



The stable long-run CAPM and
the cross-section of expected returns
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Abstract

The capital-asset-pricing model (CAPM) is one of the most popular methods of financial market analysis. But, evidence of the poor empirical performance of the CAPM has accumulated in the literature. For example, based on their empirical results regarding the relation between market Beta and average return, Fama and French (1996) conclude that the CAPM is no longer a useful tool for empirical financial market analysis. Most empirical studies of the conventional CAPM take, however, neither the fat-tails of return data nor the price relationship between an asset of interest and the bench market portfolio into account.

In the framework of a univariate Beta-model we consider a stable long-run CAPM taking account of the fat-tails of stock returns and the common stochastic trends between stock prices. Using the same data used by Fama and French (1996), the stable long-run CAPM demonstrates that Markowitz rule of the expected returns and variance of returns can (still) —without any use of firm specific variables— explain the variation of the cross-sectional average returns.

JEL Classification: G12, C21, C51

Keywords: CAPM; Stable Paretian distribution; Stochastic common trend.

Zusammenfassung

Das Capital-Asset-Pricing-Modell (CAPM) ist einer der populärsten empirischen Ansätze zur Analyse der Finanzmarktdaten. In der Literatur jedoch sind eher Gegenbeweise über seine empirische Tauglichkeit akkumuliert. Fama und French (1996) haben beispielsweise aufgrund ihrer empirischen Untersuchungsergebnisse über die Beziehung zwischen dem Markt-Beta und der Durchschnittsrendite schlussgefolgert, daß das CAPM keine nützliche Methode für empirische Finanzmarktanalyse mehr sein kann. Die meisten Arbeiten aber, die sich mit dem CAPM beschäftigen, berücksichtigen weder die ausreißerreiche empirische Renditenverteilung noch die Preisbeziehung zwischen dem einzelnen Kurs und dem Benchmark.

In der vorliegenden Arbeit wird im Rahmen univariater Beta-Modelle ein Versuch zur Spezifikation eines stabilen langfristigen CAPM gemacht, das sowohl die ausreißerreiche empirische Renditenverteilung als auch die Preisbeziehung zwischen dem einzelnen Kurs und dem Benchmark berücksichtigt. Mit dem Datensatz von Fama und French (1996) wird gezeigt, daß das stabile langfristige CAPM in der Lage ist, anhand der Markowitz'schen Mittelwert-Varianz-Regel —ohne Hinzufügen firmenspezifischer Variablen— die Variabilität durchschnittlicher Rendite in Querschnittdaten zu erklären.

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The stable long-run CAPM and the cross-section of expected returns¹

1 Introduction

Central banks are more and more concerned with the financial market because of its importance not only for their monetary policy, but also for the regulation of financial institutions regarding risk management. Despite numerous theoretical and empirical criticisms, the capital-asset-pricing model (CAPM) has been and is still one of the most popular standard tools for financial researchers and practitioners to quantify the trade-off between risk and expected return in financial markets. Markowitz (1952) argues that investors would optimally hold a portfolio with the highest expected return for a given risk. Based on the so-called mean-variance efficient portfolio, Sharpe (1965) and Lintner (1966) develop the well-known Sharpe and Lintner version of CAPM (henceforth, Sharpe/Lintner CAPM or conventional CAPM), in which the expected return of an asset must be linearly related to the covariance of its return with the return of the market portfolio. The economic theory for the CAPM is intuitively clear and the empirical implication of the CAPM is plausible, since risky assets will usually yield higher returns than investment free of risk. In line with many reports on anomalies in the 70s and 80s, however, evidence of the poor empirical performance of the conventional CAPM has accumulated in the literature. More recently, based on their empirical results regarding the relation between market Beta and average return, Fama and French (1992, 1996) have declared the end of the CAPM as an empirical tool for the analysis of trade-off relations between risk and returns on stocks. To improve the empirical performance of the conventional CAPM, some modifications are also considered in the literature. Most of the modifications are performed by including some firm specific variables such as *firm-size* in Banz (1981) and *book-to-market ratio* in Fama and French (1992). Some of the modifications are based on another probability principle (without any firm specific variables), such as the time-varying market Beta by Jagannathan and Wang (1996). Guo and Whitelaw (2000), for example, consider a structural asset-pricing model in the context of time series, and find a significant positive relationship between return and risk. See, for more debate about the CAPM, a nice summary paper of Jagannathan and McGrattan (1995).

In this paper, we consider a possible modification of the conventional CAPM to improve its empirical performance. For this modification, we examine properties of empirical financial data and take them into account:

¹The views expressed in this paper are those of the author, and not necessarily those of the Deutsche Bundesbank. I acknowledge the helpful comments of H. Herrmann.

- since the influential works of Mandelbrot (1963), the infinite-variance stable Paretian (usually α -stable) distribution² has often been considered to be a more realistic distribution for asset returns than the normal distribution, because asset returns are typically fat-tailed and excessively peaked around zero—phenomena that can be captured by α -stable distributions with $\alpha < 2$.
- Second, the conventional CAPM is based on (excess) returns and, hence, contains no information of stock prices. Note that returns are first difference of log-prices³ and the log-prices are typically assumed to be random walks (more precisely, martingale). When two stock prices (although each of them follows a random-walk process) have a common stochastic trend and build a long-run equilibrium, one can use this level information to improve the empirical performance of the CAPM.

The modified version of the conventional CAPM introduced in this paper will therefore be based on the empirical evidence for the α -stable distributed stock returns and the common stochastic trends between stock prices and, hence, may be called an (α -) stable long-run CAPM (SLCAPM). Consequently, the SLCAPM contains not only the (usual) market Beta as a *short-run* market Beta, but also a *long-run* market Beta in terms of a long-run relationship, and the returns are allowed to be non-normally distributed. An empirical application of the SLCAPM for the German Stock Index (DAX) will show how the stable long-run market Beta can be used for measuring risks. In order to demonstrate the empirical performance of the SLCAPM, we compare our model with some well-known alternative CAPMs such as the book-to-market CAPM by Fama and French (1992, 1996) (henceforth, BM-CAPM⁴) and the conditional CAPM of Jagannathan and Wang (1996) (henceforth, CCAPM). Using the same data as Fama and French (1992) and Jagannathan and Wang (1996), it turns out that the SLCAPM explains —without any firm specific variables such as firm-size, book-to-market equity and/or human labour— over 60 percent of the cross-sectional variation in average returns. This result is compared with that of the BMCAPM, which explains (with two firm specific variables, firm-size and book-to-market equity) 55.12 percent of the cross-sectional variation in average returns and the CCAPM, which captures nearly 30 percent without firm specific variables and 55.21 percent with one firm specific variable, human labour. As the main result of this paper, it turns out that the stable long-run market Beta contributes a substantial improvement in the empirical performance of the conventional CAPM *and* the SLCAPM still conforms to the original spirit of Markowitz

²A brief overview of the α -stable distribution is given in Section 2.2.

³Letting P_t denote the price of an asset at time t and assuming no dividends, the return over period $(t - 1, t]$ is typically modelled as $r_t = 100 \times \log(P_t/P_{t-1})$.

⁴This kind of modification is usually called a multifactor model in the literature.

(1952), namely the mean-variance rule.

The rest of the paper is organized as follows. In Section 2, we present empirical evidence for α -stable distributions in financial data and give a short summary of α -stable distributions. We also examine the existence of long-run relationships in empirical financial data and discuss the compatibility of the long-run relationships and market efficiency. In Section 3, we introduce the SLCAPM. Estimation of the short-run market Beta and the long-run market Beta as well as tests for the validity of the SLCAPM are discussed. Section 4 shows an empirical application of the SLCAPM. In Section 5, using the cross-sectional regression method by Fama and MacBeth (1973) a substantial improvement in the empirical performance of the conventional CAPM through the SLCAPM is presented. Section 6 summarizes the paper and contains some concluding remarks.

2 Empirical evidence

In this Section we examine the empirical properties of returns and asset prices and present evidence of fat-tails in returns and long-run relationships (common stochastic trends) in prices.

2.1 Fat-tails

It is now well-known that most financial data (high-frequency data) have thicker tails in their density than those of the normal density. For a demonstration of the phenomenon we take two data sets which we will use in our empirical analysis later. One of them is the daily returns of the DAX from June 19, 1989 to June 18, 1999, and the other monthly returns of the CRSP data. Figure 1 shows the daily stock prices, returns and the empirical density of the returns (solid line) compared with the standard normal density (dotted line). The return process shows some volatility clustering. In fact, the empirical density of the returns is excessively peaked around zero, and at the same time has thicker tails than those of the normal density. The conditional heteroscedasticity in Figure 1b results in the fat-tails in the unconditional density in Figure 1c. Figure 2 shows the same results as Figure 1. See, for more illustration and discussion on fat-tails in economic variables, Rachev, Kim and Mittnik (1998).

