NAIRU estimates for Germany: new evidence on the inflation-unemployment trade-off

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Abstract

The paper estimates the NAIRU from a Phillips curve relationship in the state-space framework. To identify the inflation-unemployment trade-off we account for a time-varying inflation trend to control for the part of inflation that is not affected by the cyclical component of unemployment. In addition we use shifts in the relative volatility of shocks to unemployment and inflation to address the simultaneity problem in Phillips curve estimations. Applying the method of Rigobon and Sack (2003) allows for a data driven identification of the contemporaneous coefficients on the unemployment gap in the Phillips curve and yields more precise estimates of the structural coefficients in the Phillips curve. This tightens the economic relation on the basis of which the NAIRU is derived.

Non-technical summary

Estimations of the so-called NAIRU - the unemployment rate which is associated with a stable inflation rate - typically yield less satisfactory results for Germany. Partly, this is reflected in a statistically insignificant (or only weakly significant) relation between inflation and unemployment; partly, estimations only allow for very imprecise statements about the level of the NAIRU. This paper aims at improving estimations of the NAIRU with a view to both weaknesses.

Our first contribution is to better measure the relevant inflation rate by incorporating a time-varying inflation trend into the Phillips curve. Allowing the first difference of the actual inflation rate to be influenced by the cyclical component of the unemployment rate neglects the fact that inflation might contain a trend component which is unaffected by cyclical variation in the unemployment rate, but rather influenced by inflation expectations and/or the inflation target of monetary policy. Therefore we model trend inflation as an additional unobserved variable and relate the cyclical component of unemployment to the cyclical component of inflation.

Second, since observations of the unemployment and inflation rate are (short-run) equilibrium points the Phillips curve is often identified by explicitly or implicitly imposing the restriction that there is no contemporaneous effect (or none at all) from the inflation rate on unemployment. In contrast, we take potential contemporaneous effects into account in a state-space system. To identify the Phillips curve we follow Rigobon and Sack's (2003, 2004) method of identification through heteroskedasticity.

We evaluate the effect of both modifications, first, by their impact on the size and significance of the coefficients on the unemployment gap in the Phillips curve. A larger absolute value of the sum of coefficients and smaller standard errors would point to an economically more meaningful relationship between the unemployment gap and the inflation gap. Second, the contributions of filter uncertainty and parameter uncertainty are compared.

The results suggest that, first, the coefficient on the unemployment gap in the Phillips curve increases in magnitude and its standard error is greatly reduced leading to a much tighter relation between inflation and unemployment. Second, the precision of the NAIRU estimates themselves are not very sensitive to the alternative identification approaches.

Nicht-technische Zusammenfassung

Die Schätzungen der sogenannten NAIRU, d.h. der Arbeitslosenrate, die mit einer stabilen Inflationsrate vereinbar erscheint, sind für Deutschland in der Regel nicht sehr befriedigend ausgefallen. Zum Teil hat sich der unterstellte Zusammenhang zwischen Inflation und Arbeitslosigkeit als nicht (oder schwach) statistisch signifikant erwiesen; zum Teil hat sich gezeigt, dass die Schätzungen nur sehr unpräzise Aussagen über die Höhe der NAIRU erlauben. In diesem Beitrag wird versucht, mit Blick auf beide Schwächen eine Verbesserung zu erreichen.

Erster Ansatzpunkt ist, die relevante Inflationsrate besser zu messen. Sofern die Veränderungsrate der Inflationsrate in Beziehung zur zyklischen Komponente der Arbeitslosenrate gesetzt wird, muss berücksichtigt werden, dass die Inflationsrate einem Trend unterliegen könnte, der unabhängig von der zyklischen Arbeitslosenrate ist. Stattdessen dürfte er von Inflationserwartungen und/oder dem Zielwert für Preisstabilität beeinflusst sein. Daher wird in diesem Diskussionsbeitrag die Trendinflationsrate als eine weitere unbeobachtete Variable modelliert und die zyklische Inflationsrate der zyklischen Arbeitslosenrate gegenübergestellt.

Darüber hinaus sind Beobachtungen der Inflations- und Arbeitslosenrate (kurzfristige) Gleichgewichtspunkte, und die Phillipskurve wird in diesem System oft identifiziert, indem explizit oder implizit angenommen wird, dass die Inflationsrate keinen zeitgleichen Effekt auf die Arbeitslosenrate ausübt bzw. gar keinen Effekt auf sie hat. Im Gegensatz dazu berücksichtigt dieser Beitrag die simultane Bestimmung beider Größen in einem Zustandsraum-Modell. Zur Identifikation der Phillipskurve wird die Methode der Identifikation durch Heteorskedastizität von Rigobon und Sack (2003, 2004) herangezogen.

Der Effekt beider Erweiterungen wird einmal beurteilt über ihre Auswirkungen auf die Größe und statistische Signifikanz der Schätzwerte für die Steigung der Phillipskurve. Ein größerer Absolutwert der Koeffizienten und geringere Standardabweichungen würden auf einen ökonomisch besser fundierten Zusammenhang zwischen der Arbeitslosen- und Inflationsrate hindeuten. Zweitens wird jeweils die Unsicherheit der NAIRU-Schätzung, die sich zusammensetzt aus der Filterunsicherheit und der Schätzunsicherheit, verglichen.

Die Ergebnisse zeigen, dass erstens der Koeffizient der zyklischen Arbeitslosenrate in der Phillipskurve einen höheren Absolutwert erreicht und seine Standardabweichung stark abnimmt. Zweitens hängt die Genauigkeit der NAIRU-Schätzung allerdings kaum von der verwendeten alternativen Identifikationsmethode.

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1 Introduction

Estimates of the non-accelerating inflation rate of unemployment (NAIRU) have become of interest again in the wake of the recent economic crisis. On the one hand, the extraordinary reduction in output and, in some countries, the rise in unemployment led observers to wonder about the consequences for wage and price inflation. On the other hand, NAIRU estimates help to shed light on the reasons behind the drop in the growth rates of potential output, which a number of countries have experienced.

Recent studies of the NAIRU employ a Kalman-filter approach in a state-space framework, where the NAIRU is modelled as an unobserved component and the difference to the actual unemployment rate is related to changes in the inflation rate while controlling for supply side shocks (e.g. Gordon, 1997; Laubach, 2001; Gruen, Pagan and Thompson, 1999; Gianella et al., 2009; Fitzenberger, Franz and Bode, 2007; Staiger, Stock and Watson, 1997a, 1997b). Unfortunately, most estimates of the NAIRU are subject to considerable uncertainty, which makes them less useful for practical application.

Our paper aims at improving the precision with which the NAIRU for Germany is estimated. We measure precision in two ways: First, we measure the error with which the Kalman filter traces out the NAIRU by empirical distributions derived from Monte Carlo replications. Second, it is defined as the economic and statistical significance with which the unemployment gap affects inflation dynamics. For the NAIRU concept to be meaningful this relationship should be clearly identified from the data.

Allowing the first difference of the actual inflation rate to be influenced by the cyclical component of the unemployment rate neglects the fact that inflation might contain a trend component which is unaffected by cyclical variation in the unemployment rate, but rather influenced by inflation expectations and/or the inflation target of monetary policy. Our first contribu-

tion is to incorporate a time-varying inflation trend into the Phillips curve. We model trend inflation as an additional unobserved variable and relate the cyclical component of unemployment to the cyclical component of inflation (cf. Berger and Everaert, 2008; Carlstrom and Fuerst, 2008a; Harvey, 2008).

Furthermore, as in any Phillips curve and NAIRU estimation a major problem is to control for shocks that move inflation and the unemployment gap in the same direction (Ball and Mankiw, 2002; Carlstrom and Fuerst, 2008b). Examples would be an unexpected temporary increase in productivity which decreases unemployment and inflation, or an unexpected increase in the mark-up increasing inflation and unemployment. A priori observations of the unemployment and inflation rate are (short-run) equilibrium points and in the bi-variate system of the unemployment gap and inflation the Phillips curve is often identified by explicitly or implicitly imposing the restriction that there is no contemporaneous effect from the inflation rate on unemployment, i.e. the unemployment gap is allowed to affect inflation contemporaneously but not vice versa. This is equivalent to instrumenting the unemployment gap by its own lags. Usually, a number of control variables such as changes in productivity, producer price inflation or imports price inflation are included.

The second contribution of the paper is to take potential contemporaneous effects into account by setting up a state-space system in reduced form, in which the inflation gap is allowed to react to the unemployment gap and vice versa. To identify the Phillips curve we follow Rigobon's (2003) and Rigobon and Sack's (2003, 2004) method of identification through heteroskedasticity by suitably defining shifts in the relative variances of shocks to unemployment and inflation. We employ their method as an alternative identification scheme to uncover the structural parameters of the slope of the Phillips curve from the non-structural form estimates.

