Currency Portfolios and Currency Exchange in a Search Economy

Ben Craig
(Federal Reserve Bank of Cleveland)

Christopher J. Waller
(University of Kentucky)

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Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main,
P.O.B. 10 06 02, 60006 Frankfurt am Main, Federal Republic of Germany

Telephone (0 69) 95 66-1
Telex within Germany 4 1 227, telex from abroad 4 14 431, fax (0 69) 5 60 10 71
Internet: http://www.bundesbank.de

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address, or by fax No. (0 69) 95 66-30 77

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Abstract

We develop a dual currency search model to study equilibrium currency exchange and the determination of nominal exchange rates. Agents hold portfolios consisting of two distinct currencies. We study equilibria in which the two currencies are identical and equilibria in which the two currencies differ according to their relative purchasing power risk. We use numerical methods to solve for the steady-state distributions of currency portfolios, nominal exchange rates and value functions. When one of the currencies is 'risky', equilibria exist in which the safe currency trades for multiple units of the risky currency with the observed ratio being the nominal exchange rate. However, due to the decentralized trading environment, we obtain a steady state *distribution* of nominal exchange rates. The mean and variance of the nominal exchange rate distribution are based on the fundamentals of the model and change in predictable ways when the fundamentals change.

Zusammenfassung

Wir entwickeln ein Modell mit zwei Währungen, um das Gleichgewicht beim Austausch der Währungen und die nominalen Wechselkurse zu bestimmen. Die Wirtschaftssubjekte halten Portfolios, die aus den zwei verschiedenen Währungen bestehen. Wir untersuchen Gleichgewichte, in denen die zwei Währungen identische Eigenschaften haben und Gleichgewichte, in denen sich die zwei Währungen hinsichtlich ihres Kaufkraftrisikos unterscheiden. Es werden numerische Methoden verwendet, um die Verteilungen der Währungsportfolios, der nominalen Wechselkurse und der Wertfunktionen im langfristigen Gleichgewicht zu bestimmen. Wenn eine der Währungen risikobehaftet ist, dann existieren Gleichgewichte, in denen die sichere Währung gegen mehrere Einheiten der risikobehafteten Währung gehandelt wird. Das Austauschverhältnis ist der nominale Wechselkurs. Da der Handel dezentral erfolgt, ergibt sich eine Gleichgewichtsverteilung der Wechselkurse. Der Mittelwert und die Varianz dieser Verteilung hängt von den Fundamentalfaktoren des Modells ab. Wenn sich die Fundamentalfaktoren ändern, dann ändert sich diese Verteilung in vorhersehbarer Weise.

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Currency Portfolios and Currency Exchange in a Search Economy*

1 Introduction

In many developing and transitional economies a ‘safe’ foreign currency like the US-Dollar or the DMark circulates as a medium of exchange alongside the ‘risky’ domestic currency, which is subject to unexpected losses of purchasing power. This phenomenon is often called „dollarization.“ Furthermore, there is active domestic currency exchange at well-defined nominal exchange rates. At first glance, the existence of these two trading patterns may not appear to be unusual but it actually creates a puzzle: if both currencies are accepted as media of exchange, then why are currencies exchanged? Currency exchange seems to be economically ‘wasteful’ in such a situation. Consequently, it must be the case that agents are heterogeneous in some dimension such that trading currencies is welfare improving for each of them. Since the home currency is risky, a natural explanation for why agents trade currencies is that they are balancing the risk exposure of their currency portfolios. Agents holding large amounts of the risky home currency may be willing to give up multiple units in return for a unit of the safe foreign currency, while agents with large amounts of safe currency are willing to increase their risk exposure by accepting the risky currency if the price is right. In this situation, the nominal exchange rate measures the relative price of altering the risk exposure of one’s currency portfolio. Our objective is to develop a model that has multiple media of exchange and active currency trading that allows us to examine how changes in domestic currency risk affect the behavior of the economy and nominal exchange rates.

In this paper we present a model of decentralized exchange with two currencies circulating as media of exchange in order to study currency exchange. To do so, we use a monetary search model of indivisible money and divisible goods as in Trejos and Wright (1995) and Shi (1995) but in which agents are allowed to hold portfolios of currencies as in Camera and Corbae (1999) and Green and Zhou (1998). We adopt a decentralized money search model to study the phenomenon described above because: i) money has a fundamental role as a medium of exchange, ii) agents are willing to accept and hold both currencies even though one is ‘dominated in rate of return’, iii) agents will have different trading histories due to decentralized trading which causes some agents to have unbalanced currency

* We would like to thank Gabriel Camera, Randy Wright, Rob Reed, Dean Corbae, Ed Green, John Duffy and participants at several seminars and conferences for their comments on our work. Some of the work on this paper was completed while Ben Craig was a visiting scholar at the Bundesbank.

portfolios, and iv) currency exchange will occur in equilibrium as agents rebalance their portfolios.

In our decentralized environment, agents are assumed to meet at random and trade bilaterally. They use currency to buy goods and services and/or the other currency. When agents meet and trade currencies, the nominal exchange rate is the ratio of the quantities of currencies exchanged. To provide agents with an incentive to engage in currency exchange, we allow the two currencies to differ by their expected rates of return and purchasing power 'risk'- the dollar is assumed to be safe while holders of the domestic currency are subject to random losses of purchasing power. As a result, agents face incentives to diversify their currency portfolios to avoid the currency risk on the domestic currency.

Due to the bilateral nature of trades in this economy, the law of one price does not hold since prices are match specific. Hence, there is an equilibrium distribution of prices. The same applies to nominal exchange rates – observed exchange rates are match specific and so we obtain an equilibrium distribution of nominal exchange rates. In short, the law of one exchange rate also fails to hold. This latter result is consistent with the casual empirical observation that exchange rates are not identical at a point in time across trading locations. We thus are able to study how fundamental changes in currency risk affects: 1) the real exchange rate, 2) the mean nominal exchange rate, 3) the cross-sectional variance of the nominal exchange rate distribution, 4) the extent to which the dollar is used in goods exchange (dollarization), and 5) the volume of currency trading. However, due to the complexity of the model, we must resort to numerical methods to study the equilibrium behavior of the economy and address these issues. Nevertheless, most of our numerical results are very intuitive and consistent with what one would expect to happen when there is a change in the relative risk of the currencies.

In our benchmark model, currencies have the same risk and relative money supplies per capita. In this case, the currencies are identical with respect to the payoffs received from holding them. Once we introduce currency risk on the domestic currency, we find that the value of the domestic currency as a medium of exchange falls – the risk acts as a 'tax' on the domestic currency. Furthermore, agents now have an incentive to diversify their portfolios by trading multiple units of the risky currency for a unit of the safe currency if the opportunity arises. At an appropriate exchange rate, sellers of the safe currency are compensated for accepting the risky currency. The nominal exchange rate observed in different matches depends on the relative portfolio positions of the currency traders who are paired together. Hence, we observe a distribution of nominal exchange rates. Not surprisingly, we find that increasing the riskiness of the domestic currency leads to a depreciation of the domestic currency relative to the foreign currency (the mean of the distribution shifts). More surprising is that the increase in currency risk can increase or decrease the variance of the nominal exchange rate distribution – if the risk is initially low, an increase in currency risk will increase the variance of the distribution; if risk is high initially, the opposite occurs. We also find that once the domestic currency risk is sufficiently large, 'dollarization' occurs as more and more traders use dollars to buy goods since sellers are willing to produce more goods for dollars. Finally, by arbitrarily shutting down currency exchange, we are able to see how the existence of a currency market affects average welfare in the economy.

The rest of the paper is organized as follows. In Section 2, we present a brief review of the literature on dual currency search models. In Sections 3 we describe the economic structure of the model and the bargaining environment and the definition of a stationary equilibrium. Section 4 contains a description of our numerical procedures. Section 5 contains the results of our numerical analysis. Finally, Section 6 summarizes our findings and suggestions for future research.

2 Literature Review of Search Models

The search theoretic model of money has become the dominant framework for studying monetary theory in the last few years. Until recently, a major drawback of the search theoretic framework has been the underlying assumption that agents can only hold one unit of currency at a time. This inventory restriction on money is imposed for analytical tractability. More recent work by Molico (1996), Green and Zhou (1998), Camera and Corbae (1999) and Taber and Wallace (1999) has relaxed the inventory assumption and these authors have studied monetary equilibrium when agents are allowed to hold multiple units of currency. These models have been used to study equilibrium price distributions, divisibility, and the welfare effects of changing the money stock.

All of the models listed above study economies in which only one currency circulates. In order to study nominal exchange rates, two or more currencies are needed. There are many papers in the search literature that look at dual currency issues.[1] Kiyotaki and Wright (1993) look at the coexistence of two currencies in a one-country model and how differing degrees of acceptability affect their relative values. Matsuyama, Kiyotaki and Matsui (1993) look at a two-country model with indivisible goods and money to study international currency equilibria. Trejos and Wright (1996a, 2000) extend the model to the

1 See Craig and Waller (2000) for a brief survey of the search literature on dual currency economies.

case of divisible goods and look at implied real exchange rates. Currency exchange does not occur in either model due to the one-unit inventory restrictions on money holdings. Li and Wright (1998) and Soller-Curtis and Waller (2000) examine how government polices affect the use of one currency over another and also its relative purchasing power. Kocherlakota and Krueger (1999) study the more fundamental question of why it may be socially optimal for more than one fiat currency to exist.

In none of the aforementioned models does currency exchange arise. Zhou (1997) introduces tastes shocks into the Matsuyama, Kiyotaki and Matsui model to generate currency exchange but, due to the one-unit inventory restrictions on money holdings, the nominal exchange rate is arbitrarily fixed at 1:1. Waller and Curtis (2001) extend Trejos and Wright (2000) to show how currency restrictions can generate currency exchange but again the nominal exchange rate is 1:1. Thus, the inventory constraint prevents us from studying equilibrium determination of nominal exchange rates at values other than 1:1. However, relaxing the unit-inventory constraint requires the equilibrium determination of the distribution of money holdings across agents, which is a complicated analytical problem. Shi and Head (2000) circumvent this problem by modeling nations as a set of households each of which has a continuum of worker/shoppers. At the end of each period all money balances are pooled within the household and redistributed evenly to all shoppers in the household. This eliminates aggregate uncertainty and distribution issues. They show that if new domestic currency transfers go to domestic households, then currency exchange can occur between shoppers from two countries in an attempt to re-balance portfolios. The model we develop in this paper compliments the work of Shi and Head; whereas they look at a two-country model where aggregation generates a degenerate distribution of money holdings, we construct a one-country model where the distribution of money holdings is determined endogenously and is not degenerate.

