

A solution to the problem of too many instruments in dynamic panel data GMM

Jens Mehrhoff

Discussion Paper Series 1: Economic Studies No 31/2009 **Editorial Board:** Heinz Herrmann

Thilo Liebig

Karl-Heinz Tödter

Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main, Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Telex within Germany 41227, telex from abroad 414431

Please address all orders in writing to: Deutsche Bundesbank, Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Internet http://www.bundesbank.de

Reproduction permitted only if source is stated.

ISBN 978-3-86558-576-9 (Printversion)

ISBN 978-3-86558-577-6 (Internetversion)

Abstract

The well-known problem of too many instruments in dynamic panel data GMM is dealt with in detail in Roodman (2009, Oxford Bull. Econ. Statist.). The present paper goes one step further by providing a solution to this problem: factorisation of the standard instrument set is shown to be a valid transformation for ensuring consistency of GMM. Monte Carlo simulations show that this new estimation technique outperforms other possible transformations by having a lower bias and RMSE as well as greater robustness of overidentifying restrictions. The researcher's choice of a particular transformation can be replaced by a data-driven statistical decision.

Keywords: Dynamic Panel Data, Generalised Method of Moments,

Instrument Proliferation, Factor Analysis

JEL classification: C13, C15, C23, C81

Non-technical summary

Dynamic panel data (DPD) models have become increasingly popular. They are characterised by two features. The first one is their dynamic structure, i.e. the model equation has at least one lagged dependent variable on the right-hand side. The second one is their panel structure, i.e. the data have both a cross-sectional and a time series dimension. Not only individual effects can be estimated with the aid of panel data – also the problem of aggregation bias can be avoided, where an aggregate regression is said to suffer from aggregation bias when the aggregate regression slope parameter does not correctly reflect the average of the individual slope parameters. However, DPD models are a source for biases themselves. The Least Squares Dummy Variables (LSDV) estimator has a non-vanishing bias for small T and large N, in particular it is downward biased because the lagged endogenous variable correlates negatively with the transformed error term.

The problem of DPD bias was solved with unbiased DPD estimators based on Generalised Method of Moments (GMM) in the 1990s: first with Difference GMM and later with System GMM. The basic idea of these estimators is that lagged levels (Difference GMM) and additionally lagged differences (System GMM) are valid instruments for the lagged endogenous variable, i.e. are uncorrelated with the transformed error term. However, one issue with regard to DPD GMM still remains problematic; the number of instruments grows quadratically with T. GMM becomes inconsistent as the number of instruments becomes too large. This begs the question: "what is the optimal set of instruments?" Currently, there are two techniques in use to reduce the instrument count. One of them is limiting the lag depth, the other one is "collapsing" the instrument set. These transformations are deterministic ones of the instrument matrix. Besides the fact that no widely accepted rule of thumb for the instrument count exists, by choosing one of the aforementioned approaches, the researcher decides which transformation is to be used. Yet, the question this paper addresses is, "can we let the data decide how the transformation matrix should look?" The answer is found here by means of principal components analysis (PCA) of the instrument set and is shown to be "yes, we can." The resulting DPD GMM estimator is characterised by both a lower bias and a lower root mean squared error (RMSE) than the standard techniques.

The results of a Monte Carlo simulation strongly suggest the use of factorised instruments as these produce the lowest bias and RMSE. This generates a set of instruments which reduces the uncertainty in the choice of instruments. Furthermore, there is a clear recommendation to collapse the instrument set prior to factorisation. Preferably, the lag depth is also limited. Most importantly, the bias of standard GMM increases due to instrument proliferation. The simulation further shows that LSDV should be applied only if the time dimension is much larger than 30, while pooled OLS has clearly sub-optimal properties for the estimation of DPD.

Nicht-technische Zusammenfassung

Modelle dynamischer Paneldaten (DPD) erfreuen sich immer größerer Beliebtheit. Sie sind durch zwei Eigenschaften charakterisiert. Die Erste ist ihre dynamische Struktur, d. h. die Modellgleichung hat zumindest eine verzögerte abhängige Variable auf der rechten Seite. Die Zweite ist ihre Panelstruktur, d. h. die Daten haben sowohl eine Querschnitts- als auch eine Zeitreihendimension. Nicht nur können mit Hilfe von Paneldaten individuelle Effekte geschätzt werden – ebenso kann das Problem der Verzerrung bei Verwendung aggregierter Daten vermieden werden, wobei eine Regression mit Aggregatdaten verzerrt ist, wenn die Regressionskoeffizienten auf Basis von aggregierten Daten den Durchschnitt der Regressionskoeffizienten auf Basis von disaggregierten Daten nicht korrekt widerspiegeln. Aber auch Schätzungen von DPD-Modellen können verzerrt sein. Der Kleinste-Quadrate-Schätzer mit Dummy-Variablen (LSDV) weist eine nicht-verschwindende Verzerrung für kleines T und großes N auf, im Speziellen ist er nach unten verzerrt, weil die verzögerte endogene Variable und der transformierte Störterm negativ miteinander korrelieren.

Das Problem dieser DPD-Verzerrung wurde mit unverzerrten DPD-Schätzern basierend auf der Verallgemeinerten Methode der Momente (GMM) in den neunziger Jahren gelöst: zuerst mit Differenzen-GMM und später mit System-GMM. Die Grundidee dieser Schätzer ist dabei, dass verzögerte Niveaus (Differenzen-GMM) bzw. zusätzlich verzögerte Differenzen (System-GMM) als valide Instrumente für die verzögerte endogene Variable fungieren, d. h. unkorreliert mit dem transformierten Störterm sind. Jedoch bleibt ein Sachverhalt bezüglich DPD-GMM weiterhin problematisch; die Anzahl der Instrumente wächst quadratisch mit T. GMM wird inkonsistent, wenn die Anzahl der Instrumente zu groß wird. Dies wirft die Frage auf: "Was ist der optimale Instrumentensatz?" Gegenwärtig sind zwei Techniken in Gebrauch, um die Anzahl der Instrumente zu reduzieren. Diese sind zum einen das Begrenzen der Anzahl der verzögerten Variablen und zum anderen das sogenannte Kollabieren des Instrumentensatzes. Diese Transformationen der Instrumentenmatrix sind deterministischer Natur. Davon abgesehen, dass keine allgemein akzeptierte Faustregel für die Anzahl der Instrumente existiert, wird durch die Wahl eines der vorstehenden Ansätze vom Forscher entschieden, welche Transformation zu verwenden ist. Dieses Papier stellt eine Methode vor, bei der die Daten selbst darüber entscheiden, wie die Transformationsmatrix aussehen soll. Dazu werden im Rahmen einer Hauptkomponentenanalyse (PCA) die verfügbaren Informationen aller infrage kommenden Instrumente in einigen wenigen Variablen zusammengefasst, die dann wiederum bei der Schätzung – anstelle des ursprünglichen Instrumensatzes – als Instrumente verwendet werden. Der resultierende DPD-GMM-Schätzer ist charakterisiert durch eine kleinere Verzerrung und eine kleinere Wurzel aus dem mittleren quadratischen Fehler (RMSE) als die Standardtechniken.

