Finite-sample distributions of self-normalized sums

Jeong-Ryeol Kim

Discussion paper 02/03

Economic Research Centre

of the Deutsche Bundesbank

January 2003

Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 95 66-1
Telex within Germany 4 1 227, telex from abroad 4 14 431, fax +49 69 5 60 10 71

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax No. +49 69 95 66-30 77

ISBN 3–935821–43–3

Contents

Tables and Figures

Finite-sample distributions of self-normalized sums *

Abstract

Logan et al. (1973) analyze the limit probability distribution of the statistic $S_n(p) = \Sigma_{i=1} X_i / \left(\Sigma_{i=1} |X_j|^p\right)^{1/p}$ as $n \to \infty$, when X_i is in the domain of attraciton of a stable law with stabilility index α. By simulations, we provide quantiles of the usual critical levels of the finite-sample distributions of the Student's t-statistic as ${}_\alpha t_\xi(n) = S_n(p) \left[(n-1)/\left(n - S_n^2(p)\right)\right]^{1/2}$ with $p = 2$. The response surface method is used to provide approximate quantiles of the finite-sample distributions of the Student's t-statistic.

Zusammenfassung

Logan u.a. (1973) untersuchen die Grenzwahrscheinlichkeitsverteilung der Statistik, $S_n(p) = \Sigma_{i=1} X_i / \left(\Sigma_{i=1} |X_j|^p\right)^{1/p}$ mit $n \to \infty$, für den Fall, dass X_i im Anziehungsbereich eines stabilen Gesetzes mit Stabilitätsindex von α liegt. Mit Hilfe einer Simulation werden Quantile der üblicherweise verwendeten kritischen Niveaus für Verteilungen endlicher Stichprobenumfänge der Studentischen t-Statistik, ${}_\alpha t_\xi(n) = S_n(p) \left[(n-1)/\left(n - S_n^2(p)\right)\right]^{1/2}$ mit $p = 2$ ermittelt. Die Antwort-Oberfläche-Methode gibt die approximierten Quantile der Verteilungen endlicher Stichprobenumfänge der *t*-Statistik in einer übersichtlichen Form an.

---

* The views expressed in this paper are those of the author and not necessarily those of the Deutsche Bundesbank.
E-Mail: jeong-ryeol.kim@bundesbank.de Tel.: +49 69 9566 4576, Fax: +49 69 9566 2982 Research support from the Alexander von Humboldt Foundation is gratefully acknowledged.

1 Introduction

Econometricians have long been aware that the tails of a great deal of economic data are thicker than that of the Gaussian distribution. Therefore, it is of interest in hypothesis testing to specify both the limiting distribution and the finite-sample distributions of the Student's t-statistic

$$ {}_{\alpha}t(n) := {}_{\alpha}S(n)\left(\frac{n-1}{n-{}_{\alpha}S^2(n)}\right)^{\frac{1}{2}}, \tag{1} $$

with

$$ {}_{\alpha}S(n) := \frac{\sum_{i=1}^{n} X_i}{\sqrt{\sum_{i=1}^{n} X_i^2}}, \tag{2} $$

where the random sequence $\{X_i\}_1^n$ is in the domain of attraction (DA) of an α-stable law with index $\alpha \in (0,2]$. The DA condition is equivalent to $P(|X| > x) = x^{-\alpha}L(x),\ x > 0$, where $L(z)$ is a slowly varying function,[1] and $\lim_{x\to\infty} \frac{P(X>x)}{P(|X|>x)} = r$; $\lim_{x\to\infty} \frac{P(X<-x)}{P(|X|>x)} = l$, for some $r,\ l \geq 0$. When $r = l$, the random variable X_i is symmetric, and thus the statistic in (2) is self-normalized in the sense that it has zero mean and unit variance. A random variable X is said to be stable if for any positive numbers A and B, there is a positive number C and a real number D such that $AX_1 + BX_2 \stackrel{d}{=} CX + D$, where X_1 and X_2 are independent random variables with $X_i \stackrel{d}{=} X, i = 1,2$; and " $\stackrel{d}{=}$ " denotes equality in distribution. Moreover, $C = (A^\alpha + B^\alpha)^{1/\alpha}$ for some $\alpha \in (0,2]$, where the exponent α is called index of stability. When $0 < \alpha < 2$, the tails of the distribution are thicker than those of the normal distribution. The tails become thicker as α decreases such that moments of order α or higher do not exist. A stable random variable, X, with index α is called α-stable. The α-stable distributions are described by four parameters denoted by $S(\alpha,\beta,\gamma,\delta)$. The shape of the α-stable distribution is determined by the stability parameter α. For $\alpha = 2$ the α-stable distribution reduces to the normal distribution, the only member of the α-stable family with finite variance; for $\alpha < 2$ it has infinite variance. Skewness is governed by $\beta \in [-1,1]$. When $\beta = 0$, the distribution is symmetric. The location and scale of the α-stable distributions are denoted by γ and δ. The standardized version of the α-stable distribution is given by $S((x-\gamma)/\delta; \alpha, \beta, 1, 0)$. The α-stable distribution is an interesting error–distribution candidate, because only the α-stable distribution can serve as a limiting distribution of sums of independent, identically distributed random variables. This is an appealing property, given that disturbances can be viewed as random variables which represent the sum of all

