Intertemporal Effects of Fiscal Policy in an RBC Model

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Summary

In the recent economic debate on the design of fiscal policy in Germany it is generally agreed that the total burden of taxes and levies has to be reduced. In this paper, arguments that form the basis of this claim are evaluated within a calibrated Real Business Cycle model. The analysis shows that reducing taxes and levies induces an increase in economic activity as well as positive welfare effects as long as government consumption has a low enough weight in the utility function of the households. Within the model the decrease in government receipts due to the reduction in taxes and levies is balanced by a reduction in government consumption according to a fiscal closure rule which guarantees a stable debt–to–output ratio.

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1 Introduction

The recent economic debate on the design of fiscal policy in Germany is characterised by different views on the principles and the scope of efficient fiscal policy. However, it is largely agreed that the burden of taxes and levies should be reduced. For instance, according to last year's report of the *Council of Experts for the Assessment of Overall Economic Trends*, which is well-known for advocating a supply-oriented economic policy, fiscal policy must "... elaborate regulations on taxes and levies that do not reduce output and investment incentives" (Sachverständigenrat (1997), item 10). This claim rests on arguments taken from neoclassical theory, according to which relative prices distorted by taxes and levies lead to a misallocation of the available resources: The intratemporal misallocation of time induced in the context of individual labour-leisure choices results in an excessively low level of employment, and the intertemporal misallocation of disposable income induced in the context of individual consumption-investment decisions leads to exceedingly low investment activity.[1]

Against the background of these arguments, this paper attempts to evaluate the intertemporal effects of a reduction in taxes and levies as requested of economic policy in a calibrated Real Business Cycle (RBC) model incorporating a government sector.[2] The model reflects the basic features of the institutional framework of the German system of taxes and levies, i.e. an income tax imposed to finance government consumption and transfer payments is supplemented by a consumption tax and levies on the wages the firms pay to the households. The government budget is assumed to be intertemporally balanced by issuing government bonds.

The model on which the analysis rests constitutes a synthesis of RBC models published in the last few years which cover a broad range of fiscal policy issues. Based on the

[1]On the analysis of fiscal policy in the neoclassical model of optimal growth by Cass (1965) and Koopmans (1965), see, for example, the papers by Becker (1985), Chamley (1986), Judd (1985, 1987) and, in particular, Lucas (1990). Another approach using the model of overlapping generations by Diamond (1965) focuses on the intergenerational redistribution effects of taxation and the design of the social security system rather than on issues of allocative efficiency. See, in particular, the book by Auerbach & Kotlikoff (1987).

[2]RBC models are quantitative dynamic general equilibrium models extending the stochastic neoclassical model of optimal growth by Brock & Mirman (1972). An overview of the development of RBC models, which were first formulated in Kydland & Prescott (1982) and Long & Plosser (1983), can be found in the book *Frontiers of Business Cycle Research* edited by Cooley (1995).

papers of Barro (1981, 1989) and Aschauer (1988), for instance, Aiyagari, Christiano & Eichenbaum (1992), Christiano & Eichenbaum (1992a) and Baxter & King (1993) study the allocative effects of government consumption. Their studies rely on the assumption of allocatively neutral financing through lump–sum taxes. The allocative distortions induced by a taxation of factor income are analysed in Judd (1989), Greenwood & Huffman (1991), Dotsey (1990), Dotsey & Mao (1994), Braun (1994), McGrattan (1991, 1994a) and McGrattan, Rogerson & Wright (1997). Cooley & Hansen (1992) and Cooley (1993) study the allocative effects of the introduction of a consumption tax, while levies on wages are addressed in Jonsson & Klein (1996). Cooley & Hansen (1992) and Dotsey & Mao (1994) introduce an intertemporal government budget ceiling, within which the government issues bonds to cover its budgetary deficits.[3,4]

Whereas Cooley & Hansen restrict the overall time path of government consumption to ensure an intertemporally balanced government budget, this paper introduces a fiscal reaction function which stabilises the government debt–to–output ratio by modelling a feedback from the development of government debt to current government consumption. In a simple way, this modelling addresses the fact that revenue shortfalls caused by a reduction in taxes and levies have to be compensated for by spending cuts in order to guarantee the sustainability of the government budget.

The paper is organised as follows. Section 2 describes the RBC model incorporating the government sector, taking into account the German system of taxes and levies. Firstly, optimal economic plans are derived for the model's economic agents, i.e. households and firms. The way they interact is determined by both the institutional structure of the markets and the government's fiscal policy. Secondly, this section demonstrates how the optimal economic plans are coordinated by market prices supporting a competitive equilibrium. In Section 3, the model is calibrated with the aim of reproducing some stylised facts of the German goods and labour markets which are summarised by simple statistics. By means of a sensitivity analysis, these stylised facts are compared with the correspon-

[3]Chari, Christiano & Kehoe (1992, 1994), Zhu (1992) and Coleman (1996) analyse problems related to optimal fiscal policy which will not be dealt with in this paper.

[4]By extending RBC models appropiately, they can also be applied to issues of foreign trade and monetary policy: For instance, extensions covering foreign trade are studied by Backus, Kehoe & Kydland (1992, 1994), Devereux, Gregory & Smith (1992), Mendoza (1991) and Stockman & Tesar (1995), while models incorporating money are studied by Christiano (1991), Christiano & Eichenbaum (1992b, 1992c), Cooley & Hansen (1989, 1991) and Fuerst (1992).

ding figures implied by the calibrated model. The model is subsequently used to simulate and to quantify the welfare effects of various fiscal policy measures designed to reduce the burden of taxes and levies as requested of economic policy. Section 4 gives a summary of the findings and draws some conclusions. The Appendix explains how the data used for the calibration of the model were obtained.

2 Description of the Model Economy

The model economy consists of a large number of identical households and identical firms which act competitively in the economy's markets, i.e. the (real) capital market, the labour market, the government bond market and the goods market, at the beginning of the periods $t = 0, 1, \ldots$:

H The households rent their capital stock in the capital market and supply labour in the labour market. They buy a homogenous good in the goods market, which they use for consumption or for investment in capital. In addition, they purchase bonds issued by the government in the bond market.

F The firms sell a homogenous good in the goods market, which they produce by using the capital borrowed in the capital market and the labour obtained in the labour market.

The economic activity of the households and firms is affected by the government's fiscal policy:

G In the goods market, the government purchases the homogenous good supplied by the firms, it makes transfer payments to the households, and it finances its spending through taxes and levies and the issuance of government bonds.

The government's fiscal policy is considered to be exogenous for the households and firms. The volume of the bonds issued by the government depends on the fiscal balance of the government budget.

The supply and demand decisions of the households and firms in the factor markets and the goods market are coordinated by the relevant market prices of period t via a Walrasian mechanism. Furthermore, the endogenous pricing of the government bonds guarantees that the volume of bonds issued by the government matches the households' demand for government bonds.

Since only the relative prices are determined in the real economy being considered here, the homogenous good is chosen as the numéraire. The supply and demand decisions in the factor markets and the goods market are then coordinated by the real factor prices, i.e. the prices for renting capital and labour expressed in units of the homogenous good. Assuming perfect competition in the markets, these prices constitute an exogenous determinant for the households and firms.

In the following subsections, the activities of the government, the households and the firms — as described under **G**, **H** and **F** — are analysed in more detail. Subsequently, the competitive equilibrium suitable for the coordination of these activities is defined.

2.1 The Government

At the beginning of the periods $t = 0, 1, \ldots$ the government purchases the amount G_t of the homogenous good Q_t offered by the firms in the goods market, and it makes transfer payments to the amount of TR_t to the households. The government uses the purchased goods for purely consumptive purposes.[5]

To finance its expenditure the government imposes:

(a) a tax of t_t^c on the households' consumption C_t,

(b) a tax of t_t^d on the households' factor income $r_t\, K_t + \tilde{w}_t\, N_t$ minus the capital depreciation $\delta\, K_t$ with $0 < \delta \leq 1$, where r_t denotes the rental rate for the capital stock K_t and $\tilde{w}_t$ the wage rate for the labour input N_t, and

(c) a levy of $2\, t_t^w$ on the wages the firms pay to the households $\tilde{w}_t\, N_t$; the households and the firms each pay for half of this levy.

The wage levy can be interpreted as a contribution to an implicit social security system which makes transfer payments to the households. The gross wage rate relevant to the firms — including the wage levies they have to pay — is $w_t \equiv (1 + t_t^w)\, \tilde{w}_t$.

[5] On models which take account of government investment, see Ambler & Paquet (1994) and Baxter & King (1993).

The government budget, comprising government expenditure and revenue, is balanced by issuing an amount B_{t+1} of government bonds with a single–period maturity and an effective return of R_t. The equation for the government budget then reads

$$G_t + TR_t + B_t \;=\; t_t^c\, C_t + t_t^d \left[(r_t - \delta)\, K_t + \frac{1}{1+t_t^w}\, w_t\, N_t \right] + \frac{2\, t_t^w}{1+t_t^w}\, w_t\, N_t + \frac{B_{t+1}}{1+R_t}. \qquad (1)$$

The discounted government bonds $B_{t+1}/(1 + R_t)$ are to be interpreted as a risk–free payment promise indicating that the government will transfer B_{t+1} worth of resources to the households in the next period.

Since a positive effective return leads to a continuous accumulation of government debt, the budget equation for the given amount of receipts proves to be dynamically unstable unless government spending is adequately restricted. In other words, the unrestricted government budget is not sustainable in the long run.

To guarantee a sustainable budget, we will now introduce a fiscal reaction function which models a stabilising feedback from the development of government debt to government consumption G_t, whose autonomous component is assumed to depend linearly on the output Q_t via the consumption rate g_t. In particular, the reaction function is specified in such a manner that deviations from a lastingly balanced debt–to–output ratio $(\bar{B}/\bar{Q})$ lead to a reduction in or expansion of government consumption,

$$G_t \;\equiv\; \left(g_t - \psi \left(\frac{B_t}{Q_t} - \left(\frac{\bar{B}}{\bar{Q}} \right) \right) \right) Q_t, \quad \psi > 0. \qquad (2)$$

Setting the parameter ψ at a sufficiently high value will then ensure the stability of the equation for the government budget.

Since the government transfers TR_t do not play an essential fiscal role in the model economy and as the assumption of identical households renders redistribution issues irrelevant, for the sake of convenience it is assumed that the transfers linearly depend on the output Q_t via the transfer rate tr_t,

$$TR_t \;\equiv\; tr_t\, Q_t. \qquad (3)$$

Owing to the linearity of the government budget equation in the aggregates C_t, K_t, N_t, B_t, B_{t+1}, G_t, TR_t — and due to equations (2) and (3) also in Q_t — these variables can be seen as per–capita variables. Such per–capita variables will turn out to be extremely useful in the definition of the economy's competitive equilibrium given below.

The vector of the exogenous fiscal policy variables $v_t \equiv (g_t, t_t^c, t_t^d, t_t^w, tr_t)'$, i.e. of the autonomous government consumption rate, the rates of taxes and levies and the transfer rate, is assumed to follow a stationary vector–autoregressive process with independent standard–normally distributed innovations $\epsilon_{v,t+1}$,

$$v_{t+1} = \nu_v + A_v \, v_t + C_{\epsilon_v} \, \epsilon_{v,t+1}. \tag{4}$$

In the calibration of the model the autoregressive transition equation of the fiscal policy variables will be fitted to fiscal data. In this connection, it is assumed that the modulus of the eigenvalues of the transition matrix A_v lies within the unit circle. In addition, the matrix C_{ϵ_v} is defined to be lower triangular. The conditional covariance matrix of v_{t+1} is $\text{Var}[v_{t+1}|v_t] = C_{\epsilon_v} \, C'_{\epsilon_v}$.

Given the above specifications, the government sector is fully characterised by the equation for the government budget (1), the government consumption according to (2), the transfers described in (3) and the autoregressive transition equation of the vector of fiscal policy variables (4).

2.2 The Households

At the beginning of each period $t = 0, 1, \ldots$ the households decide on how to use their available resources during this period. This choice is made in the context of an intertemporal decision problem under uncertainty. They decide on:

H.1 the allocation of their disposable income in period t, and

H.2 the allocation of their available time in period t.

