An Empirical Investigation of Direct and Iterated Multistep Conditional Forecasts

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Introduction

- Central banks use conditional forecasts to assess hypothetical policies. Christoffel et al. (ECB, 2008)
- Large banks have to construct conditional forecasts as part of stress testing exercises. Sarychev (BoE, 2014)
- Recent surge in conditional forecasting in academic literature:
  - Giannone et al. (NY Fed, 2014): Big VARs
  - Baumeister and Kilian (BoC, 2013): Oil
  - Aastveit et al. (NB, 2014): Break tests
  - Clark and McCracken (Feds, 2017): Inference
- All of these are VARs or near-VARs using IMS approach

Does DMS vs. IMS Matter for Forecasting?

- For unconditional point forecasts, theory says "Yes."
  - Bhansali (1997)
  - Schorfheide (2005)
- For unconditional point forecasts, empirics say "Yes."
  - Marcellino, Stock, and Watson (MSW; 2006)
- For impulse response functions, empirics say "Yes."
  - Jorda (2008)

What We Do

- We provide empirical evidence on whether the DMS vs IMS battle matters for conditional point forecasts.
  - IMS is MSE optimal from OLS VAR (Waggoner and Zha, 1999)
  - DMS is just OLS BLP (Goldberger, 1962)
- We do so by shamelessly emulating what MSW did.
  - 2000 bivariate systems used to construct MSEs
  - 150 trivariate "monetary" systems used to construct MSEs
- Scenarios are ex-post realized actuals so that scenarios actually occur and MSE is relevant.

Overview of Results

- Whether our results coincide with MSW is sample dependent
  - Over common sample, our results align with theirs: IMS generally favored with some benefits to DMS for nominals
  - For Great Moderation sample, DMS heavily favored for nominals and about the same as IMS for reals/financials
- Robustness
  - Bivariate and trivariate
  - horizons: $h = 3, 6, 12, 24$
  - lag selection: fixed at 4 or 12, or $AIC\,/\,BIC$
  - real-time vintage data

What We Don't Do

- Consider bayesian estimation.
- Consider large VARs.
- Consider univariate ARDLs (Guerrieri and Welch 2014) or alternative DMS in Jorda and Marcellino (2008).

DMS vs. IMS examples

Examples of DMS vs. IMS

- VAR(1) taking the form

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} 0 & b \\ 0 & c \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} e_t \\ v_t \end{pmatrix},$$

  with i.i.d. N(0,1) errors with contemporaneous correlation ρ.

- Use pseudo-true parameters used for forecasting
- One-step conditional forecast of y_{t+1} given x_{t+1}
- DMS model is trivial: $y_t = \gamma x_t + \eta_t$

Correct Specification

- IMS forecast is $\hat{y}^c_{t,1} = bx_t + \rho(x_{t+1} - cx_t)$
  - $E(e^{IMS}_{t,1})^2 = 1 - \rho^2$.
- DMS forecast is $\hat{y}^c_{t,1} = (bc + \rho(1 - c^2))x_{t+1}$
  - $E(e^{DMS}_{t,1})^2 = 1 - \rho^2 + (b - \rho c)^2$.
- Minimum-MSE IMS better.

VAR Misspecification of Conditional Mean

- Equation for x misspecified as $x_t = \alpha x_{t-2} + \eta_t$
- IMS forecast is $\hat{y}^c_{t,1} = bx_t + \frac{\rho}{\sqrt{1+c^2}}(x_{t+1} - c^2 x_t)$
  - $E(e^{IMS}_{t,1})^2 = 1 + \rho^2 - \frac{2\rho^2}{\sqrt{1+c^2}}$
- DMS forecast is $\hat{y}^c_{t,1} = (bc + \rho(1 - c^2))x_{t+1}$
  - $E(e^{DMS}_{t,1})^2 = 1 - \rho^2 + (b - \rho c)^2$.
- DMS better if $b = \rho^2$ and $c = \rho$.

VAR Misspecification of Residual Variance

- Conditional mean ok. Residual correlation changes from ρ_0 to ρ_1 between T and $T+1$.
- IMS forecast is $\hat{y}^c_{T,1} = bx_T + \rho_0(x_{T+1} - cx_T)$
  - $E(e^{IMS}_{T,1})^2 = 1 + \rho_0^2 - 2\rho_0\rho_1$
- DMS forecast is $\hat{y}^c_{T,1} = (bc + \rho_0(1 - c^2))x_{T+1}$
  - $E(e^{DMS}_{T,1})^2 = 1 + b^2 + \rho_0^2(1 - c^2) - 2bc\rho_1 - 2\rho_0\rho_1(1 - c^2)$.
- DMS better if $b = 0$ and $c \in (0,1)$, $\rho_0 \in (-1,0)$, and $\rho_1 \in (0,1)$.

