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Coin migration within the euro area

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Abstract:

This paper analyses how many euro coins outflow from Germany and which composition of coins is to be expected in the long run. To this end, a simple mathematical model is formulated and calibrated for €1 coins. The introduction of the euro coins in 2002 presented a unique opportunity to analyse the cross-border migration and the mixing process of coins in different euro-area countries. Based on research by Stoyan and depending on growth assumptions, the annual outflow of German €1 coins is calculated to lie somewhere between 4% and 5%. In the long run, the ratio of German €1 coins in Germany is likely to converge to around 50%.

Keywords: C61, E41

JEL-Classification: euro coins, coin volumes, mixing process

Non technical summary

Since 1 January 2002, euro banknotes and coins are legal tender in the euro area. For euro coins – in contrast to banknotes – it is relatively easy to distinguish the coins issued by the individual euro area countries, owing to their unique national side. The introduction of euro coins offered the unique opportunity to analyse the cross – border migration of coins.

The present paper analyses how many euro coins outflow from Germany and which composition of coins is to be expected in the long run. The flow of coins depends on the intensity of foreign travel and has consequences for the countries' seigniorage.

The paper exemplifies the mixing process with €1 coins. For those coins, consistent data from the online survey projects "Eurodiff" and "Euromobil" are available. Accordingly, since their introduction until the end of 2008, the share of German €1 coins in Germany dropped to about 75%. We explain the flow of coins between Germany and the other euro area countries by a simple dynamic model. The parameters of that model, outflow rates and inflow rates, are calibrated based on actual empirical data. The results depend on whether the total stock of coins is constant or growing over time. Assuming that the total stock of €1 coins is constant, the model predicts that the ratio of German coins in Germany to the coins issued by Germany would drop to merely 22% in the long run. Empirically, however, since their introduction, the stock of euro coins was growing every year, which slowed down the process of coin mixing. Extrapolating the growth rates observed in the past, the model predicts a decline of the share of German coins in Germany to 53%. Using an alternative model based on a Markov approach to explain the coin mixing yields the same results.

Nicht technische Zusammenfassung

Seit dem 1. Januar 2002 sind Euro–Banknoten und –Münzen gesetzliche Zahlungsmittel im Euro–Währungsraum. Bei den Euro–Münzen lässt sich – im Unterschied zu den Banknoten – das Herkunftsland aufgrund ihrer nationalen Seite leicht erkennen. Die Einführung der Euro–Münzen bot die einmalige Gelegenheit, die grenzüberschreitende Münzmigration zu untersuchen.

Das vorliegende Papier beschäftigt sich mit der Frage, wie viele Euro–Münzen aus Deutschland abfließen und welche Zusammensetzung der Bestände auf längere Sicht zu erwarten ist. Dies hängt von der Intensität des grenzüberschreitenden Reiseverkehrs ab und hat Auswirkungen auf den Münzgewinn (Seigniorage) der einzelnen Länder.

Die Arbeit veranschaulicht den Durchmischungsprozess beispielhaft anhand der Ein–Euro–Münzen. Hierfür sind konsistente Daten aus den Online–Umfrageprojekten „Eurodiff" und „Euromobil" verfügbar. Danach fiel der Anteil deutscher Ein–Euro–Münzen in Deutschland seit der Euro–Einführung bis Ende 2008 auf rund 75%. Der Münzfluss zwischen Deutschland und der übrigen Euro–Ländern (Ausland) wird durch ein einfaches dynamisches Modell erklärt. Die Parameter des Modells, Abfluss- und Zuflussraten, werden anhand von empirischen Daten kalibriert. Die Ergebnisse hängen davon ab, ob der gesamte Münzbestand im Zeitablauf konstant ist oder wächst. Wird angenommen, dass der gesamte Bestand an Ein–Euro–Münzen im In– und Ausland konstant ist, prognostiziert das Modell langfristig einen Rückgang des Anteils deutscher Münzen in Deutschland an den von Deutschland emittierten Münzen auf nur 22%. Empirisch war in den Jahren seit der Euro–Einführung allerdings ein Wachstum der Münzbestände zu beobachten, wodurch sich der Durchmischungsprozess verlangsamte. Werden die in den vergangenen Jahren beobachteten Wachstumsraten des Münzumlaufs extrapoliert, ergibt sich langfristig ein Rückgang des Anteils deutscher Münzen in Deutschland auf 53%. Dieselben Ergebnisse werden mit einem alternativen Modell erzielt, das den Durchmischungsprozess mit einem Markov–Ansatz beschreibt.

