aligned}$$

It thus follows that

$$Z_t - Z_{t-1} \approx Q_{t-1:1}\left[q_{t-1:2}+2q_{t-1:3}+3q_{t-1:4}+4q_{t:1}+3q_{t:2}+2q_{t:3}+q_{t:4}\right]$$

and hence

$$\begin{aligned} w_t &= \frac{Z_t - Z_{t-1}}{Z_{t-1}} \approx \frac{q_{t-1:2}+2q_{t-1:3}+3q_{t-1:4}+4q_{t:1}+3q_{t:2}+2q_{t:3}+q_{t:4}}{4+3q_{t-1:2}+2q_{t-1:3}+q_{t-1:4}} \\ &\approx \frac{1}{4}\left[q_{t-1:2}+2q_{t-1:3}+3q_{t-1:4}+4q_{t:1}+3q_{t:2}+2q_{t:3}+q_{t:4}\right] * \\ &\qquad \left(1-\frac{3}{4}q_{t-1:2}-\frac{2}{4}q_{t-1:3}-\frac{1}{4}q_{t-1:4}\right), \end{aligned}$$

which simplifies to equation (1’) in the main text.[19]

[19] Patton and Timmermann (2010) use a similar representation.

Appendix 2: Density functions and probabilities

Let the quarterly growth rates be independently $N(\omega,\sigma^2)$ distributed. Then, according to equations 6 and 7, for ω = 0, 0.25, 0.5 and σ=1 the following density functions (Figures A2a, A2b, A2c) are obtained for the carry-over effect [highest peaks], the forecast component [middle peaks] and the annual growth rate [lowest peaks].

Figure A2a: omega=0

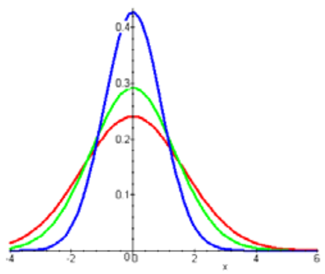


Figure A2b: omega=0.25

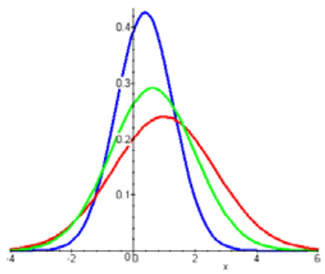
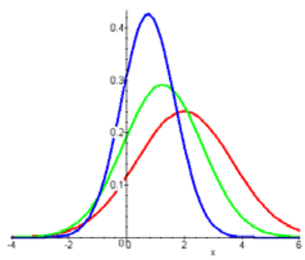


Figure A2c: omega=0.5



If ω = 0, the probability of a positive carry-over effect is 0.50. The probability that both the carry-over effect and the forecast component are positive, is 0.25. For a positive quarterly growth rate of ω=0.25, i.e. a mean annual growth rate of 1%, the probability of a positive carry-over effect increases to 0.66. The probability that both, the carry-over effect and the forecast component, are positive is 0.45.

ω = 0	P(r>0)	P(r≤0)	
P(u>0)	0.25	0.25	0.50
P(u≤0)	0.25	0.25	0.50
	0.50	0.50	1

ω = 0.25	P(r>0)	P(r≤0)	
P(u>0)	0.45	0.21	0.66
P(u≤0)	0.23	0.11	0.34
	0.68	0.32	1

ω = 0.50	P(r>0)	P(r≤0)	
P(u>0)	0.65	0.14	0.79
P(u≤0)	0.17	0.04	0.21
	0.82	0.18	1

Appendix 3: Distribution of the variance contributions

Let X be a normally distributed random variable with density $g(x;\mu,\sigma^2)$. Then the contribution of realisations *outside* the central interval $(\mu - k\sigma, \mu + k\sigma)$ to the variance is a decreasing function of k:

$$(A3.1)\quad Y(k) = \frac{\int_{-\infty}^{\mu-k\sigma}(x-\mu)^2 g(x)dx + \int_{\mu+k\sigma}^{\infty}(x-\mu)^2 g(x)dx}{\sigma^2} = 1 - \mathrm{erf}\left(\frac{k}{\sqrt{2}}\right) + \frac{2k\,e^{-\frac{k^2}{2}}}{\sqrt{2\pi}},$$

