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Forecasting German GDP using alternative factor models based on large datasets

Christian Schumacher

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Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-1
Telex within Germany 41227, telex from abroad 414431, fax +49 69 5601071

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

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Abstract:

This paper discusses the forecasting performance of alternative factor models based on a large panel of quarterly time series for the German economy. One model extracts factors by static principal components analysis, the other is based on dynamic principal components obtained using frequency domain methods. The third model is based on subspace algorithms for state space models. Out-of-sample forecasts show that the prediction errors of the factor models are generally smaller than the errors of simple autoregressive benchmark models. Among the factors models, either the dynamic principal component model or the subspace factor model rank highest in terms of forecast accuracy in most cases. However, neither of the dynamic factor models can provide better forecasts than the static model over all forecast horizons and different specifications of the simulation design. Therefore, the application of the dynamic factor models seems to provide only small forecasting improvements over the static factor model for forecasting German GDP.

Keywords: Factor models, static and dynamic factors, principal components, forecasting accuracy

JEL-Classification: E32, C51, C43

Non technical summary

Factor models based on large datasets have received increasing attention in the recent macroeconomics literature. Factor models aim at finding a few representative common factors underlying a large amount of economic activities. Compared with smaller scaled models, the factor models have the potential to consider a large amount of time series data typically available to central bank economists. Particularly for forecasting purposes, factor models based on large data sets have proven useful. For the USA, Stock and Watson (1999, 2002a, b) provide evidence for the information content of macroeconomic factors derived from hundreds of macroeconomic time series for future industrial production and inflation. This evidence highlights the potential benefits of factors derived from large data sets as indicators for monetary policy. Following the seminal work of Stock and Watson (1999, 2002a, b), a number of methodological extensions to their factor model for large data sets have been developed, so the question for the choice of the appropriate method in everyday central bank forecasting arises.

In this context, the present paper compares three factor models based on large data sets with respect to their forecasting accuracy for German GDP. The reference model is the model proposed by Stock and Watson (2002b), where the factors are obtained by static principal component analysis. A disadvantage of this model is that only static weights of factors are allowed for, so dynamic relationships between the variables are not considered explicitly. In order to take into account this issue, Forni et al. (2003a, b) propose dynamic principal component analysis in the frequency domain to estimate the factors. An alternative method proposed by Kapetanios (2004) estimates factors in a state-space framework using subspace algorithms. Both dynamic approaches allow for dynamic relationships between the variables in the model and, hence, provide a potentially useful alternative to the method of Stock and Watson (2002a). In empirical applications, however, it is not clear whether the postulated dynamics underlying the dynamic factor models can be found in the data. Moreover, the dynamic models have a more complicated structure to be estimated, which may be subject to misspecification in empirical applications.

To compare the models, different out-of-sample forecast simulations for German GDP at forecast horizons of up to four quarters are carried out. The results show that the forecasting accuracy of the dynamic approaches is not systematically better than that of the static approach. It seems to be the case that the theoretical advantages of the dynamic models cannot be fully exploited for forecasting in this empirical application. Therefore, using the static factors proposed by Stock and Watson (2002b) for forecasting purposes is not necessarily inferior to using factors estimated in a dynamic framework.

Nicht technische Zusammenfassung

In der jüngeren makroökonomischen Literatur sind Faktorenmodelle auf Basis großer Datensätze vermehrt untersucht worden. Die Faktorenmodelle verfolgen das Ziel, eine Vielzahl von ökonomischen Aktvititäten durch eine geringe Zahl von gemeinsamen Faktoren repräsentativ abzubilden. Verglichen mit kleineren Modellen haben sie den Vorteil, die großen Datenmengen, die Zentralbankökonomen heutzutage zur Verfügung stehen, simultan auswerten zu können. Insbesondere für Prognosezwecke haben sich die Faktorenmodelle in ersten Studien als nützlich erwiesen. Für die USA konnte von Stock und Watson (1999, 2002a, b) der Informationsgehalt von Faktoren auf Basis großer Datensätze für die zukünftige Entwicklung von Produktion und Inflation belegt werden. Als Ergänzung zu den ersten Arbeiten von Stock und Watson (1999, 2002a, b) wurden in der Literatur einige methodische Erweiterungen entwickelt, welche die Frage nach der Wahl des geeigneten Verfahrens in der täglichen Prognosearbeit aufwerfen.

Vor diesem Hintergrund vergleicht das vorliegende Papier drei Faktorenmodelle auf Basis großer Datensätze in Hinblick auf ihre Prognosegüte für das deutsche BIP. Als Referenzmodell fungiert das Modell von Stock und Watson (2002b), das die Faktoren unter Zuhilfenahme der statischen Hauptkomponentenanalyse extrahiert. Ein Nachteil dieses Modells ist, dass die statische Faktorenanalyse lediglich statische Faktorengewichte zwischen den Variablen zulässt, also keine dynamischen Zusammenhänge bei der Faktorenbildung explizit berücksichtigt werden. Um diesen Aspekt aufzugreifen, haben Forni et al. (2003a, b) die dynamische Hauptkomponentenanalyse, welche im Frequenzbereich durchgeführt wird, als alternative Schätzmethode für die Faktoren vorgeschlagen. Ein weiteres Verfahren von Kapetanios (2004) schätzt die Faktoren mit Zustandsraummodellen unter Anwendung von Subraumalgorithmen. Beide dynamischen Verfahren versuchen, auf unterschiedliche Art und Weise, dynamische Beziehungen zwischen den Variablen innerhalb eines Faktors zuzulassen und stellen daher eine Alternative zur rein statischen Analyse von Stock und Watson (2002b) dar. Allerdings stellt sich die Frage, ob sich die in diesen Modellen unterstellte Dynamik auch in der Realität finden lässt. Zudem sind die dynamischen Modelle auch komplexer, was in der empirischen Anwendung zu Fehlspezifikationen führen kann.

Zum Vergleich der drei Modelle werden verschiedene Prognosesimulationen ausserhalb des Schätzzeitraumes mit Prognosehorizonten von bis zu vier Quartalen durchgeführt. Die Ergebnisse zeigen, dass die Prognosegüte der dynamischen Modelle die des statischen Faktorenmodells nicht systematisch übertrifft. Die Ergebnisse deuten darauf hin, dass sich die theoretischen Vorteile der dynamischen Faktorenmodelle im Rahmen der empirischen Anwendung nur in begrenztem Umfang in geringeren Prognosefehlern niederschlagen.

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1 Introduction

In the recent applied macroeconomics literature, in particular the macroeconomic forecasting literature, factor models with large data sets have received increasing attention. In a couple of seminal papers, Stock/Watson (1999, 2002a, b) proposed a univariate dynamic forecasting model augmented with static factors obtained by static principal component analysis as a forecasting tool. In various applications, the information content of the factors for USA inflation and output is shown, see Stock/Watson (1999, 2002a). Moreover, various other applications using this type of factor model provided additional favourable evidence for the forecasting accuracy of the factors models, see, for example, Brisson et al. (2003) for Canadian data, Camacho/Sancho (2003) for Spanish data, or Artis et al. (2004) for forecasting UK time series. However, despite the early success of factor models based on large data sets, some authors cast doubts on the empirical accuracy of large factor models based on static principal components. For example, Giacomini/White (2004) find that these kinds of factor models do not always provide the best forecasting performance compared with other methods in a moving-window simulation experiment. Banerjee et al. (2003) compare static factor and single indicator forecasts for euro area aggregates and do not find improvements in the static factor models over single indicator methods. For Germany, Schumacher/Dreger (2004) do not find significant advantages of factor forecasts according to statistical tests of forecasting accuracy in a similar exercise. Against the background of these overall mixed results, the question of improvements in the estimation of factor models based on large data sets arises.

*Address: Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main, Tel.: ++49/+69-9566-2939, Fax: ++49/+69-9566-2982. E-mail: christian.schumacher@bundesbank.de. This paper represents the author's personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank. I am grateful to Oliver Bode, Jörg Breitung, Sandra Eickmeier, Rafael Gerke, George Kapetanios, Stefan Kohns, Martin Schneider and Jürgen Wolters as well as seminar participants at the Bundesbank, University of Osnabrück, DIW Berlin for helpful comments and discussions. The codes used in this paper are written in Matlab. The estimation of the dynamic factor model based on dynamic principal components was done using the codes from www.dynfactors.org/software/software.htm.

To take into account a richer dynamic structure for the factor models, various extensions to the static principal component estimators have been developed. For example, Forni et al. (2001, 2003a, b) use dynamic principal component analysis in the frequency domain to estimate large-scale factor models. Forni et al. (2003b) discuss the theoretical advantages of their proposed dynamic approach over the static approach of Stock/Watson (2002a, b). A key argument is that dynamic principal components are dynamic factors that link variables at different points in time, while only contemporaneous variables enter the static factors. Hence, temporal relationships are, in principle, better approximated by the Forni et al. (2001) approach. However, despite its theoretical advantages, the empirical success of the dynamic approach is rather mixed. In an empirical application for monthly euro area data, Forni et al. (2003a) obtain mixed results concerning the relative forecasting performance. On the other hand, den Reijer (2005) finds more favourable evidence for the superior forecasting performance of the dynamic factor model for the Dutch economy. Another dynamic factor model approach proposed by Kapetanios (2004), Camba-Mendez/Kapetanios (2004) and Kapetanios/Marcellino (2004) is based on subspace algorithms for state space models. In this framework, estimation of the factors is essentially a singular value decomposition of the regression coefficient between the data and its leads on the left-hand side of a vector equation and lags of the multivariate data on the right-hand side. Kapetanios (2004) uses this model to derive a core index for UK inflation and discusses the forecasting properties of the factors for future inflation.

Although the development of more sophisticated dynamic factor models is favourable from a theoretical point of view, Boivin/Ng (2005) have shown recently that the factor model based on static principal components is quite robust to misspecification since fewer auxiliary parameters have to be specified compared with dynamic factor models. This implies that imposing an explicit dynamic factor structure for forecasting in empirical applications could lead to a worse forecasting performance if the specification of the larger number of auxiliary parameters is prone to errors. In their simulations and empirical applications for the US, Boivin/Ng (2005) find that the static principal components serve quite well as predictors for various US time series, compared with dynamic factor estimates.