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1 Introduction

Since 1 January 2002, euro banknotes and coins have been legal tender in the euro area, which, since January 2009, comprises 16 member states. While, to the untrained eye, it is difficult to distinguish between the banknotes issued by the individual euro-area countries, distinguishing between the various euro-area coins is relatively simple owing to their unique national side. As people can clearly see in their everyday cash transactions, the national euro coins have become mixed over time. Nevertheless, it is difficult to find any reliable data on the intensity of the cross-border flow of coins. The introduction of euro coins did, however, present the unique opportunity to analyse the cross-border migration of euro coins and the mixing of coins in the various euro-area countries.

There are a number of projects being carried out at universities relating to this coin diffusion process. For example, the French project is based on representative surveys (Grasland et al., 2002), whereas the other projects are based largely on voluntary reporting. Examples of the latter are the "Euromobil" and "Eurodiff" projects in Germany, which focus exclusively on €1 coins (Stoyan, 2002; Stoyan et al., 2004), "Eurotracer", which also includes the migration of banknotes, and "Eurodiffusie" in the Netherlands and Belgium (Blokland et al., 2002).[1] Bergman et al. (2002) use an algorithm and data on travel statistics to try to determine when 50% of the coins circulating in a country are of foreign origin. Furthermore, since 2002, the European Commission has been taking random samples of coins from major towns and cities at irregular intervals to gather data on the coin mixing process. All of these studies are focused on establishing either the (net) number of foreign euro coins flowing into a

* We thank N. Bartzsch, H. Herrmann and G. Schultefrankenfeld as well as seminar participants at the Deutsche Bundesbank for valuable comments. The views in the paper are not necessarily those of the Deutsche Bundesbank.

[1] See http://www.mathe.tu-freiberg.de/inst/stoch/Stoyan/euro/euro.html and http://www.eurotracer.net.

country during a given time period or the stock of foreign euro coins at a given point in time.

Coins are a direct source of seigniorage as the difference between the nominal value of coins and their production costs is the profit of the finance ministry.[2] This process constitutes a kind of monetisation of government debt. This is also true in the euro area where the different member countries have the right to determine the amounts of coins to be issued. In Germany, this has to be done in coordination with the Deutsche Bundesbank. Furthermore, the amounts have to be approved by the European Central Bank for one year in advance. Except this approval, there is no upper limit to the issuance process. However, according to the EU Treaty, the maximum amount of coins in the stocks of the national central banks to be credited to the government must not exceed 10% of the total issue of the respective country. Therefore, if the cross-border coin flows are not symmetric or national coin demands differ, this may have consequences for the seigniorage revenues of the different countries.

In the following, we will focus on the "indirect" calculation of the flow variable which is behind the mixing process. For that purpose, we will apply a simple theoretical model to determine how many euro coins flow into other euro-area countries. Following on from this, we are going to calculate the number of coins to be expected in the long term. We will confine our calculation to Germany and €1 coins.[3] For this case, consistent data from the "Euromobil" and "Eurodiff" projects are available. Section 2 below explains the formal models and results and also puts forward an alternative approach. We will then discuss the results in section 3. Section 4 summarises and draws some conclusions.