The system of necessary conditions for this decision problem yields conditional decision functions which determine the households' supply and demand behaviour in the economy's factor markets, the goods market and the government bond market.

Assuming perfect competition in these markets, the decisions of the individual households do not affect the per–capita variables. In order to distinguish the individual households' variables from the per–capita variables, which is of vital importance for calculating the model economy's competitive equilibrium, the households' variables will subsequently be specified in small letters.

2.2.1 The Decision Problem

In period t the households receive capital and wage income. The capital income is determined by the capital stock k_t, which the households lend to the firms in the capital market for a fee amounting to the rental rate for capital utilisation r_t. The households' wage income depends on the amount of labour n_t which they offer in the labour market and which the firms hire at a wage rate of $(1+t_t^w)^{-1}w_t$. In addition, the households receive government transfer payments totalling TR_t and redeem an amount b_t of government bonds purchased in the previous period.

After deducting the capital depreciation $\delta\, k_t$, an income tax t_t^d must be paid on the income received. The income tax imposed on the households' capital income amounts to $t_t^d\,(r_t-\delta)\,k_t$, and the income tax on the wage income is $t_t^d\,(1+t_t^w)^{-1}w_t\,n_t$. In addition, a wage levy rate t_t^w is applied to the households according to which they pay the amount of $t_t^w\,(1+t_t^w)^{-1}\,w_t\,n_t$ of their wage income to the government.

The households use the disposable income left after deducting income tax and wage levies to buy the homogenous good the firms offer in the goods market. Furthermore, the households purchase the newly issued government bonds b_{t+1} discounted with a return of $1+R_t$. The homogenous good is alternatively used for consumption c_t and investment i_t. A consumption tax is imposed on their consumption c_t to the amount of the consumption tax rate t_t^c. The budget equation of the households then reads

$$\begin{aligned}(1+t_t^c)\,c_t+i_t+\frac{b_{t+1}}{1+R_t} &= (1-t_t^d)\left[r_t\,k_t+\frac{1}{1+t_t^w}\,w_t\,n_t\right] \\ &\quad +t_t^d\,\delta\,k_t-\frac{t_t^w}{1+t_t^w}\,w_t\,n_t+TR_t+b_t. \qquad (5)\end{aligned}$$

Given proportional depreciation $\delta\, k_t$, the investment i_t increases the households' capital stock in line with the capital updating equation

$$k_{t+1} = (1-\delta)\,k_t+i_t, \quad 0<\delta\le 1. \qquad (6)$$

Given the fixed initial capital stock k_0, the current capital stock k_t is the result of the past investment decisions $\{i_\tau\}_{\tau=0}^{t-1}$. We shall assume that all households have the same initial capital stock k_0 as well as the same initial stock of government bonds b_0.

As regards the labour they offer, the households must consider that their labour supply is limited by their total available time. If that time is normalised to unity and if l_t denotes leisure, i.e. the households' available time after deducting the time offered in the labour market, then

$$l_t + n_t \;=\; 1. \tag{7}$$

Since the investment chosen in the context of the allocation problem **H.1** at the beginning of period t affects the capital stock via the capital updating equation (6) and hence the income and consumption opportunities in the future periods $\tau = t+1, t+2, \ldots$, the households base their decisions on the intertemporal maximisation of their welfare.

The households' preferences regarding the sequence of current and future private and government consumption $\{ c_\tau, G_\tau \}_{\tau=t}^{\infty}$ and the sequence of current and future leisure $\{ l_\tau \}_{\tau=t}^{\infty}$ are captured in a (lifetime) utility function, which is additively separable in time,

$$V_t(\{ c_\tau, G_\tau, l_\tau \}_{\tau=t}^{\infty}) \;\equiv\; \sum_{\tau=t}^{\infty} \beta^{\tau-t} U(c_\tau + \pi\, G_\tau, l_\tau), \quad 0 < \beta < 1.$$

The parameter β is a discount factor and the aggregation parameter $\pi \geq 0$ determines the extent to which government consumption G_τ — in the sense of a public good — brings benefits to the households.[6] If $\pi = 1$, the households consider government consumption G_τ as a perfect substitute for private consumption c_τ.

The (single–period) utility function

$$U \;\equiv\; U(c_\tau + \pi\, G_\tau, l_\tau)$$

has the properties well–known from consumer theory. This means in particular that U is twice continuously differentiable, and both $U_c > 0$, $U_l > 0$ and $U_{cc} < 0$, $U_{ll} < 0$.

When formulating the intertemporal decision problem, one has to take into account that, at the beginning of period t, the households know the current factor prices r_t, w_t, the current return R_t and the current values of the fiscal policy variables t_t^c, t_t^d, t_t^w, G_t, TR_t. But there is uncertainty about the sequence of the future factor prices $\{ r_\tau, w_\tau \}_{\tau=t+1}^{\infty}$, the future returns $\{ R_\tau \}_{\tau=t+1}^{\infty}$ and the future values of the fiscal policy variables $\{ t_\tau^c, t_\tau^d, t_\tau^w, G_\tau, TR_\tau \}_{\tau=t+1}^{\infty}$. At the beginning of period t the households therefore choose a feasible allocation plan $\{ c_\tau, i_\tau, b_{\tau+1}, l_\tau, n_\tau \}_{\tau=t}^{\infty}$ which maximises the expected value of their discounted (lifetime) utility:

[6] This specification follows Barro (1981, 1989), Aschauer (1988), Aiyagari, Christiano & Eichenbaum (1992) and Christiano & Eichenbaum (1992a).

$$\max_{\{c_\tau, i_\tau, b_{\tau+1}, l_\tau, n_\tau\}_{\tau=t}^{\infty}} \mathrm{E}_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(c_\tau + \pi G_\tau, l_\tau) \right] \tag{8}$$

subject to the constraints

$$\begin{aligned} (1+t_\tau^c)\, c_\tau + i_\tau + b_{\tau+1}/(1+R_\tau) &= (1-t_\tau^d) \left[r_\tau\, k_\tau + (1+t_\tau^w)^{-1}\, w_\tau\, n_\tau \right] \\ &\quad + t_\tau^d\, \delta\, k_\tau - t_\tau^w\, (1+t_\tau^w)^{-1} w_\tau\, n_\tau + TR_\tau + b_\tau, \quad \tau = t, t+1, \ldots \quad (9) \\ k_{\tau+1} &= (1-\delta)\, k_\tau + i_\tau, \quad \tau = t, t+1, \ldots \quad (10) \\ l_\tau + n_\tau &= 1, \quad \tau = t, t+1, \ldots \quad (11) \end{aligned}$$

given the stock of government bonds b_t and the capital stock k_t.

The expectation $\mathrm{E}_t[\,\cdot\,] \equiv \mathrm{E}[\,\cdot \mid \Omega_t]$ is formed rationally, i.e. consistent with the model's structure, conditional on the information set Ω_t available at the beginning of period t with $\{r_t, w_t, R_t, t_t^c, t_t^d, t_t^w, G_t, TR_t\} \subset \Omega_t$. The assumption of rational expectations implies that the households are aware of the determination of the future factor prices r_τ, w_τ, the future returns R_τ and the future fiscal policy variables t_τ^c, t_τ^d, t_τ^w, G_τ, TR_τ.

2.2.2 The Conditional Decision Functions

The decision problem (8) – (11) can be solved by using a dynamic generalisation of the Lagrange method.[7] The Lagrangean to be maximised is

$$\begin{aligned} \mathrm{L}_t^{\mathbf{H}} &\equiv \mathrm{L}_t^{\mathbf{H}}(\{c_\tau, i_\tau, b_{\tau+1}, l_\tau, n_\tau, k_{\tau+1}, \lambda_{1,\tau+1}, \lambda_{2,\tau+1}, \mu_\tau\}_{\tau=t}^{\infty}) \quad (12) \\ &= \mathrm{E}_t \Big[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big\{ U(c_\tau + \pi G_\tau, l_\tau) - \beta\, \lambda_{1,\tau+1} \Big((1+t_\tau^c)\, c_\tau + i_\tau + b_{\tau+1}/(1+R_\tau) \\ &\quad - (1-t_\tau^d)\, (r_\tau\, k_\tau + (1+t_\tau^w)^{-1}\, w_\tau\, n_\tau) - t_\tau^d\, \delta\, k_\tau + t_\tau^w\, (1+t_\tau^w)^{-1} w_\tau\, n_\tau - TR_\tau - b_\tau \Big) \\ &\quad - \beta\, \lambda_{2,\tau+1}\, (k_{\tau+1} - (1-\delta)\, k_\tau - i_\tau) - \mu_\tau\, (l_\tau + n_\tau - 1) \Big\} \Big], \end{aligned}$$

where the Lagrange multipliers $\lambda_{1,\tau+1}$, $\lambda_{2,\tau+1}$ and μ_τ denote, respectively, the shadow prices of a marginal unit of discounted government bonds, the capital stock and leisure, each expressed in utility units.

[7] For details on how the Lagrange method is used to solve (recursive) intertemporal decision problems under uncertainty, see King, Plosser & Rebelo (1988a, 1988b), Plosser (1989), McCallum (1989) and Chow (1992, 1993).

The system of first-order necessary conditions for a maximum of the Lagrangean (12) is obtained by setting the partial first derivatives equal to zero:

(i) The derivatives with respect to c_τ, i_τ, $b_{\tau+1}$, l_τ and n_τ for $\tau = t, t+1, \ldots$ yield the conditions

$$\mathrm{E}_t[U_c(c_\tau + \pi\, G_\tau, l_\tau)] \;=\; \beta\, \mathrm{E}_t[(1+t^c_\tau)\,\lambda_{1,\tau+1}], \tag{13}$$

$$\mathrm{E}_t[\lambda_{1,\tau+1}] \;=\; \mathrm{E}_t[\lambda_{2,\tau+1}], \tag{14}$$

$$\mathrm{E}_t[\lambda_{1,\tau+1}/(1+R_\tau)] \;=\; \beta\, \mathrm{E}_t[\lambda_{1,\tau+2}], \tag{15}$$

$$\mathrm{E}_t[U_l(c_\tau + \pi\, G_\tau, l_\tau)] \;=\; \mathrm{E}_t[\mu_\tau], \tag{16}$$

$$\beta\, \mathrm{E}_t[(1 - t^d_\tau - t^w_\tau)\,(1+t^w_\tau)^{-1}\, w_\tau\, \lambda_{1,\tau+1}] \;=\; \mathrm{E}_t[\mu_\tau]. \tag{17}$$

(ii) The derivative with respect to k_τ for $\tau = t+1, t+2, \ldots$ yields a transition equation for the expected shadow price of capital

$$\mathrm{E}_t[\lambda_{2,\tau}] \;=\; (1-\delta)\,\beta\, \mathrm{E}_t[\lambda_{2,\tau+1}] + \beta\, \mathrm{E}_t[((1-t^d_\tau)\, r_\tau + t^d_\tau\, \delta)\,\lambda_{1,\tau+1}]. \tag{18}$$

(iii) The derivatives with respect to $\lambda_{1,\tau+1}$, $\lambda_{2,\tau+1}$ and μ_τ for $\tau = t, t+1, \ldots$ yield a transition equation for the expected stock of government bonds

$$\begin{aligned} \mathrm{E}_t[(1+t^c_\tau)\, c_\tau] + \mathrm{E}_t[i_\tau] + \mathrm{E}_t[b_{\tau+1}/(1+R_\tau)] \;=\; & \mathrm{E}_t[(1-t^d_\tau)\,(r_\tau\, k_\tau \\ & + (1+t^w_\tau)^{-1} w_\tau\, n_\tau)] + \mathrm{E}_t[t^d_\tau\, \delta\, k_\tau] - \mathrm{E}_t[t^w_\tau\,(1+t^w_\tau)^{-1} w_\tau\, n_\tau] + \mathrm{E}_t[TR_\tau] + \mathrm{E}_t[b_\tau], \end{aligned}$$

a transition equation for the expected capital stock

$$\mathrm{E}_t[k_{\tau+1}] \;=\; (1-\delta)\, \mathrm{E}_t[k_\tau] + \mathrm{E}_t[i_\tau]$$

and the condition

$$\mathrm{E}_t[l_\tau] + \mathrm{E}_t[n_\tau] \;=\; 1.$$

The sequence of the first-order necessary conditions (13) – (17) implies that neither the intertemporal reallocation of a marginal unit of income nor the intratemporal reallocation of a marginal unit of time can increase the expected discounted (lifetime) utility of the households at any point along the optimal allocation path.