[1] $L(z)$ is a slowly varying function as $z \to \infty$, if for every constant $c > 0$, $\lim_{x\to\infty} \frac{L(cz)}{L(z)}$ exists and is equal to 1. See Ibragimov and Linnik (1971, p. 394) for more details on slowly varying functions.

external effects ignored by the model. Therefore, the hypothesis test based on the α-stable distributed disturbances is directly related to the statistic in (1). For more details on the α-stable distributions and discussions of the role of the α-stable distribution in financial market modelling, see Zolotarev (1986), Samorodnitsky and Taqqu (1994) and McCulloch (1996).

Logan et al. (1973) consider the limit distribution of the statistic in (2). When $0 < \alpha < 2$, the statistic in (2) is a pseudo statistic because the second moment for the α-stable random variable does not exist. This pseudo statistic, however, still has a limit probability distribution. Logan et al. (1973) show that for $0 < \alpha < 2$, the limiting density, $f(y)$, is a complicated form involving an integral of a ratio of parabolic cylinder functions with $Y := n^{-\frac{1}{\alpha}} \sum_{i=1}^{n} X_i$ as:

$$f(y) = \lim_{s \to iy} \text{Real}\left[\frac{1}{\pi}\int_0^\infty \phi(t)e^{-st}dt\right], \tag{3}$$

where Real[ ] denotes the real part of a complex number and $\phi(t)$ is the characteristic function of a stable random variable as follows:

$$\phi(t) = \int_{-\infty}^{\infty} e^{iut} dP(X < u) = \begin{cases} -\delta^\alpha |t|^\alpha [1 - i\beta \ \text{sign}(t) \tan\frac{\pi\alpha}{2}] + i\gamma t, & \text{for } \alpha \neq 1, \\ -\delta |t| [1 + i\beta \frac{\pi}{2} \ \text{sign}(t) \ln |t|] + i\gamma t, & \text{for } \alpha = 1. \end{cases}$$

This limit distribution is a calculable expression, and quantiles of the usual critical levels of the distribution are given in Tables 2a-c at the end of the paper. Furthermore, the limiting distribution has the properties that the tails of the cumulated density function are Gaussian-like at $\pm\infty$ and that the density function has finite (infinite) singularities at ± 1 for $1 < \alpha < 2$ ($0 < \alpha < 1$). This is because the sums in the numerator and denominator in (2) are essentially determined by a few summands of largest modulus for $\alpha < 2$. As α increases to 2, the singularities vanish and the density function tends to the normal density. This singularity, called the bimodality for $1 < \alpha < 2$, has been conformed in several subsequent works (see, for example, Phillips and Hajivassiliou, 1987).

2 Finite-sample $_\alpha t$-distributions

In this section, we perform simulations to approximate quantiles of the distributions of the Student's t-statistic in (1) with finite degrees of freedom for α-stable variables. By doing that, we concentrate on the most empirically-relevant case, namely $1 \leq \alpha < 2$ and $\beta = \gamma = 0$, i.e., the underlying stable random variables are symmetric about zero. Specifically, we consider α-values from 1.0 to 1.9 in steps of 0.1; the degrees of freedom ranged from 1 to 30 in steps of 1. For each of the resulting 300 (α, n)-combinations, 100000 replications are generated.[2]

[2]The α-stable random variables are generated with the algorithm of Weron (1996).

Before discussing the main results, some observations regarding the finite-sample distributions are in order. First, by a given sample size, the finite-sample distributions tend more and more, as is expected, to become bimodal as the stability index α decreases. Second, by a given low α, say $\alpha < 1.5$, the bimodality of finite-sample distributions still remain as $n \to \infty$. When $1.5 < \alpha < 2$, the bimodality vanishes quickly as n increases, but the degeneration to the two points with mass at ± 1 still remains asymptotically. Third, the tails of the finite-sample densities tend to become thinner, as n increases. These phenomena are summarized in Figure 1 by the empirical distributions of the simulated ${}_\alpha t$ with $n = 2, 10$ and 30; $\alpha = 1.8, 1.5$ and 1.2.

The first column of Figure 2 shows the simulated response surfaces for percent points $\xi = 0.9$, 0.95 and 0.99, illustrating the dependence of the quantiles, denoted by ${}_\alpha t_\xi(n)$, on α and n. The difference of the quantiles of the ${}_\alpha t$-distribution from those of the usual t-distribution increases as α drops below 2, but the increase is rather smooth and well-behaved. The second column shows the fitted response surfaces for percent points $\xi = 0.9$, 0.95 and 0.99.