Die Ergebnisse einer Monte Carlo-Simulation liefern starke Evidenz für die Vorteilhaftigkeit der Verwendung von faktorisierten Instrumenten, da diese die geringste Verzerrung und den geringsten RMSE produzieren. Dies generiert einen Instrumentensatz, der die Unsicherheit bei der Wahl der Instrumente reduziert. Darüber hinaus leitet sich eine klare Empfehlung dafür ab, den Instrumentensatz vor der Faktorisierung zu kollabieren. Vorzugsweise wird auch die Anzahl der verzögerten Variablen begrenzt. Am wichtigsten ist jedoch, dass die Verzerrung von Standard-GMM durch das rasante Wachstum der Instrumente steigt. Die Simulation kommt ferner zu dem Ergebnis, dass LSDV nur angewandt werden sollte, wenn die Zeitdimension deutlich größer als 30 ist, während gepooltes OLS eindeutig sub-optimale Eigenschaften aufweist für die Schätzung von DPD.

Contents

1	Introduction	1
2	Factorisation as a solution	2
3	Performance of factorised instruments	4
4	Directions for applied research	11
A	Proof of theorems	12
В	Pseudocode	12
\mathbf{C}	Structure of transformation matrices	13
R	eferences	15

List of Figures

1	Biases from a Monte Carlo simulation	6
2	RMSEs from a Monte Carlo simulation	7
\mathbf{List}	of Tables	
1	Bias, standard error (SE) and RMSE for $\beta=.2$ and $\beta=.8$	5
2	Instrument count J and rejection frequency of valid instruments $$. $$	9
3	Fraction of explained variance ρ and measure of sampling adequacy	10
4	Instrument count I and rejection frequency of weak instruments	11

A Solution to the Problem of Too Many Instruments in Dynamic Panel Data GMM*

1 Introduction

Dynamic panel data (DPD) models have become increasingly popular in the last two decades. Nowadays the availability of micro level data, such as of firms or banks, enables researchers to identify economic relationships at a disaggregate level. Hence, the serious problem of aggregation bias (Lippi and Forni, 1990) can be avoided. However, the solution is not without a drawback: DPD bias. As Nickel (1981) has shown, the Least Squares Dummy Variables (LSDV) estimator has a non-vanishing bias for small T and large N. Anderson and Hsiao (1982) were the first to propose an unbiased DPD estimator with the notable trade-off between lag depth and sample size. It was not until Holtz-Eatkin et al. (1988) that an unbiased DPD estimator was constructed based on Generalised Method of Moments (GMM) (Hansen, 1982). The breakthrough came with Difference GMM by Arellano and Bond (1991), and System GMM by Arellano and Bover (1995) and Blundell and Bond (1998). In the meantime, Kiviet (1995) proposed a corrected LSDV estimator for balanced panels. However, one issue with regard to DPD GMM still remains unresolved; the number of instruments grows quadratically in T and GMM becomes inconsistent as the number of instruments diverges, thus begging the question "what is the optimal set of instruments?"

^{*}Detailed results and descriptions of methodology are available on request from the author. Address for correspondence: Jens Mehrhoff, Statistics Department and Research Centre, Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main, Germany, Tel: +49 69 9566-3417, Fax: +49 69 9566-2941, E-mail: jens.mehrhoff@bundesbank.de, Homepage: www.bundesbank.de. The author would like to thank Joerg Breitung, Volker Mammitzsch, Ulf von Kalckreuth and Sophia Mueller-Spahn for valuable comments. All remaining errors are, of course, the author's sole responsibility.

Roodman (2009) addresses the problem of too many instruments. Increasing the sample size causes the number of instruments to proliferate as DPD GMM generates one instrument for each time period and lag available. Currently, there are two techniques in use to reduce the instrument count. One of them is limiting the lag depth, the other one is "collapsing" the instrument set. The former implies a selection of certain lags to be included in the instrument set, making the instrument count linear in T. The latter embodies a different belief about the orthogonality condition: it no longer needs to be valid for any one time period but still for each lag, again making the instrument count linear in T. A combination of both techniques makes the instrument count invariant to T. These transformations are deterministic ones of the instrument matrix, i.e. the transformation matrix consists of zeroes and ones. Besides the fact that no widely accepted rule of thumb for the instrument count exists, by choosing one of the aforementioned approaches, the researcher decides which transformation is to be used for the data. The point in question is, "can we let the data decide how the transformation matrix should look?" The answer to this question is found by means of factor analysis of the instrument set and is shown to be "yes, we can." The resulting DPD GMM estimator is characterised by both a lower bias and a lower root mean squared error (RMSE) than the standard techniques.

The remainder of the paper is organised as follows. Section 2 introduces the new estimation technique based on factorised instruments. Monte Carlo results for this estimator are presented in Section 3. The final section concludes.

2 Factorisation as a solution

Consider an autoregressive panel model of order one for the endogenous variable $y_{i,t}$, where $\alpha_i = \alpha + \eta_i$ is a fixed effect and $\varepsilon_{i,t}$ is the error term.

$$y_{i,t} = \alpha + \beta y_{i,t-1} + \eta_i + \varepsilon_{i,t} \tag{1}$$

The standard instrument set \mathbf{Z} for the estimation of the autoregressive parameter β of Equation (1) with DPD GMM in first differences,

$$\Delta y_{i,t} = \beta \Delta y_{i,t-1} + \Delta \varepsilon_{i,t},$$

which will be treated here exclusively without loss of generality but for simplicity of exposition, consists of lagged values of the endogenous variable, which are uncorrelated with the first differences of the error term.

$$E(\mathbf{Z}'\boldsymbol{\Delta}\boldsymbol{\varepsilon}) = \mathbf{0} \tag{2}$$

First, the conditions for consistency of the aforementioned techniques, along with a whole class of transformations, to reduce the instrument count are verified in the following theorem. Unlike other authors, who derive the limited or collapsed instrument set from first principles by considering interpretable orthogonality conditions, this paper applies transformation matrices to the standard instrument set which yield the desired results (cf. Appendix C). Proofs for this and the following theorem are to be found in Appendix A.