We evaluate the effect of our modifications in two ways. First, by their impact on the sum of coefficients on the unemployment gap in the Phillips curve. A larger absolute value of the coefficients and smaller standard errors (both individually and jointly) would point to an economically more meaningful relationship between the unemployment gap and the inflation gap. Without this connection we would merely work with the Kalman filter as a statistical trend extracting device. Second, the contributions of filter uncertainty and estimation (parameter) uncertainty are compared using mean squared errors from Monte Carlo simulation exercises (Hamilton, 1986).

The results suggest that the coefficient on the unemployment gap in the Phillips curve increases in magnitude (the effect of the unemployment gap on the inflation gap is more negative) and its standard error is greatly reduced leading to a much tighter relation between inflation and unemploy-

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ment. However, the precision of the NAIRU estimates themselves are not very sensitive to the alternative identification approaches. In addition, as a by-product we get plausible results for trend inflation.

The remainder of the article is structured as follows. The next section very briefly relates our paper to the literature, section 3 presents results of a benchmark estimation of the NAIRU within a standard Phillips curve framework in state-space form, section 4 introduces first trend inflation into the standard model, and goes on to discuss the identification problem in the bi-variate reduced from system of the unemployment gap and inflation. It explains the strategy of identification through heteroskedasticity and applies it to the estimation of the NAIRU. Section 5 concludes.

2   Related literature

Improving the precision of the NAIRU estimates has been the objective of a number of papers since the seminal contribution of Staiger, Stock and Watson (1997). Their paper is the first to provide a systematic investigation into the precision of NAIRU estimates for the US and concludes that NAIRU estimates are subject to considerable uncertainty as measured by their confidence intervals. Laubach (2001) showed that uncertainty about the NAIRU is greatly reduced once one imposes additional structure on the state-space system by modelling the unobserved unemployment gap as an AR(p)-process. In addition he stresses the importance of a significant inflation-unemployment relationship, without which the NAIRU concept becomes meaningless. Additionally, Apel and Jansson (1999) put even more structure onto the state-space system by introducing an Okun's law relationship. This helps to reduce the estimation uncertainty further. Within this approach Fabiani and Mestre (2004) and Basistha and Startz (2008) compare different modelling choices and obtain estimates for NAIRU in the euro area and the US, respectively. Schumacher (2005) applies this estimation approach to German data. In this paper we take a different route and focus on the identification of the Phillips curve slope without recurring to additional observable variables. Our main contribution is to offer a more precise identification of the unemployment-inflation nexus, which could be seen as an additional, rather than an alternative device to further improve the Phillips curve relationship that some of the papers report.

Rigobon's (2003) identification method has so far been applied to e.g. models of sovereign bond yields of Latin America, the response of monetary policy to asset prices and vice versa for the US and European countries (Rigobon and Sack, 2003, 2004; Rigobon, 2003; Furlanetto, 2008; Siklos,

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Bohl and Werner, 2003) as well as to the reaction of economic activity to expectations (Grisse, 2009) and to the estimation of the returns to education (Hogan and Rigobon, 2002), however not to the estimation of the Phillips curve and NAIRU.

Since we look at German data, it is worth highlighting the relation to earlier papers on the German NAIRU. The majority of the related papers uses the Kalman filtering technique in a state-space framework and focuses on different methods to reduce the average total variance of the NAIRU estimates. Neither of the related papers account for trend inflation. Franz (2003) and Fitzenberger, Franz and Bode (2007) provide a careful discussion of potential pitfalls when estimating the NAIRU. They demonstrate that, under a variety of specification choices, the unemployment-inflation relation is subject to considerable uncertainty. Logeay and Tober (2006) estimate a NAIRU for Germany and the euro area with an emphasis on hysteresis. The lagged long-term unemployment rate is found to have a significant effect on the unobserved NAIRU variable for both Germany and the euro area. The size and significance of their coefficient on the unemplyment gap in the Phillips curve is smaller than in other studies though significant. Laubach (2001) uses German data as part of an analysis of the NAIRU in G7 countries and for some countries arrives at quite precise estimates of the NAIRU, though less so for Germany. Nevertheless, he finds a significant relationship between inflation and unemployment for Germany. Schumacher (2005) follows an estimation strategy which relies on the inclusion of a model for the output gap in the state-space system. His findings suggest that modelling the NAIRU using more observable variables helps to improve the precision of NAIRU estimates, although the effect of the unemployment gap on inflation is relatively small and less precisely estimated. Gianella et al. (2009) also obtain estimates of the German NAIRU with a view to regressing it in a second step on potential driving variables. Their confidence intervals for the German NAIRU are rather wide too, while the inflation-unemployment relation is found to be significant though relatively small.

3   A benchmark estimation of the NAIRU

In this section we set up a standard state-space model following Laubach (2001) and provide a baseline estimate of the NAIRU for Germany against which subsequent modifications can be compared. We estimate the following

reduced form model, which is presented directly in its empirical format.

$$\Delta\pi_t = \beta_1 u_{t-1}^{gap} + \beta_2 u_{t-2}^{gap} + \sum_{i=1}^{3}\beta_{2+i}\Delta\pi_{t-i} \tag{1}$$

$$+ \sum_{j=1}^{3}\beta_{5+j}\Delta\pi_{t-2-j}^{en} + \beta_9\Delta\pi_{t-2}^{imp} + \beta_{10}\Delta\pi_t^{oil} + \epsilon_t$$

$$u_t^{gap} = \gamma_1 u_{t-1}^{gap} + \gamma_2 u_{t-2}^{gap} + \eta_t \tag{2}$$

$$u_t = u_t^* + u_t^{gap} \tag{3}$$

$$u_t^* = \alpha_1 u_t^{ltu} + \tilde{u}_t \tag{4}$$

$$\tilde{u}_t = \tilde{u}_{t-1} + \mu_{t-1} + \theta d_{1991Q4} + \nu_t \tag{5}$$

$$\mu_t = \mu_{t-1} + \zeta_t \tag{6}$$

The first equation is the Phillips curve, which relates the change in inflation measured by the GDP deflator, $\Delta\pi_t$, to lags of the cyclical component of unemployment, u_{t-i}^{gap}, lags of the change in inflation and control variables. These are supposed to capture shocks to inflation, and in our specification the second to fifth lag of producer energy prices, $\Delta\pi_{t-i}^{en}$ for $i = 2, 3, 4, 5$, the second lag of the import deflator, $\Delta\pi_{t-2}^{imp}$, and the contemporaneous oil price in euros, $\Delta\pi_t^{oil}$, (all in second differences and demeaned) proved significant. These lags were chosen on the basis of individual significance to yield a parsimonious model. The second equation defines a law of motion for the unemployment gap, which has been shown to considerably improve the estimation of the NAIRU (cf. Laubach, 2001). The third equation is an identity relating the actual unemployment rate, u_t, to the NAIRU, u_t^*, and its cyclical component, u_t^{gap}. Furthermore, as an additional device to better identify the NAIRU at this stage we include the long-term unemployment rate (persons that are unemployed for longer than 12 months as a percentage of the workforce), u_t^{ltu}, in the definition of the NAIRU in equation 4. This is motivated by the hypothesis that the long-term unemployed are more likely to reduce their search effort as they become gradually discouraged, while at the same time firms may view these candidates as less suited due to the depreciation of human capital for those out of the workforce for longer. The coefficient α_1 measures the impact of the long-term unemployment rate on the NAIRU. The remainder $\tilde{u}_t$ is assumed to follow a second-order random walk, as defined in equations 5 and 6, where μ_t is a time-varying drift. The data we use refer to West Germany up to 1991 Q4, and to Germany thereafter. There is a visible jump in the unemployment rate from 3.80 percent in 1991 Q4 to 5.83 percent in 1992 Q1. All other series do not contain any obvious breaks around the date of reunification. To capture the effect of reunification on the measured

unemployment rate u_t we introduce a dummy variable d_{1991Q4} into the state equation (5), which takes on the value 1 in 1991 Q4 and zero in all other periods. The Kalman filter iteratively produces one-step ahead forecasts of the state variables to retrieve series for the unobserved variables. Therefore, taking expectations on (5), $E_{1991Q4}\left(\tilde{u}_{1992Q1}\right) = \tilde{u}_{1991Q4} + \mu_{1991Q4} + \theta$, while $E_{t-1}\left(\tilde{u}_t\right) = \tilde{u}_{t-1} + \mu_{t-1}$ in all other periods. The long-term unemployment rate also jumps up from 1.43 in 1991 to 1.87 in 1992. As a result u_t^* contains a break at 1992 Q1 and so does u_t. Therefore the unemployment gap $u_t^{gap} = u_t - u_t^*$ is not affected.

The system's specification is rather standard (except maybe for the addition of the long-term unemployment rate) and serves as a benchmark for the modifications in the following sections.

