The Eurosystem's Standing Facilities in a General Equilibrium Model of the European Interbank Market

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Abstract

We analyse the European interbank market in a general equilibrium model. Several institutional aspects of the market are taken into consideration, especially the Eurosystem's two standing facilities, reserve requirements of banks and the fact that borrowing from the Eurosystem has to be secured. We show that some characteristics of the interbank market which have been ignored in the theoretical literature on the interbank market until now can have a significant impact on the banks' recourse to the standing facilities.

Zusammenfassung

Wir analysieren den europäischen Interbankenmarkt im Rahmen eines allgemeinen Gleichgewichtmodells. Es werden verschiedene institutionelle Aspekte des Interbankenmarktes berücksichtigt, insbesondere die zwei ständigen Fazilitäten des Eurosystems, die Reserveverpflichtungen der Banken und die Tatsache, daß Kredite des Eurosystems besichert werden müssen. Wir zeigen, daß einige bisher in der theoretischen Literatur unberücksichtigt gebliebenen Eigenschaften des Interbankenmarktes einen Einfluß auf die Inanspruchnahme der Fazilitäten durch die Banken haben.

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The Eurosystem's Standing Facilities in a General Equilibrium Model of the European Interbank Market[1]

1 Introduction

The European interbank market is the market where banks trade liquidity, i.e. deposits denominated in Euros. There are three main reasons why banks need liquidity. (i) The Eurosystem (the European Central Bank and the national central banks of the countries that have adopted the Euro) requires banks to hold deposits on so called minimum reserve accounts with the Eurosystem. Banks need liquidity to fulfill these reserve requirements. (ii) Banks need liquidity to pay out cash to customers on demand. (iii) Banks need liquidity to clear transfers of their customers' deposits to other banks. The interbank market is highly regulated by the Eurosystem. The instruments used to regulate the market are required reserves, standing facilities and open market operations.

As indicated above, banks are required to hold liquidity on minimum reserve accounts with the Eurosystem. Each bank's required amount of liquidity is defined for a period of one month, the so called maintenance period. A maintenance period starts on the 24th of a month and ends on the 23rd of the following month. The minimum amount of liquidity $\overline{M}_i$ that a bank i has to hold as required reserves in a given maintenance period is determined on the basis of its balance sheet at the end of the last calendar month before the beginning of the maintenance period. Bank i does not have to hold exactly $\overline{M}_i$ on its minimum reserve account at every day of the maintenance period, but on average. The Eurosystem pays interest for holding required reserves. This interest is paid on the second day after the end of the maintenance period. No interest is paid for liquidity on a minimum reserve account that

[1]The views expressed in this paper are the views of the author and do not necessarily reflect the opinion of the Bundesbank or of the European Central Bank.

exceeds the minimum reserve requirement. The interest rate on required reserves is fixed by the Eurosystem. A bank that does not fulfill its reserve requirements is fined by the Eurosystem.

If a bank has a liquidity surplus, it can lend liquidity to the Eurosystem's so called deposit facility. The interest rate paid for lending into the deposit facility is called deposit rate and is fixed by the Eurosystem. If a bank has a liquidity deficit, it can borrow from the Eurosystem's so called marginal lending facility. The interest rate for borrowing from the marginal lending facility is the marginal lending rate and is also fixed by the Eurosystem. The marginal lending rate has always been higher than the deposit rate. All lending to and borrowing from the Eurosystem's facilities has a maturity of one day (overnight). Interest is payable with the repayment of the liquidity. Borrowing from the marginal lending facility has to be secured. A bank can lend to the deposit facility as much liquidity as it wants and borrow from the marginal lending facility as much as it can secure.

The Eurosystem conducts several forms of open market operations. In the main open market operations, the so called main refinancing operations, the Eurosystem sells liquidity by auction. To buy liquidity in a main refinancing operations means to borrow this liquidity from the Eurosystem at a rate fixed and announced by the Eurosystem before the operation starts (fixed rate tenders) or determined in the auction process (variable rate tenders). After all banks have submitted their bids simultaneously, the Eurosystem determines the amount of liquidity to be allocated. Thus, while the amount of liquidity borrowed from or lent to the facilities is mostly determined by the banks, the amount of liquidity injected into the banking sector by open market operations is determined by the Eurosystem. The Eurosystem conducts a main refinancing operation every week. The maturity is two weeks. Interest is payable with the repayment of the credit. Borrowing from the Eurosystem via open market operations has to be secured.[2]

Instead of lending to or borrowing from the Eurosystem, banks can finance liquidity deficits by borrowing from other banks or they can get rid of liquidity surpluses by lending to other banks at the interbank market. Deals at the interbank market have a maturity of one day (overnight) to several months. Both secured and unsecured deals can be observed, though most short-term deals are unsecured.[3]

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[2] A complete describtion of the ECB's regulatory instruments is in European Central Bank (2000).

[3] A detailed describtion of many institutional aspects of the European interbank market

In this paper, a general equilibrium model of the interbank market is presented. In contrast to existing models of the European interbank market, introduced for example by Quirós and Mendizábal (2001) and Välimäki (2001), we incorporate three elements in our model that will have some effects on the structure of the equilibrium. Firstly, we explicitly model the fact that borrowing from the Eurosystem has to be secured and that eligible securities may be scarce. Secondly, we assume that the facility rates of a given maintenance period are random and that banks have expectations about these rates. Thirdly, we assume that banks discount future cash flows within a given maintenance period. We show that all three assumptions can lead to a recourse to the Eurosystem's standing facilities before the last day of a maintenance period. We now give the economic intuition for these results.