Rather than simply tabulating specific quantiles of the simulated ${}_\alpha t$-distributions for selected (α, n)-combinations we employ response surface techniques to present all simulation results in a compact fashion.[3] In fact, we fit a joint response surface reflecting the dependence of the quantiles not only on α and n, but also on the percent point ξ. In the estimation, we use three percent points, namely 0.9, 0.95 and 0.99. Combinig these with the 300 (α, n)-pairs, the joint response surface is derived from simulated quantiles of 900 (α, n, ξ)-combinations. Because of the smoothness of the transition when α drops below 2, i.e., when moving from the usual t-distribution to the ${}_\alpha t$-distribution, we estimate the response surface in terms of deviations of the quantiles of the ${}_\alpha t$-distribution from those of the usually tabulated t-distribution[4] i.e.,

$$\Delta_\alpha t_\xi(n) := {}_2 t_\xi(n) - {}_\alpha t_\xi(n).$$

[3]Response surface methodology has been used in various statistical and econometric applications, see Myers et al. (1989) for more on this topic.

[4]Alternatively, one can employ the method of Peizer and Pratt (1968) to approximate the quantiles of the t-distribution

$${}_2 t_\xi(n) = \sqrt{n\left[\exp\left\{\left(n - \frac{6}{5}\right)\left(\frac{z(\xi)}{n - \frac{2}{3} - \frac{1}{10n}}\right)^2\right\} - 1\right]},$$

where $z(\xi)$ is the ξ-quantile of the standard normal distribution.

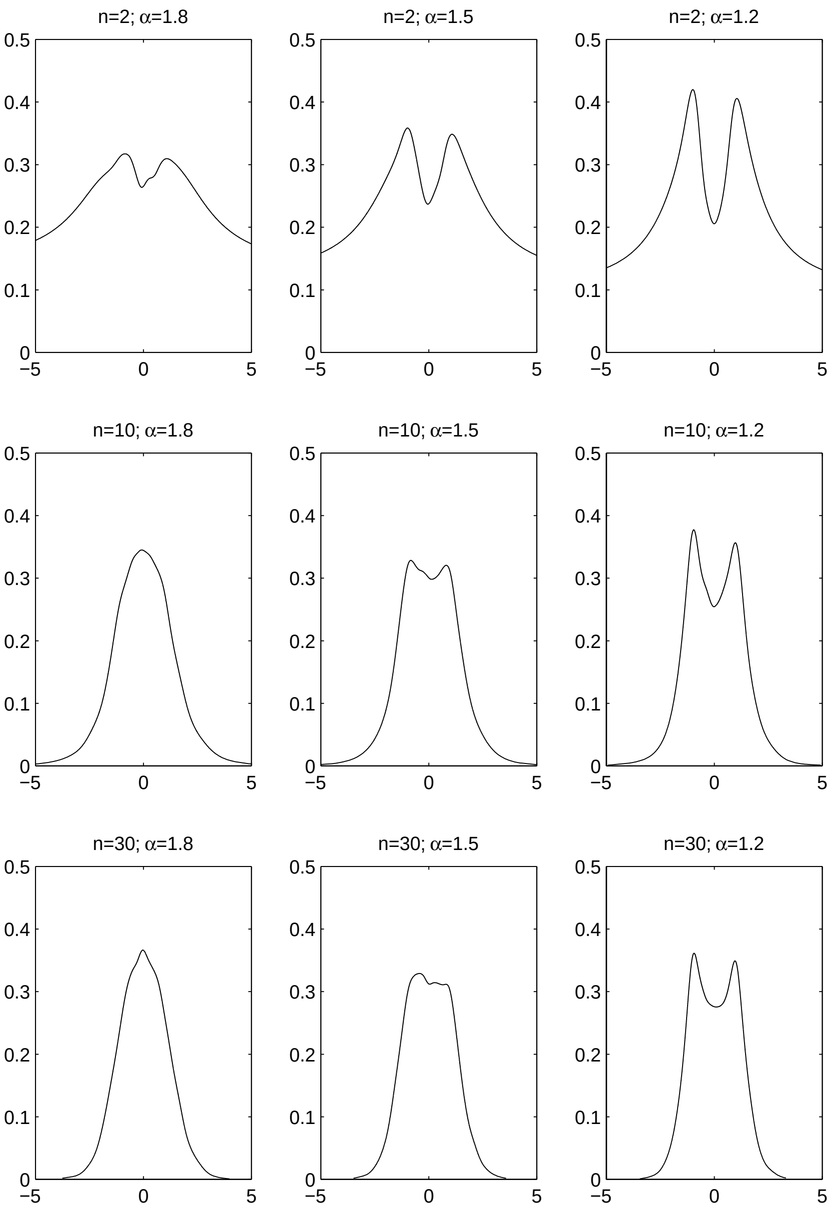

Figure 1: Simulated densities of ${}_{\alpha}t$ random variable for selected n and α

