Alternative specifications of the German term structure and its information content regarding inflation

Sebastian T. Schich

Discussion paper 8/96

Economic Research Group

of the Deutsche Bundesbank

October 1996

Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main,
P.O.B. 10 06 02, 60006 Frankfurt am Main, Federal Republic of Germany

Telephone (0 69) 95 66-1
Telex within Germany 4 1 227, telex from abroad 4 14 431, fax (0 69) 5 60 10 71

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address, or via fax No. (0 69) 95 66-30 77

ISBN 3–932002–12–1

Alternative specifications of the German term structure and its information content regarding inflation

Summary

The purpose of the present paper is twofold. First, it describes zero-coupon yield curve estimates for Germany from September 1972 to February 1996 using a variety of curve-fitting procedures. Second, these estimates are tested for their information content regarding future inflation.

The German yield curve is considered in various specifications. Zero-coupon yields are estimated using the procedures of Nelson and Siegel (1987) and Svensson (1994), both in their original form and augmented by an adjustment for the tax-induced coupon effect. These procedures provide a more accurate description of the observed data than the specification used by the Bundesbank, which is likewise taken into account. In the light of test statistics, such as the root mean squared percentage error and the (degrees-of-freedom-adjusted) coefficient of determination, the Svensson procedure is favoured. The Svensson as well as the Nelson/Siegel specifications give a more accurate account of the yield curve than the Bundesbank procedure in situations where this curve is either very steep or has a complicated form (S-shaped, U or inverted U-shaped), the incidence of such situations having increased noticeably since the beginning of the 1990s. From a monetary policy point of view, however, the quality of the estimates determined by such test statistics does not tell us very much. The decisive criterion for choosing among curve-fitting procedures should be the ultimate purpose for which the estimated yields are used. Since the Bundesbank uses the slope of the yield curve as an indicator of financial market expectations of future inflation changes, the well-known Mishkin approach to testing the information content of the yield curve is adopted here and applied to the estimates from the various procedures. Under this approach the information content is defined as the ability of the yield curve's slope to predict future changes in the inflation rate.

The paper finds that the German yield curve is informative in the sense defined above, especially in its middle segment between three and eight years. Furthermore, it finds that this result is robust with respect to these specifications of the yield curve. Thus, from a monetary policy perspective, the following conclusions may be drawn. The medium-term segment of the yield curve does indeed constitute a useful indicator of future inflation changes. As long as the interpretation of this curve is confined to simple linear inference from its slope to future changes in inflation rates, the choice of the yield curve estimation method is of minor importance. To that extent the curve fitting procedure used by the Bundesbank is appropriate in view of the uses to which the yield estimates were put in the past.

Contents

List of tables

List of figures

Alternative specifications of the German term structure and its information content regarding inflation*

I. Introduction

As price stability is the prime objective of monetary policy, it is useful for a central bank to monitor inflation indicators. This is especially true of a central bank that pursues direct inflation targeting, but also applies to a central bank that follows an intermediate-target strategy. Such information could contribute to the central bank's own inflation forecasts. These information are interesting as such. Even if such indicators do not represent accurate forecasts of the future path of inflation, they could be a useful source of information on the markets' perceptions of the stance of monetary policy, which in turn could be relevant to short-term implementation strategy.

The Bundesbank looks at several indicators of financial market expectations, among them the yield-to-maturity curve derived from the prices of government debt securities. Other central banks, in particular those that pursue direct inflation targeting, have recently intensified efforts to extract information about market expectations of future inflation from the prices of government debt securities. This includes the estimation of the term structure of interest rates.

This paper presents such term structure estimates for German government debt securities from September 1972 to February 1996 using a variety of specifications. These estimates are then tested for their information content regarding future inflation.[1] Thus the purpose of the paper is twofold; it tests whether the German term structure contains information on future inflation and whether the results of that test are robust with respect to the choice of the procedure for estimating the term structure.

The methodological approach consists of a specific interpretation of the expectations hypothesis, which states that the current slope of the term structure is related to future changes in inflation rates. This follows Mishkin (1990a, 1990b, 1991), Jorion and Mishkin (1991), Gerlach (1995) and Koedijk and Kool (1995), but differs from those authors in several respects. First, the term structure in the form of zero-coupon yields is considered, which is

* Special thanks go to Jörg Meier from the Bundesbank's Statistics Department for very helpful discussions and assistance. Jörg Clostermann, Arturo Estrella, Robert Fecht, Mark Fisher, Stefan Gerlach, Daniel Hardy, Franz Seitz and Karl-Heinz Tödter provided helpful comments. Furthermore, I would like to thank the Bank of England for their hospitality and for providing me with their yield curve estimation code and in particular Francis Breedon, John Lumsden, Vicky Read, Jim Steeley, Paul Wesson and Sanjay Yadav for helpful discussions.

1 The term structure has also been used as a predictor of future interest rates (see e.g. Shiller (1990) for a summary of the extensive literature) and of real economic activity (recently Estrella and Mishkin (1995)).

consistent with the theory. This has the advantage that it does not have to use yield-to-maturity estimates as approximations of the term structure.[2] Second, it extends the papers mentioned by applying a variety of curve-fitting approaches to the data when the information content of the estimated yield curve is examined. In addition to the estimation method used by the Bundesbank since 1981,[3] the approaches of Nelson and Siegel (1987) and Svensson (1994), both in their original form and augmented by an adjustment for the tax-induced coupon effect, are considered.[4] The original Nelson and Siegel (1987) and Svensson (1994) approaches are interesting because they have recently been implemented at a large number of central banks (see appendix table A.1). By considering various approaches the paper explicitly takes account of the fact that tests of the expectations theory are in fact tests of a joint hypothesis, namely, that the fitted yield curve gives an undistorted picture of the relevant information and, at the same time, that the specific formulation of the expectations hypothesis adopted is valid.[5] Third, the test is applied to all combinations of rates with horizons from one to ten years. This provides a better understanding of how the information content varies along the term structure. Fourth, the examination is carried out on a different sample, namely monthly data for Germany from September 1972 to February 1996.[6] This is, however, similar to Gerlach (1995).

Section II describes the relationship between bond prices and different interest rate concepts. Section III explains the basic problems of fitting yield curves from the observed prices of bonds and reviews the various approaches considered here. Section IV describes the term structure estimates. Section V reports the results of the tests on the information content for two different term structure estimates. Section VI sets out the conclusions.

---

2 Whereas Estrella and Mishkin (1995) and Gerlach (1995) use the Bundesbank's published yield-to-maturity estimates directly as approximations of zero-coupon yields, Jorion and Mishkin (1991) interpret them as par rates, that is as yields-to-maturity of bonds traded at par. Koedijk and Kool (1995) consider their own yield-to-maturity estimates, which are obtained from a regression analysis similar to the Bundesbank's approach.

3 The data are similar but not identical to those published by the Deutsche Bundesbank. The latter differ in three respects. First, bonds with remaining time-to-maturity below 12 months are excluded (here below three months). Second, two different functional approaches are used with the break occuring in January 1981 (here only the more recent approach). Third, betweentimes, specific sets of debt securities were excluded and later reintroduced (here a consistent data set is used throughout the sample period).

4 The Bank of England kindly provided me with their code for estimating German zero-coupon yield curves. In particular, I would like to thank John Lumsden and Paul Wesson for helpful advice. The Bank of England has been applying this code to German government debt securities since 1992. A description of a recent application to daily estimates for Germany can be found in Cooper and Steeley (1996), and a description of the theoretical aspects of the estimation approaches e.g. in Breedon (1995), Deacon and Derry (1994b), and Mastronikola (1991).

5 Furthermore, as has long been recognised, such tests are also tests of the joint hypothesis that the specific formulation of the expectations hypothesis holds, namely that expectations are rational and that real interest rates follow an appropriate time path.

6 The sample is dictated by the data availability, i.e. the raw data needed for estimating the term structures are only available from September 1972. Nevertheless, the sample is considerably longer than those of most other studies mentioned, which allows more precise estimates of the model. The exception is Gerlach (1995) who was able to use a slightly longer sample since he does not use the raw data but rather the Bundesbank's estimates which are available from January 1986.

II. Bond prices and interest rate concepts

Bonds can be defined as commitments by the debtor to make payments to the bondholder of predetermined amounts and at predetermined dates. Thus, the valuation of bonds is in principle easy because future payments are not contingent; instead, known payment flows can be discounted to present values. Although in practice bonds generally differ from this theoretical ideal, bonds and notes of the Federal Republic of Germany come close to the theoretical ideal because the risk of default is negligible and because these debt securities - at least since the end of the eighties - carry neither put or call options nor sinking funds. In this section the valuation of such bonds is used to explain different interest rate concepts, in particular zero-coupon yields and yields-to-maturity.

The simplest type of a bond is one which guarantees one single payment N at a predetermined future date. The price of such a bond with M years to maturity is given by its discounted cashflow:[7]

$$P=\frac{N}{(1+z_M)^M}. \tag{1}$$

The interest rate z_M denotes the zero-coupon interest rate for M years, because the bond guarantees only one payment with no intermediate coupon payments (hence zero-coupon).

The valuation of a bond which guarantees several payments is in principle the same. A bond which guarantees coupon payments of C each year m (with $m = 1, 2, ... M$) and a redemption payment N at the time to maturity M can be considered simply as a portfolio of zero-coupon bonds. If the interest rates of the various zero-coupon bonds are denoted by z_m, the price of the bond is given as:

$$P=\frac{C}{(1+z_1)}+\frac{C}{(1+z_2)^2}+....+\frac{C}{(1+z_M)^M}+\frac{N}{(1+z_M)^M}=\sum_{m=1}^{M}\frac{C}{(1+z_m)^m}+\frac{N}{(1+z_M)^M}. \tag{2}$$

The interest rates z_m for the various maturities m define the *term structure of interest rates* or zero-coupon yield curve.

There is a one-to-one correspondence between the zero-coupon yields and the implied forward rates, denoted by f_m. Whereas zero-coupon rates represent interest rates from the present time up to a specified future date, implied forward rates apply from a specified future date over a

7 For ease of exposition we assume that the maturity is an integer number of years. Of course, in the empirical estimation that follows maturities vary along a continuum.

specified period. They are termed implied because they cannot be observed directly but are derived instead from the zero-coupon yield curve. In principle, however, they can be guaranteed immediately by means of suitable spot transactions. If the length of the period over which the forward rate applies tends towards the marginal value of zero, one obtains the so-called instantaneous forward rate. This is of little relevance in practice because spot transactions with bonds whose payment dates are only marginally different are usually not concluded in view of the transaction costs. However, it is important in theory: the yield curve estimation approaches described in section III.2.2 are formulated in terms of an assumption concerning the instantaneous forward rate.

Another useful concept is the discount function, δ_m, which is simply a transformation of the appropriate spot rate, $\delta_m = (1+z_m)^{-1}$. Using this concept, the price of a bond can be expressed as the sum of the products of the payments (coupon and redemption payments) and the corresponding discount factors:

$$P=\sum_{m=1}^{M}\delta_m C+\delta_M N=\sum_{m=1}^{M}\delta_m A \ , \tag{3}$$

where A captures the payment profile (coupon and redemption payments). A useful property of the discount function is that it describes the present value of one unit payable at any time in the future. Thus, the value of a bond that provides a single payment of one unit after m years is given by the value of the discount function at that point, δ_m. Such a bond is a zero-coupon bond as described in (1), and therefore the discount factor is also referred to as the zero-coupon bond price. This concept is useful in the implementation of the zero-coupon rate estimates because it is linearly related to the price (see also sub-section III.2.2).

In practice, the price of a bond is often expressed in terms of yields-to-maturity because they are easy to interpret and to calculate. Following from (2), the price in terms of yield-to-maturity can be expressed as follows:

$$P=\frac{C}{(1+r_M)}+\frac{C}{(1+r_M)^2}+\ldots+\frac{C}{(1+r_M)^M}+\frac{N}{(1+r_M)^M}=\sum_{m=1}^{M}\frac{C}{(1+r_M)^m}+\frac{N}{(1+r_M)^M} \ . \tag{4}$$

The yield-to-maturity r_M for maturity M gives the average rate of interest of a bond assuming that all coupon payments occuring during the lifetime of the bond (i.e. at $m = 1, 2, \ldots, M$) are reinvested at exactly the same rate of interest r_M. It therefore assumes that the term structure of interest, defined in terms of z_m (with $m = 1, 2, \ldots, M$ as in (2)) is flat and always equal to z_M for that security. The yields-to-maturity r_M for various maturities M define the yield-to-maturity curve.

Only the variables *P*, *C*, *N*, and *m* (and thus *M*) are observables, the interest rates r_M, z_m and f_m and the discount function δ_m must be derived from these. The yield-to-maturity, r_M can be obtained directly from equation (4) with the help of an iterative algorithm (for example, Newton-Raphson) because it is the only unknown variable in this equation. By contrast, deriving z_m, as well as f_m and δ_m is more complicated.[8] The problems involved in estimating z_m (which we are primarily interested in) are discussed in section III. One way to circumvent the complications is to simply take the yields-to-maturity, r_M as a proxy for z_m. What are the implications of this for the yield and, of particular interest in our context, the yield spread estimates? To answer this question (2) is set equal to (4):

$$\sum_{m=1}^{M}\frac{C}{(1+z_m)^m}+\frac{N}{(1+z_M)^M}=\sum_{m=1}^{M}\frac{C}{(1+r_M)^m}+\frac{N}{(1+r_M)^M}, \qquad (5)$$

This expresses the (non-linear) relationship between the zero-coupon yields and the yield-to-maturity. For expositional purposes the case of two-period bonds with a redemption payment of one is considered. Then (5) implies:

$$\frac{C}{(1+z_1)}+\frac{C}{(1+z_2)^2}+\frac{1}{(1+z_2)^2}=\frac{C}{(1+r_2)}+\frac{C}{(1+r_2)^2}+\frac{1}{(1+r_2)^2}. \qquad (6)$$

Expanding (6) and ignoring higher-order terms which are negligible for yields ranging between 0.01 and 0.1 (on an annual basis) gives

$$r_2 \cong \omega\, z_1+(1-\omega)z_2, \text{ with } \omega=\frac{C}{2+3C}. \qquad (7)$$

This illustrates that the yield-to-maturity is a weighted average of the zero-coupon yields, where the weights depend on the payment stream (i.e. the coupon and redemption payments and the number of total payments). It also illustrates that the yield-to-maturity and the zero-coupon rate are identical if the term structure of interest rates is flat, i.e. if $z_2 = z_1$.

The spread between the one-year yield-to-maturity and the two-year yield-to-maturity, $r_2 - r_1$, expressed in terms of the corresponding zero-coupon yields is given by:

$$r_2-r_1=(1-\omega)(z_2-z_1). \qquad (8)$$

In the case of coupon bonds C is positive by definition and thus $(1-\omega)$ is strictly smaller than one. Consequently, the yield-to-maturity spread, $r_2 - r_1$, is strictly smaller than the zero-

[8] Note that absence of arbitrage opportunities implies that they are all unequivocally algebraically related. Thus, knowing any one of the three means that the other two can be readily computed.

coupon rate spread, $z_2 - z_1$. Correspondingly, the variance of the former is strictly smaller than the variance of the latter, namely by the factor $(1-\omega)^2$.

Thus the yield-to-maturity spread systematically underestimates the level and variance of the zero-coupon yield spread. Moreover, the extent of that bias might vary over time as a result of the coupon effect, signifying the dependence of the yield-to-maturity of a bond with a particular remaining time to maturity on its coupon. As can be seen from (8), other things being equal, a higher coupon implies a larger difference between the yield-to-maturity spread and the zero-coupon rate spread. This is termed here the "mathematical" coupon effect, to be distinguished from other types of coupon effects such as the tax-induced coupon effect. The yield curve-fitting approaches discussed here differ as to which curve they are fitting (the yield-to-maturity curve or directly the zero-coupon yield curve). But they also differ in other respects, as discussed in the next section.

III. Curve-fitting approaches

III.1 Basic problems of curve-fitting approaches

The term structure of interest rates is defined as the relation between the (remaining) time to maturity and the interest rate on zero coupon bonds. Ideally, we want a continuous curve, so that each maturity is assigned exactly one rate of interest and the value could be read off at any point, e.g. at maturity equals one year, two years etc.. To establish such a continuous term structure, we would really need a continuum of homogeneous default risk-free zero coupon bonds. In reality, however, only a *finite number* of bonds is traded, whose prices define only a finite number of observation points. Moreover, the bonds available are ***coupon bonds***, whereas the term structure is defined in terms of interest rates on zero-coupon bonds and therefore does not follow directly from the prices or yields on these bonds. Also, the existence of *different coupons* for different bonds has the effect that the demand for them, and hence their yields, are affected by other factors such as tax considerations, especially if different tax rates apply to (coupon) interest income and to capital gains.

An attempt must be made to construct a continuous term structure curve on the basis of the observed points. This gives rise to the question as to what shapes this curve should be allowed to take. A decision has to be taken on the trade-off between "smoothness" - the elimination of "noise" in the data - and "responsiveness" - the flexibility to accommodate genuine movements in the term structure. More flexible procedures allow a more precise description of the data and imply smaller deviations of the estimated from the observed yields-to-maturity.[9] This can be very important for those responsible for securities issues or market management operations where the interest rate structure is used as a basis for valuing financial market instruments. A smoother curve, on the other hand, is generally preferred by central banks for monetary policy purposes, e.g. if the focus is on using the term structure as an indicator of market expectations regarding future interest rate and inflation trends. Foregoing flexibility, it is possible to achieve a greater measure of simplicity in terms of the estimation methods and the interpretation of the results. Sacrificing too much flexibility in exchange for greater simplicity, however, runs the risk of constraining the curve so much that relevant information on market expectations is lost. A criterion for deciding on the appropriate trade-off is suggested in section V.

German Federal bonds are issued as coupon bonds, thus representing bundles of zero-coupon bonds as shown in (2).[10] Furthermore, the coupons C in equation (4) cannot be traded separately. Such a possibility, commonly referred to as coupon-stripping, is currently discussed in Germany, but a decision has not yet been taken. Thus, since only one price can be observed for this bundle of zero-coupon yields, estimating z_m is more complicated (see sub-section III.2.2) than placing an estimated curve directly through the yields-to-maturity. However, this latter procedure is theoretically questionable, since the concept of yield-to-maturities is an ambiguous concept suffering in particular from the "mathematical" coupon effect, as discussed in the previous sub-section.

Other types of coupon effects may arise because of preferred habitats or tax considerations. For example, a tax-induced coupon effect arises because of the fact that interest income and capital gains are subject to different tax rates. For German individuals liable to personal income tax, interest income from bonds is subject to taxation, whereas realised capital or price gains (as long as they are not realised within six months) are tax-free. The result is that there is particularly strong demand for bonds with a low coupon. Consequently they carry a premium compared with high coupon bonds. Stated differently, the latter have a systematically higher yield than the former. This effect has to be taken into account in the curve-fitting approach by either initially translating the yields-to-maturity observed into "effective after-tax rates", or by correcting the impact of this effect on interest rates as part of the estimation approach (which is the approach adopted here).

9 Remember that the only observable interest rate concept is the yield-to-maturity. Even if zero-coupon yields are estimated, the observed yields-to-maturity (or prices) provide the benchmark against which the quality of the estimates must be judged. See also sub-section III.2.2.

10 In some countries, for example the US and France, government zero-coupon bonds exist, but only for certain maturities.

III.2 Selected curve-fitting approaches

Five different curve-fitting approaches are considered here. Besides the approach to estimating a yield-to-maturity structure adopted by the Bundesbank since 1981, the term structure estimation approaches of Nelson and Siegel (1987), henceforth referred to as Nelson/Siegel, and of Svensson (1994), both in their original format and augmented by an adjustment for the tax-induced coupon effect, are considered. The latter four are implemented on the basis of an estimation programme provided by the Bank of England. In the case of all approaches the parameters are estimated for each observation date separately. That is, the parameters are allowed to change from month to month.

III.2.1 Deutsche Bundesbank

The Bundesbank approximates the term structure by placing a curve through the observed yield-to-maturities. This has the advantage that it is computationally easy to implement, but - as mentioned - can be criticised from a theoretical point of view. The Bundesbank has been aware of this. Since 1981 a mixed linear logarithmic regression approach of the following form has been used:[11]

$$r_{t,m,i} = \beta_0 + \beta_1 m_i + \beta_2 \ln m_i + \beta_3 C_i + \beta_4 \ln C_i + \varepsilon_i, \; i \in \{1, 2, \ldots, n_t\}, \tag{9}$$

where $r_{t,m,i}$ is the (empirical) yield-to-maturity of bond i calculated from its observed price at time t, m_i is the remaining time to maturity of the bond i, C_i is the coupon of the bond i and n_t is the number of securities included. The coefficients are estimated by minimising the root mean squared yield-to-maturity error. The estimated regression coefficients $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\beta}_4$ are then used to construct the yield curve for hypothetical bonds which are assumed to carry the same coupon $\overline{C}$. Thus,

$$\hat{r}_{t,m} = \hat{\beta}_0 + \hat{\beta}_1 m + \hat{\beta}_2 \ln m + \hat{\beta}_3 \overline{C} + \hat{\beta}_4 \ln \overline{C}, \tag{10}$$

where $\hat{r}_{t,m}$ denotes the yield-to-maturity estimate for maturity m at time t. The specification in terms of linear and logarithmic terms implies that the estimated yield-to-maturity curves are, as a rule, either flat, monotonicly declining or monotonicly rising (see figure 1). Owing to the

[11] For a detailed description see Deutsche Bundesbank (1983). Before 1981 the equation did not include the two terms involving the coupon. Thus the published yield data (Deutsche Bundesbank, Statistical Supplement to the Monthly Bulletin - Capital Market Statistics) is characterised by a break in terms of the functional approach used. Here, this approach is applied back to September 1972 to obtain consistent data.

logarithmic transformation, U-form shapes are possible only if there is an extreme curvature of the data.[12] Bulges or S-shapes cannot arise.

