Imperfect Competition,
Monetary Policy and
Welfare in a Currency Area
Giovanni Lombardo

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Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main, Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 95 66-1 Telex within Germany 4 1 227, telex from abroad 4 14 431, fax +49 69 5 60 10 71

Please address all orders in writing to: Deutsche Bundesbank, Press and Public Relations Division, at the above address or via fax No. +49 69 95 66-30 77

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Abstract

This paper studies the implication, in terms of welfare and monetary policy, of unequal degrees of competition across members of a currency area. We look at two ways in which the degree of competition in the market for goods can affect welfare in a currency area. One is through different average degrees of competition in the monetary union; the other is through different degrees of competition across its members. We find two interesting results. First, if prices are sticky, more competition tends to be detrimental to welfare. Second, welfare increases in proportion to the asymmetry of the union. These results imply that the monetary authority should react more strongly to inflationary pressure coming from the more competitive member of the monetary union. We compute the optimal monetary policy of the central bank of the union and compare its performance with a simple Taylor rule. We discuss the circumstances under which the Taylor rule approaches the optimal rule. The optimal inflation target for the asymmetric union is also discussed.

Keywords: Currency area; optimal monetary policy; Taylor rule; imperfect competition; sticky prices.

JEL Classification: E3, E4, E5, F4.

Zusammenfassung

Dieses Papier untersucht die Auswirkung unterschiedlicher Wettbewerbsintensitäten in den

Ländern eines Währungsraums auf die Wohlfahrt und die optimale Geldpolitik. Betrachtet

werden zwei Möglichkeiten, wie die Wettbewerbsintensität am Gütermarkt die Wohlfahrt

eines Währungsraums beeinflussen kann. Zwei interessante Schlussfolgerungen können

gezogen werden. Erstens wirkt sich bei unflexiblen Preisen ein erhöhter Wettbewerb in der

Regel negativ auf die Wohlfahrt aus. Zweitens kann die Geldpolitik effektiver sein, wenn die

Länder größere Asymmetrien aufweisen. Diese Ergebnisse legen es nahe, dass die

Zentralbank stärker auf einen Inflationsdruck reagieren sollte, der von dem Mitgliedsland

ausgeht, in dem der Wettbewerb intensiver ist. Die optimale Geldpolitik der Zentralbank der

Währungsunion wird ermittelt und mit einer einfachen Taylor-Regel verglichen. Es werden

die Voraussetzungen erörtert, unter denen die Taylor-Regel sich der optimalen Regel

annähert. Diskutiert wird auch das optimale Inflationsziel einer asymmetrischen

Währungsunion.

Schlüsseltermini: Währungsraum; optimale Geldpolitik; Taylor-Regel; unvollständiger

Wettbewerb; starre Preise.

JEL-Klassifizierung: E3, E4, E5, F4.

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Imperfect Competition, Monetary Policy and Welfare in a Currency Area¹

1 Introduction

This study addresses the question of how the monetary authority of a given currency area should react to idiosyncratic shocks when the members of the monetary union differ with respect to the degree of competition of their markets?

Two distinct issues arise in this investigation. The first pertains to the effects of competition on welfare, while the second relates to the effect on welfare of an asymmetric degree of competition across the monetary union. In analysing the welfare effects of imperfect competition, we assume that the monetary authority of the union adopts rules aimed (implicitly or explicitly) at increasing the union's welfare.

With regard to the first issue, the First Fundamental Welfare Theorem tells us that perfect competition produces the Pareto optimal equilibrium.² Nevertheless, there is no theorem to guide us in understanding the welfare implications of competition in a second best economy. In this paper we will assume that prices are not completely flexible. In such an environment, it is not obvious which way the competition-welfare relationship should go. Indeed, many authors have recently pointed out that nominal rigidities bring about an inefficient dispersion of the demand for goods across firms when the latter do not

¹I wish to thank colleagues at the Bundesbank for their comments. I also wish to thank Leo von Thadden, Alan Sutherland and Huw Dixon for their suggestions. I assume full responsibility for any remaining errors. This paper reflects the views of the author and not necessarily those of the Deutsche Bundesbank.

²In this paper we measure competition using the degree of substitutability between goods. Perfect substitutability implies perfect competition.

adjust their price simultaneously.³ The dispersion of output (and prices) is increasing in the degree of competition. Hence, from this perspective, more competition implies more inefficiency. Furthermore, some recent papers have shown that the degree of competition can affect the degree of "real rigidity" of prices (e.g. Ball and Romer (1990), Kimball (1995), Woodford (2001) and Lombardo (2001a,b)). This means that the persistence of the deviations of variables from their flexible price equilibrium, as well as their volatility, are affected by the degree of competition. Hence competition can affect welfare by affecting the volatility of the economy. In this paper we show that, in the presence of shocks, the traditional "inefficiency triangle" of imperfect competition could be more than offset by the stabilising effects of the monopolistic distortion.

The second issue pursued by our study concerns the relationship between welfare and differences in the degree of competition between members of the currency union. The Optimum Currency Area principles suggest that countries benefit from a common currency if their structures are similar (Salvatore, 2001 p 714). In this paper we do not study the optimality of a currency area. It is nevertheless surprising to see that under the assumptions that characterise our model, the member's welfare increases in the asymmetry of the degree of competition of the members of the union. The rationale behind this result lies again in the second-best nature of our economy. When prices are sticky, welfare also depends on the extent to which the terms of trade (relative aggregate prices of the union-members) deviate from their flexible price equilibrium. As pointed out by Benigno (2000), the terms of trade are independent of developments in the real economy when the union

³See for example Khan et al. (2000) and Woodford (1999a).

is symmetric. Hence they are insulated from monetary policy. Asymmetric economic structures imply different adjustment paths for domestic prices so that the terms of trade (relative prices) will be linked to the real economy, and hence – under sticky prices – to the monetary policy. The monetary authority can therefore bring about higher levels of welfare.

Here we show that the welfare effects of competition, and of different degrees of competition across the union, depend on the monetary policy pursued by the union's central bank. We consider two alternative policies: the optimal policy under commitment and the a Taylor-type of interest rate rule (Taylor (1993)). The former will serve as a benchmark rule for the second, more realistic interest rate rule. The Taylor rule has been widely discussed in the literature as a rule that closely resembles the behaviour of many central banks.⁴ As for this rule, it is important to understand which union-wide aggregates should be targeted. For example, the main objective of the European Central Bank is to maintain price stability. Price stability is measured using a union-wide price aggregate (the HICP), which consists of the average of the members' inflation rates, weighted by their consumption shares.⁵

Our analysis shows that, *ceteris paribus*, the optimal weights in the interest rule should be proportional to the degree of competition of the members of the union. That is, the central bank should respond more aggressively to the inflation which stems from the more competitive member. This fact is consistent with our earlier discussion about the welfare effects of competition. In the more competitive economy the output dispersion and

⁴See, for example, Clarida et al. (1997,1998).

⁵ See the ECB Monthly Bulletin - Euro Area Statistics, Methodological Notes, Table 4.1.

volatility effects of competition are more marked. Hence price instability in that economy is more detrimental for welfare. Since, in our model, the central bank cannot affect the steady-state level of inefficiency (the "inefficiency triangle"), its only leverage is on the reduction of the costs of the business cycle, which are higher in the more competitive economy.

Our analysis is based on a two-country model à la Obstfeld and Rogoff (1995). The main difference between our set-up and that of a pure open-economy model is that we impose a single currency, so that exchange rates do not exist. Benigno (2000) uses this set-up to study an issue very close to the one taken up in this paper: the effects of asymmetric degrees of nominal rigidity on the welfare of a currency area. Nominal rigidity (or frequency of price adjustments) and imperfect competition are clearly two distinct concepts. Yet one can reasonably imagine that the frequency of price adjustments should depend, inter alia, on the degree of competition. Carlton (1986) shows, for example, that there is empirical evidence of a positive correlation between market concentration and nominal price stickiness, although there is also evidence of a negative correlation between concentration and real rigidity (size of the adjustment given the frequency).⁶ Neither in our paper nor in Benigno's paper is this connection assumed. In this respect we see our paper as complementing Benigno's analysis. Benigno's policy prescription – that the central bank should respond more aggressively to the country with more sticky prices – could be attenuated, if not totally offset, by our policy prescription. On the one hand the stickier prices are, the more costly inflation becomes. On the other, given any positive

⁶More recently, Powers and Powers (2001) provide similar empirical evidence.

degree of price stickiness, the more competitive the markets are the more costly inflation becomes. Allowing stickiness to vary with competition produces ambiguous effects of competition (stickiness) on welfare.⁷ Consequently, the net effect of these distortions on the optimal interest rate rule remains an empirical issue. In this paper we show the welfare effects of linking stickiness and competition. Further research on the micofoundation of this link is certainly needed.

As far as we are aware, there are not many studies that address the issue tackled in our paper. We have already referred to the paper by Benigno (2000) in which a model similar to ours is studied. Along similar lines, Angelini *et al.* (2002), in an empirical paper, show that information based on national variables, as opposed to mere union-wide variables, can improve the performance of an interest rate rule.

Schmitt-Grohé and Uribe (2001) study the implication of different degrees of competition for the optimal fiscal and monetary policy of a closed economy. They use a public finance approach to simultaneously determine the optimal (non-lump sum) tax rate and the optimal inflation rate. They show that the optimal inflation rate is positive. While the volatility of inflation decreases in the degree of competition, the volatility of consumption increases, as we find in our model. These authors do not report the net effect on welfare of the degree of competition.

King and Wolman (1999) also study the optimal monetary policy in an imperfectly competitive closed economy. Among other things, they show that in the presence of an

⁷We have verified numerically that this is indeed the case in a version of the model where both stickiness and competition differ across country. The net effect depends clearly on the curvature of the "markup-stickiness relationship".

inefficient steady-state level of output, the optimal policy drives the inflation rate to zero asymptotically. We obtain the same result in our paper. These authors do not report the welfare implication of various degrees of competition.