The system of necessary conditions is completed by the transversality conditions

$$\lim_{\tau\to\infty} \beta^{\tau-t+1}\mathrm{E}_t[\lambda_{1,\tau+1}\, b_{\tau+1}/(1+R_\tau)] \;=\; 0$$

and

$$\lim_{\tau \to \infty} \beta^{\tau-t+1} \mathrm{E}_t[\lambda_{2,\tau+1}\, k_{\tau+1}] \;=\; 0$$

which imply that the discounted expected values of government bonds and capital in period $\tau+1$ — having been converted into utility units by means of their shadow prices — will disappear for $\tau \to \infty$.

Since the uncertainty regarding the factor prices r_{t+1}, w_{t+1}, the return R_{t+1} and the fiscal policy variables t^c_{t+1}, t^d_{t+1}, t^w_{t+1}, G_{t+1}, TR_{t+1} vanishes during the transition from period t to period $t+1$ — which hence yields $\{r_{t+1}, w_{t+1}, R_{t+1}, t^c_{t+1}, t^d_{t+1}, t^w_{t+1}, G_{t+1}, TR_{t+1}\} \subset \Omega_{t+1}$ with $\Omega_{t+1} \supset \Omega_t$ —, the households do, in fact, implement the planned allocation $\{c_t, i_t, b_{t+1}, l_t, n_t\}$ during period t. However, at the beginning of period $t+1$, they re–optimise the sequence of allocations planned for the rest of their lifetime $\{c_\tau, i_\tau, b_{\tau+1}, l_\tau, n_\tau\}_{\tau=t+1}^{\infty}$ based on the more comprehensive amount of information Ω_{t+1} available by then. Owing to the reoptimisation by the households, solving the intertemporal decision problem (8) – (11) can be restricted to determining the optimal allocation in period t, i.e. to determining $\{c_t, i_t, b_{t+1}, l_t, n_t\}$, which, however, depends in a forward–looking manner on the expectations about the future values of the factor prices, the return and the fiscal policy variables due to the necessary conditions (15) and (18).

Conditional on the available information Ω_t, the households' optimal allocation $\{ c_t, b_{t+1}, i_t, l_t, n_t \}$ is characterised by time–invariant conditional decision functions given their stock of government bonds b_t and their capital stock k_t. After having substituted the Lagrange multiplier μ_t and subject to the identity $\lambda_{t+1|t} \equiv \mathrm{E}_t[\lambda_{1,t+1}] = \mathrm{E}_t[\lambda_{2,t+1}]$ resulting from condition (14) the following characterisation is obtained:

H.1 The optimal allocation of the disposable income in period t is determined by a consumption function, an investment function and a demand function for government bonds

$$c_t = \mathrm{c}(r_t, w_t, R_t, \lambda_{t+1|t}, t^c_t, t^d_t, t^w_t, G_t, TR_t; b_t, k_t), \tag{19}$$

$$i_t = \mathrm{i}(r_t, w_t, R_t, \lambda_{t+1|t}, t^c_t, t^d_t, t^w_t, G_t, TR_t; b_t, k_t), \tag{20}$$

$$b_{t+1} = \mathrm{b}(r_t, w_t, R_t, \lambda_{t+1|t}, t^c_t, t^d_t, t^w_t, G_t, TR_t; b_t, k_t). \tag{21}$$

H.2 The optimal allocation of the available time in period t is determined by a leisure demand function and a labour supply function

$$l_t = \mathrm{l}(r_t, w_t, R_t, \lambda_{t+1|t}, t^c_t, t^d_t, t^w_t, G_t, TR_t; b_t, k_t), \tag{22}$$

$$n_t = \mathrm{n}(r_t, w_t, R_t, \lambda_{t+1|t}, t_t^c, t_t^d, t_t^w, G_t, TR_t; b_t, k_t). \tag{23}$$

Because of the binding budget and time constraints (5), (7) it will suffice to consider the consumption function (19), the investment function (20) and the labour supply function (23) as the solution to the allocation problems **H.1** and **H.2**. The derivation of this solution completes the description of the household sector. The following subsection will focus on the description of the firms' behaviour.

2.3 The Firms

Whereas the households make intertemporal decisions under uncertainty, the firms' decisions are based on static observation and certainty. This simplification is based on the assumption that the firms can obtain the production factors for each single period in the factor markets. Such modelling greatly simplifies the determination of a competitive equilibrium in which the households' intertemporal decisions and the firms' static decisions are to be coordinated. We will begin with a description of the firms' production technology, followed by a derivation of the conditional factor demand functions which characterise the firms' supply and demand behaviour in the economy's goods and factor markets.

2.3.1 The Production Problem

Using the production factors of capital K_t and labour N_t, the firms produce a homogenous good in the periods $t = 0, 1, \ldots$ according to a linear–homogenous neoclassical production function

$$F \equiv F(K_t, N_t).$$

The production function F has the properties well–known from the theory of the firm. In particular, F is twice continuously differentiable, and $F_K > 0$, $F_N > 0$ and $F_{KK} < 0$, $F_{NN} < 0$, $F_{KN} = F_{NK} > 0$.

Owing to the linear homogeneity of the production function, and assuming there is perfect competition in the factor and the goods markets, the firms' production problem can be formulated by using per–capita variables, which means that firm–specific variables do not need to be introduced.

In addition to the production factors K_t, N_t, the amount of output Q_t depends multiplicatively on a stochastic technology variable z_t,

$$Q_t = z_t F(K_t, N_t), \tag{24}$$

which reflects the model economy's level of technology in period t.[8]

The technology variable z_t is assumed to follow a stationary autoregressive process with independent standard–normally distributed innovations $\epsilon_{z,t+1}$,

$$z_{t+1} = (1 - \rho_z) + \rho_z z_t + \sigma_{\epsilon_z} \epsilon_{z,t+1}, \qquad |\rho_z| < 1, \quad \sigma_{\epsilon_z} > 0, \tag{25}$$

where the conditional variance of the technology variable turns out to be $\mathrm{Var}[z_{t+1}|z_t] = \sigma^2_{\epsilon_z}$.

Assuming that the firms are aware of the factor prices r_t, w_t and the realisation of the technology variable z_t at the beginning of period t, they will choose a feasible production plan $\{Q_t, K_t, N_t\}$ which is designed to maximise their current profits:

$$\max_{\{Q_t, K_t, N_t\}} Q_t - r_t K_t - w_t N_t \tag{26}$$

subject to constraint (24).

2.3.2 The Conditional Factor Demand Functions

The profit maximisation problem (26), (24) is solved with the Lagrange method. The Lagrangean to be maximised is

$$\begin{aligned} \mathrm{L}^{\mathrm{F}}_t &\equiv \mathrm{L}^{\mathrm{F}}_t(Q_t, K_t, N_t, \kappa_t) \qquad (27) \\ &= Q_t - r_t K_t - w_t N_t - \kappa_t \left(Q_t - z_t F(K_t, N_t) \right), \end{aligned}$$

where the Lagrange multiplier κ_t denotes the contribution of a marginal output unit to the profit expressed in units of the homogenous good.

The system of first–order necessary conditions for a local maximum of the Lagrangean (27) is obtained by forming the partial first derivatives with respect to the variables Q_t, K_t, N_t and κ_t which are set equal to zero:

$$1 = \kappa_t, \tag{28}$$

$$r_t = \kappa_t z_t F_K(K_t, N_t), \tag{29}$$

[8]Exogenous growth which could be modelled by integrating Harrod–neutral technological progress into the production technology (see, e.g., King, Plosser & Rebelo (1988a)) is disregarded.

$$w_t = \kappa_t z_t F_N(K_t, N_t), \quad (30)$$

$$Q_t = z_t F(K_t, N_t). \quad (31)$$

Taking condition (28) into account, conditions (29) – (31) form a system of three equations in the three unknown quantities K_t, N_t and Q_t. Owing to the linear homogeneity of the production technology, however, the production plan $\{Q_t, K_t, N_t\}$ is not determined by the firms' profit maximisation. Instead, conditional factor demand functions can be derived for a given output Q_t as a function of the factor prices r_t, w_t,

$$K_t = \mathrm{K}(r_t, w_t; Q_t), \quad (32)$$

$$N_t = \mathrm{N}(r_t, w_t; Q_t), \quad (33)$$

which fully characterise the firms' optimal behaviour in the model economy's factor markets for a given output Q_t.

2.4 The Competitive Equilibrium

After deriving the households' conditional decision functions and the firms' conditional factor demand functions which determine the individual supply and demand behaviour in the model economy's markets, we must now find out how the individual decisions are to be coordinated.

Firstly, based on our previous assumption of perfect competition in the model economy's factor and goods markets, the factor prices r_t, w_t will coordinate the economic plans of the individual households and firms and, in this manner, support a competitive equilibrium. These prices depend on the state of the model economy. This state is determined by the level of technology z_t, the fiscal policy variables g_t, t_t^c, t_t^d, t_t^w, tr_t, the per–capita stock of government bonds B_t and the per–capita capital stock K_t at the beginning of each period t. These variables are accordingly termed state variables of the model economy. It should be noted that any information about these state variables which goes beyond the factor prices is not required.

Secondly, whereas the factor prices create a balance between elastic supply and elastic demand in the factor and goods markets via a Walrasian mechanism, the rate of return R_t can be determined by a pricing function for the bonds issued by the government to cover its budget. This function ensures that the economy–wide accumulation of government bonds is consistent with the individual households' demand for government bonds.[9] Like the factor prices, the rate of return is a function of the state variables z_t, g_t, t_t^c, t_t^d, t_t^w, tr_t, B_t and K_t.

Thirdly, besides the factor prices and the rate of return, the households' intertemporal decision depends on the expected shadow price $\lambda_{t+1|t}$ of the government bonds held by the households and the households' capital stock. In this context, the variables b_t and k_t which were determined by the decisions made in the previous period are termed the state variables of the households. The expected shadow price $\lambda_{t+1|t}$ is a function of both the model economy's state variables z_t, g_t, t_t^c, t_t^d, t_t^w, tr_t, B_t, K_t and the households' state variables b_t, k_t.

Taking these functional relations into account, we can define a sequence of competitive equilibria for the model economy which satisfies the households' conditional decision functions (19), (20), (23) derived in Subsection 2.2.2 and the conditional factor demand functions of the firms (32), (33) derived in Subsection 2.3.2.

Definition: For a given level of technology z_t, given values of the fiscal policy variables g_t, t_t^c, t_t^d, t_t^w,tr_t, a given per–capita stock of government bonds B_t, a given per–capita capital stock K_t, a given stock of government bonds held by the households b_t, and a given capital stock of the households k_t, the model economy's competitive equilibrium in each period $t = 0, 1, \ldots$ is determined by the following system of functions:

E.1 the factor price functions

$$\begin{aligned} r_t &= \mathrm{r}(z_t, g_t, t_t^c, t_t^d, t_t^w, tr_t, B_t, K_t), \\ w_t &= \mathrm{w}(z_t, g_t, t_t^c, t_t^d, t_t^w, tr_t, B_t, K_t), \end{aligned}$$

E.2 the return function

$$R_t = \mathrm{R}(z_t, g_t, t_t^c, t_t^d, t_t^w, tr_t, B_t, K_t),$$

[9]The return is implicitly given by the Euler equation

$$\beta\,(1 + R_t)\,\mathrm{E}_t\left[\frac{(1+t_t^c)\,U_c(C_{t+1} + \pi\,G_{t+1}, L_{t+1})}{(1+t_{t+1}^c)\,U_c(C_t + \pi\,G_t, L_t)}\right] = 1$$

which is derived from the first–order necessary conditions (13) and (15) of the households' decision problem. The equation has to be evaluated as a function of per–capita consumption C and per–capita leisure L. Hence, it does not depend on the decisions of the individual households.