The inclusion of the coupon in (9) and (10) reflects an attempt to correct for the coupon effect. Effectively, the Bundesbank is estimating a vector of yield curves, each curve representing a different constant coupon. The yield curve that is actually chosen is the one that represents the average coupon of all bonds considered in the regression.[13] Consider an increase in the assumed coupon at this particular point in time. This causes a parallel shift in the yield curve, the size of which depends on the parameter constellation. If both $\hat{\beta}_3$ and $\hat{\beta}_4$ are positive, the size of the shift decreases with the level of the coupon (because of the logarithm). If the coefficient $\hat{\beta}_3$ is positive and the coefficient $\hat{\beta}_4$ is negative (which is typically the case in practice), the curve first shifts upwards and then downwards again. This situation is depicted in figure 2, showing the term structure estimates as a function of the assumed constant coupon.[14] The figure illustrates that the variation in the assumed constant coupon is reflected in a parallel shift of varying size.

One problem with this approach is that there is no objective criterion for deciding on the appropriate constant coupon and hence on the yield curve. The functional form (with linear and logarithmic terms) associated with it is not derived from a theoretical model. Insofar the choice of the curve is inevitably subjective. Although this obviously is a theoretical weakness, it is less of a problem if one is primarily interested in the slope of the curve and not its absolute level. The former remains unaffected by changes to the constant coupon since such changes imply parallel shifts in the yield curve. Another problem is that the coupon effect is assumed to be constant across the maturity spectrum. This assumption may be criticised on the grounds that the coupon effect depends on the slope of the yield curve (the steeper the curve the larger is the "mathematical" coupon effect) and that some categories of investors (e.g. investors in a particular tax bracket) may have preferences for a particular debt maturity. However, the "mathematical" coupon effect is relatively small for maturities up to ten years. Also, too little is known about preferences in the German capital market for this effect to be easily taken into account.[15]

[12] In the sample considered here the Bundesbank method generates only one U-shaped curve.

[13] More precisely, it is the mean of the average coupons of different maturity classes (altogether 20 classes, the first one being bonds with remaining time to maturity between 1 year and 1.25 years, then 1.25 to 1.75, 1.75 to 2.25, etc.). As a general rule, the thus calculated mean differs from the true mean of the sample, and the extent of that difference varies over time. However, the effect is quantitatively negligible.

[14] The situation is complicated by the fact that the assumed constant coupon changes over time depending on the coupons of the papers traded and included in the regression.

[15] For the UK capital market Derry and Pradhan (1993) confirm the existence of tax clienteles, i.e. groups of investors who do not regard all bonds available in the market as perfect substitutes because of tax considerations. To our knowledge similar statistical evidence has not been produced in the case of the German market.

Figure 1: Yield curve shapes according to BBK

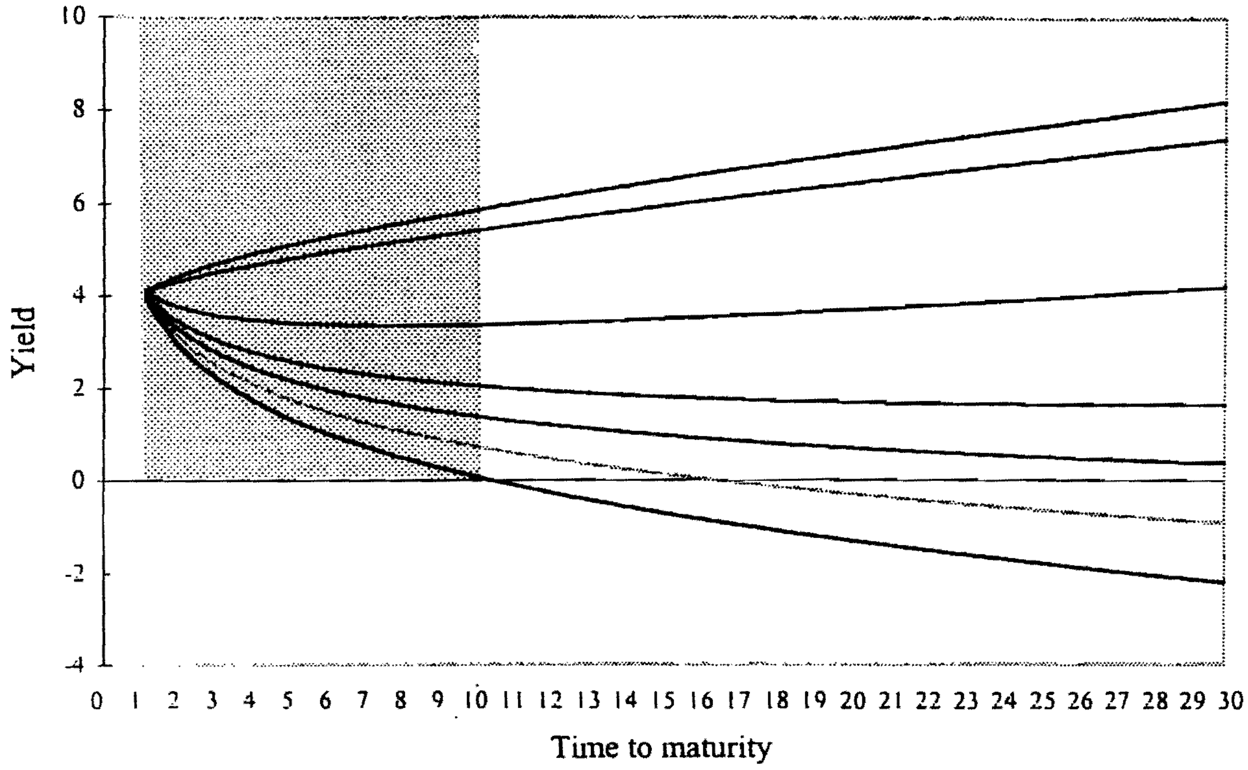
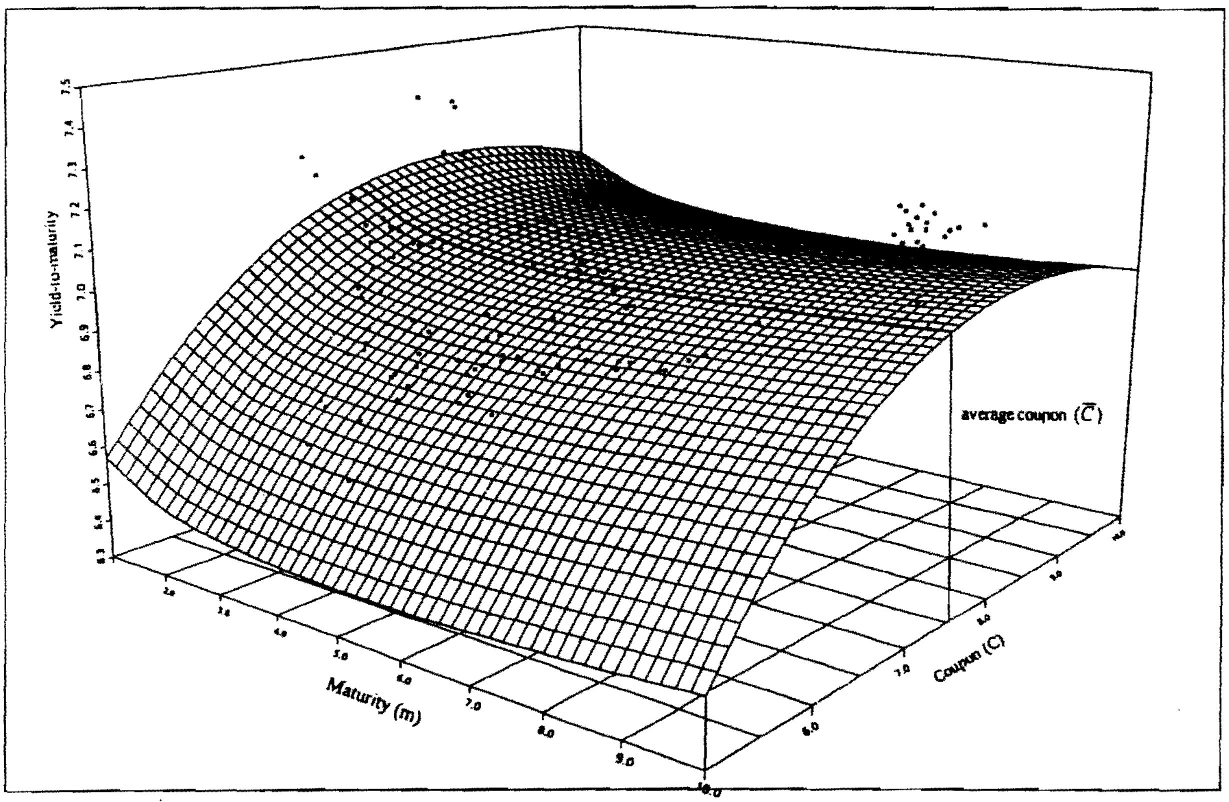


Figure 2: Yield-to-maturity estimates for different assumptions regarding the constant coupon, 1993:1



III.2.2 Nelson/Siegel and Svensson

The method developed by Nelson/Siegel and its extension by Svensson are attempts to estimate the relation between zero-coupon rates and the maturity. Both methods attempt to estimate zero-coupon rates and compare the theoretical yields-to-maturity compatible with those estimates (and the estimated discount function and forward rates) with the observed yields-to-maturity. More specifically, theoretical prices $\hat{P}_{t,i}$ for the i-th bond are obtained by discounting its payments with the corresponding discount factors:

$$\hat{P}_{t,i} = \sum_{m=1}^{M_i} A_i \delta_m(b_t), \tag{11}$$

where M_i is the number of coupon payments of the i-th security $(i=1,2,...,n_t)$, A_i is the payment profile of the i-th security (i.e. coupon and redemption payments, as explained in section II), δ_m are the discount factors and b_t is the vector of parameters to be estimated. From these theoretical prices theoretical yields-to-maturity can be obtained. The theoretical yield-to-maturity is assumed to differ from the observed yield-to-maturity by a measurement error, which is independently and identically distributed. The appropriate vector of parameters b_t, determining the zero-coupon rate estimates, is chosen by minimising the mean squared yield-to-maturity error over all n_t bonds. Yield errors are minimised instead of price errors because the primary concern here is in the yield and not price estimates. Minimising price errors could result in comparatively large yield errors for bonds with short remaining time to maturity because the prices of short-term bonds are relatively insensitive with respect to the yields.[16]

In what follows the assumptions regarding the parameters of b_t are discussed, where for notational simplicity the index i is suppressed. The (original) Nelson and Siegel approach is based on the assumption that the (instantaneous) forward rate at time t and maturity m can be described in terms of exponentials:

$$f_{t,m} = \alpha_0 + \alpha_1 \exp\left(\frac{-m}{\tau_1}\right) + \alpha_2 \frac{m}{\tau_1} \exp\left(\frac{-m}{\tau_1}\right), \tag{12}$$

with α_0, α_1, α_2 and τ_1 being the parameters to be determined. As a reason for choosing this functional form Nelson/Siegel state that it is able to capture the observable typical term structure conditions, namely, flat, U, inverted U, and S-shapes.

[16] Note that the elasticity of the price with respect to (one plus) the yield is defined as the duration, i.e. the present value-weighted average maturity of coupon payments and principal of a bond. Since the duration of short-term bonds is small, their yield errors would be penalised too little.

As explained in section II, the zero-coupon rates $z_{t,m}$ and the instanteneous forward rates are algebraically related. More specifically, the former represent an average of the latter: $(1+z_{t,m})^m = \prod_{\tau=1}^{m}(1+f_{t,\tau})$. From this an expression of $z_{t,m}$ in terms of the instantaneous forward rates can be derived, taking logarithms and using the approximation $\ln(1+x) \approx x$ gives $z_{t,m} = (1/m)\sum_{\tau=1}^{m} f_{t,\tau}$. Similarly, integrating (10) over the interval [0,*m*] and dividing by *m* yields the following function for the zero-coupon yield:

$$z_{t,m} = \left((\alpha_0 m - \alpha_1 \tau_1 \exp\left(\frac{-m}{\tau_1}\right) + \alpha_2 \tau_1 \left(1 - \exp\left(\frac{-m}{\tau_1}\right)\right) - \alpha_2 m \exp\left(\frac{-m}{\tau_1}\right) + \alpha_1 \tau_1) \right) m^{-1}. \tag{13}$$

Rearranging and substituting β_0 for α_0, β_1 for $(\alpha_1 + \alpha_2)$ and β_2 for $-\alpha_2$, another, simpler, formulation of the estimation function is obtained:

$$z_{t,m} = \beta_0 + \beta_1 \left(1 - \exp\left(\frac{-m}{\tau_1}\right)\right)\left[\frac{m}{\tau_1}\right]^{-1} + \beta_2 \exp\left(\frac{-m}{\tau_1}\right). \tag{14}$$

The shapes of the term structure which can be represented by this function are evaluated as follows. By establishing the limits of the function as *M* tends towards infinity and *M* towards zero, it can be shown that the long-term (estimated) zero-coupon yield is equal to the parameter β_0, and the short-term interest rate equal to the parameter combination $(\beta_0 + \beta_1 + \beta_2)$. If, for example, $(\beta_0 + \beta_1 + \beta_2)$ is set to zero and β_0 equal to 4 (and τ_1 is set to one), the curve can be written as a function of a single parameter β^*:

$$z_{t,m} = 4 - (1 - \beta^*)\left(\frac{1 - \exp(-m)}{m}\right) - \beta^* \exp(-m). \tag{15}$$

The possible shapes of the curve are presented in figure 3. It shows that flat, monotonicly falling or rising curves, as well as bulges and S-shaped curves, are possible. It also shows that the curve asymptotes to a constant at the very long end (for a brief discussion of this aspect see IV.2).

Svensson increased the flexibility of the Nelson/Siegel by adding a fourth form with two new parameters, α_3 and τ_2 to the forward rate equation:

$$f_{t,m} = \alpha_0 + \alpha_1 \exp\left(\frac{-m}{\tau_1}\right) + \alpha_2 \frac{m}{\tau_1} \exp\left(\frac{-m}{\tau_1}\right) + \alpha_3 \frac{m}{\tau_2} \exp\left(\frac{-m}{\tau_2}\right), \tag{16}$$

thus:

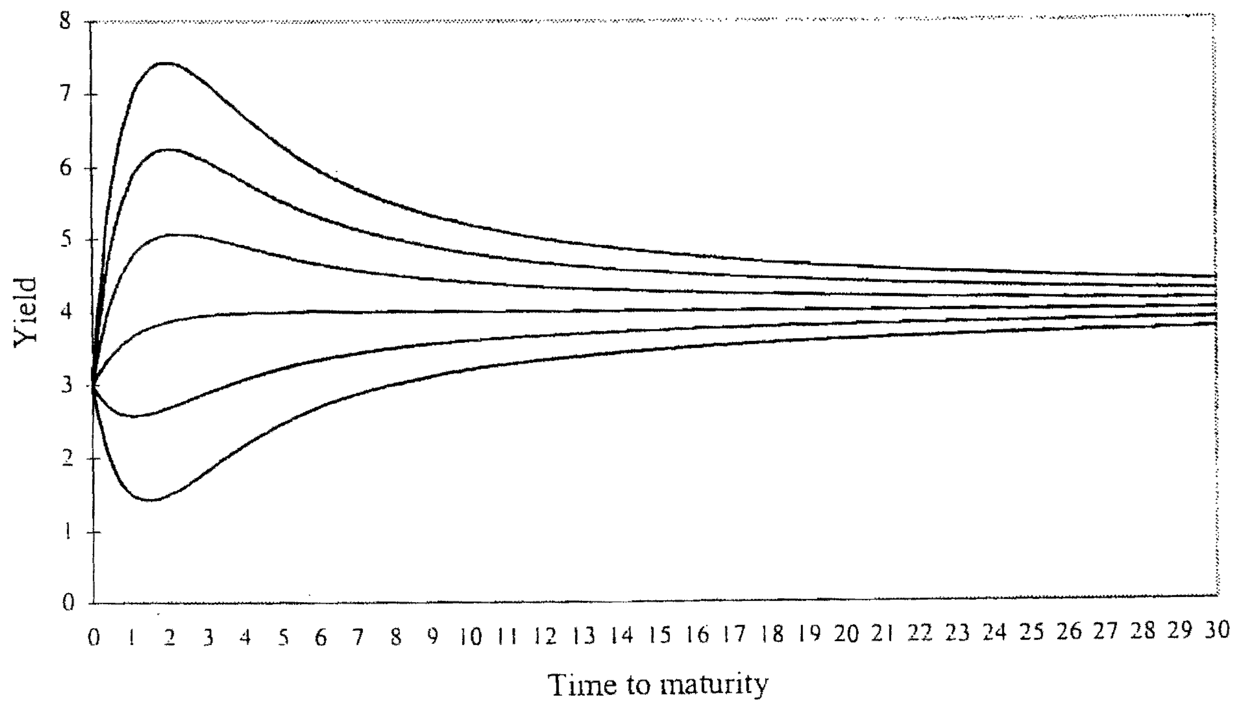
$$z_{t,m} = \beta_0 + \beta_1 \frac{1-\exp(-m/\tau_1)}{(m/\tau_1)} + \beta_2 \left(\frac{1-\exp(-m/\tau_1)}{(m/\tau_1)} - \exp(-\frac{m}{\tau_1}) \right)$$
$$+ \beta_3 \left(\frac{1-\exp(-m/\tau_2)}{(m/\tau_2)} - \exp(-\frac{m}{\tau_2}) \right) \tag{17}$$

Analogously to the procedure described for the Bundesbank approach, the (estimated) zero-coupon yields $\hat{z}_t^M$ are computed from the estimated parameters of the vector *b*, namely, $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\tau}_1$, and $\hat{\tau}_2$ (in the case of original Nelson and Siegel $\hat{\beta}_2$ and $\hat{\tau}_2$ are equal to zero) by evaluating the function below for *m*:

$$\hat{z}_{t,m} = \hat{\beta}_0 + \hat{\beta}_1 \frac{1-\exp(-m/\hat{\tau}_1)}{(m/\hat{\tau}_1)} + \hat{\beta}_2 \left(\frac{1-\exp(-m/\hat{\tau}_1)}{(m/\hat{\tau}_1)} - \exp(-\frac{m}{\hat{\tau}_1}) \right)$$
$$+ \hat{\beta}_3 \left(\frac{1-\exp(-m/\hat{\tau}_2)}{(m/\hat{\tau}_2)} - \exp(-\frac{m}{\hat{\tau}_2}) \right) \tag{18}$$

The shapes of the curve consistent with this functional form are similar to the original Nelson and Siegel approach, except that the Svensson term structure can contain an additional bulge or dip. In the literature the latter approach is often referred to as the extended Nelson and Siegel method; here, for ease of distinction it will be called the Svensson approach. Both approaches are applied to our data.

Figure 3: Yield curve shapes according to Nelson/Siegel



III.2.3 An adjustment for the tax-induced coupon effect

If interest earnings, but not realised capital gains, are liable to personal income tax (as is the case in Germany), bonds with high coupons may trade at a higher yield relative to ones with a low coupon. Ignoring this effect in the estimation of the term structure would produce a curve that is biased upwards in the areas where there are such high-coupon bonds.

Academic approaches to adjusting for tax effects have either estimated a single representative ("effective") tax rate covering all bonds and all maturities (following McCulloch (1975)) or have constructed a series of term structures from subsets of bonds which are efficiently held by "rational" investors with particular tax rates (following Schaefer (1981)). As to the former approach, it is unclear what the "effective" tax rate actually represents, presumably a kind of average tax rate faced by all investors, which would not accurately capture the tax effect across different classes of investors. The latter approach is difficult to implement in practice, especially for internationally traded bonds. It requires either the identification of all distinct tax categories of investors (which could easily change on a monthly basis) and the estimation of all their separate term structures, or it requires one to assume that one particular term structure is "representative" of the market and, in the process, to discard information from all bonds that are inefficient for that particular category of investor. This can easily lead to data shortage problems.

The Bank of England approach, which is adopted here, does not fit easily into either of the above categories.[17] It assumes that the tax effect manifests itself entirely through the bond coupons and attempts to correct for the tax effect by explicitly modelling the premium paid for low-coupon bonds. The model includes four parameters which are found simultaneously using a numerical procedure that minimises the sum of squared differences between the observed and fitted yields. Following the practice of the Bank of England (see e.g. Cooper and Steeley (1995) for an application to Germany), this tax adjustment model is combined with the original Nelson/Siegel and Svensson approaches to estimating the relation between the yield and the maturity. Thus:

$$
\begin{aligned}
z_{t,m} &= \beta_0 + \beta_1 \frac{1-\exp(-m/\tau_1)}{(m/\tau_1)} + \beta_2\left(\frac{1-\exp(-m/\tau_1)}{(m/\tau_1)} - \exp(-\frac{m}{\tau_1})\right) \\
&+ \beta_3\left(\frac{1-\exp(-m/\tau_2)}{(m/\tau_2)} - \exp(-\frac{m}{\tau_1})\right) + T(\beta_4,\beta_5,\beta_6,\beta_7,C,m),
\end{aligned}
\qquad (19)
$$

[17] For a detailed exposition see Deacon and Derry (1994b) and Mastronikola (1991).

where $T(\cdot)$ is the adjustment for the tax-induced coupon effect, which is specified as a complex function of four parameters and the coupon and time-to-maturity of the bond.[18] The zero-coupon yield estimates are computed on the basis of the estimated coefficients $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\tau}_1$, and $\hat{\tau}_2$ (in the case of original Nelson/Siegel $\hat{\beta}_2$ and $\hat{\tau}_2$ are equal to zero) using (18). Both approaches, Nelson/Siegel and Svensson, are considered in their original form as well as augmented by this tax adjustment. For easy reference, table 1 summarises the models considered in the empirical analysis.