Neiss (1999) studies the relationship between the degree of competition and the discretionary optimal inflation rate in a Barro-Gordon type of model. The optimal rate is again determined by optimising a utility-based welfare function. She shows that the optimal inflation rate is a non-monotonic function of the degree of competition. We do not consider discretionary policies here, meaning that a direct comparison is not possible. As King and Wolman (1999) show in a model similar to ours, Neiss' result would emerge if the central bank had to commit to a constant inflation rate. Without this commitment, the optimal inflation rate tends to zero asymptotically.

By introducing welfare effects of holding money, Khan et al. (2000) show that the zero inflation result of King and Wolman (1999) does not hold. Instead they find that a small deflation rate would be optimal. This deflation rate decreases in the degree of competition. Following Benigno (2000) and Woodford (1999a) we abstract from these monetary frictions.

The rest of the paper is organised as follows. Section 2 describes our model. Section 3 derives the approximation of the structural equations of the model. Section 4 derives the central bank's loss function as a second-order approximation to the households' utility function. Section 5 derives the optimal monetary policy. Section 6 shows the implications of our model by way of graphical examples. Section 7 concludes.

2 The model

The structure of our model is fully identical to that developed by Benigno (2000), apart from the degree of competition among the firms producing goods in the same country. We assume that it can vary across countries. We also assume that there are two countries, denoted by H and F.

2.1 Preferences

Households in country H and country F have identical preferences over leisure, real money balances and bundles of consumption goods produced at home and abroad. These bundles can be represented by the following homothetic aggregators⁸

$$C_H^j = \left[n^{\theta - 1} \int_0^n c_H^{\theta} \left(z \right) dz \right]^{\frac{1}{\theta}} \tag{1}$$

$$C_F^j = \left[(1 - n)^{\theta - 1} \int_n^1 c_F^{\theta}(z) \, dz \right]^{\frac{1}{\theta}} \tag{2}$$

$$C^{j} = \frac{\left(C_{H}^{j}\right)^{n} \left(C_{F}^{j}\right)^{1-n}}{n^{n} \left(1-n\right)^{1-n}} \tag{3}$$

for j=H,F and where $\theta=\theta^H$ if j=H and $\theta=\theta^F$ when j=F.

⁸In this paper we adopt the convention that time subscripts appear only when variables evaluated at different times appear in the same equation.

The aggregator (3) implies that the current account between the two countries is always at its steady-state level, which we assume to be equal to zero.⁹

The price indices associated with these aggregators are respectively

$$P_{H} = \left[\frac{1}{n} \int_{0}^{n} p_{H}(z)^{\frac{\theta}{\theta-1}} dz\right]^{\frac{\theta-1}{\theta}}$$

$$\tag{4}$$

$$P_F = \left[\frac{1}{1-n} \int_n^1 p_F(z)^{\frac{\theta}{\theta-1}} dz \right]^{\frac{\theta-1}{\theta}}$$
 (5)

$$P = P_H^n P_F^{1-n} \tag{6}$$

Households supply a large variety of labour services, each specific to a firm producing goods in their own country: i.e. labour is immobile across countries and firms.

These aggregators and their prices imply the following homothetic demands for goods

$$c_{j}\left(z\right) = \left(\frac{p\left(z\right)}{P_{H}}\right)^{-\varepsilon^{j}} \left(\frac{P_{H}}{P}\right)^{-1} C^{j}$$

for j = H, F and where $\varepsilon = \frac{1}{1-\theta}$

⁹The computational advantage of neutralizing the current account by imposing a Cobb-Douglas aggregator of consumption has been first pointed out by Corsetti and Pesenti (1999). With a more general CES aggregator the current account reaction to shocks would depend on the degree of competition too, as discussed in Lombardo (2001a). We neglect this possibility for computational convenience.

Each household solves the following problem

$$\max_{C_s, \frac{M_s}{P_s}, l_s(z)} E_t \sum_{s=t}^{\infty} \beta^s \left[U(C_s) + \chi \log \left(\frac{M_s}{P_s} \right) + \frac{1}{n} \int_0^n V(l(z)_s) dz \right]$$
(7a)

subject to

$$M_s + B_s + P_s C_s = (1 + i_s) B_{s-1} + M_{s-1} + \frac{1}{n} \int_0^n p_s(z) y_s(z) dz + \tau_s$$
 (8)

and the transversality condition

$$\lim_{s \to \infty} E_t \left[R_{s,s+t} \left(\frac{B_{s+t} + M_{s+t}}{P_{s+t}} \right) \right] = 0 \tag{9}$$

 $R_{s,t}$ is the market discount factor, M_t is the money stock held by consumers, B_s is a nominal bond representing claims on goods produced by the other country in the union, i_t is the nominal interest rate, τ_t is a lump-sum tax/transfer and $y_t(z)$ is total real output of the z-th firm. χ is a weight on real balances in the household objective. In fact, money is irrelevant in the present model, since the monetary authority uses the nominal interest rate as the policy instrument. In addition, in the welfare evaluation we assume that χ is of negligible size so that we can abstract from the real balance term in the consumer's preferences.¹⁰

¹⁰ Indeed we could drop money altogether from our model. By introducing money explicitly in the model we can show that there is a well defined demand for money, which depends on consumption and the nominal interest rate, consistently with the "Liquidity preference" theory (see equation (12) below). As long as some money is held by the households, the central bank can control the short run interest rate.

The implied first order conditions yield

$$U_{c}(C_{t}) = \left[\beta \frac{(1+i_{t+1})}{(1+E_{t}\pi_{t+1})}\right] E_{t}U_{c}(C_{t+1})$$
(10)

$$-\frac{V_l(l(z))}{U_c(C)} = \frac{W(z)}{P} \tag{11}$$

$$\frac{M_t}{P_t} = U_c \left(C_t \right)^{-1} \left(\frac{1}{i_{t+1}} + 1 \right) \tag{12}$$

where
$$U\left(C\right)=\left(1-\sigma\right)^{-1}C^{1-\sigma}$$
 and $V\left(l\left(z\right)\right)=-\left(\zeta+1\right)^{-1}l\left(z\right)^{\zeta+1}$

2.2 Firms

There are a large number of firms in the union, which we index on the unit interval. Of these, n produce in country H and 1-n in country F. Each firm produces one type of goods so that there are as many goods as firms. Each item is an imperfect substitute for the other goods, as indicated by the household's preferences. Each firm uses the same technology, that is

$$y(z) = Af(l(z), k) = Al(z)^{1-\alpha} \bar{k}^{\alpha}$$
(13)

where A_t is a technology shock parameter, l(z) is the specific labour input and \bar{k} is any other factor (possibly capital) used in production. We assume that these other factors are

fixed and normalised to one.

2.2.1 Calvo contracts

We assume that firms adjust prices only at random intervals, in accordance to the mechanism described in Calvo (1983). That is, in any period of time there is a probability ω that the firm does not adjust the price.

Each firm therefore chooses the optimal price by solving the following problem

$$\max_{p_t} E_t \sum_{s=t}^{\infty} (\omega \beta)^s \left[\frac{p_t}{P_s} y_s - \frac{TC_s}{P_s} \right]$$

where TC denotes total costs of production and β is the household discount factor.¹¹

The solution to this problem yields

$$p_t = \frac{E_t \sum_{s=t}^{\infty} (\omega \beta)^s \, \mu^j m c_s y_s}{E_t \sum_{s=t}^{\infty} (\omega \beta)^s \, \frac{y_s}{P_s}}$$
(14)

for j=H,F, where mc is the real marginal cost and $\mu^j=\frac{\varepsilon^j}{\varepsilon^j-1}$ is the mark-up.

Due to the factor specificity, the marginal cost of the firm is increasing in its own output, i.e.

$$mc\left(z^{j}\right) = \frac{W\left(z^{j}\right)}{P} \frac{1}{s^{j}\left(1-\alpha\right)} y\left(z^{j}\right)_{t}^{\frac{\alpha}{1-\alpha}} A_{j,t}^{\frac{-1}{1-\alpha}}$$

$$\tag{15}$$

where $z^H \in [0, n]$, and $z^F = (n, 1]$ and $s^j > 1$ is a country-specific subsidy for production

¹¹More appropriately we should use the market interest rate. In the linearization of the model the appropriate discount factor will be the steady state market rate, which in turn will be the household discount factor.

that will counter the monopolistic distortion. As a consequence, firms charging different prices will face different demands and hence different marginal costs of production. The technology shock A_t is country-specific but is identical across firms within each country.

3 Approximation of the model

Following Woodford (1999a) and Benigno (2000) we derive the objective of the central bank from the second-order approximation of a population-weighted sum of the households' utility function. The structural model is non-linear and can be solved only in its approximated form. As Woodford (1999a) explains, a first-order approximation of the structural equations of the model suffices for the evaluation of welfare if the second-order approximation of the welfare function does not have linear terms.¹² As we show later on, in the steady-state all linear terms in the welfare function cancel out when the economy's steady-state is an efficient equilibrium (e.g. thanks to subsidies). Our study consists of evaluating the welfare implications of various policy rules under different degrees of competition. In this regard it would not make sense to assume an efficient steady-state. We therefore need to use a second-order approximation to the structural equations of our model. Sutherland (2001) shows that there is a simple solution to this problem. A drawback of this approach is that the algebra becomes more cumbersome. Fortunately, our results based on the first order approximation are qualitatively similar to those we obtained using a second order approximation. We will therefore present the results based on the

¹²This point was first raised by Kim and Kim (2000). They show that the log-linearisation can bring about spurious "welfare reversals". This is clearly relevant to our model. The authors propose a "bias adjustment" to reduce the inaccuracy of the approximation. Moreover, they show that a quadratic approximation can reduce the inaccuracy significantly. We adopt this latter solution.

first-order solution and only discuss the implications of the second-order approximation.

We consider a union with zero cross-country bond holding. In this case aggregate consumption in the two countries is always identical, i.e. $C_t^H = C_t^F = C_t$. That is, there is perfect risk-sharing in consumption.

