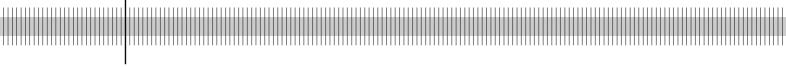


# Inflation Targeting Rules and Welfare in an Asymmetric Currency Area

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## Abstract

This paper studies the effect on monetary policy of a non-homogeneous degree of competition across the (two) members of a monetary union. In particular, we assess the welfare loss brought about by the use of a simple interest rate rule that does not take into account such structural differences. Our results show that, ceteris paribus, the welfare-maximizing central bank should react more aggressively to the inflation pressure generated by the more competitive economy. We extend the results of Benigno (2003) in two ways. First, we show that, if the degree of competition differs across countries, the optimal rule could involve a larger weight on the more 'flexible' country. Second, we allow for a non-unit-elastic demand for import. We show that this can alter Benigno's results even under a symmetric degree of competition. Our study suggests that the size of the welfare losses due to the neglect of these asymmetries crucially depends on their actual combination. Furthermore, we show that if information on the true extent of the asymmetries is incomplete, the symmetric rule could outperform the optimal rule.

**Keywords**: currency area; asymmetries; monetary policy rules; imperfect competition; sticky prices; second-order approximation.

JEL Classification: E3, E4, E5, F4.

## Non-technical Summary

The creation of the European Monetary Union has raised new challenges for the European policy makers. The European Central Bank (ECB) is now in charge of the monetary policy for the (currently) twelve members of the monetary union. A single interest rate is now set by the central bank in response to the inflationary pressures that stem from these twelve countries. This fact opens the question as to how a single monetary policy can be designed to respond to the – potentially idiosyncratic – inflation disturbances. The optimal design of a single monetary policy could be heavily complicated by the existence of differences in the functioning of the twelve economies.

A number of recent papers have addressed the question of the optimal design of a monetary policy rule in an "asymmetric" currency area. The present paper contributes to this literature by looking at the optimal response of the single interest rate to the inflation pressure that stems from two hypothetical countries that form a monetary union.

In particular, we look at two sources of asymmetry across countries, although we suggest that similar results are bound to emerge in the presence of other cross-country differences. The first asymmetry consists of a different degree of nominal rigidity in the prices for goods. That is, we assume that price adjustments are not synchronized across firms and, furthermore, on average the prices adjust more quickly in one country than in the other. If the central bank is concerned about the welfare of the members of the union (as assumed in our paper), an infrequent and asynchronous adjustment of the prices will be seen as detrimental, per se: the allocation of production is no longer dictated by the relative efficiency of production but rather by the different pace of price adjustments. The central bank will counter this by eliminating the reason for price changes, i.e. by pursuing a zero inflation policy. Yet, if the average speed of price adjustment varies across countries, different anti-inflationary policies would be recommended for each country. Since this is not feasible under a single monetary policy, the best policy of the central bank should be to react more strongly to the inflation pressure that originates from the country with less flexible prices. In practical terms this amounts to giving relatively more weight to the inflation component that originates from the "less flexible" country. This policy rule would yield a larger level of welfare compared to a rule that weights the inflation components simply by the size of the country (as done in the HICP measure of inflation used by the ECB).

This problem has already been addressed in the literature which also provided this "reasonable" policy prescription. However, we show that this recommendation is sensitive to some crucial aspects of the models used. In particular, we show that the trade interactions between the members of the union, as well as the type of financial markets that operate in the union, might alter these results. It is shown that the dynamics of the components of aggregate inflation that can be ascribed to each single country are sensitive to the degree of price rigidity in those countries. The more rigid the prices, the smaller the reaction of inflation to shocks. So, while on the one hand inflexible prices are detrimental for welfare because they bring about an inefficient allocation of output, inflexible prices tend to mitigate the response of inflation to shocks. Furthermore, we show that the sensitivity of inflation to the degree of price rigidity increases with the elasticity of the

current account with respect to the countries' relative prices. We show that if the foreign demand is sufficiently sensitive to the terms of trade, it is optimal for firms to adjust prices only in small amounts so that the moderating effect of the nominal rigidity on inflation will outweigh (in welfare terms) the distortional effects of price rigidity. This result was missing in the related literature due to the assumption of a constant trade balance typical made in this type of models.

The second source of asymmetry that we consider consists of differences in the degree of competition in the market for goods. Standard microeconomic theory suggests that welfare and degree of competition are positively correlated: perfect competition being the best scenario and perfect monopoly being the worst. Nevertheless, in a dynamic general equilibrium model with staggered price adjustments, the degree of competition has a "second best" quality. This can be seen by looking at the effect of a symmetric (positive) productivity shock. The firms that happen to adjust their price in the period of the shock will reduce their price to match the reduction in the cost of production, while the other firms are assumed to leave their price unchanged (despite having experienced the same change in costs of production). The demand for the different goods will "switch" from the latter group of firms to the former in proportion to the elasticity of substitution. The smaller this elasticity (small degree of competition) the smaller the "switching" and hence the smaller the inefficient reallocation of production. Due to this effect (and other things equal) the central bank should react more aggressively to the inflation pressure coming from the "more competitive" economy. In order to fully understand this result, the reader should notice that (in our model) the central bank is unable to affect the average degree of competition. Hence, the proposed rule should not be seen as aimed at reducing the degree of competition. The latter lies outside the realm of monetary policy.

When both asymmetries are present, it could be that the central bank reacts more aggressively to the inflation pressure coming from the country with more flexible prices, if the latter's market for goods is sufficiently more competitive. We also show that ambiguous outcomes emerge when the central bank has an incomplete information regarding the type and extent of the asymmetries. In particular a symmetric response to inflation (à la HICP) is not necessarily sub-optimal even in the presence of structural asymmetries.

Finally, we tentatively quantify the welfare losses that would be produced by a symmetric rule in the face of structural asymmetries. The quantified losses, as well as the uncertainty about the actual size and nature of the asymmetries, suggest that the symmetric approach (à la HICP) might be close to the first best. Nevertheless, a precise estimation of these losses requires further empirical investigation.

## Zusammenfassung

Die Gründung der europäischen Währungsunion stellt die geldpolitischen Instanzen Europas vor neue Aufgaben. Für die Geldpolitik der (derzeit) zwölf Mitgliedstaaten der Währungsunion ist jetzt die Europäische Zentralbank (EZB) zuständig. Die Zentralbank setzt nun einen einheitlichen Zinssatz fest, mit dem sie auf den Inflationsdruck aus diesen zwölf Ländern reagiert. Dies wirft die Frage auf, wie eine einheitliche Geldpolitik beschaffen sein muss, damit sie den - potenziell idiosynkratischen - inflatonären Störungen wirksam begegnen kann. Die optimale Ausgestaltung einer einheitlichen Geldpolitik könnte durch die vor-handenen Unterschiede in der Funktionsweise der zwölf Volkswirtschaften erheblich erschwert werden.

Eine Reihe neuerer Forschungsbeiträge befasst sich mit der Frage der optimalen Ausgestaltung einer geldpolitischen Regel in einem "asymmetrischen" Währungsraum. Auch der vorliegende Aufsatz leistet hier einen Beitrag, indem er untersucht, wie der einheitliche Zinssatz optimal auf den Inflationsdruck aus zwei gegebenen Ländern, die eine Währungsunion bilden, reagiert.

Wir werden insbesondere zwei Ursachen für Asymmetrien unter den Ländern betrachten, gehen aber davon aus, dass andere Unterschiede zwischen den Staaten zu ähnlichen Ergebnissen führen. Die erste Art der Asymmetrie besteht darin, dass die Güterpreise einen unterschiedlichen Grad an nominaler Rigidität aufweisen. Wir nehmen also an, dass Preisanpassungen von Unternehmen zu Unternehmen nicht gleichzeitig durchgeführt werden und dass die Preise zudem in einem Land schneller angepasst werden als in dem anderen. Wenn der Zentralbank am Wohlstand der Teilnehmer an der Währungsunion gelegen ist (wie in unserem Beitrag unterstellt), wird eine seltene und asynchrone Anpassung der Preise per se als Nachteil betrachtet: die Allokation der Produktion wird nicht mehr durch die relative Produktionseffizienz bestimmt, sondern durch das unterschiedliche Tempo der Preisanpassungen. Die Zentralbank wird dem begegnen, indem sie den Grund für die Preisänderungen beseitigt, d. h. sie wird eine Politik der Null-Inflation verfolgen. Wenn aber die durchschnittliche Geschwindigkeit, mit der die Preise angepasst werden, von Land zu Land abweicht, wäre für jeden Staat eine andere anti-inflationäre Politik angezeigt. Da dies im Rahmen einer einheitlichen Geldpolitik nicht möglich ist, wäre das beste Vorgehen der Zentralbank, stärker auf den Inflationsdruck aus dem Land mit den weniger flexiblen Preisen zu reagieren. Praktisch läuft dies darauf hinaus, der Inflationskomponente des Landes mit den weniger flexiblen Preisen relativ mehr Gewicht beizumessen. Diese Regel würde zu einem höheren Wohlstandsniveau führen als eine Regel, die die Inflations-komponenten lediglich nach der Ländergröe gewichtet (wie bei der von der EZB verwendeten HVPI-Inflation).