We estimate the system of equations (1) to (6) by maximum likelihood and arrive at an estimate of the unobserved component of the NAIRU, $\tilde{u}_t$, using the Kalman filter. One common problem that occurs in the state-space estimation of an unobserved state variable that is assumed to follow a non-stationary process is the "pile-up"problem. It biases the estimate of the variance of the shock to the level of $\tilde{u}_t$, σ_ν, towards zero. Solutions to this problem are the method of the median unbiased estimation of coefficient variance by Stock and Watson (1998), prescribing sufficiently informative priors on the variances of the state variables or simply fixing the signal-to-noise ratio at some appropriate value. Selection criterion for the last method is mostly the plausibility of the variance of the unobserved states compared to observed variables. We opted for the third approach following Laubach (2001) and fixed the variances of ν_t and ζ_t at the values $\sigma_\nu = 0.1$ and $\sigma_\zeta = 0.006$.[2] This restriction is maintained throughout the paper and does not impair the comparability of the different approaches. Another issue is to set the starting values for the parameters and the intial values of the state variables and their prior variances. We use coefficient estimates of a simple OLS regression of the system omitting any unobserved variable. The initial values for the NAIRU is derived from an estimation of a constant NAIRU over the first 8 quarters.[3] The initial value of the drift is set to zero. Prior variances of the state variables are set to 100 suggesting rather uninformative starting values. An alternative approach to initializing the system could be

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[2] We experimented with Stock and Watson's (1998) method, which yields plausible results which, were very similar to those obtained with a fixed ratio. However, one would have to re-estimate the signal-to-noise ratio each time a new version of the model is estimated, thereby increasing the computational burden.

[3] This is done by regressing the change in the inflation rate on a constant and two lags of the unemployment rate. The constant NAIRU is retrieved by dividing the regression constant by the (negative of the) sum of the coefficients on the lags of unemployment.

preliminary estimates using HP-filtered values for the NAIRU. This, however, runs the risk of biasing the results towards the HP-filter results and it requires a number for the smoothing parameter. Our results are derived under less restrictive choices and yield plausible results.

Variable	Coefficient	Standard error	p-value
Phillips curve eq. (1)			
u_{t-1}^{gap}	-0.32	0.19	0.08
u_{t-2}^{gap}	0.30	0.19	0.10
$\Delta\pi_{t-1}$	-0.70	0.07	0.00
$\Delta\pi_{t-2}$	-0.51	0.08	0.00
$\Delta\pi_{t-3}$	-0.44	0.06	0.00
$\Delta\pi_{t-3}^{en}$	0.03	0.01	0.08
$\Delta\pi_{t-4}^{en}$	0.05	0.02	0.00
$\Delta\pi_{t-5}^{en}$	0.05	0.02	0.00
$\Delta\pi_{t-2}^{imp}$	0.03	0.02	0.08
$\Delta\pi_{t}^{oil}$	0.003	0.001	0.01
σ_{ϵ}	0.42	0.03	0.00
Unemployment gap eq. (2)			
u_{t-1}^{gap}	1.87	0.05	0.00
u_{t-2}^{gap}	-0.90	0.05	0.00
σ_{η}	0.08	0.01	0.00
Dummy for break in u_t in 1992 Q1			
d_{1991Q4}	1.80	0.21	0.00
Long-term unemployed eq. (4)			
u_{t}^{ltu}	0.28	0.14	0.05
Log-likelihood			4.99
Akaike information criterion			0.14

Table 1: Estimation results of the benchmark state-space system.

Table 1 presents the estimation results. The Jarque-Bera statistic for the Phillips curve is $JB = 1.42$ (p-value 0.49), and the estimated residuals $\hat{\varepsilon}_t$ show no autocorrelation. Note that the sum of the coefficients on the lagged unemployment gap is only marginally negative (-0.02). While both lags are individually significant at the 10%-level, a Wald test of joint significance can't reject the null of zero sum coefficients with a Chi-square test statistic of 0.19 and two degrees of freedom (p-value 0.66). As such there is only weak support for a meaningful Phillips curve relationship. Moreover, in the unemployment gap equation the sum of coefficients on its lags is 0.97, suggesting a highly persistent process, which is typically found in comparable studies. Furthermore, the signs of the control variables are positive, although the coefficients are quantitatively negligible. These values might represent

the average reaction of prices to mark-up shocks over the sample period. The effect of the long-term unemployment rate on the NAIRU is estimated to be roughly $\frac{1}{3}$. The Kalman filter provides estimates of the unobserved component $\tilde{u}_t$. Figure 1 plots the smoothed estimated NAIRU ($\tilde{u}_t + \hat{\alpha}_1 u_t^{ltu}$) together with the confidence bands.
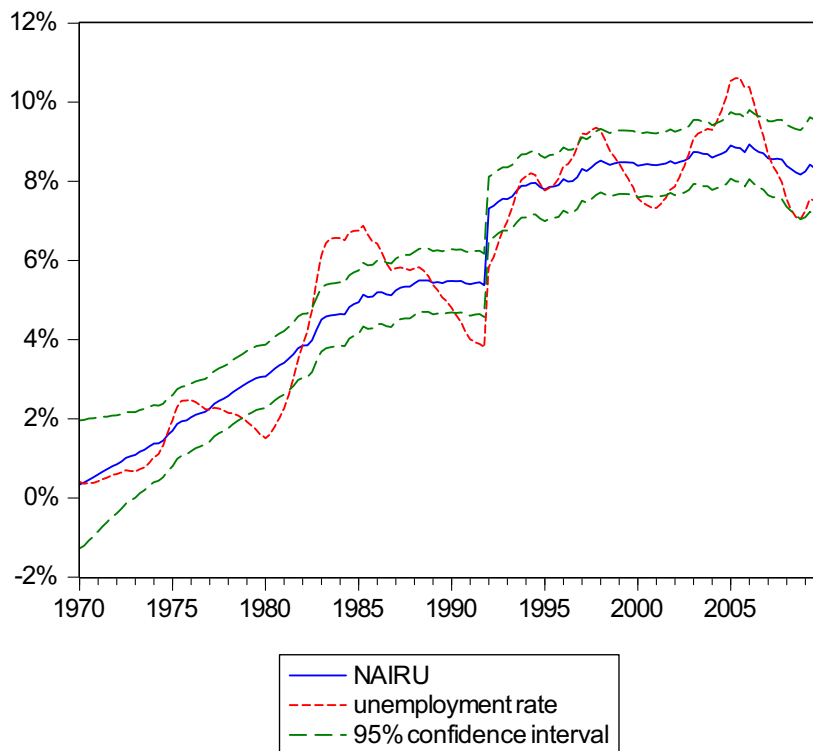

Figure 1: Smoothed estimated NAIRU from benchmark model, 95%-confidence intervals and unemployment rate. Confidence intervals exclude parameter uncertainty.

The plotted confidence bands in figure 1 neglect the uncertainty stemming from the estimation error of the coefficients. Therefore we conduct a Monte Carlo exercise à la Hamilton (1986) and as described in Schumacher (2008) to take this source of uncertainty into account. We run 500 replications of the Kalman filtered state variable while stochastically varying the estimated cofficients on the basis of a multivariate normal distribution with mean and covariance from the estimated values. The variance of the smoothed state variable across the replications approximates the parameter uncertainty. Figure 2 plots again the NAIRU, this time with confidence bands that include parameter uncertainty. As a measure of the precision
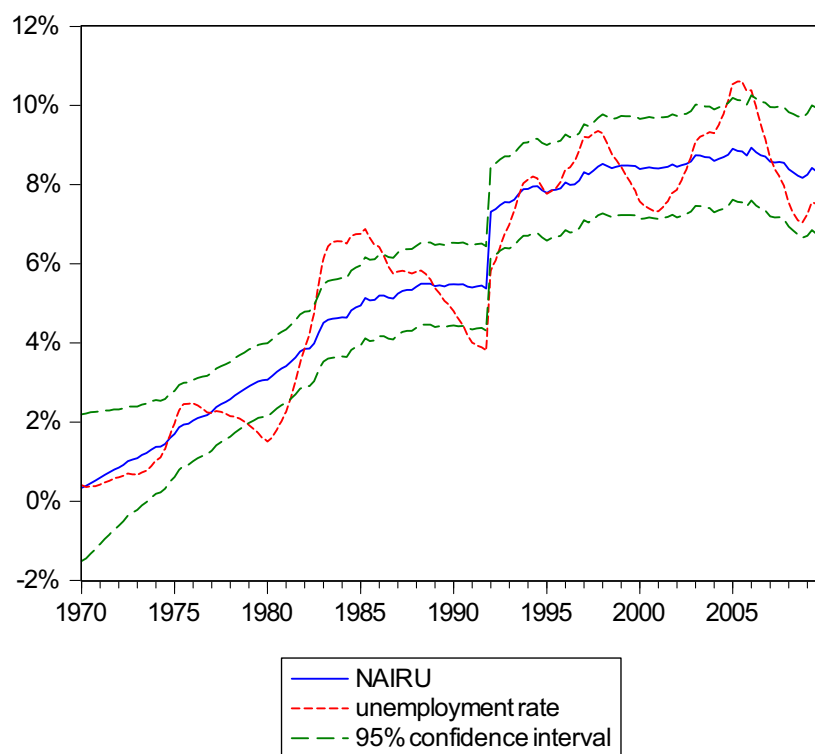
Figure 2: Smoothed estimated NAIRU from benchmark model, 95%-confidence intervals and unemployment rate. Confidence intervals include parameter uncertainty.

of the NAIRU estimate we compute the average of the mean squared error over the sample period (average variance) of smoothed standard errors. The average variance in the first case is 0.21, while it is 0.38 including parameter uncertainty.