E.3 the shadow price function

$$\lambda_{t+1|t} = \lambda(z_t, g_t, t_t^c, t_t^d, t_t^w, tr_t, B_t, K_t; b_t, k_t),$$

E.4 the conditional decision functions of the households

$$c_t = \mathrm{c}(z_t, g_t, t_t^c, t_t^d, t_t^w, tr_t, B_t, K_t; b_t, k_t),$$
$$i_t = \mathrm{i}(z_t, g_t, t_t^c, t_t^d, t_t^w, tr_t, B_t, K_t; b_t, k_t),$$
$$n_t = \mathrm{n}(z_t, g_t, t_t^c, t_t^d, t_t^w, tr_t, B_t, K_t; b_t, k_t),$$

which solve the sequence of intertemporal decision problems (8) – (11) for given factor price functions, a given return function and a given shadow price function,

and

E.5 the conditional factor demand functions of the firms

$$K_t = \mathrm{K}(z_t, g_t, t_t^c, t_t^d, t_t^w, tr_t, B_t, K_t),$$
$$N_t = \mathrm{N}(z_t, g_t, t_t^c, t_t^d, t_t^w, tr_t, B_t, K_t),$$

which solve the sequence of static profit maximisation problems (26), (24) for given factor price functions and a given production technology,

such that the following conditions are satisfied:

C.1 market clearing in the factor markets

$$K_t = k_t,$$
$$N_t = n_t,$$

C.2 market clearing in the government bond market

$$B_t = b_t,$$

C.3 market clearing in the goods market

$$Q_t = C_t + I_t + G_t$$

with $C_t = c_t$ and $I_t = i_t$,

and

C.4 a balanced government budget as given by (1) taking into account government consumption as in (2) and government transfers as in (3).

Given an identical initial capital stock $K_0 = k_0$, the equality of per–capita investment I_t and the households' investment i_t implies the consistency of the updating equation for the per–capita capital stock K_t and the updating equation for the capital stock of the individual households k_t.

The transition of the model economy from the competitive equilibrium in period t to the competitive equilibrium in period $t+1$ is then determined by the transition equation of the technology variable z_t, the transition equation of the vector of the exogenous fiscal policy variables $v_t = (g_t, t_t^c, t_t^d, t_t^w, tr_t)'$, the consistent updating equations for the per–capita capital stock K_t and the households' capital stock k_t as well as by the equation for the government budget.

In order to determine the sequence of competitive equilibria numerical methods have to be applied since analytical solutions are not known for the families of parametric utility and production functions which will be considered below. In particular, we rely on the numerical methods suggested by McGrattan (1994b) and Anderson, Hansen, McGrattan & Sargent (1996) which allow computing linear competitive equilibria for linear–quadratic model economies but require appropriately approximating non–linear economies beforehand.[10]

3 Calibration and Simulation of the Model Economy

The RBC model described in the previous section provides a consistent framework for analysing the intertemporal effects of fiscal policy. Within this framework, the intertemporal effects reflect optimal behaviour of households and firms supporting a sequence of competitive equilibria. In order to quantify these effects the model economy must be appropriately calibrated. In the following subsection, this calibration will be carried out with the aim of reproducing selected stylised facts of the German goods and labour markets by means of the stochastically simulated model economy.

[10]For computational details see Coenen (1997), Chapter 4.

3.1 Calibration of the Model Economy

Prior to calibrating the model economy, we have to choose parametric families for the utility function $U(c_t + \pi\, G_t, l_t)$ and the production function $F(K_t, N_t)$. Then, values must be assigned to these functions' parameters and to the discount parameter β, the aggregation parameter π, the depreciation parameter δ, the transition equation's parameters for the technology variable z_t, the parameter ψ of the fiscal reaction function and the transition equation's parameters for the vector of fiscal policy variables $v_t = (g_t, t_t^c, t_t^d, t_t^w, tr_t)'$.

3.1.1 Preferences and Technology

Consumption $c_t + \pi\, G_t$ and leisure l_t can be seen as an aggregate $A(c_t + \pi\, G_t, l_t)$ within the utility function which will be easily obtained by Cobb–Douglas aggregation as

$$A(c_t + \pi\, G_t, l_t) \;\equiv\; (c_t + \pi\, G_t)^{\phi}\, l_t^{1-\phi}, \quad 0 < \phi < 1.$$

The households' preferences regarding $c_t + \pi\, G_t$ and l_t will be described parametrically using the family of isoelastic utility functions with

$$U(c_t + \pi\, G_t, l_t) \;=\; \begin{cases} \dfrac{1}{1-\gamma}\left[\left((c_t + \pi\, G_t)^{\phi}\, l_t^{1-\phi}\right)^{1-\gamma} - 1\right] & \text{for} \quad \gamma > 0, \;\; \gamma \neq 1 \\[2ex] \phi\, \ln(c_t + \pi\, G_t) + (1-\phi)\, \ln(l_t) & \text{for} \quad \gamma = 1 \end{cases}$$

(see, for instance, Prescott (1986)).

If a household's willingness to intertemporally substitute a marginal unit $A_t \equiv A(c_t + \pi\, G_t, l_t)$ for a marginal unit $A_{t+1} \equiv A(c_{t+1} + \pi\, G_{t+1}, l_{t+1})$ is expressed as the ratio of their discounted marginal utilities

$$\beta\, \frac{U_A(c_{t+1} + \pi\, G_{t+1}, l_{t+1})}{U_A(c_t + \pi\, G_t, l_t)} \;=\; \beta \left(\frac{A_{t+1}}{A_t}\right)^{-\gamma},$$

the expression

$$\eta_{A_t, A_{t+1}} \;\equiv\; \frac{d \ln\left(\frac{A_{t+1}}{A_t}\right)}{d \ln\left(\beta\, \frac{U_A(c_{t+1} + \pi\, G_{t+1}, l_{t+1})}{U_A(c_t + \pi\, G_t, l_t)}\right)} \;=\; -\frac{1}{\gamma} < 0$$

can be interpreted as the elasticity of intertemporally substituting A_t for A_{t+1}. As will be shown below, the parameter γ, i.e. the inverse of the absolute value of the intertemporal elasticity of substitution $\eta_{A_t, A_{t+1}}$, is crucial to the dynamic properties of the sequence of the model economy's competitive equilibria.

As regards the production technology, we shall assume that it is determined by a linear–homogeneous Cobb–Douglas function

$$F(K_t, N_t) \equiv K_t^{\alpha} N_t^{1-\alpha}, \qquad 0 < \alpha < 1$$

with partial first derivatives $F_K(K_t, N_t) = \alpha\, K_t^{\alpha-1} N_t^{1-\alpha}$ and $F_N(K_t, N_t) = (1-\alpha)\, K_t^{\alpha} N_t^{-\alpha}$.

3.1.2 Choice of Parameter Values

Selecting the parametric utility function and the parametric production function rounds out the general specification of the model economy. The households' preferences are parametrised by the discount factor β, the aggregation parameter π and the parameters γ and ϕ of the isoelastic utility function. Technology is parametrised by the elasticity parameter α of the Cobb–Douglas production function, the depreciation parameter δ of the capital updating equation and the parameters ρ_z and σ_{ϵ_z} of the transition equation for the technology variable z_t. Fiscal policy is fully characterised by the parameter ψ of the fiscal reaction function and by the parameters ν_υ, A_υ, C_{ϵ_υ} of the transition equation for the vector of fiscal policy variables $\upsilon_t = (g_t, t_t^c, t_t^d, t_t^w, tr_t)'$.

Table 1: Parameter Estimates[a] of the Autoregressive Model for $\upsilon = (g, t^c, t^d, t^w, tr)'$

$\hat{\nu}_\upsilon$	$\hat{A}_\upsilon$					$\hat{C}_{\epsilon_\upsilon}$				
.0284	.8065	-.0421	.0697	.0298	-.0057	.0035				
(.0218)	(.0668)	(.1077)	(.0473)	(.0700)	(.0480)	(.0002)				
.0917	-.1546	.3678	-.0414	-.0019	.0549	-.0007	.0025			
(.0165)	(.0506)	(.0817)	(.0359)	(.0531)	(.0364)	(.0002)	(.0003)			
.0366	-.1509	-.2004	.6120	.3739	-.0725	.0004	.0000	.0043		
(.0270)	(.0826)	(.1334)	(.0586)	(.0867)	(.0594)	(.0004)	(.0004)	(.0003)		
.0185	-.0535	.0069	.0073	.9047	.0624	.0002	.0000	-.0003	.0022	
(.0137)	(.0420)	(.0679)	(.0298)	(.0441)	(.0302)	(.0003)	(.0002)	(.0002)	(.0001)	
-.0064[b]	.0537	-.1445	.0296	.1186	.8753	.0014	.0001	.0001	.0004	.0028
(—)	(.0620)	(.1000)	(.0439)	(.0650)	(.0445)	(.0004)	(.0002)	(.0002)	(.0002)	(.0002)

[a] Estimated standard errors in parentheses. [b] Parameter value which is calibrated such that $(\bar{B}/\bar{Q}) = 0.60$ holds.

The preference and technology parameters and the fiscal policy parameter ψ are calibrated conditional on the parameters ν_υ, A_υ, C_{ϵ_υ} of the autoregressive model (4) given in Subsection 2.1, which describes the evolution of the fiscal policy variables v_t over time. Estimates of these parameters are shown in Table 1.[11] Note that the constant of the transition equation for the transfer rate tr has been calibrated in such a way that the government debt–to–output ratio is sustained at 60% in the steady state of the calibrated model.

The subsequent calibration of the model economy follows the literature reviewed in the introduction or, if that does not contain any information about the choice of parameter values, it is based on considerations of feasibility and stability. The choice of parameter values will ultimately be evaluated against the ability of the stochastically simulated model to approximate selected stylised facts of the German goods and labour markets.

In line with the literature on this subject, the values 0.99 and 1/3 are assigned to the preference parameters β and ϕ respectively, and the values 0.36 and 0.025 are assigned to the technology parameters α and δ respectively. In the quarterly analysis carried out below, the value selected for the discount factor β results in an annual rate of return of roughly 4% on the capital lent by the households as well as on the government bonds purchased by the households, while the value selected for the depreciation parameter δ implies a roughly 10% annual depreciation of the existing capital stock. Owing to the linear homogeneity of the Cobb–Douglas production function and the compensation of the production factors according to marginal productivity, the value selected for the technology parameter α results in a wage rate of 0.64. The value chosen for the preference parameter ϕ implies that in the steady state of a corresponding economy which is free from allocative distortions caused by fiscal policy, the households' labour time amounts to slightly less than one–third of their available time.

At this stage, the value of the aggregation parameter π is set at 1.00, making government consumption a perfect substitute for private consumption. The value 0.10, which proves to be sufficiently high to guarantee the stability of the equation for the government budget, is assigned to the parameter ψ of the fiscal reaction function. In order to limit the positive correlation between government consumption and output — as will be explored below in more detail — we shall assume that the innovations in the autonomous government consumption rate ϵ_g correlate negatively with the innovations in the technology variable ϵ_z by fixing the correlation coefficient $r_{\epsilon_g,\epsilon_z}$ at -0.75.[12]

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[11]See Appendix A.2 for the construction of the time series of the fiscal policy variables.

[12]The introduction of this negative correlation proved to be necessary since the autonomous component of government consumption is specified to depend linearly on output within the model, whereas the data

We will set the values of the technology parameters ρ_z and σ_{ϵ_z} at 0.90 and 0.0075, respectively, for which the volatility and persistence of the output time series generated by the stochastically simulated model are largely consistent with the corresponding figures of the empirical time series.[13] Finally, illustrating the impact of the households' willingness to intertemporally substitute on the dynamic properties of the model, the preference parameter γ will be calibrated within a sensitivity analysis which is carried out for the alternative values of 0.50, 1.00, 3.00 and 5.00 in the following subsection.

The values of the preference, technology and fiscal policy parameters used in this sensitivity analysis are summarised in Table 2.