Theorem 1. Let Equation (2) be valid. Then $E(\mathbf{Z}^{*'}\Delta\varepsilon) = \mathbf{0}$ with $\mathbf{Z}^{*} = \mathbf{Z}\mathbf{F}$ for any deterministic transformation matrix \mathbf{F} .

It follows from Theorem 1 that limiting the lag depth, collapsing the instrument set or both are valid transformations for consistent estimation of the parameter of interest. Moreover, any transformation, no matter if it lacks a sensible interpretation, satisfies the conditions of the theorem as long as it is deterministic.

Second, the aim of this paper is to introduce a new technique rather than to evaluate standards already in use. Hence, the focus here lies on stochastic transformations instead of deterministic ones. In order to solve the problem of instrument proliferation, this paper suggests the application of factor analysis – more precisely for the case in hand – principal components analysis (PCA) to the instrument set. PCA extracts the largest eigenvalues of the estimated covariance matrix of \mathbf{Z} and assembles the corresponding eigenvectors in the matrix of component loadings \mathbf{F}^* , the transformation matrix. In this case, the transformation matrix is stochastic and Theorem 1 is no longer applicable. However, Theorem 2 provides a solution.

Theorem 2. Let $y_{i,t-1-\ell}$, $\ell = 1, 2, ...$ (the elements of the **Z** matrix) and $\Delta \varepsilon_{i,t}$ be independent random variables for all i and t. Then $E(\mathbf{Z}^{**'}\Delta \varepsilon) = \mathbf{0}$ with $\mathbf{Z}^{**} = \mathbf{Z}\mathbf{F}^{*}$, where \mathbf{F}^{*} is the matrix of component loadings from PCA of $\widehat{\text{Var}}(\mathbf{Z})$.

Theorem 2 is both more general and more specific than Theorem 1. The fact that it also holds true for deterministic $\mathbf{F}^* = \mathbf{F}$ makes it more general. It is more specific in the sense that it requires independence of $y_{i,t-1-\ell}$ and $\Delta \varepsilon_{i,t}$ which is a stronger property than uncorrelatedness. This assumption is not too strong if the error term is thought of as being an exogenous shock.

3 Performance of factorised instruments

Judson and Owen (1999) provide Monte Carlo evidence that GMM is superior to other estimation techniques when it comes to DPD. Among others, their findings are: OLS produces biased estimates even for large T, the bias of LSDV decreases with T but may still be up to 20% of the true value even when T=30, and also that the LSDV bias increases with the true value of the autoregressive parameter. Additionally, OLS is upward biased while LSDV is downward biased. Windmeijer (2005) adds to this list that GMM becomes more efficient when the lag depth is limited, and thus fewer instruments are employed in the estimation.

Table 1 and Figures 1 and 2 present biases and RMSEs from a Monte Carlo simulation of a one-step estimation of Equation (1) with parameter values of β in the range from close to zero to close to one. $\varepsilon_{i,t}$ is assumed to be standard normal, as is α_i . N is fixed at 100, T is 10, 20 and 30, respectively (large N, small T). The pre-sample period length is 30. The standard instrument set is either taken as it is, limited, collapsed or both, and additionally PCA has been applied to all four variants. The experiment is repeated 1,000 times (cf. Appendix B).

The results confirm the findings of Judson and Owen (1999) and Windmeijer (2005). In addition, factorised instruments outperform all other techniques by having both a lower bias and RMSE, however, there are a few exceptions when T=10. In general, factorisation of the limited and collapsed instrument set results in the lowest bias, while factorisation of the collapsed but unlimited instrument set yields the lowest RMSE. Biases are zero to the second decimal place or in relative terms less than 1%, RMSEs are zero to the first decimal place.

Table 1: Bias, standard error (SE) and RMSE for $\beta = .2$ and $\beta = .8$

Table 1: Bias, standard error (SE) and RMSE for $\beta = .2$ and $\beta = .8$					
		T = 10	T = 20	T = 30	
Method	Statistic	$\beta = .2 \ \beta = .8$	$\beta = .2 \ \beta = .8$	$\beta = .2 \ \beta = .8$	
Least Squares					
OLS	Bias	+.477 +.180	+.477 +.180	+.477 +.180	
	SE	.001 .000	.001 .000	.001 .000	
	RMSE	.478 .180	.478 .180	.478 .180	
LSDV	Bias	136243	064 111	042 070	
	SE	.001 .001	.001 .001	.001 .000	
	RMSE	.140 .245	.068 .113	.045 $.071$	
Standard GMM					
Untransformed	Bias	080539	146624	199681	
	SE	.002 .004	.001 .002	.001 .001	
	RMSE	.101 $.555$.151 .628	.201 .683	
Limited (Ltd.)	Bias	061506	114580	157633	
	SE	.002 $.005$.001 .002	.001 .002	
	RMSE	.089 $.528$.121 .585	.160 $.635$	
Collapsed (Col.)	Bias	014373	017296	017 257	
	SE	.002 .007	.001 .004	.001 .003	
	RMSE	.070 $.435$.047 $.325$.039 $.275$	
Ltd. & Col.	Bias	001172	007159	007 137	
	SE	.002 .008	.001 .004	.001 .003	
	RMSE	.071 $.297$.044 $.205$.036 .166	
<u>Factorised GMM</u>					
Untransformed	Bias	329709	464827	502857	
	SE	.014 .018	.014 .015	.011 .013	
	RMSE	.555 $.916$.633 .946	.607 $.949$	
Limited (Ltd.)	Bias	178587	308664	400 762	
	SE	.009 .018	.011 .014	.010 .013	
	RMSE	.324 $.825$.456 $.799$.503 .863	
Collapsed (Col.)	Bias	+.005015	+.003005	+.004 .000	
	SE	.002 .006	.001 .002	.001 .002	
	RMSE	.059 $.179$.035 $.075$.029 .048	
Ltd. & Col.	Bias	+.003 +.005	+.002002	+.003 .000	
	SE	.002 .007	.001 .003	.001 .002	
	RMSE	.067 .217	.037 .084	.031 .055	

Note: For the sake of brevity, results for values of the autoregressive parameter other than $\beta = .2$ and $\beta = .8$ are not displayed here. The results obtained for these values are similar to those presented above.