Table 1: Summary table of curve-fitting approaches

Curve-fitting approach	Curve estimated	Functional form	Total number of parameters
Bundesbank with coupon adjustment (BBK)	yield-to-maturity	linear and logarithmic terms	5
Nelson and Siegel original (NSO)	zero-coupon yield	exponential terms	4
Nelson and Siegel with coupon adjustment (NST)	zero-coupon yield	exponential terms and tax effect parameters	8
Svensson original (SVO)	zero-coupon yield	as NSO but one additional exponential term	6
Svensson with coupon adjustment (SVT)	zero-coupon yield	as SVO but in addition tax effect parameters	10

[18] The specific functional form of the last term is complicated and its explanation would require too much space. The interested reader is referred to Mastronikola (1991) or Deacon and Derry (1994b, pp. 42 - 51).

IV. Applying the curve-fitting approaches to German data

IV.1. Data

The choice of the debt instruments used in constructing the yield curve is of particular importance since it affects the estimates considerably. A decision has to be taken on the trade-off between "homogeneity" and the availability of sufficient observations at each range of the maturity spectrum. There is no objective criterion available for determining the optimal choice of the data. The following paragraphs describe an attempt to find a compromise solution to these problems.

The available set of data comprises end-of-month observations of the officially quoted prices ("amtlich festgestellte Kassakurse"), remaining maturities and coupons of a total of 523 listed public debt securities from September 1972 to February 1996.[19] They include bonds issued by the Federal Republic of Germany (Anleihen der Bundesrepublik Deutschland), bonds issued by the Federal Republic of Germany - "German Unity" Fund (Anleihen der Bundesrepublik Deutschland - Fonds "Deutsche Einheit"), bonds issued by the Federal Republic of Germany - ERP Special Fund (Anleihen der Bundesrepublik Deutschland - ERP-Sondervermögen), bonds issued by the Treuhand agency (Anleihen der Treuhandanstalt), bonds issued by the German Federal Railways (Anleihen der Deutschen Bundesbahn), bonds issued by the German Federal Post Office (Anleihen der Deutschen Bundespost), five-year special Federal bonds (Bundesobligationen), five-year special Treuhand agency bonds (Treuhandobligationen), special bonds issued by the German Federal Post Office (Postobligationen), treasury notes issued by the German Federal Railways (Schatzanweisungen der Deutschen Bundesbahn), treasury notes issued by the German Federal Post Office (Schatzanweisungen der Deutschen Bundespost), and Federal treasury notes (Schatzanweisungen des Bundes).[20] The vast bulk of the securities have a fixed maturity and an annual coupon. There are a few bonds with semi-annual coupons[21] and special terms, such as debtor right of notice and sinking funds.[22]

---

[19] No data are available for May 1982. The May 1982 term structure estimates are proxied by the average of the estimates for April and June 1982. In principle all these debt securities are used to construct the yield estimates published in the Bundesbank's *Statistical Supplement to the Monthly Bulletin: Capital Market Statistics*. However, in the case of the latter various debt securities had been excluded betweentimes (it is not clear which ones). As a result of this practice and the slightly different implementation of the Bundesbank procedure considered here (which is explained in this sub-section) the Bundesbank yield estimates calculated here and the ones published in the above bulletin and used in previous research (e.g. Gerlach (1995) and Estrella and Mishkin (1995)) are not identical.

[20] For information on individual securities issued after 1984 see Deutsche Bundesbank (1995), pp. 81 - 88.

[21] The differing coupon payment frequencies (annually - semi-annually) are taken into account in the calculation of yields. Bonds with semi-annual coupon payments were issued until the end of December 1970; they matured not later than December 1980.

[22] The debtor right of notice gives the issuer the right to redeem (or call) the loan prematurely after expiry of a fixed (minimum) maturity, therefore these bonds are refered to as callable bonds. Such bonds were issued until September 1973 and were traded until November 1988. Bonds with a sinking fund may be redeemed

In order to obtain a more homogeneous set of data, bonds with special terms and those issued by the German Federal Railways and the German Federal Post Office were eliminated from the original set.[23] The yields of these debt securities are characterised by additional premia compared to debt securities on standard terms issued by the Federal Republic of Germany. For example, the price of a bond with a debtor right of notice can be interpreted as the price of a standard bond minus the price of a call option on that bond. Since this call option has a positive value as long as the volatility of interest rates is positive, the price of the bond with the debtor right of notice is lower and its yield higher than that of a standard bond. As for bonds issued by the German Federal Railways and the German Federal Post Office, they have a rating disadvantage compared to bonds issued by the Federal Republic of Germany because the perceived default risk is marginally higher.[24] In practise, the bonds of the former carry a spread with respect to the bonds of the latter, and this spread varies over time.

The final data set comprises (standard) bonds issued by the Federal Republic of Germany (170 issues), five-year special Federal bonds (116 issues), and Federal treasury notes (17 issues), making altogether 303 issues. The debt securities available for each month vary considerably over time (especially until the mid-80s), as can be seen in figure 4. For example, only a few observations are available at the beginning of the 1970s, the smallest set being September 1972 with just 15 observations. The number of debt securities available grows sharply during the 1970s, increasing (almost) monotonicly to more than 80 observations in 1983. During the rest of our sample period the number of observations available varies between 80 and almost 100.

The observations are in general spaced equally over the maturity range from zero to ten years as can be seen from figure 5 which depicts the distribution of debt securities available at each observation date in terms of their residual maturity across the sample period. The interpretation is as follows. The available debt securities are indicated by dots and the vertical axis shows the representation of bonds across the maturity spectrum. The graph illustrates that there are a few gaps in the maturity spectrum at the beginning of the 1970s. There are no observations for May 1982. Furthermore, the overwhelming majority of newly issued bonds have a maturity of ten years and there are only a few bonds with very long (30 years) and none

prematurely and in part after a fixed (minimum) maturity. They were issued until December 1972 and have been traded until December 1984.

[23] Another possible source for variation among the yields is that, since the introduction of the Deutsche Terminbörse (DTB), some bonds and Special Federal Notes can be delivered under the Bund future contracts. In particular, bonds with a remaining time to maturity of between 8.5 and 10 years and Special Federal Notes with a remaining time to maturity of between 3.5 and 5 years are candidates for delivery under the future contracts. However, a close inspection of the data did not reveal significant differences due to these characteristics.

[24] This is supported by simple statistical tests. Regressing separately for various dates the yields of the final set of securities, on the one hand, and of the omitted securities, on the other, on the coupons and maturities, the null hypothesis of equality of the estimated coefficients (Wald test) can be rejected, with the coefficients obtained from the omitted securities being generally higher.

Figure 4: Number of debt securities in data set, 1972:9 to 1996:2

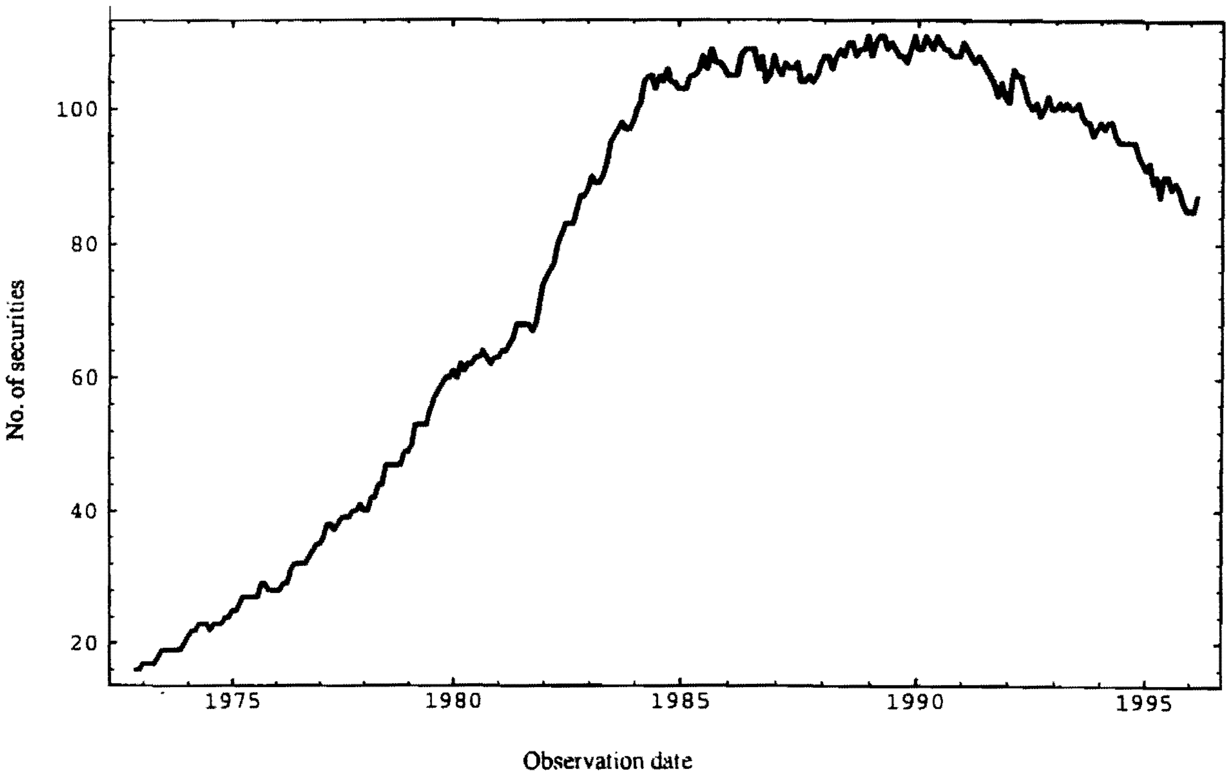
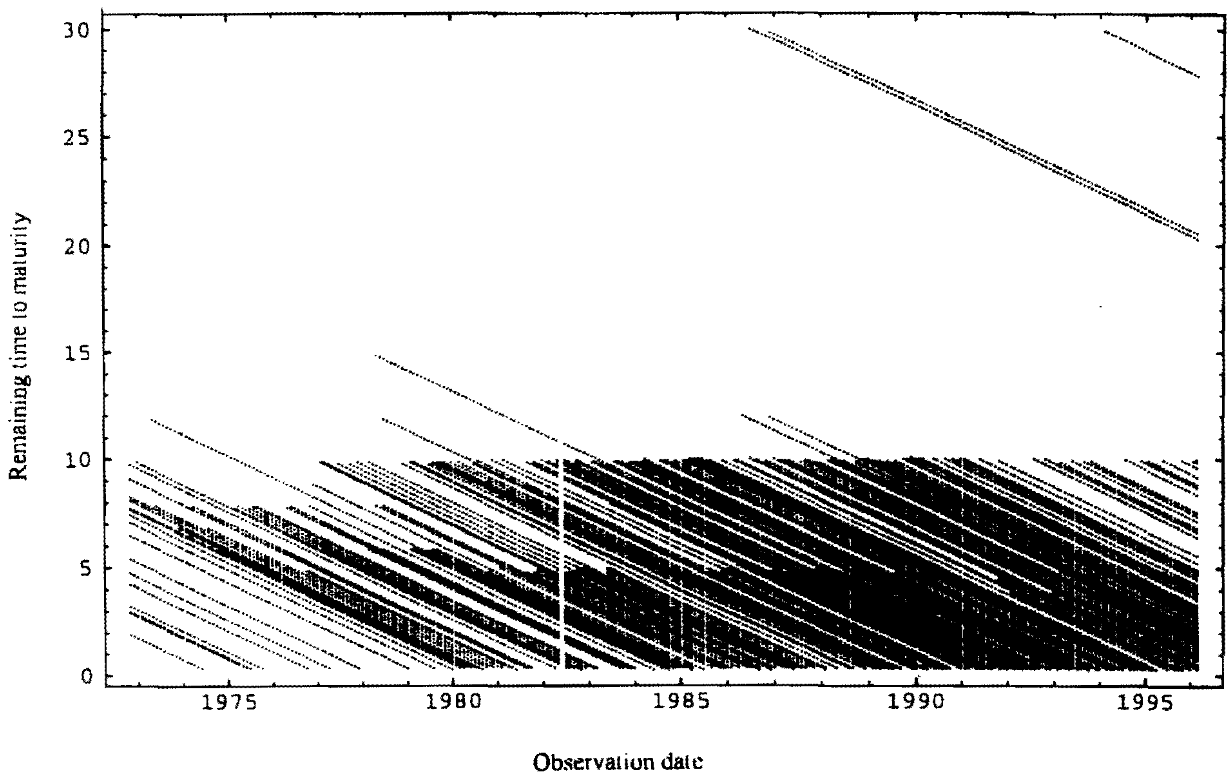


Figure 5: Distribution of debt securities in data set, 1972:9 to 1996:2



with short original time-to-maturity. Nevertheless, the short end of the yield curve is well represented by medium- and long-term issues with small residual maturities.

This leads on to the question of the maturity spectrum used. We adopt the Bank of England approach and consider all bonds with a remaining time-to-maturity above three months.[25] This is at variance with the Bundesbank's practice of excluding bonds with a residual time-to-maturity below one year. Although this exclusion improves the overall fit of all methods in terms of the deviations between observed and estimated yields, we do not adopt that strategy here because it implies very imprecise estimates for the one-year yields. Since observations of exactly one year and slightly higher than one year are regularly missing, the estimate of the one-year rate essentially becomes an out-of-sample forecast. All methods exhibit problems when used for such forecasts. For example, since the Bundesbank method essentially extrapolates linearly, it does not account for curvature in the data around one year. The Svensson approach could produce a "spoon-effect" whereby the curve flips up at the short end, thus resulting in unrealistically high estimates for the one-year rate. As the one-year rate is of special concern to policy makers and is also one of the most frequently cited interest rates in reports on the capital market, these properties are particularly undesirable. Thus, bonds with a remaining time-to-maturity of between three months and one year are included.

Another issue is whether or not the three bonds at the very long end of the maturity spectrum should be included. There is a case for leaving them out because not all of them appear to be very actively traded. However, owing to its limited flexibility the Bundesbank approach sometimes tends to produce very unrealistic "forecasts" of the ten-year yields when no observations are available beyond ten years and when the curve is otherwise very steep. In such situations the observations at the long end help to tie down the ten-year estimates. Since excluding these bonds would result in very distorted estimates using the Bundesbank approach, we follow the practice employed at the Bundesbank (and the Bank of England) and include the long-term bonds as well.

Reducing the sample to the 303 issues improved the fit of the estimates in terms of the deviation between observed and estimated yields, but the improvement was only small. The improvement of the fit varied over time; on average it amounted to about 0.5 to one basis point, depending on the curve-fitting approach. It should be noted that the reduction of the sample also rendered convergence of the estimates more difficult. For some methods convergence could not be assured in some instances. For example, in the case of NSO there was no convergence in six instances and in the case of SVO in one instance. In these cases convergence was achieved by using the original (larger) data instead. There were no such

[25] The yields of bonds with residual maturities below three months appear to be significantly influenced by their low liquidity and may therefore not be very reliable indicators of market expectations.

convergence problems for SVT, NST, and BBK. Thus, the sample of 303 issues seems to offer a good compromise between homogeneity and efficiency in estimation.

IV.2 Comparing curve-fitting approaches

Which model is preferable? As regards the curve estimated (zero-coupon yield or yield-to-maturity), the approaches to directly estimating zero-coupon yields are clearly preferable because they capture the information on expectations more precisely.

As to the functional form considered (exponential with three or four terms, or linear-logarithmic) theory does not provide a definitive answer since none of the approaches considered is derived from a theoretical model. Certain plausibility criteria have been used, nevertheless, as reference variables for assessing the models, but that does not provide any clear-cut answer either. For example, based on plausibility considerations, Siegel and Nelson (1988) have criticised estimation approaches whose estimates could lead to infinitely large or even negative yields in the case of long-term extrapolations. Shea (1984) provides an empirical demonstration of this phenomenon. An example of such an approach is the functional form considered by the Bundesbank which includes terms that are linear in maturity. In contrast, the estimation approach suggested by Nelson/Siegel and Svensson has the advantage that the estimates asymptote to a constant at the long end. Essentially, their approach allows the term structure to be "abnormal" at the short but not at the long end ("abnormal" denoting a situation in which markets anticipate a change in the expected variables in more than one direction, see Russell (1992)). That raises the question of why the so-called abnormal term structure forms are permitted only at the short end and not everywhere. This question will not be answered here, but is raised to illustrate the fact that plausibility considerations can be used to a limited extent only for assessing various estimation approaches.

Another criterion for assessing different curve-fitting approaches is how well they can represent the observable information. The errors between fitted and observed yield-to-maturities or prices have regularly been used as a criterion for assessing different curve-fitting approaches (e.g. Anderson et. al. (1996), Bekdache and Baum (1994), Bliss (1994)). Such an examination is carried out in the next sub-section.

IV.3 Observed and fitted yields across different approaches

As mentioned, the yield estimates are obtained by minimising the root mean squared yield-to-maturity error. Three simple summary statistics are suggested here for comparing the estimates obtained from the different methods with the observed yields-to-maturity. The root mean squared error (RMSE),

$$RMSE_t = \sqrt{\left(\sum_{i=1}^{n_t} \left(y_{i,t} - \hat{y}_{i,t}\right)^2\right) / n_t} * 100,$$

the coefficient of determination R^2,

$$R_t^2 = 1 - \left(\sum_{i=1}^{n_t} \left(y_{i,t} - \hat{y}_{i,t}\right)^2\right) \Big/ \left(\sum_{i=1}^{n_t} \left(y_{i,t} - \bar{y}_t\right)^2\right),$$

and the R^2 adjusted for degrees of freedom, $\bar{R}^2$,

$$\bar{R}_t^2 = 1 - \left[(n_t - 1)/(n_t - k)\right]\left(1 - R_t^2\right),$$

where n_t is - as defined before - the number of observations through which the curve is fitted at each month t (monthly observations from 1972.9 to 1996.2), $y_{i,t}$ the observed (redemption) yield of bond i, $\hat{y}_{i,t}$ the estimated yield, $\bar{y}_t$ the average observed yield and k the number of parameters. The scaling by 100 means that the RMSE is expressed in terms of basis points. The RMSE is the minimisation criterion used in the estimates (the mean squared yield error) and is thus the key criterion for comparing different estimates. The R^2 sets the squared yield error for each bond in relation to the variation of its yield from the mean. It thus places less weight upon yield errors in areas where the variation in the observed yields is relatively high. An important determinant of the flexibility of a curve-fitting approach is the number of parameters included. Other things being equal, in particular the functional forms used, one would expect the models with the higher number of parameters to produce better fits (the number of parameters are shown in table 1). For example, a perfect fit could be guaranteed by a model with as many parameters as observations. Therefore, $\bar{R}^2$, which corrects the R^2 for the degrees of freedom is considered as well. For all three summary statistics the mean and standard deviation are calculated over the whole observation period.

Flexibility is understood here to mean the degree to which the estimated curve accommodates the movements in the observed yields, high "flexibility" being measured by a small mean of the RMSE and a high mean of the R^2 and $\bar{R}^2$. The standard deviations give some indication of how constant the RMSE, R^2 or $\bar{R}^2$ are across the sample of (monthly) observations. For a *given* mean RMSE, R^2 or $\bar{R}^2$, a lower standard deviation is arguably desirable if the term structure estimates and the changes in these estimates are used to draw inferences for monetary policy purposes. Too much volatility in the estimates might suggest that some of it is attributed to the "unreliability" of the estimation procedure rather than genuine movements in market expectations. Table 2 shows the values obtained for the different curve fitting approaches.

Table 2: Summary statistics for the curve-fitting approaches

	BBK	NSO	NST	SVO	SVT
Mean of RMSE in basis points	16.4	12.9	11.1	11.8	9.9
Standard deviation of RMSE in basis points	6.4	5.3	4.2	5.3	4.0
Mean of R^2	0.75	0.82	0.84	0.84	0.87
Standard deviation of R^2	0.27	0.23	0.20	0.21	0.17
Mean of $\overline{R}^2$	0.72	0.81	0.80	0.82	0.82
Standard deviation of $\overline{R}^2$	0.30	0.24	0.27	0.23	0.28

Explanation: RMSE - Root mean squared error, R^2 is the coefficient of determination and $\overline{R}^2$ is the degrees-of-freedom-corrected coefficient of determination. BBK - Bundesbank; NSO - Nelson and Siegel original; NST - Nelson and Siegel with coupon adjustement; SVO - Svensson original; SVT - Svensson with coupon adjustment.

Looking at the summary statistics, the SVT model is favoured by the tests. Not only does it produce the lowest mean value of RMSE and the highest value of R^2 and $\overline{R}^2$, it also appears, on average, to be the most reliable.[26] The BBK model shows the worst performance in terms of both flexibility and reliability, according to any criterion considered here.[27] Furthermore, while the differences in the performances of NSO, NST, SVO, and SVT are relatively small, all four models exhibit a considerably better performance than BBK.