2.2 A short summary of α -stable Distributions

α -stable distributions, as a generalization of the normal distribution, are described by four parameters (α, b, δ, μ) with $\alpha \in (0, 2]$, $b \in [-1, 1]$, $\delta \in (0, \infty]$ and $\mu \in$

$(-\infty, \infty)$.⁵ The shape of the α -stable distribution is determined completely by the stability (or tail-thickness) parameter α , when $b = 0$. Skewness is governed by the skewness parameter, b , the symmetric case corresponding to $b = 0$. The scale and location parameter of the distributions are denoted by δ and μ . For $\alpha = 2$, the α -stable distribution reduces to the normal distribution with variance $2\delta^2$ and, for $\alpha = 1$ and $b = 0$, Cauchy distribution. For $\alpha < 2$ all moments of order α or higher are infinite and tails become thicker, i.e., the magnitude and frequency of outliers increase as α decreases. Thus, a stable distribution has no finite variance except $\alpha = 2$ (i.e., the normal distribution is the only member of the α -stable family with finite variance), and no finite mean when $\alpha \leq 1$. Closed-form expressions of α -stable distributions exist only for a few special cases.⁶ However, the logarithm of the characteristic function of the α -stable distribution can be written as

$$\ln \varphi(t) = \begin{cases} -\sigma^\alpha |t|^\alpha [1 - ib \operatorname{sign}(t) \tan \frac{\pi\alpha}{2}] + i\mu t, & \text{for } \alpha \neq 1, \\ -\sigma |t| [1 + ib \frac{\pi}{2} \operatorname{sign}(t) \ln |t|] + i\mu t, & \text{for } \alpha = 1. \end{cases}$$

All probability density functions of stable distributions are continuous (Gnedenko and Kolmogorov, 1954) and unimodal (Ibragimov and Linnik, 1970). Moreover, the support of all stable distributions is $(-\infty, \infty)$, except if $\alpha = 1$ and $b = \pm 1$, where the support is $(-\infty, 0)$ for $b = -1$ and $(0, \infty)$ for $b = 1$ (Feller, 1971). For more details on the α -stable distributions see Zolotarev (1986) and Samorodnitsky and Taqqu (1994). It is worth noting that a strong argument in favour of the α -stable distribution for financial modelling over any other fat-tailed distributions such as t -distribution is that only the α -stable distribution can serve as limiting distribution of sums of independent identically distributed random variables (Zolotarev, 1986).

The estimated stability parameter for the daily DAX in Figure 1 is 1.81, 1.84 and 1.80, and for the monthly CRSP in Figure 2 1.82, 1.87 and 1.88, according to the maximum-likelihood estimation (DuMouchel, 1973), the Hill estimation (Hill, 1975) and the quantile estimation (McCulloch, 1986), respectively.

2.3 Common stochastic trends

To check possible common trends in asset prices, we test for integration in each asset price, including DAX, and for co-integration in each univariate Beta model. For the unit roots test, Dickey/Fuller test (Dickey and Fuller, 1979) and KPSS test (Kwiatkowski, Phillips, Schmidt and Shin, 1992) are applied, where the Dickey/Fuller test assumes non-stationarity under the null, whereas the KPSS test stationarity. For co-integration test, we apply the usual t -test, i.e. t -test of the loading coefficient

⁵To avoid confusing with the market Beta, β , of the CAPM, b denotes the skewness parameter instead of the usually used β .

⁶The normal distribution ($\alpha = 2$), Cauchy distribution ($\alpha = 1$, $b = 0$) and Lévy distribution ($\alpha = 0.5$, $b = 1$) belong to the special cases.

for the co-integration residuals pre-estimated and the so-called t_{ECM} -test, considered in Banerjee, Dolado and Mestre (1998).

The first two columns of Table 1 show the result of tests for unit roots. The results of tests for co-integration are summarized in the last two columns. None of the 30 stock prices can be regarded as a stationary process by the two tests used.⁷ The two tests for co-integration show somewhat different results. About one-half of the individual series is co-integrated with DAX at 90% significant level by the t -test, while about two-thirds by the t_{ECM} -test. Note that in a dynamic setting the t_{ECM} -test is a more powerful test than the usual t -test (see, for more details on this topic, Kremers, Ericsson and Dolado, 1992). 7 assets are not co-integrated with DAX according to any of the tests. As the result shows, the empirical evidence for co-integration is, however, not very strong. A possible explanation is given: Shiller and Perron (1985) explore the power of unit root tests with respect to span and frequency of observation, and conclude that the test power depends more on the span of the data than on the number of observations. They also summarize that if the span is held fixed as the number of observations is increased, power tends towards the size of test. This means that the high frequency data with a short span (this is the case for our empirical analysis, namely 252 observations during *only* one year, which corresponds, for example, to 12 observations for monthly data) are not long enough to detect a long-run relationship.

2.4 The common stochastic trend and the efficient market hypothesis

The analysis of co-integration, introduced by Engle und Granger (1987) in financial data, is not new, and there are many empirical works which report the existence of co-integrating relationships in financial data; see, for example, Corhay, Rad and Urbain (1993). According to the co-integration theory, many economic time series are non-stationary (integrated), but a certain linear combination of them can be stationary, i.e., co-integrated. If more than two economic time series are co-integrated, one can use this stationary property of the relation among the time series and perform a forecasting exercise with a finite forecast error. I.e., the existence of co-integration can generally be interpreted as predictability, which is not compatible with the efficient market hypothesis (EMH) in the sense of Samuelson (1965) and Fama (1970). Samuelson (1965) argues that, in an informationally efficient market, price changes must be unpredictable if they fully incorporate the expectations and information of all market participants. Based on this idea, Fama (1970) develops the concept of market efficiency designated in the literature as EMH. The alleged inconsistency

⁷The result remains unchanged when using the critical values taking non-normality into account in Mittnik, Kim and Rachev (1997).

between co-integration and EMH has been critically debated by many authors in recent years. Three viewpoints on this issue are in order.

The first group of authors argues that the findings of co-integration in financial markets imply a violation of market efficiency. Baillie and Bollerslev (1989, 1994), for example, find (fractional) co-integration in nominal dollar spot exchange rates and MacDonald and Power (1993) in monthly prices for the shares of 40 companies in the UK. Using the monthly averaged stock-price indexes of nine major industrial countries, Masih and Masih (2001) find a causal transmission among the indexes in the context of error-correction models. In a slightly different setting, Bollerslev and Engle (1993) analyze co-persistence in the conditional variance between more than two return processes, indicating a direct predictability of the conditional variance. The authors in this group, therefore, believe in the predictability of financial data and ignore the EMH.

The second group of authors asserts the impossibility of co-integration in financial market theoretically and/or shows no co-integration empirically. Based on the incompatibility between the predictability of co-integration and the unpredictability of EMH, Granger (1992) and Diebold, Gardeazabel and Yilmaz (1994) deny the existence of co-integration relationships among the financial data. Granger (1992, p. 3) wrote that “... *then price changes would be consistently predictable, and so a money machine is created and indefinite wealth is possible*”. The argument of Granger is based on the logic that co-integration is a causal relationship which contains at least one exogenous variable and, hence, co-integration would necessarily imply predictability. Diebold et al. (1994) find a contradictory result in comparison with that of Baillie and Bollerslev (1989), who used the same data and found evidence of co-integration.⁸ Based on a vector autoregressive error-correction model, Barkoulas and Baum (1997) present empirical evidence of no co-integration of foreign exchange markets.

Note that the EMH must not only be defined by unpredictability, and that the existence of co-integration must not necessarily imply predictability, either. The third group of authors emphasizes, therefore, the consistency of the EMH and co-integration. Fama (1991) argues that the predictability of stock returns from dividend does not in itself yield evidence for or against market efficiency. Using the moving-average representation of co-integrated variables, Dwyer and Wallace (1992) demonstrate that co-integration in financial markets can be consistent with market efficiency, and argue that there is no general equivalence between market efficiency and co-integration, or a lack of co-integration. Engle (1996) also discusses predictability in an efficient market, and concludes that co-integration has nothing to do with EMH.

⁸In a later paper of Baillie and Bollerslev (1994), this contradiction is resolved through the introduction of fractional co-integration.

We likewise claim that co-integration is compatible with EMH. Co-integration does not necessarily enable us to predict variables in a co-integrated model when data-generating processes of asset prices are driven by fundamentals. Regarding predictability, Crowder (1994) emphasizes the role of exogeneity and causality in a co-integrating relationship. Caporale and Pittis (1998) correctly argue that (given one-directional causality and exogeneity, which is usually assumed for a co-integrating relationship) k variables can be predicted in an n -dimensional co-integrating system with k co-integrating relationships. When both the price of market portfolio P_t and the price of an individual asset p_t are driven by fundamentals which are not specified in a co-integrating relationship, the direction of causality between P_t and p_t cannot be observed in one direction, but maybe (detected by tests) partly in one direction and partly in the other. This is a very plausible scenario because each individual asset price is contained in the market price. Moreover, it is not easy in high-frequency data to distinguish cause from effect and vice versa. The causal structure in this case will then be detected by the usual tests, as if they have an instantaneous causality. We examine this kind of causality in the empirical data using Geweke's measure (Geweke, 1982) of causality⁹, which is also adopted in Bracker, Docking and Koch (1999), and the cross-correlation test of Hong (2001). The result of Geweke's measure of causality for the DAX is summarized in Table 2. The first column of Table 2 shows the causality of DAX to each individual asset. Few of them are significantly caused by the DAX. The second column shows the causality of the reverse direction. Few individual assets are causal to the DAX,

⁹This test is a likelihood-ratio test performing usually bivariate case, say x and y , assumes no causality under the null, and causality of one variable to the other under the alternative. The likelihood value, under the restriction σ_u^2 , come from the restricted model

$$x_t = a_0 + \sum_{i=1}^p a_i x_{t-i} + u_t,$$

and, under no restrictions, σ_e^2 results from the unrestricted model

$$x_t = a_0 + \sum_{i=1}^p a_i x_{t-i} + \sum_{j=1}^q b_j y_{t-j} + e_t.$$

The test statistic is formulated as the likelihood ratio of the restricted and unrestricted variance $LR = n \ln(\sigma_u^2/\sigma_e^2)$, which is asymptotically $\chi^2(q)$ -distributed. For a large value of the likelihood ratio, y is causal to x . If x and y in the equations above are rotated, one tests whether x is causal to y under the alternative hypothesis. In the framework of seemingly unrelated regression equations, one can also detect instantaneous causality based on the likelihood-ratio statistic. For this case, the likelihood value under restriction is the product of two σ_u^2 s resulting from two restricted models of x and y ; and the determinant of the residual covariance matrix of the seemingly unrelated regression equations under no restriction. This residual covariance matrix is usually estimated with the feasible generalized ordinary least squares method (Zellner, 1962), which reduces to ordinary least squares under null hypothesis.

either. The Granger-causality between DAX and its individual assets seems to be very weak. The third column shows instantaneous causality. All individual assets are instantaneously causal to the DAX. This can be seen as empirical evidence of the co-existence of common stochastic trends and market efficiency. For this case, co-integration does not imply the predictability of any variables in the system, although the forecasting of the stationary equilibrium error with a finite forecasting error is still possible. Stock and Watson (2001) examine empirical evidence of the forecasting ability of asset prices and conclude that *some asset prices predict ... in some countries in some periods. Which series predicts what, when and where is, however, itself difficult to predict.* Most empirical evidence, as summarized in Stock and Watson (2001), shows that *... a significant Granger causality statistic contains little or no information about whether the indicator has been a reliable predictor.* In line with that conclusion and the empirical evidence of Stock and Watson (2001), we conclude the co-existence of co-integration and MEH, because predictability does not necessarily mean *creating a money machine*, as Granger (1992) worries.

