

# Derivatives and Interest Rate Risk Management by Commercial Banks

Guillaume Vuillemey\*

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## Abstract

How do derivatives markets affect corporate decisions of financial intermediaries? I introduce interest rate swaps into the capital structure model of a bank. First, derivatives are a substitute to financial flexibility for risk management. Second, I show the existence of three distinct motives to engage in interest rate risk management. Together, they imply that both increases and decreases in the short rate can be optimally hedged. Third, the use of derivatives induces a “procyclical but asymmetric” lending policy. Derivatives users are better able to exploit transitory lending opportunities in good times, but do not cut lending proportionally more during either monetary contractions or real recessions. Finally, despite attractive insurance properties of derivative contracts, not all banks take derivative positions, as in the data. The model’s predictions jointly match a number of yet unexplained stylized facts. New testable predictions are obtained.

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\*HEC Paris, Finance Department. Email: [vuillemey@hec.fr](mailto:vuillemey@hec.fr). Tel: +33 660 20 42 75. A large part of this paper was written while visiting Harvard University. I am grateful to Tim Adam, Nicolas Coeurdacier, Andrea Gamba, Denis Gromb, Samuel Hanson, Augustin Landier, Philippe Martin, Erwan Morellec, Guillaume Plantin, Adriano Rampini, Jean-Charles Rochet, David Thesmar, Boris Vallée, Vish Viswanathan, Pierre-Olivier Weill and Toni Whited for their comments. I also thank seminar participants at Sciences Po, the Bank of France, CREST, the London Financial Intermediation Theory Network, HKU, HKUST, HEC Paris, Imperial College, Stockholm School of Economics, the ECB, ESMT, HEC Lausanne, the Federal Reserve Board, the Federal Reserve Bank of New York, Indiana University, Cass Business School. An earlier version of this paper was titled “Derivatives and Risk Management by Commercial Banks”.

# 1 Introduction

The interest rate derivatives market is the largest market globally, with an aggregate notional exposure totalling 505 trillion USD as of December 2014 (BIS, 2015). Yet, despite the size and rapid growth of this contract class over the past 15 years, as depicted in figure 1, there is only a limited theoretical or empirical understanding of how hedging through interest rate derivatives affects other corporate decisions of firms. In the case of financial intermediaries, which represent more than 97% of gross exposures worldwide, how does hedging using interest rate derivatives affect bank risk management or the dynamics of lending?

I introduce interest rate derivatives in the capital structure of a bank faced with both stochastic lending opportunities and with a stochastic short rate. Long-term assets are financed with short-term debt and equity. Financing frictions imply that internal and external funds are not perfect substitutes. Thus, the bank optimally engages in risk management: it aims to secure internal funds for states in which their marginal value is high, i.e. when large lending is optimal and financing constraints may lead to under-investment. Risk can be managed either by preserving debt capacity (“financial flexibility”) or by using derivatives.

The analysis yields three main contributions. First, I show that the preservation of debt capacity and derivatives hedging are substitutes for risk management. Derivatives users keep less financial flexibility and less cash than non-users. They are more leveraged and issue equity less often. The increase in equity value, however, is fairly small for our calibrated parameter values, but in line with several empirical estimates.

Second, I show that banks may optimally take derivative exposures to hedge either increases or decreases in the short rate. Both pay-fixed and pay-float derivatives positions may be taken. This result follows from the existence of three distinct and opposite incentives to engage in interest rate risk management, which the model features. The fact that banks take pay-float positions (i.e. take exposure to interest rate spikes) has often been considered earlier as evidence of speculation. I show it is consistent with hedging. The model yields detailed predictions about bank characteristics associated with either position type.

As a third contribution, I show that the ability to hedge using interest rate derivatives affects the dynamics of bank lending (“investment”) in three related ways. First, derivatives users are better able than non-users to exploit transitory lending opportunities in periods of monetary easing. Second, there are also differences in lending between users and non-users of interest rate derivatives in response to real (“productivity”) shocks, even if these shocks are uncorrelated with the short rate. Interest rate derivatives users increase lending more than non-users in response to good real shocks. This is consistent with empirical evidence by Brewer et al. (2000). There is thus an increased procyclicality of bank lending by derivatives users. Third, these effects are asymmetric in response to positive or negative shocks. Differences in lending between users and non-users are more significant in good times, and they are either non-significant or smaller in magnitude in bad times. Thus, the use of derivatives induces a *procyclical but asymmetric* lending policy. Derivatives users are better able to exploit transitory lending opportunities in good times, but do not cut lending proportionally more during either monetary contractions or real recessions.

The model structure builds upon that by [Hennessy and Whited \(2005\)](#). Key to the model structure is the existence of financing frictions: (i) A collateral constraint limits the bank’s debt capacity, and (ii) issuing equity is costly. As in [Froot et al. \(1993\)](#) these frictions induce effective risk aversion even for risk-neutral firms. Risk management makes it possible to mitigate the under-investment problem associated with these frictions. Earlier papers using a similar model structure ([Hennessy and Whited, 2005](#); [Gamba and Triantis, 2008](#); [Riddick and Whited, 2009](#); [DeAngelo et al., 2011](#)) do not consider a stochastic interest rate and do not introduce derivatives. While interest rate risk management may be neglected for non-financial firms, it is a first-order concern for banks, due to their role in maturity transformation (e.g. [Freixas and Rochet, 2008](#)), which is here modelled explicitly.

The bank’s exposure to interest rate risk comes from three sources. Together, they give rise to three distinct motives for interest rate risk management, which could not be modelled earlier in settings with a constant interest rate. First, on the liabilities side, the short-term interest rate determines the cost of debt financing. The risk that the cost of debt financing is high in states in which profitable lending opportunities are numerous gives rise to a *financing motive* for risk management, whereby the bank optimally wants to transfer resources from future states where the short rate is low to states where it is high.

Second, on the assets side, decreases in the short rate positively affect the bank’s cash flows, i.e. optimal lending is higher when the short rate is lower. This creates an *investment motive* for risk management, as the bank optimally preserves funds to meet these lending opportunities. Third, the short rate also affects the bank’s discount factor. When the short rate decreases, equity holders are increasingly willing to forego present dividend distributions to exploit current lending opportunities, i.e. optimal lending is higher. Both the investment motive and this *discount motive* are such that the bank is optimally willing to transfer resources from future states where the short rate is high to states where it is low (i.e. the opposite incentive as that provided by the financing motive).

As a first instrument for risk management, derivatives—modelled as interest rate swaps—make it possible to swap, in future periods, a fixed rate against a floating rate (“pay-fixed” position), or the contrary (“pay-float” position). Derivatives transfer funds across future states associated with particular realizations of the short-term interest rate, thus with different marginal values of funds for the bank. Consequently, they are valuable for hedging, even though they have a zero expected payoff.

The second instrument for risk management, widely studied in the literature (e.g. [Gamba and Triantis, 2008](#)), is financial flexibility. The bank optimally does not take on short-term debt up to its borrowing limit. It may optimally forego present lending and preserve debt capacity for next-period lending. A key difference between the two instruments is that derivatives provide state-contingent payoffs (i.e. that depend on the realized short rate), while financial flexibility provides funds in a non-state-contingent way. When preserving debt capacity, the bank keeps internal funds that will be available in all states next period, regardless of whether the marginal value of funds is high or low. While derivatives make it possible to transfer funds across states in future periods, financial flexibility transfers funds across periods, regardless of states.

As an important result, the difference between the types of transfers made possible by each instrument gives rise to a substitution between derivatives and the preservation of debt capacity for risk management. The preservation of debt capacity is costly due to its non-state contingent nature: present lending opportunities have to be foregone for sure, while the benefits of increased debt capacity next period are valuable only if lending opportunities are indeed large at that date. By providing insurance in a state-contingent way, swaps can be more efficiently used to transfer funds towards future states where their marginal value is high, thus reducing the need to preserve debt capacity. Derivatives users are more leveraged than non-users, but still preserve some debt capacity. This is because future investment states are not fully known *ex ante*, due to the existence of a second (real) shock.

Regarding risk management, another outcome of the model is that both pay-fixed and pay-float positions can be taken by the bank, i.e. both increases and decreases in interest rates can be hedged. This arises from the existence of opposite incentives shaping the hedging policy. To support this prediction, I document a large heterogeneity between swap positions held in the cross-section of U.S. banks. While traditional expositions of interest rate risk for banks engaged in maturity mismatching (Freixas and Rochet, 2008; Fabozzi and Konishi, 1994) consider that banks should insure against increases in the short rate, I show that decreases in interest rates can also be hedged, absent any speculative motive.

Which type of position is optimally held? To describe the bank's hedging policy, I draw a distinction between the *cross-state* and the *cross-period* dimensions of hedging. In the cross-state dimension, hedging is motivated by the existence of financing constraints in future periods. Depending on which of the three motives (financing, investment and discount) prevails, derivatives can be used to address this motive. The cross-period dimension of hedging arises from the fact that both debt and swaps are collateralized. Taking swap positions may either increase or decrease the bank's present debt capacity. If present debt capacity is very valuable (because the lending opportunities are large or the bank is constrained), then its optimal hedging policy does not only depend on its expected *future* constraints, but also on its *present* marginal value of funds, thus on present lending opportunities. Which of the two dimension matters most is key to formulate predictions about banks that hedge or not, and about the type of swap positions being taken.

Intertemporal aspects of hedging also imply an endogenous sorting between users and non-users of derivatives. Despite the fact that derivatives are valuable by allowing banks to better achieve their optimal lending policy, not all banks use them. In the U.S., only 12% of commercial banks use interest rate derivatives for hedging. In the model, the cross-sectional and time series distribution of users and non-users is endogenous. This feature is valuable to obtain sharp testable predictions on the characteristics associated with either bank type. Sorting is driven by the use of collateral for both derivatives hedging and debt financing. When cash flows from assets-in-place do not depend on the interest rate, hedging reduces the bank's debt capacity. The opportunity cost of hedging is foregone financing. As is Rampini and Viswanathan (2010), banks which are financially more constrained, or more profitable at the margin (such as small banks), hedge less or abstain from hedging. In contrast, when cash flows do depend on the interest rate, derivatives may at times be used to increase a

bank's debt capacity.

The introduction of derivatives has important implications for the dynamics of bank lending. Derivatives users increase lending more than non-users during booms and do not cut lending proportionally more during recessions or monetary contractions. Their lending policy is “procyclical but asymmetric”. The increased procyclicality of bank lending comes from two sources. First, derivatives are used to transfer funds to future states in which large lending is optimal. Derivatives users have more net worth when it is most valuable. Second, hedging future states may be used by the bank to increase its present debt capacity. Doing so when it is severely constrained is valuable and makes it possible to mitigate underinvestment problems. The model's predictions are consistent with empirical evidence by [Brewer et al. \(2000\)](#) and hold both for periods of monetary easing or of real growth. The fact that users and non-users of interest rate derivatives respond differently to real shocks, even when they are uncorrelated with interest rate shocks, is also an important result of the paper.

The fact that differences in bank lending between users and non-users are smaller in magnitude and less significant during downturns follows from the partial substitutability between derivatives and debt capacity. During downturns, all banks downsize and aim to restore their lending capacity for future periods. They all do so by cutting lending and preserving debt capacity. Derivatives users, however, can also restore their future lending capacity by using swaps to transfer funds to future investment states. They do not cut lending as much as would otherwise be the case. This yields asymmetric effects between booms and busts.

