Time-varying uncertainty and predictability of exchange rates

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Motivation - Exchange rates are difficult to predict!

- Meese and Rogoff (1983) identified that exchange rate fluctuations are difficult to predict using standard economic models
  - Random walk (RW) is frequently found to generate better exchange rate forecasts than economic models (Meese and Rogoff puzzle).
- Rossi (2013) surveys the literature and points out that exchange rate predictability is affected by:
  - Choice of the predictor
  - Forecast horizon
  - Forecasting model
  - Methods for forecast evaluation
- The predictive power is specific to some countries in certain periods.
  - Signals the presence of instability in the models' forecasting performance

Contribution of this paper - What we do

- We propose a density combination approach to exchange rate models which accounts for several sources of uncertainty
  - Time-varying weights
  - Model set incompleteness
  - Combination weight uncertainty and learning
- In an empirical exercise forecasting exchange rates for 7 countries we find
  - Large relative gains in terms of both point and density forecasting performance from our combination approach
    - Outperforms RW, all individual models and alternative combination approaches.
  - Accounting for model incompleteness and weight uncertainty is important.

Combined Exchange Rate Forecast

We assume that at a generic point in time t, the forecaster has available N different models to predict exchange rate at time $t+h$, each model producing a predictive density $p\left(s_{t+h}|\, M_i, \mathcal{D}^t\right)$, $i = 1, ..., N$.

The composite predictive density $p(s_{t+h}|\mathcal{D}^t)$ is given by:

$$p\left(s_{t+h}|\,\mathcal{D}^t\right) = \int\int p(s_{t+h}|\widetilde{\mathbf{s}}_{t+h}, \mathbf{w}_{t+h}, \mathcal{D}^t)\, p(\mathbf{w}_{t+h}|\widetilde{\mathbf{s}}_{t+h}, \mathcal{D}^t)\, p\left(\widetilde{\mathbf{s}}_{t+h}|\,\mathcal{D}^t\right) d\widetilde{\mathbf{s}}_{t+h} d\mathbf{w}_{t+h} \quad (1)$$

- $p(s_{t+h}|\widetilde{\mathbf{s}}_{t+h}, \mathbf{w}_{t+h}, \mathcal{D}^t)$ denotes the combination scheme based on the N predictions $\widetilde{\mathbf{s}}_{t+h}$ and the combination weights $\mathbf{w}_{t+h} \equiv (w_{1,t+h}, \ldots, w_{N,t+h})'$,
- $p(\mathbf{w}_{t+h}|\widetilde{\mathbf{s}}_{t+h}, \mathcal{D}^t)$ denotes the posterior density of the combination weights $\mathbf{w}_{t+h}$.
- We aggregate the N predictive densities $\left\{p\left(s_{t+h}|\, M_i, \mathcal{D}^t\right)\right\}_{i=1}^N$ into the pdf $p\left(\widetilde{\mathbf{s}}_{t+1}|\,\mathcal{D}^t\right)$.

Combination scheme

Gaussian combination that allows for model incompleteness:

$$p(s_{t+h}|\widetilde{\mathbf{s}}_{t+h}, \mathbf{w}_{t+h}, \sigma_\kappa^{-2}) \propto \exp\left\{-\frac{1}{2}\left(s_{t+h} - \widetilde{\mathbf{s}}'_{t+h}\mathbf{w}_{t+h}\right)' \sigma_\kappa^{-2}\left(s_{t+h} - \widetilde{\mathbf{s}}'_{t+h}\mathbf{w}_{t+1}\right)\right\} \quad (2)$$

where $\mathbf{w}_{t+h}$ is a vector containing the N values for the combination weights and $\widetilde{\mathbf{s}}_{t+h}$ contains the N predicted values from a distribution with density $p\left(\widetilde{\mathbf{s}}_{t+h}|\,\mathcal{D}^t\right)$

The combination disturbances, defined as κ_{t+h}, are estimated and their distribution provide a probabilistic measure of the incompleteness of the model set. The model in equation (2) is:

$$s_{t+h} = \widetilde{\mathbf{s}}'_{t+h}\mathbf{w}_{t+h} + \kappa_{t+h} \quad (3)$$

with $\kappa_{t+h} \sim \mathcal{N}(0, \sigma_\kappa^2)$.

Combination weights and individual models

Combination Weights have a probabilistic distribution in the unit interval and they are nonlinear/logistic transforms for all N models, given as

$$w_{i,t+h} = \frac{\exp\{z_{i,t+h}\}}{\sum_{l=1}^N \exp\{z_{l,t+h}\}}, \quad i = 1, \ldots, N \quad (4)$$

where $\mathbf{z}_{t+h} \equiv (z_{1,t+h}, \ldots, z_{N,t+h})'$, is a vector of latent processes.