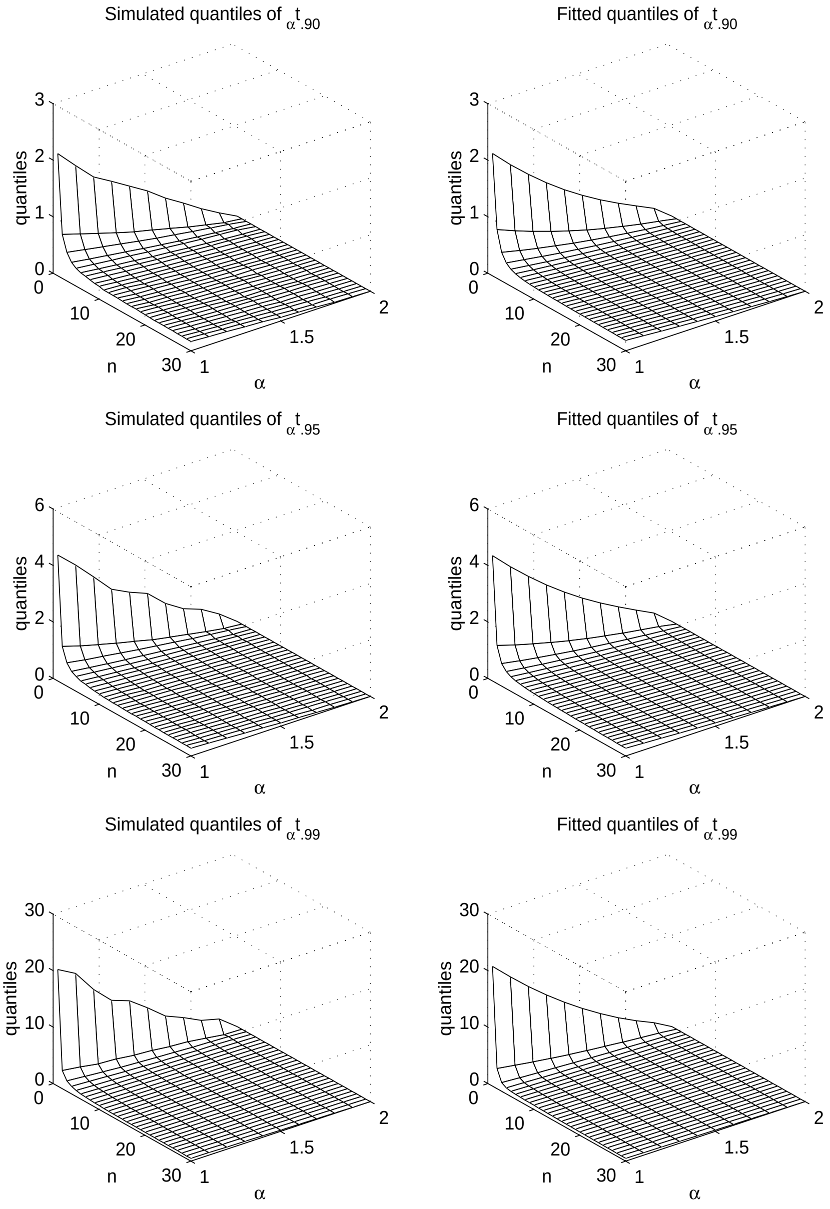

Figure 2: Simulated and fitted quantiles of ${}_{\alpha}t$-distributions

For the response surface, we specify the polynomial

$$\Delta_\alpha t_\xi(n) = \sum_{i=0}^{2}\sum_{j=1}^{4}\sum_{k=1}^{4} a_{ijk}(-\ln(1-\xi))^i(2-\alpha)^{\frac{j}{2}}n^{-\frac{k}{2}} + u_{\alpha,n,\xi}, \tag{4}$$

which ensures that $\Delta_\alpha t_\xi(n) \to 0$ as $\alpha \to 2$ and/or $n \to \infty$. When $n \to \infty$, the response surface is specified as

$$\Delta_\alpha t_\xi = \sum_{i=0}^{2}\sum_{j=1}^{4} b_{ij}(-\ln(1-\xi))^i(2-\alpha)^{\frac{j}{2}} + u_{\alpha,\xi}, \tag{5}$$

which ensures that $\Delta_\alpha t_\xi(n) \to 0$ as $\alpha \to 2$.

Estimating (4) and (5) with the least squares method, it turns out that only a subset of regressors is needed to fit the simulated quantiles. We selected the subset by maximizing the adjusted-R^2 value. The estimated coefficients are presented in Table 1a for the regression (4) and 1b for the regression (5) at the end of the paper. The adjusted-R^2 values for the regression (4) and (5) are 0.9902 and 0.9959, respectively. Various additional measures of goodness of fit, namely the standard deviation (0.1170 for (4), 0.0061 for (5)) and the absolute mean value (0.0317 for (4), 0.0047 for (5)) of the residuals, also suggest a good fit.