There are a few things to note about the baseline approach. First when we look at the raw data for inflation measured by the GDP deflator and depicted in figure 3, a downward trend since the beginning of the sample is discernible. Since the unemployment gap should be stationary by definition, it should only affect the deviation of inflation from its trend. Even though German inflation is much less trended than in other countries over the same period, it will prove important to account for the trend. Moreover, in order to give the above specification a structural interpretation it is often assumed that there is no effect of inflation on the unemployment gap. This assumption is necessary to identify the Phillips curve, however it is not innocuous with respect to the estimates of the Phillips curve as we will show in the next
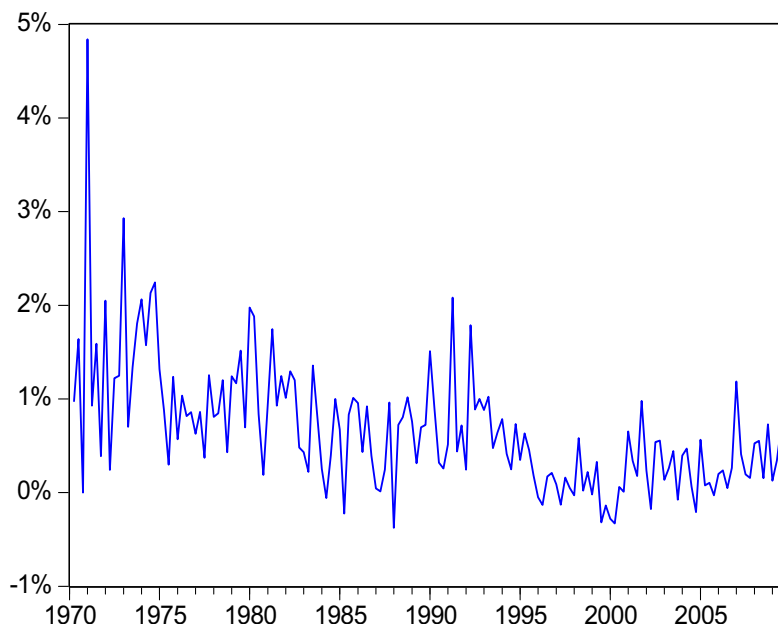
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sections.

Figure 3: Quarterly percentage change of GDP deflator.

4 Extending the standard approach

4.1 Trend inflation and the NAIRU

To demonstrate the individual contributions to increasing the precision of the NAIRU estimate of both including an inflation trend and employing an alternative identification strategy, we proceed by first estimating the system in section 3 including an inflation trend, and subsequently employing our alternative identification scheme. Note that in the following all basic assumptions with respect to starting values, initial means and variances of the states as well as the restrictions on the state variances are kept in order to preserve comparability. The notation changes slightly because instead of the changes in the rate of inflation we now use the deviation of the rate of inflation of its trend, the inflation gap. Correspondingly the control variables are now written in rates of change of the respective price index, corrected for their mean rates of change. Trend inflation is modelled as an additional

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unobserved state variable, which follows a random walk (eq. 14), and an additional observation equation that decomposes the rate of inflation in its trend and cyclical component is added (eq. 9).

$$
\begin{aligned}
\pi_t^{gap} &= \beta_1 u_{t-1}^{gap} + \beta_2 \pi_{t-4}^{gap} & (7) \\
&\quad + \beta_3 \pi_{t-4}^{en} + \beta_9 \pi_{t-2}^{imp} + \beta_{10} \pi_t^{oil} + \varepsilon_t \\
u_t^{gap} &= \gamma_1 u_{t-1}^{gap} + \gamma_2 u_{t-2}^{gap} + \varpi_t & (8) \\
\pi_t &= \pi_t^* + \pi_t^{gap} & (9) \\
u_t &= u_t^* + u_t^{gap} & (10) \\
u_t^* &= \alpha_1 u_t^{ltu} + \tilde{u}_t & (11) \\
\tilde{u}_t &= \tilde{u}_{t-1} + \mu_{t-1} + \theta d_{1991Q4} + \nu_t & (12) \\
\mu_t &= \mu_{t-1} + \zeta_t & (13) \\
\pi_t^* &= \pi_{t-1}^* + \delta_t & (14)
\end{aligned}
$$

Note that we dropped the second lag of the unemployment gap and the first three lags of the inflation gap as well as the third and fifth lag of the energy price change because their coefficients did not turn out significant.[4] The system was estimated with the restriction on the signal-to-noise ratio as before, while the variance of the unobserved inflation trend was left unrestricted. Table 2 presents the results.

The residuals from the Phillips curve equation are normally distributed with a Jarque-Bera statistic of $JB = 0.29$ (p-value 0.87) and there is no auto-correlation. The coefficient on the lagged unemployment gap is now about five times larger (in absolute terms) than in the specification without inflation trend. In addition, its significance has increased considerably. Figure 4 plots the smoothed estimated NAIRU along with the confidence bands including parameter uncertainty. The latter was again derived from 500 Monte Carlo replications. The average variance including parameter uncertainty is now 0.39 and virtually unchanged compared to before (when it was 0.38). In contrast the average variance neglecting parameter uncertainty is 0.21. However, accounting for trend inflation leads to an effect of the unemployment gap on the inflation gap that is absolutely larger and more significant than without the inflation trend. Figure 5 plots the estimated inflation trend along with

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[4]The control variables were not trend-adjusted because simple unit root tests reject the null of a unit root in the inflation rates of energy, imports and oil (whereas the null is not rejected for the rate of change of the GDP deflator). Moreover, an alternative specification using controls that were demeaned and adjusted for a deterministic trend yielded virtually the same results.
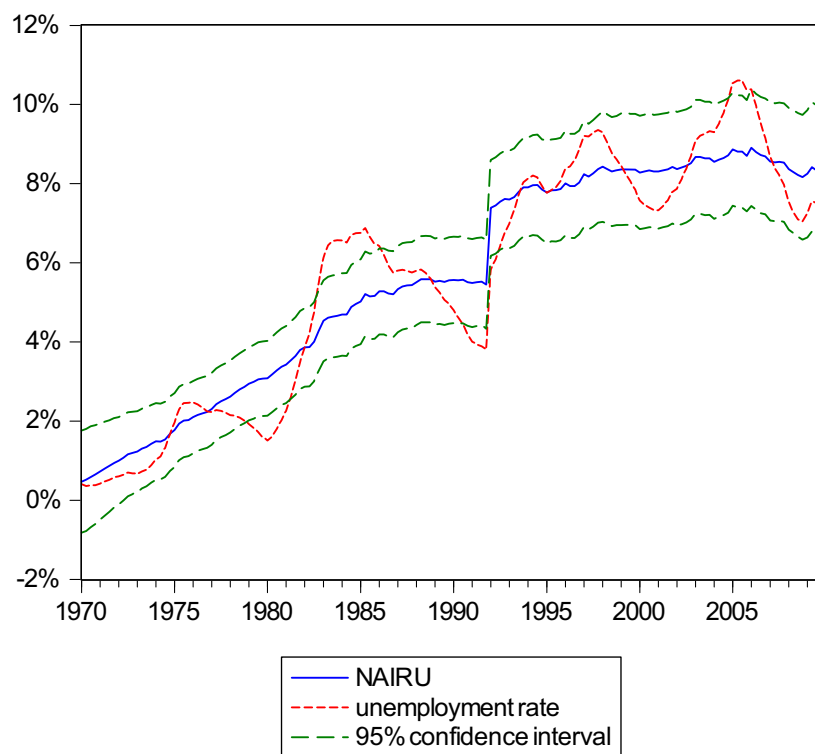
Figure 4: Smoothed estimated NAIRU from model with trend inflation, 95%-confidence intervals and unemployment rate. Confidence intervals include parameter uncertainty.