2 For different seigniorage concepts see Baltensperger and Jordan (1997).

3 In the case of the smaller-denomination euro cent coins, the results would certainly be distorted owing to a considerable amount of lost coins, whereas in the case of €2 coins, special-issue coins would have to be considered as well. In the US, it was even observed up until recently that the demand for coins went up perceptibly in the wake of surging commodity prices, creating an incentive to melt coins (Velde, 2007). Melting has since been prohibited by the US Treasury and does not seem to be an issue for euro coins owing to their different metallic composition.

2 The model

2.1 Constant volume of coins

In the following, $N_{DD}(t)$ denotes the number of German €1 coins in Germany and $N_{DA}(t)$ the number of German €1 coins abroad, both at time *t*. In this section, we assume that the total number of German €1 coins N_D and the total number of foreign €1 coins N_A remain constant. Thus, the inflow of coins to Germany and the outflow of coins from Germany cancel each other out. As Deutsche Bundesbank (2003, 208f.) notes, there is little to suggest that, despite a traditional foreign travel account deficit, Germany will encounter a net outflow of euro coins. Provided we are concerned with coins for transaction purposes, it can be assumed that tourists take a similar amount of coins with them when travelling both from and to their home country.

At time *t*, the German coins are either still in Germany ($N_{DD}(t)$) or have migrated abroad ($N_{DA}(t)$):

(1) $$N_{DD}(t) + N_{DA}(t) = N_D .$$

As a result of foreign travel (tourism, business trips) a certain amount of German €1 coins drifts to other countries. We assume that this amount is proportional to the volume at the start of the period: $\alpha N_{DD}(t\text{-}1)$. Conversely, coins also flow from other euro-area countries to Germany. This includes German coins which already drifted abroad in previous periods. Here, too, we assume that this amount is proportional to the number of coins abroad at the beginning of the period: $\beta N_{DA}(t\text{-}1)$. This results in the following dynamic equation for the number of German €1 coins in Germany:

(2) $$N_{DD}(t) = N_{DD}(t-1) - \alpha N_{DD}(t-1) + \beta N_{DA}(t-1) .$$

Since we are primarily concerned with the share of German coins in Germany

(3) $$n_{DD}(t) \equiv N_{DD}(t) / N_D ,$$

using (1) and (3), equation (2) is reformulated as follows:

(4) $$n_{DD}(t) = \beta + (1 - \alpha - \beta)\, n_{DD}(t-1) .$$

At the time of the first issue, all German coins were located in Germany. Therefore, the starting value is $n_{DD}(0) = 1$. In the long run, the share of German coins in Germany converges towards

$$(5) \qquad n^*_{DD} = \frac{\beta}{\alpha + \beta}.$$

If no outflow occurs ($\alpha = 0$), the share of German coins in Germany remains at $n^*_{DD} = 1$. Conversely, if no backflow occurs ($\beta = 0$), all German €1 coins will, in the long run, migrate abroad $(n^*_{DD} = 0)$. Together, (4) and (5) yield an equation for the evolution of coin shares over time :

$$(6) \qquad n_{DD}(t) = n^*_{DD} + (1 - \alpha - \beta)^t [1 - n^*_{DD}].$$

(6) shows a dynamic adjustment process whereby the share of German coins in Germany decreases monotonically and converges towards the equilibrium value (n^*_{DD}). The larger the outflow rate (α) and the backflow rate (β), the quicker the coins are mixed.

An equivalent equation can also be derived for the share of foreign €1 coins abroad (n_{AA}):

$$(7) \qquad n_{AA}(t) = n^*_{AA} + (1 - \alpha - \beta)^t [1 - n^*_{AA}], \quad \text{where } n^*_{AA} = \frac{\alpha}{\alpha + \beta}.$$

In order to determine the flow rates α and β, we take the number of €1 coins in circulation in the euro area at the end of 2008 (t = 7) of around 6,000 million (see table 1). Of this amount, 1,340 million were German €1 coins and 4,660 million were foreign €1 coins.[4] Statistical research carried out by Stoyan suggests that, at the end of 2008, 75% of all German €1 coins were located in Germany (n_{DD} = 75%), which is equivalent to 1,005 million coins, as around 25% of all the €1 coins observed in Germany at that time were of foreign origin.[5] This would mean that a total of 335 million coins have already migrated abroad. This is equivalent to around 48 million coins per year, or

[4] In statistical terms, this corresponds to approximately 16 €1 coins per capita of the German population and approximately 20 €1 coins per capita in the rest of the euro area.