Table 2: Calibrated Preference, Technology and Fiscal Policy Parameters

preferences				technology				fiscal policy	
β	γ	ϕ	π	α	δ	ρ_z	σ_{ϵ_z}	ψ	$r_{\epsilon_g,\epsilon_z}$
0.99	{0.50, 1.00, 3.00, 5.00}	1/3	1.00	0.36	0.025	0.90	0.0075	0.10	-0.75

3.1.3 A Sensitivity Analysis

In order to assess the sensitivity of the model's dynamics with respect to the households' preference parameter γ, a set of 100 replications of time series having a sample size of 140 were simulated for the variables of the goods and labour markets Q, C, I, G, w and N. For each replication the simulations were initialised in the model economy's steady state. The standard deviations and contemporaneous correlations of the simulated time series are reported in Table 3. In this table, they are contrasted with the corresponding empirical figures of the German goods and labour markets which were gathered for a sample period ranging from the first quarter of 1960 to the fourth quarter of 1994.

In line with the literature, the empirical and simulated time series have been detrended by means of the Hodrick–Prescott (HP) filter before computing the standard deviations and the contemporaneous correlations. In this context it should be noted that the standard deviations measure the average relative deviation of the time series from their trend component.[14]

show a negative correlation between output and government consumption.

[13]In this context, see Prescott (1986), McCallum (1989) and Cooley & Prescott (1995).

[14]See Appendix A.1 for a description of how the time series of the variables of the goods and the labour

Table 3: Stylised Facts of the Goods and the Labour Market[a]

	goods market							labour market		
	output	priv. consumption		investment		gov. consumption		real wage	employment	
	σ_Q	σ_C	$r_{C,Q}$	σ_I	$r_{I,Q}$	σ_G	$r_{G,Q}$	σ_w	σ_N	$r_{N,w}$
data:[b]										
1960:1 – 1994:4	1.54	1.37	0.59	4.03	0.84	1.82	-0.13	1.55	1.24	0.59
	(0.14)	(0.12)	(0.16)	(0.40)	(0.18)	(0.25)	(0.15)	(0.16)	(0.09)	(0.13)
model:[c]										
$\gamma = 0.50$	1.79	1.60	-0.31	6.99	0.99	4.13	0.38	0.92	1.69	-0.16
	(0.24)	(0.50)	(0.17)	(0.95)	(0.01)	(1.62)	(0.21)	(0.14)	(0.22)	(0.18)
$\gamma = 1.00$	1.67	1.45	-0.16	6.10	0.99	3.82	0.34	0.90	1.49	-0.09
	(0.22)	(0.47)	(0.18)	(0.82)	(0.00)	(1.45)	(0.21)	(0.14)	(0.19)	(0.19)
$\gamma = 3.00$	1.52	1.38	0.10	4.96	0.99	3.57	0.28	0.89	1.21	0.02
	(0.21)	(0.57)	(0.17)	(0.66)	(0.00)	(1.54)	(0.20)	(0.13)	(0.15)	(0.19)
$\gamma = 5.00$	1.48	1.34	0.18	4.68	0.99	3.37	0.26	0.89	1.12	0.06
	(0.20)	(0.83)	(0.16)	(0.63)	(0.00)	(1.50)	(0.20)	(0.13)	(0.14)	(0.19)

[a] Deviations from the HP trend component. [b] Nonparametrically estimated standard errors in parentheses. [c] The values given are the means of the standard deviations and correlations computed for the simulated time series of sample size 140 within 100 replications. The values in parentheses denote the corresponding standard deviations within the 100 replications.

The standard deviations and contemporaneous correlations reported for the simulated time series are the means of the standard deviations and contemporaneous correlations computed for each of the replications. The values in parentheses, in turn, are the standard deviations of the values resulting from the whole set of replications. They indicate the uncertainty which is due to conducting the simulation experiment. The sample uncertainty related to the empirical series is measured by their estimted standard errors given in parentheses.

For a preference parameter value of $\gamma = 3.00$, the standard deviation of the simulated output time series σ_Q and the standard deviation of the simulated private consumption time series σ_C which amount to 1.52% and 1.38%, respectively, almost match the variability of 1.54% and 1.37% estimated for the empirical time series. However, at 4.96% and 3.57%, respectively, the standard deviation of the simulated investment time series σ_I and of the time series for government consumption σ_G prove to be too high compared with the empirical figures of 4.03% and 1.82%.

market are constructed as well as for the determination of their HP trend component.

In terms of the theoretical model, the low variability of private consumption compared with the variability of investment is due to the households' limited willingness to intertemporally substitute consumption: The households absorb cyclical fluctuations of production, and hence of income, by adjusting their investment activity. That means the simulated time series of private consumption exhibit less pronounced cyclical fluctuations compared with the simulated output time series, whereas the simulated investment time series show more violent cyclical fluctuations.

Lower preference parameter values of $\gamma = 1.00$ and $\gamma = 0.50$, respectively, which imply a gradually increase in the households' willingness to intertemporally substitute, induce an increase in the cyclical fluctuations of the simulated goods market time series. A higher preference parameter value of $\gamma = 5.00$ leads to a decrease in the variability of the simulated time series, which reduces the compatibility of the simulated time series of output and private consumption with the empirical time series.

There is a positive contemporaneous correlation between output and private consumption as well as between output and investment for both the simulated and the empirical time series. Compared with the correlations computed for the empirical time series, however, the correlations implied by the model prove to be too low for private consumption and too high for investment: For $\gamma = 3.00$, these correlations amount to $r_{C,Q} = 0.10$ and $r_{I,Q} = 0.99$, respectively. An increase in the households' willingness to intertemporally substitute, i.e. decreasing values for γ, leads to implausible negative correlations for private consumption.

The model's positive contemporaneous correlation between government consumption and output — for instance, $r_{G,Q} = 0.28$ holds for $\gamma = 3.00$ — proves to be incompatible with the negative correlation of -0.13 as measured for the empirical time series. This lack of compatibility may be due to the specification of the government's consumption function, according to which the autonomous component of government consumption depends linearly on current output. Even if a negative correlation between the innovations of the technology variable ϵ_z and the innovations of the autonomous government consumption rate ϵ_g of $r_{\epsilon_g,\epsilon_z} = -0.75$ is taken into account while simulating the model economy, this is not sufficient to contain the excessive positive correlation.

Looking at the standard deviations and the contemporaneous correlation of the labour market variables for $\gamma = 3.00$, we find for the simulated model economy that the standard deviation of the real wage σ_w of 0.89% is much lower than the empirical value of 1.55%.

Furthermore, at $r_{N,w} = 0.02$, the contemporaneous correlation implied by the models is too low compared with the empirical correlation of 0.59. The standard deviation of the real wage increases with a gradually rising willingness to make intertemporal substitutions, i.e. for decreasing values of γ, but, at the same time, the contemporaneous correlation of both variables decreases.

At first sight, the discrepancies between the empirical data and the simulated data may appear relatively large.[15] However, one must bear in mind that both the empirical data and the simulated data are subject to uncertainty. If confidence intervals were to be introduced for the empirical and simulated figures when assessing the compatibility of the model with the data, the discrepancies between them would turn out to be less obvious, particularly regarding the standard deviations of the goods market variables. Furthermore, considering the high degree of abstraction of the model's structure, this result should not come as a surprise at all.

Nevertheless, without using a formal statistical criterion, the simulated model appears to match the measured data best for a preference parameter value of $\gamma = 3.00$.[16] Hence, this value is used for simulating the intertemporal effects of fiscal policy in the following subsection.

3.2 Simulation of Fiscal Policy Scenarios

The vector of steady-state values of the exogenous fiscal policy variables are given by the unconditional expectation of the vector-autoregressive process describing their evolution over time with $\bar{v} = (I_5 - A_v)^{-1}\nu_v$, where $\bar{v} \equiv (\bar{g}, \bar{t}^c, \bar{t}^d, \bar{t}^w, \bar{tr})'$. Substituting the estimates $\hat{A}_v$ and $\hat{\nu}_v$ given in Table 2 above yields the estimated vector of steady state values $\hat{\bar{v}} = (I_5 - \hat{A}_v)^{-1}\hat{\nu}_v = (0.20, 0.10, 0.13, 0.21, 0.15)'$.

If changes in these steady-state values are considered to be parameter changes, the long-run effects of various fiscal policy measures on the models's endogenous variables Q, C, I, G, B, K, N, w, r and R can easily be assessesd within a comparative-static analysis. Furthermore, the adjustment of the endogenous variables to their new steady-state values computed beforehand in the comparative-static analysis can be explored by means of the

[15]In this context see Hansen & Wright (1992) and McGrattan (1994c) who survey the limitations of replicating the stylised facts of the goods and, in particular, the labour markets by means of RBC models.

[16]On the application of simulation-based indirect inference methods for an empirical evaluation of RBC models, see Coenen (1997).

model's impulse responses to the permanent changes in the fiscal policy variables. These impulse responses are realisations of a sequence of competitive equilibria describing the optimal behaviour of households and firms.

3.2.1 Comparative–Static Analysis

Table 4 shows the long–run effects of alternative fiscal policy scenarios which aim at permanently reducing the burden of taxes and levies. Since the revenue shortfalls due to permanent reductions in taxes and levies induce an increase in government debt $\bar{B}$ which exceeds the increase in output $\bar{Q}$, it will be necessary to reduce government spending in order to maintain the long–run government debt–to–output ratio $(\bar{B}/\bar{Q})$ at 60%. This spending cut will be obtained by appropriately adjusting the autonomous government consumption rate $\bar{g}$.

In the long run, a decrease of the consumption tax rate $\bar{t}^c$ by one percentage point (Scenario I) reduces the relative price of the consumption good which leads to a 1.57% rise in consumption. Furthermore, the households' substitution decisions involve a curtailing of leisure demand and a complementary increase in labour supply. In the new steady state, the labour input and the capital stock used for production both increase by 0.65%. Owing to the uniform increase in the input of both the production factors, output likewise increases by 0.65%.[17] Since the ratio of the production factors employed remains the same, there is no change in factor prices. The reduction of the consumption tax rate results in revenue shortfalls, so that, via a reduction in the government consumption rate by 0.51 percentage points, government consumption must be reduced by 1.92% in order to maintain the government debt–to–output ratio at 60%.

The incentive to invest due to lowering the income tax rate $\bar{t}^d$ by one percentage point (Scenario II) leads to a 1.02% increase in capital accumulation. Labour input increases by 0.45%, and output by 0.66%. Since labour is used relatively scarce in production, the wage rate rises by 0.20%, whereas the capital rental cost diminishes by 0.36%. The growth in the households' income induced by the increased input of the production factors and the lowering of the income tax rate leads to a 1.59% rise in private consumption. To stabilise the government debt–to–output ratio, government consumption must be cut by 2.39%.

[17]This result is determined by the choice of the functional forms for the households' preferences and the firms' production technology. It can be proved that $\bar{N} = \mathrm{f}(\bar{t}^c, \bar{t}^d, \bar{t}^w)$ and $\bar{K} = \mathrm{g}(\bar{t}^d)\,\bar{N}$ holds in the steady state.