Methodology

VAR-based IMS Conditional Forecasts

- $Z_t = (Y_t, X_t)'$ in levels or log-levels. Let $z_t = (y_t, x_t)'$ denote the stationary transformation: level or differences.
- OLS estimate of VAR for $z_t = (y_t, x_t)'$:

$$z_t = C + A(L)z_{t-1} + e_t$$

used to produce h-step conditional forecasts of the form

$$\hat{y}^c_{t,h} = \hat{y}^u_{t,h} + \textstyle\sum_{1 \leq i \leq h} \hat{\gamma}_{i,t}(x_{t+i} - \hat{x}^u_{t,i})$$

for constants $\hat{\gamma}_{i,t}$ that are known functions of $\hat{A}_{i,t}$, and $\hat{\Sigma}_t$.

- $\hat{Y}^c_{t,h} = \left\{ \begin{array}{cl} \hat{y}^c_{t,h} & \text{if } Y_t \text{ is } I(0) \\ Y_t + \sum_{i=1}^h \hat{y}^c_{t,i} & \text{if } Y_t \text{ is } I(1) \\ Y_t + h\Delta Y_t + \sum_{i=1}^h \sum_{j=1}^i \hat{y}^c_{t,j} & \text{if } Y_t \text{ is } I(2) \end{array} \right\}.$

ARDL-based DMS Conditional Forecasts

- Define

$$y_t^{(h)} = \left\{ \begin{array}{cc} Y_t & \text{if } Y_t \text{ is } I(0) \\ Y_t - Y_{t-h} & \text{if } Y_t \text{ is } I(1) \\ Y_t - Y_{t-h} - h\Delta Y_t & \text{if } Y_t \text{ is } I(2) \end{array} \right\}.$$

- OLS estimate of ARDL for $z_t = (y_t, x_t)'$:

$$y_t^{(h)} = \alpha + \textstyle\sum_{j=0}^{p-1} \beta_j y_{t-h-j} + \sum_{j=0}^{p-1} \delta_j x_{t-h-j} + \left(\sum_{1 \le i \le h} \gamma_i x_{t-h+i}\right) + \varepsilon_t$$

used to produce h-step conditional forecasts of the form

$$\hat{y}_{t,h}^{c(h)} = \hat{\alpha}_t + \textstyle\sum_{j=0}^{p-1} \hat{\beta}_{j,t} y_{t-j} + \sum_{j=0}^{p-1} \hat{\delta}_{j,t} x_{t-j} + \left(\sum_{1 \le i \le h} \hat{\gamma}_{i,t} x_{t+i}\right)$$

- $\hat{Y}_{t,h}^c$ computed relative to order of integration of Y.

Choices

- Lag selection: 4, 12, or updated AIC/BIC
- Horizons: 3, 6, 12, 24
- Paths: Future x known throughout forecast horizon
- In-sample: Start in 1959:01 or 1984:01
- Out-of-sample: 1979:01+h – 2002:12 or 2002:12+h – 2016:12

Bivariate Data

- Bivariate results use June 2017 vintage of 121 monthly series taken from FRED-MD (McCracken and Ng, 2016).
  - Series mapped into the 5 groups MSW have:
    a) Income, output, sales, and capacity utilization
    b) Employment and unemployment
    c) Construction, inventories, and orders
    d) Interest rates and asset prices
    e) Nominal prices, wages, and money
  - MSW have 170 series: differences mostly come from groups (a) and (c) above
- The 2,000 bivariate systems are obtained at random by selecting pairs (y, x) from distinct groups (200 from each of the 10 possible group pairs). From these, 4,000 forecasts made for each lag-selection method, horizon, and forecasting approach (IMS or DMS).

Trivariate Data

- For trivariate results we use 6 real, 5 nominal, and 5 financial series

	Real
1	real personal income
2	IP
3	employment
4	unemployment rate
5	avg. weekly mfg. hours
6	real personal consumption

	Nominal
1	avg. hourly mfg. earnings
2	PPI
3	oil price
4	CPI
5	PCEPI

	Financial
1	M1 money stock
2	fed funds rate
3	10-year treasury
4	trade-weighted US dollar index
5	S&P 500

- Most results from June 2017 vintage but some from real-time data
- The 150 trivariate systems provide 450 forecasts for each lag-selection method, horizon, and forecasting approach (IMS or DMS).