where $\mathrm{erf}(x) = (2/\sqrt{\pi})\int_{t=0}^{x} e^{-t^2}dt$ is the error function. Note, that Y(k) is a function of k alone, it is independent of the parameters of the underlying normal distribution. The first derivative of the distribution function (A3.1) with respect to k,

$$(A3.2)\qquad \upsilon(k) \equiv \frac{\partial[1-Y(k)]}{\partial k} = \sqrt{\frac{2}{\pi}}\; k^2 e^{-k^2/2}\ ,\quad k = 0..\infty$$

may be interpreted as <u>volatility density</u>. This density function has no free parameters. Its moments are:

Mean: $2\sqrt{2/\pi} \approx 1.596$; Median: ≈ 1.538; Mode: $\sqrt{2} \approx 1.414$

Variance: $(3\pi - 8)/\pi \approx 0.454$

Skewness: 0.486

Kurtosis: 3.108

JB statistic: 0.0398 n

The largest contribution to the variance comes from realisations in the neighbourhood of 1.41 times the standard deviation. The distribution is right-skewed and has a slight excess kurtosis. At the 5% significance level, the Jarque-Bera (JB) test would reject the null hypothesis of normality only with n>150 observations.

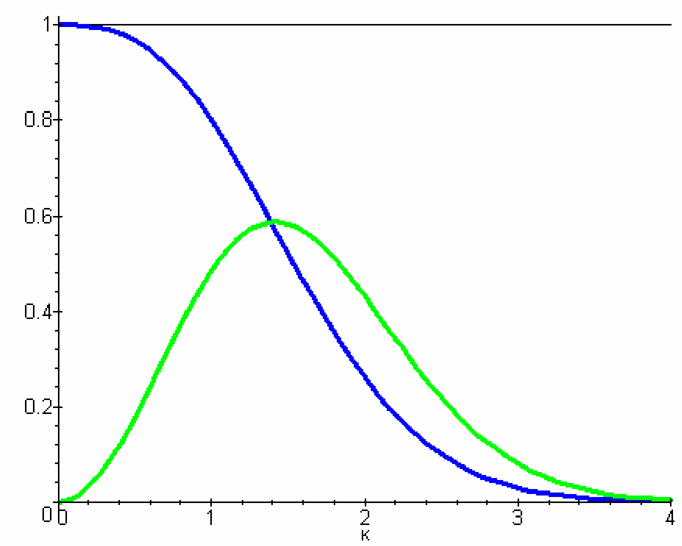
Interestingly, the density A3.2 is a special case of the **Maxwell-Boltzmann** distribution (for $\psi = 1$):

(A3.3) $$h(k;\,\psi) = \sqrt{\frac{2}{\pi}}\,\frac{k^2}{\psi^3}\,e^{-\frac{k^2}{2\psi^2}}, \quad k \geq 0,\ \psi > 0$$

This is the velocity density of a randomly chosen gas molecule in a closed container. It has mean $E(k) = 2\psi\sqrt{2/\pi}$ and variance $V(k) = \psi^2(3\pi - 8)/\pi$. In physics ψ equals $\sqrt{Tk_B/m}$, where k_B is the Boltzmann constant, T is temperature in degrees Kelvin, and m is the mass of the molecule. If the quarterly growth rates of GDP are seen as normally distributed 'objects' which move around their average at certain volatilities, the relationship between the volatility density of gas molecules and the distribution of variance contributions becomes apparent.

For the normal distribution, **Figure A3** shows the variance contribution of realisations *outside* the central interval [Y(k)] und the volatility density [υ(k)].