3.1 Flexible price equilibrium: the natural rates

The objective of the central bank is to maximise the welfare of the union. The latter is suboptimal due to monopolistic distortion and, in the presence of shocks, to the stickiness of prices. The central bank cannot directly address any of these imperfections. Nevertheless, thanks to price stickiness, the central bank can manoeuver the real economy in such a way as to reduce the welfare cost of the business cycle.

Abstracting from the deadweight loss,¹³ the first best is clearly achieved with price flexibility. In this case, prices reflect precisely the marginal cost of production and the preferences of the households. The central bank will aim at reducing the deviations of the economy from the flexible price equilibrium. Indeed, Goodfriend and King (1997, 2001) show that, under a wide range of possible assumptions, the optimal monetary policy is that which reproduces price flexibility.

It should first be noted that with flexible prices, firms set their price as a mark-up over marginal costs. Therefore having defined the terms of trade as $T = \frac{P_F}{P_H}$, we obtain

 $^{^{13}\}mathrm{By}$ this we refer exclusively to the distortion that would also be present under flexible prices, i.e. the "inefficiency triangle".

for country H, say,

$$\frac{p(z)}{P} = \frac{P_H}{P} = \mu^H m c^H
= \frac{\mu^H}{s^H} \frac{1}{1 - \alpha} \frac{W(z)^H}{P} A_H^{\frac{-1}{1 - \alpha}} \left(T^{1 - n} C^W \right)^{\frac{\alpha}{1 - \alpha}}
= \Phi_H^{-1} \frac{1}{1 - \alpha} \left(T^{1 - n} C^W \right)^{\frac{\zeta + \alpha}{1 - \alpha}} A_H^{\frac{-(\zeta + 1)}{1 - \alpha}} C^\sigma$$
(16)

where we have made use of the labour demand implied by the production function (equation (13)), the labour supply equation (11) and where $\Phi_H = s^H / \mu^H$.

An equation similar to equation (16) applies to country F. We can therefore derive the flexible price terms of trade as

$$\frac{P_F}{P_H} = T^N = \left(\frac{\Phi_H}{\Phi_F}\right)^{\frac{1-\alpha}{\zeta+1}} \frac{A_H}{A_F} \tag{17}$$

If we raise equation (16) to the power of n and the foreign counterpart by (1-n) and multiply the two resulting equations, we obtain

$$C^{N} = \left[\Phi_{W} \left(1 - \alpha\right)\right]^{\frac{1 - \alpha}{\zeta + \alpha + (1 - \alpha)\sigma}} A_{W}^{\frac{\zeta + 1}{\zeta + \alpha + (1 - \alpha)\sigma}}$$

$$\tag{18}$$

where $A_W = A_H^n A_F^{1-n}$ and where $\Phi_W = \Phi_H^n \Phi_F^{1-n}$. Then the log deviation of the natural rate from its steady-state is $\hat{C}^N \equiv \ln \left(C^N / C_{ss}^N \right) = (\zeta + 1) \left(\zeta + \alpha + (1 - \alpha) \sigma \right)^{-1} \hat{A}_W$.

It is finally worth deriving the *natural interest rate*, or the Wicksellian interest rate (Woodford, 1999b), as (in log deviation terms) $i_t^N = \sigma \left[E_t \hat{C}_{t+1}^N - \hat{C}_t^N \right]$.

It is important to note that consumption levels in the two countries are always identical

irrespective of the symmetry of the underlying shocks. Nevertheless, the natural terms of trade fluctuate only to the extent that shocks are asymmetric $(A_H \neq A_F)$.

3.2 Sticky prices

The derivation of the fundamental dynamic equations of our model require some tedious algebra, which we prefer to present in the appendix. Here we show only the final outcome.

Let us first define the log deviation of the inflation rate as $\pi_t^W \equiv \log(P_t/P_{t-1})$ for union-wide inflation, and $\pi_t^j = \log(P_{j,t}/P_{j,t-1})$, $j = \{H, F\}$.

The union-wide inflation is a version of the New-Keynesian Phillips curve, i.e.

$$\pi_{t}^{W} = \left(n\kappa_{C}^{H} + (1-n)\kappa_{C}^{F}\right)\left(\hat{C}_{t}^{W} - \hat{C}_{t}^{W,N}\right) +$$

$$+ n\left(1-n\right)\left(\kappa_{T}^{H} - \kappa_{T}^{F}\right)\left(\hat{T}_{t} - \hat{T}_{t}^{N}\right) + \beta E_{t}\pi_{t+1}^{W}$$
(19)

where ^ denotes log deviations, while the inflation differential is given by

$$\pi_t^R \equiv \pi_t^F - \pi_t^H = -\left(n\kappa_T^F + (1-n)\kappa_T^H\right)\left(\hat{T}_t - \hat{T}_t^N\right) + \left(\kappa_C^F - \kappa_C^H\right)\left(\hat{C}_t^W - \hat{C}_t^{W,N}\right) + \beta E_t \pi_{t+1}^R$$
(20)

The dynamics of the terms of trade is described by the following equation

$$\beta E_{t} \left(\hat{T}_{t+1} - \hat{T}_{t+1}^{N} \right) = \left(1 + \beta + \left(n \kappa_{T}^{F} + (1 - n) \kappa_{T}^{H} \right) \right) \left(\hat{T}_{t} - \hat{T}_{t}^{N} \right) +$$

$$- \left(\hat{T}_{t-1} - \hat{T}_{t-1}^{N} \right) - \left(\kappa_{C}^{F} - \kappa_{C}^{H} \right) \left(\hat{C}_{t} - \hat{C}_{t}^{N} \right) +$$

$$+ \left(1 + \beta \right) \hat{T}_{t}^{N} - \hat{T}_{t-1}^{N} - \beta E_{t} \hat{T}_{t+1}^{N}$$
(21)

If we assume identical degrees of competition across countries, the terms of trade become insulated from the rest of the economy: the terms of trade gap can be zero only if the natural terms of trade are zero at all times. In this case inflation and the output gap can be reduced simultaneously to zero.

Equations (19) and (21) suggest instead that asymmetry can bring about a *trade-off* between output, inflation and terms of trade stabilization, as Benigno (2000) points out.

We now turn to the derivation of our measure of welfare.

4 The welfare function

The appendix shows how to take a second-order approximation to the households' utility function in our model. The central bank aggregates the welfare function of each household across the union attaching the same weight to each of them. For the sake of simplicity we show the aggregate welfare function when $\Phi_H = \Phi_F = \Phi$ (e.g. thanks to production subsidies). This assumption allows us to express the welfare function in a much more

compact form, namely

$$\sum_{t=0}^{\infty} \beta^{t} L_{t} \equiv -2 \sum_{t=0}^{\infty} \beta^{t} \mathcal{W}^{W} = \Omega \sum_{t=0}^{\infty} \beta^{t} \left\{ -2 \left(1 - \Phi \right) \hat{x}_{t} - \left(1 - \sigma \right) \left(1 - \Phi \right) \hat{C}_{t}^{2} + \Phi \left(\sigma + \eta \right) \hat{x}_{t}^{2} + \Phi \left(1 - \eta \right) \left(1 + \eta \right) \hat{\Upsilon}_{t}^{2} + \Phi \left(\psi^{H} + \psi^{F} \right) \left(\gamma \left(\pi_{t}^{H} \right)^{2} + \left(1 - \gamma \right) \left(\pi_{t}^{F} \right)^{2} \right) \right\}$$
(22)

where
$$\hat{x} = \hat{C} - \hat{C}^N$$
, $\hat{\Upsilon} = \hat{T} - \hat{T}^N$ and where $\Omega = U_c Css = [\Phi(1 - \alpha)]^{\frac{(1 - \alpha)(1 - \sigma)}{\zeta + \alpha + (1 - \alpha)\sigma}}$, $\gamma = \psi^H / (\psi^H + \psi^F)$, $\psi^H = n \frac{\omega^2 (\varepsilon^H)^2 (1 - \theta^H + \eta)}{(1 - \omega)(1 - \omega\beta)}$, $\psi^F = (1 - n) \frac{\omega^2 (\varepsilon^F)^2 (1 - \theta^F + \eta)}{(1 - \omega)(1 - \omega\beta)}$.

We use the assumption of equal steady-state inefficiency across the union to present the numerical results later on. This is a particularly convenient means of deriving the optimal policy. Relaxing this assumption would reinforce our results concerning the optimal weights in a Taylor-type of interest rule. This can easily be verified by inspecting equation (22) and taking it as referring to one country only, say country H. In this case, the coefficient on H-inflation increases in the elasticity of substitution between H-produced goods: the cost of inflation increases. Furthermore, Φ_H increases in the elasticity, making all the "bad" arguments of the welfare function more costly. The intuition is clear: if the steady-state is efficient, $\Phi = 1$, deviating from it is more costly than if the steady-state is inefficient.

Equation (22) shows that under $\Phi = 1$, the first-best solution would be to completely eliminate the fluctuations in inflation, the output gap and the terms of trade gap. Nevertheless, equation (20) together with equation (19) and equation (21) imply that the terms of trade gap cannot be eliminated in the presence of shocks. Therefore, the efficient

outcome cannot be achieved. 14

The result is therefore that, with unequal degrees of competition – as with unequal degrees of nominal rigidities (Benigno, 1999) – the central bank faces a *trade-off* in the choice of the volatility of these three variables.

Given unequal degrees of competition, as in the case of unequal degrees of nominal rigidity, there is a tension between the need to eliminate the inefficiency due to price stickiness and the need to allow relative prices across countries to adjust in accordance with productivity differentials.

Equation (22) shows also that the difference in the degrees of competition enters welfare only through the marginal "disutility" of the country-specific inflation. If each household in the union receives the same weight, equation (22) suggests that the welfare-maximizing central bank should react more strongly to the inflation of the country with *higher* competition. The rationale for this result is that countries with higher competition experience a higher dispersion of production, which, given identical technologies, is a source of inefficiency.

Under $\Phi < 1$ there is extra tension in the objective of the policymaker. On the one hand, a higher output gap would appear to increase welfare. As shown in King and Wolman (1999), despite the inefficient steady-state, the optimal policy still calls for the elimination of the output gap in the long run.¹⁵ On the other, the volatility of consumption

¹⁴This is equivalent to proposition 7 in Benigno (2000).