To begin with, assume that in the tender operations of the Eurosystem only little liquidity has been allotted so that a liquidity deficit is likely. If the banks did not have to secure borrowings from the Eurosystem, they could wait till the last day of the maintenance period and then go to the marginal lending facility to get the liquidity needed to fulfill all reserve requirements. But since they have to secure their recourse to the marginal lending facility, this may not always be possible. The liquidity deficit of the banking sector at the last day of the maintenance period could be higher than the value of the eligible securities if they do not go to the lending facility earlier. Some banks could be unable to fulfill their reserve requirements so that they get penalized by the Eurosystem. In this case, the demand for liquidity could be so high that the interbank rate could exceed the Eurosystem's marginal lending rate. It is clear that some banks would go to the lending facility before the last day of the maintenance period if this is necessary to avoid such a situation. This is exactly one result of our paper.

Next, assume that the market is expecting the Eurosystem to raise the facility rates within the current maintenance period. Since the interbank rate is normally somewhere between the facility rates, this implies that the market is also expecting that the market rate will be rising. Anticipating this, some banks may go to the lending facility some days before a possible rise of the rates hoping to get liquidity cheaper now than later. If instead the market is expecting a decrease of the facility rates and thus the market rate, an early recourse to the deposit facility may occur. These effects of expectations will be shown in our model.

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can be found in Hartmann, Manna and Manzanares (2001).

Finally, assume that it is already quite clear that there will be a liquidity surplus in the current maintenance period so that the market rate at the last day of the maintenance period is expected to be close to the deposit rate. Moreover assume that a change in the facility rates is unlikely. In this case it is optimal for the banks to lend a liquidity surplus some days before the end of the maintenance period to the Eurosystem's deposit facility instead of using it to fulfil reserve requirements. The reason is that - as mentioned above - liquidity lent to the deposit facility pays out interest the following day so that this interest can immediately be used to gain more interest. Liquidity on a minimum reserve account pays out interest after the end of the maintenance period so that the interest on required reserves is useless within the current maintenance period. If banks discount future cash flows, they take this effect into consideration. This is also shown by our model.

We now present the assumptions and the main implications of our model. Some further discussion can be found in the concluding remarks. An appendix contains the proofs of the propositions of the paper.

2 A general equilibrium model

The federal funds market, the US interbank market, has been object of several general equilibrium analyses. One example is Ho and Saunders (1985). Since the institutional framework of the federal funds market differs from that of the European interbank market, more relevant for us are the general equilibrium models of the European market introduced by Ayuso and Repullo (2000) in an appendix, by Quirós and Mendizábal (2001) and Välimäki (2001). The latter two papers assume that at every day the interbank market closes before the access to the standing facilities is closed and that there is a liquidity shock between the closing of the interbank market and the closing of the facilities. We will assume that the interbank market and the facilities close at the same time. In all three papers listed above and in contrast to our paper, banks do not discount cash flows within the maintenance period, borrowings from the Eurosystem do not have to be secured and the facility rates are constant.

2.1 Assumptions

In this section, we formally describe the assumptions we use to analyze the European interbank market. We assume that there are n banks $i \in I = \{1, ..., n\}$ and consider only one maintenance period, lasting from day $t = 1$ to the last day of the maintenance period $t = T$, i.e. in the Eurosystem's current system the 23rd calendar day of a month. All T days are assumed to be trading days. A day in our model will be thought of as a point of time rather than a period of time. Our model is a stochastic model. The state space is denoted by Ω and will be explained below.

Let $\overline{M}_i$ be the daily average minimum reserve requirement of bank i for the maintenance period under consideration and $A_{i,t} \geq 0$ the reserves held by bank i from t to $t+1$ ($t = 1, ..., T$) at the Eurosystem. Let

$$m_{i,t} = \overline{M}_i T - \sum_{\tau=1}^{t-1} A_{i,\tau} \tag{1}$$

for all $t = 1, ..., T$. We assume that no bank holds access reserves, i.e. $A_{i,T} \leq m_{i,T}$, since the Eurosystem pays no interest for excess reserves on minimum reserve accounts as mentioned in the introduction. Bank i completely fulfills its reserve requirements in some state $\omega \in \Omega$, if and only if $A_{i,T}(\omega) = m_{i,T}(\omega)$. If $A_{i,T}(\omega) < m_{i,T}(\omega)$, bank i is punished by the central bank.[4] The punishment will be specified below. Interest for holding required reserves $A_{i,t}$ from t to $t+1$ is payable in $T+1$. The related interest rate is r_M. [5]

The liquidity lent by bank i to the Eurosystem's deposit facility from t to $t+1$ is $D_{i,t} \geq 0$, the liquidity borrowed overnight from the Eurosystem via the marginal lending facility is $S_{i,t} \geq 0$. The deposit rate is $r_{D,t}$ and the marginal lending rate is $r_{S,t}$ ($r_{S,t} > r_{D,t}$). The Eurosystem rates $r_{S,t}$ and $r_{D,t}$ are exogenous in our model and random, but known in t.

Each bank can lend liquidity to and borrow liquidity from other banks at the interbank market, where all deals are assumed to be overnight. Let

[4] For some (exogeneous or endogeneous) random variable Y, we write $Y(\omega)$ for the realization of Y in state ω. Thus, if we for example write $A_{i,T}(\omega) < m_{i,T}(\omega)$, then we just say that in state ω the reserves held by bank i in T are not high enough to fulfil bank i's reserve requirements. If we instead for example write $A_{i,T} \leq m_{i,T}$, then we say that $A_{i,T}(\omega) \leq m_{i,T}(\omega)$ for all $\omega \in \Omega$.