[5] See http://www.mathe.tu-freiberg.de/Stoyan/euro/en/euro.html

131,000 coins per day. As the same amount of foreign €1 coins have also flowed to Germany, this implies that the share of foreign coins still abroad is n_{AA} = 92.8%.

Table 1 Coin volumes

End of 2008 (in millions)	In Germany	Abroad	Total
German coins	N_{DD} = 1005	N_{DA} = 335	N_D = 1340
Foreign coins	N_{AD} = 335	N_{AA} = 4325	N_A = 4660
Total	N_D = 1340	N_A = 4660	N = 6000

Source: Deutsche Bundesbank, European Central Bank and Stoyan (see footnote 5)

These data can be used to empirically calibrate the flow rates α and β by solving equations (6) and (7) for α and β at t = 7. This yields $\hat{\alpha} = 0.041925$, $\hat{\beta} = 0.012056$. According to these estimates, per year 4.2% of German 1€ coins flow to other countries, and 1.2% of foreign 1€ coins flow to Germany. These estimates reflect the unequal number of issued coins in Germany compared to other EMU countries. Thus, the share of German coins in Germany, which currently stands at 75%, would fall to $n^*_{DD} = 22.3\%$ in the long run, whereas the share of foreign coins abroad would decline from 92.8% at present to $n^*_{AA} = 77.7\%$. This means that a perfect mixture of coins would have been achieved as these shares correspond to the percentages of issued €1 coins. The key factors in this result are the unequal amounts of German and foreign €1 coins issued and the assumption that the total number of coins *(N_D, N_A)* does not grow.

2.2 Growing volumes of coins

One of the fundamental assumptions in the above model, the constancy of the total number of coins N_A and N_D, is too restrictive and needs to be modified. Indeed, the number of coins in circulation has increased since the euro was introduced, with the number of German coins issued between 2002 and 2008 growing by almost 4% annually and the number of foreign €1 coins issued by as much as 10% per year.[6] The

[6] The latter figure may be slightly overstated because it includes the new euro area members Slowenia, Malta and Cyprus.

increase in the number of coins in circulation affects the coin mixing process: the more coins are newly circulated each year, the more slowly the coins become mixed. In order to take account of the effect of the increase in the volume of coins on the coin mixing process, we modify equation (2) as follows:

$$(8) \qquad N_{DD}(t) = N_{DD}(t-1) - \alpha N_{DD}(t-1) + \beta N_{DA}(t-1) + g_D\, N_D(t-1).$$

For the sake of simplification, we assume that the annual growth rate (g_D) of German €1 coins remains constant. The last term in (8) is the quantitative increase of German €1 coins. Dividing (8) on both sides by $N_D(t\text{-}1)$, and using the definition $n_{DD}(t) \equiv N_{DD}(t)/N_D(t)$ for the ratio of German coins in Germany to the total number of coins issued by Germany, yields

$$(9) \qquad n_{DD}(t) = \frac{\beta + g_D}{1+g_D} + \frac{1-\alpha-\beta}{1+g_D} n_{DD}(t-1).$$

Neglecting growth ($g_D=0$) results in (4) as a special case. In the long run, the share of German coins in Germany converges towards:

$$(10) \qquad n^*_{DD} = \frac{\beta + g_D}{\alpha+\beta+g_D}.$$

For the share of German coins abroad, this gives us:

$$(10a) \qquad n^*_{DA} = 1 - n^*_{DD} = \frac{\alpha}{\alpha+\beta+g_D}.$$

These two shares depend on the flow rates α and β and on the growth rate g_D. The smaller the flow rate (α) and the greater the backflow rate (β) as well as the growth rate of German coins (g_D), the greater is the share of German coins that will remain in Germany in the long run (and therefore the smaller the share of German coins abroad). This yields the following dynamic equation for the share of German coins in Germany:

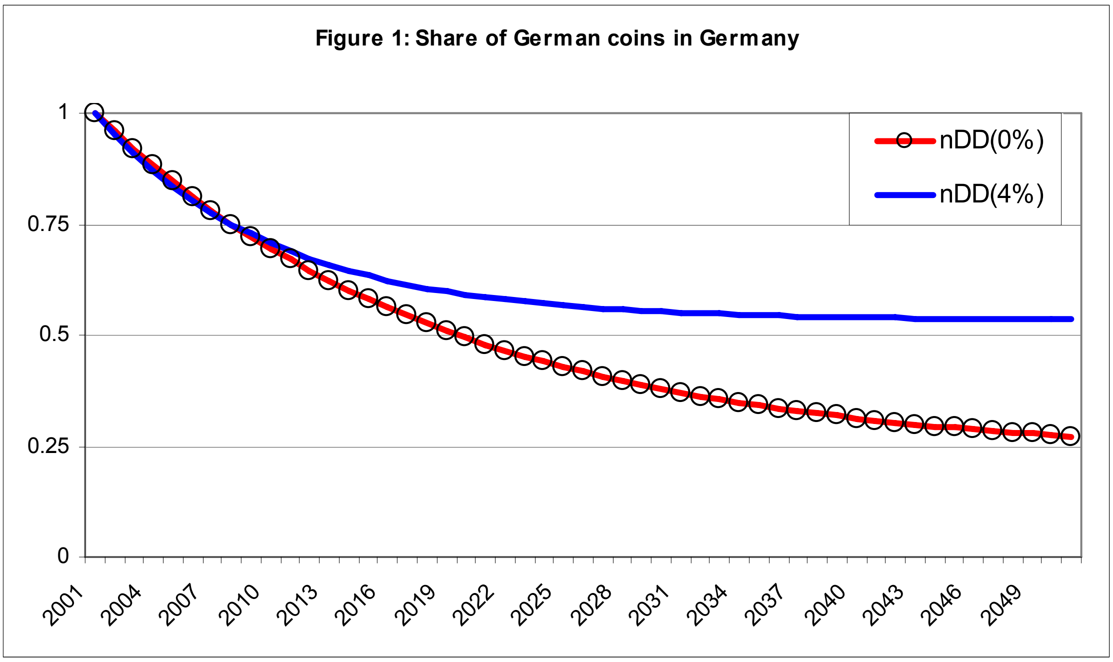
$$(11) \qquad n_{DD}(t) = n^*_{DD} + \left(\frac{1-\alpha-\beta}{1+g_D}\right)^t [1-n^*_{DD}].$$

In turn, an analogous equation applies to the number of foreign coins in circulation abroad:

$$(12) \qquad n_{AA}(t) = n_{AA}^* + \left(\frac{1-\alpha-\beta}{1+g_A}\right)^t [1-n_{AA}^*], \quad \text{where} \quad n_{AA}^* = \frac{\alpha+g_A}{\alpha+\beta+g_A}.$$

Once more, we revert to the "observed" coin volumes at the end of 2008 given in Table 1, now reflecting both, the original issue and the subsequent growth of coins. We use the average growth rates g_D = 4% and g_A = 10% and for t = 7 (end of 2008) we again set n_{DD} = 75% and n_{AA} = 92.8%. The flow rates α and β can be empirically calibrated from equations (11) and (12). This yields $\hat{\alpha} = 0.050386, \hat{\beta} = 0.017590$. Not surprisingly, the estimated flow rates with growth (5.0% and 1.8%) are larger than those obtained assuming constant volumes: When more coins are issued, the chance of a specific coin to flow abroad falls, requiring higher flow rates to obtain the mixing given in Table 1.