By modelling banks' optimal management of interest rate risk, the paper speaks to a topical issue. Most of the recent literature on risk in banking has focused on solvency risk (e.g. [Adrian and Shin, 2014](#)) or liquidity risk (e.g. [Acharya and Naqvi, 2012](#)). In contrast, interest rate risk has been relatively neglected. In an environment where both short-term and long-term interest rates have been at historically low levels for close to six years ([Krishnamurthy and Vissing-Jorgensen, 2011](#)), a well-managed interest rate risk is arguably a first-order concern for financial intermediaries ([Bednar and Elamin, 2014](#)). [Begenau et al. \(2015\)](#) quantify banks' exposure to interest rate risk. [Landier et al. \(2015\)](#) show the impact of banks' exposure to interest rate risk for the transmission of monetary policy. An early contribution by [Flannery and James \(1984\)](#) investigates the effect of interest rate changes on banks' stock returns. Regarding the use of interest rate swaps in the corporate sector, [Jermann and Yue \(2013\)](#) study an equilibrium model of production and financing with corporate default. To my knowledge, this paper provides the first model aimed at understanding, in a comprehensive way, which financial intermediaries hedge interest rate risk, what type of position is taken, how derivatives interact with other instruments for risk management and affect the provision of bank loans.

Other dynamic models of bank capital structure have focused on different questions. [Sundaesan and Wang \(2014\)](#) study the optimal liability structure of a bank. [De Nicolo et al. \(2014\)](#) and [Hugonnier and Morellec \(2015\)](#) study capital and liquidity requirements. [Gornall and Strebulaev \(2013\)](#) explain the high leverage of banks. With respect to dynamic models of corporate leverage, this paper also has a different focus. [Hennessy and Whited \(2005\)](#),

DeAngelo et al. (2011) and Gamba and Triantis (2008) restrict attention to risk management using financial flexibility. Riddick and Whited (2009) also consider cash balances. The introduction of a stochastic short rate and of derivatives is novel in this class of models.

The remainder of the paper is structured as follows. The model is introduced in section 2. Section 3 solves for the optimal policy. Section 4 describes the mechanics of interest rate risk management. Sections 5 and 6 study how derivatives hedging affects the bank’s capital structure and the dynamics of bank lending.

## 2 The model

This section introduces the model.

### 2.1 Long-term assets and cash flows

Time is discrete and the horizon infinite. The bank’s managers take decisions upon (i) lending, (ii) financing and (iii) hedging to maximize the wealth of equity holders, determined by risk-neutral security pricing in the capital market. Any variable measurable with respect to date  $t + 1$  is denoted with a prime.

#### 2.1.1 Cash flows

At the beginning of any date  $t$ , the bank holds assets-in-place  $a$  and receives two shocks  $\{z, r\}$ , where  $z$  is a real (or “productivity”) shock to the bank’s asset portfolio and  $r$  the one-period risk-free rate. Operating cash flows before financing, lending and hedging decisions take place are denoted  $\pi(a, z, r)$ . The bank pays a proportional tax  $\tau \in [0, 1]$  upon receipt of  $\pi(\cdot)$ , i.e. after-tax cash flows are  $(1 - \tau)\pi(\cdot)$ .

**Assumption 1.**  $\pi(a, z, r)$  is continuous with  $\pi(0, z, r) = 0$ ,  $\lim_{a \rightarrow \infty} \pi_a(a, z, r) = 0$  and satisfies (A1.1)  $\pi_a(a, z, r) > 0$ , (A1.2)  $\pi_{aa}(a, z, r) < 0$ , (A1.3)  $\pi_z(a, z, r) > 0$  and (A1.4)  $\pi_r(a, z, r) \leq 0$ . It takes the functional form

$$\pi(a, z, r) = ze^{\gamma(r^* - r)}a^\theta, \quad (1)$$

where  $r^*$  is the unconditional expectation of  $r$  (see equation 4),  $\theta \in [0; 1]$  and  $\gamma \geq 0$ .

Absent the intermediate term,  $e^{\gamma(r^* - r)}$ , the cash flow function is the neoclassical production function, a standard choice in the investment literature (e.g. Hennessy and Whited, 2005). Together, (A1.1) and (A1.2) ensure that cash flows are increasing and concave in the asset size. Empirically, the concavity of cash flows captures the decreasing creditworthiness of the marginal borrower when lending increases (Dell’Ariccia et al., 2012). (A1.3) reflects the fact that a bank’s borrowers are better able repaying their loans when real economic conditions are better. Innovations to  $z$  can be thought of as changes in real conditions affecting the bank’s non-performing loans ratio.

Assumption (A1.4), reflected in the term  $e^{\gamma(r^*-r)}$ , captures in reduced form the fact that borrowers are better off repaying their loans, for a given  $z$ , when the short rate  $r$  is lower (Friedman and Schwartz, 1982; Fuerst, 1992). This is particularly true if borrowers hold adjustable-rate loans, such as adjustable-rate mortgages, which tend to default more when interest rates rise (Bajari et al., 2008; Campbell and Cocco, 2014).  $\gamma$  captures the sensitivity of the bank’s cash flows to the short rate. If  $\gamma = 0$ , as I assume in most of the paper, cash flows do not depend on  $r$ . Whenever  $\gamma > 0$ , an increase in  $r$  decreases the bank’s operating cash flows from assets-in-place. While not essential to most of the main results,  $\gamma > 0$  introduces an additional source of exposure to interest rate fluctuations and yields a richer dynamics, discussed below. The fact that the unconditional expectation of  $r^* - r$  is zero implies that the average impact of  $r$  on cash flows is zero.

### 2.1.2 Lending

Long-term lending  $a$  is modelled under the simplifying assumption that a constant share of loans,  $\delta \in [0, 1]$ , matures each period. The assumption that the stock of loans decays geometrically is similar to that in Bianchi and Bigio (2014). Investment  $i$  (i.e. incremental bank lending—both terms being used interchangeably) is defined as

$$i \equiv a' - (1 - \delta)a. \quad (2)$$

The average loan maturity is  $1/\delta$ , and  $\delta < 1$  ensures that the average maturity of the assets exceeds that of the one-period debt, i.e. that the bank engages in maturity mismatching. The price of one long-term loan unit is normalized to one.

## 2.2 Shocks

The two shocks  $\{z, r\}$  are modelled following Assumption 2.

**Assumption 2.** *The shocks  $z$  and  $r$  take values in compact sets  $\mathcal{Z} \equiv [\underline{z}; \bar{z}]$  and  $\mathcal{R} \equiv [\underline{r}; \bar{r}]$  respectively. They jointly follow a first-order Markov process satisfying the Feller property. Both are AR(1) processes given by*

$$\ln(z') = \rho_z \ln(z) + \epsilon'_z \quad (3)$$

$$r' = r^* + \rho_r r + \epsilon'_r, \quad (4)$$

where  $r^*$  is the unconditional mean of  $r$ .

Restricting attention to compact sets of shocks ensures that debt can be fully collateralized, by defining a lower boundary on the future bank value. Innovations  $\epsilon_z$  and  $\epsilon_r$  are jointly normal and possibly correlated, i.e.

$$\begin{pmatrix} \epsilon_z \\ \epsilon_r \end{pmatrix} \sim \mathcal{N} \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_z^2 & \rho\sigma_z\sigma_r \\ \rho\sigma_z\sigma_r & \sigma_r^2 \end{bmatrix} \right). \quad (5)$$



The conditional distribution at date  $t$  of future shocks  $\{z', r'\}$  is denoted  $g(z', r'|z, r)$  and is common knowledge. While  $r$  is an aggregate shock,  $z$  can be thought of as having a systematic and an idiosyncratic component. For explanatory purposes, it is treated as an aggregate shock until section 4.4, where cross-sectional heterogeneity is modelled.

## 2.3 Debt financing

The bank has five sources of funds: internally generated cash flows  $\pi(\cdot)$ , internal savings in the form of cash, one-period debt, derivatives and external equity. There is no long-term debt, i.e. a maturity mismatch between assets and liabilities is assumed. The optimality of maturity mismatching for banking firms has been demonstrated on theoretical grounds (e.g. [Diamond and Dybvig, 1983](#); [Calomiris and Kahn, 1991](#)) and is here assumed (as is also the case in [Jermann and Yue, 2013](#)).

Net debt, i.e. debt minus cash, is denoted  $b$ . It is debt if  $b > 0$  and cash otherwise (i.e. gross cash is  $c = -\min\{0, b\}$ ). Cash earns the risk-free rate  $r$ . Debt comes in the form of discount bonds, i.e., upon choosing  $b'$  at  $t$ , the bank obtains funds  $b'/(1+r)$  and repays  $b'$  at  $t+1$ .

Debt benefits from a tax advantage. Interests paid on debt are not taxed, while interest earned on cash is taxed. The tax advantage, obtained at date  $t$  on interests paid between dates  $t-1$  and  $t$  equals  $\tau br_{-1}/(1+r_{-1})$ , where  $r_{-1}$  is the realized short rate at  $t-1$ . This tax structure implies that holding both positive debt and positive cash at the same time cannot create value for the bank, consistent with the restricted focus on net debt.

## 2.4 Interest rate derivative contracts

Interest rate derivatives are modelled as interest rate swaps, i.e. the most widely used contract in the data. A one-unit swap contract traded at date  $t$  mandates the payment at date  $t+1$  of a fixed swap rate (known at  $t$ ) in exchange for the variable rate  $r'$  (realized at  $t+1$ ). This contract resembles real-world interest rate swap (e.g. [Titman, 1992](#)).

Interest rate swap are provided by risk-neutral dealers, willing to take any derivative position with a zero net present value.<sup>2</sup> They are priced so as to prevent arbitrage opportunities for risk-neutral agents, i.e. the present expected value of the fixed leg and of the floating leg of the contract are equal. The swap rate equals the short rate at  $t$  plus a (positive or negative) premium  $p$  solving

$$r + p = \mathbb{E}_t [r'|r]. \quad (6)$$

The notional amount of swap contracts traded at  $t$  is  $d'$ . Whenever  $d' > 0$ , the bank has a

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<sup>2</sup>The presence of risk-neutral dealers is assumed. The intermediation of long and short swap contracts is not endogenized. The model is thus fit for representing non-dealer commercial banks, but not dealers managing an inventory of derivatives. The distinction between dealers and end-users in derivatives markets is neat, implying that the assumption is granted empirically. In the CDS market, where the network structure is best documented, there are 14 dealers concentrating most intermediary activities between hundreds of end-users ([Peltonen et al., 2014](#)). Similar suggestive evidence is put forth by [Fleming et al. \(2012\)](#) for the interest rate derivatives market.



pay-fixed position, i.e. commits at  $t$  to deliver  $d'(r+p)$  at  $t+1$ , and to receive  $d'r'$ . It has a pay-float position if  $d' < 0$ . When taking a pay-fixed (resp. pay-float) position, the bank is insuring against increases (resp. decreases) in  $r$ : it is a net receiver of funds on its swap exposure at  $t+1$  when  $r$  is high (resp. low), and a net payer if it is low (resp. high).

In many pricing models, derivatives are redundant securities, i.e. their payoffs can be replicated by a combination of other model securities, such as bonds (Oehmke and Zawadowski, 2015). Here, derivatives are not redundant because the structure of the contracts is taken as given. For banks, the non-redundancy of interest rate derivatives can be thought of as resulting from the (partial) illiquidity of long-term loans (e.g. Diamond and Dybvig, 1983). If long-term assets are illiquid, it may be less costly for a bank to manage its interest rate risk using derivatives rather than by constantly reshuffling its underlying bond or loan portfolio. Differences in bond and derivatives liquidity also motivate the non-redundancy of CDS in the model by Oehmke and Zawadowski (2015).

## 2.5 Collateral constraint

Attention is restricted to risk-free debt and risk-free derivative contracts. A justification for riskless debt contracts follows from Stiglitz and Weiss (1981), who show that lenders may prefer credit rationing over higher required interest rates when faced with adverse selection or asset substitution problems. The use of short-term collateralized debt by banks is widespread, e.g. in the form of repurchase agreements (Acharya and Öncü, 2010). On derivatives markets, the collateralization of exposures to mitigate counterparty risk is also a widespread market practice (e.g. Duffie et al., 2015).