Against the background of this discussion, the relative forecasting accuracy of three alternative factor models introduced above is discussed in this paper. In particular, we discuss whether the more sophisticated dynamic factor models proposed by Kapetanios (2004) and by Forni et al. (2003a, b) can outperform the factor model based on static principal components proposed by Stock/Watson (2002a, b).

This paper expands on results by Schumacher/Dreger (2004), where the comparative advantages of the static factor model over the smaller forecasting models are discussed. For the comparison of the factor models, out-of-sample forecast simulations are carried out, since to the best of my knowledge, there are not yet any out-of-sample comparisons incorporating both the Kapetanios/Marcellino (2004) and factor models used by Forni et al. (2003a).[1] Moreover, alternative simulation designs are used in order to check the robustness of the results. For example, we employ a recursive as well as a rolling simulation scheme for out-of-sample forecasting. Additionally, performance-based model selection as well as information criteria are used for model specification. This extends the empirical literature on factor forecasting where often either performance-based model selection, for example in Forni et al. (2003a), or information criteria as in Stock/Watson (2002a, b) are employed. For the model selection using information criteria, we use alternative criteria by Bai/Ng (2002) for the number of static factors and, amongst other things, the recently developed criteria by Bai/Ng (2005) and Breitung/Kretschmer (2005) for the number of dynamic factors.

To compare the three factor models, their forecast accuracy with respect to German GDP is investigated. For the German economy, often only small forecasting models, such as single indicator forecasting equations, are analysed in the literature.[2] A comparison of different factor models has not been carried out yet and, hence, may be worth investigaton.[3]

The paper proceeds as follows: the following section contains a discussion of the factor models and how they can be estimated by the various methods. In section 3, the single-equation method for forecasting using the factors as plug-ins is described. In section 4, the German data set is described briefly, the out-of-sample forecasting simulation is introduced, and the factor models are applied to predict German GDP. Section 5 concludes.

2 Estimation of the alternative factor models

As stressed by Boivin/Ng (2005), factor forecasting depends on how the factors are determined and how these factors are used for prediction. And for both estimation and forecasting many possible implementations exist, for example, different methods for estimating factors with large data sets. Given the factor estimates, the forecast implementation requires decisions on how the forecast comparisons are undertaken, for example, whether recursive or rolling out-of-sample simulations are used, or how the auxiliary parameters of the forecasting models are specified. Due to these differences, it is important to eliminate differences in implementing the forecasting step when a comparison of alternative methods of estimating the factors is at the centre of investigation. To highlight the difference between estimation of the factors and forecasting using the factors, Boivin/Ng (2005) divide the process of factor forecasting into two steps: the factor estimation step and the forecasting step. This two-step procedure is used in the presentation of the methods below.

[1] An in-sample comparison for USA data is provided in Kapetanios/Marcellino (2004).

[2] See Kirchgässner/Savioz (2001), Breitung/Jagodzinski (2002), Fritsche/Stefan (2002), Hüfner/Schröder (2002) and Bandholz/Funke (2003).

[3] In Schumacher/Dreger (2004), only the static factor model is used for forecasting German GDP.

Factor model representation In factor models, variables are represented as the sum of two mutually orthogonal unobservable components: the common component and the idiosyncratic component. The common component is driven by a small number of factors common to all of the variables in the model. The idiosyncratic component on the other hand is driven by variable-specific shocks. Let X_t be the $(N \times 1)$ dimensional vector of stationary time series with observations for $t = 1, \dots, T$, and it is assumed that the series have zero mean and covariance Γ_0. The factor model representation is given by

$$X_t = \chi_t(F_t) + \xi_t, \tag{1}$$

where $\chi_t(F_t)$ are the common components solely driven by factors F_t, and ξ_t are the idiosyncratic components for each of the variables. How the factors are related to the common components χ_t and how they can be estimated depends on the respective method and will be described below. The idiosyncratic component is that part of X_t not explained by the common components. The idea behind the factor model is that a small number r of factors F_t should be able to explain most of the variance of the data, $r \ll N$. The question is now how to determine the factors, and three alternative methods will be discussed below.

Determining the factors according to Stock/Watson (2002b) The factor model proposed by Stock/Watson (2002b) uses static principal component analysis to derive the factors. The goal of static principal component analysis is to choose the parameters and factors of the model

$$X_t = CF_t + \xi_t \tag{2}$$

in order to maximise the explained variance of the original variables for a given, small number r of factors F_t. The static factors in this model can be estimated using static principal component analysis.[4] Let $\widehat{\Gamma}_0 = (1/T)\sum_{t=1}^{T} X_t X_t'$ be an estimate of the contemporaneous variance-covariance matrix of the vector of time series X_t. The aim is to find r linear combinations of the time series data $\widehat{F}_{j,t} = \widehat{S}_j' X_t$ for $j = 1, \dots, r$ that maximise the variance of the factors $\widehat{S}_j'\widehat{\Gamma}_0\widehat{S}_j$. The number of factors should be sufficiently small compared with the total number of time series, $r \ll N$. As a restriction, Stock/Watson (2002a) impose the normalisation $\widehat{S}_j'\widehat{S}_i = 1$ for $i = j$ and 0 for $i \neq j$. The maximisation problem can be reformulated as an eigenvalue problem

$$\widehat{\Gamma}_0\widehat{S}_j = \widehat{\mu}_j\widehat{S}_j, \tag{3}$$

where $\widehat{\mu}_j$ denotes the j-th eigenvalue and $\widehat{S}_j$ its $(N \times 1)$ corresponding eigenvector. As before, after the calculation of the maximum of N eigenvalues, they are ranked in decreasing order

[4] See Bai/Ng (2002), pp. 197-8.

of magnitude and the eigenvectors according to the r largest eigenvalues are the weights of the static factors

$$\widehat{F}_t^{SW} = \widehat{S}' X_t, \tag{4}$$

where $\widehat{S}$ is the $(N \times r)$ matrix of stacked eigenvectors $\widehat{S} = (\widehat{S}_1, \ldots, \widehat{S}_r)$. Note that only contemporaneous variables are combined to obtain factors, and dynamics are not considered at this point. To derive the factors, only one auxiliary parameter, namely r, is needed. The asymptotic properties of the static factors are analysed in Bai/Ng (2002), Stock/Watson (2002b) and Bai (2003).

Determining the factors according to Forni et al. (2003a) The model proposed by Forni et al. (2003a, b) aims at identifying a dynamic structure of the factor model. The dynamic factor model is given by

$$X_t = \chi_t + \xi_t = B(L) U_t + \xi_t, \tag{5}$$

where $\chi_t = B(L)U_t$ are the common components, U_t is the $(q \times 1)$ vector of dynamic factors which has to be determined from the data. Similar to the factor model above, it is assumed that the dimension of the dynamic factors is lower than the dimension of the data, so $q \ll N$. $B(L) = I + B_1 L + \ldots + B_s L^s$ is a lag polynomial with non-negative power of the lag operator L, with $Lx_t = x_{t-1}$. Hence, lags of the factors are allowed to affect the current movement of the variables. ξ_t is again the $(N \times 1)$ idiosyncratic component of the variables. Note that for a given finite lag order s, the model can be rewritten in a static way as $X_t = CF_t + \xi_t$, where $F_t = \left(U_t', \ldots, U_{t-s}'\right)'$ is a $r = q(s+1)$ dimensional vector of stacked dynamic factors.[5] The $(N \times r)$ dimensional parameter matrix C contains the coefficients of $B(L)$. The dynamic method proposed by Forni et al. (2003 a, b) aims at estimating the dynamic factors U_t in a first step, and the static factors F_t are derived given the dynamic factors in a second step.

To estimate the dynamic factors and their covariances, Forni et al. (2000) propose dynamic principal component analysis in the frequency domain. The dynamic principal components are derived in order to maximise the common components' variance under orthogonality restrictions. The optimisation leads to a dynamic eigenvalue problem of the spectral density matrix of the vector of observed variables. The spectral density of the vector of observed variables $\Sigma(\theta)$ is estimated at frequency θ for $-\pi < \theta < \pi$. The estimated spectrum includes the information of all autocorrelations across the vector of variables and hence provides a summary of their dynamic relationships. Let $\widehat{\Gamma}_k$ be the k-lag estimated autocovariance of the panel of time series X_t and X_{t-k}. The estimated spectral density of

[5] See Stock/Watson (2002a), p. 148, and Boivin/Ng (2005), p. 3.

the vector of observables is then given by

$$\widehat{\Sigma}(\theta_h) = \sum_{k=-M}^{M} \widehat{\Gamma}_k \left(1 - \frac{|k|}{M+1}\right) e^{-ik\theta_h}, \tag{6}$$

at frequency $\theta_h = 2\pi h/(2M+1)$, for $h = 0, \ldots, 2M$. The spectral density matrix is estimated with a Bartlett lag window of size M which determines the weights in brackets above. Given the spectral density estimate, it can be decomposed by an eigen decomposition. For each frequency, the dynamic eigenvalues and eigenvectors of $\widehat{\Sigma}(\theta_h)$ are computed. The eigenvalues are arranged in decreasing order of magnitude. The $(N \times 1)$ eigenvectors $\widehat{P}_j(\theta_h)$ are collected for $j = 1, \ldots, q$ corresponding to the q largest eigenvalues. By inverse discrete Fourier transform, the eigenvectors in the time domain are given by

$$\widehat{P}_j(L) = \sum_{k=-M}^{M} \widehat{P}_{j,k} L^k, \text{ with } \widehat{P}_{j,k} = \frac{1}{2M+1} \sum_{h=0}^{2M} \widehat{P}_j(\theta_h) e^{ik\theta_h}, \tag{7}$$

for $k = -M, \ldots, M$, and $j = 1, \ldots, q$. The weights $\widehat{P}_j(L)$ determine the j-th dynamic principal component according to $\widehat{U}_{j,t} = \widehat{P}_j'(L) X_t$. The factor loadings are two-sided with lead-lag order of M. Hence, the resulting time-domain factors allow for dynamic relationships between the variables at different times. A projection of X_t on leads and lags of those factors gives estimators of the common components $\widehat{\chi}_t$, which converge to the true common component χ_t in (5).[6] Although these estimators of the dynamic common components have desirable statistical properties concerning consistency, the reliance on spectral based estimators leads to two-sided filters. Hence, at the beginning and end of the sample of finite time series, no estimators of the factors are available due to the lead-lag truncation for the estimation of the spectral density. This is a serious drawback for forecasting and real-time estimation.