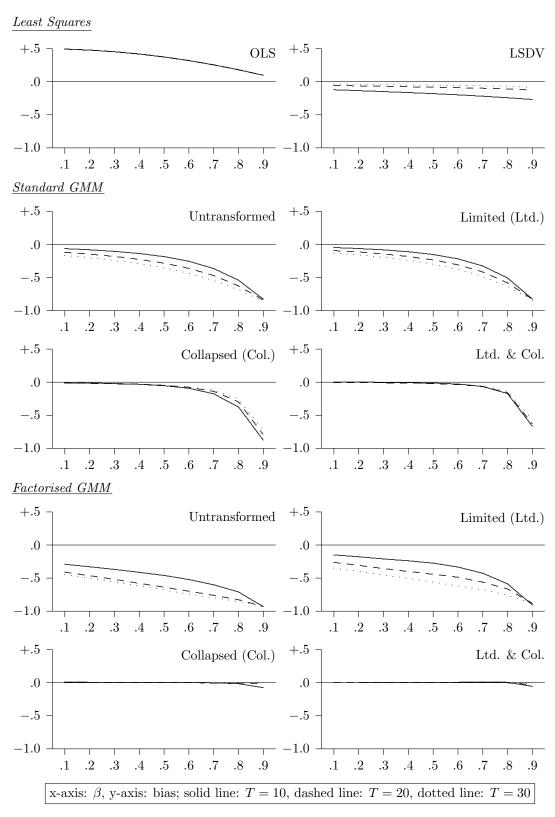


Figure 1: Biases from a Monte Carlo simulation

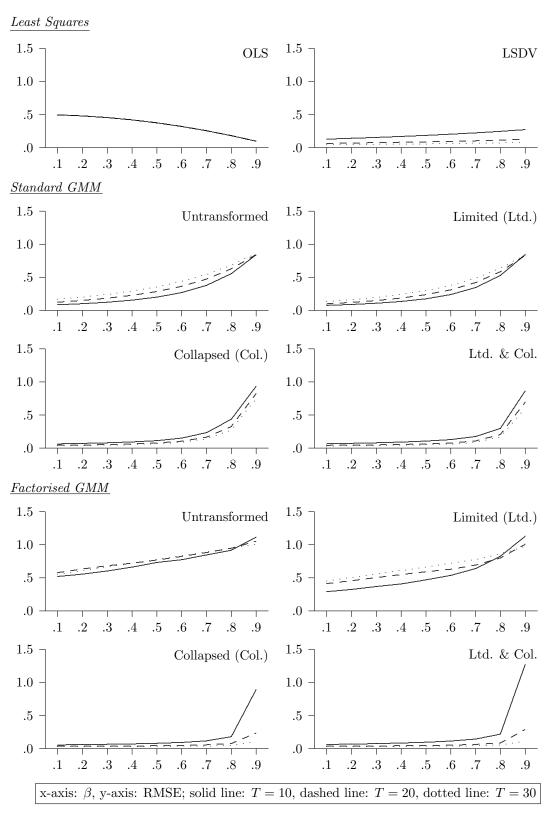


Figure 2: RMSEs from a Monte Carlo simulation

The advantage of factorised instruments over standard ones is the condensation of the informational content of the instrument set into a much lower number of instruments employed in the estimation thus lowering the risk of overfitting endogenous variables but retaining almost all information. The next best approach is standard GMM with the instrument set being both limited and collapsed. Acceptable results can also be derived from a collapsed but unlimited instrument set in standard GMM. Limiting the lag depth on the one hand is a good idea as even if the autoregressive parameter is high, serial correlation will be low after a few periods and deeper lags are weak instruments, adding almost no new information for estimation. Collapsing the instrument set on the other hand also condenses the information in the instrument set into a lower number of instruments. The techniques most frequently used in applied DPD research, the untransformed instrument set and the limited one in standard GMM, are the worst choices, that is apart from the factorised variants of them. Both techniques are significantly downward biased (which becomes even worse, the higher T is), although the estimate still has the correct sign. Performance of their factorised variants is unacceptable; not even the correct sign can be expected.

Explanations for the failure of the standard techniques can be found with recourse to the Sargan (1958) test of overidentifying restrictions (cf. Table 2). The failure of the factorised variants can be traced back to PCA and the Kaiser-Meyer-Olkin (Kaiser, 1970) measure of sampling adequacy (MSA) (cf. Table 3). Through testing for weak instruments according to Staiger and Stock (1997), more evidence is found why both the standard techniques and the factorised variants do not perform particularly well (cf. Table 4).

Table 2 shows the number of instruments employed in the estimation for each of the methods used and the proportions for which the validity of the overidentifying restrictions have been rejected at the nominal 5% significance level. It should be borne in mind that the power of the test is not weakened by many instruments. For limited instrument sets, the number of lags employed is set to be half of the available lags; for factorised instrument sets, the number of retained components has been fixed. Both choices are to a certain extent arbitrary.

Table 2: Instrument count J and rejection frequency of valid instruments

				1				
	T =	10		T=2	20		T=3	80
Method	$J \beta = .2$	$\beta = .8$	\overline{J}	$\beta = .2$	$\beta = .8$	\overline{J}	$\beta = .2$	$\beta = .8$
Standard GMM								
Untransformed	36 .103	.203	171	.176	.400	406	.318	.606
Limited (Ltd.)	26 .096	.182	126	.140	.365	301	.228	.569
Collapsed (Col.)	8 .091	.166	18	.077	.169	28	.093	.185
Ltd. & Col.	4 .047	.097	9	.069	.096	14	.074	.099
Factorised GMM								
Untransformed	3 .080	.076	4	.064	.057	5	.070	.065
Limited (Ltd.)	3 .103	.114	4	.063	.067	5	.075	.072
Collapsed (Col.)	2 .000	.000	3	.000	.000	4	.000	.000
Ltd. & Col.	2 .000	.001	3	.000	.000	4	.000	.000

Standard GMM with the untransformed or limited instrument set generates invalid overidentifying restrictions in an unacceptably high number of cases. This is due to the impossibility of fulfilling all restrictions simultaneously owing to the large number of instruments and the resulting overfitting of endogenous variables. Probabilities of rejection increase with β as well as with T. As it is known a priori that the null hypothesis of valid instruments or overidentifying restrictions is true in all cases, severe size distortions of the test become visible. While the test of the factorised variants of the collapsed (and limited) instrument set is undersized, rejecting the null hypothesis in virtually none of the cases, all tests of other instrument sets are oversized, some rather heavily.