For both debt and swap contracts to be risk-free, lenders require the bank to be able to repay all contracts outstanding in all future states. Full enforcement is assumed, i.e. the bank owners cannot abscond with part of the existing cash flows or asset stock. All contracts being one-period contracts, the ability of the bank to access both the debt and the swap market is limited at  $t$  by the lowest possible value of the bank's cash flows at  $t+1$ . Given a set of controls at  $t$ , the short rate associated with the lowest realization of bank's cash flows at  $t+1$  is denoted  $\hat{r}$ , where

$$\hat{r} = \arg \min_{r' \in [\underline{r}, \bar{r}]} \pi(a', \underline{z}, r') - d'((r+p(r)) - r'). \quad (7)$$

By monotonicity of the two terms in  $r'$ ,  $\hat{r}$  is well-defined. The collateral constraint writes as

$$b' + d'((r+p(r)) - \hat{r}) \leq (1-\tau)\pi(a', \underline{z}, \hat{r}) + \kappa a'. \quad (8)$$

The first term on the right-hand side is the lowest possible after-tax cash flow at  $t+1$ , upon choosing  $a'$  at  $t$ . The second term is the bank's liquidation value, where  $\kappa \in [0, 1]$  is the liquidation value of one unit of the long-term asset.  $\kappa < 1$  follows from Asquith et al. (1994), who show that asset liquidation is a common response to financial distress.

The collateral constraint puts an upper bound on the amount of debt and swaps. When the bank has a pay-float position ( $d' < 0$ ), future states in which cash flows from assets in

place are low and states where it is a net swap payer are the same. Thus,  $\hat{r} = \bar{r}$ . The bank is set to receive the swap rate  $r + p(r)$  at  $t + 1$  and must hold enough funds to pay both its debt and the worst-case floating rate on swaps,  $r' = \bar{r}$ , if realized. When the bank holds a pay-fixed position ( $d' > 0$ ),  $\hat{r} \leq \bar{r}$ . On its swap position, the bank is worse off if the short rate is low. In such states, however, cash flows are greater by Assumption (A1.4). Which effect dominates depends on the size of  $d'$  and on  $\partial\pi(\cdot)/\partial r$ .

## 2.6 Equity financing

The last source of funds is external equity. It is denoted  $e(\cdot) \equiv e(a, a', b, b', d, z, r, r_{-1})$  and determined jointly with lending, debt financing and swap hedging through the flow identity

$$e(\cdot) = (1 - \tau)\pi(a, z, r) - (a' - (1 - \delta)a) + \frac{\tau b r_{-1}}{1 + r_{-1}} + \frac{b'}{1 + r} - b + d(r - (r_{-1} + p_{-1})), \quad (9)$$

where  $p_{-1}$  is the swap premium at  $t - 1$  (solved for using equation 6), and where we note that swap contracts traded at  $t$ , i.e.  $d'$ , do not enter contemporaneous payoffs (i.e. they are not used directly for inter-temporal transfers of funds).

Equation (9) states that the surplus or shortage of funds after financing, lending and hedging decisions have been made is either distributed as dividend or obtained through equity issuance. When  $e(\cdot) > 0$ , the bank is distributing a dividend. It issues equity if  $e(\cdot) < 0$ .

Together with the collateral constraint, the second main friction is a cost of issuing external equity. It may arise from flotation and tax costs (Smith, 1977), from informational asymmetries (Myers and Majluf, 1984) or agency problems (Myers, 1977). It implies that internal funds and external funds are not perfect substitutes, which creates an incentive to engage in risk management, as discussed below.

**Assumption 3.** *The cost of issuing external equity, denoted  $\eta(e(\cdot))$  satisfies  $\eta(e(\cdot)) > 0$  if  $e < 0$  and  $\eta(e(\cdot)) = 0$  otherwise. It is increasing and convex in the equity amount being issued. It takes the linear-quadratic form*

$$\eta(e(\cdot)) = \mathbb{1}_{\{e(\cdot) < 0\}} \left( -\eta_1 e + \eta_2 e^2 \right), \quad (10)$$

with  $\eta_1 \geq 0$  and  $\eta_2 \geq 0$ .

Net equity distribution equals  $e(\cdot) - \eta(e(\cdot))$ . The tax structure, with taxes being paid on internal savings, ensures that strictly positive dividends are distributed in at least some periods. With  $\tau = 0$ , the bank would be better off preserving any additional funds (once optimal lending is financed) in the form of internal cash, rather than distributing it. This is because equity may need to be re-issued at a cost at a later date. Strictly positive dividend distributions could alternatively result from an agency cost of holding cash, as in DeAngelo et al. (2011).

## 2.7 Value function

There are three control variables. The bank chooses lending  $a'$ , financing  $b'$  and hedging  $d'$  each period to maximize the expected value of future distributions to equity holders. Future dividends are discounted by a factor  $1/(1+r)$  capturing the opportunity cost of funds.

The bank's problem can be written with one state variable, by defining *net worth*  $w$  as

$$w(a, b, d) = (1 - \tau) \pi(a, z, r) + (1 - \delta)a + \frac{\tau b r_{-1}}{1 + r_{-1}} - b + d(r - (r_{-1} + p_{-1})). \quad (11)$$

Net worth corresponds to the bank's resources at hand after assets and swaps in place have paid off, after maturing debt has been repaid, but before decisions on  $a'$ ,  $b'$  or  $d'$  have been made.

The Bellman equation writes as

$$V(w, z, r) = \sup_{a', b', d'} \left\{ e(\cdot) - \eta(e(\cdot)) + \frac{1}{1+r} \int \int g(z', r' | z, r) V(w', z', r') dz' dr' \right\}, \quad (12)$$

subject to the collateral constraint (8). The first two terms are the equity distribution/infusion, net of issuance costs. The last term is the discounted continuation value.

The model yields a unique policy function  $\{a', b', d'\} = \Gamma(w, z, r)$ . The policy function gives the optimal response to the trade-off between the cost of increased lending and expectations about future productivity and short rate, while optimally balancing current and future financing needs when equity issuance is costly. The policy function also balances the costs and benefits of hedging.

## 2.8 Solution and simulations

I solve numerically for the policy function  $\Gamma$  using value function iteration. Calibrated parameter values are discussed in Appendix A. Details on the numerical solution are provided in Appendix B.

Simulations of the model are also used in the remaining part of the paper. Using the baseline calibration of Table 1, a series of random shocks  $\{z, r\}$  satisfying equations (3) and (4) is simulated for 10,200 periods. The bank's optimal controls are obtained. The first 200 periods are dropped. To assess the role of derivatives, I also simulate the model in the absence of interest rate swap contracts, i.e. by restricting  $d' = 0$  in all periods. Moments of the bank capital structure, with and without swaps, computed from simulated data using the baseline calibration are summarized in Table 2. Moments computed for alternative values of the calibrated parameters are in tables 3 and 4.

## 3 Capital structure policy

This section discusses the capital structure policy of the bank. The optimality conditions of the model are derived, under the assumption that the value function is once differentiable.

This assumption is not needed for the existence of a solution or for that of an optimal policy function (Stokey and Lucas, 1989).

### 3.1 Lending policy

Differentiating equation (12) with respect to  $a'$  and using the envelope condition yields the investment Euler equation.

$$1 - \eta_e(e(\cdot)) = \frac{1}{1+r} \int \int g(z', r' | z, r) (1 - \eta_e(e'(\cdot))) \left[ (1 - \tau) \pi_a(a', z', r') + 1 - \delta \right] dz' dr' \quad (13)$$

When making an optimal choice, the bank is indifferent at the margin between increasing long-term lending by one unit at date  $t$  and waiting to lend it at date  $t+1$ . The marginal cost of present lending is the price of the long-term asset. It is larger if equity has to be issued ( $e < 0$ ). The shadow value of a marginal loan unit at  $t$  (right-hand side of equation (13)) equals the same marginal cost—discounted by both  $1/(1+r)$  and  $(1-\delta)$ —plus the foregone marginal product of assets in place captured by  $(1-\tau)\pi_a(\cdot)$ . Foregoing these future funds is more costly on expectation if equity is more likely to be issued at  $t+1$ , i.e.  $e' < 0$ .

### 3.2 Financing policy

Let  $\lambda$  be the Kuhn-Tucker multiplier on the collateral constraint (8). The first-order condition for the optimal debt policy is given by

$$\left(1 + \frac{\tau r}{1+r}\right) (1 - \eta_e(e(\cdot))) = - \int \int g(z', r' | z, r) [V_b(a', b', z', r') + \lambda'] dz' dr' \quad (14)$$

Rewriting using the envelope theorem,

$$\left(1 + \frac{\tau r}{1+r}\right) (1 - \eta_e(e(\cdot))) + \lambda = \int \int g(z', r' | z, r) [-\eta_e(e'(\cdot)) + \lambda'] dz' dr' \quad (15)$$

The optimal debt policy is such that the marginal benefit of a unit of debt (left-hand side of equation (15)) equals its marginal cost (right-hand side). Debt is valuable (*i*) because of the tax benefit it gives and (*ii*) because equity issuance is costly. Debt is more valuable whenever  $e(\cdot) < 0$ , because an additional unit of debt, in such cases, enables saving the marginal cost of equity financing, for a given level of investment.

The cost of an extra unit of debt is the interest rate to be paid next period. It also depends on present expectations over  $e'(\cdot)$  and  $\lambda'$ . The first term in the expectation implies that one additional unit of debt today is more costly if the bank is more likely to issue costly equity next period. The second term implies that debt is more costly at  $t$  if the collateral constraint is more likely to bind at  $t+1$ .

Together, these extra costs highlight the rationale for risk management in the model. The existence of frictions by which (*i*) debt is capped by a collateral constraint and (*ii*) external equity financing is costly makes the bank effectively risk-averse with respect to next-period cash flows (as in Froot et al., 1993). Increasing debt at  $t$  implies that interest payments will

absorb a larger share of the bank's cash flows at  $t + 1$ . Free cash flows available for lending will be less abundant. If investment opportunities are large at  $t + 1$ , the bank becomes more likely to resort to equity financing and to pay the associated cost, thus to under-invest. Extra debt capacity would be particularly valuable in such states. Consequently, the debt policy implies that there are benefits from preserving free debt capacity, or *financial flexibility* for next-period investment: The bank may optimally choose to reduce debt and forego present lending opportunities, so as to be better able to exploit lending opportunities at  $t + 1$ .

### 3.3 Hedging policy

Together with financial flexibility, interest rate swaps are the other margin for risk management. Swap contracts are valuable because they make it possible, in future periods, to transfer funds from states in which the marginal value of funds is low to states in which it is high. Conditional on date- $t$  controls  $\{a', b'\}$ , they can be used to transfer funds at  $t + 1$  from states  $\{z', r'\}$  where no external equity will be optimally issued to finance investment to states where external equity financing would, absent swap hedging, be optimally needed (or needed to a larger extent), i.e. from “low lending” to “high lending” states.

Deriving the first-order condition with respect to  $d'$ , the optimal hedging policy satisfies

$$\begin{aligned} & \frac{1}{1+r} \int \int g(z', r' | z, r) [r' - (r + p(r))] (1 - \eta_e(e'(.))) dz' dr' \\ & = \lambda \frac{\partial}{\partial d'} [(1 - \tau) \pi(a', z, \hat{r}(d')) - d' ((r + p(r)) - \hat{r}(d'))], \end{aligned} \quad (16)$$

where  $\hat{r}$  is denoted  $\hat{r}(d')$  for clarity. Equation (16) equalizes the expected marginal costs and benefits of an additional swap unit. As can be seen on the left-hand side, swap contracts derive value because equity issuance is costly. In case  $\eta(e(.)) = 0$  when  $e < 0$ , there would be no expected benefit from swap hedging, as the expression would simplify to zero, using the pricing equation (6). A necessary condition for hedging to create value is that internal and external funds are not perfectly substitutable.