To the extent that the better performance is attributable to the higher number of parameters, our prior expectations and the results of previous comparative analyses of different methods of estimating yield curves (see e.g. Anderson et al. (1996) and Bliss (1994)) are confirmed. Indeed, we find that SVT (10 parameters) performs better than NST (8 parameters), which again performs better than SVO (6 parameters) in terms of their mean RMSE. Comparing the R^2 and the $\overline{R}^2$, we find that the better performance can indeed be partly explained by the number of parameters. However, a notable exception is BBK (5 parameters) which produces a

[26] Note, however, that if a penalty for the number of parameters is considered, as in $\overline{R}^2$, SVT appears to be less reliable than SVO.

[27] It should also be mentioned that BBK - unlike the other methods - produces at times negative R^2-values (not shown in the table). This means that in these instances the regression produces results inferior to just assuming that the yield curve is flat and equal to the mean of the observed yields.

worse fit than NSO (4 parameters). This finding is consistent with the simulations shown in figures 1 and 3, suggesting that even the simple Nelson/Siegel approach can represent a larger number of shapes than the linear-logarithmic (BBK) approach. This points to the conclusion that the linear-logarithmic terms are not appropriate.

It is informative to look at the fit of the models over time. For example, figure 6 shows the root mean squared errors of SVT and BBK between 1972.9 and 1996.2. It illustrates that the BBK errors are almost always above the ones from SVT and that the former exhibit a considerably higher variation over time than the latter. The differences in the performance between SVT and BBK (in the sense defined above) are considerable between 1982 and 1985, in 1987/1988 and since the end of 1992. While the root mean squared errors from SVT gradually decrease from the end of 1992, the ones from the BBK model become both larger and more volatile. The reason is the increase in the number of instances in which the data either represents relatively complicated forms such as S-shapes or very steep curves. These shapes cannot be reproduced well by BBK because of its limited flexibility.

Anecdotal evidence supporting this hypothesis is provided in figure 7, which shows an example of a complicated term structure, observed in January 1994. It shows the observed yields-to-maturity (as dots) and the Bundesbank yield-to-maturity estimates (thick line). The graph illustrates that the Bundesbank approach is not flexible enough to represent the S-shape of the data. For reference it also shows the SVT zero-coupon rate estimates (dotted line), which replicate that shape quite well.[28] BBK produces a curve that cuts through the middle of the data at a remaining time-to-maturity of around six years. Between one and a half and six years it systematically overestimates and over six years it systematically underestimates the data. As a consequence the estimated yield spread is underestimated.

A simple regression analysis confirms that the inferior performance of BBK compared to SVT occurs in situations where the term structure is very steep. The "excess error" (defined as the BBK root mean squared error minus the SVT root mean squared error) is regressed on the spread between the 10 and 1 year zero-coupon yields, the squared spread, a constant and a number of AR terms. The spread and the squared spread are both positive and highly significant (at the 1.6% and 0.001% level) and explain, together with the constant and the AR terms, 70% of the variation in the excess error.[29] However, this interpretation should not be taken too far since the distribution of residuals exhibited too thick tails to qualify as normal. The reason is that some of the extreme variations in the excess errors are not captured by this simple regression.

[28] Note that these should *not* be interpreted as attempts to fit a curve through the dots.

[29] We experimented with a few other factors, but without any sucess. For example, no significant relationship could be found between the excess error and the average coupon.

Figure 6: Root mean squared yield-to-maturity errors of SVT and BBK, 1972:9 to 1996:2

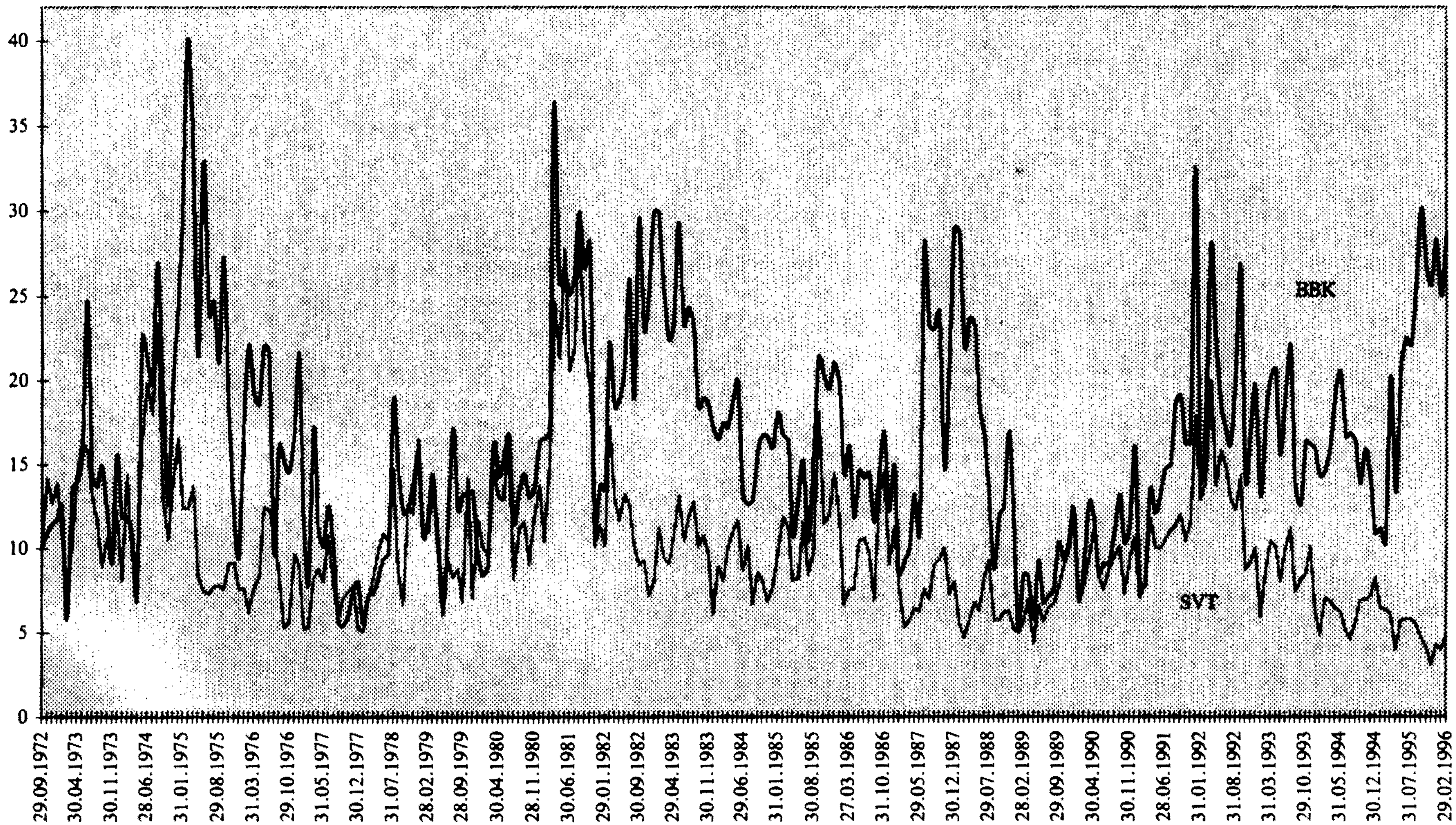


Figure 7: Observed and fitted (BBK) yields-to-maturity and SVT zero-coupon yield estimates, 1994:1

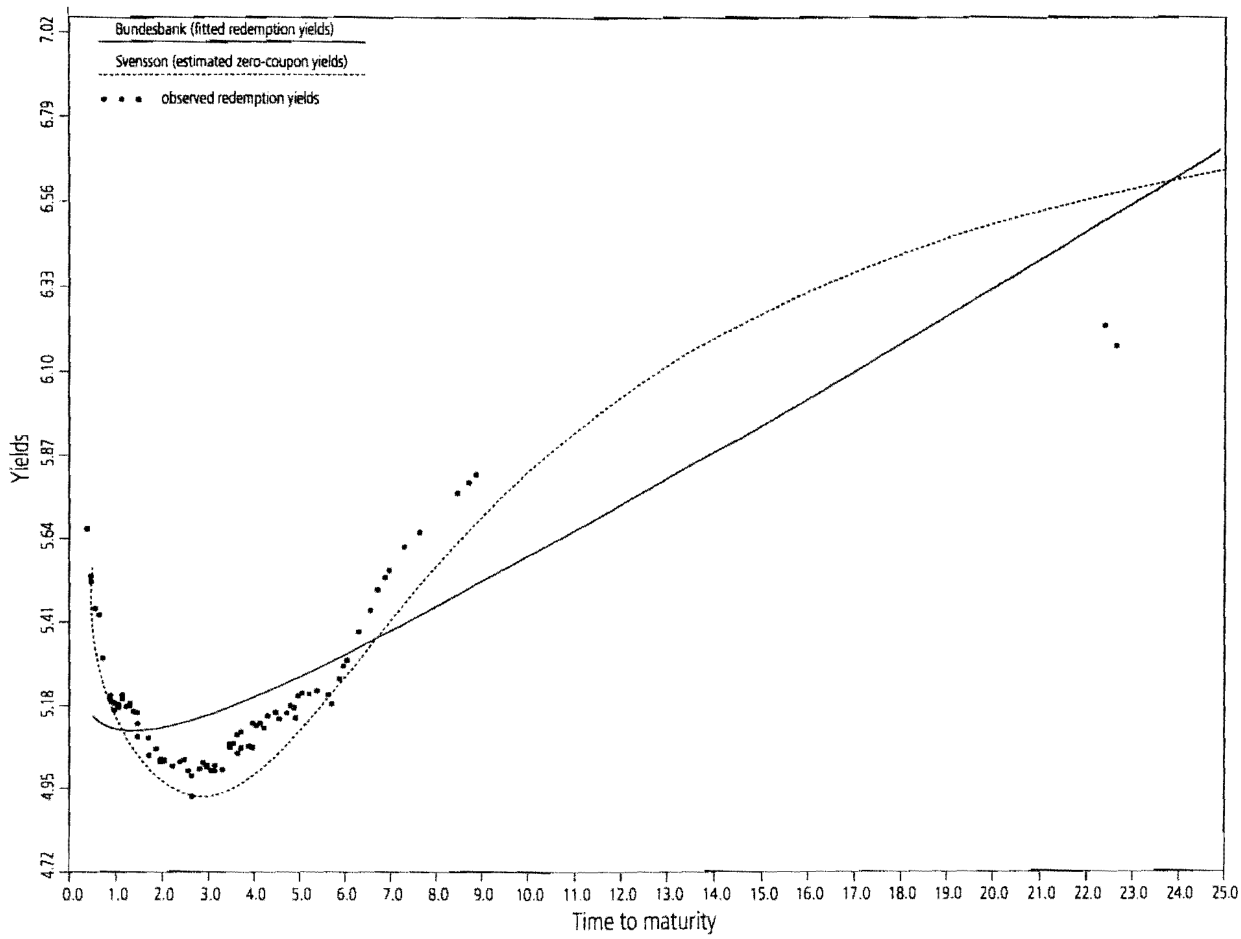
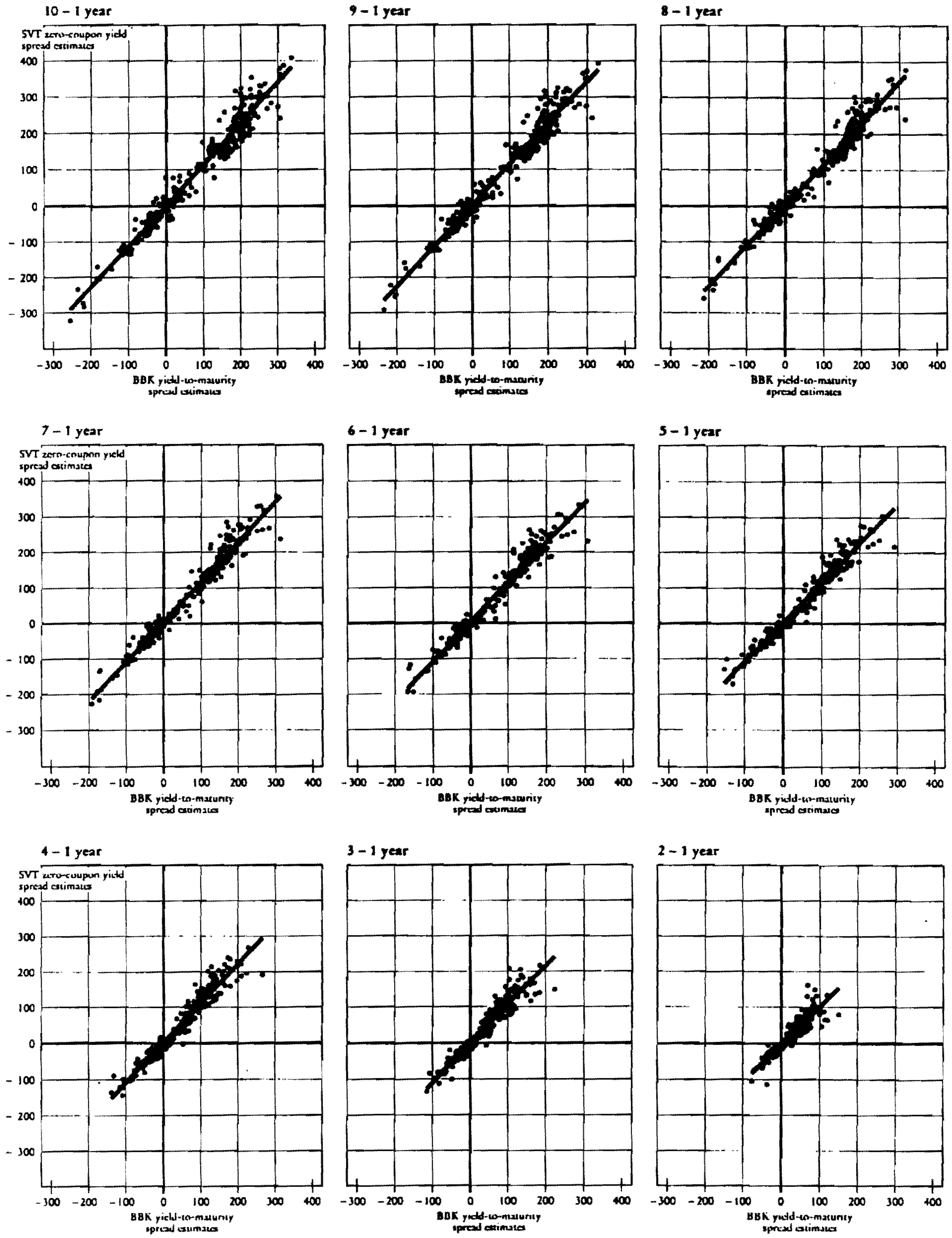


Figure 8: Correlation between SVT zero-coupon yield spread estimates and BBk yield-to-maturity spread estimates, 1972:9 to 1996:2



IV.4 Yield and yield spread estimates across different approaches

This section presents some statistics on the estimated yield and yield spread estimates. The exposition is confined to a comparison of BBK and SVT, the least and the most flexible methods. The estimates obtained from the other three methods are so similar to the latter that their separate description does not give any additional insights.

On average the yield and yield spread estimates obtained from the two methods are similar. Measured over the full period, the correlation between the yield estimates is very high, the correlation coefficient ranging from 0.97 to 0.99, depending on the maturity. The correlation between yield spreads is generally lower for spreads where the short and the long maturity are only one year apart; the correlation coefficient in these instances is often only around 0.80. Figure 8 shows some scatter plots of the two spread estimates with k equal to one. The "average yield curves" and the "average yield spreads" (with k equal to one year) are shown in the left-hand and right-hand panel of figure 9, respectively. The left-hand panel illustrates that the BBK average yield curve is consistently higher and flatter than the SVT curve. The higher levels are due in part to the coupon adjustment in the case of the BBK method, which results in an upward shift of the yield curve. The right-hand panel illustrates that the SVT yield spread estimates are almost always higher than the BBK yield spread estimates. This reflects in particular the greater flexibility or scope for differentiation across maturities of the SVT approach. It is also compatible with our considerations in section II, according to which the yield-to-maturity spreads are always smaller than the zero-coupon yield spreads.

The time-series properties of the SVT and BBK yield spread estimates are somewhat different. Simple measures of the time-series properties are the standard deviations, minima and maxima of (month-to-month) changes, as shown in figure 10. The left-hand panel of that figure illustrates that the standard deviations of monthly changes are consistently higher in the case of the SVT estimates compared to the BBK estimates. Furthermore, the extreme changes, either positive (denoted by maxima in the right-hand panel of figure 10) or negative (denoted by minima), are mostly higher in the case of the SVT estimates as compared to the BBK estimates. Another measure of the time-series properties that takes due account of the changes from one month to another is the coefficient of the first-order correlation. Almost all BBK yield spread estimates are characterised by a higher first-order correlation than the corresponding SVT yield spreads. Caution in interpreting this coefficient is warranted due to the complication of overlapping observations.

Thus, the BBK method produces yield spread estimates that are lower on average and less volatile than the SVT yield spread estimates. One may argue that a method that implies a greater persistence in the estimates is desirable, based on the belief that expectations do not

Figure 9: Average yield curve and yield spreads, 1972:9 to 1996:2
September 1972 to February 1996

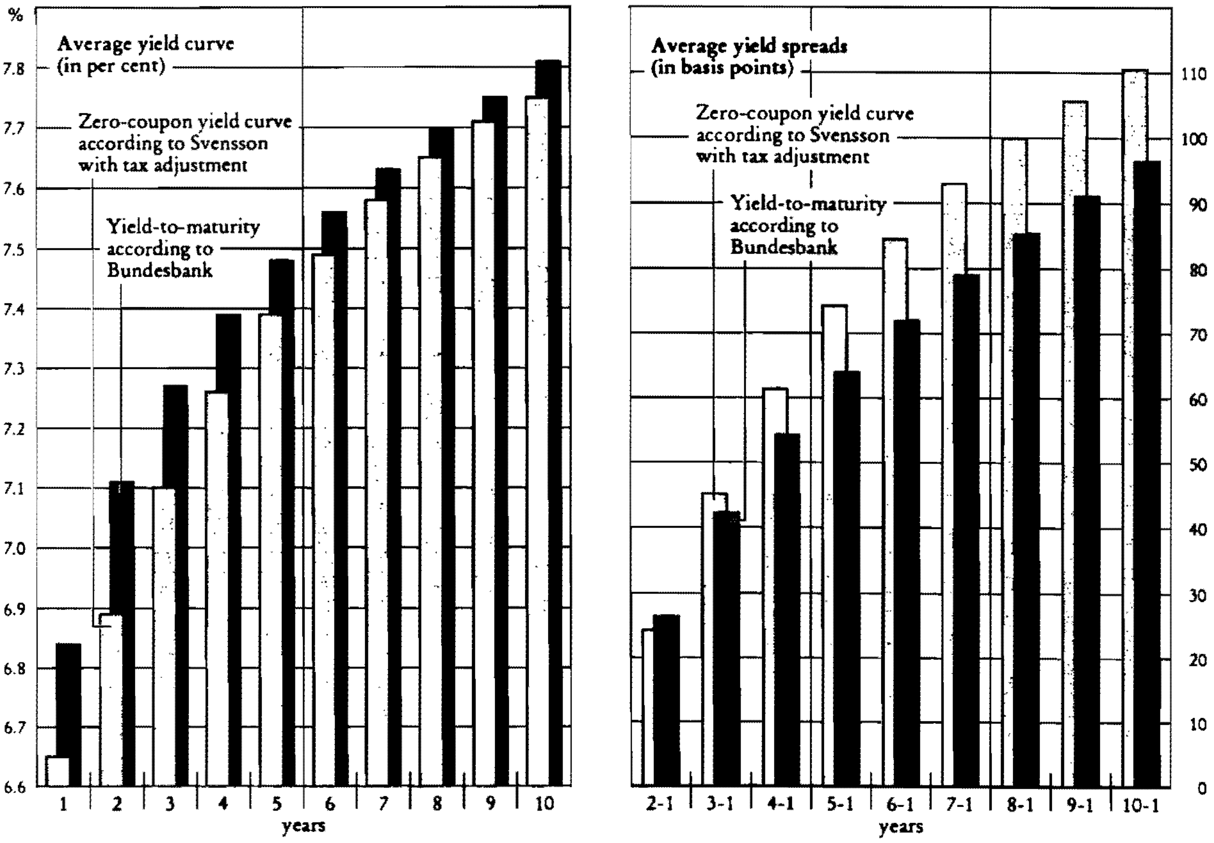


Figure 10: Standard deviations, minima and maxima of monthly changes in yield spreads, 1972:9 to 1996:2