Table 4: Intertemporal Effects of Fiscal Policy

		changes in taxes and levies neutral to the debt–to–output ratio [a]				
		I	II	III	IV	V
endogenous variables [b]		$\Delta\,\bar{t}^c = -0.01$, $\Delta\,\bar{g} = -0.0051$	$\Delta\,\bar{t}^d = -0.01$, $\Delta\,\bar{g} = -0.0060$	$\Delta\,\bar{t}^w = -0.01$, $\Delta\,\bar{g} = -0.0075$	$\Delta\,\bar{t}^d = -0.01$, $\Delta\,\bar{t}^c = 0.01$, $\Delta\,\bar{g} = -0.0010$	$\Delta\,\bar{t}^w = -0.01$, $\Delta\,\bar{t}^c = 0.01$, $\Delta\,\bar{g} = -0.0025$
output	$\bar{Q}$	0.65	0.66	0.77	0.02	0.13
priv. consumption	$\bar{C}$	1.57	1.59	2.14	0.04	0.58
investment	$\bar{I}$	0.65	1.02	0.77	0.38	0.13
gov. consumption	$\bar{G}$	-1.92	-2.39	-3.03	-0.48	-1.10
gov. debt	$\bar{B}$	0.65	0.66	0.77	0.02	0.13
capital stock	$\bar{K}$	0.65	1.02	0.77	0.38	0.13
employment	$\bar{N}$	0.65	0.45	0.77	-0.19	0.13
real wage	$\bar{w}$	0.00	0.20	0.00	0.20	0.00
rental cost	$\bar{r}$	0.00	-0.36	0.00	-0.36	0.00
rate of return	$\bar{R}$	0.00	0.00	0.00	0.00	0.00
welfare [c,d]	$\Delta^{\bar{C}}$	(1.22, 0.30, -0.23)	(1.35, 0.30, -0.31)	(1.72, 0.36, -0.43)	(0.13, -0.00, -0.08)	(0.51, 0.06, -0.20)

[a] The reduction of government receipts is balanced by a decrease in $\bar{g}$ such that $(\bar{B}/\bar{Q}) = 0.60$ holds again in the new steady state. [b] Changes in the steady–state values in percent. [c] Consumption equivalent in percent. [d] The values are obtained for $\pi = (0.00, 1.00, 2.00)$.

Lowering the wage levy rate $\bar{t}^w$ by one percentage point (Scenario III) leads to a 0.77% increase in employment. Firstly, this result reflects the fact that the reduction of the wage levy rate half to to be paid by the firms leads to a reduction of labour costs in production. Secondly, the stimulated labour demand of the firms is accompanied by an increase in the households' willingness to work because the reduction of the wage levies they have to pay leads to lower deductions from the households' wage income. As in Scenario I, capital and employment rise to the same amount, which means the wage rate and the capital rental cost do not change. The rise in disposable income caused by both the increased input of the production factors and the reduction of the wage levy rate leads to a 2.14% increase in private consumption. The increase in government debt caused by the wage levy reduction necessitates a 3.03% cut in government consumption.

Scenario IV investigates the effects of a fiscal policy measure which combines a one percentage point reduction of the income tax rate $\bar{t}^d$ with an one percentage point increase in the consumption tax rate $\bar{t}^c$. To a certain degree, this scenario pays tribute to the debate on a change in the structure of the German tax system, according to which direct taxes should be lowered, whereas indirect taxes should be raised in order to provide stronger incentives to invest. As to be expected, the resulting change in the factor price ratio in favour of capital yields an 0.38% increase in the capital stock. However, the growing capital stock goes hand in hand with a substitution of labour which leads to a reduction in labour input of 0.19%. Output only increases by 0.02%. In view of the fact that merely an 0.10% cut in government consumption rate is required to stabilise the government debt–to–output ratio, the combined measure proves to be almost self–financing.

Finally, Scenario V studies the effects of a reduction in the wage levy rate $\bar{t}^w$ by one percentage point which is (partly) offset by raising the consumption tax rate $\bar{t}^c$ to the same amount. The 0.13% increase in output is accompanied by a uniform increase in employment and the capital stock. The implied revenue shortfalls require a 1.10% cut in government consumption in order to stabilise the government debt–to–output ratio.

When assessing the long–run welfare effects of the alternative fiscal policy scenarios, one must take into account the fact that the isolated changes in consumption consisting of a private and a government component are not a suitable yardstick for evaluating their overall welfare implications. Besides changes in consumption, a comprehensive assessment must also include the changes in welfare induced by leisure changes. A suitable indicator is the consumption equivalent, which equates the single–period utility of the households in the initial steady state with their single–period utility in the new steady state implied by the particular fiscal policy measure. The (steady state) consumption equivalent $\Delta^{\bar{C}}$ is calculated using the single–period utility function U by solving the equation

$$U((1+\Delta^{\bar{C}})(\bar{C}+\pi\bar{G}),\bar{L}) \quad = \quad U(\tilde{C}+\pi\tilde{G},\tilde{L}),$$

where $(\bar{C}+\pi\bar{G},\bar{L})$ and $(\tilde{C}+\pi\tilde{G},\tilde{L})$ denote the equilibrium values of consumption and leisure in the initial and in the new steady state, respectively.

In the case of Scenarios I to III, assuming $\pi = 1.00$, the reductions in taxes and levies lead to welfare increases of 0.30, 0.30 and 0.36%, respectively, as reported in the last line of Table 4. Hence, the increase in private consumption overcompensates in terms of welfare for the reduction of leisure and the cut in government consumption needed to stabilise the government debt–to–output ratio.

To shed some light on the sensitivity of these results with regard to the aggregation parameter π, alternative consumption equivalents are calculated for $\pi = 0.00$ and $\pi = 2.00$, i.e. assuming that government consumption is either not beneficial or even more beneficial than private consumption.[18] Compared to the baseline case with $\pi = 0$, welfare increases (decreases) for $\pi = 1.00$ ($\pi = 2.00$). This result is due to the fact that the reduction in government consumption required in order to consolidate the government budget does diminish the utility of the households not at all (to a greater extent).

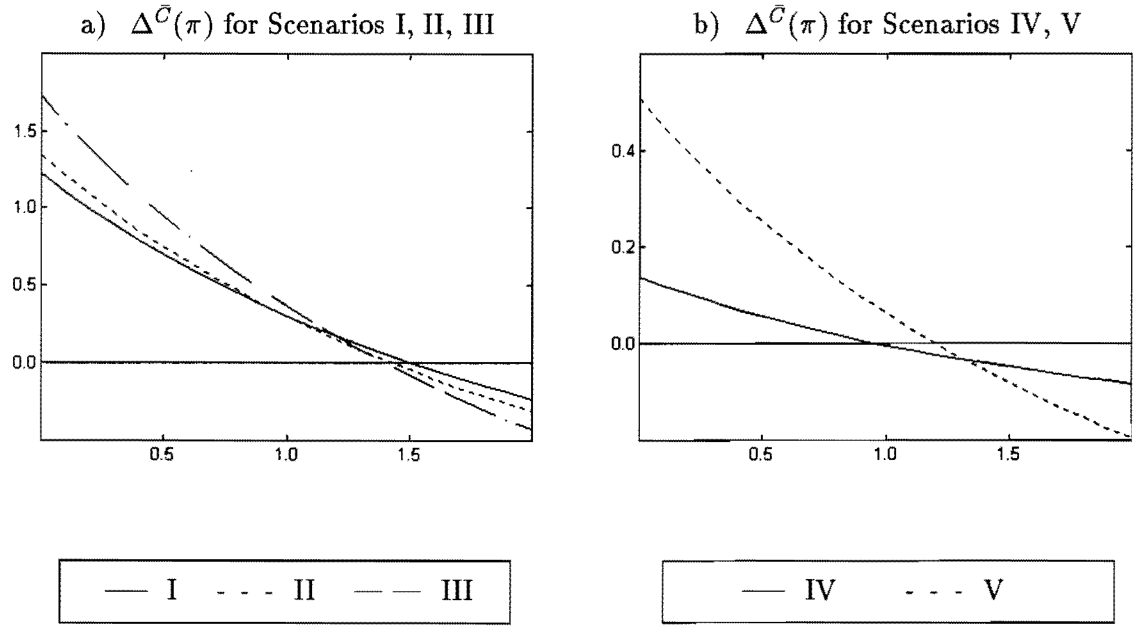
In Scenarios IV and V, assuming $\pi = 1.00$, the positive welfare effects resulting from the reduction of the income tax and the wage levies are (largely) offset by the negative welfare effects generated by the increase in the consumption tax and the cut in government consumption required to finance the reductions. This assessment changes if government consumption does not increase utility, i.e. if $\pi = 0.00$. In that case, the consumption equivalents amount to 0.13% and 0.51%, i.e. both the changeover from direct to indirect taxation and the lowering of the labour costs of the firms, the latter being financed by raising indirect taxes, imply positive welfare effects.

Since, ultimately, the analysis of the theoretical model does not provide a satisfactory basis for the choice of the parameter value for π, the final assessment of the welfare effects generated by the alternative fiscal policy scenarios must be left to the reader's judgement. This judgement may be based on Figure 1 which shows the consumption equivalent of the

[18]This calculation is feasible because the steady–state values of the endogenous variables do not depend on π owing to the utility function's separability.

fiscal policy scenarios depending on the aggregation parameter π. For below-unity values of π, any of the scenarios under consideration proves to be welfare-increasing. However, there is a critical value for each scenario. Exceeding this value will yield a negative welfare effect.

Figure 1: Consumption Equivalents of Fiscal Policy Scenarios



Yet if we consider that the model economy only features a single homogenous good, and that, on the basis of our model, the argument that government consumption may increase the households' utility more than private consumption is hence not very convincing, it then appears justified to use $\pi = 1.00$ as the upper benchmark value for the assessment of the welfare effects. As a consequence, positive welfare effects are to be expected from the fiscal policy measures under review.

3.2.2 Dynamic Analysis

As a complement to the comparative–static analysis of the fiscal policy measures, we will now turn to the model economy's dynamic adjustment to its new steady state implied by the permanent reduction in taxes and levies. For Scenarios I to III the adjustment paths, i.e. the impulse response functions of the endogenous variables, are illustrated in Figure 2. The impulse responses are expressed as percentage deviations from the initial steady–state values.