Dieses Problem wurde bereits in der einschlägigen Literatur behandelt, von der auch dieses "vernünftige" geldpolitische Rezept stammt. Wir weisen allerdings nach, dass diese Empfehlung stark von einigen zentralen Aspekten der verwendeten Modelle beeinflusst wird. Wir zeigen vor allem, dass die Handelsbeziehungen zwischen den Mitgliedern der Währungsunion sowie auch die Art der darin vorhandenen Finanzmärkte diese Ergebnisse verändern könnten. Es wird zudem deutlich, dass die Dynamik der Komponenten der aggregierten Inflation, die jedem einzelnen Land zugeschrieben werden kann, auf den Grad

der Preisrigidität in diesen Ländern reagiert. Je starrer die Preise sind, desto weniger reagiert die Inflation auf Schocks. Während demnach inflexible Preise dem Wohlstand einerseits abträglich sind, weil sie eine ineffiziente Produktionsallokation mit sich bringen, haben sie andererseits tendenziell die Neigung, die Reaktion der Inflation auf Schocks abzuschwächen. Wir legen darüber hinaus dar, dass die Sensitivität der Inflation gegenüber dem Grad der Preisrigidität mit der Elastizität der Leistungsbilanz in Bezug auf die relativen Preise des Landes zunimmt. Auerdem zeigen wir auf, dass es für Unternehmen, wenn die Auslandsnachfrage hinreichend stark von den Terms of Trade beeinflusst wird, am besten ist, ihre Preise nur in kleinen Schritten anzupassen, sodass die abschwächende Wirkung der nominalen Rigidität auf die Inflation gröer ist als die verzerrenden Effekte der Preisrigidität (im Hinblick auf den Wohlstand). Diese Erkenntnis fehlte bislang in der entsprechenden Literatur, da diese Modelle zumeist von der Annahme einer konstanten Handelsbilanz ausgehen.

Die zweite Ursache für Asymmetrien, die wir untersuchen, liegt in einem unterschiedlichen Wettbewerbsgrad am Gütermarkt begründet. Die klassische mikroökonomische Theorie geht davon aus, dass Wohlstand und Wettbewerbsgrad positiv miteinander korrelieren. Dabei gilt vollständiger Wettbewerb als günstigster, ein absolutes Monopol als ungünstigster Fall. In einem dynamischen allgemeinen Gleichgewichtsmodell mit zeitlich gestaffelten Preisan-passungen ist der Wettbewerbsgrad dessen ungeachtet nur zweitrangig. Dies wird ersichtlich, wenn man die Auswirkungen eines symmetrischen (positiven) Produktivitäts-schocks betrachtet. Unternehmen, die ihren Preis zufälligerweise in der Zeit des Schocks ändern, werden ihren Preis senken, um ihn an die niedrigeren Produktionskosten anzupassen, während die anderen Unternehmen ihren Preis unverändert lassen (obwohl sie von derselben nderung der Produktionskosten betroffen sind). Die Nachfrage nach den verschiedenen Gütern verschiebt sich proportional zur Substitutionselastizität von der zweiten Gruppe von Unternehmen zur ersten. Je geringer die Elastizität (geringer Wettbewerbsgrad), desto geringer die Verschiebung und desto geringer damit die ineffiziente Umverteilung der Produktion. Aufgrund dieses Effekts (bei ansonsten gleichen Bedingungen) sollte die Zentralbank aggressiver auf den Inflationsdruck aus der wettbewerbsintensiveren Volks-wirtschaft reagieren. Zur Erläuterung dieses Ergebnisses weisen wir darauf hin, dass die Zentralbank (in unserem Modell) nicht in der Lage ist, den durchschnittlichen Wettbewerbsgrad zu beeinflussen. Die vorgeschlagene Regel sollte deshalb nicht in dem Sinn miverstanden werden, dass die Zentralbank den Wettbewerbsgrad reduzieren will.

Wenn beide Asymmetrien gegeben sind, könnte es sein, dass die Zentralbank aggressiver auf den Inflationsdruck aus dem Land mit den flexibleren Preisen reagiert, falls dessen Gütermarkt hinreichend stärker wettbewerbsorientiert ist. Zu uneindeutigen Ergebnissen kann es kommen, wenn der Zentralbank unvollständige Informationen über Art und Ausma der Asymmetrien vorliegen. Insbesondere muss eine symmetrische Reaktion auf Inflation (wie beim HVPI) selbst bei strukturellen Asymmetrien nicht automatisch suboptimal sein. Abschlieend quantifizieren wir näherungsweise den Wohlstandsverlust, den eine symmetrische Regel bei strukturellen Asymmetrien mit sich bringen würde. Die quantifizierten Verluste wie auch die Unsicherheit über das tatsächliche Ausma und die Art der Asymmetrien lässt den Schluss zu, dass der symmetrische Ansatz (wie beim HVPI) nahe am

Optimum liegen könnte. Eine genaue Schätzung diese empirischer Untersuchungen.	er Verluste bedarf allerdings weiterer

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## Inflation Targeting Rules and Welfare in an Asymmetric Currency Area<sup>1</sup>

## 1 Introduction

This paper studies the effect on monetary policy of a non-homogeneous degree of competition and a non-homogeneous degree of nominal rigidity across members of a monetary union. In particular, we assess the welfare losses brought about by the use of a simple monetary policy rule that does not take into account such structural differences.<sup>2</sup>

The creation of the European Monetary Union (EMU) has stimulated new research in the theoretical aspects of the design of a monetary policy for a currency area. For example, Benigno (2003) has shown that the stabilization policy of a welfare-maximizing central bank should take into account differences in the degree of nominal rigidity across the countries participating in the monetary union. The reason why non-homogeneous degrees of price flexibility could be of concern for the monetary authority of the union is that these differences produce differences in the path of producers' inflation and, ultimately, differences in the degree of (inefficient) 'output dispersion' (Woodford (2003) and Khan et al. (2000)). In our paper we show that a similar type of inefficient output dispersion can be brought about by different degrees of imperfect competition even when the degree of nominal rigidity is identical across countries.

We show that the benevolent central bank of a monetary union should respond more aggressively to the inflation pressure coming from the more competitive economy, other things being equal. The rationale for this result is simple and parallels Benigno's result that the central bank should react more aggressively to the inflation generated in the country with less flexible prices. When prices are set in a staggered fashion, the amount of 'output dispersion' generated by a given deviation of prices from their average depends positively on the elasticity of substitution between goods (i.e. the degree of competition in our model). Therefore, the country with the largest degree of competition is the one that generates the greatest cost of inflation.

Our paper also extends the results of Benigno (2003) by finding that the optimal interest rule should react more aggressively to the *more* 'flexible' economy when the latter is sufficiently more competitive than the less flexible economy. Furthermore, by allowing the degree of substitutability between imported and domestic goods to differ from one, we show that (even under a symmetric degree of competition) the optimal rule could involve a relatively larger weight on the more 'flexible' country. The rationale of this result lies in

<sup>&</sup>lt;sup>1</sup>I wish to thank colleagues at the Bundesbank for their comments. I also wish to thank Thomas Werner, Leo von Thadden, Alan Sutherland, Huw Dixon, Robert Kollmann and Roel Beetsma for their suggestions and comments. I assume full responsibility for any remaining errors. This paper reflects the author's personal views and does not necessarily reflect the views of the Deutsche Bundesbank. Correspondence: Deutsche Bundesbank, Economic Research Centre, Frankfurt am Main, D-60431 - Germany. E-mail: Giovanni.Lombardo@bundesbank.de. Tel.: ++49-69-95662382. Fax: ++49-69-95663082

<sup>&</sup>lt;sup>2</sup>For example, the European Central Bank uses a measure of inflation (the euro-area HICP) that weights the individual countries' inflation by their consumption shares (see Table 4.1 of the Methodological Notes of the ECB Monthly Bulletin).

the dynamics of inflation (i.e. in the Phillips curve). When prices are very sticky, inflation does not vary very much. This effect counterbalances the effect that the degree of price rigidity has on the cost of inflation (i.e. on the degree of output dispersion).

The extent of the welfare gains from optimally weighting inflation depends on the actual combination of the various asymmetries. For example, it is likely that more competition brings about more price flexibility. In this case the effect of competition highlighted in this paper will tend to offset the nominal rigidity effect studied by Benigno (2003).<sup>3</sup>

A more general result of our paper is that imperfect competition, in a dynamic model with sticky prices, should be seen in a different way from the mere source of 'Harberger triangles'. In the presence of sticky prices it acquires a second-best quality: the less competition, the smaller the negative welfare effects of nominal price rigidity.

Our model modifies Benigno's two-country model by allowing the price elasticity of the demand for goods, as well as the degree of nominal rigidity of prices, to differ across countries. Furthermore, we do not restrict the preferences for domestic and imported goods to a Cobb-Douglas function. That is, we allow for current account imbalances. Like Benigno (2003), we study the problem of monetary policy from a welfare point of view (à la Woodford (2003)). The objective of the benevolent policymaker consists of taking the weighted average of the utility functions of the households living in the monetary union. The rest of the model is a typical New-Keynesian two-country model (e.g. Obstfeld and Rogoff (1995)) with staggered price-setting à la Calvo (1983).

Since in this paper we focus on the welfare cost of adopting a simple policy rule that neglects the structural differences across the members of the monetary union, we do not discuss the fully optimal rule.<sup>4</sup> The problem of optimally weighting the 'indicator variables' for monetary policy is a typical issue in the design of simple instrument rules (see Svensson (1999), and Svensson and Woodford (2003)). Here we focus, in particular, on the problem of aggregating the country-specific information on inflation when the degree of competition differs across countries.

From a technical point of view, our paper departs from the analysis of Benigno (2003) and of Beetsma and Jensen (2002) in that we take a full second-order approximation of the model in order to assess welfare.<sup>5</sup> As explained in Woodford (2003), the linear-quadratic solution to the policy problem is accurate only if the reduced-form welfare function does not contain first moments of the variables. In closed economy models this is generally the case when the non-stochastic steady state, around which the model is approximated, is a

<sup>&</sup>lt;sup>3</sup>From a theoretical point of view, there is no clear connection between price flexibility and monopoly power. Some empirical evidence seems to suggest that the intervals between price adjustments increase with market concentration (e.g. Carlton (1986) and Powers and Powers (2001)).

<sup>&</sup>lt;sup>4</sup>In an earlier version of this paper we followed more closely the work of Benigno and derived both the optimal policy with commitment as well as the optimal Taylor-type rule (Lombardo, 2002a). The approach followed here allows for a more compact and focused exposition besides requiring less restrictive assumptions.