3 The Search Environment

Our model is a standard random matching monetary model with divisible, non-storable goods in which goods prices are determined via bilateral bargaining. There is a continuum of agents uniformly distributed on the unit circle. Agents specialize in the production and consumption of goods. An agent located at point i on the unit circle is assumed to produce good i but consume goods in the interval $(i, i + x]$. For $x < 1/2$, double coincidences of wants cannot occur and thus barter trades do not arise. This restriction on x greatly simplifies the model and will be maintained throughout the paper. Hence a trading equilibrium requires the existence of a medium of exchange. Given the matching process described below, x is the probability that a buyer meets a seller who produces a good in the buyer's desired consumption interval.

3.1 Preferences and Costs of Production

Agents are assumed to receive utility $u(q)$ from consumption of q units of their desired good and incur a utility loss of $c(q)$ from producing q units of their good. Both $u(q)$ and $c(q)$ are continuous, twice differentiable with $u' > 0, c' > 0, u'' \leq 0,$ and $c'' \geq 0$ with at least one of the inequalities holding with strict inequality. Also assume that $u(0) = c(0) = 0$ with $u'(0) > c'(0)$ and there is a positive value of q, $\bar{q}$, such that $u(\bar{q}) = c(\bar{q})$. For the remainder of the paper we will assume that the cost function is linear and given by $c(q) = q$.

3.2 Media of Exchange and Currency Portfolios

In our economy, two fiat currencies can circulate as media of exchange. Let currency f denote the foreign currency (dollars) and currency h denote the home currency. One or both of the currencies are allowed to circulate in trade. Following Camera and Corbae (1999) we assume that agents are able to hold up to N total units of currency at zero storage cost. These N units can be held in any combination of home and foreign currency and the support of N is given by the set $\mathbf{N}=\{0,1,2,\ldots,N\}$. Consequently, agents are able to hold portfolios of currencies whose support is the simplex $S^1 = \{n^i_{jt} \in \mathbf{N}: n^i_{ft} + n^i_{ht} \leq N\}$ where n^i_{jt} denotes the units of currency $j = h,f$ held by individual i at time t. An individual's portfolio at time t is thus an ordered pair (n^i_{ft},n^i_{ht}) on S^1.

3.3 Aggregate Money Stocks

Let $m_t(n_f,n_h)$ denote the proportion of the population holding currency portfolio (n_f,n_h) at time t. The per-capita foreign money stock is then given by

$$(1) \qquad M_f = \sum_{n_f=0}^{N} \sum_{n_h=0}^{N-n_f} n_h m_t(n_f,n_h)$$

while the per-capita home money stock is given by

$$(2) \qquad M_h = \sum_{n_h=0}^{N} \sum_{n_f=0}^{N-n_f} n_h m_t(n_f,n_h) \quad .$$

In a stationary steady state, $m_t(n_f,n_h) = m(n_f,n_h)$ for all t, n_f and n_h.

3.4 Government Behavior

We distinguish the two currencies by their respective purchasing power risk. Purchasing power risk is modeled as in Li (1995). We assume that there is an atomless government that consumes goods but does not produce goods. Its main purpose is to confiscate and issue units of the domestic currency. Upon meeting a private agent, the government confiscates all of their home currency holdings with probability $0 \leq \mu \leq 1$. Thus, confiscation corresponds to a random loss of purchasing power. We refer to this potential loss of purchasing power as currency risk. We interpret $\mu = 0$ as corresponding to the case where the domestic currency's purchasing power risk is zero. We assume that the government destroys the confiscated currency. In order to have a stationary equilibrium with a positive stock of home currency in circulation, we need an inflow of the home currency to offset the outflow of currency arising through confiscation. We do this in the following manner. If the government does not confiscate an agent's domestic currency, then with probability $0 \leq \eta \leq 1$, he makes a take-it-or-leave it offer of 1 unit of domestic currency for some quantity of goods to all agents holding $N-1$ units of currency or less.[2] In equilibrium, these offers are accepted and so the government agent issues a new unit of currency. Obviously, if $\mu = 0$, then $\eta = 0$ for a steady state to exist. While clearly stylistic, we think this formulation captures the idea that for some reason – political, financial, or arising from 'bad' policy actions – the domestic currency in developing currencies is prone to sudden losses of purchasing power.

3.5 Matching Process

Agents meet at random according to a Poisson process with arrival rate α. With probability x, a single coincidence of wants occurs and one of the agents becomes a seller and the other a buyer. The buyer and seller may hold both currencies but trade requires that the buyer hold at least 1 unit of currency and that the seller hold less than N units of currency. When a single coincidence of wants occurs there are two possible types of trades: 1) the buyer gives money (foreign, domestic, or some of both) to the seller for some amount of goods, and 2) the buyer gives the seller one currency and the seller gives the buyer goods and some of the opposite currency. For example, a buyer could give the seller 3 units of

2 Government agents are assumed to like all goods. We could have modeled the government agents as using confiscated currency to buy goods rather than destroying it. However, this would have required us to solve for the distribution of portfolio holdings held by government agents in steady state. Therefore, modeling the government's behavior as we have greatly simplifies the numerical routines.

foreign currency and receive in return some goods and 1 unit of the domestic currency. We refer to these latter types of trades as ‘making change’ trades. We examine these types of trades later in the paper.[3]

When no coincidence of wants occurs, traders may still gain from trade via a pure financial transaction. Since the domestic currency is 'risky', traders may decide to diversify their portfolios by trading currencies. For example, a trader with some dollars and no home currency may meet a trader with many units of home currency and no dollars. By swapping dollars for home currency, the home currency trader gets rid of some of the risky currency and acquires some units of the safe currency. If the dollar trader gets enough rubles per dollar, he will be willing to take on a greater risk position in order to increase his total currency holdings. The amount that they trade will determine the nominal exchange rate. In general, the nominal exchange rate that occurs will be a function of the composition of the traders' current portfolios and the underlying trading values of the current portfolios. As a result, the nominal exchange rate occurring in these financial trades will vary across matches.

3.6 Bargaining

When a single coincidence of wants occurs, we assume that the buyer makes a take-it-or-leave-it offer to the seller. As a result, the buyer will offer a trade of goods for currency such that he extracts the entire trading surplus from the seller. The seller is indifferent between accepting and rejecting this offer and thus accepts the offer.[4] The buyer's offer, d, is a pair of foreign and home currency transfers $d = (d_f, d_h)$ in return for goods. If $d_f > 0$ and $d_h = 0$, the buyer offers to pay with the foreign currency while the reverse is true if $d_f = 0$ and $d_h > 0$. If both are greater than zero, than the buyer offers to pay the seller with a mixed bundle of foreign and home currency in return for goods. These are 'money for goods' trades. If $d_f > 0, d_h < 0$, the buyer offers d_f units of foreign currency in return for goods and d_h units of domestic currency. If the inequalities are reversed, the buyer offers domestic currency units in return for goods and units of the foreign currencies. When $d_f >$ $(<) 0, d_h < (>) 0$, we call these 'making change' trades since in most of these trades the buyer is giving the seller a valuable currency and receiving goods and some ‘change’ back in the form of the less valuable currency. These types of trades help overcome the

3 These types of trades can create belief equilibria in which one currency is viewed as more valuable than the other for non-fundamental reasons. As a result currency trades will occur even though nothing distinguishes the currency but their color.

4 Since q is continuous, the buyer can offer to take an infinitesimally smaller amount of q to induce the seller to accept the offer.

indivisibility of money and allow agents to trade smaller quantities of goods in a single coincidence match. Finally, we assume that the government agent who decides to issue a unit of currency also makes a take-it-or-leave-it offer to the seller.

When a single coincidence of wants does not occur a currency swap may be beneficial to both parties. Thus, let $y_f(n_f, n_h, n_f^i, n_h^i)$ denote the quantity of the currency *f* acquired by currency trader with portfolio (n_f, n_h) from an arbitrary currency trader *i* holding portfolio (n_f^i, n_h^i) in return for giving up $y_h(n_f, n_h, n_f^i, n_h^i)$ units of currency *h*. If $y_f < 0$, then $y_h > 0$, which means the (n_f, n_h) trader is giving up currency *f* for currency *h*.

3.7 Value Functions

In a stationary steady-state, the returns to search for an agent with money holdings (n_1, n_2) is

$$rV(n_f, n_h) = \alpha x \sum_{n_f^s \in \Omega} \sum_{n_h^s \in \Omega} [u(q(n_f, n_h, n_f^s, n_h^s))$$

$$+ V(n_f - d_f(n_f, n_h, n_f^s, n_h^s), n_h - d_h(n_f, n_h, n_f^s, n_h^s)) - V(n_f, n_h)] m(n_f^s, n_h^s)$$

$$+ \alpha x \sum_{n_f^b \in \Psi} \sum_{n_h^b \in \Psi} [-c(q(n_f^b, n_h^b, n_f, n_h))$$

$$+ V(n_f + d_f(n_f^b, n_h^b, n_f, n_h), n_h + d_h(n_f^b, n_h^b, n_f, n_h)) - V(n_f, n_h)] m(n_f^b, n_h^b) \tag{3}$$

$$+ \alpha(1 - 2x) \sum_{n_f^k \in K} \sum_{n_h^k \in K} \{[V(n_f + y_f(n_f, n_h, n_f^k, n_h^k), n_h - y_h(n_f, n_h, n_f^k, n_h^k)) - V(n_f, n_h)]$$

$$- \alpha\mu[V(n_f, n_h) - V(n_f, 0)] + \alpha(1 - \mu)\eta[-c(q(n_f^s, n_h^s) + V(n_f, n_h + 1) - V(n_f, n_h)]$$

where Ω denotes the set of feasible sellers, ψ denotes the set of feasible buyers and K denotes the set of feasible currency traders. The first summation term of (3) is the expected payoff from being a buyer and trading the currency bundle $(d_f(n_f, n_h, n_f^s, n_h^s), d_h(n_f, n_h, n_f^s, n_h^s))$ for the quantity $q(n_f, n_h, n_f^s, n_h^s)$ from a seller with portfolio (n_f^s, n_h^s). The second summation term is the expected payoff from being a seller and producing $q(n_f^b, n_h^b, n_f, n_h)$ for a buyer who pays a currency bundle $(d_f(n_f^b, n_h^b, n_f, n_h), d_h(n_f^b, n_h^b, n_f, n_h))$. The third double summation term captures the return from currency exchange with another trader holding portfolio (n_f^k, n_h^k). The last line is the expected loss of running into a government agent and having the domestic currency confiscated and the expected gain of selling $q(n_f, n_h)$ to the government agent for a unit of the domestic currency.