Based on the estimates in (4) the response surfaces for percent points $\xi = 0.9, 0.95$ and 0.99 are illustrated in the second column in Figure 2. For selected α-values response surface approximations for the 0.9–, 0.95–, and 0.99–quantiles of the ${}_\alpha t$-distribution are reported in Tables 2a–c at the end of the paper. The first column of each table corresponds to the usual t-distribution, and the last row of each table corresponds to the limit probability distribution in (3). Due to the singularity effect, absolute quantiles of the percentage points outside of ± 1 decrease as α decreases.

3 Summary

We presented an extension of the usual t-distribution for normally distributed variables to ${}_\alpha t$-distributions for heavy-tailed random variable Quantiles of finite degrees-of-freedom distributions for α-stable variables were simulated and compactly summarized in terms of a fitted response surface. The approximated critical values can be used to perform t-type tests when residuals are α-stable distributed.

References

Ibragimov, I.A. and Y.V. Linnik (1971) *Independent and Stationary Sequences of Random variables.* Wolters-Noordhoff, Groningen.

Logan, B.F., C.L. Mallows, S.O. Rice and L.A. Shepp (1973) Limit distributions of self-normalized sums. *Annals of Probability* 1, 788-809.

McCulloch, J.H. (1996) Financial applications of stable distributions. in *Statistical Methods in Finance (Handbook of Statistics)* Vol. 14, Maddala G.S. & C.R. Rao (ed.), Elsevier Science.

Myers, R.H., I. Khuri & W.-H. Carter, Jr. (1989) Response surface methodology: 1966-1988. *Technometrics* 31, 137-157.

Peizer, D.B. & J.W. Pratt (1968) A normal approximation for binormial, F, beta, and other common, related tail probabilities I. *Journal of the American Statistical Association* 63, 1416-1456.

Phillips, P.C.B. & V.A. Hajivassiliou (1987): Bimodel t-ratios. Cowles Foundation Discussion Paper No. 842, Yale University.

Samorodnitsky, G. and M.S. Taqqu (1994) *Stable Non-Gaussian Random Processes.* New York: Chapman & Hall.

Weron, R. (1996) On the Chambers–Mallows–Stuck method for simulating skewed stable random variables. *Statistics and Probability Letters* 28, 165-171.

Zolotarev, V.M. (1986): *One-dimensional Stable Distributions.* Translations of Mathematical Monographs, American Mathematical Society, Vol. 65, Providence.

T able1a.

P arameterEstimates of Fitted Response Surface[a]

i	j \ k	1	2	3	4
		a_{ijk}			
0	3	–	–	-25.720 (-4.920)	41.699 (7.570)
1	1	–	–	–	-0.282 (-2.706)
	3	1.415 (5.229)	-10.047 (-5.604)	41.057 (8.769)	-45.278 (-11.688)
2	1	–	–	-0.154 (-2.332)	0.390 (5.671)
	2	0.109 (5.308)	–	–	–
	3	-0.332 (-4.614)	2.815 (6.229)	-10.530 (-10.752)	10.500 (15.364)
	4	–	–	0.301 (4.573)	–

[a] t-value are reported in parentheses.

T able 1b.

P arameterEstimates of Fitted Response Surface[a]

i \ j	1	2	3	4
	b_{ij}			
0	0.410 (1.839)	-1.132 (-2.368)	1.214 (2.024)	-0.746 (-2.592)
1	-0.296 (-2.393)	0.481 (3.249)	–	–
2	0.067 (3.747)	-0.098 (-4.466)	–	0.025 (6.554)

[a] t-value are reported in parentheses.

T able2a.