3 The stable long-run CAPM

3.1 The model

Let r denote the (excess) return on an asset and R be the return on the market portfolio of all assets in an economy. The Black version of the Sharpe/Lintner CAPM (Black, 1972) accounting for the absence of the risk-free asset is formulated¹⁰

$$E[r] = \gamma_0 + \gamma_1\beta, \quad (1)$$

where β is defined as

$$\beta = \text{Cov}[r, R]/\text{Var}[R]. \quad (2)$$

The fundamental, R , is usually specified as a benchmark portfolio for financial market analysis in practice. Based on the empirical evidence surveyed above, the conventional CAPM can be modified by the following assumptions.

Assumption 1 *The asset return r is symmetric α -stable ($S_\alpha S$) distributed with $\alpha \in (1, 2]$.*

The assumption 1 is a (stable) generalization of the traditional distributional assumption of normality on financial returns since Bachelier (1900). The restriction, $\alpha \in (1, 2]$, is needed because there exists no first moment when $\alpha \leq 1$. This restriction, however, does not describe a real restriction for empirical financial analysis, because the stability parameters of most financial returns lie between 1.5 and 1.9.

¹⁰For simplicity, the firm index is suppressed.

The restriction of symmetry ($b = 0$) is needed to avoid complications for estimations and tests of the ex-post SLCAPM. The following Lemma follows assumption 1 immediately.

Lemma 1 *The asset price, p_t , follows a Lévy motion with an increment between t and s $p(t) - p(s) \sim S(\alpha, 0, (t - s)^{1/\alpha}, 0)$ for $0 \leq s < t < \infty$.*

The Lévy motion is also a (stable) generalization of Brownian motion, whose increments are normally distributed. For more details on properties of Lévy motion, see Resnick (1986, p. 72).

Assumption 2 *Each pair of the two returns, r_t and R_t , is bivariate symmetric distributed.*

Assumption 2 implies that the two Lévy motions have the same α . This means that all assets in a benchmark, including the benchmark itself, must have the same stability parameter.¹¹ Assumption 2 ensures the linearity of the relationship between expected return and risk under assumption 1. See, for more details on the relation between symmetry and linearity, Theorem 4.1.2 in Samorodnitsky and Taqqu (1994, p. 175). The next assumption concerns the relationships between asset prices.

Assumption 3 *For each pair of the two Lévy motions, p_t and P_t , there exists a constant c , so that the linear combination*

$$p_t - cP_t \sim z_t,$$

where z_t is an α -stable distributed stationary process.

Assumption 3 means a stationary relationship between level variables, and, hence, implies the explanatory power of the correction of the disequilibrium in level for the expected return. Under assumption 3, equation 1 can be written as

$$E[r] = \gamma_0 + \gamma_1\beta^s + \gamma_2\beta^l, \tag{3}$$

where β^s and β^l are defined as

$$\beta^s = [r, R]_\alpha / \delta_R^\alpha \quad \text{and} \quad \beta^l = [p, P]_\alpha / \delta_P^\alpha \tag{4}$$

¹¹For a small sample, it is actually more or less restrictive. But there is no reason—either economically or statistically— why each asset has different α 's just as all assets have the same α . All assets in a benchmark are influenced by the same market events, so that ‘the same α ’ is more possible. On the other hand, one can also argue that the firm-specific influence is stronger than the general market trend, so that ‘the different α ’ is more possible. If two variables in the bivariate Beta model have different α s, the statistical dealing of the bivariate SLCAPM is much more complicated (but tractable).

respectively. Above, $[r, R]_\alpha$ ($[p, P]_\alpha$) is covariation of r and R (p and P), and δ_R (δ_P) is variation of R (P). Covariation (variation) is a stable generalization of the concept of covariance (variance) of the normally distributed random variables. See, for more details on covariation and variation, Samorodnitsky and Taqqu (1994, p. 87).

The ex-post version of the SLCAPM can also be written as an single-equation error-correction form, in which an error-correction term takes account of the price information, i.e. the long-run covariance risk.

$$r_t = \beta^s R_t + \bar{b}[p - \beta^l P]_{t-1}, \quad (5)$$

where \bar{b} is a loading coefficient for the so-called error-correction term of the long-run equilibrium. This form is an extension of the conventional CAPM, which is consistent with any arbitrary relation between the two prices in the framework of a univariate Beta model. In contrast to the conventional CAPM, the SLCAPM is consistent with a certain relation between the two prices, namely a long-run equilibrium relationship when it exists. According to the SLCAPM, the risk of an investment is measured by both the usual (short-run) market Beta and the long-run market Beta, and the relations between expected return and the two market Betas are linear.

The theoretical background for the stable CAPM was originated by Samuelson (1967), who shows the existence of mean-variance (variation) efficiency under the non-existence of second moment. This means that the Markowitz portfolio theory still works under Assumption 1, namely α -stable distributed returns. This provides the SLCAPM with an economic foundation which is still based on the Markowitz theory, namely the rule of expected returns and variance (variation) of returns. Similarly to the CCAPM of Jagannathan and Wang (1996), the SLCAPM is, therefore, a generalized form of the probability principle of mean-variance efficiency. This is a decidedly different aspect of the SLCAPM from some mostly used modified CAPMs, such as BMCAPM of French and Fama (1992, 1996), whose modification is performed by including firm-specific variables.

3.2 Econometric Estimation and Test of the SLCAPM

The econometric models for estimating the two unknown market Betas in (3) are given as follows:

$$r_t = \beta^s R_t + u_t^s, \quad (6)$$

$$p_t = \beta^l P_t + u_t^l, \quad (7)$$

where, in the framework of regression analysis, both $R_t(P_t)$ and $u_t^s(u_t^l)$ are typically assumed to be independently α -stably distributed.

For the estimation of the short- and long-run market Beta in (6) and (7), we apply the best unbiased (BU) estimator of Blattberg and Sargent (1971). The BU-estimators for β^s and β^l in (6) and (7) are given as

$$\begin{aligned} {}_{BU}\hat{\beta}^s &= \frac{\sum_{t=1}^T |R_t|^{1/(\alpha-1)} \text{sign}[R_t] r_t}{\sum_{t=1}^T |R_t|^{\alpha/(\alpha-1)}}, \\ {}_{BU}\hat{\beta}^l &= \frac{\sum_{t=1}^T |P_t|^{1/(\alpha-1)} \text{sign}[P_t] p_t}{\sum_{t=1}^T |P_t|^{\alpha/(\alpha-1)}}, \end{aligned}$$

respectively, for $\alpha \in (1, 2]$, which is equivalent to ordinary least squares (OLS), when $\alpha = 2$. The asymptotic distribution of the BU-estimator is again a function of α -stable random variables. See, for more details on the asymptotic analysis of the BU-estimator, Kim and Rachev (1998). For the empirical application of the BU-estimator, the tail-thickness parameter α must be known. When α is unknown, some estimates of α are usually substituted for the true α .

Alternatively, equation 7 can be estimated by the so-called fully-modified (FM) OLS method introduced by Phillips (1991).

$${}_{FM}\hat{\beta}^l = \left(\sum_{t=1}^T P_t P_t' \right)^{-1} \left(\sum_{t=1}^T p_t^+ P_t' - \hat{\Lambda}_{21}^+ \right),$$

with

$$p_t^+ = p_t - \hat{\Omega}_{12} \hat{\Omega}_{22}^{-1} R_t, \quad \hat{\Lambda}_{21}^+ = \hat{\Lambda}_{21} - \hat{\Lambda}_{22} \hat{\Omega}_{22}^{-1} \hat{\Omega}_{21},$$

$$\begin{aligned} \hat{\Omega} &= T^{-1} \left[\sum_{t=1}^T \hat{\eta}_t \hat{\eta}_t' + \sum_{s=1}^l w(l, s) \sum_{t=s+1}^T (\hat{\eta}_{t-s} \hat{\eta}_t' + \hat{\eta}_t \hat{\eta}_{t-s}') \right] = \begin{bmatrix} \hat{\Omega}_{11} & \hat{\Omega}_{12} \\ \hat{\Omega}_{21} & \hat{\Omega}_{22} \end{bmatrix}, \\ \hat{\Lambda} &= T^{-1} \left[\sum_{t=1}^T \hat{\eta}_t \hat{\eta}_t' + \sum_{s=1}^l w(l, s) \sum_{t=s+1}^T \hat{\eta}_{t-s} \hat{\eta}_t' \right] = \begin{bmatrix} \hat{\Lambda}_{11} & \hat{\Lambda}_{12} \\ \hat{\Lambda}_{21} & \hat{\Lambda}_{22} \end{bmatrix}. \end{aligned}$$

Above, $\hat{\eta}_t = [{}_{BU}\hat{u}_t^l R_t]$ and $w(\cdot)$ is a weight function, which yields positive semi-definite estimates, where l is a lag-truncation parameter. The FM-estimator is designed originally for normally distributed (co-)integrated variables, but the semi-parametric correction of endogeneity of the exogeneous variable,¹² and serial correlation in the residuals still works for errors with infinite-variance.

Testing the null hypothesis of the conventional CAPM against the alternative of the SLCAPM is of interest, namely:

$$H_0 : CAPM \quad vs \quad H_1 : SLCAPM.$$

¹²Actually, the assumption of independence between $R_t(P_t)$ and $u_t^s(u_t^l)$ barely holds in the ex-post version of CAPM because of the construction of a benchmark portfolio. Note that, by its construction, a benchmark portfolio contains the residual process, including the endogenous variable, in the framework of the univariate Beta model.