3.8 Bargaining

When a single coincidence match occurs, the buyer makes a take-it-or-leave-it offer to the seller. This offer is a triplet (q,d_f,d_h) that satisfies

(4)
$$\max_{q,d_1,d_2}[u(q(n_f,n_h,n_f^s,n_h^s))+V(n_f-d_f(n_f,n_h,n_f^s,n_h^s),n_h-d_h(n_f,n_h,n_f^s,n_h^s))-V(n_f,n_h)]$$

$$s.t.\quad V(n_f+d_f(n_f,n_h,n_f^s,n_h^s),n_h+d_h(n_f,n_h,n_f^s,n_h^s))-V(n_f,n_h)\geq c(q(n_f,n_h,n_f^s,n_h^s))$$

and the constraint that d_f and d_h are feasible transfers of currency given the buyer's and seller's portfolios. When a trade actually occurs, the buyer's offer extracts the full surplus from the seller such that the constraint in (4) is satisfied with equality. This is possible because q is a continuous variable. As pointed out by Camera and Corbae, in general, there will be matches with a single coincidence of wants but no offer can be made which satisfies (4). Since the value functions are concave in money holdings, a 'rich' seller (someone holding lots of currency) may not be willing to give up enough of the good for another unit of currency from a 'poor' buyer (someone with little money). The high price of the good makes the buyer willing to wait for a better deal than to trade now. Since the seller receives no surplus when a goods trade occurs, the second double summation term in (3) will be zero. As a result, the quantity traded only depends on the seller’s currency holdings. Furthermore, the payoff from selling to the government agent for a unit of domestic currency is also zero. When a trade does occur, the monetary price observed in the match is difficult to calculate in more than one currency is exchanged for goods. In trades in which only one currency or the other changes hands, then the monetary price observed in the match is simply $d_i/q_i(n_f^s,n_h^s)$ when currency i is traded for goods.

Since agents without any currency units can only be sellers and sellers receive zero net surplus from trade, the returns to search for an agent without any currency units is

(5) $rV(0,0)=0$.

When potential currency swaps exist, we assume that agents resort to symmetric Nash bargaining to determining the quantity of currencies exchanged. The quantities traded (y_f,y_h) solve

(6) $$\max_{y_f,y_h}[V(n_f+y_f,n_h-y_h)-V(n_f,n_h)][V(n_f^i-y_f,n_h^i+y_h)-V(n_f^i,n_h^i)]$$

subject to the constraints that the proposed portfolio changes are feasible. The discreteness of the currency units means that the first-order condition to this problem will not be satisfied in general. In this case we choose the value that yields the highest surplus (which can be zero). A problem with indivisible currencies is that there are possible currency trades, which are beneficial to both sides but cannot be consummated due to the discreteness of the currencies. For example, trading 3 units of the risky currency for 2 units of the safe currency may not be enough compensation for the safe currency trader and a 2:1 trade may be too costly for the risky currency trader. It is possible that if they could trade 1.84 units of the risky currency for a unit of safe currency, a Pareto improving currency trade could occur. But if their portfolios are such that no combination of their currencies yields a ratio of 1.84, a trade does not occur. Consequently, in our model, some potentially Pareto improving currency trades do not occur due to the indivisibility the currencies. As a result, currency exchange is welfare improving in our model but incomplete.[5]

3.9 The Distribution of Portfolios

Let $F_t(n_f,n_h)$ denote the probability at time t that an individual agent has a portfolio that is smaller than or equal to (n_f,n_h). Thus, $F_t(n_f,n_h)$ is given by

$$(7) \quad F_t(n_f, n_h) = \sum_{a=0}^{n_f} \sum_{b=0}^{nh} m_t(a,b)$$

A stationary distribution of portfolios has $F_t(n_f,n_h) = F(n_f,n_h)$ for all t, n_f and n_h.

At each point in time there are flows of agents into and out of each portfolio state. In steady state, the flows of agents out of a particular portfolio state must be matched by an inflow of agents into that portfolio state. Writing down these flow equations for each possible portfolio state is complicated. Nevertheless, it is relatively easy to describe what

5 The discreteness of the currencies could be overcome through the use of lotteries as in Berentsen, Molico and Wright (2000). Using lotteries, the traders could choose two possible currency trades that straddle each trader's threat point and then choose a probability such that the *expected* nominal exchange rate satisfies both traders' incentive compatibility constraints. As a result, both traders expect to gain from the currency trade ex ante but ex post one of them would lose. On average however, welfare would increase. With regards to the nominal exchange rate distribution, we would observe a wider range of exchange rates and increased variance in the nominal exchange rate distribution as a result of the lotteries. Due to the complexity of the computation algorithm needed to do this for all possible currency trades, we do not pursue this modeling strategy. Consequently, while we will show that currency exchange raises average welfare, it should be remembered that our calculations *underestimate* the true value of currency exchange to society since our currency market is incomplete.

happens regarding matches at time t. For a given portfolio state (n_f,n_h), agents with this portfolio who are either buyers, sellers or currency traders move out. The inflows into this state are buyers from larger portfolio states, sellers from smaller portfolio states, and currency traders. In steady state, the inflows must equal the outflows of agents for each portfolio state.

Agents may meet another private agent or the government. In meetings with private agents, there is either a single coincidence of wants or no-coincidence of wants. Each single coincidence match involves a trade of money for goods and a flow into a new currency holding state and a flow out of the old currency state for both seller and buyer. For a transaction that involves the buyer paying $d_f(n_f, n_h, n_f^s, n_h^s)$ and $d_h(n_f, n_h, n_f^s, n_h^s)$ to use the notation above, the proportions $m(n_f - d_f, n_h - d_h)$ and $m(n_f^s + d_f, n_h^s + d_h)$ both increase by the amount $m(n_f, n_h)m(n_f^s, n_h^s)$, because of the new currency holdings. The old proportions, $m(n_f, n_h)$ and $m(n_f^s, n_h^s)$, are decreased by the same amount. Matches without a single coincidence of wants do not cause a change in portfolio positions unless there is a currency trade. When currency exchange occurs, the traders portfolio state changes and so the flow equations for each portfolio position must account for these pure currency trades. A steady-state equilibrium is achieved when the flows out of a given currency state are equal to the flows into it.

Meeting the government either involves no action, an outflow of currency or an inflow of currency. With regards to the inflows and outflows of the domestic currency, the government meets an agent at state (n_f,n_h) with probability α and confiscates n_h units of their domestic currency holdings with probability μ. Thus, the total outflow of domestic currency per capita is given by

$$\alpha\mu \sum_{n_f=0}^{N-1} \sum_{n_h=1}^{N-n_f} m(n_f, n_h) n_h$$

If confiscation does not occur and $n_f + n_h < N$, then with probability η the agent receives 1 unit of the domestic currency in return for producing goods for the government. Thus, the inflow of currency per capita is given by

$$\alpha(1-\mu)\eta \sum_{n_f=0}^{N-1} \sum_{n_h=0}^{N-1-n_f} m(n_f, n_h) .$$

In steady state, this outflow of the domestic currency must be equal to the inflow of domestic currency so

(8) $$\mu \sum_{n_f=0}^{N-1} \sum_{n_h=1}^{N-n_f} m(n_f, n_h) n_h = (1-\mu)\eta \sum_{n_f=0}^{N-1} \sum_{n_h=0}^{N-1-n_f} m(n_f, n_h) .$$

Note that the left-hand side of this expression reduces to μM_h while the right-hand side reduces to $(1-\mu)\eta(1-\delta_N)$ where δ_N is the measure of agents holding portfolios of size N (regardless of the portfolio composition). Thus we have

(9) $$\mu M_h = (1-\mu)\eta(1-\delta_N)$$

Since δ_N is determined by the probability flow conditions, this extra equation requires that one of the three parameters, μ, η and M_h must be determined endogenously. Under the buyer-take-all bargaining assumption, the parameters η and M_h do not appear in the value functions so for computational ease we let one of these two parameters vary in our numerical model while we fix μ. We fix M_h and let η be determined endogenously in order to keep per capita money stocks constant.

3.10 Equilibrium

We can define a stationary equilibrium as a set of functions $V(n_f,n_h)$, $q(n_f, n_h, n_f^s, n_h^s)$, $d_f(n_f, n_h, n_f^s, n_h^s)$, $d_h(n_f, n_h, n_f^s, n_h^s)$, $y_f(n_f, n_h, n_f^s, n_h^s)$, $y_h(n_f, n_h, n_f^s, n_h^s)$, $F(n_f,n_h)$ such that (1)-(8) are satisfied.

In general it is not possible to solve this model analytically. Consequently, in order to study the steady state properties of this model we must resort to numerical methods.

4 Numerical Methods

Our numerical strategy is as follows:

1. Starting from an arbitrary initial distribution and set of value functions, find a steady-state equilibrium for a given set of parameter values when the currency risk is zero.
2. Using the steady-state values under identical currencies as the initial values, increase the amount of domestic currency risk from zero. Find the new steady-state equilibrium values.
3. Repeat.

More formally, numerical solution of the system of equations follows a classic fixed-point procedure. With assumed initial value functions, V_0, currency exchange functions, d_{f0} and

d_{h0}, and probability distributions, $M_0 = (M_{0f}, M_{0f})$, we recursively calculate the economy where:

$d_{f,t+1}\ d_{h,t+1}$ = argMax(for the buyer){Feasible Bargain Conditions (V_t,M_t)}

M_{t+1} = Implied new Probability Distribution ($d_{1,t+1}\ d_{2,t+1}$,M_t)

V_{t+1} = Value from the Search Conditions ($d_{f,t+1}\ d_{h,t+1}$,M_{t+1})

and where convergence is achieved if the maximum difference between $d_{f,t+1}\ d_{h,t+1}$, M_{t+1}, V_{t+1}, and $d_{f,t}\ \ d_{h,t}$, M_t , V_t is under a specified tolerance. (In general the results did not differ for a wide range of tolerances.) The search conditions were rewritten in the numerical work to ensure a contraction mapping was as likely as possible. Under most initial conditions that we tried, the routine converged. In addition, we took the maximum operation globally for the bargaining conditions over all feasible bargains, which is allowed by the discrete number of currency units. Thus, we did not need to strongly specify local conditions in order to achieve an optimum.

With regards to the initial conditions, our default is a uniform probability distribution over money holdings and value functions, which are skewed such that the risky currency is more valuable than the safe currency. This is done to ensure that the numerical results were not driven by the choice of initial value functions. When conducting our steady-state comparative static analysis, we use the previous equilibrium as the initial condition to try and ensure that our new equilibrium lies in the neighborhood of the previous equilibrium.

Since we cannot solve the model analytically, we do not have a benchmark solution to compare our numerical results to. However, for μ=0, the two currencies are identical, hence we can choose parameter values that yield the Camera-Corbae (1999) solution where buyers only spend one unit of currency when trading. Consequently, we choose parameter values that yield the equilibrium transaction pattern ($d_f = 1, d_h = 0$) or ($d_f = 0, d_h = 1$) as our initial steady-state equilibrium.

It should be noted that this equilibrium transaction pattern does not allow for making change trades, which can exist here even if both currencies are identical except for color. Aiyagari, Wallace and Wright (1996) show that equilibria exist such that one currency is believed to be more valuable even though the two currencies are fundamentally identical. In these equilibria, making change trades are optimal in single coincidence matches. In order to see how this type of belief equilibrium affects our results, we conduct two numerical experiments in the large portfolio economy considered below. We first study the model without making change trades to see how ‘fundamentals’ affect the behavior of the economy. We then allow making change trades to occur and compare them to the ‘no making change' equilibrium. Although the initial trading patterns are very different, we show that the steady state comparative static results are qualitatively the same across both economies.