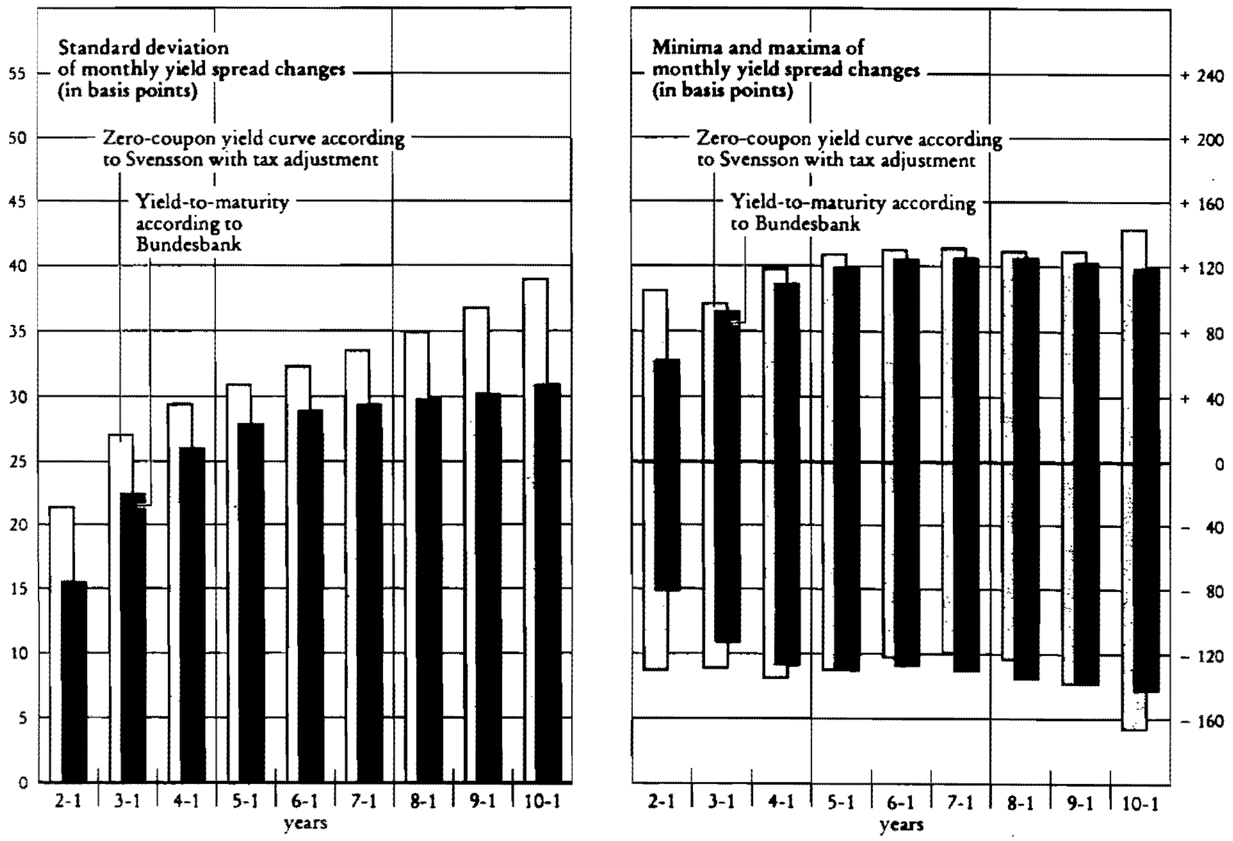
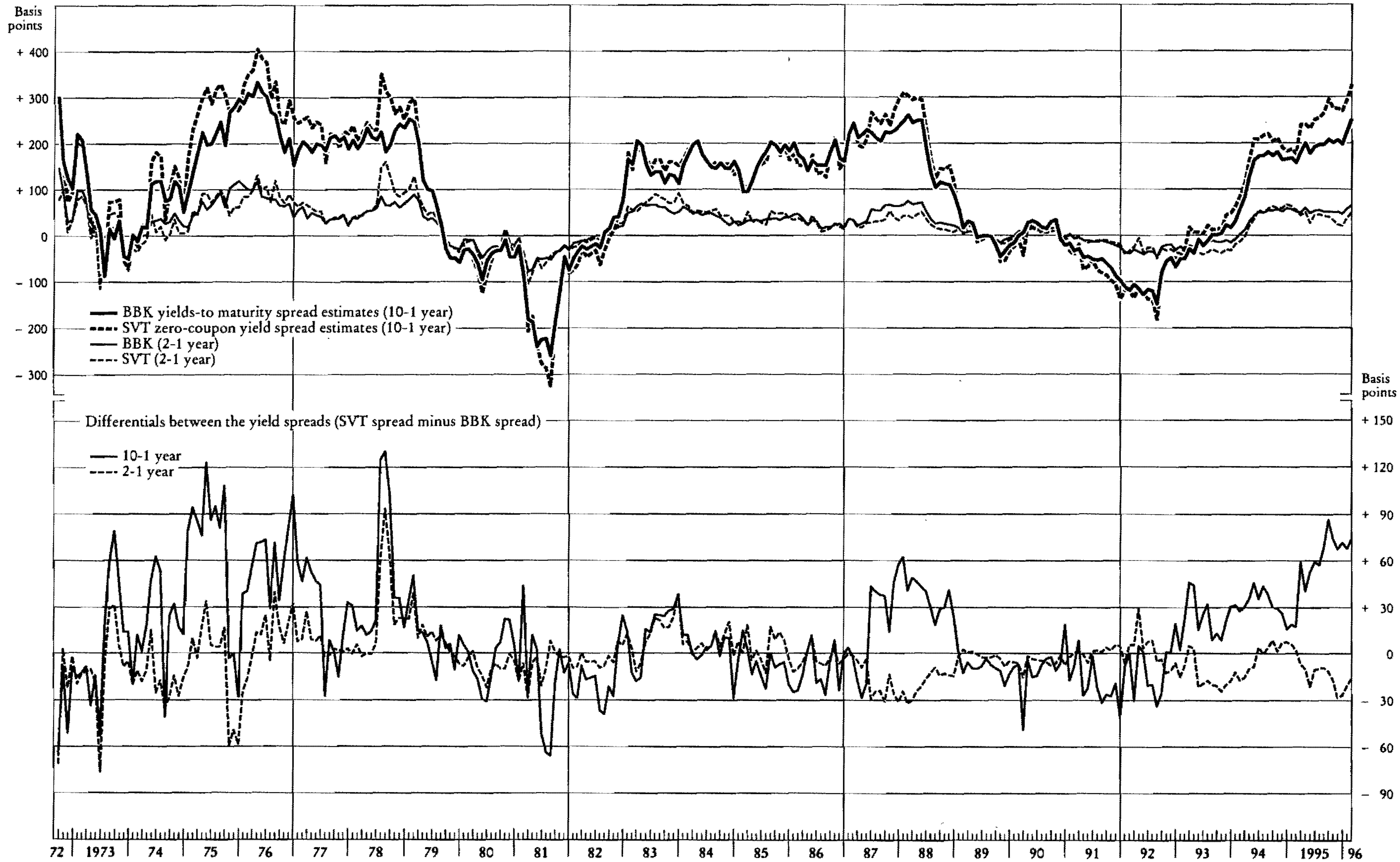
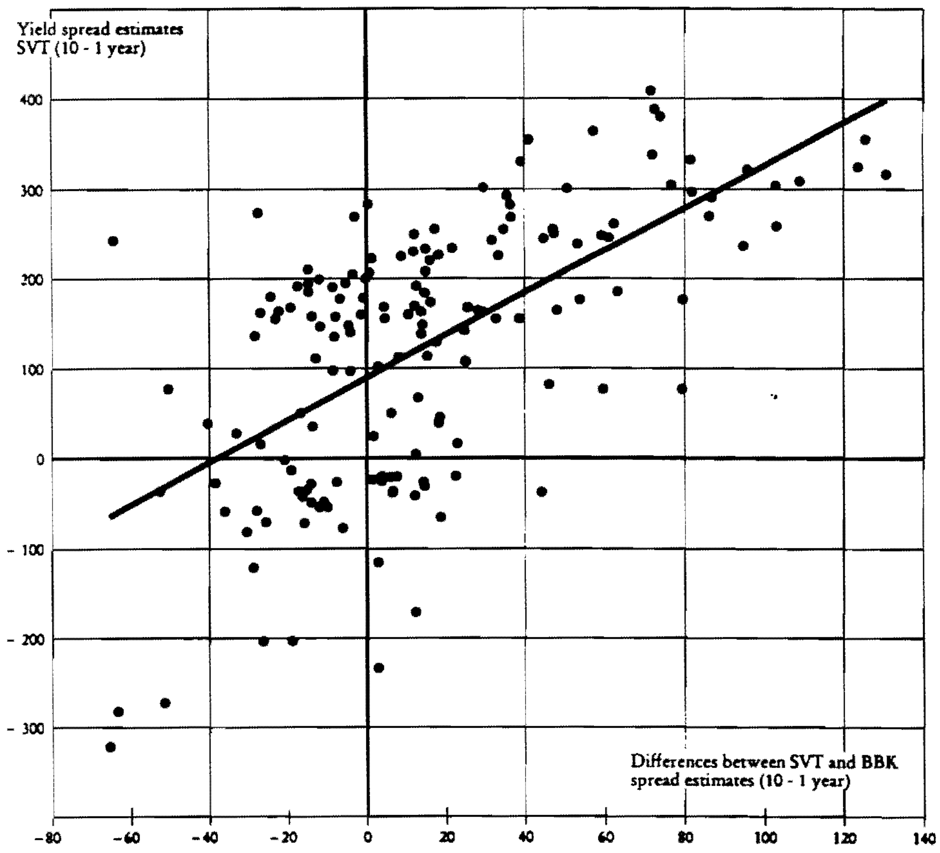


Figure 11: BBK yield-to-maturity spread estimates and SVT zero-coupon yield spread estimates, 1972:9 to 1996:2



PO/FR1X29#E

Figure 12: Steepness of the yield curve and differences between SVT and BBK yield spreads



change much either. However, this is not a convincing argument since the persistence appears to represent a systematic failure of the BBK method to represent very steep or complicated term structures (see previous sub-section). Whenever the term structure is not very complicated or not very steep, the estimates of SVT and BBK are similar. But in other situations the BBK method appears to systematically underestimate the slope of the yield curve. Indeed, most noticeable discrepancies between BBK and SVT yield spread estimates are concentrated in periods that are characterised by a high incidence of such shapes. Figure 11 illustrates the development of the yield spread estimates (10 minus 1 year and 2 minus 1 year) of the BBK and the SVT methods over the sample period. It shows that the considerable differences in the estimates (the BBK estimates being considerably smaller than the SVT estimates) are concentrated in three periods, namely, from 1973 to 1978, during 1987/88, and from 1992 to 1996. These periods are characterised by very steep yield curves (see upper panel of figure 11) and, as discussed in the previous sub-section, by a considerably worse performance of BBK compared to SVT in terms of the root mean squared errors. The connection between the steepness of the term structure and the divergence of BBK and SVT yield spread estimates is confirmed by the scatter diagram in figure 12. It demonstrates the positive correlation between the steepness of the yield curve (measured in terms of the SVT zero-coupon yield spreads) and these differences.

V. The information content of the term structure

V.1 Methodology

The previous section used the root mean squared yield errors as a measure of success of the different curve-fitting approaches. However, *per se*, the quality of the estimates determined in this way does not tell us much, since the decisive factor should be the ultimate purpose for which the estimated yields are used (Steeley (1991)). More specifically, the method should not ignore information that is relevant for the ultimate purpose for which the yields are used.

The purpose of the present section is twofold. First, it asks whether the German term structure of interest rates contains information with respect to the future path of inflation. To our knowledge this investigation has not been performed before with properly calculated zero-coupon rates. Second, it asks whether the information content differs when the Bundesbank's yield-to-maturity estimates are considered instead. We suggest the test of the information content as another criterion for assessing different curve-fitting approaches. It is interesting because the Bundesbank uses the slope of the yield curve as an indicator of the financial markets' inflation expectations.

The information content is investigated using the methodology of Mishkin (1990a, 1990b, 1991), Jorion and Mishkin (1991) and Gerlach (1995). This interprets the "information content of the term structure" quite narrowly. Information in the term structure about the path of future inflation refers only to the ability of the slope to predict the change in the inflation rate.

The starting point is a Fisher decomposition of the nominal zero-coupon yield:

$$z_t^j = rr_t^j + E_t\left[\pi_t^j\right], \tag{20}$$

where z_t^j is the j-year zero-coupon yield, E_t is the expectations operator conditional on information available at time t, rr_t^j is the j-year (ex ante) real interest rate and π_t^j is the realised forward inflation rate over the next j years, which is computed as $\left(\left(P_{t+12j} / P_t\right)^{1/j} - 1\right) * 100$ with P_t denoting the price index in month t. In other words, the nominal j-year-zero-coupon yield at time t equals the sum of the (ex ante) real interest rate and the expected rate of inflation.

Assuming rational expectations the realised inflation rate over the next j years can be written as the expected inflation rate plus a serially uncorrelated, zero-mean error ε_t^j ,:

$$\pi_t^j = E_t\left[\pi_t^j\right] + \varepsilon_t^j, \tag{21}$$

where $\varepsilon_t^j = \pi_t^j - E_t\left[\pi_t^j\right]$ is the expectation error of inflation. Substituting for $E_t\left[\pi_t^j\right]$ from equation (20) we obtain

$$\pi_t^j = z_t^j - rr_t^j + \varepsilon_t^j. \tag{22}$$

Hence the realised inflation rate over the next k (with $k< j$) years can be expressed as:

$$\pi_t^k = z_t^k - rr_t^k + \varepsilon_t^k. \tag{23}$$

Subtracting (23) from (22), we obtain an expression of the changes in the realised inflation rate between the two periods j and k:

$$\pi_t^j - \pi_t^k = -\left(rr_t^j - rr_t^k\right) + \left(z_t^j - z_t^k\right) + \left(\varepsilon_t^j - \varepsilon_t^k\right). \tag{24}$$

It will be assumed that the differences between the term premia for maturities j, k equal some constant $\alpha^{j,k}$ plus a zero mean random variable $\upsilon_t^{j,k}$, thus $-\left(rr_t^j - rr_t^k\right) = \alpha^{j,k} + \upsilon_t^{j,k}$. Equation (24) can then be written in estimatable form:

$$\pi_t^j - \pi_t^k = \alpha^{j,k} + \beta^{j,k}\left(z_t^j - z_t^k\right) + u_t^{j,k}, \tag{25}$$

where $u_t^{j,k} = \upsilon_t^{j,k} + \left(\varepsilon_t^j - \varepsilon_t^k\right)$. Under rational expectations the expectation errors, ε_t^j and ε_t^k, are orthogonal to the right-hand regressors; in other words, expectation errors are indeed unexpected, conditional on all available information at time t including z_t^j and z_t^k.[30] If the slope of the real term structure, $\left(rr_t^j - rr_t^k\right)$, is constant, then $\upsilon_t^{j,k}$ is equal to zero and the OLS estimates of $\beta^{j,k}$ have a probability limit of one and are consistent. If the slope of the real term structure is not constant, because of variable term premia, then the nominal term structure, $\left(z_t^j - z_t^k\right)$, may still contain information about future changes in the inflation rate but is not as accurate a predictor of $\left(\pi_t^j - \pi_t^k\right)$ because $\upsilon_t^{j,k}$ is predictably different from zero.[31]

This suggests (see Mishkin (1990a)) the following tests on the $\beta^{j,k}$ coefficient. If the null hypothesis $\beta^{j,k} = 0$ is rejected statistically, then the slope of the term structure, $z_t^j - z_t^k$,

[30] It is also assumed that financial market participants do not anticipate a shift in the inflation regime at some future date. As Evans and Wachtel (1993) point out, this could imply systematic forecasting errors even in the case of rational expectations.

[31] If the term premium followed an ARMA process, a better estimate could be obtained by taking that into account. Furthermore, if $\upsilon_t^{j,k}$ is correlated with $\left(z_t^j - z_t^k\right)$, the OLS estimate of $\beta^{j,k}$ will have a probability limit different from one and will not be consistent.

contains significant information about the change in inflation between j and k years ahead. If the null hypothesis $\beta^{j,k} = 1$ is rejected, then the empirical evidence indicates that the real interest rate varies over time, arguably because of time-varying term premia. The aforementioned tests are implemented using, alternatively, the SVT zero-coupon yield spread estimates and the BBK yield-to-maturity spread estimates as proxies for the (exogenous) zero-coupon yields, $\left(z_t^j - z_t^k\right)$.

The extent of discrepancies in the information content between the two yield spread estimates can be evaluated on the basis of the estimated R^2 and the significance levels at which the hypothesis $\beta^{j,k} = 0$ is rejected. A direct (one-sided) test for the equality in the information content would be to include the difference between the zero-coupon yield spread estimates and the yield-to-maturity yield spread estimates as an additional regressor in (25) when $\left(z_t^j - z_t^k\right)$ is proxied by the zero-coupon yield spread estimates. However, multicollinearity between the zero-coupon yield spread estimates and the difference between the zero-coupon yield spread estimates and the yield-to-maturity spread estimates (see subsection IV.3) makes it difficult to identify significant differences. Therefore, we suggest a t-test of the somewhat weaker hypothesis that the coefficient estimate obtained from the SVT data is equal to the point estimate obtained from the BBK data.[32] We carry out the test as follows. Subtracting $\beta^*\left(z_t^j - z_t^k\right)$ from both sides of (25) and then multiplying both sides by - 1 we obtain

$$\beta^*\left(z_t^j - z_t^k\right) - \left(\pi_t^j - \pi_t^k\right) = \delta^{j,k} + \left[\beta^* - \beta^{j,k}\right]\left(z_t^j - z_t^k\right) + v_t^{j,k} \tag{26}$$

where $\delta^{j,k} = -\alpha^{j,k}$, $v_t^{j,k} = -u_t^{j,k}$ and β^* is any constant value. Let $\beta_{BBK}^{j,k}$ denote the coefficient in the regression of (25) using the BBK yield-to-maturity spreads as independent variables, and $\beta_{SVT}^{j,k}$ the coefficient in the regression using the SVT zero-coupon yield estimates. Then the formulation in (26) permits a direct t-test of the hypothesis that $\beta_{SVT}^{j,k}$ is equal to the point estimate for $\beta_{BBK}^{j,k}$. Namely, we run regression (25) using the BBK yield-to-maturity spreads, store the coefficient estimate of $\beta_{BBK}^{j,k}$ and substitute it for β^* when regression (26) is estimated using the SVT zero-coupon yield estimates. Then one calculates the t-statistic for the estimated coefficient on $\left(z_t^j - z_t^k\right)$.

Note also that by substituting 1 for β^* in (26) we can test the hypothesis that $\beta^{j,k} = 1$, as suggested by Mishkin (1990a):

$$\overline{rr}_t^j - \overline{rr}_t^k = \delta^{j,k} + \left[1 - \beta^{j,k}\right]\left(z_t^j - z_t^k\right) + v_t^{j,k}, \tag{27}$$

32 This is not a symmetric test. The results are checked by testing that the coefficient estimate obtained from the BBK data is equal to the point estimate obtained from the SVT data. The results are similar.

where $\overline{rr}_t^j$ is the *ex post* real interest rate or realised interest rate on a j-year security at time t, i.e from t to $t+j$.

V.2 Econometric results

Before turning to the estimates it is useful to look at simple scatter diagrams relating the yield spreads (calculated from the SVT zero-coupon yield estimates) to the corresponding changes in realised inflation rates, denoted by forward inflation rates in the graphs. Figure 13 illustrates that for some maturities there is a remarkably positive correlation (indicated by the regression line). Furthermore, figure 14 shows that this correlation strengthens as the shorter rate of the spreads is increased from one year.

We estimate the inflation rate regression (equation (25)) and the real rate regression (equation (27)), letting j vary from two to ten and k from one to nine with the sample period starting in 1972:9 and ending, depending on the choice of j, between 1986:2 (for $j = 10$) and 1994:2 (for $j = 2$). As has been pointed out before (e.g. Mishkin (1991)), the use of overlapping data induces serial correlation in the regression errors of order MA($12j$ -1). For instance, with $j =$ 10 years, the residuals would exhibit a MA(119) process. Furthermore, the residuals are likely to be heteroscedastic. To correct for these problems, a heteroscedasticity and autocorrelation-consistent estimator (Newey and West (1987) with Bartlett weights) is used. However, although the corrected standard errors are valid asymptotically, they may produce misleading inferences in small samples (see e.g. Bekaert, Hodrick and Marshall (1996) and Smith and Yadav (forthcoming)). The small sample bias can be especially serious when j is ten and k is nine. Therefore, in addition to the above theoretical standard errors, empirical probability values are reported, which are calculated using a bootstrapping technique. Bootstrapped probability values are chosen here instead of Monte Carlo probability values because the former do not assume a particular distribution, such as Gaussian in the Mishkin studies (see e.g. Brock, Lakonishok and LeBaron (1992)).

The empirical probability values are calculated as follows.[33] First, univariate AR-models are fitted to the inflation rate (including eleven seasonal dummies) as well as the relevant interest rate spread. The order of the AR models is determined using the Akaike and Schwartz criteria, where due attention is paid to the residuals being white noise. Based on these residuals and the estimated AR structures, 1000 artificial samples for the inflation rate and the relevant interest rate spread are generated by means of bootstrapping, where the starting values are set equal to zero. The reshuffling of residuals in the bootstrapping procedure ensures that any temporal

33 I am grateful to Stefan Gerlach for providing a RATS programme that includes the calculation of empirical probability values by means of bootstrapping.

interdependence between the residuals of the two series is eliminated and that the sample paths of the inflation rate and the relevant interest rate spread series are independent. Finally, the inflation-rate-changes regressions are estimated for each of the 1000 samples, which yields 1000 bootstrapped probability values for the regression coefficients under the null hypothesis that the sample paths of the inflation rate and interest rate spreads are independent. The fraction of times the historical probability values (estimated from the original data) fall below these bootstrapped probability values defines the empirical probability values.