Equivalently, an appealing test of whether the data are consistent with the SLCAPM can be performed simply by the usual t -test (called t_{ECM} in Kremers, Ericsson and Dolado, 1992) on the loading coefficient in (5) as:

$$H_0 : \bar{b} = 0 \quad vs \quad H_1 : \bar{b} < 0.$$

Kim (2000) provides finite-sample distributions for the t -statistic.

4 An empirical application

For our empirical work, we use the daily DAX and its 30 assets from the Karlsruher Kapitalmarktdatenbank for the period June 19, 1998 to June 18, 1999 (252 observations), where the composition of the DAX was not changed.

A pre-test needed for estimating a SLCAPM is the checking symmetry (Assumption 2) of each pair in the bivariate SLCAPM, which ensures the linearity of the regressions in (6) and (7). This can be done by the test of bivariate symmetry of Heathecote, Rachev and Cheng (1995), as implemented in Kim (1999).¹³ The results of the test of bivariate symmetry are reported in Table 3. The null hypothesis

¹³The test procedure is based on the tail estimators of the spectral measure. It is assumed that $X_t = [X_{1t}, X_{2t}]$, $t = 1, \dots, n$, are two-dimensional, mutually independent α -stable random vectors ($0 < \alpha < 2$).

- i. For every pair of the observation $[x_{1t}, x_{2t}]$ of X_t , a corresponding polar coordinate $\rho_t := \sqrt{x_{1t}^2 + x_{2t}^2}$ and an inverse tangent $\tilde{\theta}_t := \arctan(x_{1t}/x_{2t})$ are calculated.
- ii. Let k be a sequence of integers satisfying $1 \leq k \leq n/2$. Derive the estimator for the normalized spectral measure, $\phi_n(\theta)$, by

$$\phi_n(\theta) = \frac{1}{k} \sum_{t=1}^n \mathbb{I}_{\{\theta_t \leq \theta, \rho_t \geq \rho_{n-k+1:n}\}}, \quad \theta \in (0, 2\pi],$$

where \mathbb{I}_{Ω} is the usual indicator function and $\rho_{i:n}$ denotes the i -th order statistic. Above, parameter θ_t is defined as

$$\theta_t = \begin{cases} \tilde{\theta}_t, & \text{for } x_{1t}, x_{2t} \geq 0, \\ \pi - \tilde{\theta}_t, & \text{for } x_{1t} < 0, x_{2t} \geq 0, \\ \pi + \tilde{\theta}_t, & \text{for } x_{1t}, x_{2t} < 0, \\ 2\pi - \tilde{\theta}_t, & \text{for } x_{1t} \geq 0, x_{2t} < 0. \end{cases}$$

In practice, one may take grid $(\theta_1, \dots, \theta_d)$, $\theta_1 = 2\pi/d$, $\theta_d = 2\pi$, where d is the number of grid points and $2\pi/d$ the step width.

- iii. Under some regularity conditions, one can use the sample supremum of $\phi(\theta)$ in region $0 < \theta \leq \pi$, namely

$$\Phi_n := \sup_{0 < \theta \leq \pi} \sqrt{k} \frac{|\phi_n(\theta) - \phi_n(\theta + \pi) + \phi_n(\pi)|}{\sqrt{2\phi_n(\theta)}},$$

as the test statistic.

of bivariate symmetry cannot be rejected at 5% significance level for any pair of the returns. The result shows that evidence of bivariate symmetry is strong enough for the univariate Beta, which is assumed to be a linear relation between the returns of individual assets and the benchmark portfolio in the SLCAPM.

The estimates of the short- and long-run Betas are reported in Table 4. The first column of Table 4 shows the average returns and the second the Beta-coefficient in the conventional CAPM in (2), while the third and the fourth show the estimated short- and long-run market Beta for the SLCAPM in (4). The differences between the two estimates of the Beta in the conventional CAPM and of the short-run Beta in the SLCAPM are moderate for most regressions. The long-run Beta-coefficient show how large the normalized covariance between an asset price and the price of the market portfolio may be. The high positive β^l (greater than one) means an over-proportional development of the price level of an asset in comparison with the market price. The low β^l means that the price of an asset develops under-proportionally in comparison with the market price, or partly in an opposite direction to the market price in the *long run*. If one takes information on the Beta-coefficient from the conventional CAPM (in the second column of Table 4), Bayer (0.71) has almost the same Beta as Preussag (0.72). But Bayer has lost in this period from 3.83 to 3.70, like the DAX, while Preussag has gained from 3.53 to 3.93 in the same period. Bayer has a large long-run Beta (1.17), but Preussag a small one (0.62). What is discussed above is summarized in Figure 3. Figure 3 shows three asset prices, compared with the DAX, of which the first one (SAP) has a high β^l (1.55), the second one (Hoechst) a moderate β^l (0.97), and the last one (Preussag) has a low β^l (0.62).¹⁴ In the period considered, the DAX lost from 8.65 to 8.60 points (i.e. $\bar{R}_i = -0.05$). The SAP which shows the highest positive correlation with the DAX in the long run, namely 1.55, lost over-proportionally from 6.43 to 5.99 ($\bar{r}_i = -0.44$). This is also the case for the Degussa. The Hoechst, which has a long-run Beta of about one (0.97) lost almost proportionally to the DAX from 3.78 to 3.77 ($\bar{r}_i = -0.01$). The Preussag, however, whose stable long-run Beta is small (0.62), gained from 3.53 to 3.93 ($\bar{r}_i = 0.40$). This is also the case for the Deutsche Telekom.

Decisions on an investment based on the SLCAPM look similar to those based on the conventional CAPM. One advantage is that one can use —besides the usual market Beta— stable long-run information additionally. To sum up, the decision rule based on the SLCAPM can be summarized as follow: for a short-run investment, the conventional rule is still valid, namely, when $\beta^s = 1.0$, the return tends to mirror

From the functional limit theorem for ϕ_n , one can easily verify that $\hat{\Phi}_n$ follows asymptotically a standard normal distribution. Assuming sufficiently large values of n and k , we reject the null hypothesis of bivariate symmetry at significance level γ , when $\Phi_n > z_{\gamma/2}$, where $z_{\gamma/2}$ is the $100(1 - \gamma/2)$ percentile of the standard normal distribution.

¹⁴For a better comparison, a level-shift for the illustration is performed.

the return on the market; when $\beta^s > (<)1.0$, the return tends to be greater (smaller) than the return on the market. For a long-run investment, however, the long-run risk factor plays an important role: when $\beta^l > 1(< 1)$, the return tends to be positive (negative) for the case of increasing market price, and *vice versa*. Actually, one can obtain information on both the risk factors from the SLCAPM.

5 Empirical Performance

In this Section, using the cross-sectional regression of Fama and MacBeth (1973), we perform a test to check the empirical performance of the SLCAPM. For doing this, we use the same data used in Jagannathan and Wang (1996), i.e., the stock data of nonfinancial firms listed on the New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX) covered by the Center for Research in Security Prices (CRSP) alone. Jagannathan and Wang (1996) create 100 portfolios of NYSE and AMEX stocks, as in Fama and French (1992), which we also use.¹⁵ This is a time series of monthly returns for the period July 1963 to December 1990 (330 observations). Using the same data ensures a powerful comparison among the models.

In the last few decades, many works have reported the lack of empirical support for the conventional CAPM and, at the same time, many modifications have been studied to improve the empirical performance of the CAPM. Fama and French (1992) empirically tested the following regression model, considered in Fama and MacBeth (1973)

$$E[r_i] = \gamma_0 + \gamma_1\beta_i, \quad (8)$$

where $E[r_i]$ is the expected return on the asset i , β_i the corresponding market Beta and γ_0 and γ_1 denote the expected return on a zero-beta and the market-risk premium, respectively. In empirical tests $E[r_i]$ is usually replaced by the mean value of the observed returns. In a widely-cited paper of Fama and French (1992), the poor relationship between market Beta and average return is reported. They show that the γ_1 is insignificantly different from zero and the correlation coefficient, R^2 , is very low (1.35%). That is to say, the relation between market Beta and average return is *flat*, as Fama and French (1992) conclude. Jagannathan and Wang (1996) extend the (unconditional) conventional CAPM to a conditional CAPM, relaxing the assumption of constant Beta, and show, using the same data in Fama and French (1992), that the R^2 can be substantially improved to 29.32%.

¹⁵See Jagannathan and Wang (1996) for details on creating the data set. I thank Prof. Wang for kindly sending the data to me. They can now be obtained via the Internet, <http://www.gsb.columbia.edu/faculty/zwang/exchange>.

We now empirically test the SLCAPM as follows:

$$E[r_i] = \gamma_0 + \gamma_1\beta_i^s + \gamma_2\beta_i^l, \quad (9)$$

where γ_2 is the long-run risk premium and β_i^s and β_i^l are the short- and long-run Beta for i -th asset. The results of bivariate symmetry tests and the estimated short- and long-run Betas are given in Tables 5 and 6, respectively, where, in the first block of Table 6, the average returns are reported. For estimating the short-run Betas, the BU-estimation and the long-run Betas, both the BU- and FM-estimations are applied. Next, based on the cross-sectional regression, we test the empirical performance of the SLCAPM. This can be done by regressing the estimated short- and long-run market Betas on the average returns as estimates of the unobservable expected returns given in the first block of Table 6. The cross-sectional regression for the SLCAPM is then given as

$$\bar{r} = \gamma_0 + \gamma_1\hat{\beta}^s + \gamma_2\hat{\beta}^l + \epsilon,$$

where $\bar{r} = [\bar{r}_1, \dots, \bar{r}_N]$, with $\bar{r}_i = T^{-1} \sum_{t=1}^T r_{it}$, $\hat{\beta}^s = [\hat{\beta}_1^s, \dots, \hat{\beta}_N^s]$ and $\hat{\beta}^l = [\hat{\beta}_1^l, \dots, \hat{\beta}_N^l]$. For our case, $N = 100$ and $T = 330$.