Empirical Results

Bivariate Results: Conditional on Full Path, Sample Start 1959:01, Out-of-Sample Period 1979:01+h to 2002:12

		(A) All variables			(B) Pairs excl. PWM			(C) PWM variables		
	Forecast Horizon:	3	12	24	3	12	24	3	12	24
AR(4)	Mean	1.00	1.04	1.11	1.00	1.05	1.12	0.98	0.92	0.96
	Median	1.00	1.02	1.07	1.01	1.04	1.09	0.98	0.91	0.94
	IMS better	3.2%	7.5%	11.2%	3.5%	8.6%	12.8%	0.8%	0.0%	1.0%
	DMS better	5.7%	8.2%	5.9%	5.2%	4.3%	2.8%	10.5%	27.3%	19.6%
AR(12)	Mean	1.02	1.08	1.18	1.02	1.08	1.18	1.01	1.03	1.09
	Median	1.01	1.06	1.13	1.01	1.06	1.13	1.01	1.03	1.06
	IMS better	5.3%	10.4%	15.3%	4.8%	11.5%	16.7%	6.4%	6.4%	4.5%
	DMS better	0.9%	1.3%	2.0%	0.8%	0.9%	1.3%	1.6%	3.3%	5.3%
BIC	Mean	0.95	0.96	1.04	0.98	0.99	1.09	0.84	0.72	0.76
	Median	0.99	0.98	1.04	0.99	0.99	1.06	0.83	0.70	0.73
	IMS better	3.7%	6.3%	9.6%	4.5%	7.0%	11.6%	0.5%	0.1%	0.8%
	DMS better	17.7%	16.1%	11.3%	12.6%	8.3%	4.4%	46.3%	53.5%	41.0%

Bivariate Results: Conditional on Full Path, Sample Start 1959:01, Out-of-Sample Period 2002:12+h to 2016:12

		(A) All variables			(B) Pairs excl. PWM			(C) PWM variables		
	Forecast Horizon:	3	12	24	3	12	24	3	12	24
AR(4)	Mean	1.00	1.00	1.06	1.01	1.00	1.04	0.98	0.88	0.88
	Median	1.00	0.99	1.00	1.00	1.00	1.01	0.98	0.85	0.80
	IMS better	6.3%	3.4%	5.7%	7.7%	3.7%	5.9%	0.4%	0.4%	1.6%
	DMS better	4.8%	7.6%	9.6%	5.4%	5.5%	7.3%	5.5%	18.9%	23.0%
AR(12)	Mean	1.01	1.01	1.07	1.01	1.03	1.07	0.99	0.90	0.90
	Median	1.01	1.01	1.03	1.01	1.03	1.05	0.99	0.90	0.84
	IMS better	4.6%	5.2%	8.3%	5.3%	5.5%	6.4%	2.8%	2.0%	4.6%
	DMS better	1.9%	3.0%	5.1%	1.9%	3.3%	5.3%	2.1%	2.4%	7.1%
BIC	Mean	0.96	0.94	0.99	0.97	0.95	1.01	0.88	0.70	0.68
	Median	0.99	0.97	0.98	0.99	0.97	0.98	0.88	0.62	0.54
	IMS better	4.2%	3.0%	5.3%	4.3%	2.2%	4.9%	0.6%	0.4%	1.6%
	DMS better	10.2%	9.0%	10.4%	11.5%	5.6%	7.0%	13.1%	25.4%	28.9%

Bivariate Results: Conditional on Full Path, Sample Start 1984:01, Out-of-Sample Period 2002:12+h to 2016:12

		(A) All variables			(B) Pairs excl. PWM			(C) PWM variables		
	Forecast Horizon:	3	12	24	3	12	24	3	12	24
AR(4)	Mean	1.00	0.98	1.01	1.00	1.00	1.04	0.98	0.82	0.78
	Median	1.00	0.97	0.95	1.00	0.99	0.99	0.98	0.80	0.73
	IMS better	2.5%	2.3%	4.3%	2.9%	2.5%	5.5%	0.1%	1.0%	1.3%
	DMS better	1.5%	3.8%	7.0%	1.8%	2.2%	4.0%	2.0%	12.3%	21.6%
AR(12)	Mean	1.01	1.00	1.03	1.02	1.03	1.06	0.99	0.83	0.82
	Median	1.01	1.01	0.99	1.02	1.03	1.03	0.99	0.85	0.77
	IMS better	4.0%	4.1%	5.5%	4.5%	4.8%	6.8%	0.8%	0.3%	1.5%
	DMS better	0.6%	1.6%	3.6%	1.0%	1.4%	3.1%	0.0%	3.4%	8.0%
BIC	Mean	0.97	0.92	0.94	0.98	0.98	1.01	0.89	0.59	0.55
	Median	0.99	0.95	0.93	0.99	0.97	0.97	0.85	0.53	0.43
	IMS better	2.3%	2.7%	3.5%	2.6%	3.0%	4.1%	0.8%	0.9%	1.9%
	DMS better	5.5%	8.4%	10.3%	5.6%	2.0%	3.4%	9.8%	34.9%	39.9%