<sup>&</sup>lt;sup>5</sup>Angelini et al. (2002) also consider the problem of monetary policy in an asymmetric currency area. Our work is different from their study in several respects. Our focus is mainly theoretical and is meant to extend on existing works on monetary policy in a currency area with two specific asymmetries, i.e. competition and the degree of nominal rigidity (Benigno (2003) and Beetsma and Jensen (2002)). Furthermore, the solution technique adopted is different.

Pareto-efficient equilibrium. In our two-country model this cannot be easily done. First, we study welfare outcomes under different degrees of competition. Imposing subsidies that would offset any existing degree of monopolistic distortion would make it harder to assess the welfare implications of the degree of competition. Second, in this paper we relax the assumption of a balanced current account in every period that is used in Benigno (2003) and Beetsma and Jensen (2002). This implies that the welfare function depends also on first moments, so that a second-order approximation of the structural equations is required. In this paper, for the sake of simplicity we assume that the two countries can exchange assets only in the form of bonds. We leave for future research the task of assessing the effects of alternative financial arrangements on welfare and monetary policy.

Following Sutherland (2001), we solve a 'quadratic-quadratic' policy problem. That is to say, we take a second-order approximation of the structural equations of the model as well as of the welfare function. This approach has been recently adopted by Sutherland (2002) and by Lombardo and Sutherland (2003) in open economy models with prices predetermined for one period.<sup>6</sup>

A number of other recent papers touch upon the relationship between imperfect competition and monetary policy, although none (so far) has studied the problem in a non-homogeneous currency area.

Schmitt-Grohé and Uribe (2001a) study the implication of different degrees of competition for the optimal fiscal and monetary policy of a closed economy. They show that the optimal inflation rate is positive. While the volatility of inflation decreases in the degree of competition, the volatility of consumption increases, as we find in our model. These authors do not report the net effect on welfare of the degree of competition.

Neiss (1999) studies the relationship between the degree of competition and the discretionary optimal inflation rate in a Barro-Gordon type of model. She shows that the optimal inflation rate is a non-monotonic function of the degree of competition. As King and Wolman (1999) show in a dynamic New-Keynesian model, the result obtained by Neiss would emerge if the central bank had to commit itself to a constant inflation rate. Without this commitment, the optimal inflation rate tends to zero asymptotically.

By introducing a welfare effect of holding money, Khan et al. (2000) show that the zero inflation result of King and Wolman (1999) does not hold. Instead, they find that a small deflation rate would be optimal. This deflation rate decreases in the degree of competition. Following Benigno (2003), Beetsma and Jensen (2002), Woodford (2003) and most of the recent New Open Economy literature (e.g. Obstfeld and Rogoff (2002), Corsetti and Pesenti (2001) and Devereux and Engel (2001)), we abstract from these monetary frictions.

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 discusses the assumptions regarding the financial market. Section 4 derives the objective of the policy maker and discusses the relationship between welfare, monetary policy and differences in the degree of competition and in the degree of nominal rigidity across

<sup>&</sup>lt;sup>6</sup>Recently, Benigno and Woodford (2003) have solved analytically an optimal monetary and fiscal policy problem in a closed economy using a second-order approximation technique very similar to the one used here. Given the objective of our paper, deriving the analytical solution to the policy problem would be overly involved algebraically and with little benefit in terms of intuition.

countries. Section 5 presents the numerical results. Section 7 concludes.

## 2 The model

We assume that the world economy consists of two countries (home country, H, and foreign country, F). These countries share the same currency and hence the same monetary authority. As described below, the two countries are entirely symmetric except for the degree of nominal rigidity of goods prices and the degree of competition in the markets for goods. The only stochastic disturbances are represented by uncorrelated productivity shocks.

## 2.1 Preferences

Households in country H and country F have identical preferences over leisure, real money balances and a bundle of consumption goods produced at home and abroad. This bundle can be represented by the following homothetic functions<sup>7</sup>

$$C^{j} = \left[ n^{1-\nu} C_{H}^{\nu} + (1-n)^{1-\nu} C_{F}^{\nu} \right]^{\frac{1}{\nu}}$$
 (1)

$$C_H^j = \left[ n^{\theta - 1} \int_0^n c_H^{\theta} \left( z \right) dz \right]^{\frac{1}{\theta}} \tag{2}$$

$$C_F^j = \left[ (1 - n)^{\theta - 1} \int_{n}^{1} c_F^{\theta}(z) \, dz \right]^{\frac{1}{\theta}} \tag{3}$$

where j = H, F and where  $(1 - \theta)^{-1}$  is the elasticity of substitution between domestically produced goods (henceforth denoted by  $\varepsilon^j$ ) and  $(1 - \nu)^{-1}$  is the elasticity of substitution between home and foreign goods (henceforth denoted by  $\varphi$ ). Furthermore,  $\theta = \theta^H$  if j = H and  $\theta = \theta^F$  when j = F.

The price indices associated with these aggregators are respectively

$$P = \left[ n P_H^{\frac{\nu}{\nu - 1}} + (1 - n) P_F^{\frac{\nu}{\nu - 1}} \right]^{\frac{\nu - 1}{\nu}} \tag{4}$$

$$P_{H} = \left[ \frac{1}{n} \int_{0}^{n} p_{H}(z)^{\frac{\theta}{\theta - 1}} dz \right]^{\frac{\theta - 1}{\theta}}$$

$$(5)$$

$$P_F = \left[\frac{1}{1-n} \int_n^1 p_F(z)^{\frac{\theta}{\theta-1}} dz\right]^{\frac{\theta-1}{\theta}}.$$
 (6)

Households supply a large variety of labour services, each specific to a firm producing goods in their own country: i.e. labour is immobile across countries and firms (as in Woodford (2003)).

<sup>&</sup>lt;sup>7</sup>To save on notation, we show the time subscripts only when different timing appears in the same equation or when otherwise convenient.

These aggregators and their prices imply the following homothetic demands for goods

$$c_{j}\left(z\right) = \left(\frac{p\left(z\right)}{P_{H}}\right)^{-\varepsilon^{j}} \left(\frac{P_{H}}{P}\right)^{-\varphi} C^{j}$$

where j = H, F.

Each household, say in country H, solves the following problem<sup>8</sup>

$$\max_{C_s, \frac{M_s}{P_s}, l_s(z)} E_t \sum_{s=t}^{\infty} \beta^s \left[ U\left(C_s\right) + \chi \log \left(\frac{M_s}{P_s}\right) + \frac{1}{n} \int_0^n V\left(l\left(z\right)_s\right) dz \right]$$
 (7)

subject to

$$M_s + B_s + D_s + P_s C_s = I_{s-1} \mathcal{P}(B_{s-1}) B_{s-1} + R_{s-1} D_{s-1} + M_{s-1} + \frac{1}{n} \int_0^n p_s(z) y_s(z) dz + \tau_s$$
(8)

and the transversality condition

$$\lim_{s \to \infty} E_t \left[ \bar{R}_{s,t} \left( \frac{D_{s+t} + B_{s+t} + M_{s+t}}{P_{s+t}} \right) \right] = 0.$$
 (9)

 $\bar{R}_{s,t} = \prod_{i=t+1}^s R_i^{-1}$  is the market discount factor such that  $\bar{R}_{t,t} = 1$ ,  $R_s$  is the gross interest rate on domestic assets,  $I_s$  is the gross return on foreign assets (the policy instrument),  $M_t$  is the money stock held by consumers,  $B_s$  is a nominal bond representing claims on goods produced by the other country in the union,  $D_s$  is a domestic nominal bond in zero net supply,  $\tau_t$  is a lump-sum tax/transfer and  $y_t(z)$  is total real output of the z-th firm. l(z) is labour supply to the z-th firm.  $\chi$  is a weight on real balances in the household objective. In fact, money is irrelevant in the present model since the monetary authority uses the nominal interest rate as the policy instrument. In addition, we assume in the welfare evaluation that  $\chi$  is of negligible size so that we can abstract from the real balance term in the consumer's preferences (see Woodford (2003)).  $\mathcal{P}(B_{s-1})$  is an interest rate risk 'premium' that depends on the aggregate net foreign asset position of the household's country. This premium will eliminate the unit root in the trade balance that would otherwise be induced by the presence of an international bond market. The role of this premium in our model is discussed in more detail in section 3.

The implied first order conditions relative to the choice of consumption, labour and real balances are respectively

$$U_c(C_t) = \left[\beta \frac{R_t}{(1 + E_t \hat{\pi}_{t+1})}\right] E_t U_c(C_{t+1})$$

$$\tag{10}$$

$$-\frac{V_l(l(z))}{U_c(C)} = \frac{W(z)}{P} \tag{11}$$

$$\frac{M_t}{P_t} = U_c \left( C_t \right)^{-1} \left( 1 - R_t^{-1} \right)^{-1} \tag{12}$$

<sup>&</sup>lt;sup>8</sup>A similar set of equations can be derived for country F.

where  $\frac{W(z)}{P}$  is the real wage paid by the  $z^{th}$  firm,  $U(C) = (1 - \sigma)^{-1} C^{1-\sigma}$  and  $V(l(z)) = -(\zeta + 1)^{-1} l(z)^{\zeta+1}$ .

A similar set of equations can be derived for the foreign country.

#### 2.2 Firms

There are a large number of firms in the union, which we index on the unit interval. Of these, n produce in country H and 1-n in country F. Each firm produces one type of good with the result that there are as many goods as firms. Each item is an imperfect substitute for the other goods, as indicated by the households' preferences. Each firm uses the same technology, that is (say for country H)

$$y(z) = A_H f(l(z), k) = A_H l(z)^{1-\alpha} \bar{k}^{\alpha}$$
(13)

where  $A_{H,t}$  is a country-specific technology shock identical across firms within each country, l(z) is the specific labour input and  $\bar{k}$  is any other factor (possibly capital) used in production. We assume that these other factors are fixed and normalized to one. This functional form implies that firms face increasing marginal costs and this, in turn, induces an effect of the degree of competition on the degree of real rigidity of prices (see the discussion in Kimball (1995) and Woodford (2003) and further below).

#### 2.2.1 Calvo contracts

We assume that firms adjust prices only at random intervals, in accordance with the mechanism described in Calvo (1983). That is, in any period of time there is a probability  $\omega$  that the firm does not adjust the price.