Figure 4 illustrates a set of sequential correlation coefficients, i.e., the sample size is reduced recursively from 330 (July 1963 – December 1990) to 131 (February 1980 – December 1990) and the corresponding correlation coefficient is obtained sequentially for each time length. Figure 4 shows the following:

- The correlation coefficient of the SLCAPM for the full sample is 60.90 %, whereas that of the conventional CAPM is 1.35 %, as reported in Jagannathan and Wang (1996).
- The average correlation coefficient for the SLCAPM is 63.25 %, with a standard deviation of 7.99 %, whereas that of the conventional CAPM is 17.04 % with a standard deviation of 10.36 %. The highest (lowest) correlation coefficient for the SLCAPM is 82.53 % (46.30 %), whereas that of the conventional CAPM are 41.20 % (1.18 %).
- One comment on the empirical result in Fama and French (1992): the empirical performance of the conventional CAPM is surely not persuasive, but it should be noticed that the empirical result in Fama and French (1992) is the second worst (lowest) correlation coefficient from the entire 200 sequential samples.

The result from the sequential correlation coefficient of the SLCAPM seems to be promising for applying the SLCAPM to empirical work. Since the SLCAPM is based on the economic foundation, like the Sharpe/Lintner-CAPM without any firm specific variable, the result also means that the mean-variance efficiency is still

valid as an economic hypothesis, describing the behaviour of investors in financial markets. Lastly, we compare the empirical performance of the SLCAPM with that of the recently published two modified CAPMs. The first one is the BMCAPM by Fama (1992, 1996), which contains two firm-specific variables, firm-size and book-to-market equity. The other is the CCAPM by Jagannathan and Wang (1996). Table 7 shows the comparison. The first row of Table 7 shows the result for the conventional CAPM, as also given in Fama and French (1992) and Jagannathan and Wang (1996). The coefficient for the (usual) market Beta is insignificant, and R^2 is very low (1.35%). That is to say, the conventional CAPM can be rejected for the underlying data. The BMCAPM explains —with two firm-specific variables, firm-size and book-to-market equity— 55.12% of the cross-sectional variation in average returns and the CCAPM, which captures nearly 30% without firm-specific variables and 55.21% with one firm-specific variable, human labour. The SLCAPM improves R^2 slightly more than the two, namely to 60.90%, based on the BU-estimation, and 68.17%, based on the FM-estimation, and all three coefficients are significant for the FM-estimates.

6 Conclusion

In this paper we examined a possible modification of the conventional CAPM. Two restrictive assumptions of the conventional CAPM, under which the relation between average return and Beta is very weak, namely normality of returns and single-period static property, are generalized to the α -stable Paretianity of returns and the long-run (dynamic) setting. With an empirical application, we demonstrated that the average returns are highly correlated with the long-run information that can be captured by the SLCAPM. Furthermore, it turned out that the SLCAPM improves the empirical performance of the CAPM, and still corresponds to the Markowitz portfolio theory, mean-variance rule, and, hence, demonstrates that the CAPM is alive and well.¹⁶

More recently, Lettau and Ludvigson (1999) empirically test a conditional version of the consumption CAPM with consumption-wealth ratio as a conditioning variable and find a substantial improvement in the performance of the CAPM.¹⁷ It would also be interesting to consider for future research —in line with the work of Jagannathan and Wang (1996)— a conditional stable long-run CAPM, taking the time-varying property of the market Beta into account.

¹⁶This is the title of an earlier version of the paper of Jagannathan and Wang (1996), as a light-hearted answer to the title of a paper by Fama and French (1996), “The CAPM is wanted, dead or alive”.

¹⁷With a different data set, the conditional version of the consumption CAPM presents a high R^2 of 71%.

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Table 1: Test for integration and co-integration

Data	Test	DF-Test ^a	KPSS ^b	<i>t</i> -test ^c	<i>t</i> _{ECM} -Test ^d
Adidas-Salomon		-2.74*	2.51	-2.64***	-2,25
Allianz		-2.33	0.41*	-1.60	-5,06***
BASF		-1.31	1.38	-1.33	-2,96*
Bayer		-1.93	1.31	-2.46***	-4,47***
BMW		-2.04	1.39	-1.96**	-6,68***
Commerzbank		-2.12	1.28	-2.19**	-12,10***
DaimlerChrysler		-1.73	0.76	-0.88	-5,93***
Degussa		-1.84	2.73	-1.95**	-1,87
Dresdner Bank		-2.05	1.83	-1.12	-4,20***
Deutsche Bank		-2.06	2.41	-1.83*	-3,58**
Deutsche Telekom		-0.84	4.92	-1.09	-0,68
Henkel		-3.15**	1.39	-2.90***	-7,29***
Hoechst		-2.13	1.63	-2.41**	-5,37***
HypoVereinsbank		-1.91	6.15	-1.20	-1,14
Karstadt		-2.33	1.42	-2.18**	-3,65**
Linde		-1.94	1.37	-2.68***	-7,64***
Lufthansa		-1.83	5.37	-1.25	-6,95***
MAN		-1.89	1.83	-2.11**	-2,95*
Mannesmann		-0.77	2.98	-1.05	-0,83
Metro		-1.56	3.24	-1.57	-2,92*
Münchener Rück		-2.31	1.72	-1.22	-2,56
Preussag		-0.85	3.93	-1.14	-0,73
RWE		-3.04**	4.17	-3.01***	-5,85***
SAP		-1.83	3.11	-1.15	-0,78
Schering		-1.71	4.31	-1.66*	-4,07***
Siemens		-1.43	1.55	-1.22	-3,28**
Thyssen		-1.84	1.12	-1.53	-8,09***
VEBA		-2.61*	0.82	-3.31***	-10,87***
VIAG		-1.37	4.43	-0.64	-0,71
Volkswagen		-2.11	1.85	-1.59	-3,68**

*, ** and *** significant at the 0.10, 0.05 and 0.01, respectively. ^aCritical values for DF-Test: -2.57 at 90%; -2.88 at 95%; -3.46 at 99%, see Fuller (1976). ^bCritical values for KPSS-Test: 0.38 at 90%; 0.46 at 95%; 0.74 at 99%, see Kwiatkowski et al. (1992). ^cOne-sided standard *t*-test. ^dCritical values for the *t*_{ECM}-Test: -2,92 at 90%, -3,25 bei 95%, -3,90 bei 99%, siehe Banerjee, Dolado und Mestre (1998).

Table 2: Test for causality

Causality Data	$R \rightarrow r_i$	$r_i \rightarrow R$	$R \leftrightarrow r_i$
Adidas-Salomon	1.26	0.40	57.46***
Allianz	9.42***	2.99*	279.18***
BASF	0.01	2.55	147.35***
Bayer	1.66	0.85	124.43***
BMW	0.01	0.04	141.76***
Commerzbank	0.11	0.74	224.73***
DaimlerChrysler	1.18	0.33	260.63***
Degussa	1.93	0.01	27.95***
Dresdner Bank	0.00	0.85	172.00***
Deutsche Bank	1.06	0.48	186.14***
Deutsche Telekom	1.63	0.31	153.93***
Henkel	0.15	0.91	91.84***
Hoechst	0.00	0.27	110.84***
HypoVereinsbank	1.25	0.89	100.05***
Karstadt	0.54	2.37	61.39***
Linde	0.23	0.42	67.31***
Lufthansa	4.81**	5.56**	155.25***
MAN	0.14	0.10	77.35***
Mannesmann	0.99	0.00	162.71***
Metro	2.02	0.09	79.77***
Münchener Rück	7.69***	2.22	215.95***
Preussag	0.12	0.56	77.28***
RWE	0.00	0.02	58.27***
SAP	2.71*	0.75	128.80***
Schering	0.01	0.85	91.46***
Siemens	0.87	1.42	119.76***
Thyssen	8.06***	0.40	105.36***
VEBA	0.20	0.02	94.30***
VIAG	2.15	0.04	109.66***
Volkswagen	2.64	0.00	224.95***

^aCritical values for the likelihood ratio test: 2.71 at 90%; 3.84 at 95%; 6.63 at 99%.

Table 3: Test of bivariate symmetry^a

Statistic	$\hat{\Phi}_n$	Significance level
Data		
Adidas-Salomon	1.11	0.27
Allianz	0.71	0.48
BASF	1.22	0.22
Bayer	1.06	0.29
BMW	1.00	0.32
Commerzbank	1.41	0.16
DaimlerChrysler	1.63	0.10
Degussa	1.42	0.15
Dresdner Bank	0.50	0.62
Deutsche Bank	0.71	0.48
Deutsche Telekom	0.80	0.42
Henkel	1.49	0.14
Hoechst	1.28	0.20
HypoVereinsbank	0.71	0.48
Karstadt	1.58	0.11
Linde	1.18	0.24
Lufthansa	1.07	0.29
MAN	1.54	0.12
Mannesmann	0.75	0.45
Metro	1.22	0.22
Münchener Rück	0.58	0.56
Preussag	1.37	0.17
RWE	1.22	0.22
SAP	0.82	0.41
Schering	1.70	0.09
Siemens	0.71	0.48
Thyssen	1.67	0.10
VEBA	0.51	0.61
VIAG	1.58	0.11
Volkswagen	0.55	0.58

^aFor the estimation $k = 50$ and $d = 20$.