Trivariate Results: Conditional on Full Path, Sample Start 1959:01, Out-of-Sample Period 1979:01+h to 2002:12

		(A) Real			(B) Nominal			(C) Financial		
Forecast Horizon:		3	12	24	3	12	24	3	12	24
AR(4)	Mean	1.03	1.12	1.24	1.00	0.97	1.01	1.02	1.06	1.08
	Median	1.02	1.09	1.19	1.00	0.95	0.98	1.01	1.06	1.04
	IMS better	13.0%	11.7%	19.3%	0.0%	0.0%	1.0%	5.7%	12.7%	19.3%
	DMS better	0.0%	0.0%	0.0%	1.7%	6.7%	3.7%	0.0%	4.7%	7.0%
AR(12)	Mean	1.04	1.22	1.48	1.01	1.05	1.12	1.01	1.08	1.15
	Median	1.03	1.22	1.42	1.01	1.06	1.06	1.01	1.08	1.08
	IMS better	9.7%	32.0%	29.0%	4.7%	4.7%	2.0%	11.7%	9.0%	20.0%
	DMS better	0.0%	1.3%	0.0%	0.0%	0.3%	1.3%	0.0%	0.0%	1.0%
BIC	Mean	1.00	1.07	1.15	0.76	0.63	0.64	0.98	0.96	0.99
	Median	1.00	1.06	1.09	0.77	0.62	0.64	1.01	1.02	1.00
	IMS better	4.7%	6.7%	11.7%	0.0%	0.0%	0.0%	5.0%	6.7%	17.7%
	DMS better	2.0%	1.7%	0.3%	77.7%	75.7%	55.0%	11.3%	18.3%	12.3%

Trivariate Results: Conditional on Full Path, Sample Start 1959:01, Out-of-Sample Period 2002:12+h to 2016:12

		(A) Real			(B) Nominal			(C) Financial		
Forecast Horizon:		3	12	24	3	12	24	3	12	24
AR(4)	Mean	1.00	1.03	1.09	1.00	0.85	0.86	1.02	1.08	1.18
	Median	1.01	1.03	1.09	0.99	0.85	0.85	1.01	1.02	1.05
	IMS better	5.7%	10.7%	10.3%	1.0%	0.0%	0.0%	13.3%	8.7%	9.7%
	DMS better	1.3%	0.3%	1.7%	0.0%	17.7%	17.7%	0.0%	1.3%	1.0%
AR(12)	Mean	1.03	1.10	1.10	1.00	0.91	0.88	1.03	1.08	1.23
	Median	1.02	1.09	1.10	1.00	0.88	0.85	1.01	1.04	1.09
	IMS better	3.3%	6.0%	11.3%	0.7%	0.3%	1.7%	2.0%	0.7%	5.3%
	DMS better	1.3%	0.0%	2.3%	2.0%	2.3%	6.0%	0.3%	0.0%	0.3%
BIC	Mean	0.95	1.05	1.10	0.82	0.55	0.52	0.99	0.99	1.09
	Median	0.97	1.03	1.09	0.83	0.54	0.51	1.00	0.96	1.05
	IMS better	2.0%	6.0%	10.7%	0.0%	0.0%	0.0%	7.7%	9.0%	8.3%
	DMS better	1.7%	0.3%	2.3%	11.3%	64.3%	64.7%	0.7%	4.0%	5.7%

Trivariate Results: Conditional on Full Path, Sample Start 1984:01, Out-of-Sample Period 2002:12+h to 2016:12