[5] Here, we do not exactly model the ECB's current system where all interest on holdings of required reserves is paid on the *second* day after the end of the maintenance period.

$F_{i,t}$ be the liquidity bank i lends from t to $t+1$ to the interbank market and r_t the corresponding interest rate.[6] We assume that transactions at the interbank market do not have to be secured.

We denote the liquidity bank i can dispose of at day t by $L_{i,t}$. Assuming that no bank holds cash over night, we have

$$\begin{aligned} L_{i,t} &= A_{i,t-1} + (1 + r_{D,t-1})D_{i,t-1} - (1 + r_{S,t-1})S_{i,t-1} \\ &\quad +(1 + r_{t-1})F_{i,t-1} + g_{i,t} \end{aligned} \tag{2}$$

for $t = 2, ..., T + 1$.The term $g_{i,t}$ is an exogenous random liquidity influx (or liquidity drain, if $g_{i,t}(\omega) < 0$ for some $\omega \in \Omega$) to bank i in t. If for example customers of bank i pay in (withdraw) money, $g_{i,t}$ increases (decreases). Moreover, $g_{i,t}$ comprises for example liquidity drains because of dividend payouts, payouts of factor income and real investments. From the expenditure side, we can write

$$L_{i,t} = A_{i,t} + D_{i,t} - S_{i,t} + F_{i,t} \tag{3}$$

for $t = 1, ..., T$.

In $t = 1$, the Eurosystem conducts an open market operation. The amount of liquidity injected by this operation is P. No other open market operation is conducted in the maintenance period. We assume that the Eurosystem can perfectly control P. Since we do not model the decision making of the Eurosystem, P is exogeneous. The amount of liquidity bank i receives in the operation is P_i, i.e. $\sum_{i=1}^{n} P_i = P$. Bank i's initial endowment of liquidity after the open market operation has been conducted is $L_{i,1} = \overline{L}_i + P_i$, where $\overline{L}_i$ is exogenous. We assume that the liquidity borrowed in the open market operation and the interest for this borrowing is payable in $T + 1$. Repayments relating to the open market operations are assumed to be in $g_{i,T+1}$.

Borrowing from the Eurosystem has to be secured. Bank i is assumed to have assets $\overline{S}_i > 0$ that can be used to secure borrowing from the Eurosystem. Bank i's initial endowment of eligible assets after the open market operation is $S_i = \overline{S}_i - P_i > 0$. Each bank i has to satisfy $S_{i,t} \leq S_i$.

Each bank is assumed to maximizes its in $T + 1$ disposable liquidity, i.e. its final value. This is because we assume that no endogenous liquidity drains

[6] Note that all interest rates $r_{M,t}$, r_t, $r_{D,t}$ and $r_{S,t}$ are very small numbers, since these are daily rates. If for example the annual interest rate for borrowing from the marginal lending facility is 10%, $r_{S,t}$ is at approximately $0,1/360 < 0,0003$.

occur between $t = 1$ and $t = T$, but the whole liquidity surplus is reinvested according to equation 3. Thus, we assume in the remainder of this paper that bank i's objective at day t, $t = 1, ..., T$, is to maximize

$$E_t[\pi_i] = E_t[L_{i,T+1} + r_M \sum_{t=1}^{T} A_{i,t} - \alpha(m_{i,T} - A_{i,T})]$$

The parameter α is a penalty for insufficient holdings of required reserves. In accordance with the sanctions the Eurosystem can impose on a bank that fails to fulfill its reserve requirements we assume $\alpha > r_{S,T}$.[7]

Our analysis is based on the traditional assumptions of the general equilibrium theory. All banks take the interbank interest rates and expected interbank rates as given. At each day $t = 1, ..., T$, bank i chooses $A_{i,t}$, $D_{i,t}$, $S_{i,t}$ and $F_{i,t}$ to maximize the in t expected liquidity $E_t[\pi_i]$. An equilibrium in t is defined by

$$\sum_{i=1}^{n} F_{i,t} = 0 \tag{4}$$

Finally, to ensure that our assumptions are free of contradictions, we assume that under all conditions, we have

$$\sum_{i=1}^{n} L_{i,t} + S_i > 0 \tag{5}$$

for all $t = 1, ..., T$. If equation 5 did not hold at some day t, the banks would need to borrow more liquidity from the Eurosystem than they could secure in order to avoid a negative balance on their accounts with the Eurosystem. Thus, there would be a situation like after a euro-area wide bankrun. Since we do not want to discuss a possible role of the Eurosystem as a lender of last resort, we assume that equation 5 holds throughout.

Note that in our model, there are $n(T+1) + 2T$ exogeneous random variables, namely $g_{i,t}$ for all $i \in I$ and $t = 1, ..., T+1$ and the two facility rates for T different days. A state $\omega \in \Omega$ is thus a vector of $n(T+1) + 2T$ numbers.

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[7] See European Central Bank (2000), p. 56.