${}_{\alpha}t_{.90}$-quantiles for selected α-v alues

α / n	2	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.0
1	6.31	6.07	5.92	5.77	5.60	5.42	5.21	4.98	4.73	4.45	4.15
2	2.92	2.86	2.80	2.74	2.67	2.59	2.50	2.40	2.30	2.18	2.06
3	2.35	2.32	2.29	2.25	2.20	2.16	2.11	2.05	1.99	1.92	1.85
4	2.13	2.11	2.08	2.05	2.02	1.99	1.95	1.91	1.86	1.82	1.77
5	2.02	2.00	1.97	1.95	1.92	1.89	1.86	1.83	1.79	1.75	1.72
6	1.94	1.93	1.90	1.88	1.86	1.83	1.80	1.78	1.74	1.71	1.68
7	1.89	1.88	1.86	1.84	1.81	1.79	1.77	1.74	1.71	1.68	1.65
8	1.86	1.84	1.82	1.80	1.78	1.76	1.74	1.71	1.68	1.66	1.63
9	1.83	1.82	1.80	1.78	1.76	1.74	1.71	1.69	1.66	1.64	1.61
10	1.81	1.80	1.78	1.76	1.74	1.72	1.70	1.67	1.65	1.62	1.60
11	1.80	1.78	1.76	1.74	1.72	1.70	1.68	1.66	1.63	1.61	1.58
12	1.78	1.77	1.75	1.73	1.71	1.69	1.67	1.65	1.62	1.60	1.57
13	1.77	1.76	1.74	1.72	1.70	1.68	1.66	1.64	1.61	1.59	1.56
14	1.76	1.75	1.73	1.71	1.69	1.67	1.65	1.63	1.61	1.58	1.56
15	1.75	1.74	1.72	1.70	1.69	1.67	1.64	1.62	1.60	1.58	1.55
16	1.75	1.73	1.72	1.70	1.68	1.66	1.64	1.62	1.59	1.57	1.55
17	1.74	1.73	1.71	1.69	1.67	1.65	1.63	1.61	1.59	1.57	1.54
18	1.73	1.72	1.70	1.69	1.67	1.65	1.63	1.61	1.58	1.56	1.54
19	1.73	1.72	1.70	1.68	1.66	1.64	1.62	1.60	1.58	1.56	1.53
20	1.72	1.71	1.70	1.68	1.66	1.64	1.62	1.60	1.58	1.55	1.53
21	1.72	1.71	1.69	1.68	1.66	1.64	1.62	1.60	1.57	1.55	1.53
22	1.72	1.70	1.69	1.67	1.65	1.63	1.61	1.59	1.57	1.55	1.53
23	1.71	1.70	1.69	1.67	1.65	1.63	1.61	1.59	1.57	1.55	1.52
24	1.71	1.70	1.68	1.67	1.65	1.63	1.61	1.59	1.57	1.54	1.52
25	1.71	1.70	1.68	1.66	1.65	1.63	1.61	1.59	1.57	1.54	1.52
26	1.71	1.69	1.68	1.66	1.64	1.63	1.61	1.59	1.56	1.54	1.52
27	1.70	1.69	1.68	1.66	1.64	1.62	1.60	1.58	1.56	1.54	1.52
28	1.70	1.69	1.67	1.66	1.64	1.62	1.60	1.58	1.56	1.54	1.52
29	1.70	1.69	1.67	1.66	1.64	1.62	1.60	1.58	1.56	1.54	1.51
30	1.70	1.69	1.67	1.65	1.64	1.62	1.60	1.58	1.56	1.54	1.51
∞	1.65	1.64	1.63	1.62	1.60	1.59	1.56	1.55	1.54	1.52	1.50

T able 2b.

${}_{\alpha}t_{.95}$-quantiles for selected α-v alues

α / n	2	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.0
1	12.71	12.20	11.88	11.56	11.21	10.82	10.40	9.93	9.42	8.87	8.27
2	4.30	4.18	4.09	3.99	3.88	3.76	3.63	3.48	3.32	3.14	2.95
3	3.18	3.12	3.06	3.00	2.94	2.86	2.78	2.69	2.60	2.50	2.39
4	2.78	2.73	2.69	2.64	2.58	2.53	2.47	2.40	2.33	2.25	2.17
5	2.57	2.53	2.49	2.45	2.40	2.35	2.30	2.24	2.18	2.12	2.05
6	2.45	2.41	2.38	2.34	2.29	2.25	2.20	2.15	2.09	2.04	1.98
7	2.36	2.33	2.30	2.26	2.22	2.18	2.13	2.09	2.04	1.98	1.93
8	2.31	2.28	2.24	2.21	2.17	2.13	2.09	2.04	1.99	1.94	1.89
9	2.26	2.23	2.20	2.17	2.13	2.09	2.05	2.01	1.96	1.91	1.86
10	2.23	2.20	2.17	2.14	2.10	2.06	2.02	1.98	1.94	1.89	1.84
11	2.20	2.18	2.14	2.11	2.08	2.04	2.00	1.96	1.92	1.87	1.83
12	2.18	2.15	2.12	2.09	2.06	2.02	1.98	1.94	1.90	1.86	1.82
13	2.16	2.14	2.11	2.07	2.04	2.01	1.97	1.93	1.89	1.85	1.81
14	2.14	2.12	2.09	2.06	2.03	1.99	1.96	1.92	1.88	1.84	1.80
15	2.13	2.11	2.08	2.05	2.02	1.98	1.95	1.91	1.87	1.83	1.79
16	2.12	2.10	2.07	2.04	2.01	1.97	1.94	1.90	1.87	1.83	1.79
17	2.11	2.09	2.06	2.03	2.00	1.97	1.93	1.90	1.86	1.82	1.78
18	2.10	2.08	2.05	2.02	1.99	1.96	1.93	1.89	1.86	1.82	1.78
19	2.09	2.07	2.04	2.02	1.99	1.95	1.92	1.89	1.85	1.81	1.78
20	2.09	2.06	2.04	2.01	1.98	1.95	1.92	1.88	1.85	1.81	1.77
21	2.08	2.06	2.03	2.01	1.98	1.95	1.91	1.88	1.84	1.81	1.77
22	2.07	2.05	2.03	2.00	1.97	1.94	1.91	1.88	1.84	1.81	1.77
23	2.07	2.05	2.02	2.00	1.97	1.94	1.91	1.87	1.84	1.81	1.77
24	2.06	2.04	2.02	1.99	1.96	1.93	1.90	1.87	1.84	1.80	1.77
25	2.06	2.04	2.02	1.99	1.96	1.93	1.90	1.87	1.84	1.80	1.77
26	2.06	2.04	2.01	1.99	1.96	1.93	1.90	1.87	1.84	1.80	1.77
27	2.05	2.03	2.01	1.98	1.96	1.93	1.90	1.87	1.83	1.80	1.77
28	2.05	2.03	2.01	1.98	1.95	1.93	1.90	1.87	1.83	1.80	1.77
29	2.05	2.03	2.00	1.98	1.95	1.92	1.89	1.86	1.83	1.80	1.77
30	2.04	2.02	2.00	1.98	1.95	1.92	1.89	1.86	1.83	1.80	1.77
∞	1.96	1.94	1.93	1.92	1.89	1.87	1.82	1.80	1.78	1.74	1.72

