A Severity Function Approach to Scenario Selection

Frieder Mokinski (Deutsche Bundesbank)

Workshop on Forecasting, 8-9 September 2017, HV Frankfurt

1 Research Question

How to come up with scenarios for financial sector stress testing that are "severe yet plausible"?

- severe = scenario should be expected to have an adverse stress test impact, should it materialize
- plausible = non-negligible probability of actual materialization

2 Method

- Two inputs to the severity function approach (SFA)
  1. probabilistic forecasting model $\rightarrowtail$ used to assess the plausibility of alternative scenarios, and
  2. user-specified severity function $\rightarrowtail$ used to measure how well a scenario fits with the user's idea of a severe scenario
- SFA scenario solves maximization problem

$$\widehat{\mathbf{Y}}_{f^*} = \text{argmax}_{\widehat{\mathbf{Y}}}\, s(\widehat{\mathbf{Y}}) \text{ s.t. } f_Y(\widehat{\mathbf{Y}}) = f^*$$

or equivalently

$$\widehat{\mathbf{Y}}_{s^*} = \text{argmax}_{\widehat{\mathbf{Y}}}\, f_Y(\widehat{\mathbf{Y}}) \text{ s.t. } s(\mathbf{Y}_{T+h}) = s^*,$$

where

1. $f_Y(\mathbf{Y}_{T+h})$ is a multivariate & multi-horizon predictive density for the "risk factors" (i.e. scenario variables),
2. $\mathbf{Y}_{T+h} := \left[y'_{T+1} \ \dots \ y'_{T+h}\right]'$, and
3. $s(\mathbf{Y}_{T+h})$ is the severity function that maps each scenario candidate to a scalar measure of its severity

- Interpretation: SFA finds scenario with the highest severity among a set of equally plausible scenarios
  $\rightarrowtail$ operationalization of "severe yet plausible".
- Special case with analytical solution to max. problem: linear severity function $(s(\mathbf{Y}_{T+h}) = \mathbf{Y}'_{T+h}\beta)$ & multivariate normal predictive density $(\mathbf{Y}_{T+h} \sim \text{N}(\mu, \Sigma))$

$$\rightarrowtail \widehat{\mathbf{Y}}_{s^*} = \mu + \Phi^{-1}(\alpha)\ (\beta'\Sigma\beta)^{-1/2}\ \Sigma\ \beta,$$

where α is implicitly defined through $\Pr\left[s(\mathbf{Y}_{T+h}) < s^*\right] = \alpha$.

3 The SFA vs other approaches of selecting scenarios

Purely judgmental scenario generation:

- Anything is possible. Historical experience no constraint as in data-based approaches.
- Severe? Potentially, yes!
- Plausible? It depends.
- Consistency of scenarios over time is an issue $\rightarrowtail$ rescaling through SFA possible

Conditional forecasting

- What if ... monetary policy would keep the short-term interest rate at 0.0 for the next 8 quarters / an oil supply shock would reduce the global flow volume by 10 percent?
- Severe stress test impact? Unclear.
- Plausible? Depends on conditions.

4 Implementation of approach

1. Predictive density $f(Y_{T+h})$ can, for example, come from an empirical time series model or an estimated DSGE model
2. Severity function, i.e. mapping from scenarios to a metric of how well a scenario fits with the user's idea of a severe scenario, can be obtained by
   - Guesstimation, i.e. by guessing the parameters;
   - Empirical estimation of the functional relationship between the risk factors and a severity metric;
   - Simulation-based estimation of the functional relationship based on test runs of the stress test.

5 Application

- Stress test scenario for the German banking sector
- Predictive density from a medium-sized BVAR with an informative Litterman prior, assumed multivariate normal
- $s(\cdot) = \sum_{h=1}^{12} \frac{1}{h^2}(1.5\ \text{I3M}_{T+h} - 0.5\ \text{I10Y}_{T+h} - \text{RGDP}_{T+h} + \text{UNEMP}_{T+h})$ (in appropriately standardized variables)
  - favors inversion of yield curve & rise in level of interest rates
  - favors depressed real economic activity
  - $1/h^2$ factor favors early shocks