If the above growth rates in the number of coins in circulation also hold in the future, the share of German coins in Germany would only fall to $n_{DD}^* = 53.3\%$ in the long run (and not to 22.3% as in the case with constant volumes). Alternatively, assuming at t = 7 n_{DD} = 70% (80%), we obtain $n_{DD}^* = 49.4\% \; (58.1\%)$. The share of foreign coins abroad would decline merely to $n_{AA}^* = 89.5\%$. According to this estimation, in the long run, about one in two €1 coins issued in Germany is likely to stay in Germany. Figure 1 shows the forecasted share of German €1 coins in Germany for both cases, i.e. without growth and assuming a growth rate of 4% which was recorded since the introduction of euro coins in 2002.



So far, we focused on the ratio of German 1€ coins in Germany to the total number of 1€ coins issued by Germany *($n_{DD}(t)$)*. This ratio converges to a well defined equilibrium (n^*_{DD}) and the same holds for the ratio of foreign 1€ coins to the total number of coins issued abroad (n^*_{AA}). However, with different growth rates, what happens to the ratio of German coins in Germany to the number of coins circulating in Germany?

With coin volumes growing at constant but different (positive) rates, the ratio of 1€ coins issued abroad to those issued by Germany,

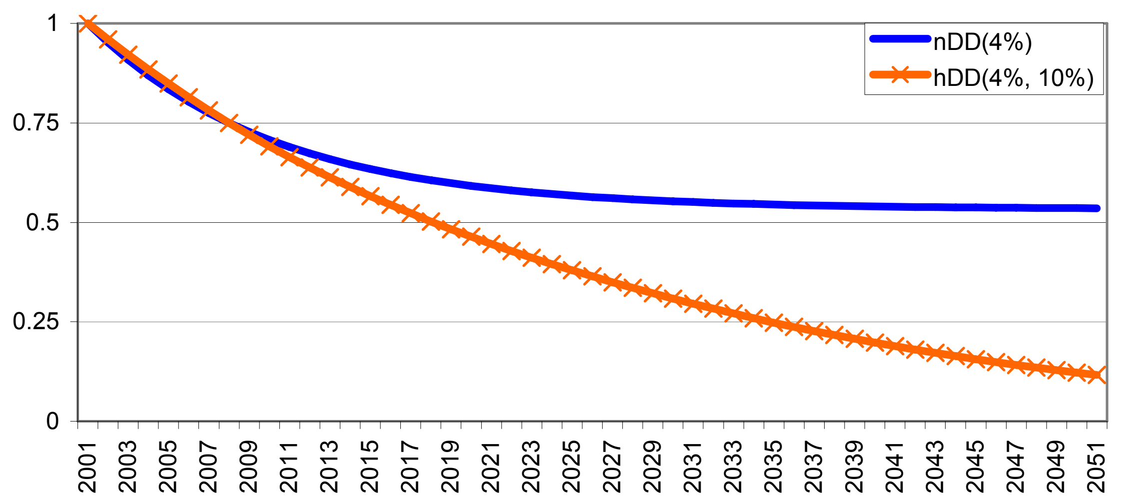
$$(13) \quad \psi(t) \equiv \frac{N_A(t)}{N_D(t)} = \frac{N_A(O)}{N_D(O)} \left(\frac{1+g_A}{1+g_D} \right)^t ,$$

converges to a constant in the long run only for $g_A = g_D$. If $g_A > g_D$ ($g_A < g_D$), the ratio tends to infinity (zero). This has important consequences for the ratio of German 1€ coins in Germany to the total number of coins circulating in Germany:

(14)
$$h_{DD}(t) \equiv \frac{N_{DD}(t)}{N_{DD}(t)+N_{AD}(t)} = \frac{N_{DD}(t)}{N_{DD}(t)+N_A(t)-N_{AA}(t)}$$
$$= \frac{n_{DD}(t)}{n_{DD}(t)+[1-n_{AA}(t)]\psi(t)}$$