Each firm therefore chooses the optimal price by solving the following problem

$$\max_{p_t} E_t \sum_{s=t}^{\infty} (\omega)^{s-t} \bar{R}_{s,t} \left[ \frac{p_t}{P_s} y_s - \frac{TC_s}{P_s} \right]$$

where TC denotes total costs of production.

The solution to this problem yields

$$p_t = \frac{E_t \sum_{s=t}^{\infty} (\omega)^s \bar{R}_{s,t} \mu^j m c_s y_s}{E_t \sum_{s=t}^{\infty} (\omega)^s \bar{R}_{s,t} \frac{y_s}{P_s}}$$
(14)

where j = H, F, where mc is the real marginal cost,  $\mu^j = \frac{\varepsilon^j}{\varepsilon^j - 1}$  is the mark-up and where market clearing requires  $y_t = n c_{H,t} + (1 - n) c_{F,t}$ .

Owing to the factor specificity, the marginal cost of the firm is increasing with its own output, i.e.

$$mc\left(z^{j}\right) = \frac{W\left(z^{j}\right)}{P} \frac{1}{s^{j}\left(1-\alpha\right)} y\left(z^{j}\right)^{\frac{\alpha}{1-\alpha}} A_{j}^{\frac{-1}{1-\alpha}}$$

$$\tag{15}$$

where  $z^H \in [0, n]$ , and  $z^F \in (n, 1]$  and  $s^j \ge 1$  is a country-specific subsidy that, in principle, could counter the monopolistic distortion.

## 2.3 Monetary policy

In this work we assess the welfare losses due to the adoption of inflation targeting rules (à la Taylor (1993) or Rotemberg and Woodford (1998)) when the country-specific inflation components are weighted by the size of the countries without taking account of their structural differences.

Following Söderlind (1999), for a given vector of endogenous variables  $X_t$  such a simple rule for the interest rate can be written as

$$I_t = \overline{\mathbf{f}}' X_t$$

where  $\bar{\mathbf{f}}$  is a conformable vector of parameters. The problem of finding an optimal simple rule amounts to finding the elements of the vector  $\bar{\mathbf{f}}$ .

Thanks to the second-order solution method adopted in this paper, the elements of  $\bar{\mathbf{f}}$  that maximize the welfare function to a second order of approximation can be obtained by means of a solution algorithm for *linear* systems.

Within the family of simple rules, inflation targeting rules have gained particular attention in recent years.<sup>9</sup> This is also the focus of our paper.

Let us then define the vector of endogenous targets as  $X_t = [\hat{\pi}_t^H, \hat{\pi}_t^F]'$ , where a on a variable denotes its log-deviation from the non-stochastic steady state. Extending this vector to include the output gap would not alter our results regarding the optimal weighting of inflation.

Further below we show that the optimal parameters  $\bar{\mathbf{f}}_1$  and  $\bar{\mathbf{f}}_2$  are generally not symmetric when the two countries' degree of competition (or degree of nominal rigidity) differs. We then compare the performance of the optimal interest rate rule with the performance of a rule that imposes symmetry. We also assume that the interest rule is inertial, i.e.

$$\hat{I}_t = 0.9\,\hat{I}_{t-1} + \overline{\mathbf{f}}'\,X_t.$$

This assumption facilitates the numerical computation of the optimal weights without affecting our results. $^{10}$ 

The central bank commits itself to follow the stated rule. This rule is implemented by supplying the necessary quantity of money that clears the market. In each period the following constraint holds for the central bank

$$M_t - M_{t-1} = \tau_t.$$

<sup>&</sup>lt;sup>9</sup>See also Svensson (1999); Woodford (2003); Clarida et al. (2000). Somewhat in contrast to the taxonomy of rules suggested by Svensson (1999), here we call an inflation targeting rule a rule that (optimally or not) links the nominal interest rate to the inflation rate: i.e. what Svensson calls an instrument rule.

<sup>&</sup>lt;sup>10</sup>In order to obtain second moments we need a solution for the price level dynamics. It is well known that an interest rate rule would not be sufficient to anchor the value of the price level: the nominal quantities would display a unit root. While this is not relevant for welfare in our model, since the latter depends on the inflation rate and not on the price level, it constitutes a problem for the derivation of the second moments. As customary in interest rate rule models (e.g. Corsetti and Pesenti (2001)), we augment the interest rate rule by a small feedback from the nominal producers' prices (of magnitude 10<sup>-4</sup>).

#### 2.4 Forcing process

Productivity shocks are the only stochastic shocks discussed in this paper. Productivity is modelled like an AR(1) process, i.e.  $\hat{A}_{H,t} = \rho_H \hat{A}_{H,t-1} + \epsilon_{H,t}$ , where  $\epsilon_{H,t}$  is an  $iid(0, \sigma_H^2)$ . A similar equation applies to the foreign country's shock. Furthermore, we assume that  $\rho_H = \rho_F$  and  $\sigma_H = \sigma_F$ .

## 3 The international financial market

In this paper we adopt the second-order approximation method of Sutherland (2001), to solve the optimal policy problem as well as to evaluate the welfare loss due to a policy rule that neglects the asymmetries of the two economies. This method amounts to taking a second-order Taylor expansion of the welfare function (à la Woodford (2003)), as well as of the structural equations of the model, around the non-stochastic steady state. The second-order approximation of the structural equations allows us to obtain first moments that are accurate to a second order of approximation. Therefore, in order to evaluate welfare up to a second order of approximation (as done among others in Woodford (2003), Benigno (2003), Beetsma and Jensen (2002) and Sutherland (2002)) we do not need to impose efficiency of the non-stochastic steady state.<sup>11</sup>

In the general case of a CES aggregator of home and foreign consumption goods, the international financial market plays an important role since current account imbalances will be financed through international transfers of wealth. Obviously, there are various ways in which the international financial market can be modelled (see Obstfeld and Rogoff (1996, chap. 5)). For reasons of space it would not be possible to consider here the implications of different assumptions regarding the financial market for the issue addressed in this paper. Therefore, we restrict our attention to the case of a bond economy (as in Obstfeld and Rogoff (1995)), where the two countries' households can hold foreign assets only in the form of bonds (as described by the budget constraint (8)).

It is well known that the assumption of trade in bonds brings about a unit root (Schmitt-Grohé and Uribe, 2001b) in foreign assets and the trade balance. This fact is problematic for the accuracy of the approximation of the model around the non-stochastic steady state, since the economy is non-stationary around this equilibrium. Nevertheless, it is well known that introducing a wedge between the interest rate faced by the two countries can re-establish the uniqueness of the stationary equilibrium (Schmitt-Grohé and Uribe, 2003). This asymmetry can be produced by financial market frictions: e.g. 'iceberg' transaction costs in the asset market. We follow this strategy in order to eliminate the non-stationarity of our model economy (as in Lombardo (2002b)). For the sake of simplicity we assume that the premium decreases in the aggregate net foreign asset position of each country. That is, the domestic interest rate premium is not affected by the decision of a

<sup>&</sup>lt;sup>11</sup>This point was first raised by Kim and Kim (2003). They show that the log-linearisation can bring about spurious 'welfare reversals'. See Woodford (2003, chap. 6) for a discussion of this problem.

<sup>12</sup>This 'premium' is generally calibrated so that its overall effect on the dynamics of the system is minimized. Schmitt-Grohé and Uribe (2003) discuss alternative ways to deal with this problem for the case of small open economies. They show that these alternatives produce very similar results.

single household.

The first order conditions relative to the choice of foreign assets yields

$$R_t = I_t e^{-\rho_B \frac{B_t}{P_{ss}C_{ss}}} \tag{16}$$

$$R_t^* = I_t e^{-\rho_B \frac{B_t^*}{P_{ss}C_{ss}^*}} \tag{17}$$

where  $\rho_B$  is a constant that will be calibrated in order to eliminate the unit root in foreign assets and where the subscript ss denotes steady-state values (see Appendix A).

The second-order approximation to the bond holding yields for the home country

$$\widetilde{B}_{t} = \beta^{-1} \widetilde{B}_{t-1} + \hat{P}_{H,t} + \hat{Y}_{H,t} - \hat{P}_{t} - \hat{C}_{t} + \frac{1}{2} \left( \hat{P}_{H,t} + \hat{Y}_{H,t} \right)^{2} - \frac{1}{2} \left( \hat{P}_{t} + \hat{C}_{t} \right)^{2} - \frac{\rho_{B} \beta^{-1}}{2} \widetilde{B}_{t-1}^{2} + \mathcal{O} \left( \|\xi\|^{3} \right)$$

$$(18)$$

and a similar expression for the foreign country.  $^{13}$ 

## 4 Welfare

In this section we derive the second-order approximation to the household utility function. This will be assumed to be the objective function of the benevolent central bank. Notice first that the aggregate of the labour supply in the utility function, together with the production function, the demand equation faced by the firm and the Calvo distribution of firms imply

$$v(y) \equiv \frac{1}{n} \int_0^n \frac{(l_t(i))^{\zeta+1}}{\zeta+1} di = \frac{(A_t Y_{H,t})^{\frac{\zeta+1}{1-\alpha}} X_t}{\zeta+1}$$
 (19)

where

$$X_{t} = (1 - \omega^{H}) \left(\frac{p_{i,t}}{P_{H,t}}\right)^{-\varepsilon^{H} \frac{\zeta+1}{1-\alpha}} + \omega^{H} X_{t-1} \left(\pi_{t}^{H}\right)^{\varepsilon^{H} \frac{\zeta+1}{1-\alpha}}.$$
 (20)

Then the second-order approximation to the household utility function is simply

$$W^{H} = U_{c}C_{ss}\left(\hat{C} + \frac{1-\sigma}{2}\hat{C}^{2}\right) - v(y_{ss})\left(\hat{v}(y) + \frac{1}{2}\hat{v}(y)^{2}\right) + \mathcal{O}\left(\|\xi\|^{3}\right)$$
(21)

for the home country and a similar one for the foreign country  $(\mathcal{W}^F)$ .