¹⁵King and Wolman refer to this result as a "modified monetary golden rule". They show that in a model where pricing decisions follow the Calvo (1983) structure, a central bank that had to choose a *constant* inflation rate, would place it slightly above zero. This would erode the firm's market power. However, a credible disinflationary policy can increase consumption in the short run and, given a positive discount rate, this is valued more than the long-run suboptimal level of consumption. The same effect is at work in

affects welfare. This is true even if the output gap is eliminated in each period. In this event, the volatility of consumption has a negative effect on welfare only if the degree of risk aversion is sufficiently high $(\sigma > 1)$.¹⁶

5 Optimal policy

In this section we characterise the optimal policy with commitment for a central bank which seeks to minimise the loss function (22) subject to the aggregate Phillips curve (equation (19)) and the dynamics of the terms of trade (equation (21)). We use the optimal policy as a benchmark for the more plausible Taylor-type interest rate rule. The advantage of using such a benchmark is that it provides a metric for the performance of the Taylor rule. The drawback is that we evaluate the optimal rule only on the basis of a first-order approximation of the structural equations of the economy and under an identical steady-state degree of inefficiency. A study of the Taylor rule alone, based on a second-order approximation and different degrees of steady-state inefficiency, produced qualitatively identical results.

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our model. A word of caution is in order in this regard. The modified golden rule hinges on the existence of price contracts (e.g. à la Calvo). These types of price contracts are not reasonable under high levels of inflation or in periods of regime changes: e.g. a strong disinflationary policy.

¹⁶Notice that with $\sigma = 0$ agents are risk neutral. The squared term in the welfare function that still appears under $\sigma = 0$, is indeed part of the mean of C_t when C_t is log-normally distributed, as assumed in this paper.

The Lagrangian associated with the central bank's problem is 17

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ L_t + 2\phi_{1,t} \left[\pi_t^W - \kappa_C^W \hat{x}_t - n (1 - n) \kappa_T^R \hat{\Upsilon}_t \right] - \beta \pi_{t+1}^W \right\} + 2\phi_{2,t} \left[\left(1 + \beta + \kappa_T^W \right) \hat{\Upsilon}_t - \beta \hat{\Upsilon}_{t+1} - \left(\hat{\Upsilon}_{t-1} \right) + \kappa_C^R \hat{x}_t \right] + t.i.p. \right\}$$

where
$$\kappa_C^W = \left(n\kappa_C^H + (1-n)\kappa_C^F\right)$$
, $\kappa_C^R = \left(\kappa_C^F - \kappa_C^H\right)$, $\kappa_T^W = \left(n\kappa_T^F + (1-n)\kappa_T^H\right)$, $\kappa_T^R = \left(\kappa_T^H - \kappa_T^F\right)$.

The policy maker minimises the Lagrangian with respect to \hat{C} , π^W and \hat{T}_t .

Noting that $\pi^H = \pi^W - (1-n)\pi^R$ and that $\pi^F = n\pi^R + \pi^W$, the solution to this problem yields

$$\hat{x}_{t} = \hat{x}_{t-1} + \frac{k_{c}^{R}}{\left[\Phi\left(\sigma + \eta\right) - (1 - \sigma)\left(1 - \Phi\right)\right]} \left(\phi_{2,t} - \phi_{2,t-1}\right) + \frac{(1 - \sigma)\left(1 - \Phi\right)}{\Phi\left(\sigma + \eta\right) - (1 - \sigma)\left(1 - \Phi\right)} \left(C_{t}^{N} - C_{t-1}^{N}\right) - \frac{\Phi\left(\psi^{H} + \psi^{F}\right)}{\Phi\left(\sigma + \eta\right) - (1 - \sigma)\left(1 - \Phi\right)} k_{c}^{W} \left(\pi_{t}^{W} + (n - \gamma)\pi_{t}^{R}\right)$$
(23)

The constant Ω is neglected since it would not change the results. The Lagrangian multipliers are also re-scaled for the sake of convenience.

$$\beta \phi_{2,t+1} = \frac{n(1-n)k_T^R(1-\Phi)}{k_c^W} + \left(1+\beta + k_T^W + \frac{n(1-n)k_T^Rk_c^R}{k_c^W}\right)\phi_{2,t} - \phi_{2,t-1} + \frac{(1-\sigma)(1-\Phi)}{k_c^W}C_t^N + \Phi\left(\psi^H + \psi^F\right)(n-\gamma)\left[\pi_t^W - \beta\pi_{t+1}^W\right] + \Phi\left(\psi^H + \psi^F\right)\left[\gamma(1-n)^2 + (1-\gamma)n^2\right]\left[\pi_t^R - \beta\pi_{t+1}^R\right] + \Phi n(1-n)(1+\eta)\hat{\Upsilon}_t - \frac{\Phi\left(\sigma + \eta\right) - (1-\sigma)(1-\Phi\right)}{k_c^W}\hat{x}_t$$
(24)

Equations (23) and (24) together with equation (19), (21) and (20) fully describe the dynamics of our economy.

Assuming, for the sake of simplicity, that $\Phi = 1$, if the two countries are symmetric (not necessarily in size), the first-order conditions imply the following system of equations

$$x_t = \frac{1}{\beta} x_{t-1} - \frac{\psi^H k_c^H}{n (\sigma + \eta)} \pi_t^W$$
$$E_t \pi_{t+1}^W = \frac{1}{\beta} \pi_t - \frac{k_c^H}{\beta} x_t$$

together with equation (21), purged of the output gap, i.e.

$$\beta E_t \hat{\Upsilon}_{t+1} = (1 + \beta + \kappa_T^H) \hat{\Upsilon}_t - \hat{\Upsilon}_{t-1} + (1 + \beta) \hat{T}_t^N - \hat{T}_{t-1}^N - \beta E_t \hat{T}_{t+1}^N$$
 (25)

This result confirms that the zero inflation and zero output gap are optimal and feasible and that the terms of trade gap is out of control of the monetary authority. That is, the first-best solution cannot be achieved.

The model is solved using a technique similar to Sims (2000) and Klein (2000). As suggested by Woodford (1999a), the appropriate measure of welfare loss is

$$E\left\{E_0 \sum_{t=0}^{\infty} \beta^t L_t\right\} \tag{26}$$

where the unconditional expectation operator takes account of all possible initial states at the time of the policy decision. Therefore, we use unconditional (asymptotic) variances of output, inflation and the terms of trade to measure the welfare loss under the assumption that the innovations to the technology process are normally distributed with unit variance and zero mean. We considered two alternatives for the mean of the "technology" process: mean zero and mean equal to minus half its variance. The latter ensures that the technology shock in levels has mean one. Our results concerning monetary policy are invariant to these alternatives.

Most of the parameters that we use in our sensitivity analysis affect the steady-state of the economy. Therefore, we measure total welfare as the non-monetary part of the utility function (equation (7a)) evaluated in the steady-state minus half the welfare loss measure of equation (22).

6 Numerical results

The numerical exercises proposed below compare the performance – in welfare terms – of the Taylor rule and of the optimal rule, under asymmetric productivity shocks and under different assumptions about the degree of competition.

The Taylor rule takes the form $i_t = \lambda_\pi \left(\lambda^H \pi_t^H + \left(1 - \lambda^H\right)\right) \pi_t^W + \lambda_x \hat{x}_t$, where, unless differently specified, $\lambda_\pi = 1.5$ and $\lambda_x = 0.5 = \lambda^H$. We take the Taylor rule as a simple feedback rule where the output gap and inflation rate of the members of the union are weighted by the size of the country. While this type of rule has often been associated with observed monetary policies in various countries (see, for example, Clarida *et al.*, 1997), it serves here only as a "reasonable" example of a policy that neglects the structural differences of the various regions of the monetary union. This rule might not be "operational" (in the sense of McCallum and Nelson (1999)). It requires the central bank to have knowledge of the current inflation rate, the current level of output and the current size of the natural level of output. The study of more operational rules is beyond the scope of this paper.¹⁸ Nevertheless, it is worth noting that replacing expected values on the right-hand side of the rule for actual values does not produce any qualitative change in the relative performance of the rule.

We start our numerical analysis by looking at the effect of different degrees of competition in a symmetric union. There are three important ways in which the degree of competition can affect welfare in our model. The first is the traditional deadweight loss produced by the monopolistic distortion: higher prices and lower output.

The second channel through which the degree of competition affects welfare is through the inefficient dispersion of output across firms.¹⁹ Through equations (43) and (51) the elasticity of the demand for goods enters the welfare function as a weight on the inflation

¹⁸See Rotemberg and Woodford (1998) and McCallum and Nelson (1999) for a detailed discussion of operational rules.

¹⁹The relationship between competition and the output-dispersion distortion is discussed also in Khan et al. (2000 p 31).

rate: the higher the elasticity, the larger the weight. We will call this channel the "weights effect" of competition. This channel implies then that, abstracting from the first channel, welfare decreases in the degree of competition.

The latter channel is only partially affected by the assumption of specific factors of production. The third channel that we highlight here is instead very sensitive to the slope of the marginal cost curve. Kimball (1995) shows that, theoretically, a higher elasticity of demand increases the degree of real rigidity of prices, if the marginal cost of the firm is increasing in its own output.²⁰ This third channel affects welfare indirectly through the volatility of the variables rather than through the weights attached to those volatilities in the welfare function. We will call this channel the "volatility effect" of competition.

No further information is necessary to understand that the net effect of competition on welfare depends on the variance of the underlying shocks. With zero variance the model is de facto static and non-stochastic. Therefore, the effect of competition on welfare is fully described by the steady-state welfare function: it is the traditional "inefficiency triangle". The model is solved in a linear (or linear-quadratic) form. Hence the moments of the variables are pure linear (continuous) transformations of the moments of the underlying shock. Hence, for sufficiently small values of the shock variance, the steady-state component of welfare must dominate the "dynamic" component. It is evident that there must be a critical value of the shock variance beyond which the negative contribution of competition stemming from the dynamics of the economy prevails over the static effect.