While swaps pay off at  $t + 1$  only, they have a present cost or benefit. This is because they have to be collateralized. Taking swap positions may either tighten or relax the present collateral constraint (to which the Lagrange multiplier  $\lambda$  is associated), depending on the sign of the right-hand side term. Which case prevails as a function of the model parameters is further discussed in section 4.2.2.

### 3.4 Policy function

To provide greater insights into the dynamics of the model, Figure 2 plot the optimal lending, debt and equity distribution as a function of the shocks  $z$  (Panel A) and  $r$  (Panel B). The policy function is evaluated at the steady state asset stock.<sup>3</sup>

<sup>3</sup>Following [Strebulaev and Whited \(2012\)](#), the steady state asset stock is defined as the value of  $a$  to which the bank would converge if it were to receive no shocks, i.e.  $\epsilon_z = 0$  and  $\epsilon_r = 0$ , for a

### 3.4.1 Dynamics with respect to $z$

The policy function with respect to  $z$ , in Panel A of Figure 2, illustrates two of the cyclical features of financing and lending in the model. First, optimal lending rises with the productivity shock, as lending opportunities become more profitable. Increased lending is funded from two sources. As a first source of financing, dividend distributions to equity holders are cut, and a larger share of internal funds is used for investment. The bank foregoes present dividend distributions, which are traded off against future dividends expected to be paid out of current investment's proceeds. Second, the bank optimally takes on more short-term debt. The debt-to-assets ratio—which can be interpreted either as leverage or as maturity mismatching—increases with  $z$ . Equity is issued in a few states where large lending is optimal (high  $z$ ) and both internal funds and free debt capacity are insufficient to meet funding needs.

The second element of the model dynamics that figure 2 illustrates is the preservation of free debt capacity. The collateral constraint does not bind in all states. The bank optimally foregoes current lending opportunities so as to be able to exploit future lending opportunities. Doing so is more costly when current investment is highly profitable ( $z$  high). The bank exhausts its debt capacity in such states. It instead preserves more debt capacity when  $z$  is low, i.e. free debt capacity is counter-cyclical and under-investment may be larger in downturns. Finally, due to the concavity of  $\pi(\cdot)$  in  $a$ , preserving debt capacity is also more costly for a small bank.

### 3.4.2 Dynamics with respect to $r$

Turning to the model dynamics with respect to  $r$  (Figure 2, Panel B), optimal lending is increasing when  $r$  decreases. This effect is driven by three forces. First, cash flows  $\pi(\cdot)$  increase as the short rate decreases, provided  $\gamma > 0$ . Lending becomes more profitable. Second, for a given profitability of assets, more investment can be sustained if debt financing is cheaper ( $r$  lower), because fewer debt capacity needs to be preserved. A lower short rate implies that more present debt can be obtained for a given asset stock at  $t + 1$ . A third effect is through the discount factor. As  $r$  decreases, equity holders value future dividend distributions relatively more. The bank trades off present versus future dividend distributions and optimally invests more at date  $t$ , out of current internal resources. Dividend distributions are consequently lower.

There are two main effects of shocks to the short rate on the bank's capital structure. First, a decrease in the short rate increases long-term lending, thus the bank's size. Second, the bank loads on short-term debt and increases maturity mismatching/leverage. Equity is issued only when debt capacity is exhausted and internal funds are not sufficient to meet sustained period of time. The steady state asset stock  $a^*$  equals

$$a^* = \left[ \frac{r^* + \delta}{(1 - \tau)\theta} \right]^{\frac{1}{\sigma-1}}. \quad (17)$$

optimal investment expenses. For high realizations of the short rate, the bank may find it optimal to hold cash ( $b' < 0$ ). It thus keeps resources for future investment.

## 4 The mechanics of risk management

This section discusses the mechanics of risk management and the results related to it. There are two dimensions of hedging: a *cross-state* and a *cross-period* dimension, which I discuss separately. In the cross-state dimension, the main question is which future states will be associated with the highest marginal value of funds. Derivatives can be used to transfer funds to such states. The cross-period dimension of hedging arises from the fact that both debt and swaps are collateralized. Because taking derivatives at  $t$  may increase or decrease a bank's present debt capacity, its optimal hedging policy does not only depend on its expected value of funds next period, but also on its present marginal value of funds, thus on present lending opportunities and financial constraints. Together, these two dimensions make it possible to obtain predictions about (i) whether banks use derivatives or not and, conditional on hedging, on (ii) the type of position being taken.

### 4.1 Hedging in the cross-state dimension: Motives

I first turn to the cross-state dimension of hedging. From the perspective of date- $t$  decision-making, the question is which states at  $t + 1$  will be associated with the highest marginal value of funds. There are three sources of exposure to the short rate: (i) through the cost of debt financing, (ii) through the discount factor, and (iii) through the sensitivity of cash flows  $\pi(\cdot)$  to  $r$ . Each source of exposure gives rise to a distinct motive for interest rate risk management.

#### 4.1.1 Financing motive

On the liability side, a stochastic short rate affects the cost of debt financing. There is a risk that the future cost of debt financing will be high in states where profitable lending opportunities are large. This may cause future lending opportunities to be foregone. When  $r$  rises, the bank loses debt capacity, because the amount it can borrow is capped by the collateral constraint. This effect is possibly large. To see this, denote  $\bar{c}(a', d')$  the maximum amount of debt  $b'$  that the bank can commit to repay in the worst state at  $t + 1$ , given constraint (8) and controls  $\{a', d'\}$ . A marginal increase in  $r$  leads to a decrease in debt capacity by a quadratic factor

$$\frac{\partial \left( \frac{\bar{c}(a', d')}{1+r} \right)}{\partial r} = -\frac{\bar{c}(a', d')}{(1+r)^2}. \quad (18)$$

This effect gives rise to a *financing motive* for risk management. The financing motive implies that the bank optimally wants to transfer funds from future states where  $r$  is low to states where  $r$  is high, because funding costs are higher in such states. When driven by



the financing motive, interest rate risk management in the swap market requires the bank to take pay-fixed positions ( $d' > 0$ ), which pay off when the short rate is high.

#### 4.1.2 Discount motive

A second channel through which the bank is exposed to the short rate is through the discount factor,  $1/(1+r)$  in equation (12). As  $r$  decreases, foregoing present dividend distributions is less costly for equity holders, implying that the optimal investment at date  $t$  is greater. As the discount factor changes, the bank trades off present versus future dividend distributions, and consequently present versus future lending. Because the optimal lending is larger when  $1/(1+r)$  is larger (i.e.  $r$  is lower), this *discount motive* for risk management provides incentives to transfer funds from future states where  $r$  is high to states where it is low—as opposed to the incentive provided by the financing motive.

#### 4.1.3 Investment motive

The third motive for risk management arises from the asset side. It exists only if  $\gamma > 0$ , i.e. if a decrease in the short rate positively affects the bank’s cash flows from assets-in-place. When this is the case, optimal lending is greater when  $r$  is lower—for a given discount factor. This gives rise to an *investment motive* for risk management: for a given cost of debt financing, the bank optimally wants more funds in states where  $r$  is lower, to meet higher profitable investment opportunities in such states. In terms of the transfers to be realized, the *investment motive* for risk management reinforces the discount motive and opposes the financing motive. When driven by the investment or the discount motive, risk management in the swap market requires the bank to take pay-float positions ( $d' < 0$ ), which pay off when the short rate is low.

#### 4.1.4 The “natural hedge” case and the role of $z$

Apart from  $r$ , the second stochastic factor in the model is  $z$ . Absent a stochastic short rate, risk management would be purely driven by the need to optimally exploit investment opportunities arising from changes in  $z$ , as in [Hennessy and Whited \(2005\)](#). A natural question is how interest rate risk management operates in the absence of  $z$ , and what role the productivity shock plays in the model.

If  $z$  were to be a constant, the bank would benefit from a *natural hedge*. Both its debt capacity and its lending opportunities are decreasing in  $r$ . Thus, the bank would have a high debt capacity in states in which high lending is optimal. This does not imply that there is no longer a rationale to engage in interest rate risk management—because the elasticity of debt capacity and investment opportunities with respect to the short rate need not be the same—, but the need to do so is reduced. In particular, the relative magnitude of the financing motive is reduced.

The introduction of  $z$  implies that the bank loses part of this natural hedge. A stochastic productivity level implies that lending opportunities may be large at times the short rate is high. The financing motive for risk management is thus relatively more important, as the

bank becomes more likely to face an under-investment problem when  $r$  is high. The extent to which the natural hedge is lost also depends on the correlation between shocks to  $z$  and  $r$ , as well as on the persistence and volatility of  $z$ . The impact of these parameters is further discussed below.

## 4.2 Hedging in the cross-period dimension

While derivatives hedging is most often thought of as providing insurance against future states (i.e. its cross-state dimension), it also has an important cross-period dimension. Intertemporal trade-offs arise because both debt and swaps are collateralized. The optimality conditions (15) and (16) make it clear that financing and hedging policies are dynamically related, as  $\lambda$  appears in both. Using interest rate swaps at  $t$  to hedge states at  $t + 1$  may either increase or decrease the bank's present debt capacity. Whether it finds it optimal to hedge, and the type of position it takes, does not only depend on the bank's expected value of funds at  $t + 1$ , but also on its *present* value of funds, i.e. on its present net worth and investment opportunities. In the cross-period dimension, present financing constraints may affect both whether a bank hedges future states or not, and the type of swap position it takes to do so.

### 4.2.1 A particular case: $\gamma = 0$

The case in which cash flows from assets-in-place do not depend on  $r$ , i.e. when  $\gamma = 0$ , is an interesting particular case. First, as already outlined, the investment motive is absent. In the cross-state dimension, which swap position is taken (if any) depends solely on the relative magnitude of the financing and of the discount motives.

In the cross-period dimension,  $\gamma = 0$  implies that derivatives hedging *always reduces* the bank's debt capacity. This is because future pledgeable cash flows (the right-hand side term in the collateral constraint 8) do not depend on  $r$ , so that there cannot be offsets between future cash flows from assets-in-place and from swaps. Whenever the bank takes a swap position, collateral is required to do so, and can no longer be pledged to obtain debt financing. The bank's debt capacity is reduced accordingly. Whether the bank takes a pay-fixed or a pay-float position does not make a difference in this respect. In the cross-period dimension, collateral constraints affect the *extent* to which banks hedge (depending on how they value present debt capacity) but not the *sign* of the positions being taken, conditional on hedging. The case with  $\gamma = 0$  is thus well-suited to study how the relative strength of each motive for risk management (i.e. the cross-state dimension) depends on other model parameters, as discussed below.

### 4.2.2 Swap hedging and debt capacity

In the more general case where  $\gamma \geq 0$ , in contrast, taking swap positions may either decrease *or increase* a bank's present debt capacity. In such cases, the choice of pay-fixed or pay-float swap positions is driven not only by the relative strength of each motive in anticipation of

date  $t + 1$ , but also by concerns related to the bank's present debt capacity. Both the extent of hedging and the position signs are affected by collateral constraints. The interaction between the cross-state and the cross-period dimensions of hedging is thus richer.

The impact of swap hedging on debt capacity can be seen by differentiating, for a given choice  $a'$ , the collateral constraint in the absence of derivatives, i.e.

$$b' \leq (1 - \tau) \pi(a', z, \bar{r}) + \kappa a', \quad (19)$$

with that with swaps (equation 8). Denoting  $A(\cdot)$  this difference, one gets

$$A(a', d', \hat{r}) = (1 - \tau) \pi(a', z, \bar{r}) - (1 - \tau) \pi(a', z, \hat{r}) + d' (r + p - \hat{r}). \quad (20)$$

The bank's debt capacity with swap hedging is larger than that without swap hedging whenever  $A(\cdot) < 0$ . For the bank's debt capacity to increase when hedging, it has to be the case that swap positions provide additional funds in the worst state of the world at  $t + 1$ , against which debt is collateralized.