Variable	Coefficient	Standard error	p-value
Phillips curve eq. (7)			
u_{t-1}^{gap}	-0.10	0.04	0.02
π_{t-4}^{gap}	0.30	0.08	0.00
π_{t-4}^{en}	0.04	0.01	0.01
π_{t-2}^{imp}	0.05	0.02	0.01
π_{t}^{oil}	0.004	0.001	0.00
σ_ε	0.38	0.03	0.00
Unemployment gap eq. (8)			
u_{t-1}^{gap}	1.87	0.04	0.00
u_{t-2}^{gap}	-0.90	0.04	0.00
σ_ϖ	0.08	0.01	0.00
Dummy for break in u_t in1992 Q1			
d_{1991Q4}	1.79	0.19	0.00
Long-term unemployed eq. (11)			
u_t^{ltu}	0.28	0.14	0.05
Inflation trend eq. (14)			
σ_δ	0.05	0.02	0.03
Log-likelihood			6.76
Akaike information criterion			0.07

Table 2: Estimation results of the state-space system including an unobserved inflation trend.

the 95%-confidence bands (excluding parameter uncertainty). The trend declines from quarterly rates of about 1% in the 1970s to almost zero around 2000, hovering around 0.2% per quarter from thereon. We take the plausible estimates of trend inflation (which is derived without restrictions on its variance) as support for our hypothesis that it should be accounted for in a Phillips curve estimation.

4.2 Identifying the Phillips curve slope through shifts in volatility

4.2.1 The identification problem in a state-space system of inflation and unemployment

Consider the following generalized version of the above system. It is more general in the sense that we do not restrict the contemporaneous coefficients on the output gap and inflation gap, while keeping most of the remaining
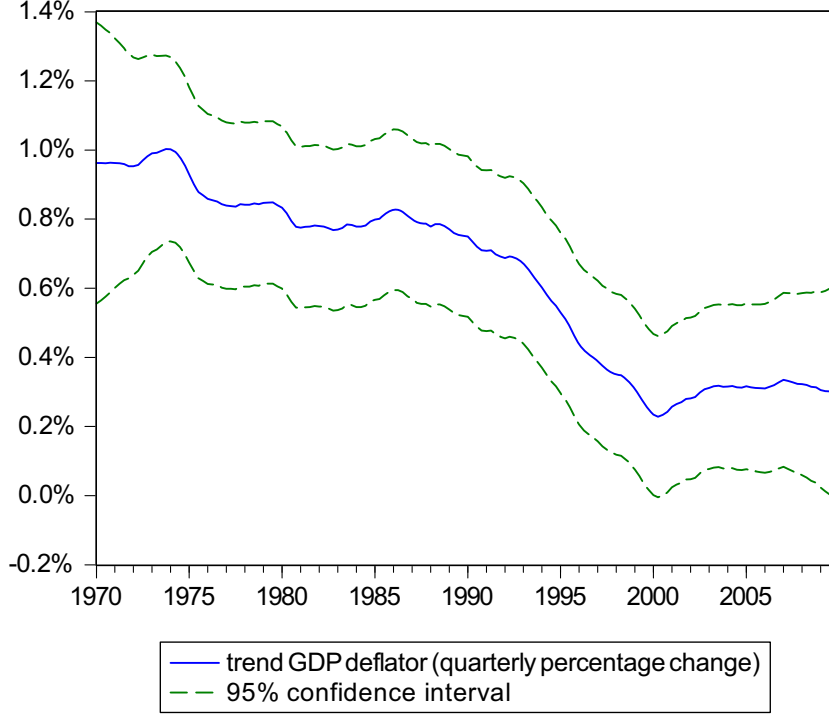
Figure 5: Smoothed estimated trend inflation as measured by the quarterly percentage change in the GDP deflator. Confidence intervals exclude parameter uncertainty.

specification choices.

$$\pi_t^{gap} = \beta_0 u_t^{gap} + \beta_1 u_{t-1}^{gap} + \beta_2 u_{t-2}^{gap} + \sum_{i=1}^{3} \beta_{2+i} \pi_{t-i}^{gap} \qquad (15)$$

$$+ \beta_6 \pi_{t-4}^{en} + \beta_7 \pi_{t-2}^{imp} + \beta_8 \pi_t^{oil} + e_t$$

$$u_t^{gap} = \gamma_1 u_{t-1}^{gap} + \gamma_2 u_{t-2}^{gap} + \gamma_3 \pi_t^{gap} + \sum_{i=1}^{3} \gamma_{3+i} \pi_{t-i}^{gap} \qquad (16)$$

$$+ \gamma_7 \pi_{t-4}^{en} + \gamma_8 \pi_{t-2}^{imp} + \gamma_9 \pi_t^{oil} + u_t$$

$$\pi_t = \pi_t^* + \pi_t^{gap} \qquad (17)$$

$$u_t = u_t^* + u_t^{gap} \qquad (18)$$

$$u_t^* = \alpha_1 u_t^{ltu} + \tilde{u}_t \qquad (19)$$

$$\tilde{u}_t = \tilde{u}_{t-1} + \mu_{t-1} + \theta d_{1991Q4} + \nu_t \qquad (20)$$

$$\mu_t = \mu_{t-1} + \zeta_t \qquad (21)$$

$$\pi_t^* = \pi_{t-1}^* + \delta_t \qquad (22)$$

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Again the control variables are written in the rates of change of the respective price index corrected for their sample mean, and an additional observation equation that decomposes the rate of inflation in its trend and cyclical component is added. The structural error terms e_t and u_t are contemporaneously uncorrelated. Note that both π_t^{gap} and u_t^{gap} are allowed to be affected by the same set of variables in order to capture the potential endogeneity between π_t^{gap} and u_t^{gap}. We incorporate Rigobon and Sack's VAR(X)-based procedure into a state-space model, which requires some modifications to keep the model tractable. To the best of our knowledge we are the first to apply their identification method in a state-space framework. Typically, the method involves estimating a reduced form VAR(X) and using the residual variances to define regimes for the structural variances. We adopt a sequential approach to identification, which corresponds to the fashion in which a state-space system is estimated. Based on starting values for the parameters the Kalman filter retrieves the unobserved variables of the system, which are in the next step used as inputs for the estimation of the parameters by maximum likelihood. There are therefore two kinds of errors; those that appear in the Kalman filtering rounds and which refer to the error terms of the unobserved trend variables, π_t^* and $\tilde{u}_t$; and those that refer to the ML estimation of the parameters for given state variables. We base our definition of the regimes on the variances of the gap-equations (15) and (16) for given preliminary estimates of trend inflation and the NAIRU. Shocks to π_t then translate into shocks to π_t^{gap} for given trend inflation. This approach reduces the identification problem to the covariance matrix of the gap-equations since the trend and NAIRU variables are taken as given. The preliminary estimates for trend inflation and the NAIRU are taken from an estimation of a reduced form of the system (15) to (22), where in (15) and (16) the contemporaneous right-hand variables are substituted out. It can be demonstrated that the preliminary estimates of trend inflation and the NAIRU from the non-structural model are indeed close, though not identical, to their final estimates using the alternative identification scheme.

To illustrate the resulting identification problem consider the variance-covariance matrix $\hat{\Omega}$ of the system for given values of trend inflation and the NAIRU.

$$\hat{\Omega} = \frac{1}{(1-\beta_0\gamma_3)^2} \begin{bmatrix} \beta_0^2\sigma_u^2 + \sigma_e^2 & \beta_0\sigma_u^2 + \gamma_3\sigma_e^2 \\ . & \sigma_u^2 + \gamma_3^2\sigma_e^2 \end{bmatrix} \tag{23}$$

The identification problem is that $\hat{\Omega}$ provides only three equations, the reduced form variance of u_t^{gap}, the reduced form variance of π_t^{gap} and the reduced form covariance between the two, while there are four unknowns,

β_0, γ_3, σ_u^2 and σ_e^2. Most existing studies of the NAIRU in a state-space-framework explicitly or implicitly impose restrictions on $\hat{\Omega}$. E.g. Gordon (1997) and Staiger, Stock and Watson (1997) omit the equation for the unemployment gap, which requires the assumption that the reduced form covariance of u_t^{gap} and π_t^{gap} is zero, $\beta_0\sigma_u^2 + \gamma_3\sigma_e^2 = 0$, whereas the two standard approaches in section 3 could be interpreted as if imposing $\gamma_3 = \beta_0 = 0$, while keeping the equation for the unemployment gap without control variables and lags of inflation. Essentially, these assumptions imply that the structural parameters are directly estimated.