To a second-order approximation, the objective of the central bank of the currency area is assumed to be the following  $^{14}$ 

$$\mathcal{W}^{W} = E_0 \sum_{t=1}^{\infty} \beta^t \left( n \mathcal{W}^H + (1-n) \mathcal{W}^F \right) + \mathcal{O}\left( \|\xi\|^3 \right)$$
 (22)

 $<sup>\</sup>overline{^{13}}$ We denote the approximation error by  $\mathcal{O}(\|\xi\|^3)$ .  $\xi$  is the vector of exogenous disturbances.

<sup>&</sup>lt;sup>14</sup>The computation of second-order accurate conditional moments raises some numerical difficulties. For the solution of these problems I benefited from discussions with Alan Sutherland. Clearly any remaining error would be my sole responsibility.

#### 4.1 Inflation and welfare

It is convenient at this point to make explicit the relationship between inflation, output dispersion and welfare.

For this purpose we need to take a second-order approximation of equation (20), i.e.

$$\hat{X}_{t} = \omega^{H} b \, \hat{\pi}_{t}^{H} - b(1 - \omega^{H}) \, (\hat{p}_{i,t} - \hat{P}_{H,t}) + \omega^{H} \hat{X}_{t-1} + \frac{(1 - \omega^{H})}{2} \, \omega^{H} \, \left( b \, \hat{\pi}_{t}^{H} + b \, (\hat{p}_{i,t} - \hat{P}_{H,t}) + \hat{X}_{t-1} \right)^{2} + \mathcal{O} \left( \|\xi\|^{3} \right) \tag{23}$$

where  $b = \varepsilon^H \frac{\zeta+1}{1-\alpha}$ .

In the Appendix B we show that

$$\hat{p}_{i,t} - \hat{P}_{H,t} = \frac{\omega^H}{1 - \omega^H} \hat{\pi}_t^H - \frac{\omega^H}{2} \frac{(1 - \varepsilon^H)}{(1 - \omega^H)^2} (\hat{\pi}_t^H)^2 + \mathcal{O}(\|\xi\|^3).$$
 (24)

By combining equation (24) and (23) we obtain

$$\hat{X}_{t} = \frac{b\omega^{H}}{2(1-\omega^{H})} \left(1 - \varepsilon^{H} + \varepsilon^{H} \frac{\zeta + 1}{1-\alpha}\right) \left(\hat{\pi}_{t}^{H}\right)^{2} + \omega^{H} \hat{X}_{t-1} + \frac{\left(1 - \omega^{H}\right)}{2} \omega^{H} \left(\frac{2b}{1 - \omega^{H}} \hat{\pi}_{t}^{H} \hat{X}_{t-1} + \hat{X}_{t-1}^{2}\right) + \mathcal{O}\left(\|\xi\|^{3}\right).$$
(25)

Notice that to a first-order approximation, the auxiliary variable  $\hat{X}_t$  depends only on its past value, i.e. it has no forcing process attached to it. Hence, to a first-order approximation it is constant over time. Its variance and its covariance with inflation are zero. Therefore, we can rewrite equation (25) as

$$\hat{X}_{t} = \frac{\varepsilon^{H} (\zeta + 1)\omega^{H}}{2(1 - \omega^{H})(1 - \alpha)} \left( 1 - \varepsilon^{H} + \varepsilon^{H} \frac{\zeta + 1}{1 - \alpha} \right) \left( \hat{\pi}_{t}^{H} \right)^{2} + \omega^{H} \hat{X}_{t-1} + \mathcal{O} \left( \|\xi\|^{3} \right). \tag{26}$$

Since  $\left(1 - \varepsilon^H + \varepsilon^H \frac{\zeta + 1}{1 - \alpha}\right)$  is positive, the dispersion of output and, hence, the disutility of labour increase with the variance of inflation. Importantly, this relationship is stronger the smaller  $(1 - \alpha)$ , i.e. the smaller the share of labour in production. Furthermore, the degree of competition and the degree of nominal rigidity interact in a complex way.

From equations (26), (19), (21) and (22) it should now be clear that the optimizing central bank, in choosing the optimal rate of inflation in the two countries, will take into account the relative degree of competition as well as the relative degree of nominal rigidity.

account the relative degree of competition as well as the relative degree of nominal rigidity. In particular, since  $\frac{\partial^2 \hat{X}_t}{\partial (\pi_t^H)^2 \partial \omega^H} > 0$ , the optimizing central bank – other things being equal – will react more aggressively to the country with less flexible prices. Similarly, since  $\frac{\partial^2 \hat{X}_t}{\partial (\pi_t^H)^2 \partial \varepsilon^H} > 0$ , the optimizing central bank – other things being equal – will react more aggressively to the country with a more competitive domestic market.

Note, nevertheless, that the degree of competition and the degree of nominal rigidity contribute to the determination of the dynamics of inflation and, hence, to its variance.

To a second order of approximation the variance of inflation depends only on first order terms. Appendix C shows that the first order approximation to the dynamic equation of inflation is given by

$$\hat{\pi}_t^H = \kappa_1^H \left( \kappa_3 \left( 1 + \varphi \frac{\zeta + \alpha}{1 - \alpha} \right) \hat{T}_t + \hat{C}^W + \hat{\phi}_t \right) + \beta E_t \left( \hat{\pi}_{t+1}^H \right) + \mathcal{O} \left( \|\xi\|^2 \right)$$
 (27)

where  $\phi_t$  depends on home consumption and on the home productivity shock. The parameter  $\kappa_3$  depends on the size of the countries and decreases in  $\varepsilon^H$  if  $\varphi > 1$ .<sup>15</sup> The other key parameter in this equation is  $\kappa_1$  (see the Appendix). This parameter decreases in  $\omega^H$ : if prices do not vary at all, there cannot be any inflation. It also decreases in the degree of competition (i.e. in  $\varepsilon^H$ ). This effect has been described by Kimball (1995). The more competitive the market the larger the degree of real rigidity of prices: i.e. the firms that adjust the price will do this by small amounts.<sup>16</sup> These counteracting effects of the degree of competition and of the degree of nominal rigidity have implications for monetary policy. We explore these numerically in the following sections.

## 5 Results: the case of $\varphi = 1$

In this section we study numerically the effect on monetary policy and welfare of different degrees of competition and of nominal rigidity, under the assumption of  $\varphi = 1$  (i.e. the case studied in Benigno (2003) and Beetsma and Jensen (2002)). Welfare changes are transformed into percentages of steady state consumption.<sup>17</sup>

In the first subsection we consider the effect of competition on welfare under the optimal simple rule in a symmetric union. In the second subsection we measure the gains from optimally weighting inflation when the degree of competition differs across countries. Finally, in the third subsection we extend the results of Benigno (2003) by showing the combined effect of different degrees of competition and different degrees of nominal rigidity across countries on monetary policy and welfare.

Except for the parameters discussed in the following three subsections, we fix the value of some parameters to numbers that are within ranges used in the related literature (in particular Benigno (2003) and Beetsma and Jensen (2002)). We set  $\beta=0.99$ ,  $\sigma=6$ ,  $\zeta=2$ , and  $\omega^H=\omega^F=0.5$  unless differently specified. As for the labour share,  $1-\alpha$ , we follow Galí et al. (2001) by setting it equal to  $0.75/\bar{\mu}$  (unless differently specified), where  $\bar{\mu}=1.15$  is the benchmark value of the mark-up. The innovation to the productivity shocks is assumed to have a variance of  $\sigma_H^2=\sigma_F^2=0.01$ . Our welfare measures will be proportional to this variance.

<sup>&</sup>lt;sup>15</sup>More precisely, if  $\varphi > 1$ ,  $\kappa_3$  decreases in the steady-state terms of trade. The latter increase in  $\varepsilon^H$  (see equation (31)).

<sup>&</sup>lt;sup>16</sup>This fact has some empirical support. Carlton (1987), among others, provides some evidence of a positive relationship between the degree of market concentration and the size of price adjustments.

<sup>&</sup>lt;sup>17</sup>If we denote such percentages by c%, then c solves  $\mathcal{W}^W = \frac{\left(nC_{ss}^{(1-\sigma)} + (1-n)C_{ss}^{*}^{(1-\sigma)}\right)\left(c^{(1-\sigma)} - 1\right)}{(1-\beta)(1-\sigma)}$  so that c = 100% means that the equivalent loss (gain) in steady state consumption is zero.

The degree of persistence of the productivity shock, besides co-determining the variance of the technology shock, affects welfare in a special way. The correlation between output and the shock enters with a positive sign in the welfare function and is increasing in  $\rho_H$  (see equation (19)). The household gains if the demand is high when the productivity is high. Our main result is not qualitatively affected by the value of the persistence of the shocks so that we set it at  $\rho_H = 0.8$ .

Finally, we set the risk premium parameter  $\rho_B$  to  $10^{-6}$ .

## 5.1 Competition and welfare under the optimal simple rule

In this section we study the effect of the degree of competition on welfare as well as on the optimal monetary policy. In order to isolate this effect from the consequences of asymmetric structures, we assign the same degree of competition and of nominal rigidity to both countries. We further assume that there are no subsidies offsetting the monopolistic distortion (i.e.  $s^H = s^F = 1$ ).

For convenience we redefine the policy vector as  $\overline{\mathbf{f}} \equiv a \, \mathbf{f}$  where  $\mathbf{f_2} = (1 - \mathbf{f_1})$  and a is a constant. Under symmetry  $\mathbf{f_1} = 0.5$ . Therefore, in this section we study the relationship between the average elasticity of demand  $(\varepsilon^W \equiv n \, \varepsilon^H + (1 - n) \, \varepsilon^F)$ , welfare and the policy parameter a.