The results of our empirical tests indicate that there is significant information in the German term structure about the future path of inflation. This confirms the results of previous empirical studies (Gerlach (1995), Jorion and Mishkin (1991), Koedijk and Kool (1995) and Levin (1996)).[34] Two summaries of the results are given in tables 3 and 4. They show the results of the test for $\beta^{j,k} = 0$ for various maturity segments, i.e. combinations of j and k, obtained from the SVT and BBK data, respectively. The tables A.2 to A.7 in the appendix contain a more exhaustive account of these results. Four results are singled out for special attention.

The first result is of a more technical nature. Namely, as others have found there are large differences between the asymptotical and empirical probability values (see e.g. Freedman and Peters (1984)). Both are reported in the appendix tables A.2 to A.7, so that the results can be compared with those of other studies that rely on the asymptotical probability values (see e.g. Estrella and Mishkin (1995), Koedijk and Kool (1995), Levin (1996)). In general, the empirical probability values are higher than the asymptotical ones, thus implying a lower probability of rejecting the null hypothesis. For example, consider the null hypothesis that the spread contains no information about future inflation rates, i.e. H_0: $\beta^{j,k} = 0$. Theoretical probability values (shown in the upper square brackets of appendix table A.2) imply that for $k = 1$ and $j > 2$ this hypothesis could always be rejected at the 5% level and for $k = 2$ or 3 and any j even at the 1% level. However, empirical probability values (shown in the lower square brackets) imply that this hypothesis could not be rejected at conventional significance levels in several of these instances, namely it could not be rejected at the 5% level of significance for $k = 1$ and $j = \{9, 10\}$, for $k = 2$ and $j = 10$ and for $k = 3$ and $j = \{8, 9, 10\}$. It can be observed that the difference between the empirical and the asymptotical probability values increases as longer time horizons are considered. This reflects that fact that the empirical distribution has thicker tails, so that there is an area where the asymptotical distribution converges faster

[34] Koedijk and Kool (1995) obtain results that are very similar to the other studies mentioned when they consider their full data sample for Germany (e.g. in the case of the 5-minus-1-year spread 76:4 to 87:9). However, they lay more emphasis on their findings from a much smaller sub-sample (82:1 to 87:9), in which there appears to be less evidence supporting this formulation of the expectations hypothesis. These inferences may, however, be unreliable since they are based solely on asymptotical standard errors. These may be misleading in such small samples, as explained above.

Figure 13: Correlation between yield spread estimates and forward inflation rate spreads (short rate equal to one year)

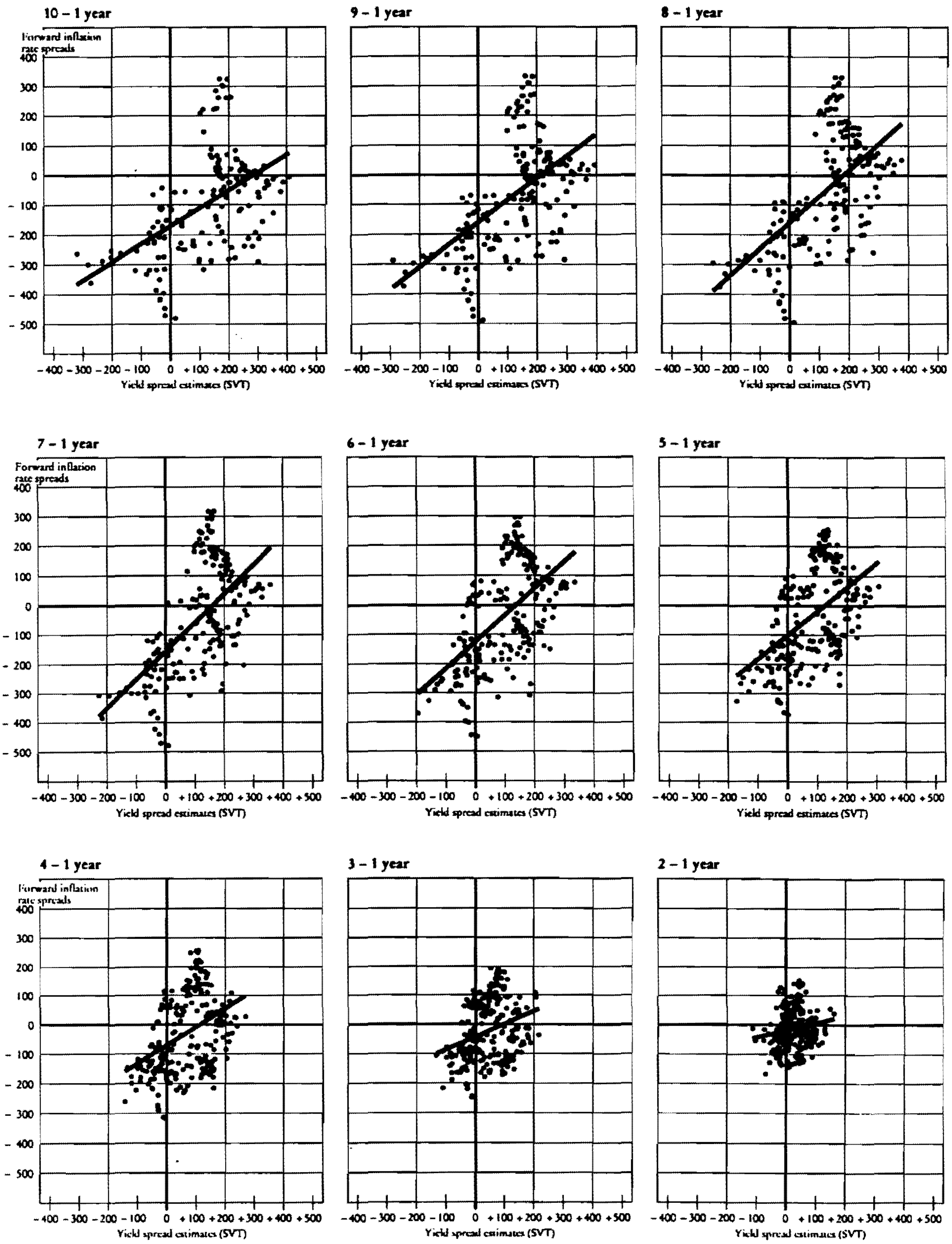
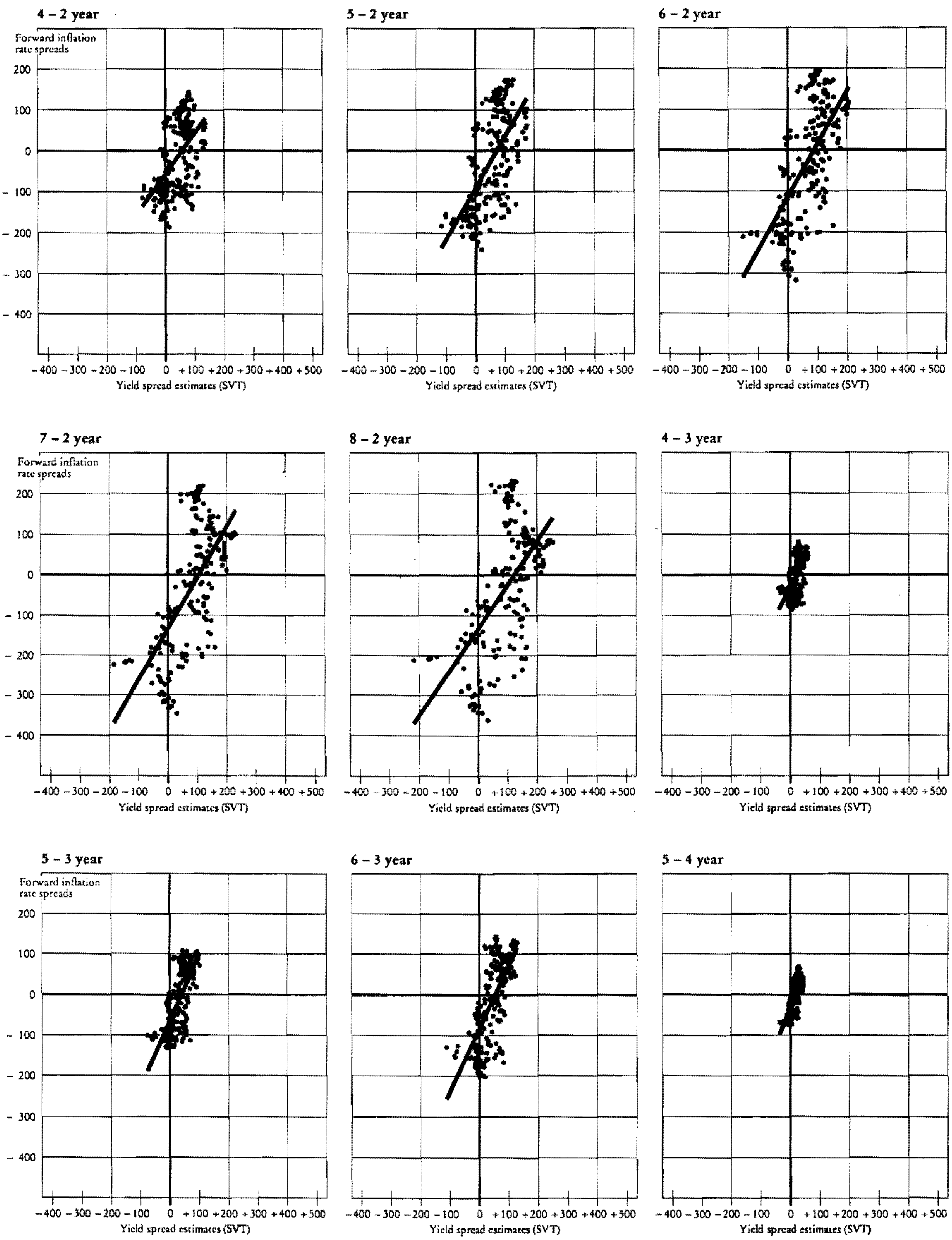


Figure 14: Correlation between yield spread estimates and forward inflation rate spreads (various short rates)



towards zero than the empirical one. The evaluation of the significance levels in the following is based on the empirical probability values.

Second, and most importantly, the medium term segment of the term structure contains significant predictive content about future changes in inflation rates. Consider table 4. The following pattern can be observed. For a constant k, as j increases from 2, the estimate of the slope parameter increases and becomes significant. As j increases further, its significance level peaks and then starts to fall again. Eventually it could fall to negative values and become insignificant. For example, consider $k = 1$ and let j vary. The slope parameter estimate increases and becomes significant at the 10% level as j increases from 2. It continues to increase and becomes significant at the 1% level as j increases to 4 and peaks at $j = 7$. As j increases further the coefficient estimates and the significance levels fall. For example, at $j = 9$, the slope parameter estimates are significant only at the 10% level. Similarly, when j is held constant and k is increased, the estimated slope parameter first rises and its significance level increases, but ultimately falls again and becomes insignificant. A similar pattern can be observed for the coefficient of determination.[35] The R^2 rises with both j and k - the other being held constant - peaks and then falls again. Thus, the information content varies with the maturity range considered. According to the significance levels at which the hypothesis $\beta^{j,k} = 0$ can be rejected and to the level of the R^2, the yield curve is most informative in its mid range. This includes the longer rate, j, varying between 4 and 8 years and the shorter rate, k, varying between 1 and 3 years. For several maturity combinations about half of the total variation in future inflation rates can be explained by the zero-coupon yield spreads.

To analyse why the estimated slope parameter varies with maturity we follow Mishkin (1990a), who shows that the point estimate of the slope parameter $\beta^{j,k}$ can be written as

$$\beta^{j,k} = \left(\sigma^2 + \rho\sigma\right) / \left(1 + \sigma^2 + 2\rho\sigma\right), \tag{28}$$

where

$\sigma^2 = Var\left(E_t\left(\pi_t^j - \pi_t^k\right)\right) / Var\left(E_t\left(rr_t^j - rr_t^k\right)\right)$ is the ratio of the variance of the expected inflation change to the variance of the slope of the real term structure, and

$\rho = Corr\left(E_t\left(\pi_t^j - \pi_t^k\right), \left(rr_t^j - rr_t^k\right)\right)$ is the correlation between the expected inflation change and the slope of the real term structure.

[35] A broadly similar pattern can be observed for the estimate of the constant, $\alpha^{j,k}$, which turns increasingly negative as j increases, reaches a minimum and then increases slightly again. The finding that it turns increasingly negative as maturity is increased is compatible with the notion that the yield curve is (on average) upward sloping because the mean term premium increases with maturity (see e.g. Hicks (1939) and Meiselman (1962)).

To calculate estimated values of σ and ρ, the procedure outlined in Mishkin (1990b) is followed. The *ex post* real interest rate differential is regressed on the current zero-coupon yield spreads and the lagged inflation rate changes to the extent that their values were available at time *t*. The thus fitted *ex post* real interest rate is then interpreted as the expected real interest rate. This is then subtracted from the (nominal) zero-coupon yield to obtain expected inflation rates. The values of σ and ρ are then obtained from (27). Figure 15 shows the combinations of σ, ρ, and $\beta^{j,k}$ obtained here.[36]

At shorter maturities the variability of expected inflation changes is dominated by the variability of the real term structure slopes, which in turn implies small estimates of $\beta^{j,k}$ (see also Gerlach (1995) and Jorion and Mishkin (1991)).[37] The higher variability of the real term structure over short maturities may be attributed to considerable variation in the term premium over time, such variation being particularly important at the short end (Fama and Bliss (1987)).

By contrast, for the medium-term segment studied here, $\beta^{j,k}$ is substantially above zero because the variability of the expected inflation changes is greater than the variability of the real term structure slopes ($\sigma > 1$). The estimates of σ are indeed greater than one and as high as 1.6. As can be seen from figure 15, these high values of σ lead to high estimates of $\beta^{j,k}$.

Third, our results confirm the finding of Jorion and Mishkin (1991) and Gerlach (1995) that, in the case of Germany, the null hypothesis that $[1 - \beta^{j,k}]=0$ can never be rejected on the basis of the empirical probability values. This holds for both, the BBK and the SVT yield spread estimates. This implies that the hypothesis of a constant real term structure cannot be rejected and that the nominal term structure does not contain information about the real term structure.

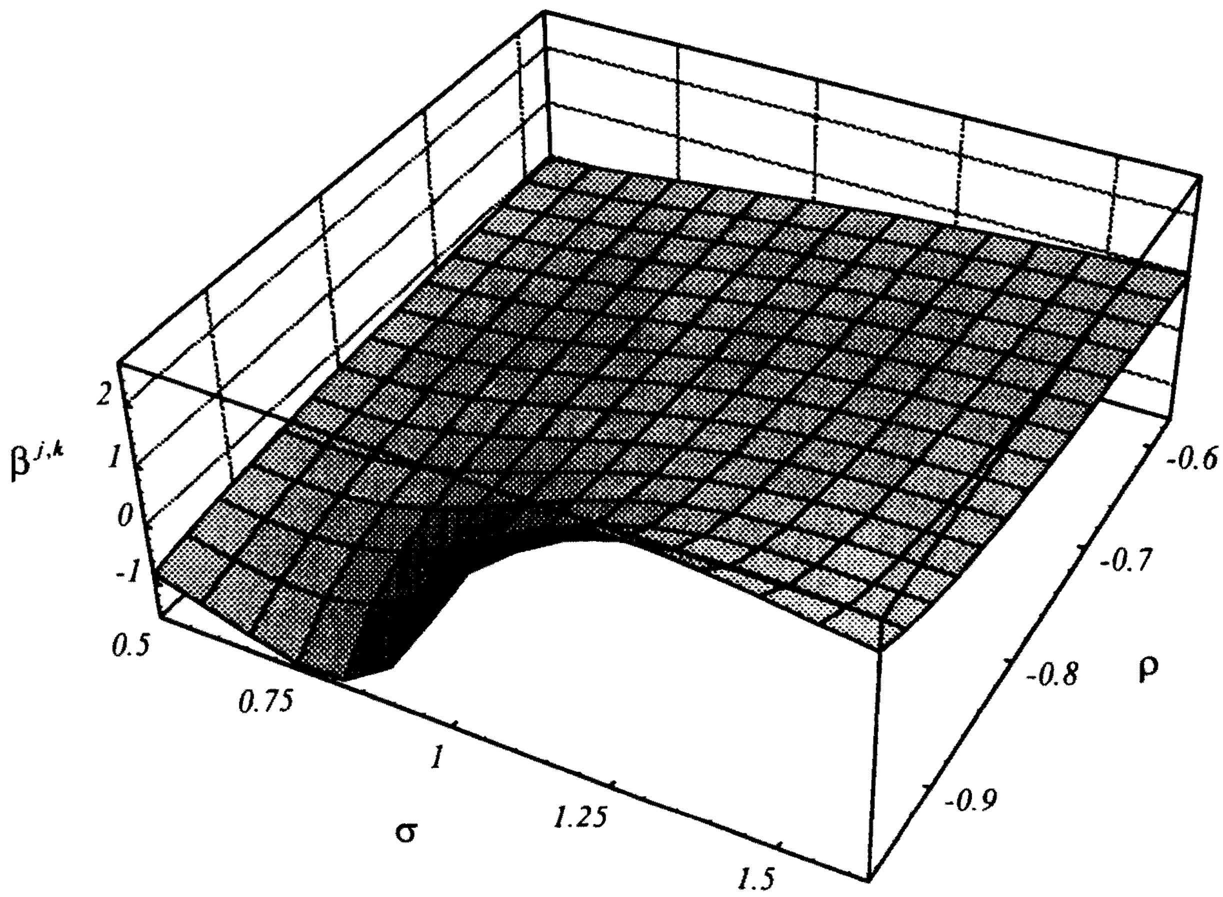
Fourth, as shown in the appendix tables A.2 to A.7, the results of the test of $\beta^{j,k}=0$ differ across the two data sets. The estimates of the slope parameter using the BBK spreads are always higher than (or equal to) the ones obtained from the SVT spreads. This reflects the higher variance of the SVT spreads as opposed to the BBK spreads. However, as far as the significance levels are concerned at which the null hypothesis of $\beta^{j,k}=0$ can be rejected, there are no systematic differences between the two sets of spreads. For both data the null hypothesis can be strongly rejected for the mid-term range of the term structure. There are only very few cases in which the null hypothesis is rejected at different levels of significance,

[36] The estimates of σ and ρ are very similar to Gerlach (1995) and broadly similar to Mishkin (1990b).

[37] The figure also explains why negative estimates of the slope parameter are obtained over short horizons at the long end of the yield curve. Specifically, the estimates of σ become very small, while the estimates of ρ vary only little, ranging around - 0.90. As a result, the term $\rho\sigma$ dominates σ^2.

or rejected in one case but not in the other.[38] Furthermore, the R^2 are similar as well (see tables 3 and 4). If anything, the R^2 obtained from the BBK data are higher, though not much. Also, the t-test of the null hypothesis that $\beta^{j,k}_{SVT} = \hat{\beta}^{j,k}_{BBK}$, i.e. that the coefficient estimate obtained using the SVT spreads is equal to the point estimate obtained from using the BBK spreads, can never be rejected at conventional significance levels.[39] In that sense the choice of the curve-fitting approach does not matter much for the information content of the (estimated) slope parameter for the future changes in inflation rates.

Figure 15: Variation of $\beta^{j,k}$ as a function of σ and ρ



[38] For example, only in two cases ($j = 10$ and $k = 2$ and $j = 9$ and $k = 2$) is the null hypothesis rejected at the 10% level using the SVT spreads, but not rejected using BBK spreads. In two other cases the null is rejected at different levels of significance.

[39] Note that this conclusion is based on the empirical probability values, which are shown in the lower square brackets in the last column of the appendix tables A.2 to A.7.

Table 3: Summary results of tests for $\beta^{j,k} = 0$ using SVT yield spreads

j\k	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6	k = 7	k = 8	k = 9
j = 2	0.27 (0.19) R^2 = 0.04								
j = 3	0.43 (0.21) R^2 =0.09	0.69 (0.30)* R^2=0.15							
j = 4	0.62 (0.16)** R^2 =0.17	0.98 (0.20)*** R^2=0.28	1.39 (0.16)*** R^2 =0.41						
j = 5	0.80 (0.15)*** R^2 =0.27	1.22 (0.16)*** R^2=0.42	1.59 (0.11)*** R^2 =0.54	1.73 (0.13)*** R^2 =0.51					
j = 6	0.89 (0.16)*** R^2 =0.35	1.26 (0.15)*** R^2=0.47	1.49 (0.12)*** R^2 =0.50	1.41 (0.15)*** R^2 =0.37	1.01 (0.18)** R^2 =0.16				
j = 7	0.95 (0.13)*** R^2 =0.40	1.22 (0.11)*** R^2=0.45	1.28 (0.13)*** R^2 =0.38	1.01 (0.21)* R^2 =0.20	0.49 (0.25) R^2 =0.04	- 0.02 (0.24) R^2 =0.00			
j = 8	0.86 (0.14)** R^2 =0.37	1.05 (0.11)*** R^2=0.37	0.99 (0.11)** R^2 =0.25	0.64 (0.17) R^2 =0.09	0.09 (0.20) R^2 =0.00	- 0.43 (0.18) R^2 =0.03	- 0.90 (0.18)** R^2 =0.15		
j = 9	0.73 (0.14)* R^2 =0.31	0.87 (0.14)* R^2=0.28	0.74 (0.14) R^2 =0.14	0.33 (0.20) R^2 =0.02	- 0.23 (0.24) R^2 =0.01	- 0.74 (0.23) R^2 =0.11	- 1.14 (0.22)* R^2 =0.26	- 1.32 (0.22)** R^2 =0.36	
j = 10	0.58 (0.10)* R^2 =0.28	0.83 (0.12)* R^2=0.28	0.43 (0.12) R^2 =0.06	- 0.02 (0.18) R^2 =0.00	- 0.54 (0.20) R^2 =0.07	- 0.97 (0.19) R^2 =0.22	- 1.26 (0.18) R^2 =0.38	- 1.37 (0.16)** R^2 =0.46	- 1.35 (0.13)** R^2 =0.46

Explanation: Coefficient estimates and autocorrelation and heteroscedasticity consistent standard errors in parentheses from OLS regressions of (25). Asterisks denote the significance levels at which the null hypothesis is rejected, the significance levels being determined with bootstrapped empirical probability values (* = 10%, ** = 5%, *** = 1% etc). The exact probability values are contained in appendices A.2 to A.7.