To circumvent this problem, Forni et al. (2003a, 2003b) propose one-sided estimates of the factors. For this purpose, the dynamic factor model is rewritten in a static way as in (1), and $r = q(s+1)$ factors can be determined using static principal components methods where the covariances of idiosyncratic and common components obtained in the steps before can be exploited. The estimation of the model aims at maximising the variance of the common components, and hence the minimisation of the variance of the idiosyncratic components. These matrices are given by[7]

$$\widehat{\Sigma}_\chi(\theta) = \widehat{P}(\theta)\Lambda(\theta)\widehat{P}^*(\theta), \text{ and } \widehat{\Sigma}_\xi(\theta) = \widehat{\Sigma}(\theta) - \widehat{\Sigma}_\chi(\theta), \tag{8}$$

where a star denotes complex conjugates, $\Lambda(\theta)$ is a $(q \times q)$ diagonal matrix with the largest q

[6] See Forni et al. (2004) for further asymptotics.
[7] See Forni et al. (2003a), p. 1253.

dynamic eigenvalues on the main diagonal, and the $(N \times q)$ matrix $\widehat{P}(\theta) = (\widehat{P}_1(\theta), \ldots, \widehat{P}_q(\theta))$ contains the corresponding eigenvectors at frequency θ. To obtain time-domain autocovariances of the common components, inverse discrete Fourier transform gives

$$\widehat{\Gamma}_{\chi,k} = \frac{1}{2M+1} \sum_{h=0}^{2M} \widehat{\Sigma}_{\chi}(\theta_h) e^{ik\theta_h}, \tag{9}$$

for $k = -M, \ldots, M$, and the covariance of the idiosyncratic component $\widehat{\Gamma}_{\xi,k}$ can be obtained accordingly. The aim is to find r linear combinations of the time series data $\widehat{F}_{j,t} = \widehat{Z}_j' X_t$ for $j = 1, \ldots, r$ that maximise the variance explained by the common factors $\widehat{Z}_j' \widehat{\Gamma}_{\chi,0} \widehat{Z}_j$. As a restriction, Forni et al. (2003b) impose the normalisation $\widehat{Z}_j' \widehat{\Gamma}_{\xi,0} \widehat{Z}_i = 1$ for $i = j$ and 0 for $i \neq j$. $\widehat{\Gamma}_{\chi,0}$ and $\widehat{\Gamma}_{\xi,0}$ are the contemporaneous variance-covariances of the dynamic common and idiosyncratic components, respectively.[8] This maximisation problem can be reformulated as a generalised eigenvalue problem

$$\widehat{\Gamma}_{\chi,0} \widehat{Z}_j = \widehat{\mu}_j \widehat{\Gamma}_{\xi,0} \widehat{Z}_j. \tag{10}$$

$\widehat{\mu}_j$ denotes the j-th generalised eigenvalue and $\widehat{Z}_j$ its $(N \times 1)$ corresponding eigenvector. After the calculation of the maximum of N eigenvalues, they are ranked in decreasing order of magnitude. According to the static factor model (1), the largest $r = q(s+1)$ eigenvalues should be used to determine the number of static factors. These factors are obtained as the product of the r eigenvectors corresponding to the largest eigenvalues and the vector of observable variables X_t:

$$\widehat{F}_t^{FHLR} = \widehat{Z}' X_t, \tag{11}$$

where $\widehat{Z} = (\widehat{Z}_1, \ldots, \widehat{Z}_r)$ is the $(N \times r)$ matrix of the stacked eigenvectors. Note that although the first steps to obtain the covariance matrix of the common components are essentially dynamic, the final step of the estimation of the factors is finding a linear combination of contemporaneous variables. Asymptotics for the dynamic factor model are discussed in Forni et al. (2003b). Auxiliary variables to be determined by the user are M, q and r.

Determining the factors according to Kapetanios/Marcellino (2004) The time-invariant state space model for estimating factors from large data sets proposed by Kapetanios/Marcellino (2004) is given by the prediction error representation

$$X_t = CF_t + E_t, \tag{12}$$

$$F_{t+1} = AF_t + KE_t, \tag{13}$$

[8]The off-diagonal elements of the covariance matrix of the idiosyncratic components are forced to be zero in order to improve the forecasting properties of the model. See Forni et al. (2003b), p. 16.

where X_t is again the N dimensional data vector, which is determined by the r factors F_t, and the innovations E_t.[9] The factors as well as the system matrices A, C and the Kalman gain K are assumed as unknown.[10] Kapetanios/Marcellino (2004) now suggest the application of subspace algorithms to estimate the factors from data where the cross section is large. In general, subspace algorithms aim at determining the number of factors and the factors themselves without specifying and identifying the full state space model, for example the system matrices.[11] For the estimation of the factors or states, the state space model can be written as a vector equation, which can be obtained by solving for the innovations. This leads to the equation

$$X_t^f = \mathcal{O}\mathcal{K}X_t^p + \mathcal{E}E_t^f, \tag{14}$$

which is central to all subspace algorithms.[12] The variables are stacked according to $X_t^f = (X_t', X_{t+1}', X_{t+2}', \ldots)'$ and $X_t^p = (X_{t-1}', X_{t-2}', X_{t-3}', \ldots)'$ and $E_t^f = (E_t', E_{t+1}', E_{t+2}', \ldots)'$. The coefficient matrices are given by

$$\mathcal{O} = \left[C', A'C', (A^2)'C', \ldots\right]', \tag{15}$$

$$\mathcal{K} = \left[K,\, (A - KC)K,\, (A - KC)^2K,\, \ldots\right], \tag{16}$$

$$\mathcal{E} = \begin{pmatrix} I & 0 & \cdots & 0 \\ CK & I & \ddots & \vdots \\ CAK & \ddots & \ddots & 0 \\ \vdots & \cdots & CK & I \end{pmatrix}. \tag{17}$$

Compared with the state space representation above, the factors or states are defined as $F_t = \mathcal{K}X_t^p$. Hence, the goal of the analysis is to estimate the matrix $\mathcal{K}$, which in turn gives an estimate for the factors. To obtain the factors, the model user has to decide on the model order r, the number of factors in our case, and approximate the regression coefficient $\mathcal{O}\mathcal{K}$ by a matrix with a reduced rank r.[13] For rank reduction in subspace algorithms, an SVD is typically used.[14] Up to now, the state space model is written in infinite dimensional vectors, and a truncation is necessary for empirical applications with finite data sets. To replace X_t^f

[9]See, for example, Bauer (1998), p. 2 and Bauer (2005).
[10]See Van Overschee/De Moor (1996), pp. 61-2.
[11]See Van Overschee/De Moor (1996), pp. 11-2.
[12]For a detailed description of the equation, see Bauer (1998), pp. 45-7.
[13]See Bauer (1998), p. 46.
[14]See, for example, Van Overschee/De Moor (1996), p. 75.

and X_t^p, the truncated matrices

$$X_{s,t}^f = (X_t', X_{t+1}', X_{t+2}', \dots, X_{t+s-1}')' \text{ and } X_{b,t}^p = (X_{t-1}', X_{t-2}', X_{t-3}', \dots, X_{t-b}')' \tag{18}$$

are used. Furthermore, since the factors are to be used for forecasting purposes, leads in the data matrices are ruled out by Kapetanios (2004), which implies $s = 1$ and $X_{s,t}^f = X_{1,t}^f = X_t$. Given these stacked data matrices, an estimator of the coefficient matrix $\mathcal{OK}$ would be obtained by regressing $X_{s,t}^f$ onto $X_{b,t}^p$.[15] However, when the data set to be used is large, there is a rank deficiency in the variance $X^{p\prime}X^p$ of the regressors, where X^p contains all the available stacked data according to $X^p = (X_{b,1}^p, \dots, X_{b,T}^p)'$. To overcome the rank deficiency, Magnus/Neudecker (1999) propose an estimate of $X^p(\mathcal{OK})'$ instead of only $\mathcal{OK}$.[16] From equation (14), this estimator of $X^p(\mathcal{OK})'$ is given by

$$\widehat{X^p(\mathcal{OK})'} = X^p(X^{p\prime}X^p)^+X^{p\prime}X^f, \tag{19}$$

where $X^f = (X_{s,1}^f, \dots, X_{s,T}^f)'$, and A^+ is the Moore-Penrose inverse of a matrix A. However, if the row dimension of X^p is smaller than its column dimension, the projection matrix becomes the identity matrix, $X^p(X^{p\prime}X^p)^+X^{p\prime} = I$, and the estimate reduces to $\widehat{X^p(\mathcal{OK})'} = X^f$. Note that this can easily be the case if relatively more time series are available than time series observations. Following the subspace literature to estimate the state space, a decomposition based on this estimate simply leads to an SVD of the contemporaneous data (for $s = 1$) for estimating the factors. To obtain an essentially dynamic factor estimate, Kapetanios/Marcellino (2004) propose an alternative estimator based on the coefficient $(X^{p\prime}X^p)^+X^{p\prime}X^f$, which is the right part of equation (19). This coefficient is decomposed by SVD according to

$$(X^{p\prime}X^p)^+X^{p\prime}X^f = \widehat{U}\widehat{S}\widehat{V}', \tag{20}$$

and the factors are then defined as $X^p\widehat{U}_r\widehat{S}_r^{1/2}$, or in time indexed variable notation

$$\widehat{F}_t^{KM} = \widehat{\mathcal{K}}X_t^p, \tag{21}$$

where $\widehat{\mathcal{K}}' = \widehat{U}_r\widehat{S}_r^{1/2}$, and $\widehat{U}_r$ denotes the first r columns of the left singular vector matrix $\widehat{U}$, and $\widehat{S}_r^{1/2}$ is the $(r \times r)$ upper left square matrix of the square root of the singular value matrix $\widehat{S}$ containing the largest singular values in descending order.[17] Note that this method incorporates lead/lag relationships between the variables and is essentially dynamic compared with the other methods. The auxiliary parameters of this procedure to be chosen are the

[15] Note that the coefficient matrix $\mathcal{OK}$ is now also truncated compared with equation (14). For details, see Bauer (1998), p. 46.