Proper identification of the structural coefficients on the unemployment gap matter because they are the economic foundation on which the NAIRU concept rests. Without this relationship it would not be possible to estimate a NAIRU but merely some trend unemployment rate, which could conceivably done more easily using a statistical filtering procedure like the HP-filter (Franz, 2003). Furthermore the coefficient on the unemployment gap is one determinant of the precision with which the NAIRU is estimated (Hamilton, 1994, pp.377). In a reduced form model, the coefficient on the unemployment gap in the Phillips equation is a composite of the effect of unemployment and inflation shocks. It is likely to be biased because unemployment shocks trace out the Phillips curve in the data, while inflation shocks trace out the labour demand curve. This is equivalent to saying that the reduced form slope of the Phillips curve is likely to be flat because it combines shocks to unemployment and the mark-up. Principally, one tries to account for mark-up shocks by including appropriate control variables in the estimation. However, it is very unlikely that one is able to control for all possible mark-up shocks using proxy variables.

As an alternative to exclusion restrictions the identification through heteroskedasticity procedure splits the sample in two subgroups $s \in \{1, 2\}$ with two different reduced form covariance matrices. Under the assumption that the coefficients of interest, β_0 and γ_3, are constant over the whole sample we get six equations and exactly six unknowns.

$$\hat{\Omega}_s = \left[\begin{array}{cc} \omega_{11,s} & \omega_{12,s} \\ . & \omega_{22,s} \end{array} \right] = \frac{1}{(1 - \beta_0\gamma_3)^2} \left[\begin{array}{cc} \beta_0^2\sigma_{u,s}^2 + \sigma_{e,s}^2 & \beta_0\sigma_{u,s}^2 + \gamma_3\sigma_{e,s}^2 \\ . & \sigma_{u,s}^2 + \gamma_3^2\sigma_{e,s}^2 \end{array} \right] \quad (24)$$

The six equations imply that β_0 and γ_3 are the solutions to the following

system of equations (Rigobon, 2003).

$$\beta_0 = \frac{\omega_{12,s} - \gamma_3 \omega_{11,s}}{\omega_{22,s} - \gamma_3 \omega_{12,s}} \tag{25}$$

$$0 = \left(\omega_{11,1}\omega_{12,2} - \omega_{12,1}\omega_{11,2}\right)\gamma_3^2 - \left(\omega_{11,1}\omega_{22,2} - \omega_{22,1}\omega_{11,2}\right)\gamma_3 \tag{26}$$
$$+ \left(\omega_{12,1}\omega_{22,2} - \omega_{22,1}\omega_{12,2}\right)$$

The two solutions for γ_3 (and for β_0) correspond to the two ways the structural form can be written, i.e. inflation or the unemployment gap on the left-hand side of the Phillips curve and the same for the aggregate labour demand curve.

Summing up, the requirements to identify the system using shifts in the volatility in the error terms are that i) the contemporaneous coefficients of the system are constant, that ii) there are at least as many linearly independent equations in the reduced form covariance matrix as there are unknowns, and that iii) we can use preliminary estimates of trend inflation and the NAIRU to yield the non-structural error variances. The second requirement can be tested. For proofs and derivations of these results refer to Rigobon (2003), Rigobon and Sack (2003) and Rigobon and Sack (2004). One interesting aspect of this identification method as opposed to exclusion restrictions is that we can test for the signifcance of the restrictions on the contemporaneous cofficients. The contemporaneous coeffcients can then be used to recover the structural parameters of the reduced form system. We can then use the Kalman filter on the structural equations to get estimates of the unobserved state variables (the NAIRU and the inflation trend) and their confidence intervals.

4.2.2 Results

We begin by estimating the system (15) to (22) in non-structural form, where the contemporaneous right-hand gap-variables are substituted out.[5] Next, the error varinaces of the gap equations are used to define regimes of shifts in their relative variances. As long as the there are six linearly independent equations in the two covariance matrices, identification can be achieved. What is important for identification are shifts in the variances of the structural shocks, for which changes in the reduced form variances are proxies. To identify the regimes we compute the variances of the reduced form error terms over moving windows as well as their correlation. We chose a window of eight quarters, the first of which starts in 1970 Q1. A given quarter

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[5] The results of this first step estimation exercise are available from the author upon request.

is defined to be in the high volatility regime whenever the variance of the error term over the previous eight quarters is one standard deviation above its average over the whole sample (cf. Rigobon and Sack, 2003). Figure 6 plots the distribution of the high volatility regimes along with the correlation between the errors.Since two regimes are sufficient for our purposes we focus
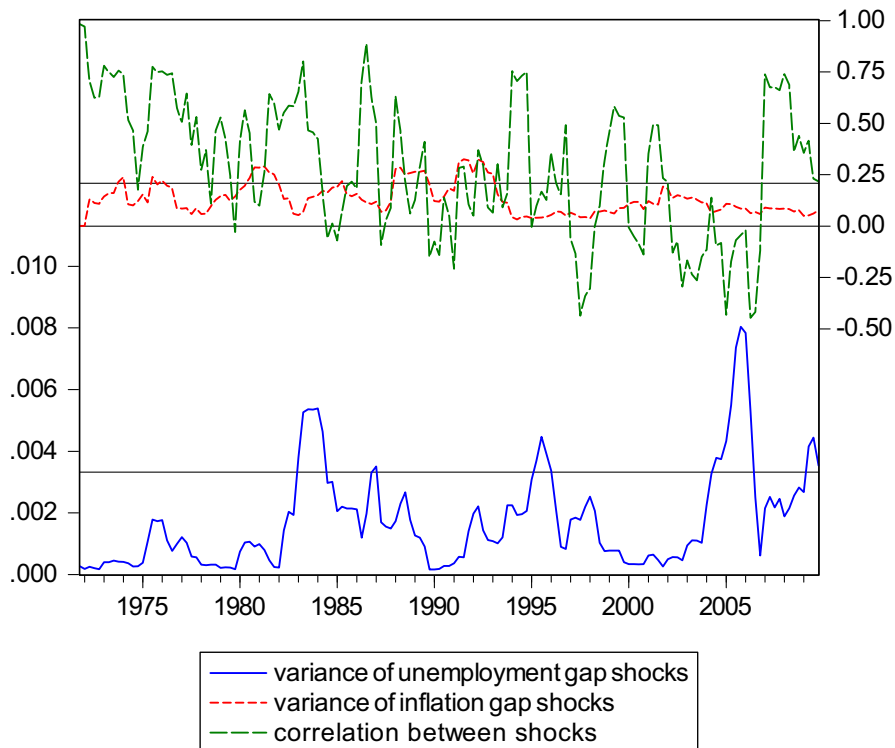

Figure 6: Variances of shocks to unemployment and inflation gap over moving windows of eight quarters and their correlation. Horizontal lines are threshold for high volatility regime of inflation shocks, zero line for correlation between shocks and threshold for high volatility regime of unemployment shocks (from top to bottom).

on periods when shocks to unemployment dominate vs. periods with low volatility in both types of shocks. This yields the following results in table 3.

Regime 1 contains all periods in which the variance of unemployment shocks exceeds the threshold while the variance of inflation shocks is below the threshold. This characterises a situation in which unemployment shocks dominate and trace out the Phillips curve. Regime 2 is made up of all other periods. The variance of unemployment shocks is indeed twice as large

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	Variance of unemployment shocks	Variance of inflation shocks	Correlation of unemployment and inflation shocks	Frequency of observations
Regime 1	0.004	0.086	-0.10	0.14
Regime 2	0.002	0.135	0.27	0.86

Table 3: Volatility regimes of shocks to the unemployment and inflation gap.

in regime 1 as in regime 2, while inflation shocks differ only little in their variance across regimes. In addition, the correlation between unemployment and inflation shocks is negative in regime 1 as would be expected, while it is positive in regime 2. Finally, 14% of all periods fall into regime 1 and 86% into regime 2. The test for linear independence of the equation in the covariance matrices is passed.

From (25) and (26) one can now compute the contemporaneous coefficients β_0 and γ_3. In addition we compute 500 bootstrap replications of the reduced form covariance matrix using draws of the errors from their empirial distribution. This yields a distribution of the contemporaneous coefficients, the summary statistics of which are presented in table 4.

	Coefficient	
	β_0	γ_3
Point estimate	-0.70	20.42
Mean of distribution	-0.90	77.82
Standard deviation of distribution	4.81	564.34
Mass below zero	94.4%	4.6%

Table 4: Summary statistics of the distribution of the contemporaneous structural coefficients. Obtained from 500 bootstrap replications of the reduced form covariance matrices.

The point estimate for β_0 is significantly different from zero as only about 5% of all realisations in the bootstrap exercise exceed zero. Similarly, the point estimate of γ_3 is clearly larger than zero. In addition, both the point estimate and the mean of distribution take on rather large values and the standard deviation of γ_3 is quite large too. However, our emphasis is on the contemporaneous coefficient in the Phillips curve, the sign and size of which seems reasonable. From the contemporaneous coefficients it is now possible to recover the structural parameters on the remaining variables and to apply the Kalman filter to the structural form to arrive at an estimate of the NAIRU.