Table 4: Summary results of tests for $\beta^{jk} = 0$ using BBK yield spreads

j\k	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6	k = 7	k = 8	k = 9
j = 2	0.33 (0.24) $R^2 = 0.04$								
j = 3	0.53 $(0.25)^{*}$ $R^2 = 0.11$	0.96 $(0.32)^{**}$ $R^2 = 0.19$							
j = 4	0.78 $(0.32)^{**}$ $R^2 = 0.20$	1.33 $(0.19)^{***}$ $R^2 = 0.34$	1.84 $(0.24)^{***}$ $R^2 = 0.44$						
j = 5	1.00 $(0.16)^{***}$ $R^2 = 0.31$	1.59 $(0.12)^{***}$ $R^2 = 0.47$	2.05 $(0.21)^{***}$ $R^2 = 0.58$	2.21 $(0.27)^{***}$ $R^2 = 0.52$					
j = 6	1.10 $(0.16)^{***}$ $R^2 = 0.39$	1.63 $(0.12)^{***}$ $R^2 = 0.52$	1.92 $(0.18)^{***}$ $R^2 = 0.53$	1.85 $(0.27)^{***}$ $R^2 = 0.41$	1.41 $(0.35)^{*}$ $R^2 = 0.20$				
j = 7	1.17 $(0.14)^{***}$ $R^2 = 0.46$	1.57 $(0.11)^{***}$ $R^2 = 0.50$	1.66 $(0.19)^{***}$ $R^2 = 0.42$	1.39 $(0.29)^{*}$ $R^2 = 0.24$	0.81 (0.38) $R^2 = 0.07$	- 0.25 (0.39) $R^2 = 0.01$			
j = 8	1.09 $(0.17)^{**}$ $R^2 = 0.44$	1.35 $(0.17)^{**}$ $R^2 = 0.42$	1.29 $(0.22)^{*}$ $R^2 = 0.29$	0.92 (0.33) $R^2 = 0.12$	0.31 (0.40) $R^2 = 0.01$	- 0.25 (0.39) $R^2 = 0.01$	- 0.76 (0.37) $R^2 = 0.08$		
j = 9	0.96 $(0.18)^{*}$ $R^2 = 0.40$	1.15 (0.22) $R^2 = 0.35$	0.99 (0.28) $R^2 = 0.20$	0.58 (0.13) $R^2 = 0.06$	0.00 (0.45) $R^2 = 0.00$	- 0.50 (0.44) $R^2 = 0.04$	- 0.91 (0.42) $R^2 = 0.13$	- 1.08 (0.39) $R^2 = 0.19$	
j = 10	0.79 $(0.13)^{*}$ $R^2 = 0.37$	0.88 (0.19) $R^2 = 0.27$	0.66 (0.27) $R^2 = 0.11$	0.21 (0.37) $R^2 = 0.01$	- 0.35 (0.43) $R^2 = 0.02$	- 0.81 (0.41) $R^2 = 0.11$	- 1.15 (0.37) $R^2 = 0.22$	- 1.30 (0.31) $R^2 = 0.29$	- 1.35 $(0.22)^{*}$ $R^2 = 0.32$

Explanation: Coefficient estimates and autocorrelation and heteroscedasticity consistent standard errors in parentheses from OLS regressions of (25). Asterisks denote the significance levels at which the null hypothesis is rejected, the significance levels being determined with bootstrapped empirical probability values (* = 10%, ** = 5%, *** = 1% etc). The exact probability values are contained in appendices A.2 to A.7.

VI. Concluding remarks

The present paper examines various procedures for estimating yield curves. No clear-cut decision can be made on theoretical grounds alone as to which approach is preferable, although the Nelson/Siegel and Svensson approaches are theoretically more plausible than the approach currently adopted by the Bundesbank. As far as empirical performance is concerned, the former approaches dominate the latter. However, ultimately the choice of the curve-fitting approach should be geared to the precise purpose for which the estimated yield curves are to be used. A key criterion is that the estimation approach should be flexible enough to represent the information in which the monetary policy makers are interested.

If the focus is on the pricing of debt securities, none of the methods considered here seems to be wholly appropriate. All of them represent parametric approaches that impose a specific structure on the data. They smooth out most kinks in the curves, thus making them not the most ideal way to detect yield or price abnormalities. At the same time, the methods are not very sensitive with respect to small variations in yields of individual debt securities, smoothing these effects out as well. This smoothing is arguably desirable from a "monetary policy" point of view.

However, there is a risk of oversmoothing. Relevant information could be suppressed. For example, the Bundesbank approach seems to systematically underestimate the slope of the term structure in situations where it is very steep. It also ignores S-shapes. This is reflected in a poor empirical performance in terms of the root mean squared yield errors of the estimates. Yet, *per se*, the quality of the estimates determined in this way does not tell us much. More importantly, we would like to know whether relevant information is being suppressed.

The interpretation of the term structure for monetary policy purposes focuses on efforts to obtain information on market expectations of future inflation (and interest rates). Thus, an alternative criterion for comparison between procedures is suggested here. Specifically, the well-known Mishkin approach to testing the information content of the term structure is adopted and applied to the term structure estimates from the various procedures. Under this criterion the information content is defined as the ability of the yield curve's slope to predict future changes in the inflation rate.

Our empirical tests applying that criterion yields the following results for Germany. 1) The term structure is informative in the sense defined above, especially in its middle segment between three and eight years. 2) The information content does not vary significantly between the yield curve estimates obtained using different procedures, provided that significance is assessed on the basis of empirical probability values (and not the Newey-West probability values that are valid only asymptotically). Thus, from a monetary policy perspective, the following conclusion may be drawn. The medium-term segment of the term structure does indeed constitute a useful indicator of market expectations in respect of future inflation. As long as the interpretation of the term structure is confined to simple linear inference from its slope to future changes in inflation rates, the choice of the term structure estimation method is of minor importance. To that extent the curve fitting procedure used by the Bundesbank is appropriate in view of the uses to which the yield estimates were put in the past.

The first result confirms the findings of the empirical tests by Jorion and Mishkin (1991) and by Gerlach (1995), which are based on the Bundesbank's yield-to-maturity estimates. Although these practices may be criticised on theoretical grounds, the second result of our paper suggests that, for the specific tests in these papers, the choice of the method is not very relevant. However, this result can possibly be explained by the low levels and the limited variation of coupons of German government debt securities' (in our sample between 5 and 11 per cent). In situations where the coupon levels vary considerably the result may be different. The result may also be due to the relatively large sample period considered here. This implies that the cases in which the estimates are considerably different across methods have only a small weight, which makes it difficult to identify significant differences across the procedures.

Another issue is how one could make optimum use of the term structure to forecast changes in inflation rates. The Mishkin approach may imply a loss of relevant information, since it considers average concepts (zero-coupon rates). More recently, many central banks have made increased use of implied forward rates. These represent a marginal concept and thus arguably provide more precise information on expectations. If the focus is on these rates, it is preferable to use the Nelson/Siegel and Svensson procedures because implied forward rates can be calculated directly from the estimated zero-coupon yields.

References

Anderson, N., F. Breedon, M. Deacon, A. Derry and G. Murphy (1996), *Estimating and interpreting the yield curve*, John Wiley Series in Financial Economics and Quantitative Analysis, John Wiley & Sons, New York.

Bang, B.K. (1993), "International evidence on the predictive power of interest spreads", Ph.D. thesis, Leonard N. Stern Graduate School of Business Administration, New York University, July.

Bekaert, G., R.J. Hodrick and D. Marshall (1996), "On biases in tests of the expectations hypothesis of the term structure of interest rates", Federal Reserve Bank of Chicago Research Department, Working Paper Series Issues in Financial Regulation No. 3, January.

Bekdache, B. and C.F. Baum (1994), "Comparing alternative models of the term structure of interest rates", Boston College, Department of Economics, Working Paper in Economics No. 271, June.

Bliss, R.R. (1994), "Testing term structure estimation methods", Indiana University, Finance Department, mimeo, April 4.

Breedon, F. (1995), "Bond prices and market expectations of inflation", *Bank of England Quarterly Bulletin*, May, pp. 1 - 6.

Brock, W, J. Lakonishok, and B. LeBaron (1992), "Simple technical trading rules and the stochastic properties of stock returns", *The Journal of Finance*, XLVII, 5, December, pp. 1731 - 1764.

Bußmann, J. (1989), "Die Bestimmung der Zinsstruktur am deutschen Kapitalmarkt. Eine empirische Untersuchung für den Zeitraum 1978 bis 1986", *Kredit und Kapital*, 22, 1, pp. 117 - 137.

Campbell, J.Y (1995), "Some lessons from the yield curve", *Journal of Economic Perspectives*, 9, 3, Summer, pp. 129 - 152.

Coleman, T.S.L. Fisher and R. Ibbotson (1992), "Estimating the term structure of interest from data that include the prices of coupon bonds." *The Journal of Fixed Income*, 2, September, pp. 85 - 116.

Cooper, N. and J. Steeley (1996), "G7 yield curves", *Bank of England Quarterly Bulletin*, May, pp. 199 - 208.

Dahlquist, M. and L.E.O. Svensson (1996), "Estimating the term structure of interest rates for monetary policy analysis", *Scandinavian Journal of Economics*, 98, 2, pp. 163 - 183.

Deacon, M. and A. Derry (1994a), "Deriving estimates of inflation expectations from the prices of UK government bonds", Bank of England Working Paper No. 23, July.

Deacon, M. and A. Derry (1994b), "Estimating the term structure of interest rates", Bank of England Working Paper No. 24, July.

Derry, A.J. and M. Pradhan (1993), "Tax specific term structures of interest rates in the UK government bond market", Bank of England Working Paper No 11, April.

Deutsche Bundesbank (1983), "Interest rate movements since 1978", *Monthly Report of the Deutsche Bundesbank*, 35, 1, January, pp. 13 - 25.

Deutsche Bundesbank (1995), *The market for German Federal securities*, Frankfurt a. M., July.

Estrella, A. and F.S. Mishkin (1995), "The term structure of interest rates and its role in monetary policy for the European central bank", NBER Working Paper No. 5279, September.

Estrella, A. and F.S. Mishkin (1996), "The predictive power of the term structure of interest rates in Europe and the United States: implications for the European Central Bank", Federal Reserve Bank of New York, mimeo, July.

Evans, M. and P. Wachtel, "Inflation regimes and the sources of inflation uncertainty", *Journal of Money, Credit, and Banking*, 25, 3, August, pp. 475 - 511.

Fama, E.F. (1990), "Term-structure forecasts of interest rates, inflation, and real returns, " *Journal of Monetary Economics*, 25, 1, January, pp. 21, 59 - 76.

Fama, E.F. (1984), "The information in the term structure", *Journal of Financial Economics*, 13, December, pp. 509 - 28.

Fama, E.F., and R.R. Bliss (1987), "The information in long-maturity forward rates", *American Economic Review*, LXXVII, pp. 680 - 92.

Frankel, J.A. (1995), *Financial Markets and Monetary Policy*, MIT Press, Cambridge Massachusettes.

Frankel, J.A. and C.S. Lown (1994), "An indicator of future inflation extracted from the steepness of the interest rate yield curve along its entire length", *The Quarterly Journal of Economics*, 109, 437, May, pp. 517 - 530.

Freedman, D.A. and S.C. Peters (1984), "Bootstrapping a regression equation: some empirical results", *Journal of the American Statistical Association*, 79, 385, March, pp. 97 - 106.

Gerlach, S. (1995), "The information content of the term structure: evidence for Germany", BIS Working paper No. 29, September.

Hassler, U. and D. Nautz (1995), "The term structure of interest rates as an indicator of German monetary policy?", Humboldt-Universität Berlin Discussion Paper No. 64 (Sonderforschungsbereich 373), October.

Hesse, H. and G. Roth (1992), "Die Zinsstruktur als Indikator der Geldpolitik?", *Kredit und Kapital*, 35, 1, pp. 1002 - 1023.

Hicks, J.R. (1993), *Value and capital*, 2nd ed., Oxford University Press.

Issing, O. (1994), "Zinsstruktur oder Geldmenge?", *Wirtschaftspolitik in offenen Volkswirtschaften*, Festschrift für H. Hesse zum 60. Geburtstag, Herrmann Sautter (ed.), Göttingen, pp. 3 - 21.

Jorion, P. and F. Mishkin (1991), "A multicountry comparison of term-structure forecasts at long horizons", *Journal of Financial Economics*, 29, pp. 59 - 80.

Klein, W. (1990), "Forward rates and the expectations theory of the term structure: tests for the Federal Republic of Germany", *Empirical Economics*, 15, 3, pp. 245 - 265.

Kirchgässner, G. and M. Savioz (1995), "Is the interest rate spread a valid predictor for real and nominal economic developments? - An empirical investigation for the Federal Republic of Germany", paper presented at the Tenth Annual Congress of the European Economic Association, Pargue, September 1 - 4.

Koedijk, K.G. and C.J.M. Kool (1995), "Future inflation and the information in international term structures", *Empirical Economics*, 20, pp. 217 - 242.

Langfeldt, E. (1994), "Die Zinsstruktur als Frühindikator für Konjunktur und Preisentwicklung in Deutschland", Kiel Working Paper No. 615, February.

Lassak, G. (1993), *Bewertung festverzinslicher Wertpapiere am deutschen Rentenmarkt*, Physica Verlag Heidelberg.

Levin, F. (1996), *Die Erwartungstheorie der Zinsstruktur - Eine empirische Untersuchung für die Bundesrepublik Deutschland im Zeitraum von 1974 bis 1988*, Peter Lang Verlag, Frankfurt a.M.

Mastronikola, K. (1991), "Yield curves for gilt-edged stocks: a new model", Bank of England Discussion Paper (Technical Series) No. 49, December.

McCulloch, J.H. (1971), "Measuring the term structure of interest rates", *Journal of Business*, 44, pp. 19 - 31.

McCulloch, J.H. (1975), An estimate of the liquidity premium, *Journal of Political Economy*, 83, pp. 95 - 119.

Meiselman, D. (1962), *The term structure of interest rates*, Englewood Cliffs, Prentice Hall.

Mishkin, F.S. (1988), "The information in the term structure: some further results", *Journal of Applied Econometrics*, 3, pp. 307 - 314.

Mishkin, F.S. (1990a), "What does the term structure tell us about future inflation?", *Journal of Monetary Economics* 25, pp. 77 - 95.

Mishkin, F.S. (1990b), "The information in the longer maturity term structure about future inflation", *The Quarterly Journal of Economics*, CV, August, pp. 815 - 828.

Mishkin, F.S. (1991), "A multi-country study of the information in the shorter maturity term structure about future inflation", *Journal of International Money and Finance*, 10, 1, March, pp. 2 - 22.

Nelson, C.R., and A.F. Siegel (1987), "Parsimonious modeling of yield curves", *Journal of Business*, 60, 4, pp. 473 - 89.

Neumann, M.J.M. (1968), "Yield-curve analysis: eine Methode zur empirischen Bestimmung der Zinsstruktur am Rentenmarkt", *Jahrbücher für Nationalökonomie und Statistik*, 182, pp. 193 - 203.

Newey, W.K. and K.D. West (1987), "A simple, positive definite, heteroscedasticity and autocorrelation consistent covariance matrix", *Econometrica*, 55, pp. 703 - 708.

Pagan, A. (1984), "Econometric issues in the analysis of regression with generated regressors", *International Economic Review*, 25, pp. 226 - 247.

Reinhardt, V. (1992), "Understanding the simple algebra of forward rates", mimeo, Federal Reserve Board, April.

Ricart, R. and P. Sicsic (1995), "Estimating the term structure of interest rates from French data", *Banque de France Bulletin Digest*, 22, October, pp. 47 - 57.

Russell, S. (1992), "Understanding the term structure of interest rates: the expectations theory", *Federal Reserve Bank of St. Louis Review*, 74, 4, July/August, pp. 36 - 50.

Rudebusch, G.D. (1995), "Federal Reserve interest rate targeting, rational expectations, and the term structure", *Journal of Monetary Economics*, 35, pp. 245 - 274.

Schaefer, S. (1981), "Measuring a tax-specific term structure of interest rates in the Market for British Government Securities", *Journal of Political Economy*, 83, pp. 95 -119.

Shea, G. (1984), "Pitfalls in smoothing interest rate term structure data: equilibrium models and spline approximations", *Journal of Financial and Quantitative Studies*, 19, 3, pp. 253 - 269.

Shiller, R.J. (1990), "The term structure of interest rates", *Handbook of Monetary Economics*, Vol 1, B.M. Friedman and F.H. Hahn (eds.), Elsevier Science Publishers, B.V.

Siegel, A.F. and C.R. Nelson (1988), "Long-term behavior of yield curves", *Journal of Financial and Quantitative Analysis*, 23, 1, March, pp. 105 - 110.

Smith, J. and S. Yadav, "A comparison of alternative covariance matrix estimators", forthcoming in *Journal of International Money and Finance*.

Steeley, J.M. (1991), "Estimating the gilt-edged term structure: basis splines and confidence intervals", *Journal of Business Finance and Accounting*, 18, 4, June, pp. 513 - 529.

Subrahmanyam, M.G. (1995), "The term structure of interest rates: alternative paradigms and implications for financial risk management", *Geneva Risk Lecture* held at the 22nd Seminar of the European Group of Risk and Insurance Economists, University of Geneva, September 18 - 20.

Svensson, L.E.O. (1993), "Term, inflation, and foreign exchange risk premia: a unified treatment", IIES Seminar Paper No. 548 (also as NBER Working Paper No. 4544).

Svensson, L.E.O. (1994), "Estimating and interpreting forward interest rates: Sweden 1992 - 1994", IMF Working Paper No. 114, September.

Woodford, M. (1994), "Nonstandard indicators for monetary policy: can their usefulness be judged from forecasting regressions"?, N.G. Mankiw, (ed), *Monetary Policy*. NBER and the University of Chicago Press: Chicago.

Appendix A1: Background information

Table A.1: Summary of term structure estimation approaches at selected central banks

Central bank	Estimation method	Curve fitted
Canada	Third-order polynomial with coupon adjustment	Yield-to-maturity
Finland	Svensson	Zero coupon yields
France	Nelson/Siegel; Svensson	Zero coupon yields
Germany	Linear-logarithmic regression with coupon adjustment	Yield-to-maturity
Italy	Cubic splines; Cox, Ingersoll & Ross one and two factor model; Swap rate yield curve	Zero coupon yields
Japan	5th order spline	Zero coupon yields
Norway	Cubic spline; Nelson/Siegel	Zero-coupon yields
Spain	Nelson/Siegel; Svensson	Zero-coupon yields
Sweden	Svensson	Zero-coupon yields
Switzerland	Svensson	Zero-coupon yields
United Kingdom	Svensson; Nelson/Siegel; Cubic splines	Zero-coupon yields
United States	Nelson/Siegel; Svensson; Smoothing splines	Zero-coupon yields

Source: Handout to and background notes for the "Meeting on the estimation of zero-coupon yield curves" at the BIS, June 5, 1996.