Table (1) shows the optimal response to inflation as well as the welfare outcome for different deviations of the average elasticity from the benchmark case (i.e.  $\varepsilon^W = \Delta \varepsilon^W + 7.66$ ). The table shows that welfare decreases in the degree of competition.<sup>18</sup> The table also shows that the central bank reacts more strongly to inflation for larger average degrees of competition. Both these results are consistent with the analytical description of the relationship between inflation and welfare given in subsection 4.1.<sup>19</sup>

#### 5.2 Gains from the optimal rule

In the previous section we saw that competition can indeed have a negative effect on welfare in a stochastic model with staggered price setting. In this section we turn to the case of an asymmetric monetary union. In order to simplify the exposition of our results we fix the average degree of competition at  $\varepsilon^W = 7.66$  which amounts to a mark-up of 15% (the value chosen by Beetsma and Jensen (2002)). We then study the relationship between the optimal policy parameters ( $\mathbf{f}$ ) and the difference in the degree of competition between the two countries (henceforth,  $\varepsilon^R \equiv \varepsilon^F - \varepsilon^H$ ) as well as differences in the degree of nominal rigidity (henceforth,  $\omega^R \equiv \omega^H - \omega^R$ ).

<sup>&</sup>lt;sup>18</sup>One should recall that the magnitude of the welfare losses/gains is proportional to the variance of the shocks. Furthermore, we abstract from the steady-state effect of the monopolistic distortion on welfare since in our model this cannot be affected by the monetary policy (the focus of our paper). Nevertheless, it is worth noting that under a sufficiently small variance of the shocks (as must be assumed in our approximated model) the steady-state inefficiency will unambiguously dominate, though with no consequences for monetary policy.

 $<sup>^{19}</sup>$ It is worth mentioning that the welfare function is rather flat in the parameter a, so that there are no big losses associated with rules 'close' to the optimal.

First we note from Table (2) that welfare decreases in the difference between the degree of competition in the home and foreign countries when the central bank neglects the structural asymmetry (i.e. when it weights the country-specific measures of inflation by the relative size of the countries and sets a to the optimal value for the symmetric scenario). The reduction in welfare is not substantial. Nevertheless, it should be noted that, under the optimal rule, welfare increases in the 'competition gap'. This can be seen by reading the first column of Table (3). The reason for this result is that in a symmetric currency area the central bank cannot control the terms of trade (as shown by Benigno (2003)). The terms of trade in a symmetric two-country model depend (symmetrically) on the developments of the two economies and vice versa. Since the central bank has only one instrument, it can only affect the two (identical) economies symmetrically. For example, if the monetary authority induces an increase in the producers' price in one country, it must induce the same increase in the producers' price level in the other country: The terms of trade remain unaffected. By contrast, if the two countries differ, so that for example an increase in the nominal interest rate has different effects on the two countries' price level, the central bank can affect the terms of trade. That the latter are relevant for welfare can again be gauged by inspecting equation (19). Home production (and effort) depends on the demand for home goods. A change in the terms of trade shifts the demand (and the burden of production) from one country to the other. Furthermore, the terms of trade affect the dynamics of inflation (equation (27)) If the central bank acquires some control on the terms of trade, it can improve its influence on the determinants of welfare. The gap between the performance of the symmetric rule and the performance of the optimal rule widens in the degree of asymmetry. The relative loss associated with the symmetric rule more than offsets the gain associated with the improved control of the terms of trade.

Equations (19) and (26) show that the disutility of labour effort decreases in  $(1 - \alpha)$ . Table (2) shows that this effect can be quantitatively very large. From this result we can infer that, were the returns to labour different across countries, the central bank would have a strong incentive to take into account this difference in responding to the county-specific inflation rate. It is also clear that the correct assessment of the welfare effects of alternative monetary policies will depend on the correct assessment of this potential source of asymmetry. For reasons of space we do not refer further to this aspect in this work.

#### 5.3 Two asymmetries: nominal rigidity and competition

The effect on monetary policy of an asymmetric degree of nominal rigidity in a monetary union is thoroughly studied in Benigno (2003). Here we suggest that differences in the degree of competition could either exacerbate or offset the effect of an asymmetric degree of nominal price rigidity. In which of these two directions the interaction goes depends on the relationship between the degree of competition and the degree of nominal rigidity and therefore remains an empirical issue upon which we touch briefly further below.

We start first by looking at the effect on welfare of different degrees of nominal rigidity and different degrees of competition when the central bank optimally chooses the interest rate rule parameters. Table (3) shows that welfare, under the optimal rule, increases in both the gaps. As explained in the previous section, this is due to the fact that the central bank's control of the terms of trade increases in the degree of asymmetry of the two economies.

The main result of our work is shown in Table (4). The entries correspond to the optimal parameter  $\mathbf{f_1}$  for various values of the 'competition gap' ( $\varepsilon^R$ ) and of the 'nominal rigidity gap' ( $\omega^R$ ). As  $\omega^R$  increases, the home (foreign) country's price flexibility is reduced (increased). As  $\varepsilon^R$  increases the home (foreign) country becomes less (more) competitive. So, for example, according to the evidence provided by Carlton (1986) the two 'gaps' should move in the same direction. One can easily see that, for any given degree of competition,  $\mathbf{f_1}$  increases in the nominal rigidity gap. Similarly, for any given degree of nominal rigidity,  $\mathbf{f_1}$  decreases in the competition gap. In the table we have highlighted the initial values of  $\mathbf{f_1}$  equal or larger than 0.5: i.e. when monetary policy starts reacting more aggressively to the home inflation. Strikingly, for sufficiently wide competition gaps the central bank should react more strongly (i.e. a coefficient larger that 0.5) to the country with less nominal rigidity. This result suggests that incomplete information about the true economic structure of the 'two' economies might lead to the design of sub-optimal instrument rules.

Table (5) shows this case in more detail. In this table we show, as an example, the difference between the welfare function evaluated under a rule optimized only on the basis of information regarding the degree of nominal rigidity and the welfare function evaluated under a symmetric rule.<sup>20</sup> As one would expect, when the degree of competition differs substantially across the two countries and the nominal rigidity gap is not too extreme, the symmetric rule outperforms the optimal rule under incomplete information. We leave for future research the study of rules for currency areas that are 'robust' to imperfectly measurable asymmetries.

It is clear from Table (4) that the result in Benigno (2003) – that the less 'flexible' countries should receive a larger weight in the policy rule – could be altered by a positive relationship between price flexibility and competition. For example, if we start from  $\omega^R = 0$  and  $\varepsilon^R = 0$ , whether the optimal weight should increase or decrease as  $\omega^R$  increases, depends crucially on whether (and how quickly)  $\varepsilon^R$  increases or decreases.<sup>21</sup>

Owing to the very scant cross-country evidence on the degree of price rigidity and on the degree of competition, it is difficult to infer how large the gains from optimally weighting inflation could be in reality, e.g. in the European Monetary Union. It would be tempting to use labour market data on wage rigidity and unionization (e.g. Nickell (1997)) as proxies for the degree of price rigidity and of competition in goods markets.<sup>22</sup>

 $<sup>^{20}</sup>$ In the optimal rule, both a and  $f_1$  are chosen optimally. The symmetric rule uses values that are optimal under symmetry.

<sup>&</sup>lt;sup>21</sup>This result stresses the importance of micro-foundations. Our model, as is mostly the case in the current macroeconomic literature, falls short of pinning down the endogenous determination of all the agents' decisions. For example, prices are set at exogenous random intervals and – the point at hand – the frequency of price adjustments and the degree of competition are assumed to be independent and exogenous.

<sup>&</sup>lt;sup>22</sup>Benigno (2003) and Beetsma and Jensen (2002) use this evidence only as a proxy for the degree of nominal price rigidity.

Nevertheless, such a parallel runs the risk of being misleading, since the structure of labour markets typically differs markedly from the structure of the markets for goods.<sup>23</sup> Instead, there is some evidence that the degree of price rigidity differs significantly across sectors and that it is greater in less competitive sectors (e.g. Carlton (1987), Powers and Powers (2001)). Furthermore, some evidence supports the conjecture that the degree of price rigidity varies across countries within the same sector as well as across sectors (as shown in Bils and Klenow (2002) and Hoffmann et al. (2003) for the USA and Germany respectively). We take this evidence as suggestive of the existence of asymmetries in the degree of nominal rigidity and in the degree of competition across countries, although it is insufficient to quantify these asymmetries.

Despite the lack of reliable measures of the degree of price rigidity and of the degree of competition, we can gauge from Table (6) the order of magnitude of the welfare gains from optimally weighting inflation. The table shows the difference between the welfare change under the optimal weighting and that under the symmetric weighting for various values of the relative degree of nominal rigidity and of the competition gap. Two important results emerge from the graph. First, the relationship between welfare and nominal rigidity differentials is monotone in the rigidity gap only if the competition gap is either very small or very large. This result extends the analysis of Benigno (2003) in the sense that his work focuses only on the first row of Table (6). The non-monotonicity of welfare in the Calvoprobability can be seen by looking at equation (26) and (27). The first relationship (the measure of the disutility of effort), say for the home country, increases in  $\omega^H$ . The square of the second relationship determines the variance of home producers' price inflation. From the parameter  $\kappa_1$  we see that the variance of inflation decreases in  $\omega^{\bar{H}}$ . Hence, the overall cost of inflation is potentially non-monotone in  $\omega^H$ . From equation (27) one can also see that the degree of substitutability between domestic and imported goods  $(\varphi)$  can amplify the variance of inflation by strengthening the effect of terms of trade variations on the inflation rate. In the next section we explore this possibility.

A second and important result that can be drawn from Table (6) is that, at least under our calibration, the welfare losses from a suboptimal weighting of inflation are not negligible. Up to about 0.80% per quarter of steady state consumption could be lost due to the adoption of a suboptimal rule.

# 6 Results: the case of $\varphi \neq 1$

So far we have studied the case of a Cobb-Douglas aggregator for domestic and foreign goods. This is the case mostly studied in the literature, and in particular in Benigno (2003) and Beetsma and Jensen (2002). Nevertheless, our solution technique allows us to study the more general case of a CES aggregator. For reasons of space, though, we reconsider only a few of the issues discussed under  $\varphi = 1$ .