4.2.3 The NAIRU and confidence intervals

We run the Kalman filter on the following (slightly modified) system of (7) to (14) with the coefficient value of $\hat{\beta}_0$ set to its estimated value in the identification procedure of the previous section. Again, the first to third lag of the inflation gap turned out insignifcant and were omitted. Moreover, we found the second lag of the unemployment gap to come out significant in the estimation.

$$
\begin{align}
\pi_t^{gap} &= \hat{\beta}_0 u_t^{gap} + \beta_1 u_{t-1}^{gap} + \beta_2 u_{t-2}^{gap} + \beta_3 \pi_{t-4}^{gap} \tag{27} \\
&\quad + \beta_4 \Delta \pi_{t-4}^{en} + \beta_5 \Delta \pi_{t-2}^{imp} + \beta_6 \Delta \pi_t^{oil} + \varepsilon_t \\
u_t^{gap} &= \gamma_1 u_{t-1}^{gap} + \gamma_2 u_{t-2}^{gap} + \varpi_t \tag{28} \\
\pi_t &= \pi_t^* + \pi_t^{gap} \tag{29} \\
u_t &= u_t^* + u_t^{gap} \tag{30} \\
u_t^* &= \alpha_1 u_t^{ltu} + \tilde{u}_t \tag{31} \\
\tilde{u}_t &= \tilde{u}_{t-1} + \mu_{t-1} + \theta d_{1991Q4} + \nu_t \tag{32} \\
\mu_t &= \mu_{t-1} + \zeta_t \tag{33} \\
\pi_t^* &= \pi_{t-1}^* + \delta_t \tag{34}
\end{align}
$$

Additionally, we account for the shifts in volatility by dummy variables that take on the value of one in regime 1 and zero in regime 2 in order to account for the variance shifts in the sample that are used to identify the structural parameters. However, both dummies turn out insignificant and are dropped from the estimation. Table 9 in the appendix contains the results including the dummy variables. The estimation results without dummies are presented in table 5.

The sum of coefficients attached to the unemployment gap is -0.14 with both the first and the second lag being individually significant. In addition, a Wald test for the significance of the sum of the first two lags rejects the null of a zero sum at the 1%-significance level (Chi-square statistic 124.30, p-value 0.00). A second Wald test for the restriction that the sum of the first two lags of the unemployment gap equals 0.70 (minus the effect of the contemporaneous unemployment gap) rejects the null at the 1%-level (Chi-square statistic 7.81, p-value 0.01). To sum up, restricting the contemporaneous coefficient on the unemployment gap to a value of -0.70, which is derived from the data, yields a much more meaningful relationship - both economically and statistically - between the unemployment gap and the inflation gap than any of the approaches discussed in section 3.

Figure 7 shows the resulting NAIRU along with the 95%-confidence bands including parameter uncertainty. Figure 8 presents the estimate of the in-

Variable	Coefficient	Standard error	p-value
Phillips curve eq. (15)			
u_t^{gap}	-0.70	-	-
u_{t-1}^{gap}	1.26	0.28	0.00
u_{t-2}^{gap}	-0.70	0.28	0.01
π_{t-4}^{gap}	0.14	0.07	0.05
σ_ε	0.38	0.03	0.00
Unemployment gap eq. (16)			
u_{t-1}^{gap}	1.87	0.05	0.00
u_{t-2}^{gap}	-0.90	0.05	0.00
σ_ϖ	0.08	0.01	0.00
Dummy for break in u_t in 1992 Q1			
d_{1991Q4}	1.81	0.21	0.00
Long-term unemployed eq. (19)			
u_t^{ltu}	0.27	0.15	0.06
Inflation trend eq. (22)			
σ_δ	0.05	0.02	0.03
Log-likelihood			-5.79
Akaike information criterion			0.25

Table 5: Estimation results from the structural state-space model with trend inflation. Note: Coefficient on contemporaneous unemployment gap restricted to value obtained in identification procedure. Control variables were included but are not presented for the sake of brevity.

flation trend from the structural model.The average variance ignoring parameter uncertainty is 0.22, while it is 0.46 including parameter uncertainty. Table 6 summarizes the results for the significance of the inflation-unemployment trade-off and the average variance of the NAIRU from the different models. Judging the models according to the size and significance of the impact of the unemployment gap in the Phillips curve, the structural model with trend inflation yields the best results. In terms of the average variance of the estimated NAIRU the models do roughly equally well when ignoring parameter uncertainty. Looking at the results including parameter uncertainty the simple model without trend inflation does best. This is however due to the additional unobserved state variable in the other two models that has to be estimated from the same data. Therefore it is not surprising that the average variances increase in the specifications with trend inflation.
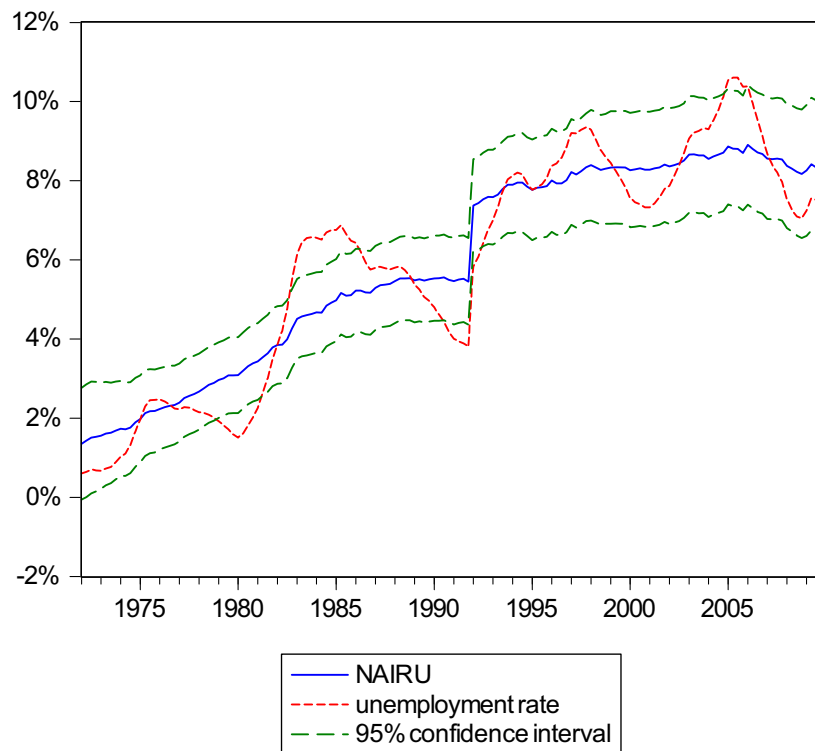
Figure 7: Smoothed estimated NAIRU from structural model with trend inflation, 95%-confidence intervals and unemployment rate. Confidence intervals include parameter uncertainty.
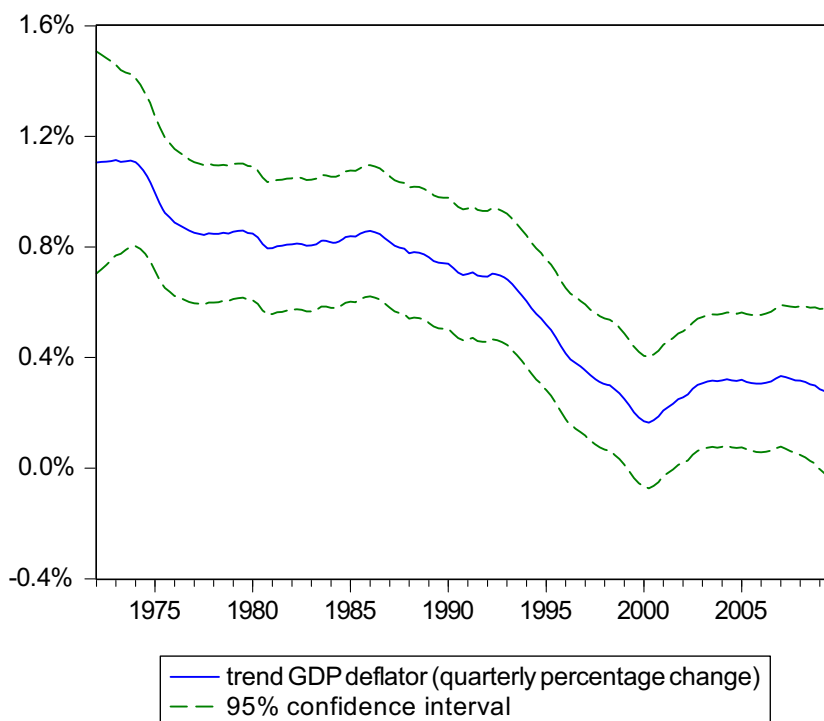
Figure 8: Smoothed estimated trend inflation as measured by quarterly percentage change of GDP deflator from structural model. Confidence intervals exclude parameter uncertainty.