When the bank holds pay-float swaps ( $d' < 0$ ),  $\hat{r} = \bar{r} > r + p$ . Therefore,  $A(\cdot) > 0$  and the bank's debt capacity is reduced. Intuitively, cash flows  $\pi(\cdot)$  are low in states where the bank is a net swap payer. Thus, when increasing swap hedging, the cash flows that can be pledged against such states to debt holders are lower. Hedging using pay-float swaps (i.e. motivated by investment and discount motives) creates a trade-off between hedging and financing. The opportunity cost of collateral pledged on swaps is foregone present debt capacity.

When the bank takes a pay-fixed swap position ( $d' > 0$ ),  $A(\cdot)$  can be either positive or negative. There are two opposite forces. First, cash flows  $\pi(\cdot)$  reach their lower level when  $\bar{r}$  is realized. Second, the bank receives swap payments in such states. Whether cash flows after swap payments are higher or lower when  $\bar{r}$  is realized depends on the size of the swap position  $d'$  and on the sensitivity of cash flows to the short rate,  $\gamma$ . The relative importance of these two forces is illustrated graphically in figure 3, which plots  $\hat{r}$  for various choices  $d'$  at the steady state asset stock  $a^*$ . When  $\gamma = 0$ , as discussed above,  $\hat{r} = \underline{r} < r + p$ , and  $d' > 0$  implies that  $A(\cdot)$  is positive and that the bank's debt capacity is reduced. This is because cash flows do not provide any offset to swap payments in this case. For swap hedging to increase the bank's debt capacity,  $\gamma$  must be sufficiently high, while  $d' > 0$  must be such that  $A(\cdot)$  is strictly negative.

Theoretically, the case with  $\gamma = 0$  is close in spirit to that featured in [Rampini and Viswanathan \(2010\)](#) and [Rampini and Viswanathan \(2013\)](#). A clear dynamic trade-off between financing and hedging exists. When  $\gamma > 0$ , in contrast, a different prediction arises because debt capacity may either increase or decrease when banks hedge. This is because banks borrow against the worst possible state next period, and because hedging may improve cash flows in this particular state. In [Rampini and Viswanathan \(2010\)](#), in contrast, debt and derivative contracts are written against particular future states. Empirically, the fact that hedging may increase debt capacity has been demonstrated by [Campello et al. \(2011\)](#), who show that hedging firms can commit to a lower cost of financial distress and enhance

their ability to invest. Due to our collateral constraint, a similar effect occurs here even in the absence of financial distress.

### 4.3 Pay-fixed and pay-float positions

The co-existence of several distinct motives for risk management yields a first result: both increases and decreases in the interest rate can be hedged. Thus, both pay-fixed and pay-float positions coexist. The relative magnitude of each motive for risk management, however, changes over time and in the cross-section of banks. This yields predictions regarding bank characteristics associated with either position type.

#### 4.3.1 The coexistence of position types

With the baseline calibration, conditional on using swaps, 74.5% of the positions taken are pay-fixed, and 25.5% pay-float (Table 2, rows 19 and 20). Traditional expositions of interest rate risk in banking (Freixas and Rochet, 2008) or practitioners' textbooks (Fabozzi and Konishi, 1994) consider that hedging should primarily be concerned with increases in the interest rate. This is because the duration of banks' liabilities is shorter than that of assets, so that a bank may be stuck with low-yield assets while the cost of rolling over short-term debt increases. The fact that bank value is negatively related to the short rate has also been documented empirically by Flannery and James (1984) and English et al. (2012). Thus, banks should primarily take pay-fixed positions.

Consistent with the model, banks in the data use both pay-fixed and pay-float positions for hedging, as appendix C demonstrates. According to our interpretation, the fact that they take pay-float positions (which pay off when the short rate is low) need not imply they are engaged in speculation.

#### 4.3.2 Predictions in the cross-state dimension

To obtain predictions about whether pay-fixed or pay-float positions are used, we study how the motives for interest rate risk management are affected by the model parameters. To do so, attention is restricted to the case in which  $\gamma = 0$ . In this case, taking a pay-fixed or a pay-float position always decreases a bank's debt capacity. The observed positions then do not reflect incentives to increase one's present debt capacity, as can be the case when  $\gamma > 0$ . Furthermore, the investment motive is absent, so that the observed positions only reflect the relative strength of the financing and of the investment motives.

The structure of the shocks drives the relative magnitude of each motive to an important extent. First, the financing motive is more acute as the correlation between  $\epsilon_z$  and  $\epsilon_r$ , i.e.  $\rho$ , is positive and high.  $\rho > 0$  (as in the data) implies that the cost of debt financing is more likely to be high in states where lending opportunities driven by real factors are likely to be large. The financing motive is larger and banks become more willing to hedge increases in the short rate, thus to take pay-fixed positions. This can be seen in Table 3, which compares

the baseline case ( $\rho = 0$ ) with cases in which  $\rho \in \{-0.6; 0.6\}$ . When  $\rho > 0$  pay-fixed positions are taken to a larger extent (rows 21 and 22).

The financing motive is also relatively more important when the autocorrelation of shocks to either  $z$  or  $r$  is lower. A high autocorrelation implies that good realizations of  $z$  (resp.  $r$ ) are more likely followed by other good realizations of  $z$  (resp.  $r$ ). As a good shock occurs, the bank value increases to reflect the fact that other good shocks are likely to follow in subsequent periods. Expectations about the discounted bank value in future periods increase more than when the autocorrelation of shocks is higher. Consequently, the discount motive is relatively larger when either  $\rho_z$  or  $\rho_r$  are higher, while the financing motive is larger when either of them is low. Banks take relatively more pay-fixed positions when  $\rho_z$  or  $\rho_r$  are low (Table 3, rows 10 to 11 and 21 to 22).

Finally, a high standard deviation of  $z$  implies that large lending outlays are more likely. Missing transitory investment opportunities is more costly, and the financing motives becomes relatively more important. Pay-fixed positions are taken more often (row 11), while investment is larger on average (row 1) and more volatile (row 2).

### 4.3.3 Relative magnitude of cross-state versus cross-period concerns

I depart from the case in which  $\gamma = 0$ . When cash flows from assets-in-place do depend on the realized short rate, i.e.  $\gamma > 0$ , then collateral constraints affect both the decision to hedge and the sign of the positions being taken. This is because, in such cases, taking swap positions may either increase or decrease the bank's present debt capacity. Allowing for  $\gamma > 0$  makes it possible to obtain richer predictions on the sign of the swap positions being taken. Moments from the model with several values of  $\gamma \in [0; 5]$  are in Table 4, which yields two results.

First, pay-fixed positions become relatively more attractive in the cross-period dimension when  $\gamma > 0$ . They are used more often (Table 4, row 19). This is because they make it possible to increase the bank's present debt capacity. By taking pay-fixed swaps, i.e. by hedging increases in the short rate, the bank relaxes its collateral constraint and can increase present investment. In the above subsection with  $\gamma = 0$ , the fact that pay-float positions are used much more often than pay-fixed positions could be seen as a source of concern. This concern disappears for reasonable values of  $\gamma > 0$ . The case in which  $\gamma = 0$  neglects a key mechanism through which interest rate swaps may be used by banks to increase their present debt capacity.<sup>4</sup>

Second, the ability for a bank to use swaps to relax its collateral constraint is particularly valuable when it is severely constrained, i.e. when net worth is low relative to lending opportunities. Thus, the bank uses more pay-fixed swaps when its collateral constraint binds or is close to bind. This can be seen through an econometric approach, as Panel A of Table

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<sup>4</sup>There is also another opposite effect at play. When  $\gamma$  increases, the investment motive becomes more important, i.e. the bank is increasingly willing to hedge decreases in the short rate by taking pay-float positions. In our simulations, the incentive to relax the collateral constraint overrides this second effect when  $\gamma$  is positive but low. When  $\gamma$  increases, banks use more and more pay-float positions in relative terms.

5 reports. A dummy variable equal to one if the bank takes a pay-fixed position is regressed on financial constraints, measured as the percentage of debt capacity used in the current period. A positive and statistically significant coefficient is obtained across specification, both unconditionally (regardless of whether derivatives are used) and conditional on using non-zeros swaps.

Relatedly, given investment opportunities, banks are more likely to take pay-fixed positions if their net worth is low. This is illustrated in figure 4, which plots the hedging policy given shocks  $\{z = 0, r = r^*\}$  as a function of  $w$ . For low levels of net worth, taking pay-fixed positions enables banks to increase their debt capacity by transferring funds to future states in which their cash flows would otherwise be low.

## 4.4 Cross-sectional sorting between derivatives users and non-users

This section addresses one last question related to risk management. If derivatives create value for equity holders and make it possible for banks to better achieve their optimal lending policy, why is it the case that not all banks use derivatives? In the U.S., only 12% of commercial banks use interest rate derivatives for hedging. Furthermore, hedging for banks that use derivatives is usually incomplete. Even derivatives users keep (possibly large) exposure to interest rates, as documented by Landier et al. (2015). Part of the limited participation in derivatives markets can be explained by fixed costs (Brown, 2001; Guay and Kothari, 2003). It is unlikely, however, that fixed costs explain most of the cross-sectional sorting between derivatives users and non-users, as many banks in the highest percentiles of the size distribution also do not use derivatives (Purnanandam, 2007).

Regardless of the position signs, derivatives in the model are not used in all periods (Table 2, row 18), and the percentage of periods in which the bank uses non-zero derivatives varies with the model parameters (Table 3, rows 9 and 20). The results of Table 3 can be given a cross-sectional interpretation, as in DeAngelo et al. (2011), if banks differ along some of the structural model parameters. While derivatives have attractive insurance properties, banks optimally choose to use them to a larger or smaller extent. In this respect, the model is one where sorting between users and non-users arises endogenously.

I study two questions. First, how changes in the model parameters affect the extent to which derivatives are used, as captured by the fraction of periods in which non-zero positions are taken? Second, given a set of structural parameters, what aggregate and bank-level characteristics explain the fact that derivatives positions are traded at particular dates?

### 4.4.1 The use of derivatives: Cross-sectional results

Changes in the properties of the shocks imply that the incentive of the bank to engage in *any type* of risk management changes, i.e. either through financial flexibility or through interest rate derivatives. Banks use more swaps on average for calibrations in which they also preserve more financial flexibility on average.

When  $z$  is highly volatile ( $\sigma_z$  high) or persistent ( $\rho_z$  high), banks keep more debt capacity (Table 3, rows 3 and 8). Their collateral constraint binds less often (row 7). This is because a high  $\sigma_z$  implies that large funding needs may arise, while a high  $\rho_z$  implies that periods in which funding needs are large are likely followed by other periods in which funding needs are large. Banks exposed to more volatile or more persistent real shocks (possibly due to the composition of their portfolio) are expected to hedge more, as the value of risk management is greater for them.

Not surprisingly, there is more interest rate risk management when the short rate is less predictable, i.e. either when it is highly volatile (high  $\sigma_r$ ) or highly transitory (low  $\rho_r$ ). Derivatives are used more often in such cases (row 20). This fact, however, does not have a clear cross-sectional interpretation, because the short rate is typically the same for all banks in a given banking sector. Over time, it implies that hedging may be larger in periods of high interest rate uncertainty, all else equal.

#### 4.4.2 The use of derivatives: Within-bank results

For a given set of parameters, both the decision to use derivatives (a binary variable) and the extent of derivatives hedging (a continuous variable) vary over time. Collateral constraints are a key driver of whether derivatives are used or not. To see this, consider the case where  $\gamma = 0$ . In this case, there is a dynamic trade-off between financing and hedging. Theoretically, this trade-off has been studied by Rampini and Viswanathan (2010, 2013). Predictions similar to theirs arise.