Dynamics of Weights

$$\begin{aligned} \mathbf{z}_{t+h} &\sim p(\mathbf{z}_{t+h}|\mathbf{z}_t, \Lambda) \\ &\propto |\Lambda|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\mathbf{z}_{t+h} - \mathbf{z}_t)' \Lambda^{-1} (\mathbf{z}_{t+h} - \mathbf{z}_t)\right\} \end{aligned} \quad (5)$$

with Λ an $(N \times N)$ diagonal matrix.

General Linear Model

$$\begin{aligned} s_{t+h} = \Delta e_{t+h} &= \mu + \beta x_t + \varepsilon_{t+h}, \quad \tau = 1, ..., t-1, \\ \varepsilon_{t+h} &\sim N(0, \sigma_\varepsilon^2), \\ x_t &= \Omega_t - e_t. \end{aligned} \quad (6)$$

x_t measures the disequilibrium between the exchange rate's spot value and the level of the fundamentals.

Menu of predictors

- Uncovered Interest Rate Parity (UIP)

$$x_{t,UIP} = i_t - i_t^* + e_t \quad (7)$$

- Purchasing Power Parity (PPP)

$$x_{t,PPP} = p_t - p_t^* \quad (8)$$

- Monetary Model (MM), Mark (1995)

$$x_{t,MM} = (m_t - m_t^*) - (y_t - y_t^*) \quad (9)$$

- Symmetric Taylor Rule (TR1), Engel and West (2015)

$$x_{t,TR_1} = 1.5(\pi_t - \pi_t^*) + 0.5(y_t - y_t^*) + e_t \quad (10)$$

- Asymmetric Taylor Rule (TR2), Li et al (2015)

$$x_{t,TR_2} = 1.5(\pi_t - \pi_t^*) + 0.1(y_t - y_t^*) + 0.1(e_t + p^* - p_t) + e_t \quad (11)$$

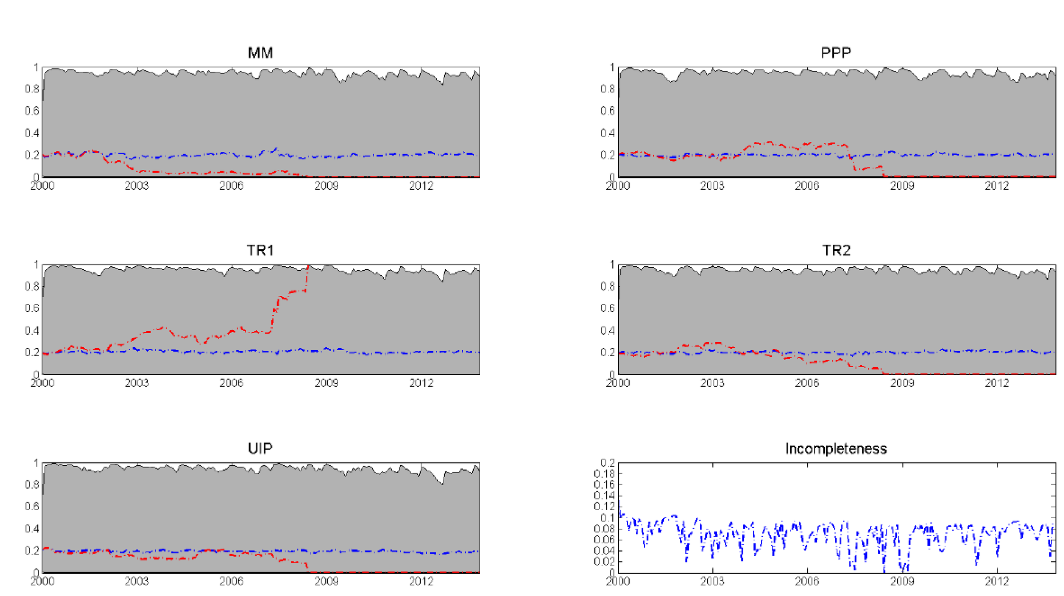
Empirical exercise

- Forecast monthly exchange rates for 7 countries
  - Countries: Australia, Canada, Norway, Euro, Japan, Switzerland and Great Britain.
  - Exchange rates measured as end-of-month exchange rate to USD
  - Forecast evaluation: RMSE, LPS, CRPS
  - Evaluation sample: 2000:M5-2014:M3
  - Forecast horizons: $h = 1, 3$
  - Consider forecasts from 5 different fundamentals-based empirical models
    - Consider models with constant coefficients, stochastic volatility and time-varying parameters with stochastic volatility.
  - Consider model 5 different model combinations
    - DeCo, BMA and equal weights