When  $\varphi \neq 1$ , a positive productivity shock, say in the home country, improves its current account and, hence, it generates a transfer of wealth in the form of bonds from the

<sup>&</sup>lt;sup>23</sup>This is certainly true for most European countries, whose labour markets are generally characterized by few trade unions as documented in Groth and Johansson (2002).

foreign to the home country. The larger  $\varphi$  the larger the current account surplus, i.e. the amount of 'consumption switching' that takes place. This type of output dispersion has different effects on welfare than those produced by the dispersion of production within the domestic economy. The former is an efficient shift of production from the less productive economy to the more productive economy. The latter is a reallocation due to inefficient price staggering.

Moreover, the more sensitive the demand for domestic goods to the terms of trade, the stronger the effect of different inflation dynamics on the allocation of output among the members of the currency area. This affects welfare through equation (19).

Table (7) shows that welfare (measured in percentages of steady-state consumption) and  $\varphi$  are linked by a non-monotonic relationship. For example, around  $\Delta \varepsilon^W = 0$  (i.e. the benchmark case) intermediate values of  $\varphi$  yield greater welfare than values that are either too small or too large. This reflects the complex way in which the terms of trade affect welfare and the dynamics of inflation. Table (8) shows that the value of  $\varphi$  has virtually no effect on the optimal policy in the symmetric currency area. This suggests that a precise knowledge of this parameter is not crucial for the determination of the optimal response to aggregate inflation.

Next we compare the performance, in terms of welfare, of the optimal interest rate rule relative to the symmetric rule when competition and nominal rigidity differ across countries and  $\varphi \neq 1$ . For the sake of conciseness, we focus only on one special case, i.e.  $\varphi = 2$ , since this order of magnitude if often assumed in calibrated models (e.g. Backus et al. (1994)).

Table (9) shows that the gains are smaller under  $\varphi = 2$  than under the Cobb-Douglas case. Now no more than 39% per quarter of steady-state consumption could be lost due the use of the symmetric rule. Interestingly, the worst case for the symmetric rule occurs under a large nominal-rigidity gap and a small competition gap. Under Cobb-Douglas preferences such a scenario yielded a smaller loss.

Finally, Table (10) shows the optimal response coefficient to the home producers' price inflation. Interestingly, even under an identical degree of competition across countries, the central bank does not always find it optimal to respond more aggressively to the 'less flexible' country. In our particular example, a competition gap equal to or larger than 2 calls for a rule that responds more aggressively to the more competitive country, regardless of the degree nominal rigidity. The rationale behind this result was discussed in the previous sections. The degree of nominal rigidity (as well as the degree of competition) produces two counteracting effects on welfare. One (negative) via the cost of inflation (equation (26)) and the other (positive) via the Phillips curve (equation (27)). Larger values of  $\varphi$  amplify the second effect.

 $<sup>^{-24}</sup>$ Also in this case we maximize welfare with respect to both a and  $f_1$ . The optimal value of a is not qualitatively different from that found under  $\varphi = 1$ .

## 7 Conclusion

In this paper we have studied the relationship between monetary policy and two sources of asymmetry across the members of a monetary union, in terms of the households' welfare. In particular, we have considered the implications of different degrees of competition as well as different degrees of nominal rigidity.

Our focus has been on the problem of optimally weighting the country-specific components of inflation in the design of an optimal interest rate rule. We have shown that, in a dynamic model with staggered price setting, the degree of competition acquires a second-best quality: less competition is associated with a smaller dispersion of output and, hence, with greater welfare. In principle, this role of the degree of competition should be taken into account by a welfare-maximizing central bank. When the central bank adopts an interest rate rule that responds to the rate of inflation, the weights attached to the country-specific components of inflation should increase in the degree of competition of the relative countries.

Our study also extends the analysis of Benigno (2003) by showing that in the presence of asymmetric degrees of competition, attaching a larger weight to the inflation of the 'less flexible' country might be suboptimal. If in reality the degree of nominal rigidity decreases in the degree of competition, the relationship between the optimal weights and competition (price rigidity) is ambiguous. Furthermore, we have shown that with non-Cobb-Douglas preferences in home and foreign consumption, Benigno's policy prescription becomes a 'special' case: it holds only under very small differences in the degree of competition and, even in this case, only if the differences in the degree of nominal rigidity are not too large.

The welfare cost due to neglecting these asymmetries crucially depends on their actual combination as well as on the degree of substitutability between domestic and imported goods. In particular we show that the symmetric rule, i.e. the rule that neglects the asymmetries, can outperform the optimal rule when the central bank has incomplete information about the number and size of the asymmetries. Future research should look at rules that are 'robust' to this type of uncertainty.

Finally, our work has focused on one particular source of shocks, i.e. productivity shocks. Future research should look at whether our results would generally apply under other types of shocks. This would be particularly important if the central bank had incomplete information about the type of shocks as well as the type of asymmetries.

## Appendix. Second-order approximations

# A The non-stochastic steady-state allocation

In this section we derive the non-stochastic equilibrium allocation for the home country. Similar relationships can be derived for the foreign country.

In the non-stochastic equilibrium  $A_H = A_F = 1$ . Moreover, firms set prices as a

mark-up over marginal costs (as they would do under flexible prices). Then, we have that

$$\frac{p(z)_{ss}}{P_{ss}} = \frac{P_{H,ss}}{P_{ss}} = \mu^H m c_{ss}^H = \Phi_H^{-1} \frac{1}{1-\alpha} \left( \left( \frac{P_{H,ss}}{P_{ss}} \right)^{-\varphi} C_{ss}^W \right)^{\frac{\zeta+\alpha}{1-\alpha}} C_{ss}^{\sigma}$$
(28)

for country H and a similar expression for country F, where we have made use of the labour demand implied by the production function (equation (13)) and the labour supply equation (11) and where  $\Phi_H = s^H / \mu^H$  and  $C_{ss}^W = nC_{ss} + (1-n)C_{ss}^*$ . We can therefore derive the steady-state terms of trade  $(T = \frac{P_F}{P_H})$  as

$$\frac{P_{F,ss}}{P_{H,ss}} = T_{ss} = \left[\frac{\Phi_H}{\Phi_F} \frac{C_{ss}^{*\sigma}}{C_{ss}^{\sigma}}\right]^{\frac{1-\alpha}{1-\alpha+\varphi(\zeta+\alpha)}}.$$
(29)

The steady state of the terms of trade can be derived by noting that in the steady state households do not hold foreign bonds. Then  $P_{ss}C_{ss} = P_{H,ss}Y_{H,ss}$ . Using the demand equation and the definition of the aggregate price index, one can show that

$$\frac{C_{ss}}{C_{ss}^*} = T_{ss}^{\varphi - 1} \tag{30}$$

Using equation (30) in equation (29) yields

$$T_{ss} = \left(\frac{\Phi_H}{\Phi_F}\right)^{\frac{1-\alpha}{(1-\alpha)(1+(\varphi-1)\sigma)+\varphi(\zeta+\alpha)}} \tag{31}$$

In the steady state we have that

$$\frac{P_{H,ss}}{P_{ss}} = \left(n + (1-n)T_{ss}^{1-\varphi}\right)^{\frac{1}{\varphi-1}}$$

$$C_{ss}^{W} = \left(n + (1-n)T_{ss}^{1-\varphi}\right)C_{ss}$$
(32)

$$C_{ss}^{W} = (n + (1 - n)T_{ss}^{1 - \varphi}) C_{ss}$$
(33)

Replacing equations (32) and (33) into equation (28) and solving for home consumption yields

$$C_{ss} = \left(\frac{P_{H,ss}}{P_{ss}}\right)^{\frac{\zeta+1}{\zeta+\alpha+\sigma(1-\alpha)}} \left((1-\alpha)\Phi_H\right)^{\frac{1-\alpha}{\zeta+\alpha+\sigma(1-\alpha)}}$$
(34)

Equation (31) together with equations (30) and (34) provide the steady-state value of foreign consumption.

#### В Second-order approximation of the aggregate price indexes and inflation dynamics

Under Calvo contracts the domestic price index at time t is a weighted sum of the prices prevailing at that time, i.e.

$$P_{H,t} = \left[ (1 - \omega^H) \, p_{i,t}^{1 - \varepsilon^H} + \omega^H \, P_{H,t-1}^{1 - \varepsilon^H} \right]^{\frac{1}{1 - \varepsilon^H}}. \tag{35}$$

It is worth noting that the first-order approximation of equation (35) yields, after few rearrangements

$$\frac{\omega^j}{1-\omega^j}\hat{\pi}_j = \hat{p}_t - \hat{P}_{j,t} \tag{36}$$

where  $\hat{x} = \log(x/x_{ss})$ . By taking the second-order expansion of equation (35), in deviation from the steady state, we obtain

$$\hat{P}_{H,t} = \omega^{H} \hat{P}_{H,t-1} + (1 - \omega^{H}) \hat{p}_{i,t} + \frac{\omega^{H}}{2} (1 - \omega^{H}) (1 - \varepsilon^{H}) \left( \hat{p}_{i,t} - \hat{P}_{H,t-1} \right)^{2} + \mathcal{O}(\|\xi\|^{3})$$

$$= \omega^{H} \hat{P}_{H,t-1} + (1 - \omega^{H}) \hat{p}_{i,t} + \frac{\omega^{H}}{2} \frac{(1 - \varepsilon^{H})}{(1 - \omega^{H})} \left( \hat{\pi}_{t}^{H} \right)^{2} + \mathcal{O}(\|\xi\|^{3}). \tag{37}$$

where we have used equation (36).

Inflation (say for the home producer price index) is simply given by  $\hat{\pi}_t^H = \hat{P}_{H,t} - \hat{P}_{H,t-1}$ , so that we can derive the firm i-th relative price as shown in equation (24).