2.2 The last day of the maintenance period

To begin with, we consider day T, the last day of the maintenance period. With the equations 1 to 3, it is easy to show that bank i's maximization problem in T is to choose $D_{i,T}$, $S_{i,T}$ and $A_{i,T}$ to maximize

$$\begin{aligned}&E_T[\pi_i] = (r_M + \alpha - r_T)A_{i,T} + (r_{D,T} - r_T)D_{i,T} - (r_{S,T} - r_T)S_{i,T}\\ &-(r_M + \alpha)m_{i,T} + (1 + r_T)L_{i,T} + E_T[g_{i,T+1}] + r_M T\overline{M}_i\end{aligned}$$

subject to
$D_{i,T} \geq 0,\ S_i \geq S_{i,T} \geq 0,\ m_{i,T} \geq A_{i,T} \geq 0$

Since transactions at the interbank market are unsecured, arbitrage is possible, if $r_T(\omega) < r_{D,T}(\omega)$. Thus, in equilibrium we have $r_T \geq r_{D,T}$. It is obvious that solving the above maximization problem yields

$$D_{i,T}(\omega)\begin{cases} \in [0,\infty], \text{ if } r_T(\omega) = r_{D,T}(\omega)\\ 0, \text{ otherwise}\end{cases}$$

$$S_{i,T}(\omega)\begin{cases} = S_i, \text{ if } r_T(\omega) > r_{S,T}(\omega)\\ \in [0, S_i], \text{ if } r_T(\omega) = r_{S,T}(\omega)\\ = 0, \text{ otherwise}\end{cases}$$

$$A_{i,T}(\omega)\begin{cases} = m_{i,T}(\omega), \text{ if } r_T(\omega) < r_M + \alpha\\ \in [0, m_{i,T}(\omega)], \text{ if } r_T(\omega) = r_M + \alpha\\ = 0, \text{ otherwise}\end{cases}$$

Finally, $F_{i,T}(\omega)$ is given by equation 3.

Let x_i be the surplus of some bank i's disposable liquidity over its remaining reserve requirements in T, i.e. x_i is defined by

$$x_i = L_{i,T} - m_{i,T}$$

Assuming that $\sum_{i=1}^n x_i \neq 0$ and $\sum_{i=1}^n x_i \neq -\sum_{i=1}^n S_i$, we easily get with equation 4

$$r_T(\omega) = \begin{cases} r_{D,T}(\omega), \text{ if } \sum_{i=1}^n x_i(\omega) > 0\\ r_{S,T}(\omega), \text{ if } \ 0 > \sum_{i=1}^n x_i(\omega) > -\sum_{i=1}^n S_i\\ r_M + \alpha, \text{ if } \sum_{i=1}^n x_i(\omega) < -\sum_{i=1}^n S_i\end{cases} \tag{6}$$

as the equilibrium rate in T.

The economic reason for this result is simple: If there is a liquidity surplus in the market ($\sum_{i=1}^{n} x_i(\omega) > 0$), then there has to be a recourse to the deposit facility. But if the market rate were higher than the deposit rate, no bank would lend to the deposit facility, since lending to the market is more profitable. Thus, the market rate has to equal the facility rate in equilibrium.

If there is a liquidity deficit that is too high to be balanced by borrowing from the Eurosystem because the banks do not have enough eligible collaterals ($\sum_{i=1}^{n} x_i(\omega) < -\sum_{i=1}^{n} S_i$), not all reserve requirements can be fulfilled. But if the market rate were lower than $r_M + \alpha$, all banks would want to borrow enough liquidity from the market to completely fulfill all reserve requirements. If instead the market rate were higher than $r_M + \alpha$, it is also higher than the lending rate (since we have assumed $r_{S,T} < \alpha$), i.e. the banks borrow as much as possible from the lending facility, but do not hold any reserves. The amount of liquidity offered to the market is then $\sum_{i=1}^{n} L_{i,T}(\omega) + S_i > 0$. Thus, the market rate has to equal $r_M + \alpha$.

Finally, in the intermediate case, there is a liquidity deficit in the market, but all reserve requirements can be fulfilled by borrowing from the marginal lending facility. If the market rate were higher than the marginal lending rate, all banks would borrow as much from the lending facility as they can secure. Since this is more than needed to fulfill all reserve requirements, there would still be a liquidity surplus at the interbank market in this case. If instead the market rate were lower than the lending rate, it would also be lower than $r_M + \alpha$, since we have assumed $r_{S,T} < \alpha$. In this case, all banks would try to completely fulfill their reserve requirements by borrowing from the market, but not from the lending facility. Thus, there would be an excess demand of liquidity at the market. The market rate therefore has to be equal to the marginal lending rate in this case.

2.3 The other days of the maintenance period

Now consider the other days of the maintenance period $t = 1, ..., T-1$. Again, arbitrage is possible if $r_{D,t}(\omega) > r_t(\omega)$. Thus, we ignore this case. It is obvious that

$$D_{i,t}(\omega) \begin{cases} \in [0, \infty], \text{ if } r_{D,t}(\omega) = r_t(\omega) \\ = 0, \text{ if } r_{D,t}(\omega) < r_t(\omega) \end{cases}$$

and

$$S_{i,t}(\omega)\begin{cases} = S_i, \text{ if } r_{S,t}(\omega) < r_t(\omega) \\ \in [0, S_i], \text{ if } r_{S,t}(\omega) = r_t(\omega) \\ 0, \text{ otherwise} \end{cases}$$

Less obvious is the following lemma. Define

$$a_t = \frac{E_t[r_T]}{1 + (T-t)E_t[r_T]}$$

Then we get

Lemma 1

$$A_{i,t}(\omega)\begin{cases} = m_{i,t}(\omega), \textit{ if } r_t(\omega) < a_t(\omega) \\ \in [0, m_{i,t}(\omega)], \textit{ if } r_t(\omega) = a_t(\omega) \\ = 0, \textit{ otherwise} \end{cases}$$