Figure 2: Impulse Responses to a Decrease in Taxes and Levies

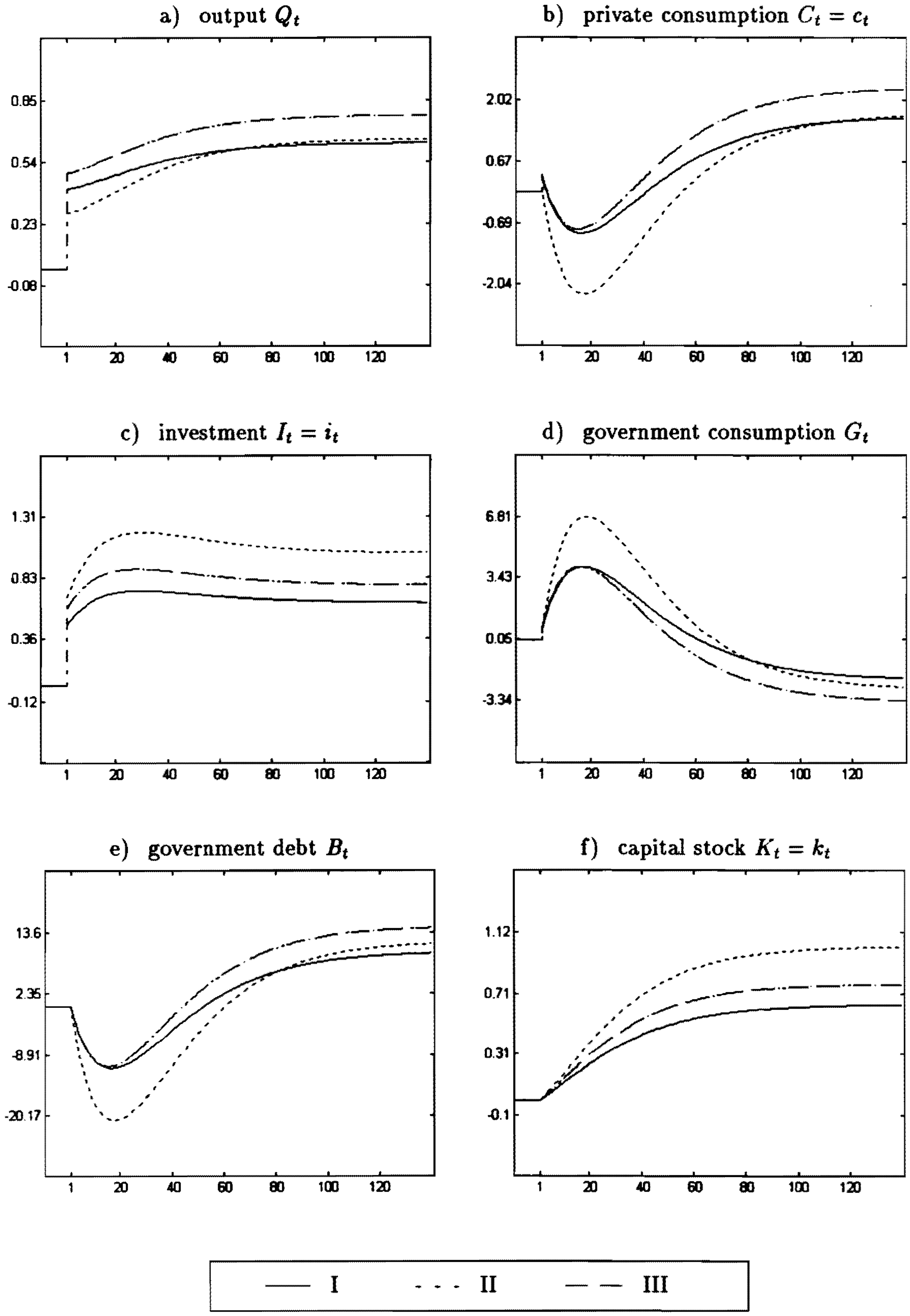
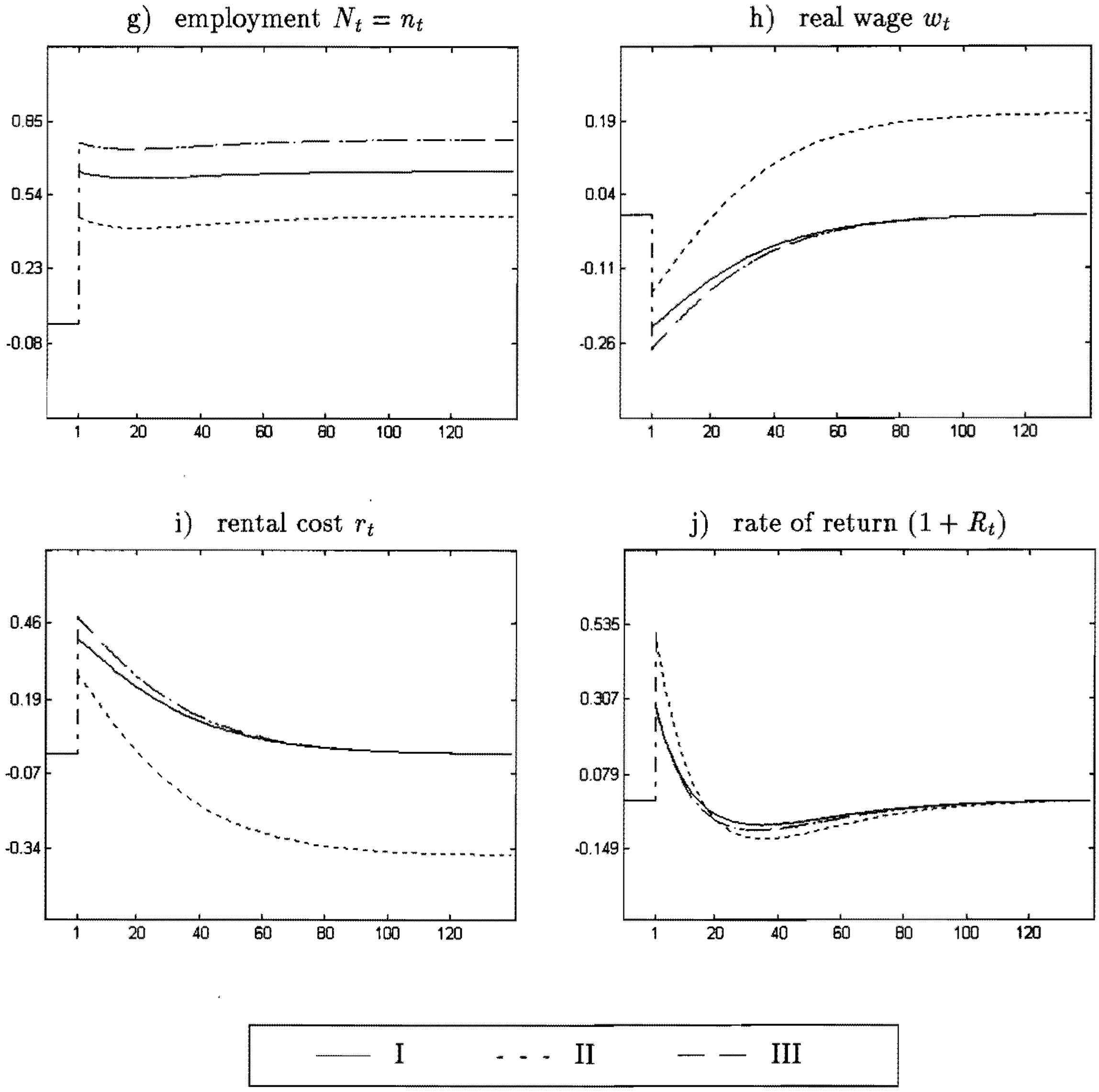


Figure 2: Impulse Responses to a Decrease in Taxes and Levies



As the figure shows, the endogenous variables follow similar paths in response to the lasting reduction of taxes and levies across the alternative scenarios. This is due to the fact that each lowering of taxes and levies induces a unidirectional reduction in the allocative distortions of the model economy. This reduction enhances the households' propensity to work and leads to a rise in overall economic activity which, in the long run, increases the welfare of the households, as demonstrated in the preceding subsection.

The fiscal policy measures clearly generate positive effects, even in the short run. Above all, these effects are the result of the immediate increase in employment which leads to an increase in output, and hence to a rise in the households' income. During the adjustment to the new steady state, the optimal choice of the households consists in temporarily restraining consumption in spite of the increase in their current income resulting from the reduction in taxes and levies and to use it for investments. With the gradual increase in the capital stock of the economy, the capacity effect caused by the boost in investment impacts on production and leads to a rise in the households' future income and, thus, to an improvement of their future consumption opportunities. Owing to the households' forward-looking optimal plans, these future consumption opportunities are already taken into account in their current decisions, which explains why the households temporarily forego consumption.

In the short run, the positive output effects make it possible to even increase government consumption. The increase is financed by a rise in tax revenues resulting from the rise in output, which, in turn, implies an increase in taxable income. In this case, the rise in tax revenues even leads to a temporary reduction in government debt. In the course of the adjustment to the new steady state, government debt increases again. However, the restraint in government consumption induced by the reduction of the autonomous government consumption rate will ensure that the government debt-to-output ratio remains at 60% over the long run.

The scenarios differ as to the extent of the short and medium-term adjustments. Compared with changes in the income tax rate, changes in the consumption tax rate and the wage levy rate will cause factor prices, employment and output to react more strongly. By contrast, a change in the income tax rate leads to higher investment and, hence, more restrained consumption. The resulting increase in government revenue and expenditure causes a greater transitory reduction in government debt, which implies a higher increase in the rate of return owing to more restrained private consumption. These phenomena reflect the fact that income taxes exert a relatively strong influence on the households' intertemporal consumption-investment decisions, whereas their intratemporal labour-leisure choices are affected to a greater extent by consumption taxes and wage levies.

As regards the above interpretation of the short and medium-term adjustments, one must consider that the Walrasian model economy under review does not include institutional rigidities: The effects of the fiscal policy measures, which are positive even in the short run, presumably hinge on the assumption of market-clearing by immediate price adjustments. Hence, an extension of the model covering lagged price adjustment mechanisms would have to analyse the extent to which the short and medium–term adjustments would be robust.

4 Summary and Conclusions

This paper has analysed the intertemporal effects of fiscal policy reforms as requested of economic policy in Germany within the framework of a calibrated RBC model. Based on alternative scenarios, the analysis demonstrated that a reduction of the burden of taxes and levies, criticised as too high in the economic policy debate, will lead to an increase in overall economic activity. In each scenario, the revenue shortfalls induced by lowering the taxes and levies are offset by a cut in government consumption, which implies that the government debt–to–output ratio can be lastingly maintained and, hence, that the sustainability of the government budget is ensured. For plausible values of the weighting of government consumption in the households' utility function, the alternative fiscal policy scenarios have positive welfare effects.

The increase in overall economic activity is a consequence of the reduction in intratemporal and intertemporal allocative distortions induced by taxes and levies. It is shown that the accumulation of capital can be stimulated, in particular, by income tax reductions. However, the analysis also reveals that a rise in the capital stock resulting from an income tax cut implies the substitution of labour which is becoming comparatively expensive. Instead, a stronger increase in employment can be achieved by reducing the levies on wage income. A partial changeover from income to consumption taxes leads to an increase in investment but also to lower employment, whereas a reduction in the levies on wage income offset by raising the consumption tax triggers an increase in both investment and employment.

Whereas the findings support a supply–oriented economic policy which aims at reducing government activities, their final assessment should take into account the fact that the model is an abstraction of a number of important factors that, in general, motivate government action. For instance, government investment providing public infrastructure is disregarded, as is the explicit introduction of a social security system which would be justified by an intra– or intergenerational redistribution objective. In addition, the model does not take into account rigidities and market imperfections which can lead to persistent imbalances — a fact that holds particularly true for the labour market.

Nonetheless, this paper highlights major obstacles on the supply side which stand in the way of sustained economic growth supported by increased investment and a higher level of employment. Thereby, analysing the arguments calling for the removal of these obstacles within a dynamic general equilibrium model and identifying the assumptions underlying these arguments, this paper may offer a disciplining analytical input to the economic policy debate on the appropriate design of fiscal policy in order to improve the investment and employment conditions and, thus, to promote overall economic activity.

A The Data

This appendix describes the construction of the data which are employed to calibrate the above RBC model. The data build on time series taken from the national accounts of the Federal Republic of Germany (West) with a sample period ranging from the first quarter of 1960 to the fourth quarter of 1994.

A.1 The Variables of the Goods and the Labour Market

To construct the per–capita variables of the goods and labour markets the following time series were taken from the German national accounts:

- Gross domestic product at 1991 prices (in DM billion)
- Private consumption at 1991 prices (in DM billion)
- Investment: fixed capital formation by firms at 1991 prices (in DM billion)
- Government consumption at 1991 prices (in DM billion)
- Compensation of employees: gross wages and salaries (in DM billion)
- Employed persons: total number of hours worked (in billion hours)
- Price index of the gross domestic product (1991 = 100)
- Resident population (in 1,000 people)

Where required, the time series are seasonally adjusted using the Census X–11 method. The seasonally–adjusted time series of the gross wages and salaries is deflated by the price index of the gross domestic product. By dividing the adjusted time series by the time series of the resident population, we obtain the time series of the real per–capita variables, i.e. of output Q, private consumption C, private investment I, government consumption G, real wage w and employment N.

Since the time series of the per–capita variables are non–stationary, a suitable trend adjustment must be carried out. In line with the literature, the Hodrick–Prescott (HP) filter is applied which presumes that the time series of a variable y (in logarithms) can be decomposed into a smooth trend component y^g plus a residual component y^r,

$$y_t = y_t^g + y_t^r, \quad t = 1, \ldots, T.$$

Further assuming that the mean of the residual component y^r equals zero, the trend component y^g is defined as the solution to the following quadratic minimisation problem:[19]

$$\min_{\{y_t^g\}_{t=1}^T} \left\{ \sum_{t=1}^{T} (y_t - y_t^g)^2 + \lambda \sum_{t=2}^{T-1} \left[(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)\right]^2 \right\}.$$

The first term of the objective function to be minimised captures the fit of the trend component to the data, whereas the second term captures its smoothness. The parameter λ controls these two conflicting properties. For $\lambda = 0$, the component y^g is perfectly fitted to the data, such that $y_t^g = y_t$ holds. If the determination of y^g only takes account of the smoothness, i.e. for $\lambda \to \infty$, y^g follows a linear trend.[20]

Figure 3 shows the time series of the per–capita variables of the goods and labour markets as well as their HP trend component for a parameter value of $\lambda = 1,600$ which is generally used for quarterly time series in the literature.