Similarly, a second-order expansion of the price index (4) yields

$$\hat{P}_{t} = \frac{nP_{H,t} + (1-n)T_{ss}^{(1-\varphi)}P_{F,t}}{(n+(1-n)T_{ss}^{(1-\varphi)})} + \frac{1}{2}n(1-n)(1-\varphi)\frac{T_{ss}^{(1-\varphi)}}{\left(n+(1-n)T_{ss}^{(1-\varphi)}\right)^{2}}\left(\hat{P}_{H,t} - \hat{P}_{F,t}\right)^{2} + \mathcal{O}(\|\xi\|^{3}). \quad (38)$$

# C Second-order approximation of Calvo prices

Let us rewrite the optimal price (say for the home country firms) as follows

$$p_{i,t}^{\left(1+\varepsilon^H\frac{\alpha+\zeta}{1-\alpha}\right)} = \mu \frac{Q_{1,t}}{Q_{2,t}} \tag{39}$$

where

$$Q_{1,t} = E_t \sum_{s=t}^{\infty} (\omega^H)^{s-1} q_{1,s,t}$$
(40)

$$Q_{2,t} = E_t \sum_{s=t}^{\infty} (\omega^H)^{s-1} q_{2,s,t}$$
(41)

where  $q_{1,s,t} = \bar{R}_{s,t} \phi_s \, \psi_s^{\frac{\zeta+1}{1-\alpha}}$ ,  $q_{2,s,t} = \bar{R}_{s,t} \, P_s^{-1} \, \psi_s$  and where  $\phi$  is the part of the marginal cost that is independent of the firm's output and  $\psi$  is the part of the demand for the firm's output that is independent of the firm's price. One can see that  $q_{1,s,t+1} = R_t \, q_{1,s,t}$ . Therefore we can rewrite equations (40) and (41) as  $Q_{j,t} = q_{j,t} + \omega^H E_t R_t^{-1} Q_{j,t+1}$  or  $R_t Q_{j,t} = R_t q_{j,t,t} + \omega^H E_t Q_{j,t+1}$ , where j = 1, 2.

The second-order expansion of equations (40) and (41) then yields

$$\hat{Q}_{j,t} + \frac{1}{2} \left( \hat{Q}_{j,t} + \hat{R}_t \right)^2 = \left( 1 - \omega^H \beta \right) \left( \hat{q}_{j,t,t} + \frac{1}{2} \left( \hat{q}_{j,t,t} + \hat{R}_t \right)^2 \right) - \omega^H \beta \hat{R}_t + \omega^H \beta E_t \left( \hat{Q}_{j,t+1} + \frac{1}{2} \hat{Q}_{j,t+1}^2 \right) + \mathcal{O} \left( \|\xi\|^3 \right)$$
(42)

where we have omitted the linear and quadratic terms in the gross interest rate since, eventually, they will cancel out in the determination of the firm's optimal price. The optimal price of the firm as written in equation (39) is linear in logs, so that we have, say for the home country  $\left(1 + \varepsilon^H \frac{\alpha + \zeta}{1-\alpha}\right) \hat{p}_{i,t} = \hat{Q}_{1,t} - \hat{Q}_{2,t}$ .

It is convenient to look at equation (42) also up to its first order of approximation. By dropping the second-order term we can write

$$\hat{Q}_{1,t} - \hat{Q}_{2,t} = (1 - \omega^H \beta) \left( q_{1,t,t} - q_{2,t,t} \right) + \omega^H \beta E_t \left( \hat{Q}_{1,t+1} - \hat{Q}_{2,t+1} \right) + \mathcal{O} \left( \|\xi\|^2 \right). \tag{43}$$

By using equation (39) and by subtracting  $\hat{P}_{H,t}$  on both sides of equation (43) we have

$$\hat{\pi}_{t}^{H} = \kappa_{1} \left( \hat{q}_{1,t,t} - \hat{q}_{2,t,t} \right) - \kappa_{2} \hat{P}_{H,t} + \beta E_{t} \left( \hat{\pi}_{t+1}^{H} \right) + \mathcal{O} \left( \|\xi\|^{2} \right)$$
(44)

where 
$$\kappa_1^H = \frac{\left(1 - \omega^H \beta\right) \left(1 - \omega^H\right)}{\left(1 + \varepsilon^H \frac{\alpha + \zeta}{1 - \alpha}\right) \omega^H}$$
 and  $\kappa_2^H = \frac{\left(1 - \omega^H \beta\right) \left(1 - \omega^H\right)}{\omega^H}$ .

Note that

$$\hat{q}_{1,t,t} - \hat{q}_{2,t,t} = \frac{\zeta + \alpha}{1 - \alpha} \left( \varepsilon^H \hat{P}_{H,t} + \varphi \kappa_3 \hat{T}_t + \hat{C}^W \right) + \hat{\phi}_t + \hat{P}_t + \mathcal{O}\left( \|\xi\|^2 \right)$$
(45)

where  $\kappa_3 = \frac{(1-n)T_{ss}^{(1-\varphi)}}{(n+(1-n)T_{ss}^{(1-\varphi)})}$ , and where  $\phi_t$  depends only on aggregate consumption and the domestic productivity shock.

By adding and subtracting  $\hat{P}_{H,t}$  to the right side of equation (45) we obtain

$$\hat{q}_{1,t,t} - \hat{q}_{2,t,t} = \left(1 + \varepsilon^H \frac{\zeta + \alpha}{1 - \alpha}\right) \hat{P}_{H,t} + \kappa_3 \left(1 + \varphi \frac{\zeta + \alpha}{1 - \alpha}\right) \hat{T}_t + \hat{C}^W + \hat{\phi}_t + \mathcal{O}\left(\|\xi\|^2\right) (46)$$

Finally, by replacing equation (46) into equation (44), we obtain equation (27)

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$\Delta \varepsilon^W$	-3	-1.5	0	1.5	3	4.5	6
a	4.11	5.52	6.93	8.35	9.76	11.19	12.6
Welfare $(c\%)$	88.41	87.88	87.44	87.05	86.7	86.37	86.05

Table 1: Competition, monetary policy and welfare: The symmetric case.

		$1-\alpha$									
		0.5	0.6	0.7	0.8	0.9	1				
	0	83.54	86.31	88.33	89.84	91	91.92				
	2	83.54	86.31	88.32	89.83	91	91.92				
$\underline{\epsilon}^R$	4	83.5	86.28	88.3	89.81	90.98	91.9				
	6	83.45	86.23	88.26	89.78	90.95	91.87				
	8	83.4	86.2	88.23	89.76	90.93	91.86				

Table 2: Welfare under the symmetric rule v.  $\varepsilon^R$  and  $1 - \alpha$ .

				$\underline{\omega^R}$		
		0	0.2	0.4	0.6	0.8
	0	87.44	87.62	88.14	88.93	89.89
	2	87.48	87.63	88.12	88.91	89.87
$\underline{\underline{\varepsilon}^R}$	4	87.59	87.71	88.16	88.92	89.88
	6	87.79	87.87	88.27	88.98	89.91
	8	88.09	88.14	88.48	89.12	89.98

Table 3: Welfare under the optimal interest rate rule v. the two asymmetries.

				$\underline{\underline{\omega}^R}$		
		0	0.2	0.4	0.6	0.8
	0	0.5	0.55	0.6	0.65	0.67
	2	0.41	0.46	0.52	0.58	0.6
$\underline{\varepsilon}^R$	4	0.33	0.37	0.43	0.5	0.52
	6	0.25	0.29	0.34	0.4	0.42
	8	0.19	0.21	0.24	0.29	0.3

Table 4: Optimal weight on home inflation  $(\mathbf{f_1})$  and the two gaps.

				$\underline{\underline{\omega}^R}$		
		0	0.2	0.4	0.6	0.8
	0	0	0.01	0.05	0.1	0.29
	2	0	-0.04	-0.02	0.04	0.23
$\underline{\varepsilon}^R$	4	0	-0.09	-0.11	-0.03	0.18
	6	0	-0.15	-0.22	-0.11	0.13
	8	0	-0.22	-0.34	-0.22	0.08

Table 5: Welfare gains from optimizing under incomplete information.

				$\underline{\omega^R}$		
		0	0.2	0.4	0.6	0.8
	0	0	0.01	0.05	0.1	0.29
	2	0.05	0.01	0.01	0.06	0.23
$\underline{\varepsilon}^R$	4	0.2	0.11	0.05	0.06	0.2
	6	0.46	0.31	0.17	0.11	0.19
	8	0.8	0.63	0.4	0.22	0.21

Table 6: Welfare gains from optimally responding to inflation.

				4	$\Delta arepsilon^W$			
		-3	-1.5	0	1.5	3	4.5	6
	1	88.41	87.88	87.44	87.05	86.7	86.37	86.05
	3	88.29	88.04	87.87	87.74	87.61	87.5	87.38
$\underline{\underline{\varphi}}$	5	88	87.8	87.7	87.62	87.57	87.52	87.47
	7	87.78	87.6	87.51	87.47	87.44	87.42	87.4

Table 7: Optimal welfare in the symmetric currency area v.  $\varphi$  and  $\varepsilon^W$ .

					$\Delta \varepsilon^W$			
							4.5	
								12.59
$\stackrel{\varphi}{=}$	7	4.11	5.52	6.93	8.35	9.76	11.18	12.6

Table 8: Optimal response (a) to aggregate inflation in the symmetric currency area v.  $\varphi$  and  $\Delta \varepsilon^W$ .

				$\underline{\omega^R}$		
		0	0.2	0.4	0.6	0.8
$\underline{\varepsilon^R}$	0	0	0	0.02	0.09	0.39
	2	0.02	0.02	0.03	0.08	0.33
	4	0.09	0.08	0.07	0.11	0.28
	6	0.19	0.18	0.16	0.16	0.27
	8	0.32	0.32	0.28	0.25	0.27

Table 9: Gains from the optimal rule when  $\varphi = 2$ .

				$\underline{\omega^R}$		
		0	0.2	0.4	0.6	0.8
$\underline{\underline{\varepsilon}^R}$	0	0.5	0.51	0.53	0.53	0.47
	2	0.42	0.43	0.45	0.46	0.4
	4	0.35	0.35	0.37	0.37	0.31
	6	0.28	0.27	0.28	0.28	0.21
	8	0.22	0.21	0.2	0.18	0.09

Table 10: The optimal weight  $\mathbf{f_1}$  when  $\varphi=2$ .

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