Figure A3: Variance contribution and volatility density



Appendix 4: Variance contributions for the Chebyshev density

The variance contributions for the Chebyshev density (15) cannot be calculated straightforwardly owing to the non-existent variance. However, we may define a truncated Chebyshev density as follows:

(A4.1) $$f(x)=\frac{\sigma^2}{K|x-\mu|^3}, \quad x\in[\mu-\Omega\sigma,\ \mu-\sigma]\cup[\mu+\sigma,\mu+\Omega\sigma].$$

where $K=1-1/\Omega^2$ and $1<\Omega\leq\infty$. For $\Omega\rightarrow\infty$ the density (15) is obtained as a special case. In A4.1 extreme events beyond $\mu\pm\Omega\sigma$ are ruled out. These events have a probability of (less than or equal to) $1/\Omega^2$. If, for instance, Ω=16, then realisations with a probability not exceeding 0.0039 are dismissed. In the case of quarterly data, such events should occur on average only once in 1,000 quarters (250 years). The capped Chebyshev density A4.1 has mean and variance

$$E(x)=\mu\ ,\ \ E(x-\mu)^2=(2\sigma^2/K)\ln(\Omega).$$

For $1\leq k\leq\Omega$, the following simple formula for the variance contributions is obtained

(A4.2) $$\upsilon(k)=\frac{\int_{\mu-\Omega\sigma}^{\mu-k\sigma}(x-\mu)^2f(x)dx+\int_{\mu+k\sigma}^{\mu+\Omega\sigma}(x-\mu)^2f(x)dx}{\int_{\mu-\Omega\sigma}^{\mu-\sigma}(x-\mu)^2f(x)dx+\int_{\mu+\sigma}^{\mu+\Omega\sigma}(x-\mu)^2f(x)dx}=1-\frac{\ln(k)}{\ln(\Omega)}$$

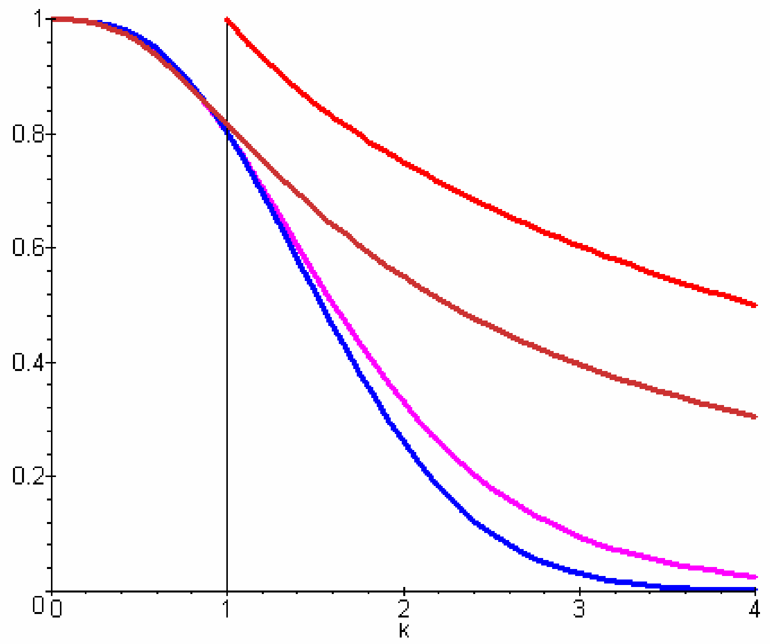
The relative variance decreases logarithmically, i.e. very slowly. Extreme realisations provide a persistently large contribution to the variance. For $\Omega\rightarrow\infty$, $\lim\upsilon(k)=1$ applies.

Figure A4 shows the contribution of extreme observations to the variance for different distributions (from lower to upper curves): (1) the normal distribution, (2) the t-distribution[20] with ν=10 degrees of freedom (df.), (3) the t-distribution with ν=3 df., and (4) the Chebyshev density A4.1, capped at Ω=16. Compared to the normal, the thicker tails of both t-distributions are reflected in a considerably larger contribution of extreme observations to the variance, in

[20] In the case of the t-distribution, the variance contribution depends on the number of degrees of freedom (ν) and, thus, on the variance $\nu/(\nu\text{-}2)$.

particular when the number of df. is small. The variance contributions of extreme observations in the case of the (truncated) Chebyshev density, however, are still significantly greater.

Figure A4: Variance contributions



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