5 Numerical Results

5.1 "Small" Portfolios

In this section we explore the behavior of equilibria when $N=2$ and both currencies are fully acceptable in trade. Furthermore, no making change trades are allowed. This is the easiest example to study and it provides some basic insight for N>2.[6] In these simulations, we examine equilibria in which there is no currency risk on either currency and we only consider trades in which currency trades for goods. We then introduce currency risk to see how the equilibrium is affected. No currency exchange occurs in this case since 2:1 is the only currency exchange possible and thus exchange cannot make both parties better off. This is because a currency trade would merely involve trading places so there cannot be mutual gains from exchange. While helpful in understanding how the model works, this case is not interesting in terms of our ultimate pursuit, which is generating equilibrium currency exchange. As a benchmark, we initially set the currency risk to zero on the domestic currency, which requires setting $\mu=\eta=0$.[7]

Letting $u(q) = q^{\sigma}$, Table 1a below shows the equilibrium solutions for the value functions, the portfolio distribution and the quantities traded for the $N=2$ economy for four different values of the confiscation probability μ.

6 Camera, Craig and Waller (2001) provide analytical conditions for existence and uniqueness of this equilibrium and determine when (d_f=1,d_h=0) or (d_f=0,d_h=1) is the optimal strategy.

7 While this benchmark is intuitively appealing as a starting point, it is characterized by a fundamental indeterminacy – there are an infinite number of portfolio distributions that support the same equilibrium value functions when the currencies are identical. The source of this problem is that buyers holding both currencies are indifferent as to which currency they use to buy goods. Thus, there are an infinite number of mixed strategies that can be used to make this decision. While this has no effect on the equilibrium value functions or quantities traded, the mixed strategy adopted fundamentally affects the flows of agents into different portfolio states.

Table 1 a
Small Economy with Differing Risk of Domestic Currency Confiscation

Parameter Values: $N=2, M_1=M_2=2/3, \sigma = .5, \alpha=1, x = .45, r=.1$

	$\mu=0$ $\eta=0$	$\mu=0.10$ $\eta=0.152$	$\mu=0.15$ $\eta=0.245$	$\mu=0.25$ $\eta=0.466$
$V(0,0)$	0	0	0	0
$V(1,0)$	0.375	0.387	0.393	0.400
$V(0,1)$	0.375	0.187	0.139	0.085
$V(2,0)$	0.632	0.653	0.662	0.673
$V(1,1)$	0.632	0.487	0.458	0.433
$V(0,2)$	0.632	0.285	0.204	0.118
$m(0,0)$	0.18086	0.18035	0.18717	0.18988
$m(1,0)$	0.15248	0.16894	0.13922	0.14168
$m(0,1)$	0.15248	0.13702	0.15310	0.14523
$m(1,1)$	0.37470	0.31788	0.22745	0.21955
$m(2,0)$	0.06975	0.08992	0.15000	0.15272
$m(0,2)$	0.06975	0.10588	0.14306	0.15094
q_{00}^{f}	0.375	0.387	0.393	0.400
q_{01}^{f}	0.257	0.300	0.318	0.347
q_{10}^{f}	0.257	0.266	0.269	0.273
q_{00}^{h}	0.375	0.187	0.139	0.085
q_{01}^{h}	0.257	0.098	0.064	0.032
q_{10}^{h}	0.257	0.100	0.064	0.032

where q_{ik}^{j} is the quantity produced by a seller with portfolio (i,k) for currency j.[8]

From Table 1a we see that the law of one price does not hold in this decentralized environment. This is the key result in Camera and Corbae (1999). Since the marginal value of an addition unit of money is falling, 'rich' sellers, i.e., those at $(1_f,0)$ and $(0,1_h)$, will not produce as much as 'poor' sellers, those at (0,0). Thus, we get price dispersion in equilibrium. What is new is that we now have price dispersion across currencies as well

8 The distribution in the first column of Table 1a was obtained by having the buyers at portfolio state $(1_f,1_h)$ spend the dollar on sellers with probability .5. This is the reason for the symmetry of the distribution.

when currency risk is not zero. For the rich sellers, we see that as the risk on the domestic currency increases, sellers holding a unit of home currency will give up far more to acquire a dollar (nearly 40% more) than sellers already holding a dollar (only 6% more). What is also interesting is that as risk increases, the simple variance of the dollar price of goods *across* matches falls while the variance of the home currency price of goods across matches rises. Thus, currency risk alters the relative amounts of price dispersion across currencies. It is important to note that as shown by Camera and Corbae, as search frictions ($r/\alpha x$) go to zero, price dispersion goes to zero, *for a particular currency*. The same will be true here, except that when $\mu>0$ there will still be price differences *across* currencies.

Looking at the value functions we see that, not surprisingly, as the home currency's risk rises, the value of holding dollar-weighted portfolios increases and the value of holding home currency denominated portfolios falls dramatically – a five fold increase in the probability of loss reduces the value of the $(0,2_h)$ portfolio to less than 1/5 of its initial value. Since the dollar buys more goods, the value of holding a dollar portfolio rises while the value of a home currency portfolio falls.

With regards to the portfolio distribution, we see that the biggest change in the distribution occurs in the marginal distribution of agents at with 2-unit portfolios. The increasing risk on the domestic currency is associated with fewer diversified portfolios and more undiversified portfolios. Finally, as the risk on the home currency increases, the quantity of goods given up for a unit of home currency also falls across all sellers. Increasing the risk on the domestic currency reduces sellers' willingness to accept it in trade at a given monetary price. Thus to induce sellers to accept it, buyers ask them to produce a lower quantity of goods.

Table 1b reports the economic data for these economies:

Table 1b
Characteristics of a Small Economy with Differing Currency Risk
Economic Data for N= 2, Risky Economy

	μ=0 η=0	μ=0.1 η=0.152	μ=0.15 η=0.245	μ=0.25 η=0.466
Percentage of matches with goods trade:	35.90%	35.88%	35.08%	34.76%
Percentage of goods trades involving:				
Currency f (Dollars) only:	41.49%	42.51%	52.64%	63.44%
Currency h (Home) only:	58.51%	57.49%	47.36%	36.56%
Expected welfare:	0.439	0.335	0.309	0.285
Expected production:	0.054	0.038	0.038	0.042

From Table 1b we see several interesting results. First, although with x = .45, 90% of all matches have a single coincidence of wants, roughly 35% of those matches actually lead to an exchange of goods. The reason is that in many of those matches, the buyer has no money or the seller is unable to produce because the inventory constraint on money holdings binds. Second, 'dollarization' occurs as the domestic currency risk rises. Dollarization rises for two reasons: 1) agents resort to using dollars when buying goods rather than home currency, 2) more matches occur between buyers with dollars and sellers with no money. By examining the transaction patterns associated with each economy we found that in the first column, the buyer at (1_f, 1_h) uses domestic currency to buy from sellers at (0,0) and at (1_f, 0). He uses the dollar to buy from sellers at (0,1_h), i.e., those already holding a unit of the domestic currency. In column two, the spending behavior of the (1_f, 1_h) buyer is exactly the same as column one, so any changes in dollarization are due to changes in the probabilities of matches occurring. However, in column three, the

(1_f, 1_h) buyer stops spending the home currency on the (0,1_h) seller and switches to dollars instead. Finally, in column four, the (1_f, 1_h) buyer is using dollars in trade with all three sellers. Thus, the change in the transaction pattern of the (1_f, 1_h) buyer in columns three and four are responsible for the large increases in dollarization that are observed.[9]

Not surprisingly, average welfare is falling steadily as risk on the domestic currency increases and the value functions associated with holding home currency declines. Average production falls initially and then rises but this simply is due to the fact that there are more home currency trades occurring with sellers at (0,0) who in turn are willing to produce the largest quantity of goods for a unit of currency. Thus, although the model cannot be solved analytically, the results above are appealing in the sense that they are consistent with what we believe will happen (and observe happening) if there is a dramatic change in the relative returns and relative risk of the two currencies.

While the N=2 economy is very useful for understanding the basic mechanics of our economy, it does not allow currency exchange to occur. Consequently, we do not have nominal exchange rates nor can we look at the welfare gains from having a currency market. To examine these issues we need to examine larger portfolio economies.

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9 Camera, Craig and Waller (2001) show that this transaction pattern arises because the net surplus from trade $u(q)$-$c(q)$ is not monotonically increasing in q. If it were monotonically increasing, then the (1,1) buyer would always spend the dollar on all sellers when μ>0. Note also that the average of the production reported in Table 1a is considerably higher than expected production in Table 1b because the reported matches in the first table are the high production matches. Most matches result in small or no production.

5.2 'Large' Portfolios

5.2.1 N = 10, No Making Change Trades

We now set N=10 and see how currency risk affects the economy and to study currency exchange. Since we cannot solve the model analytically, we do not have a benchmark model to guide us as to what parameter values we should start with for simulations. As before, we choose our parameter values such that we get the equilibrium transaction pattern (d_f=1,d_h=0) or (d_f=0,d_h=1) as our initial steady-state equilibrium. However, we do not impose this transaction pattern in our numerical routine. We merely choose appropriate parameters and let the model run to see if that transaction pattern is generated in equilibrium. The Camera Corbae equilibrium was the only one we found for those parameter values when μ=0. Furthermore, in this section, we prevent making change trades from happening.