		(A) Real			(B) Nominal			(C) Financial		
Forecast Horizon:		3	12	24	3	12	24	3	12	24
AR(4)	Mean	1.00	0.94	0.97	1.00	0.76	0.69	1.01	1.03	1.03
	Median	1.00	0.93	0.93	1.00	0.78	0.70	1.00	0.96	1.00
	IMS better	6.3%	1.7%	5.0%	0.0%	0.0%	0.0%	2.0%	2.7%	5.7%
	DMS better	0.7%	0.0%	1.7%	0.0%	23.0%	31.0%	0.0%	0.3%	4.7%
AR(12)	Mean	1.03	1.06	1.01	1.00	0.74	0.61	1.02	1.08	1.12
	Median	1.03	1.04	0.98	1.00	0.70	0.56	1.02	1.06	1.07
	IMS better	3.3%	1.0%	5.3%	0.0%	0.0%	0.0%	5.7%	7.3%	4.3%
	DMS better	1.0%	0.0%	0.0%	0.0%	3.3%	8.7%	0.0%	0.0%	2.0%
BIC	Mean	0.96	0.99	1.02	0.84	0.43	0.33	0.99	0.91	0.95
	Median	0.97	0.99	1.00	0.85	0.45	0.33	1.00	0.92	0.94
	IMS better	0.3%	4.7%	2.3%	0.0%	0.0%	0.0%	3.7%	0.3%	1.0%
	DMS better	0.3%	0.3%	0.0%	21.0%	69.3%	74.0%	1.7%	5.0%	14.0%

Trivariate Results: Unconditional, Sample Start 1959:01, Out-of-Sample Period 1979:01+h to 2002:12

		(A) Real			(B) Nominal			(C) Financial		
Forecast Horizon:		3	12	24	3	12	24	3	12	24
AR(4)	Mean	1.03	1.03	1.11	1.00	0.92	0.90	1.02	1.05	1.08
	Median	1.02	1.02	1.10	1.00	0.92	0.90	1.01	1.04	1.06
	IMS better	13.3%	10.0%	3.3%	0.0%	0.0%	0.0%	6.7%	18.7%	22.0%
	DMS better	0.0%	0.0%	0.0%	2.0%	21.3%	30.0%	0.0%	10.0%	2.7%
AR(12)	Mean	1.04	1.21	1.34	1.01	1.04	0.98	1.01	1.08	1.15
	Median	1.03	1.19	1.28	1.01	1.05	1.00	1.01	1.10	1.10
	IMS better	8.0%	24.7%	10.7%	4.0%	0.0%	1.3%	8.7%	7.3%	14.7%
	DMS better	0.0%	0.0%	0.0%	0.0%	0.7%	6.0%	0.0%	0.0%	0.0%
BIC	Mean	1.01	1.03	1.07	0.77	0.61	0.57	0.98	0.96	0.97
	Median	1.01	1.03	1.05	0.77	0.60	0.61	1.00	1.02	1.02
	IMS better	6.0%	23.3%	0.7%	0.0%	0.0%	0.0%	3.3%	2.7%	12.7%
	DMS better	2.0%	2.7%	0.0%	76.7%	87.3%	78.7%	10.0%	20.0%	8.0%

Trivariate Results: Unconditional, Sample Start 1984:01, Out-of-Sample Period 2002:12+h to 2016:12

		(A) Real			(B) Nominal			(C) Financial		
Forecast Horizon:		3	12	24	3	12	24	3	12	24
AR(4)	Mean	1.00	0.95	0.99	1.00	0.77	0.68	1.01	0.98	0.94
	Median	1.01	0.93	0.97	1.00	0.76	0.69	1.01	0.99	0.93
	IMS better	2.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.7%	0.0%	3.3%
	DMS better	0.0%	0.0%	2.0%	0.0%	40.0%	44.7%	0.0%	6.7%	16.7%
AR(12)	Mean	1.03	1.05	0.94	1.00	0.69	0.52	1.02	0.92	0.85
	Median	1.03	1.03	0.95	1.00	0.65	0.45	1.02	0.90	0.82
	IMS better	3.3%	0.7%	4.0%	0.7%	0.0%	0.0%	6.0%	1.3%	0.0%
	DMS better	0.7%	0.0%	1.3%	0.0%	0.7%	13.3%	0.0%	0.7%	5.3%
BIC	Mean	0.97	1.02	1.05	0.87	0.46	0.37	0.99	0.89	0.86
	Median	0.97	1.04	1.03	0.88	0.46	0.37	0.99	0.94	0.86
	IMS better	0.7%	13.3%	4.7%	0.0%	0.0%	0.0%	1.3%	0.0%	0.7%
	DMS better	0.7%	1.3%	0.0%	17.3%	87.3%	90.7%	0.0%	23.3%	24.0%

Conclusion

- In the context of conditional forecasts, we compare the relative accuracy of point forecasts from VAR-based IMS models to that of ARDL-based DMS models
- Somewhat to our surprise, DMS methods do quite well relative to "optimal" IMS methods
  - Despite the fact the models are obviously flawed
  - DMS seems to have improved relative to IMS over the Great Moderation
  - Large gains to forecasting nominals using DMS methods
  - On average, it is basically a push between DMS and IMS for other variables