[16] See Magnus/Neudecker (1999), pp. 261-3.

[17] See Kapetanios/Marcellino (2004), p. 21.

number of factors r, and the truncation parameters s and b.

Comparison of the factor estimation methods The factor models described above differ primarily with respect to the dynamics underlying the factors. The dynamic factor model by Forni et al. (2003a, b), for example, relies on dynamic principal component analysis in the first step. The eigen decomposition of the spectral density of the data allows for more general dynamics of the factors than using only static principal components as in Stock/Watson (2002a, b). In particular, if $M = 0$ is chosen in (6), the spectral density collapses to the covariance matrix and hence encompasses the static factor approach in this respect. So at least in this first step, more general dynamics are allowed for using the dynamic principal component analysis. However, since the resulting weights of the dynamic principal components are two-sided filters in the time domain, and hence are often regarded as inappropriate for forecasting, a static generalised eigen decomposition is proposed by Forni et al. (2003a, b), which leads to static factors weights. Hence, compared with the dynamic loadings obtained in the first step, some of the dynamics are lost in the second step. Compared with the approach of Stock/Watson (2002a, b) that relies on the covariance matrix $\widehat{\Gamma}_0$ for obtaining factor weights, the approach by Forni et al. (2003a, b) relies on the common components variance $\widehat{\Gamma}_{\chi,0}$ and idiosyncratic variance $\widehat{\Gamma}_{\xi,0}$. Thus, possible differences between variables according to their common component-idiosyncratic component variance ratios are, in principle, better taken into account when the dynamic factor model approach by Forni et al. (2003a, b) is applied. The method by Kapetanios/Marcellino (2004) provides factor estimates that are linear combinations of variables at different periods included in X_t^p, see (21). This is in contrast to the other two factor estimation methods, where factor weights only link contemporaneous variables included in X_t, see (4) and (11). Therefore, the factor estimates proposed by Kapetanios/Marcellino (2004) have the potential to take into account more complicated dynamics than the other methods.

In forecast applications, the three methods differ not only with respect to the dynamic properties of the factor estimates, but also with respect to the implementation of the auxiliary parameters. Whereas the Stock/Watson (2002a, b) factors require only the specification of the number of static factors r, the dynamic factors Forni et al. (2003a, b) additionally require the specification of the number of dynamic factors q and the truncation lag parameter for spectral estimation M. The estimation of the dynamic factors by Kapetanios/Marcellino (2004) also requires r, and the truncation parameters for stacking the data matrices s and b. Although the dynamic factor models allow for a richer structure of the factor dynamics, it is important to choose the corresponding auxiliary parameters appropriately. Misspecifications might lead to large sampling errors that could overcompensate the benefits from a richer dynamic structure. This argument is stressed in Boivin/Ng (2005), where it is shown that Monte Carlo simulations based on models without misspecification of the auxiliary parameters provide a too optimistic view on the dynamic factor models compared with the static factors. In real data applications, the auxiliary parameters might be misspecified, implying a poorer forecast performance. Therefore, it cannot be expected that the static factor forecast proposed by Stock/Watson (2002a, b) are clearly outperformed by the more sophisticated dynamic factor models.

3 Factor forecasting

To evaluate the empirical performance of the described factor estimates, they have to be implemented into a forecasting model. This will be described below. Given the forecasting model, a variety of simulation designs are considered to allow for comparisons of relative forecasting accuracy between the various factor models. Since there is obviously no "best" way to carry out the simulations, a variety of simulation designs should ensure the robustness of the results.

Forecast equation The three types of estimated factors will be used for prediction in a dynamic estimation model. For forecasting purposes, a single equation model is estimated with the dynamic or multi-step estimation approach.[18] The forecasting model is specified and estimated as a linear projection of an h-step ahead transformed variable, y_{t+h} onto t-dated predictors. In our case, the predictors are the static and dynamic factors. More precisely, the forecasting models follow the setup in Forni et al. (2003a), Kapetanios/Camba-Mendez (2004) and have the form

$$y_{t+h} = \beta \widehat{F}_t + \alpha(L) y_t + \varepsilon_{t+h}, \tag{22}$$

where $\widehat{F}_t$ are the factors determined by the three factor estimation methods.[19] β is a coefficient matrix for the factors which is estimated by OLS for each forecast horizon h. Autoregressive terms are taken into account by the coefficients $\alpha(L)$, which is a polynomial with non-negative power of the lag operator L. The variable on the left-hand side of the forecasting equation is y_{t+h}, which is defined as the growth rate of the chosen time series between period t and period $t+h$, $y_{t+h} = \log(Y_{t+h}/Y_t) = \sum_{i=1}^{h} \Delta \log(Y_{t+i})$, where the original series, Y_t, is assumed to be integrated of order one.[20] The out-of-sample forecast for y_{T+h} given information in period T is then given by the conditional expectation $y_{T+h|T} = \widehat{\beta}\widehat{F}_T + \widehat{\alpha}(L) y_T$. As a simple benchmark for the factor models, we also consider the forecasting performance of a simple autoregressive model

$$y_{t+h} = \alpha(L) y_t + \varepsilon_{t+h}, \tag{23}$$

[18] See chapter 11 of Clements/Hendry (1998) for a detailed treatment of multi-step estimation.

[19] See Forni et al. (2003a), p. 1250.

[20] See Stock/Watson (2002a), p. 149.

although the main focus of the investigation will be on the forecast comparison of the three factor models described above. The dynamic estimation approach above differs from the standard one-step ahead approach. To forecast h periods ahead within the standard approach, one estimates the model with one lag, and then iterates that model forward to obtain h-step ahead predictions.[21] Choosing the left-hand side variables specified h periods ahead of the explanatory variables as in the dynamic estimation approach has two main advantages. First, additional equations for simultaneously forecasting the indicators F_t are not needed. Second, the potential impact of specification errors in the one-step ahead model can be reduced by using the same horizon for estimation as for forecasting. However, Monte Carlo simulation results by Marcellino et al. (2005) indicate that the direct estimation approach is not generally the best method to choose, although their simulations did not take into account factor forecasts. On the other hand, in the recent investigation by Boivin/Ng (2005), the direct approach works best overall in forecast comparisons of factor models. In the forecast simulations below, we follow Boivin/Ng (2005) and also use the direct estimation approach.

Forecast simulation design: Performance-based versus information criteria model selection To evaluate the empirical performance of the factor forecasting models described, a variety of simulation experiments are considered. Common to all the models is that forecasts are out-of-sample, and only in-sample information is used to estimate the factors. We compare the models in four simulation experiments that can be distinguished according to how the models are specified and how the estimation and forecast sample are related to each other. In particular, we compare recursive versus rolling window simulations as well as simulations with model specifications chosen after inspecting the forecast performance versus specifications based on information criteria.

The performance-based model selection is based on the estimation of models with various specifications of lags and numbers of factors. For example, upper bounds for the number of dynamic factors q, the number of lags s have to be provided for the estimation of dynamic factors proposed by Forni et al. (2003a, b). For the forecasting equation, a maximum number of lags for the autoregressive parameters has to be set. For each possible combination of auxiliary parameters, the factors are estimated and the forecasts are calculated. Finally, the specification with the lowest mean squared forecast error for each forecast horizon is chosen as the appropriate specification. This strategy follows Forni et al. (2003a), who estimate their models over a broad range of parameters and choose parameter combinations which minimise a forecast error criterion. The models with these parameter values yield a minimal value of the mean squared forecast error. This mean squared error is thus the best forecasting performance obtainable, at the horizon h, via the dynamic forecast procedure based on each of the models. It eliminates possible errors due to misspecification. In contrast,

[21]For a discussion, see Clements/Hendry (1998), pp. 243-6.

the information criteria model selection chooses a configuration of the models based only on time series information from the estimation sample. For example, to evaluate the specifications of factor models with respect to the number of factors, special information criteria are available that minimise the variance of the idiosyncratic components and penalise overparameterisation.[22] An argument in favour of performance-based experiments often found in the literature is a higher robustness in case of structural change. However, according to the simulation results of Inoue/Kilian (2003), the performance-based framework is not necessarily better than information criteria selection and tends to select overparameterised models. The performance-based exercise selects a model specification, which is constant over the whole sample of recursions. Hence, it provides insights into the relative accuracy of the best specifications of different forecasting models. A selected model specification has the maximum forecasting accuracy among the set of possible constant specifications for this model, and specification errors are neglected. Moreover, all the sample information is used to detect the best model specification ex-post. On the other hand, in a real-time environment, where no out-of-sample information is available for a forecaster, the model has to be inspected for possible respecification every period.[23] Hence, applying information criteria can use only in-sample information and may lead to time-varying specifications. Therefore, the two forecasting schemes have a different information content and it is difficult to compare them. All in all, the recent results by Inoue/Kilian (2003) are more in favour of the information criteria approach. However, these simulations are not explicitly concerned with factor models, and due to the widespread use of performance-based comparisons, the application below includes both approaches.[24] The values for the maximum of the auxiliary parameters in the performance-based simulations are presented in appendix A.2. For the model selection based on information criteria, the number of static factors is determined using the criterion IC_{p2} from Bai/Ng (2002), whereas the number of dynamic factors is determined using the criterion proposed in Bai/Ng (2005). Details can again be found in appendix A.2.