Thus, if $g_A \neq g_D$. $h_{DD}(t)$ is different from $n_{DD}(t)$ and it either converges towards zero or towards one (and not to n^*_{DD}). Hence, if the growth rates of coins issued in Germany and abroad are different, German coins in Germany eventually relatively disappear or dominate.[7] Figure 2 shows the development of German coins in Germany, both, as a ratio of coins issued by Germany ($n_{DD}(t)$) and as a ratio of coins circulating in Germany ($h_{DD}(t)$), calculated for the growth rates g_D = 4% and g_A = 10%. With the ratio of German coins circulating in Germany converging towards zero (and the ratio of foreign coins circulating abroad converging towards one), this graph suggests that substantially different growth rates of euro coins are likely to be unsustainable in the long run.

Figure 2: Share of German coins in Germany (nDD, hDD)



[7] Under conditions of balanced growth (g_D=g_A=g), $h_{DD}(t)$=$n_{DD}(t)$ holds.

2.3 A Markov chain approach

Alternatively, using a Markov chain, the coin mixing process can be examined within one closed approach. To this end, we define the matrix of the number of coins as

$$M(t) = \begin{bmatrix} N_{DD}(t) & N_{DA}(t) \\ N_{AD}(t) & N_{AA}(t) \end{bmatrix}$$

and assume the following matrix of transition probabilities

$$Z = \begin{pmatrix} (1-\alpha) & \alpha \\ \beta & (1-\beta) \end{pmatrix},$$

where α denotes the probability that a €1 coin located in Germany will drift abroad over a given period (and $1\text{-}\alpha$ the probability that the coin will remain in Germany). Furthermore, β denotes the probability that a coin located abroad will migrate to Germany. The exogenous growth in the number of coins is given by $N(t) = N(t-1) + N(t-1)G$, with

$$G = \begin{bmatrix} g_D & 0 \\ 0 & g_A \end{bmatrix} \text{and} \qquad N(0) = M(0) = \begin{bmatrix} N_D(0) & 0 \\ 0 & N_A(0) \end{bmatrix}.$$

Thus, the dynamic process of coin circulation reads as

$$(15) \qquad M(t) = M(t-1)Z + N(t-1)G.$$

If, at t = 7 (end of 2008), we take the distribution of coins specified in table 1

$$M(7) = \begin{bmatrix} 1005 & 335 \\ 335 & 4325 \end{bmatrix}$$

and the average growth rates g_D = 4% and g_A = 10% observed in the past, we can use equation (15) to empirically calibrate the transition probabilities. This results in $\hat{\alpha} = 0.050386$, $\hat{\beta} = 0.017590$. Thus, the probability that a €1 coin located in Germany will flow abroad in the course of one year is 5%, and the probability that a coin located abroad will flow to Germany 1.8%. These values exactly match the flow rates that were calculated in section 2.2.

3 Discussion of the results: plausibility and robustness checks

According to information from Fraport AG, 7.2 million passengers from other euro-area countries landed at Frankfurt International Airport in 2007. This amounts statistically to around 20,000 passengers per day. Frankfurt is the biggest German airport which handles around one-third of all air passengers in Germany. Moreover, other means of transport are also used. And commuters travelling to and from Germany on a daily basis also have to be taken into account. In 2006, a (net) daily average of 90,000 persons commuted to Germany. The majority of these are likely to have originated from other euro-area countries, primarily Austria and the Netherlands (Stoyan et al., 2004, 74). Therefore, our results regarding the flow rates - about 130,000 per day - do not seem to be unrealistic.

Based on a small, yet non-representative, sample of 2,000 coins taken in Frankfurt am Main, the European Commission calculated that, for the year 2007, approximately 13% of the €1 coins in circulation in Germany were of foreign origin. This was a significant decline compared with the samples taken in previous years, when, in some cases, the share of foreign €1 coins in circulation in Germany was over 50%. Therefore, this figure is likely to have a severe downward bias. Furthermore, the results of this kind of random sampling vary substantially depending on the time of year and the place where the samples were taken. If we were to set $n_{DD} = 50\%$ at the end of 2008, a zero-growth scenario would yield $\hat{\alpha} = 0.10648, \hat{\beta} = 0.03062$. Positive growth rates (g_D=0.04, g_A=0.10) result in $\hat{\alpha} = 0.13690, \hat{\beta} = 0.04653$. In the former scenario, the long-term coin volumes would converge to 22.3%, yet would settle at 38.7% in the latter scenario.