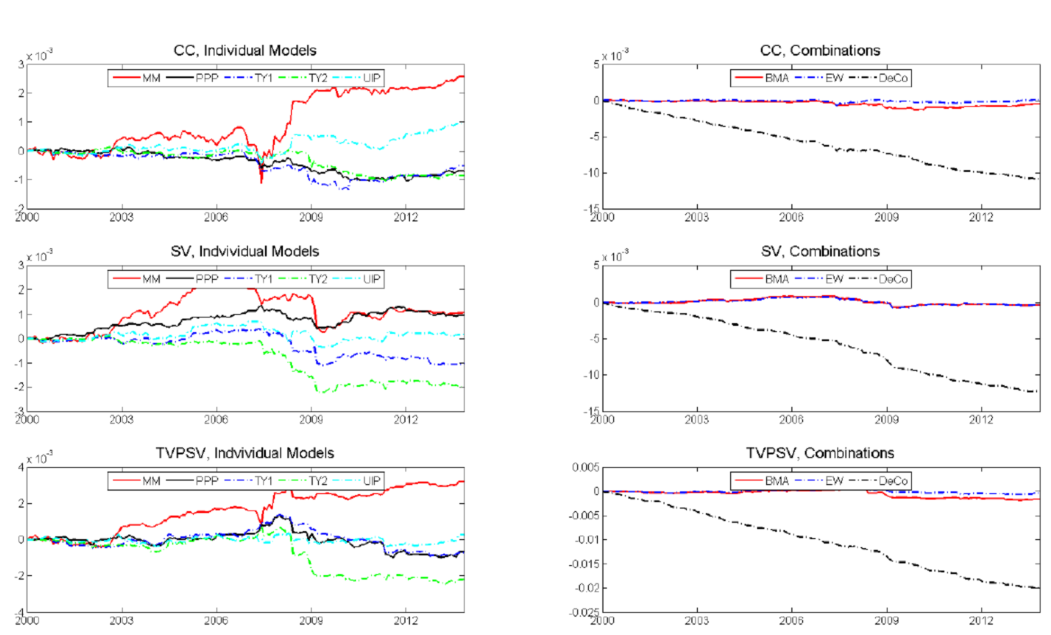
Results relative to RW, H=1

Model	AUS	CAN	NOR	EUR	JPN	CHE	GBP
MSPE							
DeCo	**0.938**	**0.845**	**0.863**	**0.827**	**0.787**	**0.824**	**0.793**
BMA	0.998	1.000	0.997	0.994	0.999	1.001	0.996
EW	0.998	1.000	0.997	0.994	0.999	1.001	0.996
MM	1.019	1.032	1.051	1.039	1.034	1.046	1.018
PPP	1.011	1.001	0.981	0.992	1.032	0.999	1.014
TR1	1.001	1.000	1.020	0.980	1.017	0.994	1.010
TR2	0.993	0.988	1.003	0.988	1.010	0.999	1.014
UIP	1.007	1.008	1.020	1.002	1.010	1.010	0.993
CRPS							
DeCo	**0.818**	**0.649**	**0.674**	**0.648**	**0.591**	**0.642**	**0.601**
BMA	1.004	1.019	1.008	1.001	0.984	1.003	0.974
EW	1.012	1.022	1.037	1.016	0.998	1.014	0.987
MM	1.020	1.048	1.058	1.068	1.087	1.072	1.088
PPP	1.017	1.039	1.028	1.049	1.078	1.050	1.084
TR1	1.008	1.036	1.040	1.043	1.073	1.041	1.080
TR2	1.005	1.030	1.038	1.043	1.069	1.049	1.082
UIP	1.013	1.038	1.045	1.052	1.066	1.055	1.075

Time-varying weights and model incompleteness, CAN



Cumulative SPE differentials, CAN



The importance of incompleteness and weight uncertainty

Model	AUS	CAN	NOR	EUR	JPN	CHE	GBP
MSPE							
DeCo	**0.923**	**0.912**	**0.910**	**0.884**	**0.821**	**0.871**	**0.834**
No Inc	0.970	0.974	0.962	0.953	0.948	0.957	0.926
No Inc and TVW	1.002	0.999	0.999	0.987	1.002	0.998	0.987
CRPS							
DeCo	**0.819**	**0.800**	**0.778**	**0.727**	**0.647**	**0.710**	**0.664**
No Inc	0.903	0.902	0.875	0.847	0.800	0.841	0.800
No Inc and TVW	1.013	1.018	1.006	0.988	0.979	0.990	0.982

Relation between range volatility and incompleteness

- For each draw we regress contemporaneous range volatility on the standard deviation of incompleteness (computed across particles)
- Report median and 95% credible interval for regression coefficients
- Results: Incompleteness is related to range volatility (which is not observed at the time of forecast)

Country	Median	95% CI
AUS	-0.285	[-0.426, -0.092]
CAN	-0.196	[-0.301 , -0.086]
NOR	-0.217	[-0.338 , -0.060]
EUR	-0.201	[-0.320 , -0.036]
JPN	-0.116	[-0.234 , 0.010]
CHE	-0.173	[-0.301 , -0.028]
GBP	-0.126	[-0.241 , 0.019]