The proof of this lemma is in the appendix. With $A_{i,t}(\omega)$, $D_{i,t}(\omega)$ and $S_{i,t}(\omega)$, $F_{i,t}(\omega)$ follows from equation 3.[8]

2.4 Further analysis of the model

We now determine under which conditions there is a recourse to the facilities in equilibrium. In order to proceed, we introduce the following definitions:

$$\widetilde{L}_t = \sum_{i=1}^{n}[L_{i,1} + \sum_{\tau=2}^{t} g_{i,\tau}]$$

and

$$\widetilde{m}_t = \sum_{i=1}^{n}[m_{i,1} - (t-1)L_{i,1} - \sum_{\tau=2}^{t-1}(t-\tau)g_{i,\tau}]$$

for all $t = 2, ..., T$. It is easy to see that $\widetilde{L}_t(\omega) = \sum_{i=1}^{n} L_{i,t}(\omega)$ and $\widetilde{m}_t(\omega) = \sum_{i=1}^{n} m_{i,t}(\omega)$, if $D_{i,\tau}(\omega) = S_{i,\tau}(\omega) = 0$ for all $i \in I$ and $\tau = 1, ..., t-1$. Thus,

[8]With the definition of a_t, we have implicitely defined $a_t(\omega)$ and thus $E_t[r_T](\omega)$. Note that $E_t[r_T](\omega)$ is the in t and ω expected market rate at day T. But it is not the expected market rate in T given the information ω which could be denoted by $E_t[r_T|\omega]$. Thus $E_t[r_T](\omega) = E_t[r_T|\omega]$ is not true in general.

we have $\widetilde{L}_T(\omega) - \widetilde{m}_T(\omega) = \sum_{i=1}^n x_i(\omega)$, if $D_{i,\tau}(\omega) = S_{i,\tau}(\omega) = 0$ for all $i \in I$ and $\tau = 1, ..., T-1$. Finally, we define $\widetilde{r}_T$ by

$$\widetilde{r}_T(\omega) = \begin{cases} r_{D,T}(\omega), \text{ if } \widetilde{L}_T(\omega) - \widetilde{m}_T(\omega) > 0 \\ r_{S,T}(\omega), \text{ if } 0 > \widetilde{L}_T(\omega) - \widetilde{m}_T(\omega) > -\sum_{i=1}^n S_i \\ r_M + \alpha, \text{ if } -\sum_{i=1}^n S_i > \widetilde{L}_T(\omega) - \widetilde{m}_T(\omega) \end{cases}$$

and $\widetilde{a}_t$ by

$$\widetilde{a}_t = \frac{E_t[\widetilde{r}_T]}{1 + (T-t)E_t[\widetilde{r}_T]}$$

It is clear from equation 6 that $\widetilde{r}_T(\omega) = r_T(\omega)$, if $D_{i,\tau}(\omega) = S_{i,\tau}(\omega) = 0$ for all $i \in I$ and $\tau = 1, ..., t-1$. Note that $\widetilde{L}_t$ and $\widetilde{m}_t$ and with this $\widetilde{r}_T$ are random variables that depend on P. Since P is exogeneous, we can treat these variables as exogeneous, too.

Our first proposition presents conditions under which there is no recourse to the standing facilities:

Proposition 2 *If $\widetilde{L}_t > 0$, $\widetilde{m}_t - \widetilde{L}_t > 0$ and $r_{D,t} < \widetilde{a}_t < r_{S,t}$ for all $t = 1, ..., T-1$, then there is an equilibrium with $r_t = a_t$, $D_{i,t} = S_{i,t} = 0$ for all $i \in I$ and $\sum_{i=1}^n A_{i,t} = \widetilde{L}_t$ for all $t = 1, ..., T-1$ and $r_T = \widetilde{r}_T$.*

As mentioned above, we have $\sum_{i=1}^n L_{i,t} = \widetilde{L}_t$, $\sum_{i=1}^n m_{i,t} = \widetilde{m}_t$ and $a_t = \widetilde{a}_t$ for all $t = 1, ..., T$, if there is no recourse to the standing facilities in any state and at any day. The proposition thus states, that there is no recourse to the facilities in equilibrium, if no recourse to the facilities implies that (i) the liquidity the banks dispose of at a given day t, i.e. $\sum_{i=1}^n L_{i,t}$, is positive, (ii) it is not higher than the remaining reserve obligations $\sum_{i=1}^n m_{i,t}$ at that day and (iii) the at some day t expected and discounted market rate at day T, i.e. a_t, is strictly between the facility rates in t.

Note that proposition 2 describes a situation where the interbank rate follows a martingale-like process $r_t = a_t$. But why is it not exactly a martingale? The reason is that the banks discount interest payments. In the equation $r_t = a_t$, the interbank rate r_t is equal to the discounted, in t expected interbank rate of day T, where the discount factor is $1 + (T-t)E_t[r_T]$. If we assumed that the banks do not discount future cash flows (as for example in Ayuso and Repullo (2000), Quirós and Mendizábal (2001) and Välimäki (2001)), we would have $r_t = E_t[r_T]$ instead.