Forecast simulation design: Rolling versus recursive simulations In addition to the alternative ways to specify the forecast models, we compare different estimation samples for the alternative models. In particular, a comparison of recursive versus rolling window simulations is carried out. The recursive simulation scheme proceeds as follows: to obtain forecast steps that can be compared with realisations, the full time series sample is shortened. The first estimation sample period covers one third of the total time series sample. Forecasts are computed with a forecast horizon of $h = 1, \dots, 4$ and forecast errors are stored. Then the sample size is increased by one period, the model is reestimated, forecasts are computed and so on. For the rolling scheme, the initial forecast is the same as in the recursive scheme, but when an additional period of data is added after the first forecast, the first period

[22]For the number of static factors, these information criteria are provided by Bai/Ng (2002).
[23]For an application, see Stock/Watson (2002b), p. 1173.
[24]A complementary use of these methods can also be found in Boivin/Ng (2005).

of the initial estimation sample is also deleted. Hence, the estimation sample size in the rolling scheme remains constant, whereas the estimation sample size in the recursive scheme increases every period. However, the number of forecasts that can be compared with the data is equal for both methods. The recursive simulation scheme has the advantage of using all the data available at a certain point in time, whereas the rolling forecast skips information. However, the rolling forecast scheme might be preferable if some sort of structural change occurs in the sample.[25]

4 Predicting German GDP

Data The data set collected for Germany, which is explained in appendix A.1, contains 124 quarterly series over the sample period 1978:1-2004:1. We choose quarterly time series because we want to discuss the empirical properties of the factor model with respect to GDP which is available at quarterly frequency. In addition, this data set enables us to describe the economy on a broad basis because sectoral supply side data can be taken into consideration. The whole data set includes GDP and its expenditure components such as consumption and fixed capital formation, as well as gross value added by sectors. It also contains industrial production, received orders and turnover, disaggregated by sectors. Labour market variables that are taken into consideration are employment, unemployment and wages. Several disaggregated price time series, interest rates and spreads are also considered. Additionally, we use ifo survey time series such as business situation and expectations, assessment of stocks and capacity utilisation, and other series.

As is typical for the empirical literature on factor estimation using large data sets, the vector of time series will be preprocessed prior to estimation. First, the time series are corrected for outliers and then seasonally adjusted as explained in appendix A.1. Moreover, since the principal component analysis requires stationary time series for estimation, non-stationary time-series were appropriately differenced.[26] Finally, the series were normalised to have sample mean zero and unit variance.

Forecasting results using performance-based model selection We first report relative mean squared errors (MSE) which are computed out-of-sample relative to the MSE of the autoregressive model. A relative MSE less than one indicates a superior forecasting performance of a model for the chosen forecast horizon $h = 1, \ldots, 4$. Table 1 shows the relative MSEs and a ranking, where smallest relative MSEs are ranked first and largest relative MSEs last. The table contains results for out-of-sample simulations with performance-based model selection as discussed in section 3. The results from both panels of the table show that on average, all models provide smaller MSEs than the simple autoregressive model. Of

[25] See Giacomini/White (2004), p. 3.

[26] See, for example, Altissimo et al. (2002), Forni et al. (2001).

Table 1: Relative MSE, performance-based model selection
A. Recursive scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
SW	0.725	0.685	0.796	0.865	3	3	3	3
FHLR	0.670	0.648	0.754	0.793	1	1	2	1
KM	0.709	0.674	0.746	0.829	2	2	1	2

B. Rolling-window scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
SW	0.727	0.657	0.779	0.883	3	2	2	2
FHLR	0.664	0.617	0.758	0.850	1	1	1	1
KM	0.712	0.690	0.832	0.927	2	3	3	3

Notes: The table shows the mean-squared errors (MSE) of the various models relative to the MSE of the autoregressive model. 'SW' denotes the Stock/Watson (2002a, b) approach forecasts, 'FHLR' is the dynamic factor model forecast from Forni et al. (2003a, b), and 'KM' denotes the subspace factor model forecast proposed by Kapetanios (2004) and Kapetanios/Marcellino (2004). The ranking on the right-hand side of the panel ranks the series with the smallest relative MSE with respect to the AR model first.

the factor models, the model proposed by Forni et al. (2003a, b) outperforms the others with only one exception. The differences in forecast accuracy between the models based on Kapetanios (2004) and Stock/Watson (2002a, b) are not clear-cut. The Stock/Watson (2002a, b) model provides better forecasts in the rolling window simulation scheme, while the Kapetanios (2004) model provides better forecasts for the recursive simulations. Hence, in this simulation exercise, only the model proposed by Forni et al. (2003a, b) can outperform the static factor forecasts proposed by Stock/Watson (2002a, b), although the overall differences between the relative MSEs are small.

Forecasting results using information criteria for model selection Table 2 shows the relative MSEs and the respective ranking when information criteria model selection is applied in the out-of-sample simulation experiment. The relative MSEs from both panels of the table show that all models provide smaller or equal MSEs than the simple autoregressive model. Among the factor models, no model clearly outperforms the other models over all forecast horizons under investigation, and the ranking changes over the forecast horizons. The differences between the relative MSEs are small. However, the Kapetanios' (2004) method provides better forecasts than the other factor models in three out of four forecast horizons for both the recursive and the rolling window schemes. The forecasts of the dynamic

Table 2: Relative MSE, information criteria model selection
A. Recursive scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
SW	0.840	0.739	0.842	0.885	3	2	3	2
FHLR	0.829	0.756	0.817	0.881	2	3	2	1
KM	0.726	0.675	0.825	0.910	1	1	1	3

B. Rolling-window scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
SW	0.727	0.736	0.809	0.996	2	3	1	2
FHLR	0.731	0.707	0.812	1.000	3	2	2	3
KM	0.707	0.660	0.867	0.915	1	1	3	1

Notes: The table shows the mean-squared errors (MSE) of the various models relative to the MSE of the autoregressive model. 'SW' denotes the Stock/Watson (2002a, b) approach forecasts, 'FHLR' is the dynamic factor model forecast from Forni et al. (2003a, b), and 'KM' denotes the subspace factor model forecast proposed by Kapetanios (2004) and Kapetanios/Marcellino (2004). The ranking on the right-hand side of the panel ranks the series with the smallest relative MSE with respect to the AR model first.

factor model by Forni et al. (2003a, b) and the factor model proposed by Stock/Watson (2002a, b) have similar accuracy as they change ranks for different forecast horizons. When using information criteria chosen, it seems to be more difficult for the dynamic factor model proposed by Forni et al. (2003a, b) to estimate the dynamic factor structure than in a simulated out-of-sample forecast comparison, where the specifications are chosen according to the forecast performance. Hence, both the performance-based simulations and the simulations using information criteria show that the Stock/Watson (2002a, b) forecast model is hardly ever the best forecasting model, although it cannot be systematically outperformed by either of the other two methods.

Tests for equal forecasting accuracy Up to now, the investigation has relied on the validity of forecast MSE rankings. However, the differences in MSE behind these rankings could not be systematic due to sampling errors. If the differences in MSE were indeed not significant or systematic, the conclusions drawn from the rankings could not be taken at face value. In particular, the preliminary conclusion that the static factor model proposed by Stock/Watson (2002a, b) is outperformed by one of the more sophisticated factor models depending on the specification scheme, would not be warranted. Therefore, for each forecast horizon, it may be interesting to investigate whether the differences between the forecasting performance of the alternative models are systematic or not. For this purpose, pairwise tests for forecast accuracy are carried out. For the recursive simulation scheme, the test on equal forecasting accuracy proposed by Diebold/Mariano (1995) and West (1996) is applied. For the rolling scheme, the asymptotic justification of the same test is given in Giacomini/White (2004), theorem 6. The detailed results of these tests are given in appendix A.3. The results show that overall, the differences in MSE between the factor forecasts are not statistically significant; there are only a few exceptions. Therefore, according to the tests, the dynamic factor models proposed by Forni et al. (2003a, b) and Kapetanios (2004) do not seem to outperform the factor forecast model by Stock/Watson (2002a, b) in a systematic way.

Robustness checks To check the robustness of the results obtained so far, we change some of the settings of the simulation design. Firstly, the information criteria for selecting the number of static and dynamic factors are changed. In addition to the criterion IC_{p2} for the number of static factors used in the simulations above, we also take into account IC_{p1}, which also performed well in the Monte Carlo analysis by Bai/Ng (2002). Additionally, alternative rules for the number of dynamic factors for the model of Forni et al. (2003a, b) are applied. For example, the information criterion proposed by Breitung/Kretschmer (2005), which is based on canonical correlations of the static factors and its lags, is applied to determine the number of dynamic factors. The detailed descriptions of the information criteria used as well as the forecast results are provided in appendix A.4. The results show that neither criterion leads to a major change of the results obtained above. The alternative selection rules for determining the number of dynamic factors, in particular, cannot improve the forecasting accuracy of the model proposed by Forni et al. (2003a, b).

As a further check for robustness, the particular contribution of the factors to the overall forecasting performance was isolated by neglecting the autoregressive terms in the forecasting equation (22). Hence, the final forecasting equation used includes only the estimated factors, $y_{t+h} = \beta \widehat{F}_t + \varepsilon_{t+h}$. The comparison between the results obtained here and the results without autoregressive terms enables us to assess whether the factor forecasts are the key sources of the forecasting performance. If the forecasting performance deteriorates too much after eliminating autoregressive terms, this would indicate only a small information content of the factors in isolation. The results of this experiment can again be found in appendix A.4. The key conclusions are not altered by this robustness check. The ranking of the forecasting methods is similar to the results obtained above, although the factor forecasts by Stock/Watson (2002a, b) perform a little bit worse compared with the results obtained above. However, neither the Kapetanios (2004) nor the Forni et al. (2003a, b) factor model can outperform the static factor forecasts at all forecast horizons and simulation designs, and MSE differences are often not significant.