Table 4: Estimates of short- and long-run Beta

Estimates Data	\bar{r}_i	$\hat{\beta}$	$BU\hat{\beta}^s$	$BU\hat{\beta}^l$
Adidas-Salomon	-0.52	0.69	0.70	1.40
Allianz	-0.04	1.15	1.17	0.76
BASF	0.01	0.74	0.74	1.16
Bayer	-0.12	0.71	0.69	1.17
BMW	-0.31	1.11	1.11	1.69
Commerzbank	-0.18	0.99	0.99	1.26
DaimlerChrysler	0.01	1.09	1.11	1.03
Degussa	-0.42	0.49	0.50	1.42
Dresdner Bank	-0.32	1.21	1.18	1.58
Deutsche Bank	-0.32	1.06	1.05	1.53
Deutsche Telekom	0.58	1.11	1.10	0.30
Henkel	-0.27	0.86	0.85	1.01
Hoechst	-0.01	0.90	0.87	0.97
HypoVereinsbank	-0.23	1.06	1.07	0.84
Karstadt	-0.04	0.66	0.66	0.47
Linde	-0.12	0.67	0.67	1.20
Lufthansa	-0.27	1.02	1.03	1.35
MAN	-0.11	0.80	0.78	1.18
Mannesmann	0.58	1.18	1.18	0.85
Metro	0.06	0.64	0.64	0.08
Münchener Rück	-0.15	1.13	1.13	0.70
Preussag	0.40	0.72	0.72	0.62
RWE	-0.27	0.66	0.64	0.57
SAP	-0.44	1.26	1.26	1.55
Schering	0.02	0.58	0.58	0.42
Siemens	0.26	0.89	0.88	0.87
Thyssen	-0.14	0.81	0.79	1.57
VEBA	-0.13	0.74	0.75	0.89
VIAG	-0.32	0.79	0.81	0.44
Volkswagen	-0.27	1.19	1.20	1.37

Table 5: Test for bivariate symmetry

1.57 (0.12)	1.57 (0.12)	1.71 (0.09)	1.92 (0.06)	1.92 (0.06)	1.71 (0.09)	1.57 (0.12)	1.89 (0.06)	1.92 (0.06)	1.71 (0.09)
1.37 (0.17)	1.63 (0.10)	1.92 (0.06)	1.60 (0.11)	1.71 (0.09)	1.77 (0.08)	1.25 (0.21)	2.04 (0.04)	1.46 (0.14)	1.42 (0.15)
1.07 (0.29)	1.64 (0.10)	1.07 (0.29)	1.12 (0.26)	1.18 (0.24)	0.71 (0.48)	1.70 (0.09)	1.15 (0.25)	1.11 (0.27)	2.00 (0.05**)
0.97 (0.33)	1.50 (0.13)	1.25 (0.21)	1.28 (0.20)	1.50 (0.13)	1.08 (0.28)	1.28 (0.20)	1.15 (0.25)	1.65 (0.10)	1.65 (0.10)
0.67 (0.50)	1.24 (0.22)	1.60 (0.11)	1.34 (0.18)	1.26 (0.21)	0.75 (0.45)	1.50 (0.13)	1.08 (0.28)	2.45 (0.01***)	1.26 (0.21)
0.71 (0.48)	1.18 (0.24)	1.22 (0.22)	1.13 (0.26)	1.44 (0.15)	1.57 (0.12)	0.87 (0.39)	1.34 (0.18)	1.65 (0.10)	1.08 (0.28)
1.43 (0.15)	1.12 (0.26)	1.15 (0.25)	1.59 (0.11)	0.95 (0.34)	0.71 (0.48)	1.08 (0.28)	1.50 (0.13)	0.71 (0.48)	1.73 (0.08)
1.30 (0.19)	0.71 (0.48)	1.73 (0.08)	1.73 (0.08)	1.50 (0.13)	1.20 (0.23)	1.15 (0.25)	1.34 (0.18)	2.24 (0.03**)	1.26 (0.21)
2.04 (0.04)	1.00 (0.32)	1.25 (0.21)	1.07 (0.29)	0.71 (0.48)	0.41 (0.68)	0.75 (0.45)	0.95 (0.34)	1.50 (0.13)	1.78 (0.07)
1.18 (0.24)	1.50 (0.13)	1.15 (0.25)	1.07 (0.29)	1.26 (0.21)	0.87 (0.39)	0.44 (0.66)	1.78 (0.07)	0.44 (0.66)	0.95 (0.34)

^a*p*-values are in parentheses.

Table 6: Average returns and estimated market-betas^a

Average returns									
1.44	1.53	1.56	1.71	1.36	1.44	1.37	1.33	1.46	1.34
1.13	1.22	1.09	1.19	1.38	1.37	1.37	1.30	1.15	0.95
1.26	1.27	1.22	1.26	1.16	1.29	1.34	1.19	1.12	0.89
1.37	1.47	1.40	1.28	1.01	1.39	1.11	1.33	1.07	0.95
0.97	1.53	1.10	1.28	1.18	1.04	1.35	1.07	1.23	0.82
1.07	1.36	1.34	1.12	1.25	1.27	0.84	0.94	0.92	0.77
0.99	1.18	1.13	1.19	0.96	0.99	1.11	0.91	0.90	0.83
0.95	1.19	1.02	1.39	1.18	1.24	0.94	1.02	0.88	1.08
0.94	0.92	1.05	1.17	1.15	1.03	1.02	0.84	0.80	0.51
1.06	0.97	1.02	0.94	0.83	0.93	0.82	0.83	0.61	0.72
$_{BU}\hat{\beta}^s$									
0.79	0.86	0.86	0.95	1.00	1.05	1.06	1.13	1.30	1.38
0.76	0.89	1.00	1.03	1.09	1.18	1.25	1.23	1.33	1.47
0.74	0.91	0.99	1.03	1.07	1.15	1.20	1.34	1.33	1.65
0.69	0.85	0.98	1.07	1.11	1.24	1.08	1.22	1.48	1.50
0.53	0.76	1.03	1.06	1.06	1.13	1.17	1.34	1.35	1.43
0.61	0.75	0.86	0.94	1.02	1.16	1.13	1.28	1.23	1.54
0.65	0.82	0.96	1.01	1.10	1.16	1.25	1.17	1.27	1.50
0.71	0.75	0.88	1.05	1.02	1.14	1.18	1.20	1.19	1.42
0.65	0.77	0.90	0.94	1.03	0.99	1.07	1.10	1.16	1.30
0.73	0.82	0.86	1.05	1.01	1.04	1.07	1.12	1.09	1.24
$_{BU}\hat{\beta}^l$									
0,94	1,03	1,05	1,15	1,19	1,24	1,22	1,31	1,45	1,54
0,86	1,02	1,11	1,14	1,19	1,29	1,33	1,39	1,46	1,61
0,81	0,95	1,11	1,11	1,19	1,27	1,29	1,39	1,41	1,67
0,78	0,94	1,07	1,15	1,20	1,33	1,24	1,32	1,53	1,58
0,59	0,82	1,10	1,11	1,13	1,20	1,27	1,42	1,44	1,51
0,64	0,79	0,90	1,02	1,09	1,26	1,20	1,33	1,31	1,54
0,66	0,86	1,02	1,08	1,15	1,21	1,26	1,25	1,29	1,53
0,65	0,75	0,92	1,06	1,07	1,17	1,21	1,19	1,22	1,48
0,64	0,79	0,89	0,98	1,04	1,05	1,13	1,16	1,20	1,31
0,70	0,78	0,81	1,00	0,97	1,01	1,04	1,08	1,07	1,26
$_{FM}\hat{\beta}^l$									
1,04	1,15	1,19	1,32	1,33	1,37	1,38	1,48	1,59	1,70
0,92	1,11	1,19	1,21	1,26	1,38	1,39	1,49	1,57	1,69
0,84	1,01	1,18	1,21	1,24	1,35	1,36	1,44	1,47	1,71
0,83	1,00	1,12	1,20	1,24	1,37	1,34	1,38	1,61	1,65
0,60	0,86	1,14	1,14	1,17	1,22	1,32	1,47	1,51	1,55
0,63	0,80	0,94	1,07	1,14	1,32	1,25	1,35	1,36	1,58
0,65	0,87	1,03	1,11	1,19	1,25	1,26	1,29	1,32	1,55
0,60	0,73	0,92	1,08	1,10	1,19	1,23	1,18	1,25	1,52
0,60	0,78	0,86	0,97	1,08	1,07	1,13	1,18	1,21	1,31
0,67	0,75	0,80	0,99	0,95	0,98	1,01	1,06	1,07	1,27

^aSee Jagannathan and Wang (1996, p. 20) for the estimates of the (usual) market Beta.

Table 7: Empirical performance of the stable long-run CAPM_a

Coefficient Model	γ_0	γ_{short}	γ_{long}	γ_{prem}	γ_{labor}	γ_{size}	$\gamma_{B/M}$	R^2
CAPM	1,24 (5,17)	-0,10 (-0,28)	–	–	–	–	–	1.35%
CCAPM	0.81 (2.72)	-0.31 (-0.87)	–	0.36 (3.28)	–	–	–	29.32%
	1.24 (5.51)	-0.40 (-1.18)	–	0.34 (3.31)	0.22 (2.31)	–	–	55.21%
Fama/French	1.39 (6.07)	-0.45 (-0.95)	–	–	–	0.33 (1.53)	0.25 (0.96)	55.12%
SLCAPM ^b	0.27 (0.64)	-0.01 (-0.02)	0.57 (3.61)	–	–	–	–	60.90%
	1.21 (4.94)	-3.72 (-2.92)	3.50 (2.61)	–	–	–	–	68.17%

^a*t*-values are reported in parentheses. ^bThe upper row presents results from the BU-estimation, the lower row, results from the FM-estimation.