Appendix A.2: Inflation rate change regression results

Table A.2: Inflation rate change regressions with n-1 year spread

	Bundesbank with coupon adjustment (BBK)				Svensson with tax adjustment (SVT)				
	$\alpha = 0$	$\beta = 0$	R^2	β-1=0	$\alpha = 0$	$\beta = 0$	R^2	β-1 = 0	β-$\hat{\beta}_{BBK}$=0
2 - 1, 73:9-94:2	- 0.18 (0.13) [0.16] [0.05]	0.33 (0.24) [0.17] [0.18]	0.04	[0.00] [0.57]	- 0.17 (0.13) [0.21] [0.06]	0.27 (0.19) [0.15] [0.21]	0.04	[0.00] [0.45]	0.06 (0.19) [0.75] [0.90]
3 - 1, 73:9-93:2	- 0.39 (0.23) [0.09] [0.09]	0.53 (0.25) [0.03] [0.10]	0.11	[0.06] [0.87]	- 0.37 (0.24) [0.13] [0.08]	0.43 (0.21) [0.04] [0.12]	0.09	[0.01] [0.81]	0.10 (0.21) [0.65] [0.94]
4 - 1, 73:9-92:2	- 0.64 (0.32) [0.04] [0.08]	0.78 (0.32) [0.00] [0.02]	0.20	[0.23] [0.96]	- 0.61 (0.34) [0.08] [0.10]	0.62 (0.16) [0.00] [0.03]	0.17	[0.02] [0.90]	0.16 (0.16) [0.32] [0.97]
5 - 1, 73:9-91:2	- 0.92 (0.39) [0.02] [0.07]	1.00 (0.16) [0.00] [0.01]	0.31	[0.98] [0.99]	- 0.88 (0.44) [0.04] [0.09]	0.80 (0.15) [0.00] [0.01]	0.27	[0.20] [0.99]	0.20 (0.15) [0.20] [0.98]
6 - 1, 73:9-90:2	- 1.14 (0.43) [0.01] [0.07]	1.10 (0.16) [0.00] [0.01]	0.39	[0.53] [0.99]	- 1.09 (0.49) [0.27] [0.11]	0.89 (0.16) [0.00] [0.01]	0.35	[0.50] [0.99]	0.21 (0.16) [0.20] [0.99]
7 - 1, 73:9-89:2	- 1.45 (0.31) [0.00] [0.02]	1.17 (0.14) [0.00] [0.01]	0.46	[0.21] [0.97]	- 1.40 (0.39) [0.00] [0.04]	0.95 (0.13) [0.00] [0.01]	0.40	[0.72] [0.99]	0.22 (0.13) [0.10] [0.99]
8 - 1, 73:9-88:2	- 1.53 (0.30) [0.00] [0.02]	1.09 (0.17) [0.00] [0.02]	0.44	[0.60] [0.99]	- 1.44 (0.39) [0.00] [0.06]	0.86 (0.14) [0.00] [0.03]	0.37	[0.33] [0.99]	0.23 (0.14) [0.11] [0.99]
9 - 1, 73:9-87:2	- 1.58 (0.29) [0.00] [0.03]	0.96 (0.18) [0.00] [0.06]	0.40	[0.80] [0.99]	- 1.45 (0.39) [0.00] [0.08]	0.73 (0.14) [0.00] [0.07]	0.31	[0.05] [0.97]	0.23 (0.14) [0.09] [0.97]
10 - 1, 73:9-86:2	- 1.68 (0.25) [0.00] [0.03]	0.79 (0.13) [0.00] [0.07]	0.37	[0.11] [0.95]	- 1.56 (0.33) [0.00] [0.07]	0.58 (0.10) [0.00] [0.07]	0.28	[0.00] [0.84]	0.21 (0.10) [0.03] [0.90]

Explanation: OLS regressions of equation (25) for the tests of $\alpha = 0$ and $\beta = 0$, of equation (27) for the tests of $\beta - 1 = 0$, and of equation (26) for $\beta - \hat{\beta}_{BBK} = 0$. Asymptotic autocorrelation and heteroscedasticty-consistent (Newey and West (1987) with Bartlett weights) standard errors in parentheses and probability values in the upper square brackets and bootstrapped empirical p-values in the lower square brackets.

Table A.3: Inflation rate change regressions with n-2 year spread

	Bundesbank with tax adjustment (BBK)				Svensson with tax adjustment (SVT)				
	$\alpha = 0$	$\beta = 0$	β-1=0	R^2	$\alpha = 0$	$\beta = 0$	R^2	β-1 = 0	β-$\hat{\beta}_{BBK}$=0
3 - 2,	- 0.23	0.96		0.19	- 0.22	0.69	0.15		0.27
74:9-93:2	(0.09)	(0.32)			(0.10)	(0.30)			(0.30)
	[0.02]	[0.00]	[0.89]		[0.03]	[0.02]		[0.31]	[0.37]
	[0.03]	[0.03]	[0.99]		[0.04]	[0.08]		[0.97]	[0.96]
4 - 2,	- 0.52	1.33		0.34	- 0.50	0.98	0.28		0.34
74:9-92:2	(0.16)	(0.19)			(0.19)	(0.20)			(0.20)
	[0.00]	[0.00]	[0.08]		[0.01]	[0.00]		[0.94]	[0.09]
	[0.01]	[0.00]	[0.79]		[0.02]	[0.01]		[0.99]	[0.97]
5 - 2,	- 0.81	1.59		0.47	- 0.81	1.22	0.42		0.37
74:9-91:2	(0.22)	(0.12)			(0.27)	(0.16)			(0.16)
	[0.00]	[0.00]	[0.00]		[0.00]	[0.00]		[0.17]	[0.02]
	[0.01]	[0.00]	[0.28]		[0.04]	[0.00]		[0.95]	[0.99]
6 - 2,	- 1.01	1.63		0.52	- 1.00	1.26	0.47		0.37
74:9-90:2	(0.28)	(0.12)			(0.36)	(0.15)			(0.15)
	[0.00]	[0.00]	[0.00]		[0.00]	[0.00]		[0.09]	[0.01]
	[0.04]	[0.00]	[0.36]		[0.06]	[0.01]		[0.96]	[0.99]
7 - 2,	- 1.22	1.57		0.50	- 1.20	1.22	0.45		0.35
74:9-89:2	(0.27)	(0.13)			(0.38)	(0.11)			(0.11)
	[0.00]	[0.00]	[0.00]		[0.00]	[0.00]		[0.05]	[0.00]
	[0.04]	[0.00]	[0.47]		[0.08]	[0.00]		[0.95]	[0.98]
8 - 2,	- 1.26	1.35		0.42	- 1.21	1.05	0.37		0.30
74:9-88:2	(0.29)	(0.17)			(0.42)	(0.11)			(0.11)
	[0.00]	[0.00]	[0.04]		[0.00]	[0.00]		[0.67]	[0.01]
	[0.05]	[0.02]	[0.87]		[0.13]	[0.01]		[0.99]	[0.98]
9 - 2,	- 1.26	1.15		0.35	- 1.17	0.87	0.28		0.28
74:9-87:2	(0.30)	(0.22)			(0.42)	(0.14)			(0.14)
	[0.00]	[0.00]	[0.50]		[0.01]	[0.00]		[0.37]	[0.05]
	[0.09]	[0.11]	[0.97]		[0.18]	[0.07]		[0.99]	[0.99]
10 - 2,	- 1.28	0.88		0.27	- 1.58	0.83	0.28		0.05
74:9-86:2	(0.30)	(0.19)			(0.36)	(0.12)			(0.12)
	[0.00]	[0.00]	[0.52]		[0.00]	[0.00]		[0.15]	[0.66]
	[0.10]	[0.16]	[0.98]		[0.12]	[0.06]		[0.99]	[1.00]

Explanation: OLS regressions of equation (25) for the tests of $\alpha = 0$ and $\beta = 0$, of equation (27) for the tests of $\beta - 1 = 0$, and of equation (26) for $\beta - \hat{\beta}_{BBK} = 0$. Asymptotic autocorrelation and heteroscedasticty-consistent (Newey and West (1987) with Bartlett weights) standard errors in parentheses and probability values in the upper square brackets and bootstrapped empirical p-values in the lower square brackets.

Table A.4: Inflation rate change regressions with n-3 year spread

	Bundesbank with tax adjustment (BBK)				Svensson with tax adjustment (SVT)				
	$\alpha = 0$	$\beta = 0$	$\beta\text{-}1 = 0$	R^2	$\alpha = 0$	$\beta = 0$	R^2	$\beta\text{-}1 = 0$	$\beta\text{-}\hat{\beta}_{BBK} = 0$
4 - 3, 75:9-92:2	- 0.29 (0.06) [0.00] [0.00]	1.84 (0.24) [0.00] [0.00]	 [0.00] [0.26]	0.44	- 0.29 (0.07) [0.00] [0.01]	1.39 (0.16) [0.00] [0.00]	0.41	 [0.01] [0.72]	0.45 (0.16) [0.00] [0.96]
5 - 3, 75:9-91:2	- 0.59 (0.10) [0.00] [0.00]	2.05 (0.21) [0.00] [0.00]	 [0.00] [0.18]	0.58	- 0.59 (0.14) [0.00] [0.02]	1.59 (0.11) [0.00] [0.00]	0.54	 [0.00] [0.33]	0.46 (0.11) [0.00] [0.93]
6 - 3, 75:9-90:2	- 0.76 (0.17) [0.00] [0.03]	1.92 (0.18) [0.00] [0.00]	 [0.00] [0.27]	0.53	- 0.76 (0.25) [0.00] [0.06]	1.49 (0.12) [0.00] [0.00]	0.50	 [0.00] [0.62]	0.43 (0.12) [0.00] [0.98]
7 - 3, 75:9-89:2	- 0.88 (0.23) [0.00] [0.05]	1.66 (0.19) [0.00] [0.01]	 [0.00] [0.54]	0.42	- 0.86 (0.35) [0.01] [0.15]	1.28 (0.13) [0.00] [0.00]	0.38	 [0.03] [0.93]	0.38 (0.13) [0.00] [0.99]
8 - 3, 75:9-88:2	- 0.86 (0.28) [0.00] [0.14]	1.29 (0.22) [0.00] [0.06]	 [0.19] [0.91]	0.29	- 0.81 (0.41) [0.05] [0.25]	0.99 (0.11) [0.00] [0.02]	0.25	 [0.96] [0.99]	0.30 (0.11) [0.01] [0.97]
9 - 3, 75:9-87:2	- 0.84 (0.29) [0.00] [0.21]	0.99 (0.28) [0.00] [0.25]	 [0.98] [0.99]	0.20	- 0.75 (0.41) [0.07] [0.32]	0.74 (0.14) [0.00] [0.13]	0.14	 [0.07] [0.98]	0.25 (0.14) [0.07] [0.97]
10 - 3, 75:9-86:2	- 0.83 (0.29) [0.00] [0.23]	0.66 (0.27) [0.01] [0.38]	 [0.21] [0.97]	0.11	- 0.72 (0.41) [0.08] [0.39]	0.43 (0.12) [0.00] [0.28]	0.06	 [0.00] [0.84]	0.23 (0.12) [0.05] [0.88]

Explanation: OLS regressions of equation (25) for the tests of $\alpha = 0$ and $\beta = 0$, of equation (27) for the tests of $\beta - 1 = 0$, and of equation (26) for $\beta - \hat{\beta}_{BBK} = 0$. Asymptotic autocorrelation and heteroscedasticty-consistent (Newey and West (1987) with Bartlett weights) standard errors in parentheses and probability values in the upper square brackets and bootstrapped empirical p-values in the lower square brackets.

Table A.5: Inflation rate change regressions with n-4 year spread

	Bundesbank with tax adjustment (BBK)				Svensson with tax adjustment (SVT)				
	$\alpha = 0$	$\beta = 0$	β-1=0	R^2	$\alpha = 0$	$\beta = 0$	R^2	β-1 = 0	β-$\hat{\beta}_{BBK}$=0
5 - 4, 76:9-91:2	- 0.28 (0.04) [0.00] [0.00]	2.21 (0.27) [0.00] [0.00]	 [0.00] [0.16]	0.52	- 0.28 (0.07) [0.00] [0.01]	1.73 (0.13) [0.00] [0.00]	0.51	 [0.00] [0.20]	0.48 (0.13) [0.00] [0.94]
6 - 4, 76:9-90:2	- 0.44 (0.13) [0.00] [0.03]	1.85 (0.27) [0.00] [0.00]	 [0.00] [0.47]	0.41	- 0.43 (0.18) [0.02] [0.10]	1.41 (0.15) [0.00] [0.00]	0.37	 [0.01] [0.76]	0.44 (0.15) [0.00] [0.98]
7 - 4, 76:9-89:2	- 0.51 (0.22) [0.02] [0.16]	1.39 (0.29) [0.00] [0.06]	 [0.00] [0.86]	0.24	- 0.46 (0.31) [0.13] [0.32]	1.01 (0.21) [0.00] [0.07]	0.20	- 0.01 (0.21) [0.97] [0.99]	0.38 (0.21) [0.07] [0.99]
8 - 4, 76:9-88:2	- 0.48 (0.27) [0.08] [0.31]	0.92 (0.33) [0.00] [0.28]	 [0.81] [0.99]	0.12	- 0.40 (0.36) [0.27] [0.46]	0.64 (0.17) [0.00] [0.17]	0.09	 [0.03] [0.94]	0.28 (0.17) [0.09] [0.95]
9 - 4, 76:9-87:2	- 0.45 (0.27) [0.10] [0.37]	0.58 (0.39) [0.13] [0.58]	 [0.28] [0.95]	0.06	- 0.35 (0.36) [0.33] [0.58]	0.33 (0.20) [0.10] [0.58]	0.02	 [0.00] [0.91]	0.25 (0.20) [0.22] [0.95]
10 - 4, 76:9-86:2	- 0.43 (0.27) [0.11] [0.44]	0.21 (0.37) [0.58] [0.84]	 [0.03] [0.92]	0.01	- 0.33 (0.36) [0.35] [0.65]	- 0.02 (0.18) [0.93] [0.98]	0.00	 [0.00] [0.83]	0.23 (0.18) [0.20] [0.74]

Explanation: OLS regressions of equation (25) for the tests of α =0 and β =0, of equation (27) for the tests of β - 1 = 0, and of equation (26) for β - $\hat{\beta}_{BBK}$ = 0. Asymptotic autocorrelation and heteroscedasticty-consistent (Newey and West (1987) with Bartlett weights) standard errors in parentheses and probability values in the upper square brackets and bootstrapped empirical p-values in the lower square brackets.

Table A.6: Inflation rate change regressions with n-5 and n-6 year spread

	Bundesbank with tax adjustment (BBK)				Svensson with tax adjustment (STNSES)				
	$\alpha = 0$	$\beta = 0$	β-1=0	R^2	$\alpha = 0$	$\beta = 0$	R^2	β-1 = 0	β-$\hat{\beta}_{BBK}$=0
6 - 5,	- 0.17	1.41		0.20	- 0.14	1.01	0.16		0.40
77:9-90:2	(0.08)	(0.35)			(0.10)	(0.18)			(0.18)
	[0.05]	[0.00]	[0.24]		[0.15]	[0.00]		[0.95]	[0.02]
	[0.18]	[0.04]	[0.80]		[0.26]	[0.02]		[0.98]	[0.94]
7 - 5,	- 0.22	0.81		0.07	- 0.17	0.49	0.04		0.32
77:9-89:2	(0.17)	(0.38)			(0.22)	(0.25)			(0.25)
	[0.21]	[0.03]	[0.63]		[0.43]	[0.05]		[0.04]	[0.20]
	[0.37]	[0.31]	[0.96]		[0.52]	[0.32]		[0.88]	[0.80]
8 - 5,	- 0.18	0.31		0.01	- 0.12	0.09	0.00		0.22
77:9-88:2	(0.23)	(0.40)			(0.28)	(0.20)			(0.20)
	[0.43]	[0.43]	[0.08]		[0.66]	[0.64]		[0.00]	[0.27]
	[0.60]	[0.70]	[0.85]		[0.76]	[0.83]		[0.69]	[0.73]
9 - 5,	- 0.16	0.00		0.00	- 0.09	- 0.23	0.01		0.23
77:9-87:2	(0.23)	(0.45)			(0.29)	(0.24)			(0.24)
	[0.48]	[0.99]	[0.03]		[0.76]	[0.32]		[0.00]	[0.32]
	[0.68]	[0.99]	[0.89]		[0.87]	[0.69]		[0.77]	[0.71]
10 - 5,	- 0.14	- 0.35		0.02	- 0.08	- 0.54	0.07		0.19
77:9-86:2	(0.24)	(0.43)			(0.29)	(0.20)			(0.20)
	[0.53]	[0.41]	[0.00]		[0.78]	[0.01]		[0.00]	[0.93]
	[0.73]	[0.76]	[0.88]		[0.88]	[0.38]		[0.70]	[0.88]
7 - 6,	- 0.07	- 0.25		0.01	- 0.04	- 0.02	0.00		- 0.23
78:9-86:2	(0.09)	(0.39)			(0.11)	(0.24)			(0.24)
	[0.44]	[0.53]	[0.06]		[0.69]	[0.93]		[0.00]	[0.33]
	[0.54]	[0.72]	[0.73]		[0.71]	[0.96]		[0.47]	[0.63]
8 - 6,	- 0.05	- 0.25		0.01	- 0.01	- 0.43	0.03		0.18
78:9-86:2	(0.15)	(0.39)			(0.17)	(0.18)			(0.18)
	[0.75]	[0.52]	[0.00]		[0.95]	[0.02]		[0.00]	[0.32]
	[0.81]	[0.76]	[0.65]		[0.97]	[0.30]		[0.34]	[0.69]
9 - 6,	- 0.04	- 0.50		0.04	0.01	- 0.74	0.11		0.24
78:9-86:2	(0.17)	(0.44)			(0.20)	(0.23)			(0.23)
	[0.83]	[0.26]	[0.00]		[0.96]	[0.00]		[0.00]	[0.31]
	[0.89]	[0.64]	[0.73]		[0.98]	[0.26]		[0.53]	[0.88]
10 - 6,	- 0.03	- 0.81		0.11	0.00	- 0.97	0.22		0.16
78:9-86:2	(0.19)	(0.41)			(0.21)	(0.19)			(0.19)
	[0.88]	[0.05]	[0.00]		[0.99]	[0.00]		[0.00]	[0.42]
	[0.93]	[0.52]	[0.76]		[0.99]	[0.13]		[0.54]	[0.99]

Explanation: OLS regressions of equation (25) for the tests of α =0 and β =0, of equation (27) for the tests of β - 1 = 0, and of equation (26) for β - $\hat{\beta}_{BBK}$ = 0. Asymptotic autocorrelation and heteroscedasticty-consistent (Newey and West (1987) with Bartlett weights) standard errors in parentheses and probability values in the upper square brackets and bootstrapped empirical p-values in the lower square brackets.

Table A.7: Inflation rate change regressions with n-7, n-8, and n-9 year spread

	Bundesbank with tax adjustment (BBK)				Svensson with tax adjustment (SVT)				
	$\alpha = 0$	$\beta = 0$	β-1=0	R^2	$\alpha = 0$	$\beta = 0$	R^2	β-1 = 0	β-$\hat{\beta}_{BBK}$=0
8 - 7, 79:9-88:2	0.01 (0.06) [0.93] [0.93]	- 0.76 (0.37) [0.04] [0.31]	1.76 (0.37) [0.00] [0.32]	0.08	0.02 (0.07) [0.00] [0.84]	- 0.90 (0.18) [0.00] [0.04]	0.15	[0.00] [0.08]	0.14 (0.18) [0.42] [0.91]
9 - 7, 79:9-87:2	0.01 (0.11) [0.93] [0.93]	- 0.91 (0.42) [0.03] [0.36]	1.91 (0.42) [0.00] [0.55]	0.13	0.03 (0.11) [0.82] [0.88]	- 1.14 (0.22) [0.00] [0.09]	0.26	[0.00] [0.29]	0.23 (0.22) [0.31] [0.98]
10 - 7, 79:9-86:2	0.01 (0.15) [0.93] [0.97]	- 1.15 (0.37) [0.00] [0.27]	2.15 (0.37) [0.00] [0.59]	0.22	0.01 (0.14) [0.93] [0.94]	- 1.26 (0.18) [0.00] [0.04]	0.38	[0.00] [0.34]	0.11 (0.18) [0.51] [1.00]
9 - 8, 80:9-87:2	0.00 (0.06) [0.98] [0.99]	- 1.08 (0.39) [0.01] [0.24]	2.08 (0.39) [0.00] [0.37]	0.19	0.00 (0.05) [0.93] [0.94]	- 1.32 (0.22) [0.00] [0.04]	0.36	[0.00] [0.14]	0.24 (0.22) [0.27] [0.99]
10 - 8, 80:9-87:2	0.00 (0.11) [0.97] [0.98]	- 1.30 (0.31) [0.00] [0.15]	2.30 (0.31) [0.00] [0.39]	0.29	- 0.01 (0.09) [0.92] [0.94]	- 1.37 (0.16) [0.00] [0.02]	0.46	[0.00] [0.16]	0.07 (0.16) [0.66] [1.00]
10 - 9, 81:9-86:2	0.00 (0.06) [0.99] [0.99]	- 1.35 (0.22) [0.00] [0.04]	2.35 (0.22) [0.00] [0.12]	0.32	- 0.01 (0.04) [0.76] [0.84]	- 1.35 (0.13) [0.00] [0.00]	0.46	[0.00] [0.03]	0.00 (0.13) [0.83] [0.99]

Explanation: OLS regressions of equation (25) for the tests of α =0 and β =0, of equation (27) for the tests of β - 1 = 0, and of equation (26) for β - $\hat{\beta}_{BBK}$ = 0. Asymptotic autocorrelation and heteroscedasticty-consistent (Newey and West (1987) with Bartlett weights) standard errors in parentheses and probability values in the upper square brackets and bootstrapped empirical p-values in the lower square brackets.

The following papers have so far been published:

May	1995	The Circulation of Deutsche Mark Abroad	Franz Seitz
June	1995	Methodology and technique for determining structural budget deficits	Gerhard Ziebarth
July	1995	The information content of derivatives for monetary policy – Implied volatilities and probabilities	Holger Neuhaus
August	1995	Das Produktionspotential in Ostdeutschland *	Thomas Westermann
February	1996	Sectoral disaggregation of German M3	Vicky Read
March	1996	Geldmengenaggregate unter Berücksichtigung struktureller Veränderungen an den Finanzmärkten *	Michael Scharnagl
March	1996	Private consumption and the interest rate in Germany **	Hermann-Josef Hansen
May	1996	Market Reaction to Changes in German Official Interest Rates	Daniel C. Hardy
Mai	1996	Die Rolle des Vermögens in der Geldnachfrage	Dieter Gerdesmeier

* Available in German only.
** Forthcoming.

August	1996	Intergenerative Verteilungseffekte öffentlicher Haushalte – Theoretische Konzepte und empirischer Befund für die Bundesrepublik Deutschland *	Stephan Boll
August	1996	Der Einfluß des Wechselkurses auf die deutsche Handelsbilanz *	Jörg Clostermann
Oktober	1996	Alternative specifications of the German term structure and its information content regarding inflation	Sebastian T. Schich

* Available in German only.
** Forthcoming.