Now we consider deviations from the situation described in proposition 2. The following proposition is trivial:

Proposition 3 *If for some* $\omega \in \Omega$ *and some* $t < T$ *we have* $\widetilde{m}_t(\omega) - \widetilde{L}_t(\omega) < 0$ *or* $\widetilde{L}_t(\omega) < 0$*, then not* $\sum_{i=1}^n D_{i,\tau} = \sum_{i=1}^n S_{i,\tau} = 0$ *for all* $\tau \leq t$.

It is clear that without recourse to the facilities at all days $\tau < t$, we get $\widetilde{L}_t = \sum_{i=1}^n L_{i,t}$ and $\widetilde{m}_t = \sum_{i=1}^n m_{i,t}$. But $\sum_{i=1}^n m_{i,t} - L_{i,t} < 0$ requires a recourse to the deposit facility in t and $\sum_{i=1}^n L_{i,t} < 0$ requires a recourse to the marginal lending facility in t. This proves proposition 3. Note that the situation described in proposition 3 has been considered in Quirós and Mendizábal (2001).

We now state our main two propositions.

Proposition 4 *If* $\widetilde{m}_t(\omega) - \widetilde{L}_t(\omega) > 0$ *and* $r_{S,t}(\omega) < \widetilde{a}_t(\omega)$ *for some* $t \leq T-1$ *and* $\omega \in \Omega$*, then not* $\sum_{i=1}^n S_{i,\tau} = 0$ *for all* $\tau = 1, ..., T-1$.

Proposition 4 gives a parameter constellation that leads to recourse to the marginal lending facility. Of special interest is the expression $r_{S,t}(\omega) < \widetilde{a}_t(\omega)$. We now present two quite extreme conditions that each imply $r_{S,t}(\omega) < \widetilde{a}_t(\omega)$. There are less extreme conditions that also imply $r_{S,t}(\omega) < \widetilde{a}_t(\omega)$, but are not so instructive.

To begin with, assume that $E_t[\widetilde{r}_T](\omega) = r_M + \alpha$, i.e. it is expected in t and ω that without recourse to the facilities before day T the liquidity deficit of the market at T, i.e. $\widetilde{m}_T - \widetilde{L}_T$, is higher than the value of the eligible assets $\sum_{i=1}^n S_i$.[9] We clearly get $\widetilde{a}_t(\omega) = \frac{r_M + \alpha}{1+(T-t)(r_M + \alpha)}$. If we additionally assume $r_{S,t}(\omega) < \frac{r_M + \alpha}{1+(T-t)(r_M + \alpha)}$, then we get $r_{S,t}(\omega) < \widetilde{a}_t(\omega)$. Thus, an expected shortage of eligible assets at the last day of the maintenance period without recourse to the facilities before T can lead to a recourse to the marginal lending facility before T. This result should not surprise. A shortage of eligible assets in T would bring r_T close to $r_M + \alpha$. If $r_M + \alpha$ is sufficiently high and $r_{S,t}$ sufficiently low, it is optimal to borrow liquidity in t from the Eurosystem's marginal lending facility and to use the liquidity to fulfill reserve requirements early in order to have a liquidity surplus in T when liquidity is lent at a very high rate. Note that it is not unlikely to get a rate $\widetilde{r}_T$ that is higher than $r_{S,T}$ if the allotments in the tender operations of the Eurosystem within the maintenance period under consideration are rather low.

Next assume that $0 < \widetilde{m}_T - \widetilde{L}_T < \sum_{i=1}^n S_i$ so that (a) even without recourse to the marginal lending facility before T there is no shortage of

[9] See section 2.2.

eligible assets, but (b) $\widetilde{r}_T = r_{S,T}$. If $E_t[r_{S,T}](\omega) = r_{S,t}(\omega)$, i.e. if there are no expectations of facility rate changes, then we still have $r_{S,t}(\omega) > \widetilde{a}_t(\omega)$. But assume that it is expected in t and ω that the marginal facility rate in T will be considerably higher than in t. Now $r_{S,t}(\omega) < \widetilde{a}_t(\omega)$ and an early recourse to the marginal lending facility is possible. Thus, an expected rise of the marginal lending rate (together with an expected moderate liquidity deficit in T if there is no recourse to the facilities before T) can lead to a recourse to the marginal lending facility before T.

Finally, note that $\widetilde{r}_T \leq r_{S,T}$ and $r_{S,t} = r_S$ for all $t = 1, ..., T$ and some number $r_S > 0$ implies $r_{S,t} > \widetilde{a}_t$ for all $t = 1, ..., T-1$. Thus, if no recourse to the facilities before T leads to no shortage of eligible assets and if the facility rate is constant over the maintenance period, then the parameter constellation that is sufficient for a recourse to the marginal lending facility according to proposition 4 never occurs.

Ignoring the unlikely case of $\sum_{i=1}^{n} L_{i,t}(\omega) = 0$, we get a proposition that describes parameter constellations that normally lead to a recourse to the deposit facility:

Proposition 5 *If $r_{D,t}(\omega) > \widetilde{a}_t(\omega)$ for some $t \leq T-1$ and $\omega \in \Omega$, then $\sum_{i=1}^{n} L_{i,t}(\omega) = -\sum_{i=1}^{n} S_{i,t}(\omega) < 0$ and $\sum_{i=1}^{n} A_{i,t}(\omega) = 0$, or not $\sum_{i=1}^{n} D_{i,\tau} = 0$ for all $\tau = 1, ..., T-1$.*