²⁰On the real rigidity see Ball and Romer (1990). Woodford (2001) re-names this effect as the "strategic complementarity in price-setting". Lombardo (2001) provides some macroeconometric evidence of this mechanism

As for the interest rule, what really matters is the effect of competition on the dynamic part of welfare: in this model the central bank can do nothing to affect the static part of welfare. Indeed, our results are *qualitatively* invariant to the variance of the shocks.²¹ Nevertheless, the first graphs show the total effect of competition on welfare as a heuristic way of verifying that, for a given shock variance, total welfare might deteriorate as competition increases.

The values imposed on the parameters in our numerical exercises are $\beta = 0.99$, n = 0.5, $\zeta = 0.5$ and $\omega = 0.5$ (unless stated otherwise). These values are not calibrated on any existing economy. Instead they are within ranges that can be found in the existing related literature. Given the purpose of our exercise this is all we need. These values imply that we consider a union made up of two equally sized countries. We also impose that the underlying productivity shocks take the form $\hat{A}_t = 0.9\hat{A}_{t-1} + \nu_t$ with $\nu \sim N(0,1)^{.22}$

6.1 Degree of competition and welfare under the optimal rule

The first set of diagrams (figure (2) to figure (4)) describes the sensitivity of our measure of welfare to the degree of competition and to the elasticity of the marginal cost function in a symmetric monetary union.

An interesting result of our study is that competition can be *bad* for welfare if prices are not flexible enough. One can easily see why this is the case by assuming that the regional governments subsidise firms to offset the monopolistic distortion. In this case,

²¹Obviously they vary *quantitatively*: with zero variance there is nothing the central bank needs to counter. The gap between total welfare under the optimal rule and total welfare under a suboptimal rule tends to zero as the variance shrinks.

 $^{^{22}\}text{Correcting}$ for the mean, i.e. $E\left(\hat{A}\right)=-0.5var\left(\hat{A}\right)$ does not alter our results.

more competition induces more output dispersion (higher weights in the loss function) as well as more volatility in the variables that matter for welfare. That is, competition operates only through the "weights effect" and the "volatility effect". Nevertheless, our numerical exercise shows that competition is bad for welfare also when we take account of the deadweight loss produced by imperfect competition.

Allowing for an inefficient steady-state level of output does not change the long-run dynamics of the log-deviation of output, inflation or the terms of trade in the first-order approximation of the model. Nevertheless, when the steady-state is inefficient, the first orderapproximation to the structural equations is inappropriate to evaluate a second-order approximation to the welfare function. In principle and under these circumstances, it would be wrong to compare outcomes based on different assumptions regarding the steady-state degree of competition. A second-order approximation to the structural equations would eliminate this inconvenience. However, deriving the optimal policy from the second-order approximation to the structural equation would be very involved algebraically. Conversely, evaluating welfare under the Taylor rule and a second-order approximation of the model is extremely simple. Hence we look only at the quadratic solution for this case. Rather than going through the whole algebraic solution, it should suffice to note that the only term, in the welfare function, affected by the second order approximation would be the mean of the output gap. Figure (1) shows how the mean of the output gap changes with the degree of competition and with the competition gap between countries. The figure is drawn under the assumption that $\omega = 0.5$ and $\sigma = 1.5$. A similar pattern is obtained with $\omega = 0.001$ and with $\sigma < 1$. A higher degree of competition reduces the mean output

gap. The underlying model is non-linear, hence the mean of the variables is, in principle, affected by the variance, which is in turn affected by the degree of competition. The same can be said of the competition gap. While this is true under the Taylor rule, we can only conjecture that a similar result would be borne out under the optimal rule. All the results presented here are robust to the second-order approximation.²³

[Figure (1) here]

Inspection of equation (22) also shows that, under an inefficient steady-state, the welfare function is augmented by a term that measures the marginal disutility of consumption volatility. This term contributes negatively to welfare only if consumers are very risk-averse ($\sigma > 1$). The reason for $\sigma = 1$ to be a threshold value is that volatility of the log deviation of consumption contributes one-to-one to the mean of consumption, on the one hand and it contributes negatively, in proportion to σ , to the volatility of consumption on the other. Therefore, under an inefficient steady-state, a higher mean of consumption improves welfare. Only when $\sigma > 1$ does the volatility effect outweigh the mean effect.²⁴

Figure (2) shows the welfare function plotted against the degree of competition and the returns to labour in a symmetric economy under the optimal rule with almost price flexibility ($\omega = 0.001$) and $\Phi = \frac{\varepsilon^W - 1}{\varepsilon^W}$. Welfare increases in competition for any value of α when $\sigma > 1$. The more distortion there is in the market, the more weight is attached

 $^{^{23}}$ To solve the model in its second-order approximation we had to use price equations rather than the Phillips curve. Under the Taylor rule, prices have a unit root so that the unconditional moments can not be computed. We therefore imposed an infinitesimal degree of price level targeting in the interest rate rule (coefficient equal to 10^{-7}).

 $^{^{24}}$ It should be noted that the higher is σ (the more concave the utility function), the less accurate the second-order approximation to the welfare function is. See Woodford (1999a, p 8) for a discussion of this problem.

 $^{^{25}\}mathrm{The}$ marked lines in the graphs are the contours of the plots.

to the mean and variance of consumption. High risk aversion means that the variance effect more than offsets the mean effect. This effect is magnified at low returns to labour because at low returns the degree of competition produces a stronger volatility effect.

[Figure (2) here]

A similar picture is produced when we allow for a competition gap between the two countries. The Taylor rule also produces this pattern, although at a lower welfare level.

[Figure (3) here]

A degree of risk aversion smaller than one yields figure (3). In this case the bigger the monopolistic distortion, the better output volatility is for welfare. The picture also shows that the returns to labour have here a partially off-setting effect.

If we increase the degree of nominal rigidity to $\omega = .1$ (i.e. 90% of the firms adjust their price in each period), we obtain figure (4). This clearly shows that a small degree of nominal rigidity is sufficient to induce a *negative* relationship between welfare and competition. The dynamic effects of competition on welfare ("volatility effect" and "weights effect") more than offset the deadweight loss of the monopolistic distortion.²⁶

[Figure (4) here]

We have claimed above that, with an upward sloping marginal cost curve, there is a stronger volatility effect of competition. The source of this effect can be seen by inspecting the Phillips curve (equation (19)). The elasticity of the marginal cost and the elasticity of demand enter symmetrically in the Phillips curve. Increasing either of these elasticities increases the real rigidity of prices (Kimball (1995)) and reduces the response of inflation

 $^{^{-26}}$ A similar pattern is produced by the Taylor rule and also in an asymmetric union where the elasticity gap $(\varepsilon^F - \varepsilon^H)$ is equal to half the average elasticity.

to the output gap. At the same time, more real rigidity means that prices respond little to monetary policy while output, consequently, responds more. Figure (5) shows that the variance of inflation *decreases* in the degree of competition while the variance of consumption *increases*. This result is consistent with the findings of Schmitt-Grohé and Uribe (2001).²⁷ The variance of the terms of trade *increases* in competition.

Increasing the degree of nominal rigidity reinforces the effect of the real rigidity of prices. Figure (6) shows the net effect of competition on the sum of the volatility of inflation, of the output gap and of the terms of trade under the Taylor rule when the average length of price contracts is two periods (a similar picture obtains under the optimal rule). One can see that the "average" volatility effect is increasing in the degree of competition.

[Figure (5) here]

[Figure (6) here]

6.2 Competition, nominal rigidity and welfare

The welfare cost of price dispersion is due, in this model, to the presence of price staggering. A higher degree of competition increases this cost. Yet many would argue that the frequency of price adjustments and the degree of competition should be linked. This is not necessarily the case. As we show in another current paper, wage stickiness can bring about price staggering even when firms update their prices every period. Nevertheless we show in this section the possible welfare effect of a link between the degree of competition

²⁷ Figure (5) shows the response under the Taylor rule. The optimal rule is extremely effective in a symmetric union in controlling inflation and output gap volatility so that the effect of competition is hardly detectable. The terms of trade are instead insulated from the monetary policy.

and the frequency of price adjustments.

In figure (7) we show the relationship between competition and welfare under various degrees of nominal rigidity in a symmetric union for $\alpha=0.2$ and $\sigma=1.5$. The figure shows the total welfare and the change in welfare. The two diagrams are almost identical, suggesting that the "static" effect of competition plays a minor role. One can see that more competition is bad under any degree of nominal rigidity. There is instead a non-monotonic relationship between welfare and nominal rigidity. The latter is bad for welfare up to a point where more rigidity is indeed good. A plausible rationale for this result is that the ability of the central bank to control the economy under almost fixed prices allows the monetary authority to reach higher levels of stability despite the output dispersion distortion that the rigidity brings about.

An interesting feature of figure (7) is that it indicates the implication of assuming that the degree of nominal rigidity is a (e.g. decreasing) function of the degree of competition. If we read the figure along a diagonal segment from left to right, we see that welfare would increase in competition only under a very "steep" relationship between nominal rigidity and competition. While we believe that nominal rigidity and degree of competition should be linked, without a theory explaining this relationship we prefer not to pursue this issue further here.²⁸

[Figure (7) here]

While the Taylor rule produces a similar pattern (not shown), the performance of the

²⁸Carlton (1986) shows some empirical evidence of a negative relationship between the degree of competition and price rigidity. He also shows that competition and "real rigidity" seem to be positively linked. In a study currently in progress we derive price staggering purely from wage rigidity so that imperfect competition and price rigidity are totally independent.

latter relative to the optimal rule is non-monotonic in the nominal rigidity. The Taylor rule performs worst under intermediate degrees of nominal rigidity. The two rules converge at the corners since they are both either quite ineffective (price flexibility) or extremely effective (price rigidity).²⁹

6.3 The asymmetric currency area

We turn now to the central issue in this paper, i.e. the effects of competition and competition differences across countries on the "optimal" interest rule. In the following we will assume that $\alpha = 0.2$, $\sigma = 1.5$ and $\omega = 0.5$. The study of the best interest rule can abstract from the steady-state level of welfare, since the latter is independent of the former.