5 Conclusion

In order to achieve a more precise and economically more meaningful Phillips curve relationship we have, first, incorporated trend inflation and, second, employed an alternative identification scheme based on regime shifts in the structural shocks to the unemployment gap and inflation gap. This was motivated by noting that for the NAIRU concept to be economically meaningful it must be based on a clearly identified inflation-unemployment trade-off. Our results suggest that introducing trend inflation in the estimation goes some way in improving the significance and magnitude of the effect of the unemployment gap on the inflation gap. Furthermore, distinguishing between periods when shocks to unemployment were relatively more pronounced than shocks to inflation allows for an even more precise estimate of the coefficients on the unemployment gap in the Phillips curve. The uncertainty with which the Kalman filter traces out the NAIRU is affected only little in all three

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Model specification	Sign and significance of unemployment gap in Phillips curve		Average variance of estimated NAIRU	
	sum of coefficients	test statistic	without parameter uncertainty	with parameter uncertainty
Standard model without trend inflation *eq. (1) to (6)*	-0.02	0.17^a (p-value 0.68)	0.21	0.38
Standard model with trend inflation *eq. (7) to (14)*	-0.10	5.11^b (0.02)	0.21	0.39
Structural model with trend inflation *eq. (27) to (34)*	-0.14	7.81^c (0.01)	0.22	0.46

Table 6: Summary of main results. The structural model with trend inflation yields the most precise and quantitatively relevant effect of the unemployment gap on the inflation gap. Notes: a: Chi-square statistic with two degrees of freedom from Wald test for joint significance. b: Chi-square statistic with one degree of freedom. c: Chi-square statistic with one degree of freedom from Wald test for joint significance.

models. Given that it is natural that the average variances are higher in the model incorporating trend inflation, the structural model with trend inflation delivers the best results overall. We conclude that even though the structural model with trend inflation yields larger confidence intervals it is nevertheless preferable since it delivers the tightest Phillips curve relationship. After all this is the economic foundation on which the NAIRU rests.

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A Appendix

A.1 Data

We use data from the German National Accounts for 1970 Q1 to 2009 Q4. Inflation is the quarter-on-quarter percentage change of the GDP deflator, the unemployment rate is measured according to the ILO concept and the oil price is the world market price for Brent in euros. Data on long-term unemployment come from the Federal Labour Agency. Before 1992 long-term unemployment is only available on an annual basis; quarterly data was obtained by linear interpolation.

A.2 Accounting for regime shifts

In section 4.2.1 we compared the results of our proposed alternative identification scheme to existing approaches to estimating the NAIRU. Since our preferred approach relies on shifts in the volatility of the error terms of the Phillips curve and the unemployment equation these shifts should be accounted for in the two previous approaches for a fair comparison. We introduce dummy variables for variance shifts as defined in section 4.2.2 in the respective regressions. Tables 7 and 8 present the estimation results of the standard model without trend inflation and the one with trend inflation as in section 3 including the same variance shifts as in section 4.2.1. None of the shift dummies prove significant when we use as starting values the differences in variances between regimes for each equation. However, with different (and more arbitrary) starting values the dummies in the Phillips curve come out significant.

Accounting for the regime shifts contributes to reducing the parameter uncertainty of the NAIRU in all three specifications (see table 10). The average variance of the estimated NAIRU including parameter uncertainty is always lower than the average variance ignoring shifts in volatility (cf. table 6) for all cases. The average variances when excluding parameter uncertainty and accounting for volatility shifts also decline. However, the basic message of the estimation exercise is not altered. The results on the sign and significance of the unemployment gap in the Phillips curve remain basically unchanged. The average variance of the structural model with trend inflation do not worsen as they did without the dummies, which even improves our earlier results. However, we caution against putting too much weight on these numbers as they are based on the inclusion of insignificant variables in the estimation. All in all we regard these results as a robustness check for the specifications in the main text.

Variable	Coefficient	Standard error	p-value
Phillips curve eq. (1)			
u_{t-1}^{gap}	-0.28	0.19	0.15
u_{t-2}^{gap}	0.25	0.20	0.20
$\Delta\pi_{t-1}$	-0.69	0.07	0.00
$\Delta\pi_{t-2}$	-0.51	0.08	0.00
$\Delta\pi_{t-3}$	-0.44	0.06	0.00
$\Delta\pi_{t-3}^{en}$	0.03	0.01	0.07
$\Delta\pi_{t-4}^{en}$	0.05	0.02	0.00
$\Delta\pi_{t-5}^{en}$	0.05	0.02	0.00
$\Delta\pi_{t-2}^{imp}$	0.03	0.02	0.11
$\Delta\pi_{t}^{oil}$	0.002	0.001	0.01
σ_{ϵ}	0.43	0.03	0.00
Dummy variable for variance shift	-0.08	0.08	0.33
Unemployment gap eq. (2)			
u_{t-1}^{gap}	1.91	0.04	0.00
u_{t-2}^{gap}	-0.94	0.04	0.00
σ_{η}	0.06	0.01	0.00
Dummy for variance shift	0.08	0.07	0.21
Dummy for break in u_t in 1992 Q1			
d_{1991Q4}	1.78	0.17	0.00
Long-term unemployed eq. (4)			
u_{t}^{ltu}	0.28	0.15	0.06
Log-likelihood			10.04
Akaike information criterion			0.11

Table 7: Estimation results of the benchmark state-space system including dummies for variance shifts in the Phillips curve and unemployment equation.

Variable	Coefficient	Standard error	p-value
Phillips curve eq. (7)			
u_{t-1}^{gap}	-0.10	0.04	0.02
π_{t-4}^{gap}	0.30	0.08	0.00
π_{t-4}^{en}	0.04	0.01	0.00
π_{t-2}^{imp}	0.05	0.02	0.01
π_{t}^{oil}	0.004	0.001	0.00
σ_{ε}	0.39	0.03	0.00
Dummy for variance shift	-0.06	0.07	0.35
Unemployment gap eq. (8)			
u_{t-1}^{gap}	1.92	0.03	0.00
u_{t-2}^{gap}	-0.95	0.04	0.00
σ_{ϖ}	0.06	0.01	0.00
Dummy for variance shift	0.09	0.06	0.11
Dummy for break in u_t in 1992 Q1			
d_{1991Q4}	1.78	0.20	0.00
Long-term unemployed eq. (11)			
u_{t}^{ltu}	0.29	0.15	0.05
Inflation trend eq. (14)			
σ_{δ}	0.05	0.02	0.02
Log-likelihood			10.63
Akaike information criterion			0.04

Table 8: Estimation results of the state-space system with trend inflation including dummies for variance shifts in the Phillips curve and unemployment equation.

Variable	Coefficient	Standard error	p-value
Phillips curve eq. (15)			
u_t^{gap}	-0.70	-	-
u_{t-1}^{gap}	1.25	0.26	0.00
u_{t-2}^{gap}	-0.69	0.26	0.01
π_{t-4}^{gap}	0.13	0.07	0.07
σ_e	0.39	0.03	0.00
Dummy variable for regime shift	-0.08	0.07	0.23
Unemployment gap eq. (16)			
u_{t-1}^{gap}	1.91	0.04	0.00
u_{t-2}^{gap}	-0.94	0.04	0.00
σ_u	0.06	0.01	0.00
Dummy variable for regime shift	0.09	0.08	0.22
Dummy for break in u_t in 1992 Q1			
d_{1991Q4}	1.76	0.16	0.00
Long-term unemployed eq. (19)			
u_t^{ltu}	0.30	0.15	0.04
Inflation trend eq. (22)			
σ_δ	0.05	0.02	0.02
Log-likelihood			0.18
Akaike information criterion			0.19

Table 9: Estimation results of the structural state-space system including dummies for variance shifts in the Phillips curve and unemployment equation.

Model specification	Sign and significance of unemployment gap in Phillips curve		Average variance of estimated NAIRU	
	sum of coefficients	test statistic	without estimation uncertainty	with estimation uncertainty
Standard model without trend inflation *eq. (1) to (6)*	-0.03[e]	0.35[d] (p-value 0.56)	0.16	0.32
Standard model with trend inflation *eq. (7) to (14)*	-0.09	5.36[f] (0.02)	0.15	0.34
Structural model with trend inflation *eq. (27) to (34)*	-0.14	7.86[g] (0.01)	0.14	0.29

Table 10: Summary of main results when including (non-significant) dummies for variance shifts in the Phillips curve and unemployment equation. The structural model with trend inflation yields the most precise and quantitatively relevant effect of the unemployment gap on the inflation gap. The average variances do not vary much across the models. Notes: d: Chi-square statistic with two degrees of freedom from Wald test for joint significance. e: Coefficients individually not significant. f: Chi-square statistic with one degree of freedom. g: Chi-square statistic with one degree of freedom from Wald test for joint significance.

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