Figure 1: Daily Prices, Returns and their empirical density of DAX

Figure 1a. Log of daily DAX

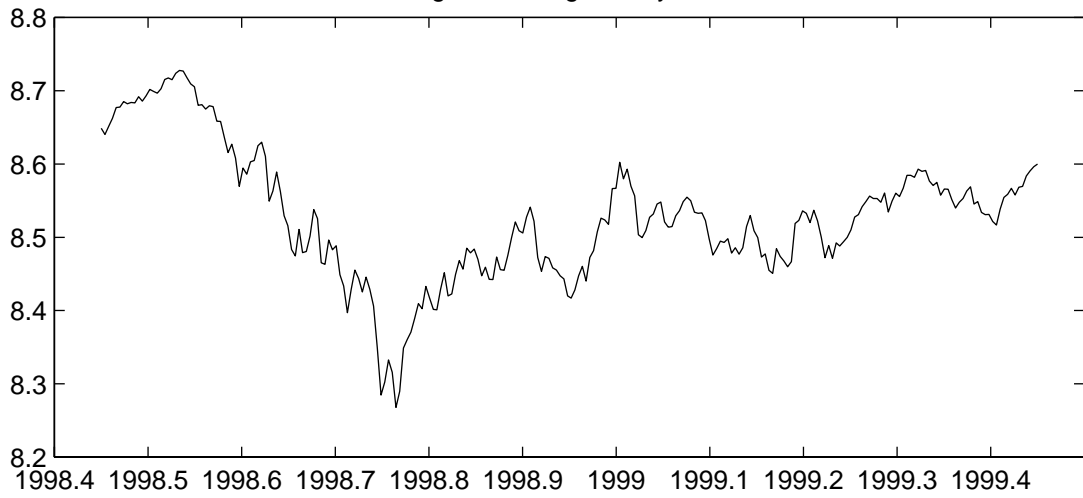


Figure 1b. Daily returns of DAX

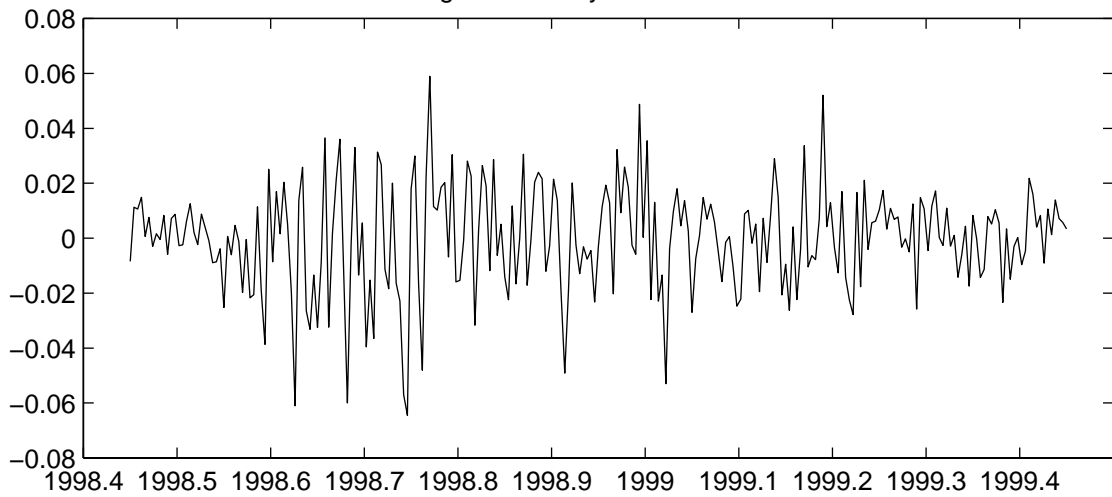


Figure 1c. Empirical density of the daily returns of DAX

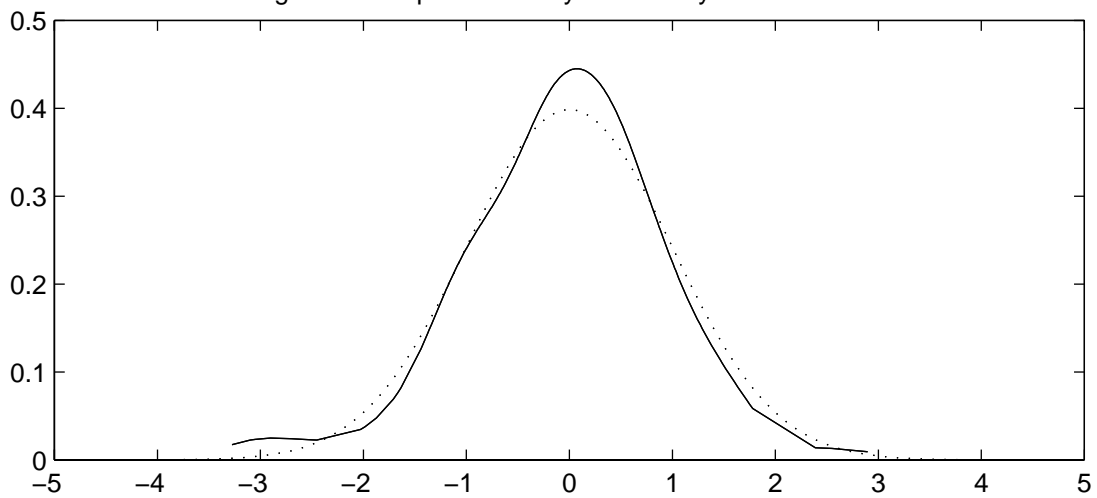


Figure 2: Monthly Returns and their empirical density of CRSP

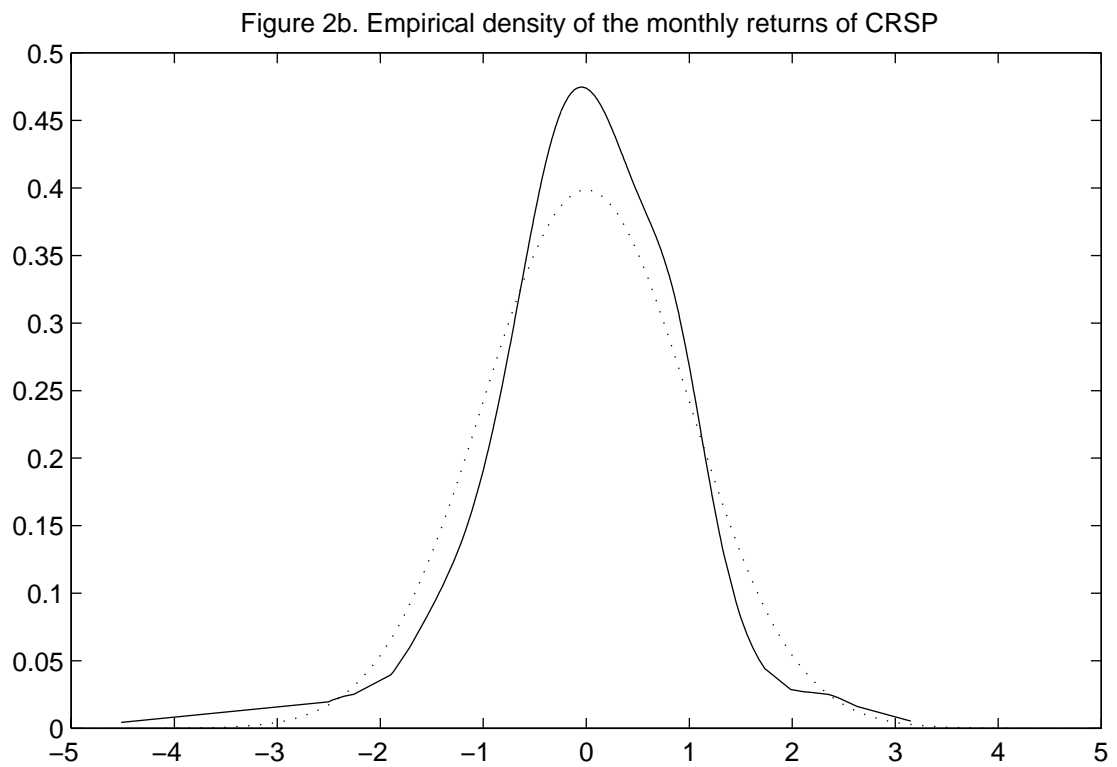
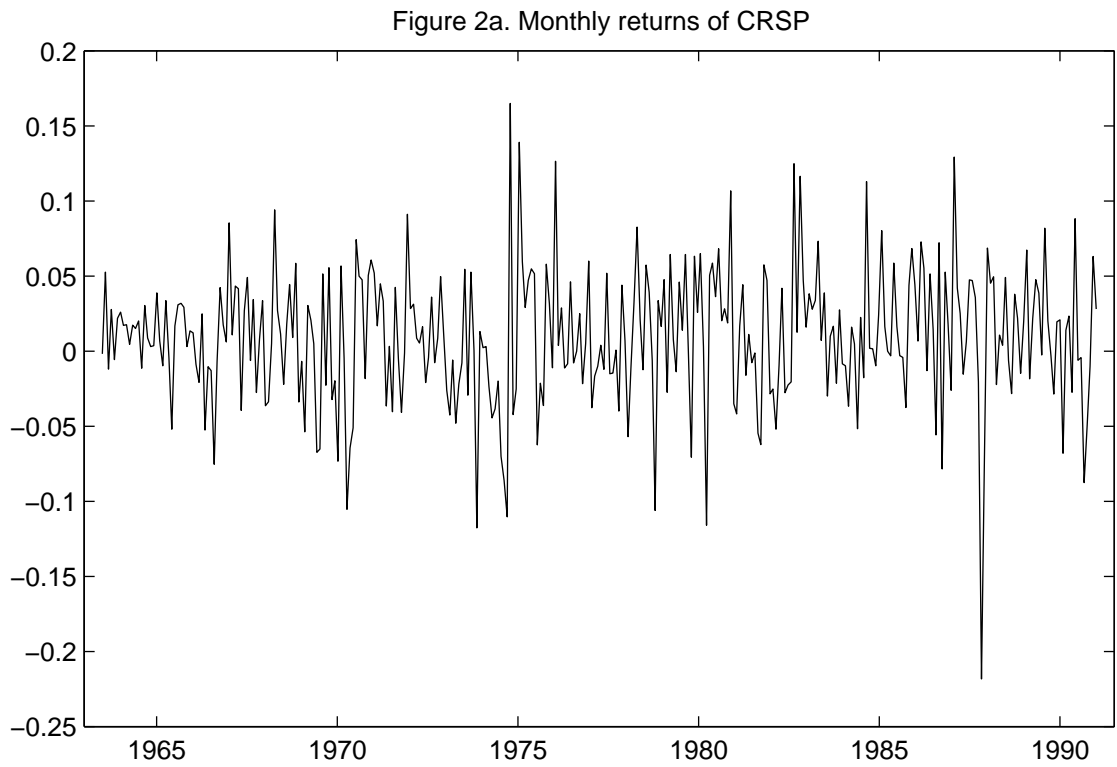