Figure (8) shows that welfare increases in the competition gap when the monetary authority implements the optimal policy.³⁰ To understand this result we have to refer to equation (21). The terms of trade cannot be controlled by the central bank when the union is symmetric. Indeed, plotting the variance of the terms of trade against the competition gap shows that the former decreases in the latter.

[Figure (8) here]

A central bank that targets the average inflation rate across the union, e.g. following a Taylor rule, cannot do as well as the optimising central bank. The (absolute) performance of the Taylor rule is indeed rather insensitive to the degree of asymmetry.³¹ In particular the degree of sub-optimality of the Taylor rule increases in the competition gap, as figure

²⁹Here the monetary union is symmetric so that the terms of trade are insulated from the monetary policy.

³⁰ In the graph the gap goes up to the size of the average elasticity.

 $^{^{31}}$ The welfare loss under the Taylor rule falls slightly in the competition gap.

(9) shows.³² The link between the terms of trade and output still exists under the Taylor rule, so that controlling output has some feedback on the terms of trade. Nevertheless, the average inflation rate is clearly the wrong target, and the more so the greater the asymmetry of the union: hence the result in figure (9).

[Figure (9) here]

Our results so far show that competition could be detrimental for welfare, and it is certainly so if one abstracts from the steady-state inefficiency. In this respect, we ought to expect the central bank to respond more aggressively to the inflation contribution of the more competitive economy. In figure (10) the relative performance of the Taylor rule is plotted against the competition gap and the weight attached to country H (λ^H) under the assumption that $\varepsilon^W = 10$. The figure shows that the "optimal" λ^H decreases in the gap between the degree of competition in country F and country H.

In our model the degree of real rigidity increases with the degree of competition.³³ The result of figure (10) is then tantamount to proposition 9 in Benigno (2000): the country with more persistence (nominal rigidity in Benigno (2000)) should be given a higher weight. However, the source of persistence in Benigno's paper differ from that shown in ours. Moreover, if one were to assume that higher competition implies lower nominal rigidity, the two results would point at potentially opposed pressures on the monetary authority. In this case, as in general, the correct "mix" would be an empirical matter.

[Figure (10) here]

 $^{^{32}}$ In reading the figures reporting ratios of welfare changes, it should be borne in mind that changes are negative: a ratio bigger than one indicates a larger reduction in welfare under the Taylor rule.

³³Given the large number of graphs already shown, we refrain from displaying this result. This result is discussed in Kimball (1995) as well as in Lombardo (2001) in a similar dynamic model.

Our calculations also show that the optimal weight attached to the overall inflation rate in the Taylor rule (λ_{π}) is not 1.5 but a much higher value. This value depends on the degree of symmetry in the union. When the union is not very symmetric (and equal weights are attached to each country), it is better not to react too strongly to inflation, as opposed the situation in a more symmetric union. In an asymmetric union the population-weighted average of the inflation rate is a poor indicator of the relevant inflation.

Finally, we find that the optimal inflation-weight for the less competitive country decreases in the weight attached to output. Consequently, in the limit, a zero weight should be attached to the less competitive country's inflation and an infinite weight should be attached to the output gap. Such an extreme feedback rule would not affect the feasibility of the rule in terms of the "zero bound" problem, since the variance of the nominal interest rate would also be smaller.³⁴ As in Rotemberg and Woodford (1998) negative values for the weights of the Taylor rule on the output gap and inflation can increase welfare. Contrary to the result of these authors, under our assumption infinite weights on inflation and the output gap bring the Taylor performance close to that of the optimal rule.³⁵

6.4 Shocks to the rule

Here we briefly discuss the optimal weights for the Taylor rule, when the rule is augmented by a random shock. We assume that the shock follows an AR(1) process with

³⁴Rotemberg and Woodford (1998) and Woodford (1999b) discuss the lower-bound problem in detail. Woodford (1999b) also shows that extending the objective of the central bank with an interest smoothing goal can help to avoid the lower bound. We have avoided this issue altogether. Consequently, we have also neglected the possibility of an interest rate feedback in the Taylor rule.

³⁵As in Rotemberg and Woodford, there is an area of indeterminacy between the two extremes. Values in that area can bring about instability through "sunspots".

autocorrelation coefficient equal 0.5.

A pure interest rate shock is a symmetric shock to the two economies. Nevertheless, each economy responds in a different way, depending on its structure. If the union is homogeneous, the response of each country's inflation contribution will be identical, hence any weight on these contributions will suffice. If, instead, the two economies have different degrees of competition, the nominal shock will induce different responses of prices and output. In this case the best response of the central bank would be to react more strongly to the inflation originating in the more competitive economy, as shown figure (11).³⁶ Contrary to the case involving productivity shocks, under interest rate shocks the optimal (Taylor-rule) response does not involve a convex-combination of the inflation contributions. Instead, the weight on the more competitive country should exceed unity, while the less competitive country should receive a negative weight.³⁷

Under idiosyncratic shocks, the asymmetry in the optimal weights is justified by the asymmetry in competition. Nevertheless, the central bank still needs to counter the country-specific shocks. This "constraint" is absent under the common interest rate shock.

[Figure (11) here]

If this result is taken literally, it would pose a problem for the formulation of an "optimal" interest rule: what is optimal under interest rate shocks is not necessarily optimal under technology shocks. Nevertheless, if the result is taken as an indication of the direction which the central bank should take, it suggests that the central bank should adopt a more active response to the inflation originating in the more competitive economy.

³⁶The figure is obtained under $\sigma = 1.5$, $\zeta = 0.5$ and $\omega = 0.5$.

 $^{^{37}}$ Weights still add up to one. The weight on total inflation is kept constant at 1.5.

7 Conclusions

In this paper we have studied the relationship between welfare, monetary policy and competition in a currency union where there might be different degrees of competition among the domestic firms of the individual members.

We have highlighted three channels through which the degree of competition can affect welfare in this economy. The first is the traditional deadweight loss of imperfect competition. The second is due to the inefficient dispersion of output across firms, while the third is due to the "real rigidity" effect of competition.

A key result of this paper is that, for a given steady-state level of welfare, competition (cum sticky prices) is detrimental for welfare. Under a sufficiently volatile economy and sufficiently sticky prices, total welfare might be higher under lower competition. As for the monetary authority, the steady-state consequences of the monopolistic distortion cannot be affected by its instrument. As a consequence, the central bank finds it optimal to counter more aggressively the inflationary pressure stemming from the more competitive country.

Our results can be compared with those obtained by Benigno (2000). Benigno's prescription for the monetary authority is that it should react more aggressively to the country with more nominal rigidity, i.e. the more sluggish economy. In our model the more competitive economy turns out to be also the more sluggish one. Hence, Benigno's prescription is similar to ours, although for different reasons. Furthermore, if the nominal rigidity of prices is thought of as negatively linked to the degree of competition, our paper shows that the central bank would be faced with two opposing factors. There is nothing to prevent that the central bank, on balance, will still find it optimal to react symmetrically to the different members of the union.

Quantifying the structural differences among countries (or regions) in a monetary union and their welfare implications therefore remains an empirical issue. If it turns out that the members of the union have very different economic structures, our analysis suggests that a simple rule that neglects this fact would perform poorly compared to a rule that takes account of these differences. Nevertheless, welfare could indeed be larger, the bigger these differences are, even under a suboptimal rule.

A Appendix

A.1 Structural equations under sticky prices

Equation (6) clearly implies that the union-wide inflation rate is a simple weighted average of the inflation rates of the domestic price indices in the two countries.

That is
$$\pi^W = n \pi^H + (1 - n) \pi^F$$

Under Calvo contracts the domestic price index at time t is a weighted sum of the prices prevailing at that time, i.e.³⁸ $P_{j,t} = (1 - \omega) \sum_{i=0}^{\infty} \omega^i p_{t-i}$, for j = H, F and where p_t is defined in equation (14), where we have omitted the index z, since all domestic firms resetting the price will choose the same price. Expressing this relationship in differences

³⁸The exact price index would weigh the various prices by a factor depending on the degree of substitutability, as equation (4) shows. Nevertheless, the first order log linearization of this price would be independent of this factor, which we therefore omit for the sake of convenience.

yields

$$P_{j,t} = \omega P_{j,t-1} + (1 - \omega) p_t \tag{27}$$

If we divide equation (27) by $P_{j,t}$ and rearrange it we obtain

$$\frac{\omega}{1-\omega}\pi_j = \hat{p}_t - \hat{P}_{j,t} \tag{28}$$

where $\hat{x} = \log(x/x_{ss})$

We can now derive a New-Keynesian Phillips curve from the firm's optimal price condition (14). We first linearise this condition around the symmetric steady-state, i.e.

$$\hat{p}_t = (1 - \omega \beta) E_t \sum_{s=t}^{\infty} (\omega \beta)^s \left(\widehat{mc}_s |_{p_t} + \hat{P}_s \right)$$
(29)

Substituting equation (28) in (29) yields

$$\omega \, \pi_{H,t} = (1 - \omega \beta) \, (1 - \omega) \, E_t \sum_{s=t}^{\infty} \left(\omega \beta \right)^s \left(\widehat{mc}_s \big|_{p_t} + \hat{P}_s - \hat{P}_t + (1 - n) \, \hat{T}_t \right) \tag{30}$$

Note that the marginal cost, conditional on price p_t , can be obtained from equation (15) after having substituted the equilibrium real wage with the labour market clearing

condition, i.e.

$$\widehat{mc}_{t}|_{p_{t}} = \left(\sigma + \frac{\zeta + \alpha}{1 - \alpha}\right) \hat{C}_{t} - \frac{\zeta + 1}{1 - \alpha} \hat{A}_{H,t} +$$

$$+ (1 - n) \frac{\zeta + \alpha}{1 - \alpha} \hat{T}_{t} - \frac{(\zeta + \alpha) \varepsilon^{H}}{1 - \alpha} \frac{\omega}{1 - \omega} \pi_{t}^{H}$$
(31)

This yields

$$\widehat{mc}_{t+u}|_{p_t} = \left(\sigma + \frac{\zeta + \alpha}{1 - \alpha}\right) \hat{C}_{t+u} - \frac{\zeta + 1}{1 - \alpha} \hat{A}_{H,t+u} +$$

$$+ (1 - n) \frac{\zeta + \alpha}{1 - \alpha} \hat{T}_{t+u} - \frac{(\zeta + \alpha) \varepsilon^H}{1 - \alpha} \frac{\omega}{1 - \omega} \pi_t^H +$$

$$+ \frac{(\zeta + \alpha) \varepsilon^H}{1 - \alpha} \sum_{i=1}^u \pi_{t+i}^H$$
(32)

where $\pi_{t+i}^H \equiv 0$ if i > u.