Note that according to proposition 5, $r_{D,t}(\omega) > \widetilde{a}_t(\omega)$ does not necessarily lead to a recourse to the deposit facility, if $\sum_{i=1}^{n} L_{i,t}(\omega) < 0$. But $\sum_{i=1}^{n} L_{i,t}(\omega) < 0$ is not possible, if $\widetilde{L}_t(\omega)$ is sufficiently high.[10] We now describe two conditions that imply $r_{D,t}(\omega) > \widetilde{a}_t(\omega)$:

Firstly assume that $r_{D,t} = r_D$ for all $t = 1, ..., T$ and some number $r_D > 0$, i.e. the deposit rate is constant over the maintenance period. Moreover assume that $E_t[\widetilde{m}_T - \widetilde{L}_T < 0](\omega) = 1$ so that $E_t[\widetilde{r}_T](\omega) = r_D$. It is clear that we now get $r_{D,t}(\omega) > \widetilde{a}_t(\omega)$. Thus, recourse to the deposit facility is possible

[10] However, $\widetilde{L}_t(\omega) > 0$ does not imply $\sum_{i=1}^{n} L_{i,t}(\omega) > 0$, since recourse to the marginal lending facility before t can reduce $\sum_{i=1}^{n} L_{i,t}(\omega)$. But it is easy to show that

$$\sum_{i=1}^{n} L_{i,t}(\omega) \geq \widetilde{L}_t(\omega) - \sum_{\tau=1}^{t-1} [r_{S,\tau}(\omega) \sum_{i=1}^{n} S_i]$$

Thus, if the right hand side of the above inequation is positive and $r_{D,t}(\omega) > \widetilde{a}_t(\omega)$, then there is recourse to the deposit facility.

even without expected changes of the facility rates. The reason for this effect is our assumption that banks discount future cash flows. Liquidity lent to the deposit facility at some day $t < T$ pays interest in $t+1$, while liquidity on a minimum reserve account in t pays interest in $T+1$. Since interest for liquidity in the deposit facility is payed earlier than interest for liquidity on minimum reserve accounts, lending to the deposit facility has advantages that will be taken into account by a bank that discounts future cash flows. Note that this would not be the case if interest related to recourses to the facilities were also payable in $T+1$.

To be complete, we finally mention that expected changes of the facility rates can also lead to an early recourse to the deposit facility. If $E_t[\widetilde{r}_T](\omega)$ is sufficiently low, thought $\widetilde{m}_T - \widetilde{L}_T$ is expected to be high, because it is expected that the facility rates will fall considerably, then we may have $r_{D,t}(\omega) > \widetilde{a}_t(\omega)$ and a recourse to the deposit facility in t.

3 Concluding remarks

In this paper, a general equilibrium model of the European interbank market has been presented. Our model deviates from other general equilibrium models of the European interbank market in three ways: Firstly, the fact that all borrowings from the Eurosystem have to be secured and that eligible assets may be scarce is explicitly modeled. Secondly, the Eurosystem's facility rates are regarded as random rather than constant parameters. Thirdly, banks are assumed to discount future cash flows. We have shown that all three deviations from previous models are important as to whether banks go to the Eurosystem's standing facilities before the last day of the maintenance period under consideration.

Moreover, it should be emphasized that our model is rather rich but at the same time not hard to analyze. It could therefore provide a framework for further investigations. The outcome of the Eurosystem's open market operations for example is in our model exogeneous. But given that the model is easily analyzed, it may be quite simple to incorporate one or several endogeneous open market operations into the model.[11]

However, one may question whether the assumptions of the general equilibrium theory are appropriate in an analysis of the interbank market. For

[11] See for example Välimäki (2001) who has incorporated open market operations into a general equilibrium model of the European interbank market.

asymmetric information and market power may be important characteristics of the interbank market in Europe and elsewhere. But the question which model is the most appropriate one can only be answered by means of empirical research. And empirical research should not be conducted before a clear understanding of competing theoretical models has been reached.

4 Appendix

Proof of lemma 1:

Consider some bank i and two feasible plans

$$[\widehat{A}_{i,\tau}, \widehat{D}_{i,\tau}, \widehat{S}_{i,\tau}, \widehat{F}_{i,\tau}]_{\tau=1,...,T}$$
$$[\widetilde{A}_{i,\tau}, \widetilde{D}_{i,\tau}, \widetilde{S}_{i,\tau}, \widetilde{F}_{i,\tau}]_{\tau=1,...,T}$$

with $\widehat{D}_{i,\tau} = \widetilde{D}_{i,\tau}$ and $\widehat{S}_{i,t} = \widetilde{S}_{i,t}$ for all $\tau = 1,...,T$, $\widehat{F}_{i,\tau} = \widetilde{F}_{i,\tau}$ for all $\tau = 1,...t-1, t+1,...,T-1$, $\widehat{F}_{i,t} = \widetilde{F}_{i,t} - a$ for some $a > 0$ and $\sum_{\tau=1}^{T} \widehat{A}_{i,\tau} = \sum_{\tau=1}^{T} \widetilde{A}_{i,\tau}$. From 2 and 3, it follows that $\widehat{A}_{i,\tau} - \widetilde{A}_{i,\tau} = 0$ for $\tau = 1,...,t-1$, $\widehat{A}_{i,t} - \widetilde{A}_{i,t} = a$ and $\widehat{A}_{i,\tau} - \widetilde{A}_{i,\tau} = -r_t a$ for $\tau = t+1,...,T-1$. Thus, $\widehat{A}_{i,T} - \widetilde{A}_{i,T} = -[1-(T-t-1)r_S]a$. With this, we get $\widehat{F}_{i,T} - \widetilde{F}_{i,T} = a-(T-t)r_t a$. Thus, $\widehat{L}_{i,T+1} - \widetilde{L}_{i,T+1} = a[r_T(1-(T-t)r_t) - r_t]$ and therefore

$$E_t[\widehat{L}_{i,T+1} - \widetilde{L}_{i,T+1}](\omega) < (>) 0 \Leftrightarrow r_t(\omega) > (<) a_t(\omega)$$