Banks that are more constrained hedge *less*. This can be seen in Panel B of Table 5. Both the decision to hedge (a binary variable that takes value one if the bank uses derivatives) and the extent of hedging (the absolute value of the derivative position  $d'$ ) are regressed on financial constraints, measured as the percentage of debt capacity used. The relation is negative and significant at a 1% level across specifications. An interesting implication follows: while hedging is motivated by the existence of financing constraints, banks which are more constrained hedge less, not more. Empirically a negative relation between hedging and financial constraints has been demonstrated for non-financial firms by Rampini et al. (2014).

A corollary is that, due to collateral constraints, complete hedging using derivatives is not optimal. At times the trade-off between hedging and financing is too acute, banks cut hedging, or optimally choose not to hedge. Relatedly, small banks also choose to hedge less, as in the data (Purnanandam, 2007). This is because they are more profitable at the margin, due to the concavity of  $\pi(\cdot)$  in  $a$ , so that foregoing debt capacity is more costly for them. Incomplete hedging is observed in the data, as shown by Landier et al. (2015). Even after accounting for their derivatives portfolio, U.S. commercial banks keep open exposure to interest rate risk, as measured by the income gap.

In case  $\gamma > 0$ , the relation between hedging and financial constraints is more subtle. This is because pay-fixed positions can be used by the bank to relax its collateral constraint and increase its debt capacity. They may thus be taken at times the bank is more constrained,



especially if  $\gamma$  is sufficiently high. The relation between financial constraints and hedging is consequently weaker. It is still the case, however, that pay-float positions are primarily taken at times the bank is less constrained.

## 5 Derivatives hedging and bank capital structure

This section compares the capital structure of banks that use or do not use interest rate derivatives.

### 5.1 Substitution between derivatives and the preservation of debt capacity

While hedging may both increase or decrease a bank's debt capacity—i.e. the maximum amount it can borrow—, a related question is how it affects the preservation of debt capacity, i.e. the distance between a bank's collateral constraint and its optimal choice of debt. This section shows that derivatives users substitute swaps for financial flexibility, i.e. take higher leverage.

#### 5.1.1 Derivatives versus financial flexibility

To answer the question whether derivatives hedging induces a bank to preserve more or less debt capacity, simulations of the model are used. Table 2 compares moments obtained from the models with and without swaps. When the moment of interest is an average, a two-sample  $t$ -test is also conducted, to test whether differences in average are statistically significant.

The most important result is that debt capacity and derivatives are *substitutes* for risk management. In the simulation results, the bank uses about 97.7% of its total debt capacity (given by the collateral constraint) on average in the model with swaps, as compared to 95.9% in the model without swaps (row 8). The bank has a higher debt-to-assets ratio on average (row 3) and finances a larger share of its expenses using debt (row 15).<sup>5</sup> These differences are statistically significant at a 1% level.

In the U.S. data, Purnanandam (2007, Table 3) provides suggestive evidence consistent with this substitution effect. In his sample, non-user banks keep more liquid assets as a percentage of total assets than user banks (35.86% versus 30.64%). They have a lower leverage, reflected in the fact that their equity ratio is higher (10.83% versus 9.53%). Derivatives users also have more loans to total assets than non-users (63.90% versus 60.86%). All these differences are statistically significant at a 1% level when performing a two-sample  $t$ -test.

In the model, the substitution between swaps and debt capacity can be explained by differences in the types of transfers they make possible. Preserving debt capacity at  $t$  makes it possible to transfer funds from  $t$  to  $t + 1$  in a non-state contingent way. As such, it is

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<sup>5</sup>In unreported simulations with alternative calibrated parameters, for which the bank's leverage is lower on average, users of derivatives keep less cash than non-users.

particularly costly if just a subset of states next period are associated with high funding needs, because present investment opportunities have to be foregone. Derivatives, which make it possible to transfer funds in a state-contingent way, can alleviate this problem. Funds can be channelled towards future states where funding needs are high, thus reducing the need to preserve debt capacity and preserving present investment to some extent. The fact that using derivatives makes it possible to increase leverage is also consistent with [Stulz \(1996\)](#).

Another implication of the fact that swaps and debt capacity do not provide the same payoffs is that the extent to which they are substitute varies over time. The benefits of swaps arise particularly when there is investment only in a few future states, i.e. when state-contingent wealth is particularly valuable. If, in contrast, high investment is likely to occur in most or all future states, then non-state contingent wealth is more valuable. In such a case, derivatives provide a poor substitute to debt capacity for risk management.

### 5.1.2 The preservation of debt capacity in the presence of swaps

The opportunity to hedge using swaps, however, does not imply that the bank no longer keeps debt capacity. There is *partial*, but not complete, substitution. Some debt capacity is still preserved in most periods ([Table 2](#), row 7). There are two main reasons for this.

First, debt capacity would still be preserved in the absence of a stochastic short rate, in order to hedge fluctuations in  $z$ . As in [Hennessy and Whited \(2005\)](#), the bank finds it optimal to preserve funds for future states in which  $z$  is high. Even when shocks to  $z$  and  $r$  are correlated, this motivates the need to preserve funds in a non-state contingent way, i.e. through debt capacity.

Second, as soon as both  $z$  and  $r$  co-exist (i.e. outside the “natural hedge” case of [section 4.1.4](#)) future investment states can be associated either with high or low realizations of  $r$ . Even though one of the three motives for interest rate risk management dominates, all three are always present. Because of the state-contingent nature of their payoffs, derivatives cannot address all three motives simultaneously, while the preservation of debt capacity can. Thus, debt capacity is optimally preserved to some extent, and derivatives are used to address the prevailing motive at each date.

## 5.2 Hedging, equity financing and bank value

When the bank optimally preserves less free debt capacity as a consequence of the use of derivatives, additional short-term debt financing can be used either to increase lending or be distributed to equity holders. This subsection focuses on the payout policy. [Section 6](#) later focuses on the interaction between derivatives hedging and bank lending.

### 5.2.1 Equity financing

The equity issuance/distribution policy is altered with the introduction of swaps. The frequency of equity issuance is lower for the simulated bank in the model with swaps ([Table 2](#),

row 11). Conditional on equity issuance, the amount being issued is also lower, even though not statistically significant (row 12). For the simulated bank, equity is issued in about 8.7% of the periods in the model without swaps and in 7.8% in the model with swaps (row 11). Even though small in magnitude, non-user banks finance a larger share of their expense with equity (row 16), the difference being significant at the 10% level.

The effect is driven by the nature of each margin for risk management. Non-user banks manage risk by preserving debt capacity, i.e. by transferring non-contingent funds to future periods. They trade off present versus future investment opportunities, and transfers to future periods occur regardless of whether equity issuance will indeed be needed at  $t + 1$ . It is thus particularly costly, because the cost (foregone investment) is paid in all present states while the benefit of increased debt capacity is obtained in some future states only. Swap payoffs, in contrast, can be targeted more precisely to future states in which equity will be optimally issued. The optimal choice of each margin for risk management is an equilibrium condition; however, for a bank that engages in risk management, swaps can be relatively less costly, due to their state-contingent payoffs. It is the case that banks in the model without swaps engage less in risk management and optimally choose to issue equity more often.

### 5.2.2 Bank value

Does hedging increase bank value? Existing evidence on whether risk management using derivatives affects firm value is mixed. [Jin and Jorion \(2006\)](#) find no relation between hedging and firm market value in the oil and gas industry. Most of the recent studies ([Allayannis and Weston, 2001](#); [Mackay and Moeller, 2007](#); [Perez-Gonzalez and Yun, 2013](#); [Gilje and Taillard, 2014](#)), however, find statistically and economically significant differences in firm value between hedgers and non-hedgers.

The average value of a bank increases by 1.0% when it uses derivatives (Table 2, row 21). This difference is statistically significant at a 1% level. It is to be compared with the empirical estimates by [Graham and Rogers \(2002\)](#), who finds that hedging increases firms' market value by 1.1%, or with those by [Mackay and Moeller \(2007\)](#), who find an increase in firm value between 2% and 3% for firms hedging concave revenues. While bank value is higher for derivatives users, the standard deviation of bank value is lower for them (row 22). Supportive evidence has been put forth by [Bartram et al. \(2011\)](#), who show that the volatility of the market value of derivatives users is lower than that of non-users.

## 6 Derivatives hedging and the dynamics of bank lending

How is lending affected by the possibility to hedge using interest rate derivatives? This section uses a regression approach to address this question. It yields three main results. First, there are significant differences between users and non-users in the response of bank lending to interest rate shocks. Second, lending by users and non-users of interest rate derivatives also responds differently to *real* shocks ( $z$ ), even if these shocks are uncorrelated

with interest rates. Third, differences in lending growth between users and non-users are larger in periods in which “good” shock realizations hit (from the equity holders’ perspective), i.e. either when the short rate decreases or when productivity rises. The use of derivatives increases the procyclicality of bank lending during booms.

## 6.1 Baseline lending regression

The regression approach uses data simulated from the model to investigate differences in bank lending between derivatives users and non-users. Our econometric model is inspired by [Purnanandam \(2007\)](#). In the absence of a closed-form solution, this approach yields further insights into the model’s dynamics. Theoretical elements from the model make it possible to interpret the empirical findings.

### 6.1.1 Econometric specification

I reproduce the two main regressions of [Purnanandam \(2007\)](#) using simulated data. First, the log change in total loans for group  $j \in \{0 \equiv \text{Non-users}, 1 \equiv \text{Users}\}$  at date  $t$  is regressed on its own four lags, and on contemporaneous and three lags of innovations to  $z$  and  $r$ :

$$\Delta \log(a)_{jt} = \alpha_0 + \sum_{k=1}^{k=4} \alpha_k \Delta \log(a)_{jt-k} + \sum_{k=0}^{k=3} \beta_k \Delta r_{t-k} + \sum_{k=0}^{k=3} \gamma_k \Delta z_{t-k} + \epsilon_{jt}, \quad (21)$$

where  $\Delta \log(a)_{jt}$  is the model equivalent of  $\Delta \log(LOAN)_{jt}$  in [Purnanandam \(2007\)](#)’s empirical work. Relatedly,  $\Delta r_t \equiv r_t - r_{t-1}$  can be seen as innovations to the Fed funds rate and  $\Delta z_t \equiv z_t - z_{t-1}$  as innovations to the log GDP in his setup. After estimating the model of equation (21), I test the null hypothesis that the sums of coefficients on innovations to the short rate ( $\sum_{k=0}^{k=3} \beta_k$ ) and on the real factor ( $\sum_{k=0}^{k=3} \gamma_k$ ) are equal to 0. From the policy function, a negative and a positive coefficient signs are expected, respectively, indicating that lending decreases with the short rate and increases with real conditions.

Second, the lending behaviour of derivatives users and non-users is compared. The difference in lending growth of the two groups (non-user minus user) in a given quarter is used as the dependent variable. The model, which provides the most stringent evidence that the lending policy of users and non-users differs, writes as

$$\begin{aligned} \Delta \log(a)_{0t} - \Delta \log(a)_{1t} &= \alpha_0 + \sum_{k=1}^{k=4} \alpha_k \left[ \Delta \log(a)_{0t-k} - \Delta \log(a)_{1t-k} \right] \\ &\quad + \sum_{k=0}^{k=3} \beta_k \Delta r_{t-k} + \sum_{k=0}^{k=3} \gamma_k \Delta z_{t-k} + \epsilon_t. \end{aligned} \quad (22)$$

Coefficients of interest include both  $\sum_{k=0}^{k=3} \beta_k$  and  $\sum_{k=0}^{k=3} \gamma_k$ . I test for the null hypothesis that they are equal to zero. For example, a negative and significant coefficient on  $\sum_{k=0}^{k=3} \gamma_k$  would indicate that, when  $z$  increases (which can be interpreted as growth in real terms), user banks’ lending volume increases significantly more than that of non-user banks. These models are estimated using 10,000 observations simulated from the model for both users and

non-users, receiving the same path of shocks. The regressions are estimated in both cases where  $\gamma = 0$  and  $\gamma = 3$  (baseline value), as the incentives to use derivatives are different in each case. The estimated coefficients for the models in equations (21) and (22) are reported in Panel A of Table 6.