Analogously, we can define the real marginal cost conditional on price p_{t+1}

$$\widehat{mc}_{t+u}|_{p_{t+1}} = \left(\sigma + \frac{\zeta + \alpha}{1 - \alpha}\right) \hat{C}_{t+u} - \frac{\zeta + 1}{1 - \alpha} \hat{A}_{H,t+u} + \left(1 - n\right) \frac{\zeta + \alpha}{1 - \alpha} \hat{T}_{t+u} - \frac{(\zeta + \alpha)\varepsilon^H}{1 - \alpha} \frac{\omega}{1 - \omega} \pi_{t+1}^H + \left(\frac{(\zeta + \alpha)\varepsilon^H}{1 - \alpha} \sum_{i=2}^u \pi_{t+i}^H \right)$$

$$(33)$$

We therefore obtain

$$\widehat{mc}_{t+u}|_{p_t} = \widehat{mc}_{t+u}|_{p_{t+1}} - \frac{(\zeta + \alpha)\varepsilon^H}{1 - \alpha} \frac{\omega}{1 - \omega} \pi_t^H + \frac{(\zeta + \alpha)\varepsilon^H}{1 - \alpha} \frac{1}{1 - \omega} \pi_{t+1}^H$$
(34)

Substituting equation (34) into (30) and rearranging yields

$$\omega \left(1 + \frac{\omega \beta \alpha \varepsilon^{H}}{1 - \alpha} \right) \pi_{t}^{H} = (1 - \omega) \left((1 - \omega \beta) \widehat{mc}_{t}|_{p_{t}} + (1 - n) \left(\hat{T}_{t} - \omega \beta \hat{T}_{t+1} \right) \right) + \omega \beta \left(\omega + \frac{(\zeta + \alpha) \varepsilon^{H}}{1 - \alpha} \right) E_{t} \pi_{t+1}^{H}$$

Then noting that $\hat{T}_{t+1} = \pi_{t+1}^F - \pi_{t+1}^H + \hat{T}_t$ and using (31), we obtain

$$\pi_t^H = \kappa_C^H \left(\hat{Y}_{H,t} - \hat{Y}_{H,t}^N \right) + (1 - n) \kappa_T^H \left(\hat{T}_t - \hat{T}_t^N \right) + E_t \beta \pi_{t+1}^H$$
 (35)

for country H, and

$$\pi_t^F = \kappa_C^F \left(\hat{Y}_{F,t} - \hat{Y}_{F,t}^N \right) - n\kappa_T^F \left(\hat{T}_t - \hat{T}_t^N \right) + E_t \beta \pi_{t+1}^F \tag{36}$$

for country F, where

$$\begin{split} \kappa_C^j &= \frac{\left(1-\omega\right)\left(1-\omega\beta\right)}{\omega\left(1+\frac{\left(\zeta+\alpha\right)\varepsilon^j}{1-\alpha}\right)} \left(\sigma + \frac{\zeta+\alpha}{1-\alpha}\right) \\ \kappa_T^j &= \frac{\left(1-\omega\right)\left(1-\omega\beta\right)}{\omega\left(1+\frac{\left(\zeta+\alpha\right)\varepsilon^j}{1-\alpha}\right)} \frac{\zeta+1}{1-\alpha} \end{split}$$

for j=H,F, and where we have used the fact that the total demand for goods from country H is

$$\hat{Y}_{H} = (1 - n)\,\hat{T} + \hat{C}^{W} \tag{37}$$

while the total demand for goods produced in country F is

$$\hat{Y}_F = -n\hat{T} + \hat{C}^W \tag{38}$$

We can then easily derive equations (19) and (20).

By using the fact that $\hat{T}_t - \hat{T}_{t-1} = \pi_t^R$, we can substitute the terms of trade growth for the inflation differential in (20), obtaining equation (21).

A.2 Second-order approximation to the welfare function

Following Woodford (1999a) we take a second-order Taylor approximation of the house-holds' utility function around the symmetric equilibrium. For this purpose we do not take account of the welfare deriving from holding real money balances.

A Taylor approximation will give a second order polynomial in deviations of the variables from the symmetric equilibrium. We then take second-order power expansions of these deviations of the type³⁹

$$\frac{x}{x_{ss}} - 1 = \log\left(\frac{x}{x_{ss}}\right) + \frac{1}{2}\left(\log\left(\frac{x}{x_{ss}}\right)\right)^2 = \hat{x} + \frac{1}{2}\hat{x}^2$$

Therefore a second order approximation of U(C) yields

$$\widetilde{U}\left(C\right) = U_c C_{ss} \left(\hat{C}_t + \frac{1 - \sigma}{2} \hat{C}^2\right) + \mathcal{O}\left(\|A\|^3\right)$$
(39)

³⁹We assume that variables are log-normally distributed. Under this assumption, the mean of a variable, say consumption, also depends on its variance. This fact is important in interpreting the log linearised expressions.

where $\mathcal{O}\left(\|A\|^3\right)$ is the approximation error of order bigger than 2. This equation is identical for both countries, since the consumption-utility function is independent of the degree of competition and since we have assumed full consumption risk-sharing.

As for the work effort function, we solve the case for country H, since the approximation for country F is identical. We first define a reduced form function that maps output into labour effort, i.e. $v(y) \equiv V(f^{-1}(y))$, then the second-order approximation yields

$$\widetilde{v}(y) = v_y y_{ss} \hat{y}_t + \frac{1}{2} \left(v_y y_{ss} + v_{yy} y_{ss}^2 \right) \hat{y}_t^2 + v_{Ay} y_{ss} \hat{A}_t \hat{y}_t + t.i.p. + \mathcal{O}\left(\|A\|^3 \right)$$
(40)

where, following Woodford (1999a), we gather all the terms independent of policy under the term t.i.p.

Defining $\eta \equiv \frac{v_{yy}}{v_y} y = \frac{\zeta + \alpha}{1 - \alpha}$ and $q_t y_{ss} \equiv \frac{v_{yA}}{v_{yy}} \hat{A}_t = -\frac{\zeta + 1}{\zeta + \alpha} y_{ss} \hat{A}_t$, and $\Phi_H = s \cdot mc_{ss} = \frac{v_y}{U_c}$, we can rewrite (40) as⁴⁰

$$\widetilde{v}(y) = U_c y_{ss} \Phi_H \left[\hat{y}_t + \frac{1}{2} (1 + \eta) \, \hat{y}_t^2 + \eta q_t \hat{y}_t \right] + t.i.p. + \mathcal{O}\left(\|A\|^3 \right)$$
(41)

Let us now define the expectation operator and the variance operator under the Calvo distribution of firms, namely $E_i(x_t(z)) = \frac{1}{n} \int_0^n x_t(z) dz$ and $var_i(x(z)) = E_i\left[x(z)^2\right] - \left[E_i(x(z))\right]^2$. Then, integrating (41) over all the variety of goods produced by the domestic

 $^{^{40}}$ Note that Φ measures the degree of efficiency of our economy. When $\Phi < 1$, the marginal cost of producing one unit of output for the household is smaller than the marginal benefit of consuming that unit. The household would be better off with more output. The presence of imperfect competition prevents the household from achieving the first best. Subsidising production can re-establish efficiency.

economy we obtain

$$\frac{1}{n} \int_0^n \widetilde{v}(y) = U_c y_{ss} \Phi \left[E_i \hat{y}_t + \frac{1}{2} (1+\eta) \left[(E_i \hat{y}_t)^2 + var_i \hat{y}_t \right] + \eta q_t E_i \hat{y}_t \right]$$
(42)

Perfect risk-sharing implies that each country's total demand for each country's output must be equal. Taking a second-order expansion of the weighted sum of the aggregator (1) and the (identical) foreign counterpart yields

$$\hat{Y}_H = E_i \hat{y}_H + \frac{1}{2} \theta var_i \hat{y}_H \tag{43}$$

The last expression can be used in (42) to eliminate $E_i\hat{y}_H$.