Figure 1: Value Functions for a Medium Sized Economy,

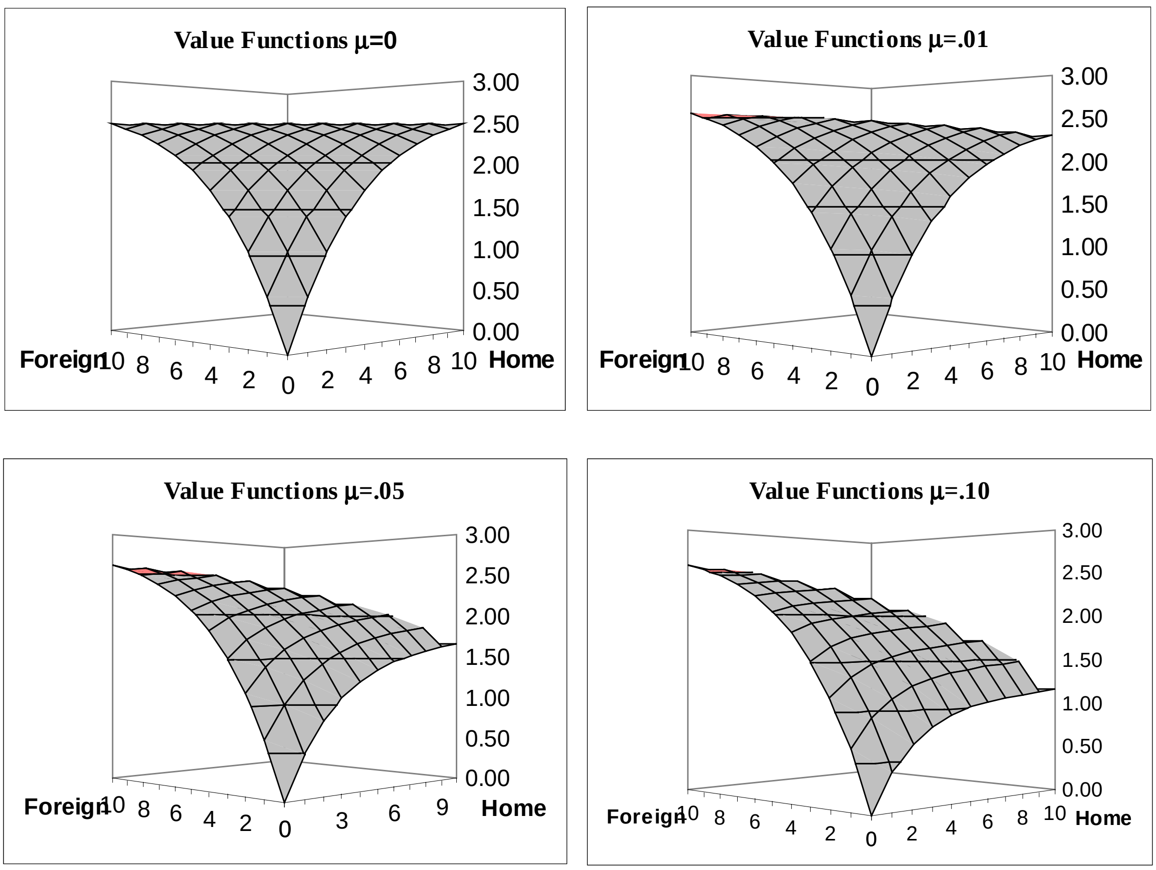


Figure 1 contains the value functions for four differnet values of μ. The value functions are concave in money holdings and in the case of $\mu = 0$, the value functions are symmetric across portfolio size. As the risk on the domestic currency increases, the value of holding a portfolio with home currency falls while holding a dollar weighted portfolio rises. Thus, the pattern observed in the N=2 economy prevails here as well. The concavity of the value functions also means that the quantities given up by 'rich' sellers will be less than for 'poor' sellers just as in the N=2 case. Consequently, we have equilibrium price dispersion within a currency and across currencies.

The economic data for these parameterizations of the economy are contained in Table 2:

Table 2
Medium Economy with Differing Currency Risk

Economic Data for N = 10, Risky Economies
Parameter Values: $N=10, M_1=M_2=3.33, \sigma=.15, \alpha=1, x=.45, r=.1$

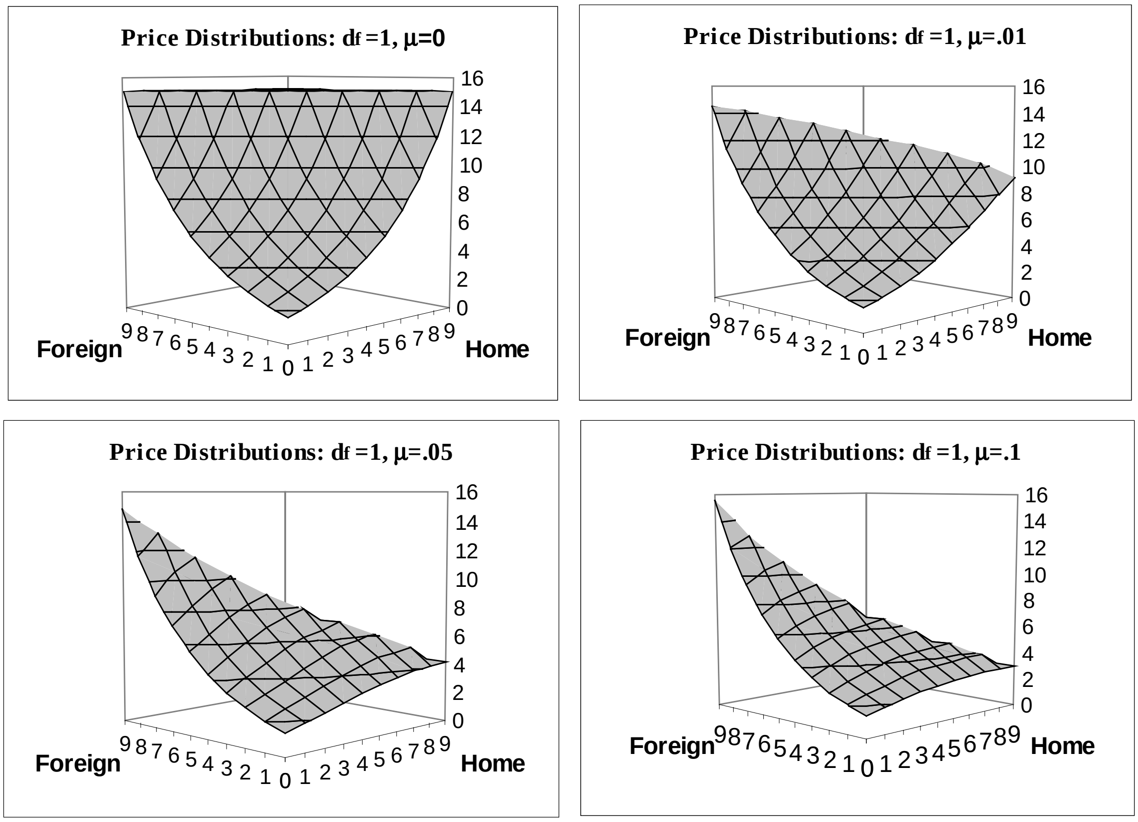
	$\mu=0.01$ $\eta=0.041$	$\mu=0.05$ $\eta=0.124$	$\mu=0.10$ $\eta=0.466$
Percentage of matches with trade:	35.97%	36.08%	35.02%
Percentage of goods trades involving:			
Currency f (dollars) only:	35.23%	36.57%	39.53%
Currency h (home) only:	64.77%	63.43%	60.29%
Expected welfare:	2.01	1.89	1.77
Expected production:	0.0628	0.059	0.055
Expected dollar price per unit of output:	7.83	6.38	5.29
Expected home price per unit of output:	9.63	13.65	23.44
Implied real exchange rate: (Units of home currency per dollar)Table 2	1.23	2.14	4.43

Other than the use of dollars in exchange, the data show a similar pattern as in the N=2 case. Dollarization again increases with an increase in domestic currency use as agents switch to using dollars when a trading opportunity arises. An increase in the domestic currency risk again leads to a reduction in welfare and production. It also leads to a decline in the dollar price of goods, a rise in the home currency price of goods and an increase in the implied real exchange rate. Finally, it should be pointed out the percentage of dollars only trades and home currency only trades does not add up to 100% in the last column since there are a small number of trades in which the buyer gives up some of both currencies in exchange for goods.

Inspection of the buyers' spending patterns (contained in the appendix) with sellers holding portfolios (0,0), $(0,7_h)$ and $(7_f,0)$ shows that when $\mu = 0$, agents only spend one unit of currency in all matches with sellers. However, as risk increases, buyers holding home currency weighted portfolios begin dumping home currency on poor sellers and they also begin to spend more than one unit of the home currency. The largest number of home currency units spent in these simulations is 3. Buyers tend to dump rubles on rich sellers holding lots of dollars. However, buyers with dollar-weighted portfolios begin spending dollars on poor sellers since they get much more for a dollar compared to a unit of home currency and because the marginal value of that dollar is smaller to them. The $(0,7_h)$ sellers conduct most of their trades in dollars since they are willing to give up a large quantity of goods to get a dollar rather than another unit of home currency. Sellers with lots of dollars, those at $(7_f,0)$, are mainly paid in units of the home currency since the relative price that they offer for a unit of home currency is much better than that offered by $(0,7_h)$ sellers.

The price distribution across sellers based on receiving one dollar is given in Figure 2:

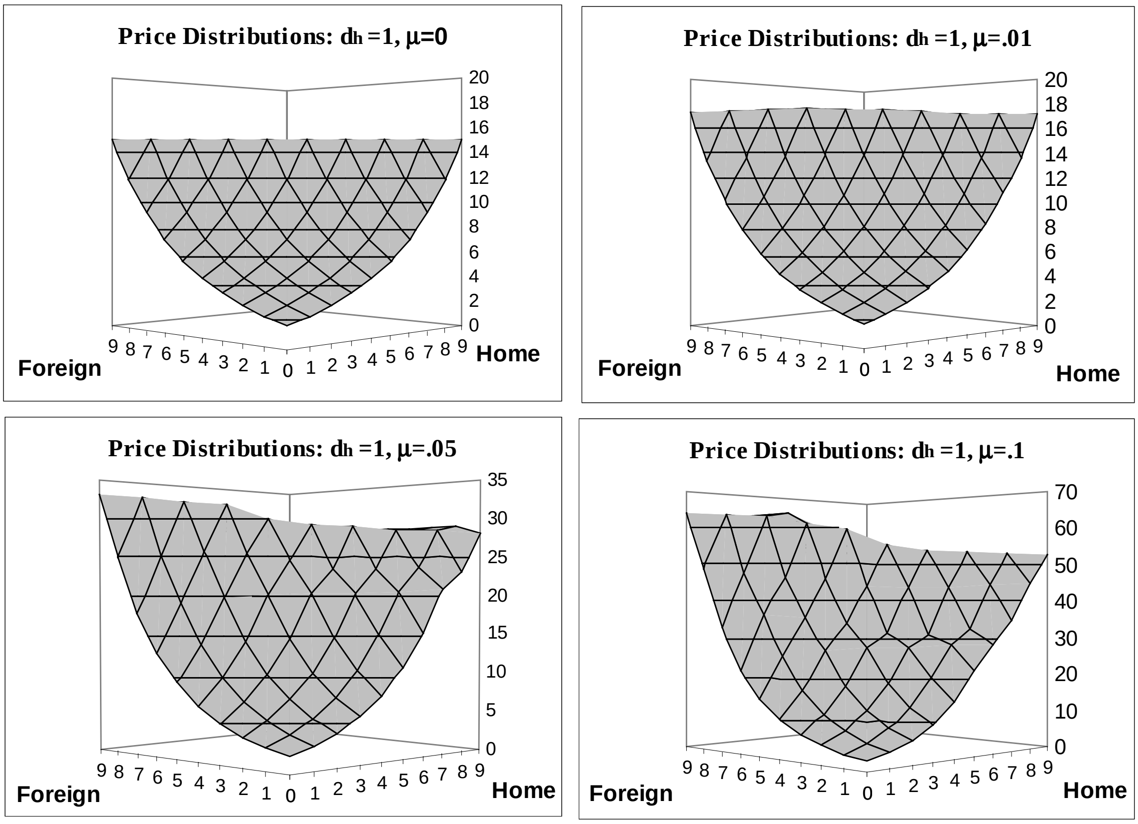
Figure 2: Price Distributions for Purchases by One Unit of Riskless Foreign Currency
$d_f = 1$



We can see that when there is no home currency risk, the price charged by sellers is increasing in portfolio size and symmetric across portfolios of a given size. Once risk begins to increase, the price charged by sellers holding units of domestic currency begins to fall with sellers holding very risky portfolios, $(0,9_h)$, giving the best price among the richest sellers for acquiring a unit of the safe currency. By flatting out the price distribution, domestic currency risk reduces the amount of *dollar* price dispersion.