Figure 3: Daily Price of SAP, Hoechst, Preussag and DAX

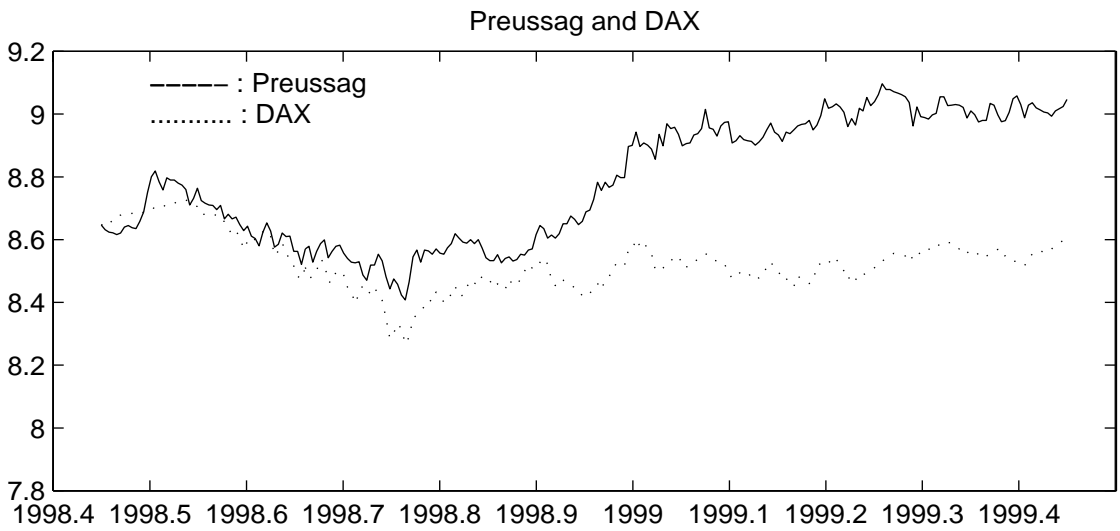
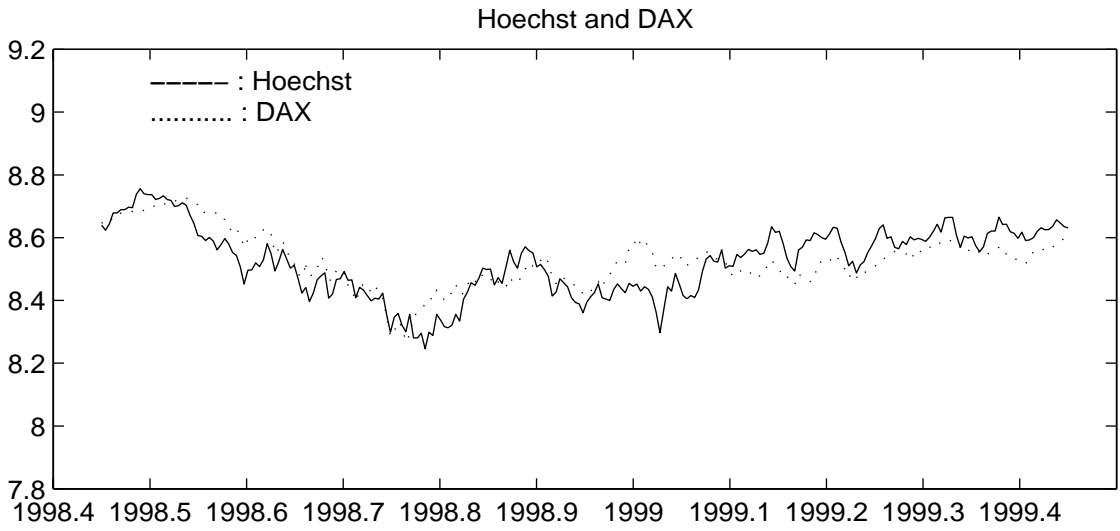
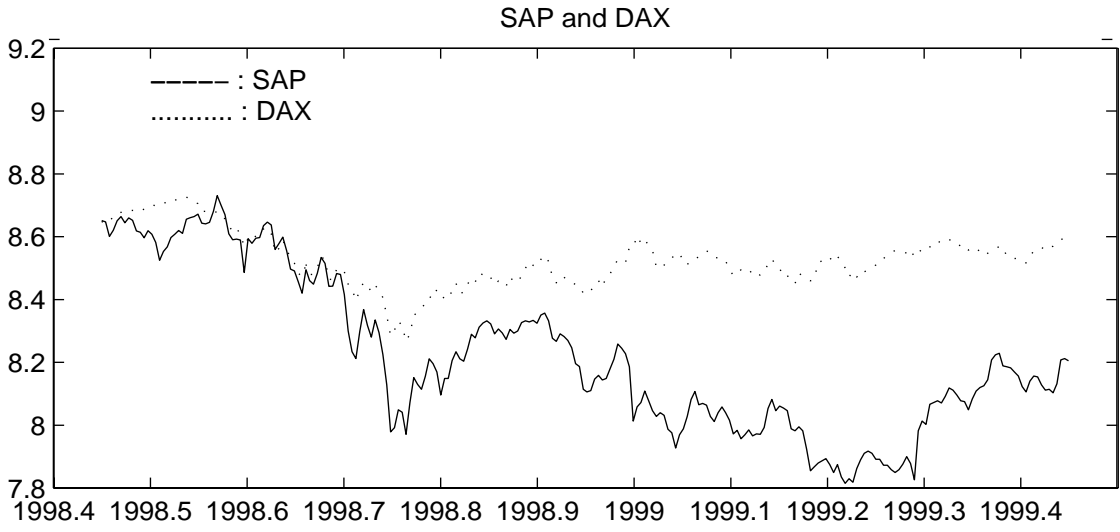
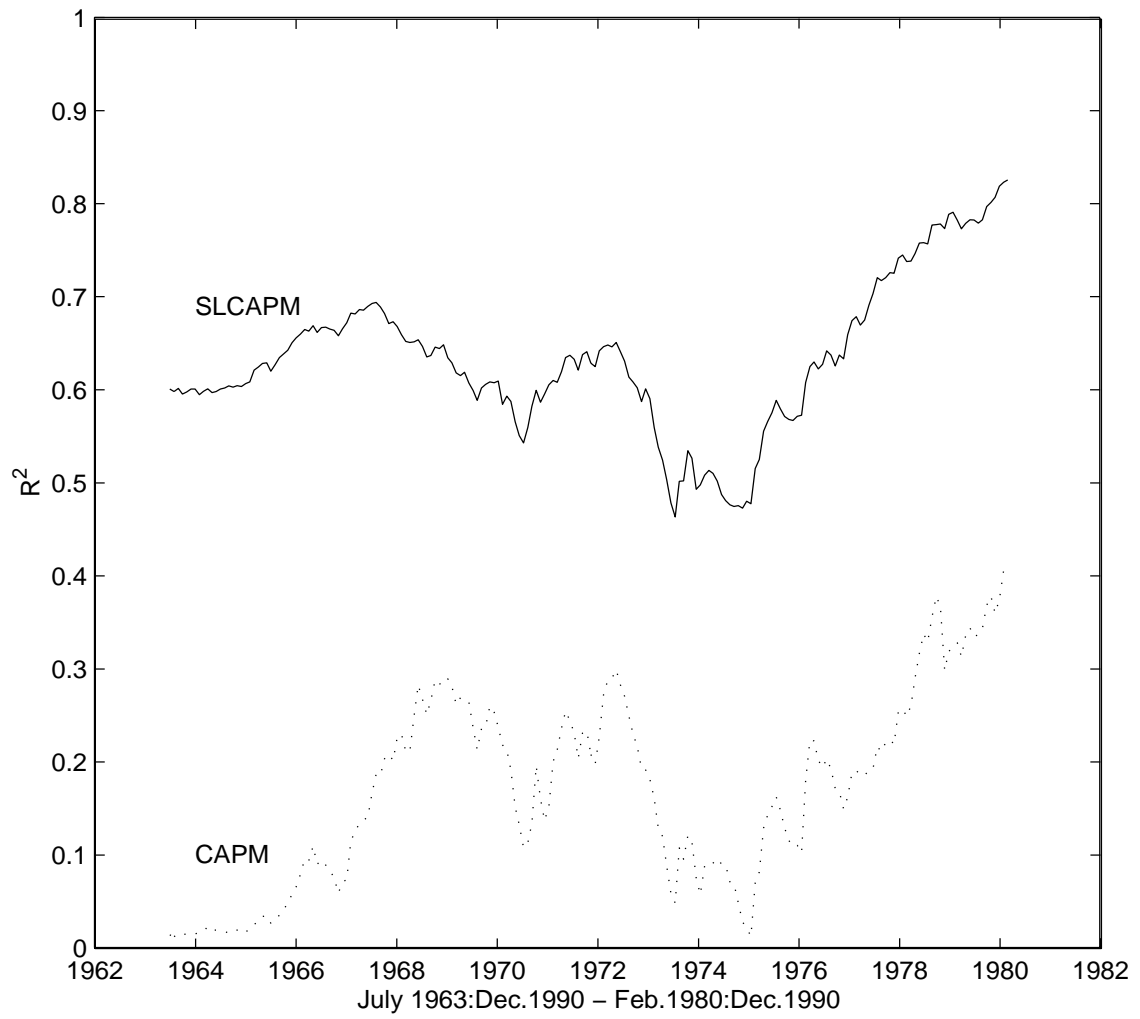


Figure 4: Sequential R² of the conventional CAPM and the SLCAPM



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