Table 2c.

$_{\alpha}t_{.99}$-quantiles for selected α-values

α / n	2	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.0
1	63.66	61.91	60.50	58.91	57.14	55.17	53.01	50.65	48.11	45.37	42.45
2	9.92	9.58	9.32	9.05	8.76	8.44	8.09	7.70	7.28	6.82	6.33
3	5.84	5.69	5.56	5.44	5.31	5.17	5.02	4.86	4.68	4.49	4.29
4	4.60	4.50	4.41	4.32	4.23	4.13	4.02	3.91	3.79	3.66	3.53
5	4.03	3.95	3.87	3.79	3.71	3.62	3.53	3.43	3.33	3.23	3.12
6	3.71	3.63	3.56	3.48	3.41	3.32	3.24	3.15	3.06	2.96	2.86
7	3.50	3.43	3.36	3.29	3.21	3.13	3.05	2.97	2.88	2.79	2.69
8	3.36	3.29	3.22	3.15	3.08	3.00	2.92	2.84	2.75	2.67	2.57
9	3.25	3.19	3.12	3.05	2.98	2.90	2.83	2.75	2.66	2.58	2.49
10	3.17	3.11	3.04	2.98	2.91	2.83	2.76	2.68	2.60	2.52	2.43
11	3.11	3.05	2.98	2.92	2.85	2.78	2.70	2.63	2.55	2.47	2.38
12	3.05	3.00	2.94	2.87	2.80	2.73	2.66	2.59	2.51	2.43	2.35
13	3.01	2.96	2.90	2.83	2.77	2.70	2.63	2.56	2.48	2.40	2.32
14	2.98	2.92	2.86	2.80	2.74	2.67	2.60	2.53	2.46	2.38	2.30
15	2.95	2.89	2.84	2.78	2.71	2.65	2.58	2.51	2.44	2.36	2.29
16	2.92	2.87	2.81	2.75	2.69	2.63	2.56	2.49	2.42	2.35	2.28
17	2.90	2.85	2.79	2.73	2.67	2.61	2.55	2.48	2.41	2.34	2.27
18	2.88	2.83	2.77	2.72	2.66	2.60	2.53	2.47	2.40	2.33	2.26
19	2.86	2.81	2.76	2.70	2.64	2.58	2.52	2.46	2.39	2.32	2.26
20	2.85	2.80	2.75	2.69	2.63	2.57	2.51	2.45	2.39	2.32	2.25
21	2.83	2.78	2.73	2.68	2.62	2.56	2.50	2.44	2.38	2.32	2.25
22	2.82	2.77	2.72	2.67	2.61	2.56	2.50	2.44	2.38	2.31	2.25
23	2.81	2.76	2.71	2.66	2.61	2.55	2.49	2.43	2.37	2.31	2.25
24	2.80	2.75	2.70	2.65	2.60	2.54	2.49	2.43	2.37	2.31	2.25
25	2.79	2.74	2.70	2.65	2.59	2.54	2.48	2.43	2.37	2.31	2.25
26	2.78	2.74	2.69	2.64	2.59	2.53	2.48	2.42	2.37	2.31	2.25
27	2.77	2.73	2.68	2.63	2.58	2.53	2.48	2.42	2.36	2.31	2.25
28	2.76	2.72	2.68	2.63	2.58	2.53	2.47	2.42	2.36	2.31	2.25
29	2.76	2.72	2.67	2.62	2.57	2.52	2.47	2.42	2.36	2.31	2.25
30	2.75	2.71	2.66	2.62	2.57	2.52	2.47	2.42	2.36	2.31	2.25
∞	2.58	2.50	2.47	2.44	2.43	2.36	2.33	2.28	2.25	2.18	2.11