If we assume that growth in the number of coins in Germany and abroad is 5% (g_D=g_A=0.05), the number of German coins that would migrate from Germany each year would be 5.20%, and 1.49% of German coins located abroad would return to Germany. The implied long-term equilibrium values of the coin shares would then be n^*_{DD} = 55.5% and n^*_{AA} = 87.2%. Again, the German share would not fall below 50%.

Equations (6) and (11) can also be used to estimate how the mixing process will develop over time. Larger values would simply have to be used for t. At $t = 14$ (in

2015) and assuming constant volumes, this would result in $n(14) = 0.58$, i.e. the share of German coins at the end of 2015 would be approximately 58%. The share of foreign €1 coins in Germany is expected to reach the 50% threshold around 2020. This contrasts significantly with figures calculated by Bergman et al., who – based on 2002 knowledge – do not expect a 50% mixing of coins until the year 2040. These different results are especially due to the assumption as to how high the volumes of German coins in Germany are at the starting point.

Our analysis is flexible enough to be adapted when new countries join the euro area. We would only need to adjust the flow rates α and β, with the other calculations left unchanged.

4 Summary and conclusions

By means of a simple model, we have tried to capture the mixing process of euro coins and to determine the share of German euro coins in Germany. The model was calibrated using data on €1 coins in Germany. This approach allows the above values to be determined indirectly. According to our calculations, a total of between 4% and 5% of German coins are likely to flow abroad each year. We believe that the most plausible value for the long-term share of German €1 coins in Germany is likely to be around 50%. The present analysis can be easily extended to other euro-area countries, provided that the required data (on, for instance, the number of foreign coins at a given point in time) are available.

The derived results are conditional on two assumptions: the growth rates of the number of German and foreign coins and the number of German coins in Germany at a given point in time. In this context, the plausibility of the various growth scenarios would need to be examined. Although, at a first glance, the positive growth scenario appears to be more plausible, it does, however, present the problem that people would have to walk around with increasingly large amounts of coins in their pockets.[8] This would probably lead to a decline in demand or to a replacement of the smaller-denomination coins by issuing a greater amount of larger-denomination coins or even a

[8] Alternative scenarios include the loss or hoarding/collecting of a growing number of coins.

changeover to banknotes. Thus, from an economic standpoint, the assumption that, in the very long term, national coin volumes will remain constant with an unchanged denomination structure is not implausible. Nevertheless, this may take a very long time to unfold, which means that, during a transitional period, it would make sense to work with positive growth rates. In any case, the effects of an increase in foreign travel would also have to be taken into account. This would entail an increase in the cross-border flow of coins. Over the 2002-2008 period, travel by Germans to other euro-area countries went up by an annual average of 4%-5%.

Another interesting route would be to abandon some of our simplifying assumptions and to review the sensitivity of the results accordingly. There are, for example, repercussions on the mints and on seigniorage in those countries which record a net outflow of coins.[9] These countries would benefit, as they would issue more coins than are actually required in their country. In the first few years following the introduction of euro coins, the mixing process is also likely to have been influenced by collectors of euro coins in the individual euro-area countries. This influence is likely to diminish over time. Nevertheless, this process will be relaunched from time to time whenever new members join the euro area. The cross-border flow of coins and collection activities could lead to growing uncertainty when forecasting coin demand (Croushore and Stark, 2002). These issues should be the subject of future research.

[9] Historically, this kind of situation occurred time and time again and, in some cases, led to a coin shortage. A common response to this in the past was to impose a ban on coin exports. A theoretical analysis of this relationship can be found in Wallace and Zhou (1997).

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