Table 3 reports the explained variance and MSA from PCA. The explained variance states the proportion of the instrument set's variance that can be explained by the retained components. MSA is a statistical criterion to judge the adequacy of the covariance matrix to be factorised; the closer it gets to one, the better. A value in the .90s is regarded as being "marvellous" in the literature (Kaiser and Rice, 1974).

The explained variance from PCA of the collapsed (and limited) instrument set is in the high .70s, low .80s for $\beta = .2$ and in the high .90s for $\beta = .8$. Almost all of the variation of the standard instrument set can be explained by much fewer components. Irrespective of β , PCAs of the untransformed or limited instrument set do not score appreciable values. This is the main reason why these procedures fail to result in plausible estimates (cf. Table 1). Although high MSAs

Table 3: Fraction of explained variance ρ and measure of sampling adequacy

						1 0	
		T =	10	T =	= 20	T =	= 30
Method	Statistic	$\beta = .2$	$\beta = .8$	$\beta = .2$	$\beta = .8$	$\beta = .2$	$\beta = .8$
Untransformed	ρ	.398	.562	.247	.363	.200	.297
	MSA	.424	.988	.250	.973	.149	.954
Limited (Ltd.)	ho	.350	.457	.197	.279	.154	.224
	MSA	.283	.979	.190	.962	.127	.940
Collapsed (Col.)	ho	.703	.892	.673	.911	.669	.923
	MSA	.936	.999	.973	1.000	.983	1.000
Ltd. & Col.	ho	.824	.961	.764	.963	.748	.967
	MSA	.939	.999	.979	1.000	.988	1.000

can be achieved for $\beta = .8$, the explained variance remains low. MSAs for the first two procedures are close to one in all instances. The collapsed instrument set is much more suitable for PCA as each instrument is non-zero for all applicable observations, unlike untransformed instruments which are non-zero for just a single observation.

Table 4 gives the proportions for which the weakness of the instruments has been rejected, along with the number of instruments employed in the estimation for each of the methods used. As a rule of thumb, the instrument set is deemed to be weak if the F-statistic from the first stage regression in two stages least squares (TSLS) is less than ten. This is an approximate test at the 5% significance level that the TSLS bias is at most 10% of the OLS bias (Stock and Yogo, 2005).

Irrespective of the instrument set used, the instruments get weaker, the higher β becomes. This is because the more the process approaches a random walk, the lower is the correlation between levels and differences. Both in standard and factorised GMM, as T rises, the untransformed or limited instrument set becomes weaker, while the collapsed (and limited) instrument set gets stronger. Partial R^2 s of deeper, uncollapsed instruments are virtually zero; thus, these add almost no new information for estimation. Moreover, it seems as if many weak instruments cause the entire instrument set to be weak even though it contains a few strong ones. Again, the factorised variants of the collapsed (and limited) instrument set perform best, while the factorised variants of the untransformed or limited instrument set are worse than their standard GMM counterparts. Factorised instruments are the only ones which are strong even for relatively high β and any T.

Table 4: Instrument count J and rejection frequency of weak instruments

		T = 1	0		T=2	0		T=3	0
Method	J β	$\beta = .2$	$\beta = .8$	\overline{J}	$\beta = .2$	$\beta = .8$	\overline{J}	$\beta = .2$	$\beta = .8$
Standard GMM									
Untransformed	36	.316	.000	171	.000	.000	406	.000	.000
Limited (Ltd.)	26	.947	.000	126	.000	.000	301	.000	.000
Collapsed (Col.)	8	1.000	.000	18	1.000	.000	28	1.000	.000
Ltd. & Col.	4	1.000	.002	9	1.000	.000	14	1.000	.000
<u>Factorised GMM</u>									
Untransformed	3	.000	.000	4	.000	.000	5	.000	.000
Limited (Ltd.)	3	.003	.000	4	.000	.000	5	.000	.000
Collapsed (Col.)	2	1.000	.039	3	1.000	.199	4	1.000	.670
Ltd. & Col.	2	1.000	.068	3	1.000	.545	4	1.000	.966

4 Directions for applied research

The Monte Carlo results strongly suggest the use of factorised instruments as these produce the lowest bias and RMSE. This generates an ultimate set of instruments and reduces the uncertainty researchers face in their choice of instruments. Furthermore, there is a clear recommendation to collapse the instrument set prior to factorisation or, if factorisation is not to be used at all, then at the very least the instrument set should be collapsed. To reiterate, this implies a deterministic transformation of the standard instrument set, and the factorised variant of this instrument set is the method of choice. Preferably, the lag depth is also limited. The lag limit should be chosen based on a priori information on the value of the autoregressive parameter, as serial correlation decreases exponentially. Most importantly, standard GMM suffers from instrument proliferation. The findings in this paper indicate that results of numerous applications of GMM in the literature may benefit from factorised instruments. LSDV should be applied only if the time dimension is much larger than 30, while pooled OLS should not be used at all in the estimation of DPD.

In applied research, the number of retained components from PCA can be derived from factor analytic criteria, such as MSA, and should be tested for their validity in the GMM framework. The methodology outlined here can be applied to System GMM or exogenous variables in a completely analogous fashion. It is reasonable to make use of the correlation between all instruments to lower the instrument count.

A Proof of theorems

Proof of Theorem 1. Using the definition of \mathbf{Z}^* in Theorem 1 and Equation (2), the proposition follows directly from the linearity property of the expectation operator: $\mathrm{E}(\mathbf{Z}^{*'}\Delta\varepsilon) = \mathrm{E}(\mathbf{F}'\mathbf{Z}'\Delta\varepsilon) = \mathbf{F}'\mathrm{E}(\mathbf{Z}'\Delta\varepsilon) = \mathbf{0}.$

Proof of Theorem 2. Per definitionem of Theorem 2, the corresponding elements of \mathbf{Z} and $\Delta \varepsilon$, meaning those which form the cross products in $\mathbf{Z}'\Delta \varepsilon$, are independent random variables, and thus Borel. For any pair $\phi(\cdot)$ and $\psi(\cdot)$ of Borel functions, the corresponding elements of $\phi(\mathbf{Z})$ and $\psi(\Delta \varepsilon)$ are also independent.

 $\widehat{\text{Var}}(\mathbf{Z})$ is a positive semi-definite symmetric matrix meaning that all eigenvalues are real and non-negative. It is well-established that the sum and product of two real-valued measurable functions are measurable. That eigenvectors can be found in a Borel measurable fashion was shown by Azoff (1974, Corollary 4).