The following papers have been published since 2002:

January	2002	Rent indices for housing in West Germany 1985 to 1998	Johannes Hoffmann Claudia Kurz
January	2002	Short-Term Capital, Economic Transformation, and EU Accession	Claudia M. Buch Lusine Lusinyan
January	2002	Fiscal Foundation of Convergence to European Union in Pre-Accession Transition Countries	László Halpern Judit Neményi
January	2002	Testing for Competition Among German Banks	Hannah S. Hempell
January	2002	The stable long-run CAPM and the cross-section of expected returns	Jeong-Ryeol Kim
February	2002	Pitfalls in the European Enlargement Process – Financial Instability and Real Divergence	Helmut Wagner
February	2002	The Empirical Performance of Option Based Densities of Foreign Exchange	Ben R. Craig Joachim G. Keller
February	2002	Evaluating Density Forecasts with an Application to Stock Market Returns	Gabriela de Raaij Burkhard Raunig
February	2002	Estimating Bilateral Exposures in the German Interbank Market: Is there a Danger of Contagion?	Christian Upper Andreas Worms
February	2002	The long-term sustainability of public finance in Germany – an analysis based on generational accounting	Bernhard Manzke

March	2002	The pass-through from market interest rates to bank lending rates in Germany	Mark A. Weth
April	2002	Dependencies between European stock markets when price changes are unusually large	Sebastian T. Schich
May	2002	Analysing Divisia Aggregates for the Euro Area	Hans-Eggert Reimers
May	2002	Price rigidity, the mark-up and the dynamics of the current account	Giovanni Lombardo
June	2002	An Examination of the Relationship Between Firm Size, Growth, and Liquidity in the Neuer Markt	Julie Ann Elston
June	2002	Monetary Transmission in the New Economy: Accelerated Depreciation, Transmission Channels and the Speed of Adjustment	Ulf von Kalckreuth Jürgen Schröder
June	2002	Central Bank Intervention and Exchange Rate Expectations – Evidence from the Daily DM/US-Dollar Exchange Rate	Stefan Reitz
June	2002	Monetary indicators and policy rules in the P-star model	Karl-Heinz Tödter
July	2002	Real currency appreciation in accession countries: Balassa-Samuelson and investment demand	Christoph Fischer
August	2002	The Eurosystem's Standing Facilities in a General Equilibrium Model of the European Interbank Market	Jens Tapking

August 2002	Imperfect Competition, Monetary Policy and Welfare in a Currency Area	Giovanni Lombardo
August 2002	Monetary and fiscal policy rules in a model with capital accumulation and potentially non-superneutral money	Leopold von Thadden
September 2002	Dynamic Q-investment functions for Germany using panel balance sheet data and a new algorithm for the capital stock at replacement values	Andreas Behr Egon Bellgardt
October 2002	Tail Wags Dog? Time-Varying Information Shares in the Bund Market	Christian Upper Thomas Werner
October 2002	Time Variation in the Tail Behaviour of Bund Futures Returns	Thomas Werner Christian Upper
November 2002	Bootstrapping Autoregressions with Conditional Heteroskedasticity of Unknown Form	Sílvia Gonçalves Lutz Kilian
November 2002	Cost-Push Shocks and Monetary Policy in Open Economies	Alan Sutherland
November 2002	Further Evidence On The Relationship Between Firm Investment And Financial Status	Robert S. Chirinko Ulf von Kalckreuth
November 2002	Genetic Learning as an Explanation of Stylized Facts of Foreign Exchange Markets	Thomas Lux Sascha Schornstein
December 2002	Wechselkurszielzonen, wirtschaftlicher Aufholprozess und endogene Realignmentrisiken *	Karin Radeck

* Available in German only.

December 2002	Optimal factor taxation under wage bargaining – a dynamic perspective	Erkki Koskela Leopold von Thadden
January 2003	Testing mean-variance efficiency in CAPM with possibly non-gaussian errors: an exact simulation-based approach	Marie-Claude Beaulieu Jean-Marie Dufour Lynda Khalaf
January 2003	Finite-sample distributions of self-normalized sums	Jeong-Ryeol Kim

Visiting researcher at the Deutsche Bundesbank

The Deutsche Bundesbank in Frankfurt is looking for a visiting researcher. Visitors should prepare a research project during their stay at the Bundesbank. Candidates must hold a Ph D and be engaged in the field of either macroeconomics and monetary economics, financial markets or international economics. Proposed research projects should be from these fields. The visiting term will be from 3 to 6 months. Salary is commensurate with experience.

Applicants are requested to send a CV, copies of recent papers, letters of reference and a proposal for a research project to:

Deutsche Bundesbank
Personalabteilung
Wilhelm-Epstein-Str. 14

D - 60431 Frankfurt
GERMANY