5 Discussion of the results and conclusions

This paper discusses the comparative forecasting accuracy of alternative types of factor models based on large data sets for German GDP. Of all the different factor models, the factor forecast model proposed by Stock/Watson (1999, 2002a, b) is hardly ever the best forecasting model in terms of relative MSE, and either the dynamic principal component model proposed by Forni et al. (2003a, b) or the dynamic factor model proposed by Kapetanios (2004) perform best in terms of forecast accuracy in most cases. However, neither of the dynamic factor models can provide better forecasts than the static model over all forecast horizons and different specifications of the simulation design. Moreover, statistical significant differences in forecast accuracy could be found in only a few cases. Therefore, despite the conceptual and theoretical advantages of the dynamic factor models proposed by Kapetanios (2004) and Forni et al. (2003a), their forecast performance in the data set used here is only slightly better than the baseline forecasting model by Stock/Watson (1999, 2002a, b) based on static principal components. These findings are in line with the mixed empirical results obtained from forecast comparisons in Forni et al. (2003a) and the conclusions drawn by Boivin/Ng (2005), who find that the dynamic factor models generally do not outperform the static Stock/Watson (2002a, b) method for estimating the factors using USA data. These overall mixed results raise the question of further methodological improvements of factor model estimation methods. Note that this paper makes no claim on the general usefulness of dynamic factor models, but, in some data sets, it seems to be the case that their theoretical advantages cannot be fully exploited for forecasting.

One direction for future research is the heterogeneity of the chosen data set, which may be worth further investigation. As shown in Boivin/Ng (2004), preselecting variables for factor estimation may improve the fit of the model because the data often does not represent a homogenous factor structure.[27] Following this discussion, data selection in the factor model context may be an important topic for future research.

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A Appendix

A.1 German data set

This appendix describes the panel of time series for the German economy. The whole data set for Germany contains 124 quarterly series over the sample period 1978:1-2004:1. The sources of the time series are the Bundesbank database, the National Accounts database of the Federal German Statistical Office, and Datastream. Some of the time series for unified Germany are available only for the time period after 1991. In order to obtain longer samples, the time series of West Germany and unified Germany were combined after rescaling the West German data to the unified German time series.[28] The national accounts data for West and unified Germany are both measured according to the ESA 95 (European System of National Accounts).

Because GDP is the reference series, all time series are quarterly or transformed by averaging into quarterly series. Moreover, natural logarithms were taken for all time series except interest rates, unemployment rates, and capacity utilisation. Stationarity was obtained by appropriately differencing the time series. Seasonal fluctuations were eliminated using Census-X12 if necessary. To eliminate scale effects, the series were centered around zero mean and standardised to have unit variance. Extreme outlier correction was done using a modification of the procedure proposed by Watson (2003). Large outliers are defined as observations that differ from the sample median by more than six times the sample interquartile range.[29] The identified observation is set equal to the respective outside boundary of the interquartile.

[28] This procedure avoids modelling regime shifts and follows numerous empirical studies based on German data. For example, the euro area-wide model proposed by Fagan et al. (2001) relies on German time series that are linked as described above, see Fagan et al. (2001), p. 52. See also Bandholz/Funke (2003), p. 295, for another application.

[29] See Watson (2000), p. 93.

Use of GDP and gross value added

1. gross domestic product
2. private consumption expenditure
3. government consumption expenditure
4. gross fixed capital formation: machinery & equipment
5. gross fixed capital formation: construction
6. gross fixed capital formation: other
7. exports
8. imports
9. gross value added: mining and fishery
10. gross value added: producing sector excluding construction
11. gross value added: construction
12. gross value added: wholesale and retail trade, restaurants, hotels and transport
13. gross value added: financing and rents
14. gross value added: services

Prices

1. consumer price index
2. export prices
3. import prices
4. terms of trade
5. deflator of GDP
6. deflator of private consumption expenditure
7. deflator of government consumption expenditure
8. deflator of machinery & equipment
9. deflator of construction

Manufacturing turnover, production and received orders

1. production: intermediate goods industry
2. production: capital goods industry
3. production: durable and non-durable consumer goods industry
4. production: mechanical engineering
5. production: electrical engineering
6. production: vehicle engineering
7. export turnover: intermediate goods industry

8. domestic turnover: intermediate goods industry
9. export turnover: capital goods industry
10. domestic turnover: capital goods industry
11. export turnover: durable and non-durable consumer goods industry
12. domestic turnover: durable and non-durable consumer goods industry
13. export turnover: mechanical engineering
14. domestic turnover: mechanical engineering
15. export turnover: electrical engineering industry
16. domestic turnover: electrical engineering industry
17. export turnover: vehicle engineering industry
18. domestic turnover: vehicle engineering industry
19. orders received by the intermediate goods industry from the domestic market
20. orders received by the intermediate goods industry from abroad
21. orders received by the capital goods industry from the domestic market
22. orders received by the capital goods industry from abroad
23. orders received by the durable and non-durable consumer goods industry from the domestic market
24. orders received by the durable and non-durable consumer goods industry from abroad
25. orders received by the mechanical engineering industry from the domestic market
26. orders received by the mechanical engineering industry from abroad
27. orders received by the electrical engineering industry from the domestic market
28. orders received by the electrical engineering industry from abroad
29. orders received by the vehicle engineering industry from the domestic market
30. orders received by the vehicle engineering industry from abroad

Construction

1. orders received by the construction sector: building construction
2. orders received by the construction sector: civil engineering
3. orders received by the construction sector: residential building
4. orders received by the construction sector: non-residential building construction
5. man-hours worked in building construction
6. man-hours worked in civil engineering
7. man-hours worked in residential building
8. man-hours worked in industrial building
9. man-hours worked in public building
10. turnover: building construction

11. turnover: civil engineering
12. turnover: residential building
13. turnover: industrial building
14. turnover: public building
15. production in the construction sector

Surveys

1. business situation: capital goods producers
2. business situation: producers durable goods
3. business situation: producers non-durable goods
4. business situation: retail trade
5. business situation: wholesale trade
6. business expectations for the next six months: producers of capital goods
7. business expectations for the next six months: producers of durable goods
8. business expectations for the next six months: producers of non-durable goods
9. business expectations for the next six months: retail trade
10. business expectations for the next six months: wholesale trade
11. stocks of finished goods: producers of capital goods
12. stocks of finished goods: producers of durable goods
13. stocks of finished goods: producers of non-durable goods
14. capacity utilisation: producers of capital goods
15. capacity utilisation: producers of durable goods
16. capacity utilisation: producers of non-durable goods

Labour market

1. residents
2. labour force
3. unemployed
4. employees and self-employed
5. employees
6. self-employed
7. volume of work, employees and self-employed
8. volume of work, employees
9. hours, employees and self-employed
10. hours, employees

11. productivity, per employee
12. productivity, per hour
13. wages and salaries per employee
14. wages and salaries per hour
15. wages and salaries, excluding employers' social security contributions
16. unit labour costs, per production unit
17. unit labour costs, per production unit, hourly basis
18. short-term employed
19. vacancies
20. unemployment rate

Interest rates, stock market indices

1. money market rate, overnight deposits
2. money market rate, 1 month deposits
3. money market rate, 3 months deposits
4. bond yields on public and non-public long term bonds with average rest maturity from 1 to 2 years
5. bond yields on public and non-public long term bonds with average rest maturity from 2 to 3 years
6. bond yields on public and non-public long term bonds with average rest maturity from 3 to 4 years
7. bond yields on public and non-public long term bonds with average rest maturity from 4 to 5 years
8. bond yields on public and non-public long term bonds with average rest maturity from 5 to 6 years
9. bond yields on public and non-public long term bonds with average rest maturity from 6 to 7 years
10. bond yields on public and non-public long term bonds with average rest maturity from 7 to 8 years
11. bond yields on public and non-public long term bonds with average rest maturity from 8 to 9 years
12. bond yields on public and non-public long term bonds with average rest maturity from 9 to 10 years
13. stock prices: CDAX
14. stock prices: DAX
15. stock prices: REX

Miscellaneous indicators

1. current account: goods trade
2. current account: services
3. current account: transfers
4. HWWA raw material price index
5. new car registrations

A.2 Model specification for forecast simulations

Information criteria model selection For the dynamic factor model proposed by Forni et al. (2003a, b), the auxiliary variables to be determined by the user are r, q and M as well as the autoregressive lag length of the forecast equation. In this paper, the number of static factors r is determined by the criterion IC_{p2} of Bai/Ng (2002), which is given by[30]

$$IC_{p2}(r) = \ln(V(r,F)) + r\left(\frac{N+T}{NT}\right)\ln(\min\{N,T\}). \tag{24}$$

The information criterion reflects the trade-off between goodness-of-fit on the one hand and overfitting on the other. The first term on the right-hand side shows the goodness-of-fit, which is given by the residual sum of squares

$$V(r,F) = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(X_{it} - C_i F_t\right)^2, \tag{25}$$

and depends on the estimates of the static factors and the number of factors. The residuals are given by $X_{it} - C_i F_t$, where C_i is a $(1 \times r)$ dimensional row vector of the parameter matrix C of the static model, see (2) in the main text. If the number of factors r is increased, the variance of the factors F_t increases, too, and the sum of squared residuals decreases. Hence, the information criteria have to be minimised in order to determine the number of factors. The penalty of overfitting, which is the second term on the right-hand side behind r in (24), is an increasing function of the cross-section size N and time series length T. In empirical applications, one has to fix a maximum number of factors, say $r_{\max}$, and estimate the model for all number of factors $r = 1, \ldots, r_{\max}$. The optimal number of factors minimises IC_{p2}. In the forecast comparison, we set $r_{\max} = 10$. The number of dynamic factors q is determined by the information criterion proposed by Bai/Ng (2005). This criterion takes the estimated static factors as given, and estimates a VAR of lag order p on these factors, where p is determined by the Bayesian information criterion (BIC). Then, a spectral decomposition of the residual covariance matrix $\widehat{\Gamma}$ is computed. Define a pseudo matrix $\widehat{\Gamma}(k)$ of $\widehat{\Gamma}$ according to $\widehat{\Gamma}(k) = \sum_{j=1}^{k} \widehat{c}_j \widehat{\beta}_j \widehat{\beta}_j'$, where $\widehat{c}_j$ is the j-th ordered eigenvalue of $\widehat{\Gamma}$, and $\widehat{\beta}_j$ is the corresponding eigenvector, and $k \leq r$. Define

$$\widehat{D}_k = \left\|\widehat{d}_{k+1} - \widehat{d}_k\right\| / \left\|\widehat{d}_0\right\|, \text{ where } \widehat{d}_k = \operatorname{vech}(\widehat{\Gamma}(k)), \tag{26}$$

and $\widehat{d}_0 = \widehat{d}_r$. $\widehat{D}_k$ is a measure of the marginal contribution to covariance when the number of dynamic factors is increased from k to $k+1$. The set of admissible numbers of dynamic factors is chosen by a boundary according to $\mathcal{K} = \{k : \widehat{D}_k < m/\min[N^{2/5}, T^{2/5}]\}$ where $m = 0.5$ is chosen following the Monte Carlo results in Bai/Ng (2005), p. 15. Finally, the number of