A.2 The Fiscal Policy Variables

Complementing the above data, the following time series from the national accounts are used for constructing the fiscal policy variables:

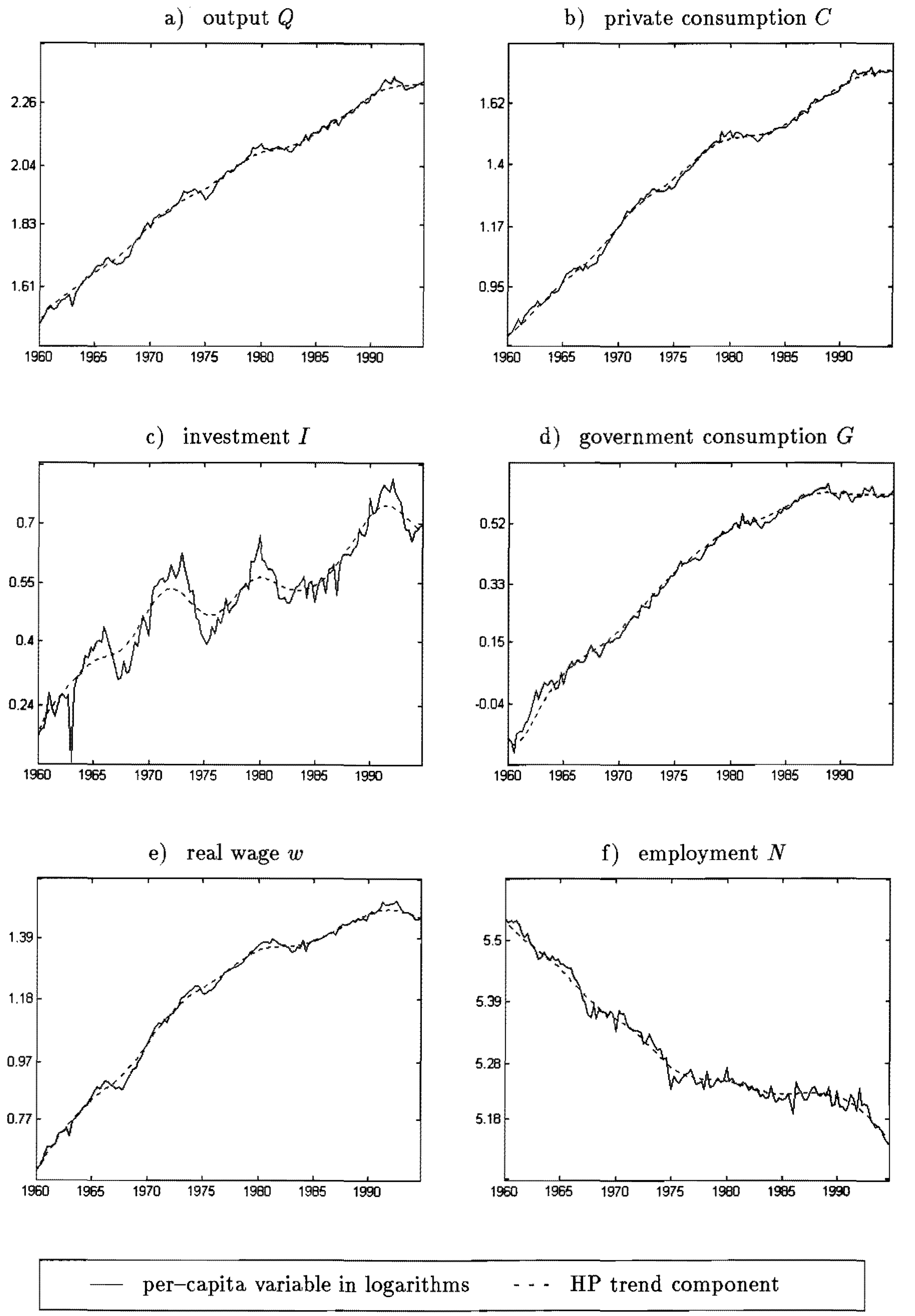
- Investment: depreciations by firms (in DM billion)
- Compensation of employees: social contributions by employers (in DM billion)
- Current transfers: direct domestic taxes paid to the government (in DM billion)
- Current transfers: social benefits the government pays to the households (in DM billion)
- Gross value added: non–deductible turnover tax (in DM billion)

The time series are seasonally adjusted using the Census X–11 method and deflated by the price index of the gross domestic product.

[19] See Hodrick & Prescott (1980), Prescott (1986) and Cooley & Prescott (1995).

[20] King & Rebelo (1993) demonstrate that the cyclical component of a time series integrated up to order $d = 4$, which is determined by applying the HP filter, is stationary. However, statistical problems arising from HP filtering non–stationary time series are addressed in Harvey & Jäger (1993), Jäger (1994) and Cogley & Nason (1995).

Figure 3: HP Trend Component of the Variables of the Goods and the Labour Market



The government consumption rate g is calculated as the share of real gross domestic product taken up by real government consumption. The consumption tax rate t^c is calculated as the ratio of the sum of real private and government consumption and the sum of real private and government consumption less government's real value–added tax revenue, minus one. Owing to the linear homogeneity of the model's production technology and the deductibility of the depreciations from the households' taxable capital income, the income tax rate t^d is defined as the share of real gross domestic product less real depreciation accounted for by real direct taxes. The wage levy rate t^w denotes the share of the employers' real social contributions to real total gross wages and salaries. The transfer rate tr is calculated as the share of real gross domestic product accounted for by the real social benefits the households receive from the government.

The fiscal policy variables g, t^c, t^d, t^w and tr are stacked to form the 5 – dimensional vector $v \equiv (g, t^c, t^d, t^w, tr)'$. Since the time series of the vector of the fiscal policy variables prove to be non–stationary too, the time series v_i are again decomposed into a smooth trend component v_i^g and a residual component v_i^r beforehand,

$$v_{i,t} \;=\; v_{i,t}^g + v_{i,t}^r, \quad t = 1, \ldots, T, \quad i = 1, \ldots, 5. \tag{34}$$

For the present we shall assume that the mean of the residual component v_i^r equals zero, but the smooth trend component v_i^g is defined as a third–order polynomial,

$$v_{i,t}^g \;=\; \beta_{i,0} + \beta_{i,1}\, t + \beta_{i,2}\, t^2 + \beta_{i,3}\, t^3, \quad t = 1, \ldots, T.$$

Figure 4 shows the time series of the fiscal policy variables and their estimated polynomial trend component.

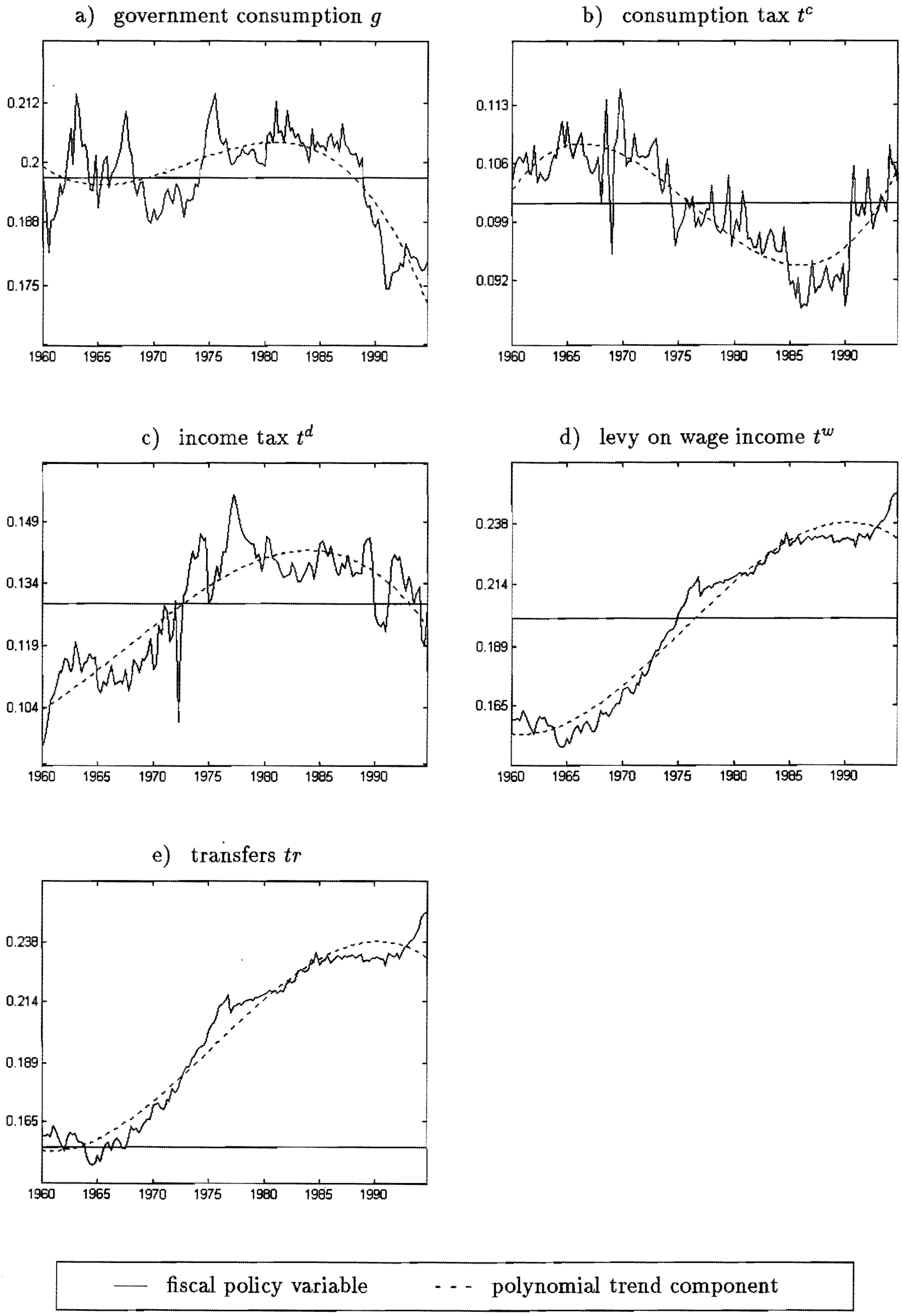
Subsequently, since the analysis of the model economy rests on information about the mean of the time series v_i, the residual components v_i^r are adjusted appropriately,

$$\tilde{v}_i^r \;\equiv\; v_i^r + \bar{v}_{i,T}\, \mathbf{i}_T \qquad \text{with} \qquad \bar{v}_{i,T} \equiv \frac{1}{T} \sum_{t=1}^{T} v_{i,t},$$

where $\mathbf{i}_T$ denotes the T – dimensional vector of ones. The means of the time series are illustrated by the horizontal lines in Figure 4.[21]

[21] The mean $\bar{v}_{5,T} = \bar{tr}$ was adjusted appropriately in order to ensure that the government debt–to–output ratio $(\bar{B}/\bar{Q})$ in the model economy is maintained at 60% in the long run.

Figure 4: Polynomial Trend Component of the Fiscal Policy Variables



A VAR(p) model, i.e. a vector-autoregressive model of order p, is then fitted to the time series of the vector of adjusted residuals $\tilde{v}^r$,

$$\tilde{v}_t^r = \nu_v + A_{v,1}\,\tilde{v}_{t-1}^r + \cdots + A_{v,p}\,\tilde{v}_{t-p}^r + u_t, \quad t = 1+p,\ldots,T$$

where the error terms u_t are assumed to be independent white noise with expectation $\mathrm{E}[u_t] = 0$ and positive–definite covariance matrix $\mathrm{Var}[u_t] = \mathrm{E}[u_t\,u_t']$. The Choleski decomposition of the covariance matrix yields the lower triangular matrix C_{ϵ_v} with $\mathrm{Var}[u_t] = C_{\epsilon_v}C_{\epsilon_v}'$.

Various information criteria calculated to determine the lag order of the VAR(p) model show $p^* = 1$ to be the optimal lag length. The P value of a portmanteau test indicates that the error terms of the VAR(1) model are free from serial correlation. The estimated parameters of the VAR(1) model are reported in Table 1 of Section 2.

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The following papers have so far been published:

May	1995	The Circulation of Deutsche Mark Abroad	Franz Seitz
June	1995	Methodology and technique for determining structural budget deficits	Gerhard Ziebarth
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