### 6.1.2 Response to interest rate shocks

Consistent with the policy function, the summed coefficients on contemporaneous and lagged innovations to the short rate  $r$  are negative and significant. This is true for both users and non-users of derivatives, regardless of the value of  $\gamma$ . The decrease is larger when  $\gamma > 0$ , since cash flows are decreasing in  $r$  in this case. While Purnanandam (2007) also finds a negative and significant coefficient for derivatives non-users, his estimated coefficient is not significant for derivatives users. In the model, optimal lending is decreasing in the short rate, so that using derivatives to offset the full increase in the short rate and preserve lending to its full extent cannot be optimal.

Whether lending growth differs between derivatives users and non-users, and whether the difference is significant, is seen by estimating equation (22). When  $\gamma = 0$ , i.e. when the bank cannot use derivatives to increase its debt capacity, the coefficient  $\sum_{k=0}^3 \beta_k$ , also reported in Table 6, is negative but non-significant. Purnanandam (2007) also finds a negative but significant coefficient, which he interprets as derivatives users shielding their lending policy against interest rate spikes. A potential explanation why our estimate is not significant is that, when  $\gamma = 0$ , derivatives are used in a rather small number of periods (Table 4, row 18).

When  $\gamma > 0$ , as also shown in Table 6, the estimated difference between users and non-users turns positive and significant at a 1% level. Thus, derivatives users increase lending more during periods of monetary easing and may cut lending more during monetary tightenings. Which effect is more important is discussed below. Even though statistically significant, the economic magnitude of this difference is small. The difference in lending response between users and non-users amounts to around 2% of the overall response of bank lending, regardless of whether derivatives are used. In this respect, the model's predictions are consistent with Landier et al. (2015), who argue that the effect of derivatives hedging on bank lending is likely small.

### 6.1.3 Response to real shocks

Turning to real shocks, bank lending is increasing in  $z$ , also consistent with the policy function. More interesting is the difference in lending response by user and non-user banks to real shocks. It is again not significant when  $\gamma = 0$  and turns negative and significant at a 5% level when  $\gamma > 0$ . This coefficient estimate suggests that derivatives users increase lending more during periods of real growth. The next section shows that this estimate is driven by periods in which  $\Delta z > 0$ , i.e. that derivatives users do not cut lending more than non-users during recessions.

It is the case that, while banks use derivatives to hedge interest rate fluctuations only, differences in lending dynamics between users and non-users also arise in response to real

shocks. This is the true even when the correlation between  $\epsilon_z$  and  $\epsilon_r$  is zero, as in Table 6. This result is consistent with empirical evidence by Brewer et al. (2000), who show that loan growth is positively related to the use of interest rate derivatives.

#### 6.1.4 Procyclical lending: Mechanism

The increased procyclicality of bank lending for derivatives users comes from two sources, in the cross-state and in the cross-period dimensions. When  $\gamma = 0$ , banks cannot increase their present debt capacity by taking swaps. Whether banks hedge or not depends on whether they expect to fall short of funds next period to meet their optimal lending level. Using derivatives to transfer funds to such states, they have higher net worth in states in which it is most valuable, and can thus exploit profit opportunities to a larger extent.

The procyclicality of lending for derivatives users is further reinforced by a second force, whenever  $\gamma > 0$ . In this case, hedging future states can increase the bank's present debt capacity. Thus, derivatives tend to be used at times internal funds are low relative to lending opportunities. Lending is consequently higher in good times. This is true regardless of whether lending opportunities arise from shocks to  $z$  or  $r$ . The fact that interest rate hedging increase the procyclicality of lending in response to real shocks, even when real shocks are uncorrelated with interest rates, should therefore not be considered a surprise.

## 6.2 Asymmetric response to shocks

A limitation of the regression model in equation (22) is that it cannot account for asymmetric effects between positive and negative shocks. For example, the estimated coefficient  $\sum_{k=0}^{k=3} \gamma_k$  (Table 6, Panel A) should be interpreted as indicating that derivatives users increase lending more than non-users during booms, and decrease lending more during busts, in similar proportions. In contrast, another outcome of the model is an asymmetric response of bank lending to positive and negative shocks, yielding a *procyclical but asymmetric* lending policy.

### 6.2.1 Regression approach

To document the differential response of lending by user and non-user banks, the model in equation (22) is re-estimated on subsets of observations when  $\Delta r$  or  $\Delta z$  are either positive or negative. Panel B in Table 6 summarizes these additional regression results, both  $\gamma = 0$  and  $\gamma > 0$ .

As earlier, estimated differences in lending policy when  $\gamma = 0$  are not statistically significant. Asymmetric effects arise when  $\gamma > 0$ . In response to good real shocks ( $\Delta z > 0$ ), the difference between users and non-users is significant at a 10% level, while it is not significant when  $\Delta z < 0$ . It is thus the case that derivatives users increase lending more than non-users but do not decrease lending more than non-users in bad times.

With respect to interest rate shocks, a positive and statistically significant coefficient is estimated both when  $\Delta r > 0$  and  $\Delta r < 0$ . Therefore, derivatives users increase lending more than non-users during periods of monetary easing, and cut lending more when the short rate

rises. However, the magnitude of the coefficient for periods in which  $\Delta r < 0$  is close to 50% larger than that estimated for periods in which  $\Delta r > 0$ . Consequently, the decrease in bank lending during periods of monetary tightening is larger for users than for non-users, but does not fully offset the difference made in good times, when  $\Delta r < 0$ . Together with the results related to shocks to  $z$ , these results point to a “procyclical but asymmetric” lending policy, by which derivatives users are better able to exploit transitory lending opportunities in good times, and where the difference is not fully offset during downturns.

### 6.2.2 Asymmetric lending: Mechanism

The fact that derivatives users are able to preserve during downturns some of the benefits obtained in good times (in the form of additional lending) can be explained by the substitution between swaps and financial flexibility for risk management. When bad shocks hit (either a low  $z$  or a high  $r$ ), the present optimal lending is low. Banks cut lending both to regain profitability and to restore their debt capacity for future investment. To regain profitability, derivatives users may cut lending more than non-users. This is because they operate at a larger scale, and are consequently less profitable at the margin, due to the concavity of  $\pi(\cdot)$  in  $a$ . When it comes to the restoration of lending capacity, however, derivatives users are better able to continue lending. At such times, non-users restore their ability to exploit future opportunities by cutting present lending and preserving debt capacity. Derivatives users also preserve debt capacity to some extent, but the availability of an extra instrument for them—which is a partial substitute—implies that they do so to a lower extent. The fact that more debt capacity is preserved during downturns implies that the substitution between financial flexibility and derivatives plays a larger role in such periods.

## 7 Conclusion

The interest rate derivatives market is the largest market worldwide, and is used extensively by financial intermediaries. This paper provides the first comprehensive model of interest rate hedging by commercial banks. It yields novel predictions about who hedges, the types of positions being taken, as well as on the effects of derivatives hedging on bank’s capital structure and lending policy. These predictions leave room for future empirical work on the use of interest rate derivatives and on their effects on intermediaries’ capital structure.

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# A Calibration

The model is calibrated and solved for numerically. This section discusses the calibrated parameter values,, which are all summarized in Table 1.

## A.1 Cash flow function

The value of  $\theta$  is inferred from Mankart et al. (2014). They estimate the convex cost to be paid by a bank that increases lending. Equating marginal cash flow when lending increases by one unit in both models (given their parameter estimate), yields a value of  $\theta$  close to  $\theta = 0.85$ , which I choose. The concavity of cash flows for banks is thus lower than for non-financial firms (see estimates by Hennessy and Whited, 2007).

In the absence of guidance on the value of  $\gamma$ , the sensitivity of the bank's cash flows to the short rate, I treat it as a free parameter. In the baseline calibration,  $\gamma = 3$  is assumed. Most results are also provided for  $\gamma = 0$ . In this case, changes in the short rate do not affect the bank's cash flows from assets-in-place. Moments of the model for several alternative values of  $\gamma$  are also contained in Table 4.

## A.2 Shocks

Parameters  $\rho_z$  and  $\sigma_z$  are calibrated following the structural estimation by Hennessy and Whited (2007), as  $\rho_z = 0.68$  and  $\sigma_z = 0.12$ , where one period is thought of a one year.  $\rho_r$  and  $\sigma_r$  are estimated from the time series of the Fed funds rate over the period from 1995 to 2014. The estimates are  $\rho_r = 0.8$  and  $\sigma_r = 0.008$ .  $r^*$  is the average Fed funds rate over that period, equal to 0.03. In the baseline model,  $\rho$ , the correlation between  $\epsilon_z$  and  $\epsilon_r$ , is assumed to be zero. Non-zero correlations are also explored.

## A.3 Other structural parameters

The parameter  $\delta$ , which captures the share of assets maturing between any two periods, is obtained from the Call reports data. For each bank, the share of loans and debt securities with a remaining maturity or next repricing data below one year is computed, as a percentage of total loans.  $\delta$  is parameterized as 0.23, which is the mean (weighted by total assets) over all sampled banks and periods.

The liquidation value  $\kappa$  is set to 0.7, following Granja et al. (2014). They show that the cost of a sold failed bank to the FDIC represents 28% of its assets. The corporate tax rate is set to  $\tau = 0.35$ , consistent with the U.S. tax code. Parameters for the issuing equity are calibrated following the structural estimation by Hennessy and Whited (2007).  $\eta_1$  is set to 0.09 and  $\eta_2$  to 0.0004.

## B Model solution

This appendix details the numerical solution method. To solve the model, I specify a finite state space for the state variable and for the shocks. Net worth  $w$ , defined in equation (11), is the only state variable. It lies within a bounded set  $[\underline{w}; \bar{w}]$ . This follows from the fact that  $a$ ,  $b$  and  $d$  all lie within bounded sets.

Values of  $b$  and  $d$  are bounded by the collateral constraint (8). Equation (8) ensures the existence of an upper limit on the amount of debt,  $\bar{b}$ . A lower limit  $\underline{b}$  on  $b'$ , even though not computable in closed form, is ensured by the tax structure if  $\tau > 0$ : when  $b'$  becomes too negative, the tax cost of holding cash becomes too large, and additional internal funds are optimally distributed to equity holders. Consequently,  $b'$  lies within a compact set  $[\underline{b}, \bar{b}]$ . Equation (8) also imposes a lower and an upper bound on  $d'$ , which thus lies within a compact set  $[\underline{d}, \bar{d}]$ .

Regarding  $a$ , it is bounded below by 0. The fact that it is bounded above follows from the fact that  $z$  and  $r$  both have bounded supports,  $[\underline{z}; \bar{z}]$  and  $[\underline{r}; \bar{r}]$ . Denote  $\bar{a}$  the value of the asset stock such that  $a > \bar{a}$  is not profitable for the bank, i.e.

$$(1 - \tau) \pi_a(\bar{a}, \bar{z}, \underline{r}) - \delta = 0. \quad (23)$$

$\bar{a}$  is well-defined by concavity of  $\pi(\cdot)$  in  $a$  and because  $\lim_{a \rightarrow \infty} \pi_a(a, z, r) = 0$ . From the definition of  $\bar{a}$ ,  $\bar{b}$  is computed as  $\bar{b} = (1 - \tau) \underline{z} (R - \bar{r})^\gamma \bar{a}^\theta + \kappa \bar{a}$ .  $\bar{d}$  is also obtained from the collateral constraint in equation (8), given  $\bar{a}$ .  $\bar{w}$  and  $\underline{w}$  cannot be computed in closed form. I let the grid values for  $w$  take 30 equally spaced values in the interval  $[-\bar{w}^a/4, \bar{w}^a]$ , where  $\bar{w}^a \equiv w(a^*, 0, 0)$  and  $a^*$  is the steady state asset. The lower and upper limits are never hit by the optimal choice of  $w'$ .