Hence, the corresponding elements of $\mathbf{Z}^{**} = \mathbf{Z}\Lambda(\mathbf{Z}) = \phi(\mathbf{Z})$, with $\mathbf{F}^* = \Lambda(\mathbf{Z})$ being the matrix of component loadings, and $\Delta \varepsilon = \psi(\Delta \varepsilon)$ are independent random variables, too. Moreover, given quadratic integrability of the elements of \mathbf{Z}^{**} and $\Delta \varepsilon$, the corresponding ones are uncorrelated. The proposition follows from the fact that this can be the case if and only if $\mathbf{E}(\mathbf{Z}^{**}'\Delta \varepsilon) = \mathbf{0}$ as $\mathbf{E}(\Delta \varepsilon) = \mathbf{0}$.

B Pseudocode

The pseudocode for the simulation reads as follows.

Pseudocode	Input	Output
Simulate data	$\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\varepsilon}, \mathbf{y_0}, N, T$	y
Generate instrument set	$\mathbf{y}_{-1-oldsymbol{\ell}}$	$\mathbf{Z}~(\mathbf{Z}^*)$
Factorise instruments	$\mathbf{Z}~(\mathbf{Z}^*)$	\mathbf{Z}^{**}
Estimate parameter	$oldsymbol{\Delta}\mathbf{y},oldsymbol{\Delta}\mathbf{y_{-1}},\mathbf{Z}^{**}$	\hat{eta}

C Structure of transformation matrices

For the sake of exposition, let T = 6 and i = 1, 2, ..., N. Note that the first observation is dropped due to differencing.

Untransformed

The standard instrument set consists of lagged values of the endogenous variable; in particular, one instrument is generated for each time period and lag available. The instrument count is J = (T-2)(T-1)/2 = 10.

Limited (L)

Limiting the maximum lag depth of $y_{i,t-1}$ to $\tau = 2$, for example, gives as transformation matrix a block matrix of identity matrices up to dimension τ (for each time period, indicated by solid lines) separated by rows of zeroes (for excluded lags, indicated by dashed lines). Using this technique reduces the instrument count to $J^L = J - (T - 2 - \tau)(T - 1 - \tau)/2 = 7$.

Collapsed (C)

The transformation matrix for collapsing the instrument set is made up of identity matrices of increasing dimension stacked one upon the other (indicated by solid lines) with blocks of zero matrices to the right (indicated by dashed lines). By collapsing the instrument count is cut to $J^C = T - 2 = 4$.

$$\mathbf{Z_{i}^{C}} = \mathbf{Z_{i}}\mathbf{F^{C}} = \mathbf{Z_{i}}\begin{bmatrix} \frac{1 & 0 & 0 & 0}{1 & 0 & 0 & 0} \\ \frac{0 & 1 & 0 & 0}{1 & 0 & 0 & 0} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ y_{i,1} & 0 & 0 & 0 \\ y_{i,2} & y_{i,1} & 0 & 0 \\ y_{i,3} & y_{i,2} & y_{i,1} & 0 \\ y_{i,4} & y_{i,3} & y_{i,2} & y_{i,1} \end{bmatrix}$$

Limited & Collapsed (LC)

When both techniques are combined, i.e. rows of zeroes from $\mathbf{F}^{\mathbf{L}}$ and stacked identity matrices (now again only up to dimension τ) from $\mathbf{F}^{\mathbf{C}}$, the instrument count becomes $J^{LC} = \tau = 2$.

$$\mathbf{Z_{i}^{LC}} = \mathbf{Z_{i}}\mathbf{F^{LC}} = \mathbf{Z_{i}}\begin{bmatrix} \frac{1 & 0}{1 & 0} \\ \frac{0 & 1}{1 & 0} \\ \frac{0 & 1}{0 & 0} \\ \frac{1}{1 & 0} \\ \frac{0}{1 & 0} \\ \frac{0}{1 & 0} \\ \frac{0}{0 & 1} \\ \frac{0}{0 & 0} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ y_{i,1} & 0 \\ y_{i,2} & y_{i,1} \\ y_{i,3} & y_{i,2} \\ y_{i,4} & y_{i,3} \end{bmatrix}$$

References

- Anderson, T.W., and Hsiao, C. (1982), "Formulation and estimation of dynamic models using panel data," *Journal of Econometrics*, 18, 47-82.
- Arellano, M., and Bond, S. (1991), "Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations," *Review of Economic Studies*, 58, 277-297.
- Arellano, M., and Bover, O. (1995), "Another look at the instrumental variable estimation of error-components models," *Journal of Econometrics*, 68, 29-51.
- Azoff, E.A. (1974), "Borel measurability in linear algebra," *Proceedings of the American Mathematical Society*, 42, 346-350.
- Blundell, R., and Bond, S. (1998), "Initial conditions and moment restrictions in dynamic panel data models," *Journal of Econometrics*, 87, 115-143.
- Hansen, L.P. (1982), "Large sample properties of generalized method of moments estimators," *Econometrica*, 50, 1029-1054.
- Holtz-Eatkin, D., Newey, W., and Rosen, H.S. (1988), "Estimating vector autoregressions with panel data," *Econometrica*, 56, 1371-1395.
- Judson, R.A., and Owen, A.L. (1999), "Estimating dynamic panel data models: a guide for macroeconomists," *Economics Letters*, 65, 9-15.
- Kaiser, H.F. (1970), "A second generation little jiffy," Psychometrika, 35, 401-415.
- Kaiser, H.F., and Rice, J. (1974), "Little jiffy, mark iv," Educational and Psychological Measurement, 34, 111-117.
- Kiviet, J.F. (1995), "On bias, inconsistency, and efficiency of various estimators in dynamic panel data models," *Journal of Econometrics*, 68, 53-78.
- Lippi, M., and Forni, M. (1990), "On the dynamic specification of aggregated models," in: Barker, T., and Pesaran, M.H. (eds.), *Disaggregation in econometric modelling*, London: Routledge.

- Nickel, S. (1981), "Biases in dynamic models with fixed effects," *Econometrica*, 49, 1417-1426.
- Roodman, D. (2009), "A note on the theme of too many instruments," Oxford Bulletin of Economics and Statistics, 71, 135-158.
- Sargan, J.D. (1958), "The estimation of economic relationships using instrumental variables," *Econometrica*, 26, 393-415.
- Staiger, D., and Stock, J.H. (1997), "Instrumental variables regression with weak instruments," *Econometrica*, 65, 557-586.
- Stock, J.H., and Yogo, M. (2005), "Testing for weak instruments in linear IV regression," in: Andrews, D.W.K., and Stock, J.H. (eds.), *Identification and inference for econometric models: essays in honor of Thomas Rothenberg*, New York: Cambridge University Press.
- Windmeijer, F. (2005), "A finite sample correction for the variance of linear efficient two-step GMM estimators," *Journal of Econometrics*, 126, 25-51.