Thus, for any plan with a positive holding of reserves at some day t in some state ω there is another plan with no holding of reserves in t so that the latter yields a higher profit than the former, if $r_t(\omega) > a_t(\omega)$. It follows that it is optimal for bank i to choose $A_{i,t}(\omega) = 0$, if $r_t(\omega) > a_t(\omega)$. If instead $r_t(\omega) < a_t(\omega)$, we get the opposite result analogously.

■

Proof of proposition 2:

If $r_t = a_t$ and $r_T = \widetilde{r}_T$, then

$$r_t = \frac{E_t[\widetilde{r}_T]}{1 + (T-t)E_t[\widetilde{r}_T]}$$

i.e. $r_{D,t} < r_t < r_{S,t}$ and thus $D_{i,t} = S_{i,t} = 0$ for all $i \in I$ and $t \leq T-1$. This implies $r_T = \widetilde{r}_T$. Moreover, $D_{i,t} = S_{i,t} = 0$ for all $i \in I$ and $t \leq T-1$ implies $\sum_{i=1}^{n} L_{i,t} = \widetilde{L}_t$ and $\sum_{i=1}^{n} A_{i,t} = \sum_{i=1}^{n} L_{i,t}$, thus $\sum_{i=1}^{n} A_{i,t} = \widetilde{L}_t$. The latter requires $r_t = a_t$. This shows that proposition 2 indeed describes an equilibrium.

■

Proof of proposition 4:

Assume $\sum_{i=1}^{n} S_{i,t} = 0$ for all $t = 1, ..., T-1$. This implies (i) $\sum_{i=1}^{n} m_{i,t} - L_{i,t} \geq \widetilde{m}_t - \widetilde{L}_t$ for all $t = 1, ..., T$, (ii) $E_t[r_T] \geq E_t[\widetilde{r}_T]$ for all $t = 1, ..., T$ and (iii) $r_{S,t} \geq r_t$ for all $t \leq T-1$ and $\omega \in \Omega$. Moreover, assume $r_{S,t}(\omega) < \widetilde{a}_t(\omega)$ for some $t \leq T-1$ and $\omega \in \Omega$. With (ii), this implies (iv) $r_{S,t}(\omega) < a_t(\omega)$. From (iii) and (iv), we get $r_t(\omega) < a_t(\omega)$, i.e. $\sum_{i=1}^{n} A_{i,t}(\omega) = \sum_{i=1}^{n} m_{i,t}(\omega)$ (see lemma 1), thus $\sum_{i=1}^{n} L_{i,t}(\omega) = \sum_{i=1}^{n} m_{i,t}(\omega) + D_{i,t}(\omega)$. With (i) and $\widetilde{m}_t(\omega) - \widetilde{L}_t(\omega) > 0$, this implies $\sum_{i=1}^{n} D_{i,t}(\omega) < \widetilde{L}_t(\omega) - \widetilde{m}_t(\omega) < 0$. This is not possible. It follows that simultaneously $\sum_{i=1}^{n} S_{i,t} = 0$ for all $t = 1, ..., T-1$, $\widetilde{m}_t(\omega) - \widetilde{L}_t(\omega) > 0$ and $r_{S,t}(\omega) < \widetilde{a}_t(\omega)$ for some $t \leq T-1$ and $\omega \in \Omega$ is not possible either. This proves proposition 4.

■

Proof of proposition 5:

Assume $\sum_{i=1}^{n} D_{i,t} = 0$ for all $t = 1, ..., T-1$. This implies (i) $E_t[r_T] \leq E_t[\widetilde{r}_T]$ for all $t = 1, ..., T$. Moreover, assume $r_{D,t}(\omega) > \widetilde{a}_t(\omega)$ for some $t \leq T-1$ and $\omega \in \Omega$. With (i), this implies (ii) $r_{D,t}(\omega) > a_t(\omega)$. Because $r_{D,t} \leq r_t$, (ii) implies $r_t(\omega) > a_t(\omega)$, i.e. $\sum_{i=1}^{n} A_{i,t}(\omega) = 0$, thus $\sum_{i=1}^{n} L_{i,t}(\omega) = -\sum_{i=1}^{n} S_{i,t}(\omega)$. This implies $\sum_{i=1}^{n} L_{i,t}(\omega) < 0$ (since we ignore the unlikely case of $\sum_{i=1}^{n} L_{i,t}(\omega) = 0$), i.e. $\sum_{i=1}^{n} S_{i,t}(\omega) > 0$. It follows that either (A) $r_{D,t}(\omega) > \widetilde{a}_t(\omega)$ and $\sum_{i=1}^{n} L_{i,t}(\omega) < 0$, i.e. $\sum_{i=1}^{n} S_{i,t}(\omega) > 0$, or $r_{D,t}(\omega) > \widetilde{a}_t(\omega)$ and not $\sum_{i=1}^{n} D_{i,t} = 0$ for all $t = 1, ..., T-1$. This proves proposition 5.

■

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