Then the total welfare for the two countries will be 41

$$W^{H} = U_{c}C_{ss} \left[\hat{C}_{t} + \frac{1-\sigma}{2} \hat{C}_{t}^{2} - \Phi_{H} \frac{1+\eta}{2} \hat{Y}_{H}^{2} - \Phi_{H} \eta q_{t}^{H} \hat{Y}_{H} - \Phi_{H} \frac{\left(1-\theta^{H}+\eta\right)}{2} var_{i} \hat{y}_{H} - \Phi_{H} \hat{Y}_{H} \right]$$

$$(44)$$

and

$$W^{F} = U_{c}C_{ss} \left[\hat{C}_{t} + \frac{1 - \sigma}{2} \hat{C}_{t}^{2} - \Phi_{F} \frac{1 + \eta}{2} \hat{Y}_{F}^{2} - \Phi_{F} \eta q_{t}^{F} \hat{Y}_{F} - \Phi_{F} \frac{\left(1 - \theta^{F} + \eta\right)}{2} var_{i} \hat{y}_{F} - \Phi_{F} \hat{Y}_{F} \right]$$

$$(45)$$

Taking a population-weighted average of equation (44) and (45), using (37) and (38)

⁴¹Note that here $\mathcal{O}(\|A\|^3)$ will include terms such as $q_t var_i \hat{y}_H$, since these are of order larger than 2.

yields the welfare function for the union, i.e.

$$\mathcal{W}^{W} = U_{c}C_{ss} \left\{ (1 - \Phi) \, \hat{C}_{t} + \frac{(1 - \sigma) - \Phi(1 + \eta)}{2} \, \hat{C}^{2} - \Phi \frac{(1 - n)n}{2} \, (1 + \eta) \, \hat{T}^{2} + \right.$$

$$\left. - \Phi \eta \left(nq_{t}^{H} + (1 - n) \, q_{t}^{F} \right) \, \hat{C} - \Phi \, (1 - n) \, n\eta \, \left(q_{t}^{H} - q_{t}^{F} \right) \, \hat{T}_{t} + \right.$$

$$\left. - \Phi \left[\frac{n(1 - \theta^{H} + \eta)}{2} var_{i} \hat{y}_{H} + \frac{(1 - n)(1 - \theta^{F} + \eta)}{2} var_{i} \hat{y}_{F} \right] \right\}$$

$$(46)$$

It proves to be convenient to express welfare in terms of deviations of consumption and the terms of trade from the *natural level*. This can easily be done after noting that $\hat{C}^N = -\frac{\eta}{\eta + \sigma} \left(n q_t^H + (1-n) \, q^F \right) \text{ and } \hat{T}^N = -\frac{\eta}{1+\eta} \left(q_t^H - q_t^F \right) \text{ so that}$

$$\mathcal{W}^{W} = -\frac{U_{c}C_{ss}}{2} \left\{ -2\left(1 - \Phi\right)\hat{x}_{t} - \left(1 - \sigma\right)\left(1 - \Phi\right)\hat{C}_{t}^{2} + \Phi\left(\sigma + \eta\right)\hat{x}_{t}^{2} + \Phi\left(1 - n\right)\left(1 + \eta\right)\hat{\Upsilon}_{t}^{2} + \Phi\left(1 - \theta^{H} + \eta\right)var_{i}\hat{y}_{H} + \left(1 - n\right)\left(1 - \theta^{F} + \eta\right)var_{i}\hat{y}_{F} \right] + t.i.p. + \mathcal{O}\left(\|A\|^{3}\right)$$

$$(47)$$

Finally, note that

$$E_{i}\hat{y}_{H,t} = \sum_{i=0}^{\infty} (1 - \omega) \omega^{i} \varepsilon^{H} (P_{H,t} - p_{t-i}) + (1 - n) \hat{T}_{t} + \hat{C}_{t}$$

$$= (1 - n) \hat{T}_{t} + \hat{C}_{t}$$
(48)

so that

$$var_{i}\hat{y}_{H,t} = E_{i}\hat{y}_{H,t}^{2} - (E_{i}\hat{y}_{H,t})^{2}$$

$$= \frac{\left(\varepsilon^{H}\omega\right)^{2}}{1-\omega} \sum_{i=0}^{\infty} \omega^{i} \left(\pi_{t-i}^{H}\right)^{2}$$

$$= \frac{\left(\varepsilon^{H}\omega\right)^{2}}{1-\omega} \left(\pi_{t-i}^{H}\right)^{2} + \omega var_{i}\hat{y}_{H,t-1}$$

$$(49)$$

Solving (49) forward, we obtain

$$var_{i}\hat{y}_{H,t} = \omega^{t+1}var_{i}\hat{y}_{H,0-1} + \frac{\left(\varepsilon^{H}\omega\right)^{2}}{1-\omega}\sum_{s=0}^{t}\omega^{t-s}\left(\pi_{s}^{H}\right)^{2}$$

$$(50)$$

Then, by taking a discounted sum of (50) yields

$$\sum_{t=0}^{\infty} \beta^{t} var_{i} \hat{y}_{H,t} = \frac{\left(\varepsilon^{H}\omega\right)^{2}}{\left(1-\omega\right)\left(1-\omega\beta\right)} \sum_{t=0}^{\infty} \beta^{t} \left(\pi_{t}^{H}\right)^{2} + t.i.p.$$
 (51)

where t.i.p. stands for the variance of the firms' output at a date -1, i.e. before the policy decision is taken.

We can finally combine equation (51) and its counterpart for country F (not shown) with equation (47) to obtain equation (22).

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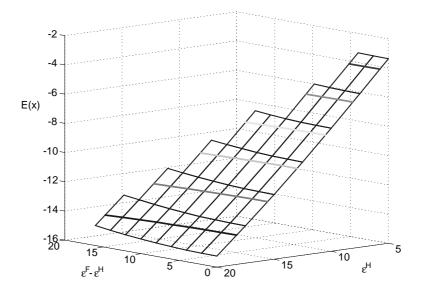


Figure 1: Unconditional mean of the output gap derived from the second order approximation, as a function of competition.

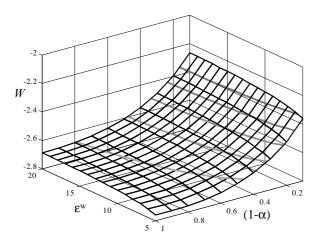


Figure 2: Welfare in a symmetric union under the optimal policy, $\omega=0.001$ and $\sigma=1.5$.

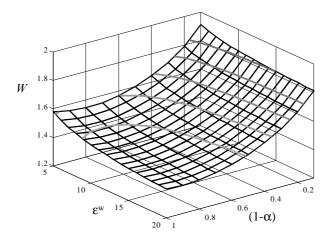


Figure 3: Welfare in a symmetric union under the optimal policy, $\omega=0.001$ and $\sigma=0.5$

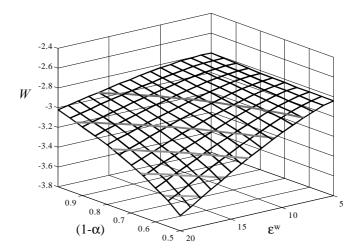


Figure 4: Welfare in a symmetric union under the optimal policy, $\sigma=1.5$ and $\omega=0.1$.

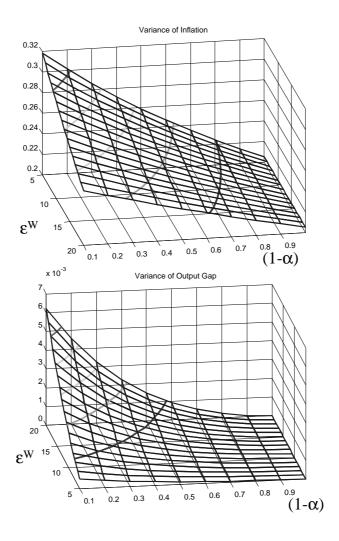


Figure 5: Variance of the inflation rate and the output gap in a symmetric union under the Taylor-rule: $\omega=0.1$ and $\sigma=1.5$.

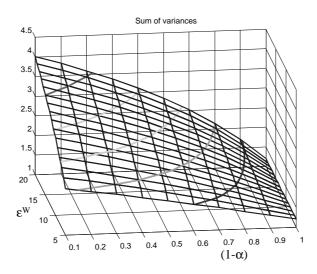


Figure 6: Sum of the variance of the inflation rate, of the output gap and of the terms of trade in a symmetric union under the Taylor rule: $\omega = .5$ and $\sigma = 1.5$.

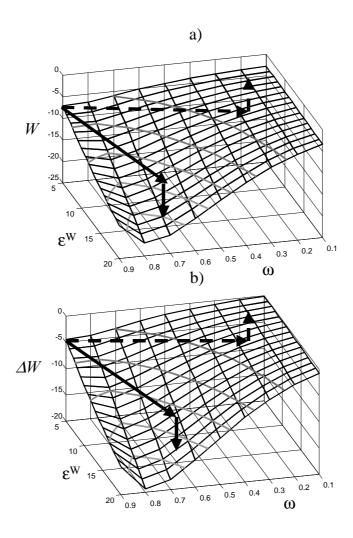


Figure 7: Welfare (a) and Welfare change (b) in a symmetric economy under the optimal rule, $\sigma=1.5$ and $(1-\alpha)=.8$.

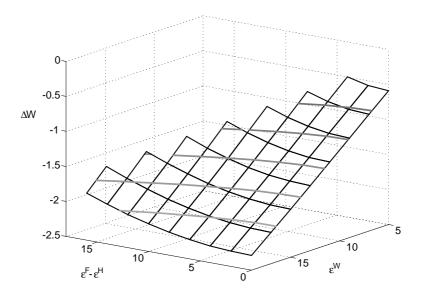


Figure 8: Welfare under the optimal rule at varying degrees of competition and competition gap $\frac{1}{2}$

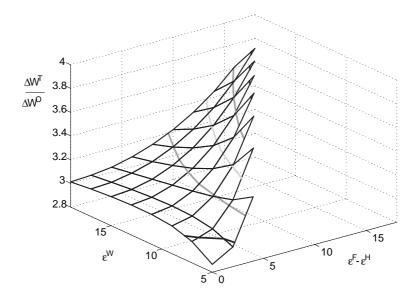


Figure 9: Ratio of Welfare change under the Taylor rule and the optimal rule, versus competition and competition gap

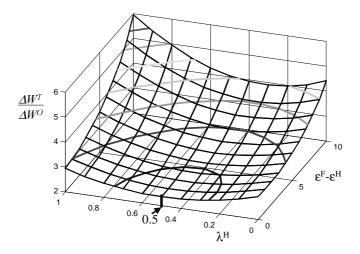


Figure 10: Relative performance of the Taylor rule and the optimal rule under different weighting schemes and competition gaps.

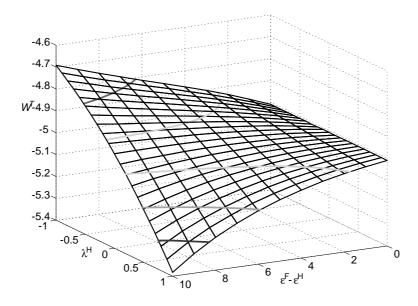


Figure 11: Taylor-rule shocks and the relative weight on countries' inflation.

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