The following Discussion Papers have been published since 2008:

Series 1: Economic Studies

01	2008	Can capacity constraints explain asymmetries of the business cycle?	Malte Knüppel
02	2008	Communication, decision-making and the optimal degree of transparency of monetary policy committees	Anke Weber
03	2008	The impact of thin-capitalization rules on multinationals' financing and investment decisions	Buettner, Overesch Schreiber, Wamser
04	2008	Comparing the DSGE model with the factor model: an out-of-sample forecasting experiment	Mu-Chun Wang
05	2008	Financial markets and the current account – emerging Europe versus emerging Asia	Sabine Herrmann Adalbert Winkler
06	2008	The German sub-national government bond market: evolution, yields and liquidity	Alexander Schulz Guntram B. Wolff
07	2008	Integration of financial markets and national price levels: the role of exchange rate volatility	Mathias Hoffmann Peter Tillmann
08	2008	Business cycle evidence on firm entry	Vivien Lewis
09	2008	Panel estimation of state dependent adjustment when the target is unobserved	Ulf von Kalckreuth
10	2008	Nonlinear oil price dynamics – a tale of heterogeneous speculators?	Stefan Reitz Ulf Slopek
11	2008	Financing constraints, firm level adjustment of capital and aggregate implications	Ulf von Kalckreuth

12	2008	Sovereign bond market integration: the euro, trading platforms and globalization	Alexander Schulz Guntram B. Wolff
13	2008	Great moderation at the firm level? Unconditional versus conditional output volatility	Claudia M. Buch Jörg Döpke Kerstin Stahn
14	2008	How informative are macroeconomic risk forecasts? An examination of the Bank of England's inflation forecasts	Malte Knüppel Guido Schultefrankenfeld
15	2008	Foreign (in)direct investment and corporate taxation	Georg Wamser
16	2008	The global dimension of inflation – evidence from factor-augmented Phillips curves	Sandra Eickmeier Katharina Moll
17	2008	Global business cycles: convergence or decoupling?	M. Ayhan Kose Christopher Otrok, Ewar Prasad
18	2008	Restrictive immigration policy in Germany: pains and gains foregone?	Gabriel Felbermayr Wido Geis Wilhelm Kohler
19	2008	International portfolios, capital accumulation and foreign assets dynamics	Nicolas Coeurdacier Robert Kollmann Philippe Martin
20	2008	Financial globalization and monetary policy	Michael B. Devereux Alan Sutherland
21	2008	Banking globalization, monetary transmission and the lending channel	Nicola Cetorelli Linda S. Goldberg
22	2008	Financial exchange rates and international currency exposures	Philip R. Lane Jay C. Shambaugh

23	2008	Financial integration, specialization and systemic risk	F. Fecht, H. P. Grüner P. Hartmann
24	2008	Sectoral differences in wage freezes and wage cuts: evidence from a new firm survey	Daniel Radowski Holger Bonin
25	2008	Liquidity and the dynamic pattern of price adjustment: a global view	Ansgar Belke Walter Orth, Ralph Setzer
26	2008	Employment protection and temporary work agencies	Florian Baumann Mario Mechtel, Nikolai Stähler
27	2008	International financial markets' influence on the welfare performance of alternative exchange rate regimes	Mathias Hoffmann
28	2008	Does regional redistribution spur growth?	M. Koetter, M. Wedow
29	2008	International financial competitiveness and incentives to foreign direct investment	Axel Jochem
30	2008	The price of liquidity: bank characteristics and market conditions	Falko Fecht Kjell G. Nyborg, Jörg Rocholl
01	2009	Spillover effects of minimum wages in a two-sector search model	Christoph Moser Nikolai Stähler
02	2009	Who is afraid of political risk? Multinational firms and their choice of capital structure	Iris Kesternich Monika Schnitzer
03	2009	Pooling versus model selection for nowcasting with many predictors: an application to German GDP	Vladimir Kuzin Massimiliano Marcellino Christian Schumacher

04	2009	Fiscal sustainability and policy implications for the euro area	Balassone, Cunha, Langenus Manzke, Pavot, Prammer Tommasino
05	2009	Testing for structural breaks in dynamic factor models	Jörg Breitung Sandra Eickmeier
06	2009	Price convergence in the EMU? Evidence from micro data	Christoph Fischer
07	2009	MIDAS versus mixed-frequency VAR: nowcasting GDP in the euro area	V. Kuzin, M. Marcellino C. Schumacher
08	2009	Time-dependent pricing and New Keynesian Phillips curve	Fang Yao
09	2009	Knowledge sourcing: legitimacy deficits for MNC subsidiaries?	Tobias Schmidt Wolfgang Sofka
10	2009	Factor forecasting using international targeted predictors: the case of German GDP	Christian Schumacher
11	2009	Forecasting national activity using lots of international predictors: an application to New Zealand	Sandra Eickmeier Tim Ng
12	2009	Opting out of the great inflation: German monetary policy after the breakdown of Bretton Woods	Andreas Beyer, Vitor Gaspar Christina Gerberding Otmar Issing
13	2009	Financial intermediation and the role of price discrimination in a two-tier market	Stefan Reitz Markus A. Schmidt, Mark P. Taylor
14	2009	Changes in import pricing behaviour: the case of Germany	Kerstin Stahn

15	2009	Firm-specific productivity risk over the business cycle: facts and aggregate implications	Ruediger Bachmann Christian Bayer
16	2009	The effects of knowledge management on innovative success – an empirical analysis of German firms	Uwe Cantner Kristin Joel Tobias Schmidt
17	2009	The cross-section of firms over the business cycle: new facts and a DSGE exploration	Ruediger Bachmann Christian Bayer
18	2009	Money and monetary policy transmission in the euro area: evidence from FAVAR-and VAR approaches	Barno Blaes
19	2009	Does lowering dividend tax rates increase dividends repatriated? Evidence of intra-firm cross-border dividend repatriation policies by German multinational enterprises	Christian Bellak Markus Leibrecht Michael Wild
20	2009	Export-supporting FDI	Sebastian Krautheim
21	2009	Transmission of nominal exchange rate changes to export prices and trade flows and implications for exchange rate policy	Mathias Hoffmann Oliver Holtemöller
22	2009	Do we really know that flexible exchange rates facilitate current account adjustment? Some new empirical evidence for CEE countries	Sabine Herrmann
23	2009	More or less aggressive? Robust monetary policy in a New Keynesian model with financial distress	Rafael Gerke Felix Hammermann Vivien Lewis
24	2009	The debt brake: business cycle and welfare consequences of Germany's new fiscal policy rule	Eric Mayer Nikolai Stähler