Figure 3 below shows the price distribution across sellers when buyers spend one unit of the risky domestic currency.

Figure 3: Price Distributions for Purchases by One Unit of Risky Domestic Currency d_h =1



It is clear that as the domestic currency risk increases, the price of goods increases across all sellers. Rich sellers holding a large amount of domestic currency units offer the worst price since they already are saddled with a large amount of risky currency. Rich sellers holding lots of dollars offer the best prices for a unit of domestic currency since their currency portfolios have relatively little risk exposure.[10] Unlike the dollar price distribution, increasing domestic currency risk does not flatten the home currency price distribution but instead increases the degree of price dispersion.

10 The jaggedness at the top of the distribution in the last two figures is the result of approximation error and rounding off.

In this economy, we now have currency exchange occurring between agents. The nominal exchange rate data corresponding to Table 2 are:

Table 3
Currency Market Exchange Rates for a Medium Economy

Nominal Exchange Rate (Home per Dollar)	1	Conditional Probability 2	3
	μ = 0.01	*μ = 0.05*	*μ = 0.10*
5:4 = 1.25	0.397		
4:3 = 1.33	0.587		
3:2 = 1.50	0.023		
5:3 = 1.67			
2:1		0.663	0.046
5:2 = 2.50		0.028	
3:1		0.308	0.371
4:1		0.001	0.454
5:1			0.122
6:1			0.007
7:1			0.000
Size of Currency Market	0.16%	2.24%	1.48%
Mean	1.305	2.325	3.674
Stand. Deviation	0.05	0.46	0.77

The conditional probability gives the percentage of all currency trades that occur at a given nominal exchange rate.[11]

There are several interesting results from Table 3. First, we see that there are several observed nominal exchange rates in each economy, ranging from 3 observed exchange rates when the risk is relatively low and up to 6 observed exchange rates when the currency risk is relatively high. As it did with equilibrium quantities traded, the decentralized nature of the trading environment generates an equilibrium distribution of nominal exchange rates rather than a single exchange rate. Thus decentralization not only breaks down the law of one price, it also breaks down the law of one nominal exchange

11 If a cell shows 0.000 it means some matches produced currency exchange at this exchange rate but these matches occurred with probabilities near zero. A blank cell denotes no observed trades at the corresponding exchange rate.

rate. Second, despite the number of different exchange rates observed in economies 1 and 2, the overwhelming majority of currency trades occur at one or two exchange rates. Thus for the first two economies, we see that a little over 95% of all currency trades occur at two exchange rates. However, this is not true for economy 3; a considerable mass of trades occurs at multiple exchange rates.

From these comparative static results we find not surprisingly, that the home currency depreciates against the dollar as its currency risk increases (the mean of the distribution shifts). More interestingly, the cross-sectional variance of the nominal exchange rate also increases as the currency risk on the home currency increases. Hence, greater currency risk is not only associated with a lower exchange value of the domestic currency but also greater instability in the exchange value of the domestic currency. This pattern was found for every simulation that we have done. The only time we have observed a decrease in the variance is when the home currency becomes so risky that the required nominal exchange rate to induce currency exchange starts bumping up against the maximum possible exchange rate (N:1). Once this happens, the variance of the nominal exchange rate distribution starts to decline. With regards to the variance of the exchange rate distribution, it should be noted that the exchange rate distribution is driven by the price distribution. Thus as search frictions go to zero, the price distribution for a given currency collapses to a single price. This implies that the nominal exchange rate distribution would also collapse to a single exchange rate. Therefore, the degree of dispersion in the observed nominal exchange rates is a function of how severe the search frictions are – when trading is easy, nominal exchange rates will have a small variance and when trading is difficult, the variance in observed exchange rates will be larger.

The size of the currency 'market' shows the percentage of all meetings that end in currency exchange. For the economies above, this number is very small and has its maximum value around 2.25%. This is not surprising since pure currency trades can only occur when a single coincidence match does not occur and with x = .45; this means 90% of all meetings have a single coincidence of wants. As a result, only 10% of all meetings result in no coincidence of wants. Therefore, when the size of the currency market is 1% this means that 10% of all feasible currency trade meetings result in currency exchange. It is interesting to note that as the currency risk on the ruble increases, the size of the currency market increases but then declines. This is not too surprising; as risk increases from a low level, there are incentives to exchange currencies to reallocate risk. However, once risk becomes too large, it becomes harder to induce agents to accept the risky currency even at relatively high nominal exchange rates. As a result, the volume of currency trading falls.

Currency exchange is quite small in economy 1 and the range of observed exchange rates is small. This is due to the upper bound on money holdings and the indivisibility of currency. With N=10, the lowest possible exchange rate that can be observed is 9:8 or 1.125.[12] So we can see that in economy 1, we are bumping up against this constraint. If we relaxed the upper bound on money holdings or allowed divisibility of currency, we would observe exchange rates below 1.125 in economy 1 and a larger currency market.

5.2.2 Changes in Search Frictions

In this section we examine how changes in the probability of a single coincidence match affects the equilibrium behavior of the economy. In Table 4 below, we vary the probability of a single coincidence match starting from the high-risk economy, μ=.1, in Table 2 above:

Table 4
Medium Economy with Differing Search Frictions

Parameter Values: $N=10, M_f=M_h=3.3, \sigma = .15, \alpha=1, x = .45, r=.1, \mu=.1$

	x=.45	x=.40	x=.35
Percentage of matches with trade:	35.02%	31.18%	27.00%
Percentage of goods trades involving:			
Currency f (Dollars) only:	39.53	46.74	52.74
Currency h (Home) only:	60.29	52.07	42.28
Expected welfare:	1.77	1.61	1.42
Expected production:	0.055	0. 046	0.038
Expected dollar price per unit of output	5.29	5.96	7.13
Expected home price per unit of output	23.44	31.72	46.35
Implied Real Exchange Rate (Units of home currency per dollar)	4.43	5.32	6.50
Size of Currency Market	1.48%	1.86%	1.48%

The first column of Table 4 is the last column of Table 2. We see that as matching becomes more difficult, agents increasingly use the 'safe' currency as the main medium of exchange. This is because as matching becomes difficult, agents want to get as much consumption as possible when the opportunity arises and so they start using the more valuable currency. As a result, this alone causes a depreciation of the home currency against the dollar in terms of relative purchasing power. From Table 4 we also see that after an intial increase in the

12 This is because no trade at 10:9 will occur because one trader would have to hold more than 10 units or the traders would simply be trading places and this would not satisfy the incentive compatibility constraints on exchange.

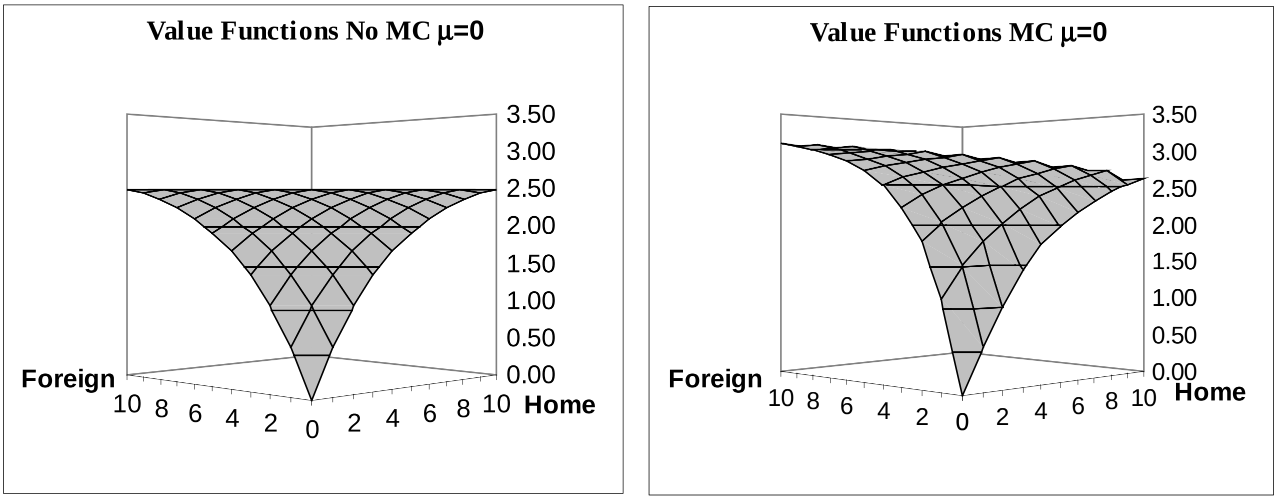
currency market size due to a greater opportunity for currency trade to occur, the percentage of currency trades actually falls when matching becomes extremely difficult. This is largely due to the greater unwillingness to give up dollars in return for the home currency. Not surprisingly, average production and welfare also decrease.

5.2.3 N=10, Making Change Trades

In this section we allow agents to engage in making change trades. If the foreign currency is more valuable than the home currency, then buyers holding dollars may prefer to receive a smaller quantity of goods if they also receive some units of the domestic currency as 'change'. This arises because of the indivisibility of money; if buyers have to spend the entire dollar they get more than they actually want. Consequently, they would like to spend a fraction of a dollar but it is not possible. Thus, receiving some units of the weak currency from the seller lowers the amount of goods they need to acquire for giving up a dollar. At the same time, these types of trades allow agents constrained by the inventory constraint to produce since they can acquire a dollar by producing goods and giving a unit of the home currency as change, thereby maintaining the size of their currency portfolio. While these types of trades are clearly beneficial, they create the possibility that one currency is viewed as being more valuable even though there is nothing fundamentally different about the two currencies. This was demonstrated by Aiyagari, Wallace and Wright (1996). If such a belief equilibrium exists, then agents will engage in making change trades and direct currency trades in order to alter their currency holdings. Consequently, when currency risk does exist on one currency it is difficult to disentangle how much of the differences in values is due to differences in beliefs versus differences in fundamentals. In order to control for this, we first examine the N=10 economy with making change trades and no currency risk. We then introduce currency risk and do steady state comparative static analysis.

Figure 4 and Table 5 compares the results for the N=10 economies when making change trades are prevented and then allowed. In both economies there is no currency risk and the parameter values are exactly the same.