[30] See Bai/Ng (2002), p. 201.

dynamic factors is given by $\widehat{q}^{BN} = \min\{k \in \mathcal{K}\}$.[31] The Bartlett lag length M for calculating the spectral density in (6) is determined by $M = \text{round}\,(\frac{2}{3}T^{1/3})$, where T is the time series sample size in the recursive or rolling-window subsample.[32] In the forecasting equation (22), the lag length is determined according to the Bayesian information criterion with a maximum number of lags to check equal to four. For the model proposed by Kapetanios/Marcellino (2004), the auxiliary variables to be determined by the user are r, s and b. The number of factors r is chosen according to IC_{p2}, and the lead truncation in (18) is fixed at $s = 1$, so the left-hand side of the subspace core equation (14) contains only contemporaneous data, as suggested for forecasting purposes by Kapetanios (2004).[33] The regressor matrix has a lag truncation of order b according to $b = \text{round}\,(\ln(T)^{1.05})$.[34] In the forecasting equation, the lag length is again determined according to the Bayesian information criterion with a maximum number of lags to check equal to four. For the model by Stock /Watson (2002a, b), the only parameter to choose is r and the lag length in the forecast equation. This specification is carried out as above for the other models.

Performance-based model selection For the performance-based model selection maximum values are used for the auxiliary parameters. For the dynamic factor model proposed by Forni et al. (2003a, b), the auxiliary variables to be determined by the user are r, q and M, as well as the autoregressive lag length of the forecast equation. From the link between the number of static and dynamic factors, which is given by $r = q(p+1)$, we can also specify q and p, see (5). Here, $q_{\max} = 4$, $p_{\max} = 4$, and $M_{\max} = 4$ are used. In the forecasting equation (22), the maximum lag length is equal to four. For all possible specifications of p, q, M and lag length in the forecast equation, the model is estimated and forecasts are computed. Finally, the best-performing model according to the forecast MSE is chosen. For the model proposed by Kapetanios/Marcellino (2004), the auxiliary variables to be determined by the user are r, s and b. We set $r_{\max} = 8$, $s = 1$, and $b_{\max} = 4$. In the static factor model proposed by Stock/Watson (2002a, b), the parameter to specify is $r_{\max} = 8$.

A.3 Testing for equal forecast accuracy

In this section investigates whether the differences between the forecasting performance of the alternative models are systematic or not. For this purpose, pairwise tests for forecast accuracy are carried out. For the recursive simulation scheme, the test on equal forecasting accuracy proposed by Diebold/Mariano (1995) and West (1996) is applied. For the rolling scheme, the asymptotic justification of the same test is given in Giacomini/White (2004), theorem 6. Note that both tests are applied to compare the MSEs, so no estimation un-

[31] See Bai/Ng (2005), p. 12, proposition 2.

[32] This rule for M is chosen by Forni et al. (2000), p. 548, according to its favourable performance in Monte Carlo simulations.

[33] See Kapetanios (2004), p. 66.

[34] See Kapetanios/Marcellino (2004), p. 23.

certainty has to be taken into account for computing the test statistics.[35] Moreover, the alternative factor models are non-nested, so the tests by Diebold/Mariano (1995) and West (1996) can be directly applied.[36] The test statistic is constructed as follows: Consider two models A and B that both produce forecasts of the variable y_t in period t. The forecasts h periods ahead are conditional on information available in period $t-h$, and the forecast is the application of the conditional expectation operator $y_{A,t|t-h}$ and $y_{B,t|t-h}$ for model A and B, respectively. In the factor model context chosen here, the forecasts are provided using equation (22) in the main text. Calculate a sequence of P forecast errors for both models $e_{i,t,h} = y_t - y_{i,t|t-h}$ for $i = A, B$ and observations $t = 1, \ldots, P$. The Diebold/Mariano (1995) and West (1996) test for equal forecast accuracy is based on the time series of differences of the squared forecast errors, $d_{t,h} = e^2_{A,t,h} - e^2_{B,t,h}$. Under the null hypothesis, the sample mean of $d_{t,h}$, $\overline{d}_h = \frac{1}{P}\sum_t d_{t,h}$, is not significantly different from zero. The statistic is defined as

$$\mathrm{DM}_h = \frac{\sqrt{P}\,\overline{d}_h}{\sqrt{(1/P)\sum_{\tau=-(h-1)}^{(h-1)}[(h-|\tau|)/h]\sum_{t=|\tau|+1}^{P}\left(d_{t,h}-\overline{d}_h\right)\left(d_{t-|\tau|,h}-\overline{d}_h\right)}}, \tag{27}$$

where the denominator includes a heteroscedasticity and autocorrelation consistent estimate of the variance of $d_{t,h}$ assuming that the h-step ahead forecast errors are at most $(h-1)$-dependent.[37] The weighting scheme of the autocovariances follows Newey/West (1987). The statistic DM_h is standard normal distributed. Harvey et al. (1997) provide simulation results that suggest using a small sample correction for the statistic DM_h. The modified statistic is defined as $\mathrm{MDM}_h = \kappa\,\mathrm{DM}_h$ with $\kappa = P^{-0.5}\left[P+1-2h+P^{-1}h\,(h-1)\right]^{0.5}$ and its critical values should be taken from the $\mathrm{t}(P-1)$ distribution rather than the normal distribution. For the tests presented below, the small sample correction is used. The results of the application of the test to the factor models are given in table 3 for both the performance-based model selection and the model selection based on information criteria. The tables show p-values for the tests. For the performance-based model selection in panel A of table 3, there are no significant differences in forecasting accuracy at the 5% level. For the model selection based on information criteria, only the method proposed by Kapetanios/Marcellino (2004) can in one case outperform the other models at the 5% level. However, the overall impression is that the differences in MSEs are not systematic as the significance levels are large in most cases.

[35] See West (1996), p. 1073, McCracken/West (2002), p. 312.

[36] See McCracken/West (2002), p. 309. For the recursive scheme, the distinction of nested or non-nested models under consideration is not relevant for the asymptotic validity of the tests. See Giacomini/White (2004), p. 3.

[37] See Diebold/Mariano (1995), p. 254.

Table 3: Testing for equal forecasting accuracy
A. Performance-based model selection

Forecast horizon		Recursive scheme SW	Recursive scheme FHLR	Recursive scheme KM	Rolling scheme SW	Rolling scheme FHLR	Rolling scheme KM
1	SW	-	0.24	0.43	-	0.27	0.42
	FHLR		-	0.36		-	0.28
	KM			-			-
2	SW	-	0.16	0.43	-	0.30	0.32
	FHLR		-	0.36		-	0.21
	KM			-			-
3	SW	-	0.31	0.26	-	0.38	0.27
	FHLR		-	0.47		-	0.11
	KM			-			-
4	SW	-	0.20	0.31	-	0.21	0.30
	FHLR		-	0.38		-	0.13
	KM			-			-

B. Information criteria model selection

Forecast horizon		Recursive scheme SW	Recursive scheme FHLR	Recursive scheme KM	Rolling scheme SW	Rolling scheme FHLR	Rolling scheme KM
1	SW	-	0.39	0.06	-	0.44	0.38
	FHLR		-	0.11		-	0.36
	KM			-			-
2	SW	-	0.29	0.19	-	0.21	0.05
	FHLR		-	0.13		-	0.15
	KM			-			-
3	SW	-	0.42	0.40	-	0.43	0.13
	FHLR		-	0.45		-	0.15
	KM			-			-
4	SW	-	0.42	0.38	-	0.41	0.12
	FHLR		-	0.37		-	0.11
	KM			-			-

Notes: The table shows p-values of the test. The null hypothesis is pairwise equal forecast accuracy. The test is symmetric. Further information about the computation of the test is given in the appendix. 'SW' denotes the Stock/Watson (2002a, b) approach forecasts, 'FHLR' is the dynamic factor model forecast from Forni et al. (2003a, b), and 'KM' denotes the subspace factor model forecast proposed by Kapetanios (2004) and Kapetanios/Marcellino (2004).

A.4 Robustness checks of the results

Using alternative information criteria for model selection: static factors In the literature, several information criteria can be found to determine the number of static factors or the number of dynamic factors. In order to check the robustness of the results, the forecast simulations are carried out with different information criteria. With respect to the number of static factors, Bai/Ng (2002) also find that the criterion IC_{p1} given by [38]

$$IC_{p1}(r) = \ln(V(r, F)) + r\left(\frac{N+T}{NT}\right)\ln\left(\frac{NT}{N+T}\right) \tag{28}$$

performs well in Monte Carlo simulations. Hence, as a check for robustness, the forecasts will be computed using this criterion IC_{p1} instead of (24). Table 4 below shows the forecast performance of the factor models in terms of relative MSE compared with the reference autoregressive model. The reference model remains the same as in the main text, so the results can be directly compared with those from table 2 in the main text. Compared with

Table 4: Relative MSE, information criteria model selection using IC_{p1}
A. Recursive scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
SW	0.835	0.731	0.780	0.781	3	1	2	2
FHLR	0.829	0.757	0.719	0.754	2	3	1	1
KM	0.797	0.747	0.853	0.894	1	2	3	3

B. Rolling-window scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
SW	0.816	0.765	0.840	1.032	2	3	3	3
FHLR	0.792	0.802	0.879	1.043	3	2	2	2
KM	0.702	0.645	0.833	0.964	1	1	1	1

Notes: The table shows the mean-squared errors (MSE) of the various models relative to the MSE of the autoregressive model. 'SW' denotes the Stock/Watson (2002a, b) approach forecasts, 'FHLR' is the dynamic factor model forecast from Forni et al. (2003a, b), and 'KM' denotes the subspace factor model forecast proposed by Kapetanios (2004) and Kapetanios/Marcellino (2004). The ranking on the right-hand side of the panel ranks the series with the smallest relative MSE with respect to the AR model first.

the results in the main text, there is less clear evidence in favour of the Kapetanios (2004)

[38] See Bai/Ng (2002), p. 201.

model using the criterion IC_{p1} for the recursive scheme, whereas the results of the main text are clearly confirmed for the rolling scheme. Again, there does not seem to be one model that consistently outperforms the others.