25	2009	Price discovery on traded inflation expectations: Does the financial crisis matter?	Alexander Schulz Jelena Stapf
26	2009	Supply-side effects of strong energy price hikes in German industry and transportation	Thomas A. Knetsch Alexander Molzahn
27	2009	Coin migration within the euro area	Franz Seitz, Dietrich Stoyan Karl-Heinz Tödter
28	2009	Efficient estimation of forecast uncertainty based on recent forecast errors	Malte Knüppel
29	2009	Financial constraints and the margins of FDI	C. M. Buch, I. Kesternich A. Lipponer, M. Schnitzer
30	2009	Unemployment insurance and the business cycle: Prolong benefit entitlements in bad times?	Stéphane Moyen Nikolai Stähler
31	2009	A solution to the problem of too many instruments in dynamic panel data GMM	Jens Mehrhoff

Series 2: Banking and Financial Studies

01	2008	Analyzing the interest rate risk of banks using time series of accounting-based data: evidence from Germany	O. Entrop, C. Memmel M. Wilkens, A. Zeisler
02	2008	Bank mergers and the dynamics of deposit interest rates	Ben R. Craig Valeriya Dinger
03	2008	Monetary policy and bank distress: an integrated micro-macro approach	F. de Graeve T. Kick, M. Koetter
04	2008	Estimating asset correlations from stock prices or default rates – which method is superior?	K. Düllmann J. Küll, M. Kunisch
05	2008	Rollover risk in commercial paper markets and firms' debt maturity choice	Felix Thierfelder
06	2008	The success of bank mergers revisited – an assessment based on a matching strategy	Andreas Behr Frank Heid
07	2008	Which interest rate scenario is the worst one for a bank? Evidence from a tracking bank approach for German savings and cooperative banks	Christoph Memmel
08	2008	Market conditions, default risk and credit spreads	Dragon Yongjun Tang Hong Yan
09	2008	The pricing of correlated default risk: evidence from the credit derivatives market	Nikola Tarashev Haibin Zhu
10	2008	Determinants of European banks' engagement in loan securitization	Christina E. Bannier Dennis N. Hänsel
11	2008	Interaction of market and credit risk: an analysis of inter-risk correlation and risk aggregation	Klaus Böcker Martin Hillebrand

12	2008	A value at risk analysis of credit default swaps	B. Raunig, M. Scheicher
13	2008	Systemic bank risk in Brazil: an assessment of correlated market, credit, sovereign and interbank risk in an environment with stochastic volatilities and correlations	Theodore M. Barnhill, Jr. Marcos Rietti Souto
14	2008	Regulatory capital for market and credit risk interaction: is current regulation always conservative?	T. Breuer, M. Jandačka K. Rheinberger, M. Summer
15	2008	The implications of latent technology regimes for competition and efficiency in banking	Michael Koetter Tigran Poghosyan
16	2008	The impact of downward rating momentum on credit portfolio risk	André Güttler Peter Raupach
17	2008	Stress testing of real credit portfolios	F. Mager, C. Schmieder
18	2008	Real estate markets and bank distress	M. Koetter, T. Poghosyan
19	2008	Stochastic frontier analysis by means of maximum likelihood and the method of moments	Andreas Behr Sebastian Tente
20	2008	Sturm und Drang in money market funds: when money market funds cease to be narrow	Stehpan Jank Michael Wedow
01	2009	Dominating estimators for the global minimum variance portfolio	Gabriel Frahm Christoph Memmel
02	2009	Stress testing German banks in a downturn in the automobile industry	Klaus Düllmann Martin Erdelmeier
03	2009	The effects of privatization and consolidation on bank productivity: comparative evidence from Italy and Germany	E. Fiorentino A. De Vincenzo, F. Heid A. Karmann, M. Koetter

04	2009	Shocks at large banks and banking sector distress: the Banking Granular Residual	Sven Blank, Claudia M. Buch Katja Neugebauer
05	2009	Why do savings banks transform sight deposits into illiquid assets less intensively than the regulation allows?	Dorothee Holl Andrea Schertler
06	2009	Does banks' size distort market prices? Evidence for too-big-to-fail in the CDS market	Manja Völz Michael Wedow
07	2009	Time dynamic and hierarchical dependence modelling of an aggregated portfolio of trading books – a multivariate nonparametric approach	Sandra Gaisser Christoph Memmel Rafael Schmidt Carsten Wehn
08	2009	Financial markets' appetite for risk – and the challenge of assessing its evolution by risk appetite indicators	Birgit Uhlenbrock
09	2009	Income diversification in the German banking industry	Ramona Busch Thomas Kick
10	2009	The dark and the bright side of liquidity risks: evidence from open-end real estate funds in Germany	Falko Fecht Michael Wedow
11	2009	Determinants for using visible reserves in German banks – an empirical study	Bornemann, Homölle Hubensack, Kick, Pfingsten
12	2009	Margins of international banking: Is there a productivity pecking order in banking, too?	Claudia M. Buch Cathérine Tahmee Koch Michael Koetter
13	2009	Systematic risk of CDOs and CDO arbitrage	Alfred Hamerle, Thilo Liebig Hans-Jochen Schropp

14 2009 The dependency of the banks' assets and Christoph Memmel liabilities: evidence from Germany Andrea Schertler

Visiting researcher at the Deutsche Bundesbank

The Deutsche Bundesbank in Frankfurt is looking for a visiting researcher. Among others under certain conditions visiting researchers have access to a wide range of data in the Bundesbank. They include micro data on firms and banks not available in the public. Visitors should prepare a research project during their stay at the Bundesbank. Candidates must hold a PhD and be engaged in the field of either macroeconomics and monetary economics, financial markets or international economics. Proposed research projects should be from these fields. The visiting term will be from 3 to 6 months. Salary is commensurate with experience.

Applicants are requested to send a CV, copies of recent papers, letters of reference and a proposal for a research project to:

Deutsche Bundesbank Personalabteilung Wilhelm-Epstein-Str. 14

60431 Frankfurt GERMANY