Figure 4
Value Functions with and without "Making Change" in Goods Trades



In Figure 4, we see that the value of holding dollars is greater than the value of holding the same size portfolio with domestic currency. Comparing the two value function graphs shows that the value of dollar portfolios increase substantially and the value of home currency increases slightly in the making change equilibrium. Thus, even though there is nothing fundamentally different about the two currencies, if agents believe that the dollar is more valuable, that belief can be supported in equilibrium.[13]

Table 5 compares the basic economic data across the two economies.

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13 The figure shows an assymetry due to alowing the participants to make change. As is well known, these equilibria choose one currency or the other to be the lower valued change, and thus the outcomes are not symetric.

Table 5

Medium Economy with and without "Making Change" in Goods Trades

Economic Data for N= 10, No Currency Risk Economies ($\mu = 0$)
Parameter Values: $N=10, M_1=M_2=3.33, \sigma = .15, \alpha=1, x = .45, r=.1$

	No Change Trades Allowed	Change Trades Allowed
Percentage of matches with trade:	70.70%	88.88%
Percentage of goods trades involving:		
Currency *f* (dollars) only:	50%	9.80%
Currency *h* (home) only	50%	47.41%
Any dollars traded	50%	52.59%
Any home traded	50%	90.02%
Expected welfare:	2.02	2.57
Expected production:	0.063	0.051
Size of the currency market:	0%	0.0025%

Table 5 reveals several interesting results. First, more goods trades occur in the making change equilibrium than the no change equilibrium. This is not surprising since more possible trades can be achieved. Furthermore, the fraction of dollar only trades falls dramatically while the number of goods trades involving any exchange of the home currency soars to 90%. This is result of all agents being able to be sellers in this economy regardless of whether the inventory constraint is binding. Agents holding 10 units of money can engage in trades that alter the composition of their currency holdings without increasing its size. quantity and gets change back from the dollar in the form of a unit of home currency.

Average welfare goes up when change trades are allowed. This is because more trades are occurring hence consumption is occurring more frequently per period of time. Furthermore, since some buyers are able to obtain quantities that are closer to their most desired amount, their welfare goes up. It is interesting that welfare rises despite the fact that average production falls. Production falls because some buyers are now engaging in smaller trades than before. This occurs for two reasons. First, many of the trades in the no change equilibrium involve buyers buying quantities that are 'too large' because of the indivisibility of money. Thus, rather than buying a dollar's worth of goods and receiving no change, a buyer now spends a dollar, receives a smaller and more desirable. Second, the richest sellers are now trading small quantities in making change trades, which lowers the average amount of production in the economy. Finally, a direct currency market exists now where agents can engage in pure currency trades when no coincidence of wants occurs. This currency trade is simply exploiting differences in the marginal valuations of the portfolios as opposed to altering the risk exposure of one's portfolio. In this example, only 1 nominal exchange rate is observed which is 2 units of home for one dollar.

Finally, an analysis of specific transaction patterns for particular buyers and sellers reveals a very different transaction pattern. The transaction patterns for a (10_f,0_h) buyer with all sellers are shown in Table 6 across the two economies. When change trades are not permitted, the parameterization yields the Camera-Corbae equilibrium in which (d_f,d_h) = (1,0) or (0,1).). The letter f in each cell means that this buyer spends one dollar on any seller he meets (in this case agents holding 10 units of currency cannot sell). However, once making change trades are allowed, the transaction pattern for this buyer changes dramatically.

Table 6
Transaction Patterns for a Foreign Currency Rich Buyer
(10_f, 0_h) Buyer with all Sellers

No Making Change

Foreign denotes foreign currency holdings of a seller
Home denotes home currency holdings of a seller

$\mu=0$

		0	1	2	3	4	5	6	7	8	9	10	Home
	0	f	f	f	f	f	f	f	f	f	f	---	
	1	f	f	f	f	f	f	f	f	f	---		
	2	f	f	f	f	f	f	f	f	---			
	3	f	f	f	f	f	f	f	---				
	4	f	f	f	f	f	f	---					
Foreign	5	f	f	f	f	f	---						
(dollars)	6	f	f	f	f	---							
	7	f	f	f	---								
	8	f	f	---									
	9	f	---										
	10	---											

Making Change

$\mu=0$

		0	1	2	3	4	5	6	7	8	9	10	Home
	0	2f	2f, -h	2f, -h	2f, -h	2f, -h	2f, -h	2f, -h	3f, -3h	3f, -3h	2f, -2h	2f, -2h	
	1	2f	2f, -h	2f, -h	2f, -h	2f, -h	2f, -h	3f, -3h	3f, -3h	3f, -3h	3f, -3h		
	2	2f	2f, -h	2f, -h	2f, -h	2f, -h	2f, -h	3f, -3h	3f, -3h	3f, -3h			
	3	f	2f, -h	2f, -h	2f, -h	2f, -h	2f, -h	2f, -h	3f, -3h				
	4	f	2f, -h	2f, -h	2f, -h	2f, -h	2f, -h	2f, -h					
Foreign	5	f	2f, -h	2f, -h	2f, -h	2f, -h	3f, -3h						
(dollars)	6	f	2f, -h	2f, -h	2f, -h	3f, -3h							
	7	f	2f, -h	2f, -h	3f, -3h								
	8	f	2f, -h	2f, -2h									
	9	f	f, -h										
	10	---											

For example, now this buyer spends two dollars on a (0,0) seller but one dollar on a (8_f,0) seller. More interestingly, this buyer engages in making change trades with nearly every other seller he meets. In some trades he gives up two dollars in order to receive goods plus a unit of the home currency (2f, -h) while in some others he gives up two dollars for goods and two units of home (2f, -2h) or 3dollars for goods and 3 units of home currency (3f,-3h). Thus, trading bundles of money and goods is the preferred trading strategy overall for this buyer. The problem with these trades of course is determining what the ‘price’ of the good is since two different currencies are involved in most trades. Thus, it is difficult to think about price dispersion in this environment.

Table 7 shows how increasing currency risk on the home currency affects the performance of the making change economy.

Table 7

Medium „Making Change“ Economy with Differing Currency Risk

Economic Data for N= 10, Risky Economies

Parameter Values: $N=10$, $M_1=M_2=3.33$, $\sigma = .15$, $\alpha=1$, $x = .45$, $r=.1$

	$\mu=0.0001$ $\eta=0.0003$	$\mu=0.005$ $\eta=0.0178$	$\mu=0.01$ $\eta=0.037$	$\mu=0.05$ $\eta=0.211$
Percentage of matches with trade:	89.5%	89.62%	89.46%	88.52%
Percentage of goods trades involving:				
Currency *f* (dollars) only:	9.88%	8.57%	8.65%	9.05%
Currency *h* (home) only:	43.12%	37.56%	38.59%	42.48%
Any dollars traded	56.88%	62.44%	61.41%	57.52%
Any home traded	90.12%	91.43%	91.36%	90.96%
Expected welfare:	2.63	2.62	2.60	2.44
Expected production:	0.0413	0.0413	0.0447	0.0655
Size of the currency market:	0.019%	0.11%	0.074%	0%

Compared to the ‘no change trades’ economy, for the most, the qualitative behavior is the same, although the quantitative changes are quite different. The percentage of trades that involve dollars only falls initially and then rises as before. The percentage of trades involving both currencies rises initially and then falls. This is because as risk increases, agents want to diversify portfolios via making change trades. However, once the risk gets sufficiently high, agents resort to the use of a single currency in goods trade. Welfare falls as it does in the no change economy. Surprisingly, average production actually increases as risk increases. This is because the quantity given up by sellers in making change trades rises as the value of the dollar increases relative to the home currency. Although the volume of trading in the currency market is much smaller than in the no change economy, the pattern is the same as before – it initially rises, then falls and eventually shuts down completely when the home currency risk is too large.

Table 8 shows the behavior of nominal exchange rates in this economy.

Table 8
Currency Market Exchange Rates for a Medium "Making Change" Economy

Nominal Exchange Rate	**Conditional Probabilities**			
(Home per dollar)	1 ***$\mu = 0$***	2 ***$\mu = 0.0001$***	3 ***$\mu = 0.005$***	4 ***$\mu = 0.01$***
2:1	1.00			
3:1		1.00		
4:1			.373	
5:1			.627	.415
6:1				.315
7:1				.270
Size of Currency Market	0.0025%	0.019%	0.11%	0.074%
Mean	2	3	4.627	5.856
Stand. Deviation	0	0	.484	.815

The qualitative results are the same as before – an increase in domestic currency risk causes the home currency to depreciate and the variance of the nominal exchange rate distribution to increase then decrease. The main quantitative difference is the reduction in the number of observed exchange rates, which is the result of having agents diversify their currency portfolios via making change trades as opposed to pure currency trades.

5.2.4 Very Large Portfolios N = 15, No Making Change Trades

In this section we increase the inventory constraint to N=15. With N=15 there are 136 different portfolio states, 120 potential sellers and a minimum of 220 prices. Despite the substantial increase in portfolio states, the same basic patterns occur here as when N=10. We also examined the case of N=15 when agents are allowed to engage in making change trades. The results displayed the same patterns as in the N=10 case and are thus not presented here.

Table 5 shows the economic data for the no change trade economy when we varied the degree of domestic currency risk:

Table 9
Large Portfolio Economy with Differing Currency Risk

Parameter Values: $N=15$, $M_f=M_h=5$, $\sigma = .15$, $\alpha=1$, $x = .45$, $r=.1$

	$\mu=.03$	$\mu=.05$	$\mu=.1$
Percentage of matches with trade:	40.21	39.40	37.40
Percentage of goods trades using:			
Currency f (dollars) only:	51.57	52.91	54.84
Currency h (home) only:	37.88	37.90	34.91
Expected welfare:	2.25	2.17	2.02
Expected production:	0.052	0.053	0.052
Expected dollar price:	10.53	9.04	7.79
Expected home price:	22.56	31.61	62.87
Implied real exchange rate:	2.14	3.50	7.66

Table 9 shows that increasing the degree of currency risk causes the frequency of goods trading to fall. Welfare also falls, while the average production increases. Production increases because fewer trades are occurring with rich sellers (since they are having their currency confiscated), hence the average quantity traded per match rises. Again, as in the case with N=10, the home price of goods increases while the dollar price of goods falls. Consequently, the implied real exchange rate between the two currencies shows a dramatic depreciation of the home currency. With regards to the degree of dollarization that occurs, we see a similar pattern as in the N=10 case. Initially, the percentage of dollar only trades falls while the percentage of trades involving goods for a bundle of the two currencies increases. In short, buyers now start to give up some home and foreign currency units rather than using dollars only. This is consistent with idea that at low levels of risk, buyers start dumping the home currency on sellers. However, as the currency risk rises, buyers increasing resort to using dollars since the quantities purchased with the home currency fall dramatically. Thus for sufficiently high levels of risk, currency competition kicks in and dollarization occurs.