Using alternative information criteria for model selection: dynamic factors To investigate the role of the choice of the correct number of dynamic factors in the model proposed by Forni et al. (2003a, b), we additionally investigate two other rules. Namely, we investigate the heuristic rule that selects the number of dynamic factors according to their marginal contribution to the variance of the vector of time series included in the factor model. In particular, the number of dynamic factors q is determined by the individual variance contribution of a dynamic factor to the overall variance of the panel of time series, where we set the marginal contribution boundary to 10%.[39] This selection rule will be denoted as $IC^{10\%}$ below. As a second rule, we apply the information criterion proposed by Breitung/Kretschmer (2005). This criterion is based on canonical correlation analysis of the static factors obtained from principal component analysis and relies on the estimation of a VAR of the factors of lag order p which is chosen by the Bayesian information criterion. The static factors and their lags are stacked into the matrix $\widehat{G}_t^{(p)} = [\widehat{F}_t', \ldots, \widehat{F}_{t-p}']'$. Then define

$$\widehat{S}_{ij} = \sum_{t=2}^{T} \widehat{G}_{t-i}^{(p)} \widehat{G}_{t-j}^{(p)\prime}, \; i, j \in \{0, 1\}. \tag{29}$$

and solve the generalised eigenvalue problem

$$\left| \widehat{\mu} \widehat{S}_{11} - \widehat{S}_{10} (\widehat{S}_{00})^{-1} \widehat{S}_{01} \right| = 0 \tag{30}$$

to obtain the eigenvalues $\widehat{\mu}_i$ for $i = 1, \ldots, r$ in decreasing order of magnitude. The information criterion is then given by

$$IC^{BK}(q) = \left(-T \sum_{i=r-q+1}^{r} \ln(1 - \widehat{\mu}_i) \right) + (r - q)^2 \ln(T). \tag{31}$$

Asymptotic properties and Monte Carlo results for this information criterion are provided in Breitung/Kretschmer (2005). In table 5, the results for the dynamic factor models using the three alternative information criteria are shown. The table shows that the selection rule by Bai/Ng (2005), denoted as $\widehat{q}^{BN}$ and used in the main text, provides the best forecasts overall. Hence, neither criteria used for cross-checking can improve the performance of the dynamic factor model proposed by Forni et al. (2003a, b) as presented in the main text. Note that the selection rules tend to choose quite different numbers of q. Whereas the $IC^{10\%}$ tends to select between two and three dynamic factors, and the criterion by Breitung/Kretschmer (2005)

[39] See Forni et al. (2000), p. 547.

Table 5: Relative MSE of dynamic factor model using different information criteria for the number of dynamic factors

A. Recursive scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
$\widehat{q}^{BN}$	0.829	0.756	0.817	0.881	2	1	1	1
IC^{BK}	0.827	0.855	0.828	0.920	1	3	2	2
$IC^{10\%}$	0.840	0.804	0.875	0.938	3	2	3	3

B. Rolling-window scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
$\widehat{q}^{BN}$	0.731	0.707	0.812	1.000	1	1	1	1
IC^{BK}	0.755	0.756	0.872	1.083	2	3	3	3
$IC^{10\%}$	0.779	0.724	0.828	1.000	3	2	2	2

Notes: The table shows the mean-squared errors (MSE) of the dynamic factor model forecast from Forni et al. (2003a, b) using three different criteria for selecting the number of dynamic factors q. 'BN' is the selection rule proposed by Bai/Ng (2005), and 'BK' denotes the information criterion by Breitung/Kretschmer (2005). '10%' denotes choosing the number of dynamic factors with marginal variance contribution to the whole vector of time series of 10%. The ranking on the right-hand side of the panel ranks the series with the smallest relative MSE with respect to the AR model first.

between two and four, the selection rule proposed by Bai/Ng (2005) selects between four and seven dynamic factors. This is in line with the empirical results for the USA provided by Bai/Ng (2005), where the selection rule $\widehat{q}^{BN}$ chooses considerably more dynamic factors than other methods. In our case, the larger number of factors also leads to a better forecast performance when applied to the German data set.

Eliminating AR terms in the forecasting equation The results for the forecasting models where autoregressive terms are neglected, in particular using the forecast equation $y_{t+h} = \beta\widehat{F}_t + \varepsilon_{t+h}$, can be found in table 6. The benchmark autoregressive model remains the same as in the main text. The results of the table show that the factor models with no autoregressive terms still have considerable advantages over the benchmark autoregressive model with respect to their forecasting performance, when the autoregressive terms in the factor forecasting equation (22) are neglected. Among the factor models, the Stock/Watson (2002a, b) static factor forecasts never rank first. As in the main text, when performance-based model selection is used, the Forni et al. (2003a, b) factors always provide better forecasts than the Stock/Watson (2002a, b) factors. If information criteria are used, the

Table 6: Relative MSE, no autoregressive terms in the forecast equation
I. Performance-based model selection

A. Recursive scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
SW	0.833	0.805	0.854	0.865	3	3	3	2
FHLR	0.767	0.689	0.805	0.793	2	1	2	1
KM	0.709	0.697	0.746	0.909	1	2	1	3

B. Rolling-window scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
SW	0.839	0.726	0.811	0.940	3	2	2	3
FHLR	0.734	0.695	0.791	0.890	1	1	1	1
KM	0.754	0.747	0.883	0.929	2	3	3	2

II. Information criteria model selection

A. Recursive scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
SW	0.831	0.817	0.846	0.899	3	3	2	2
FHLR	0.799	0.794	0.828	0.886	2	2	1	1
KM	0.740	0.714	0.878	0.940	1	1	3	3

B. Rolling-window scheme

	Relative MSE				Ranking			
Forecast horizon	1	2	3	4	1	2	3	4
SW	0.804	0.714	0.759	0.945	2	2	2	3
FHLR	0.812	0.731	0.758	0.940	3	3	1	2
KM	0.782	0.688	0.868	0.937	1	1	3	1

Notes: The factor model forecasts are based on forecasting equations with no autoregressive terms for GDP. The table shows the mean-squared errors (MSE) of the various models relative to the MSE of the autoregressive model. 'SW' denotes the Stock/Watson (2002a, b) approach forecasts, 'FHLR' is the dynamic factor model forecast from Forni et al. (2003a, b), and 'KM' denotes the subspace factor model forecast proposed by Kapetanios (2004) and Kapetanios/Marcellino (2004). The ranking on the right-hand side of the panel ranks the series with the smallest relative MSE with respect to the AR model first.

ranking between both models is sometimes reversed in the rolling-window scheme. The ranking between the dynamic factor model proposed by Forni et al. (2003a, b) and the dynamic factor model proposed by Kapetanios (2004) changes for different forecast horizons and simulations schemes. To check whether the differences are significant or not, again the tests for equal pairwise forecast accuracy are applied. Results can be found in table 7. Compared with the forecast equation, where AR terms are included, there are a few more cases where differences are significant. However, according to the tests, most of the differences in forecasting accuracy are not significant and the results are not clear-cut over the different forecast horizons and simulation designs. Therefore, the exclusion of AR terms does not alter the key message from the main text: although the factor forecasts proposed by Stock/Watson (2002a, b) have a slightly worse performance in terms of relative MSE, the differences in forecast accuracy are small and not systematic.

Table 7: Testing for equal forecasting accuracy, no autoregressive lags in forecast equation

A. Performance-based model selection

Forecast horizon		Recursive scheme SW	Recursive scheme FHLR	Recursive scheme KM	Rolling scheme SW	Rolling scheme FHLR	Rolling scheme KM
	SW	-	0.31	0.12	-	0.10	0.13
1	FHLR		-	0.25		-	0.33
	KM			-			-
	SW	-	0.00	0.01	-	0.32	0.35
2	FHLR		-	0.45		-	0.23
	KM			-			-
	SW	-	0.32	0.20	-	0.29	0.45
3	FHLR		-	0.27		-	0.16
	KM			-			-
	SW	-	0.20	0.26	-	0.22	0.45
4	FHLR		-	0.18		-	0.36
	KM			-			-

B. Information criteria model selection

Forecast horizon		Recursive scheme SW	Recursive scheme FHLR	Recursive scheme KM	Rolling scheme SW	Rolling scheme FHLR	Rolling scheme KM
	SW	-	0.05	0.07	-	0.28	0.36
1	FHLR		-	0.18		-	0.33
	KM			-			-
	SW	-	0.13	0.07	-	0.13	0.34
2	FHLR		-	0.12		-	0.23
	KM			-			-
	SW	-	0.23	0.37	-	0.46	0.08
3	FHLR		-	0.29		-	0.07
	KM			-			-
	SW	-	0.23	0.33	-	0.39	0.46
4	FHLR		-	0.29		-	0.48
	KM			-			-

Notes: The table shows p-values of the test. The null hypothesis is pairwise equal forecast accuracy. The test is symmetric. Further information about the computation of the test is given in the appendix. 'SW' denotes the Stock/Watson (2002a, b) approach forecasts, 'FHLR' is the dynamic factor model forecast from Forni et al. (2003a, b), and 'KM' denotes the subspace factor model forecast proposed by Kapetanios (2004) and Kapetanios/Marcellino (2004).

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