Table 10 shows the nominal exchange rate distributions for the risky economies:

Table 10
Currency Market in Large Portfolio Economy with Differing Currency Risk

Nominal Exchange Rate	**Conditional Probability**		
(Home per Dollar)	1 μ=.03	2 μ=.05	3 μ=.1
3:2 = 1.5	0.012		
5:3 = 1.67	0.001		
2	0.427	0.078	
7:3 = 2.33	0.003		
5:2 = 2.5	0.120	0.039	
8:3 = 2.67	0.000		
3	0.430	0.368	0.050
7:2 = 3.5	0.000	0.013	
4	0.007	0.330	0.149
9:2 = 4.5		0.000	
5	0.000	0.149	0.228
6		0.023	0.205
7		0.000	0.188
8		0.000	0.115
9			0.048
10			0.016
11			0.000
12			0.000
Size of Currency Market	1.11%	1.07%	0.41%
Mean	2.5	3.61	5.95
Stand. Deviation	0.492	0.927	1.617

From Table 6 we see that the number of exchange rates observed increases significantly when the inventory restriction is loosened. We now observe 10 nominal exchange rates across these economies. However, as in the case with N=10, most of the mass is distributed over a couple of nominal exchange rates until the currency risk gets very large. As before, an increase in currency risk leads to a nominal depreciation of the home currency against the dollar and a significant increase in the variance of the nominal exchange rate. The standard deviation is over 3 times larger in the high-risk economy than the low risk economy. The currency market is most active when the currency risk is low; as risk increases, the volatility of the exchange rate and the risk associated with holding home currency, leads to a reduction in activity in the 'financial' market. Thus, greater risk and uncertainty leads to a reduction in financial market trading volume.

As a final exercise, we artificially closed down currency exchange in order to see what the welfare effects were from having the ability to trade currencies to arbitrage risk. In general we found that the existence of the currency market led to a 1% point increase in welfare. While not dramatic, this is in part due to the small size of the currency market and in part to the ability of agents to simply use bundles of currency to trade for goods. For a few parameterizations (particularly when N is small) we found much more dramatic effects on welfare – as much as a 4% point increase in welfare. Thus, the welfare effects of currency exchange appear to be important but small in our model.

6 Conclusions

In this paper we have developed an analyzed a model that generates currency exchange in an economy even though both currencies are accepted as media of exchange. As a result we are able to obtain equilibrium nominal exchange rates based on a model in which money is essential. Thus, starting from first principles, we can study how currency risk affects the exchange value of two distinct currencies in a single economy. We have also contributed to the growing literature involving the distribution of money and prices in search theoretic model. We have shown that monetary equilibria exist in which both currencies circulate as media of exchange. New aspects of our work that have not been studied in previous work are the issue of portfolio diversification, the endogenous determination of nominal exchange rates and the role of risk aversion in portfolio choice. Our comparative static analysis revealed sensible predictions regarding how the values of currencies change after various changes in policy regimes. They also revealed some surprising results regarding the behavior of the nominal exchange rate distributions that need to be explored further. We have also been able to study the issues of dollarization and currency substitution from first principles without having to resort to artificial restrictions on which currency is used as a medium of exchange. Dollarization is the natural outcome of optimizing agents in the face of risky domestic currencies. Consequently, it is a market response to bad government policy rather than a regime dictated by the government. Further work in this area is needed to understand the consequences of dollarization and what governments can do to prevent it. One additional easy extension to our work is to allow agents to trade in several different foreign currencies. This would allow one to analyze the effect of a new intenational currency competitor such as the Euro and the Dollar on countries with weak currencies.

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Table A1

Transaction Patterns for a Poor Seller

Buyer's transaction pattern with a (0,0) seller

$ denotes spending 1 unit of foreign currency

Numerals denote number of home currency units spent

Foreign denotes buyer's foreign currency, Home denotes buyer's home currency holdings

μ=0

		0	1	2	3	4	5	6	7	8	9	10	Home		
	0	---	1	1	1	1	1	1	1	1	1	1			
	1	$	1	1	1	1	1	1	1	1	1				
	2	$	1	1	1	1	1	1	1	1					
	3	$	1	1	1	1	1	1	1						
	4	$	1	1	1	1	1	1							
Foreign	5	$	1	1	1	1	1								
	6	$	1	1	1	1									
	7	$	1	1	1										*Prices*
	8	$	1	1										df=1	1.70
	9	$	1											dh=1	1.70
	10	$													

μ=.01

		0	1	2	3	4	5	6	7	8	9	10	Home		
	0	---	1	1	1	1	1	1	1	1	2	2			
	1	$	1	1	1	1	1	1	1	1	2				
	2	$	1	1	1	1	1	1	1	2					
	3	$	1	1	1	1	1	1	2						
	4	$	1	1	1	1	1	2							
Foreign	5	$	1	1	1	1	2								
	6	$	1	1	1	2									
	7	$	1	1	2										*Prices*
	8	$	1	2										df=1	1.67
	9	$	**1, $**											dh=1	1.77
	10	$												dh=2	2.00

μ=.05

		0	1	2	3	4	5	6	7	8	9	10	Home		
	0	---	1	1	1	1	1	1	1	2	2	2			
	1	$	1	1	1	1	1	1	2	2	2				
	2	$	1	1	1	1	1	2	2	2					
	3	$	1	1	1	1	2	2	2						
	4	$	1	1	1	2	2	2							
Foreign	5	$	1	1	2	2	2								
	6	$	1	1	2	2									
	7	$	1	2	2										*Prices*
	8	$	1	2										df=1	1.57
	9	$	**1, $**											dh=1	2.06
	10	$												dh=2	2.41

μ=.10

		0	1	2	3	4	5	6	7	8	9	10	Home		
	0	---	1	1	1	1	1	1	2	2	2	3			
	1	$	1	1	1	1	1	2	2	2	3				
	2	$	1	1	1	1	2	2	2	2					
	3	$	1	1	1	2	2	2	3						
	4	$	1	1	2	2	2	3							
Foreign	5	$	1	1	2	2	3								*Prices*
	6	$	1	2	2	3								df=1	1.59
	7	$	1	2	3									dh=1	2.45
	8	$	**1, $**	2										dh=2	2.99
	9	$	**1, $**											dh=3	3.59
	10	$													

Table A2

Transaction Patterns for a Foreign Currency Rich Seller

Buyer's transaction pattern with a (7f,0h) seller

$ denotes spending 1 unit of foreign currency

Numerals denote number of home currency units spent

Foreign denotes buyer's foreign currency, Home denotes buyer's home currency holdings

μ=0

		0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	**Home**	
	0	---	1	1	1	1	1	1	1	1	1	1		
	1	$	1	1	1	1	1	1	1	1	1			
	2	$	1	1	1	1	1	1	1	1				
	3	$	1	1	1	1	1	1	1					
	4	$	1	1	1	1	1	1						
Foreign	*5*	$	1	1	1	1	1							
	6	$	1	1	1	1								
	7	$	1	1	1									***Prices***
	8	$	1	1									df=1	9.26
	9	$	$										dh=1	9.26
	10	$												

μ=.01

		0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	**Home**	
	0	---	1	1	1	1	1	1	1	1	1	1		
	1	$	1	1	1	1	1	1	1	1	1			
	2	$	1	1	1	1	1	1	1	1				
	3	$	1	1	1	1	1	1	1					
	4	$	1	1	1	1	1	1						
Foreign	*5*	$	1	1	1	1	1							
	6	$	1	1	1	1								
	7	$	1	1	1									***Prices***
	8	$	$	1									df=1	8.98
	9	$	$										dh=1	10.32
	10	$												

μ=.05

		0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	**Home**	
	0	---	1	1	1	1	1	1	1	1	2	2		
	1	$	1	1	1	1	1	1	1	2	2			
	2	$	1	1	1	1	1	1	2	2				
	3	$	1	1	1	1	1	2	2					
Foreign	*4*	$	1	1	1	1	2	2						
	5	$	1	1	1	2	2							
	6	$	1	1	2	2								
	7	$	$	$	2									***Prices***
	8	$	$	$									df=1	9.06
	9	$	$										dh=1	17.76
	10	$											dh=2	20.82

μ=.10

		0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	**Home**	
	0	---	1	1	1	1	1	1	2	2	2	2		
	1	$	1	1	1	1	1	1	2	2	2			
	2	$	1	1	1	1	1	2	2	2				
	3	$	1	1	1	1	2	2	2					
Foreign	*4*	$	1	1	1	2	2	2						
	5	$	1	1	2	2	2							
	6	$	$	$	2	2								
	7	$	$	$	$									***Prices***
	8	$	$	$									df=1	9.28
	9	$	$										dh=1	31.76
	10	$											dh=2	37.81

Table A3

Transaction Patterns for a Domestic Currency Rich Seller

Buyer's transaction pattern with (0f,7h) seller

$ denotes spending 1 unit of foreign currency

Numerals denote number of home currency units spent

Foreign denotes buyer's foreign currency, Home denotes buyer's home currency holdings

μ=0

		0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	**Home**	
	0	---	1	1	1	1	1	1	1	1	1	1		
	1	$	$	$	$	$	$	$	$	$	1			
	2	$	$	$	$	$	$	$	$	$				
	3	$	$	$	$	$	$	$	$					
	4	$	$	$	$	$	$	$						
Foreign	***5***	$	$	$	$	$	$							
	6	$	$	$	$	$								
	7	$	$	$	$									
	8	$	$	$										***Prices***
	9	$	$										df=1	9.26
	10	$											dh=1	9.26

μ=.01

		0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	**Home**	
	0	---	1	1	1	1	1	1	1	1	1	1		
	1	$	$	$	$	$	$	1	1	1	1			
	2	$	$	$	$	$	$	$	$	$				
	3	$	$	$	$	$	$	$	$					
	4	$	$	$	$	$	$	$						
Foreign	***5***	$	$	$	$	$	$							
	6	$	$	$	$	$								
	7	$	$	$	$									
	8	$	$	$										***Prices***
	9	$	$										df=1	7.02
	10	$											dh=1	10.83

μ=.05

		0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	**Home**	
	0	---	1	1	1	1	1	1	1	1	2	2		
	1	$	$	$	1	1	1	1	1	2	2			
	2	$	$	$	$	$	$	$	$	2				
	3	$	$	$	$	$	$	$	$					
	4	$	$	$	$	$	$	$						
Foreign	***5***	$	$	$	$	$	$							
	6	$	$	$	$	$								
	7	$	$	$	$									***Prices***
	8	$	$	$									df=1	3.96
	9	$	$										dh=1	20.94
	10	$											dh=2	21.98

μ=.10

		0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	**Home**	
	0	---	1	1	1	1	1	1	2	2	2	3		
	1	$	$	$	1	1	1	2	2	2	3			
	2	$	$	$	$	$	$	$	$	2				
	3	$	$	$	$	$	$	$	$					
	4	$	$	$	$	$	$	$						
Foreign	***5***	$	$	$	$	$	$							
	6	$	$	$	$	$								***Prices***
	7	$	$	$	$								df=1	2.98
	8	$	$	$									dh=1	35.79
	9	$	$										dh=2	40.27
	10	$											dh=3	44.03

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