The shock processes for  $z$  and  $r$ , described by equations (5), (3) and (4), are transformed into discrete-state Markov chains using an extension of Tauchen (1986)'s method to multivariate and correlated AR(1) processes. They each take 10 equally spaced values in  $[-2\sigma_z; 2\sigma_z]$  and  $[-2\sigma_r; 2\sigma_r]$  respectively.

The model is solved by value function iteration. It yields a policy function  $\{a', b', d'\} = \Gamma(w, z, r)$ .

## C Pay-fixed and pay-float swap positions in the U.S. data

The prediction that both pay-fixed and pay-float swap positions can be optimally chosen is broadly consistent with the data. Measuring banks' net exposure in the derivatives market is a challenging endeavour, discussed in [Begenau et al. \(2015\)](#). In this appendix, I simply provide evidence that both types of positions co-exist in the data. A precise test of the model's predictions regarding position signs is left for future research.

I define the net swap position of a bank as

$$\text{Net swap position} = \frac{\text{Gross pay-fixed exposure} - \text{Gross pay-float exposure}}{\text{Total assets}}, \quad (24)$$

where only exposures in the interest rate swap market are considered. To compute the net swap position of U.S. commercial banks, I retrieve data from the Call Reports from 1995Q to 2013Q4. I exclude derivatives user for trading and restrict attention to derivatives used for risk management. These exposures are reported separately in the call reports, as derivatives "held for trading" or "held for purposes other than trading".<sup>6</sup> Furthermore, the net swap position can be computed only for a subset of banks. The notional amount of interest rate swaps used for hedging on which the bank pays a fixed rate is known for all banks, but both the amount on which it pays a floating rate or the total amount of swaps for hedging are unknown. The aggregate amount of swaps on which the bank pays a floating rate can be computed only for a subset of banks which use only interest rate swaps for hedging and not, for example, futures or options. These swaps represent 71.7% of all interest rate derivatives held by U.S. commercial banks. These restrictions imply that the net swap position can be computed for 4,464 bank-quarter observations, i.e. about 89 observations per quarter (which is about one fourth of the sample of banks that actively use interest rate derivatives).

The net swap position of U.S. commercial banks is plotted in figure 5. There is considerable heterogeneity in the type of positions taken by commercial banks, as both pay-fixed and pay-float positions are observed among exposures reported for risk management. Furthermore, the average net swap position is negative over the sample period, implying that pay-float positions are held on average. Absent a detailed understanding of the reasons why decreases in the interest rate may be hedged, this fact could be considered a puzzle, or as evidence of misreporting or speculation. Whether alternative theories of interest rate risk in banking can also rationalize this pattern is a question which is left for future work.

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<sup>6</sup>Derivative contracts held for trading include *(i)* dealer and market making activities, *(ii)* taking positions with the intention to resell in the short-term or to benefit from short-term price changes *(iii)* taking positions as an accommodation for customers and *(iv)* taking positions to hedge other trading activities. Derivatives for purposes other than trading thus include contracts held for hedging.



## D Tables and figures

Table 1: Calibrated values of the model parameters.

This table contains the baseline calibrated values for the model parameters. Appendix A provides justifications for the chosen parameter values. Tables 2 to 4 provide moments of the bank's capital structure obtained using alternative calibrated values for the most relevant model parameters.

Parameter	Description	Value
<i>Structural parameters</i>		
$\delta$	Share of maturing loans	0.23
$\theta$	Profit function concavity	0.85
$\eta_1$	Cost of equity financing (linear part)	0.09
$\eta_2$	Cost of equity financing (convex part)	0.0004
$\tau$	Corporate tax rate	0.35
$\kappa$	Liquidation value of an asset unit	0.7
$\gamma$	Profit sensitivity to the short rate	3
<i>Shocks</i>		
$r^*$	Unconditional mean of $r$	0.03
$\rho_z$	Persistence of $z$	0.68
$\sigma_z$	Standard deviation of $z$	0.12
$\rho_r$	Persistence of $r$	0.8
$\sigma_r$	Standard deviation of $r$	0.008
$\rho$	Correlation between $\epsilon_z$ and $\epsilon_r$	0

Table 2: Moments of the bank capital structure, with and without swaps.

This table provides moments of the bank capital structure, as obtained from simulated data with the baseline calibration. The first and second columns contain moments obtained respectively in the model without and with interest rate swaps. In the model without swaps, the restriction that  $d' = 0$  is imposed in all periods. Each model is simulated for 10,200 periods in which the bank receives stochastic productivity and interest rate shocks  $\{z, r\}$ . The first 200 periods are dropped before the moments of interest are calculated. All calibrated parameters are the same for the two models, and are given in Table 1. The simulated sequence of shocks is also the same for the two models. The third column contains the difference between the moments obtained from the model with swaps and those obtained from the model without swaps. When the moment of interest is an average over simulated observations, the fourth column provides the  $p$ -value as obtained from a two-sample  $t$ -test. \*, \*\* and \*\*\* denote respectively statistical significance at the 10%, 5% and 1% level.

		Without swaps	With swaps	Diff. (With minus without)	$p$ -value
1	Average investment ( $i/a$ )	0.361	0.364	0.003	0.715
2	Standard deviation of investment ( $i/a$ )	0.555	0.570	0.015	—
3	Average debt to assets ratio ( $b/a$ )	0.878	0.895	0.017	0.000***
4	Standard deviation of ( $b/a$ )	0.045	0.040	-0.006	—
5	Frequency of positive debt outstanding	1.000	1.000	0.000	—
6	Average cash balances to assets	0.000	0.000	0.000	1.000
7	Frequency of collateral constraint binding	0.376	0.456	0.080	—
8	Average percent of debt capacity used	0.959	0.977	0.018	0.000***
9	Average equity distribution ( $e/a$ )	0.043	0.044	0.000	0.665
10	Standard deviation of equity distribution ( $e/a$ )	0.040	0.040	-0.001	—
11	Equity issuance frequency	0.087	0.078	-0.008	—
12	Average of non-zero equity issuance/assets	0.003	0.002	0.000	0.472
	<i>Average fraction of expenses (incl. lending) funded from:</i>				
13	Current cash flow	0.265	0.261	-0.004	0.001***
14	Cash balances	0.000	0.000	0.000	1.000
15	Debt issuance	0.735	0.738	0.003	0.029**
16	Equity issuance	0.000	0.000	-0.000	0.052*
17	Swap payoffs	0.000	0.001	0.001	0.000***
18	Frequency of swaps use	—	0.576	—	—
19	Frequency of pay-fixed (cond.)	—	0.745	—	—
20	Frequency of pay-float (cond.)	—	0.255	—	—
21	Bank value for equity holders ( $V$ )	418.084	422.300	4.216	0.000***
22	Standard deviation of bank value ( $V$ )	59.045	58.702	-0.344	—

Table 3: Moments of the bank capital structure — With alternative properties of the shocks.

This table provides moments of the bank capital structure with alternative structural properties of the shocks. Panel A contains moments for alternative properties of the productivity shock  $z$  and Panel B for alternative properties of the interest rate  $r$ . All moments are computed from simulated data with  $\gamma = 0$ . The restriction that  $\gamma = 0$  ensures that the incentive to take derivatives exposures is not driven by the need to increase its debt capacity. The first column in Panel A contains moments obtained with the baseline calibration of the model with interest rate swaps. Other columns contain the same moments with alternative parameterizations of the shocks, respectively the standard deviation and persistence of the productivity shock ( $\sigma_z$  and  $\rho_z$ ) and of the interest rate shock ( $\sigma_r$  and  $\rho_r$ ), as well as the correlation between  $\epsilon_z$  and  $\epsilon_r$  (i.e.  $\rho$ ). Other calibrated parameters are set at their baseline value. For each parameter, we study high and low values, respectively  $\sigma_z \in \{0.05; 0.15\}$ ,  $\rho_z \in \{0.2; 0.85\}$ ,  $\sigma_r \in \{0.004; 0.012\}$ ,  $\rho_r \in \{0.1; 0.9\}$  and  $\rho \in \{-0.6; 0.6\}$ . In each case, the model is simulated for 10,200 periods in which the bank receives stochastic productivity and interest rate shocks  $\{z, r\}$ . The first 200 periods are dropped before the moments of interest are calculated. The simulated sequence of shocks is the same in each case.

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*Panel A: Capital structure moments with alternative properties of the productivity shock  $z$  (with  $\gamma = 0$ )*

		Baseline w/ $\gamma = 0$	$\sigma_z$		$\rho_z$	
			Low	High	Low	High
1	Average investment ( $i/a$ )	0.345	0.266	0.407	0.256	0.418
2	Standard deviation of investment ( $i/a$ )	0.522	0.279	0.689	0.239	0.721
3	Average debt to assets ratio ( $b/a$ )	0.906	0.965	0.859	0.940	0.839
4	Standard deviation of ( $b/a$ )	0.038	0.018	0.057	0.022	0.063
5	Frequency of positive debt outstanding	1.000	1.000	1.000	1.000	1.000
6	Average cash balances to assets	0.000	0.000	0.000	0.000	0.000
7	Frequency of collateral constraint binding	0.499	0.973	0.354	0.776	0.273
8	Average percent of debt capacity used	0.975	0.999	0.939	0.995	0.924
9	Frequency of swaps use	0.174	0.012	0.262	0.059	0.203
10	Frequency of pay-fixed (cond.)	0.032	0.029	0.047	0.272	0.043
11	Frequency of pay-float (cond.)	0.968	0.971	0.953	0.728	0.957

*Panel B: Capital structure moments with alternative properties of the interest rate  $r$  and of the correlation  $\rho$  (with  $\gamma = 0$ )*

		$\sigma_r$		$\rho_r$		$\rho$	
		Low	High	Low	High	< 0	> 0
12	Average investment ( $i/a$ )	0.336	0.358	0.348	0.345	0.390	0.311
13	Standard deviation of investment ( $i/a$ )	0.494	0.557	0.530	0.519	0.643	0.428
14	Average debt to assets ratio ( $b/a$ )	0.913	0.900	0.908	0.904	0.852	0.876
15	Standard deviation of ( $b/a$ )	0.020	0.050	0.022	0.053	0.045	0.036
16	Frequency of positive debt outstanding	1.000	1.000	1.000	1.000	1.000	1.000
17	Average cash balances to assets	0.000	0.000	0.000	0.000	0.000	0.000
18	Frequency of collateral constraint binding	0.570	0.388	0.409	0.492	0.269	0.554
19	Average percent of debt capacity used	0.982	0.968	0.977	0.972	0.947	0.973
20	Frequency of swaps use	0.073	0.258	0.255	0.160	0.375	0.133
21	Frequency of pay-fixed (cond.)	0.056	0.055	0.280	0.058	0.017	0.941
22	Frequency of pay-float (cond.)	0.944	0.945	0.720	0.942	0.983	0.059

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Table 4: Moments of the bank capital structure — With alternative value of  $\gamma$ . This table provides moments of the bank capital structure, as obtained from simulated data, for alternative values of  $\gamma \in \{0; 5\}$ .  $\gamma$  captures the sensitivity of the bank's cash flow from assets-in-place to the short rate  $r$ . Other calibrated parameters are set at their baseline value. In each case, the model is simulated for 10,200 periods in which the bank receives stochastic productivity and interest rate shocks  $\{z, r\}$ . The first 200 periods are dropped before the moments of interest are calculated. The simulated sequence of shocks is the same in each case.

		$\gamma = 0$	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
1	Average investment ( $i/a$ )	0.345	0.351	0.364	0.381
2	Standard deviation of investment ( $i/a$ )	0.522	0.536	0.575	0.625
3	Average debt to assets ratio ( $b/a$ )	0.906	0.910	0.901	0.895
4	Standard deviation of ( $b/a$ )	0.038	0.028	0.035	0.035
5	Frequency of positive debt outstanding	1.000	1.000	1.000	1.000
6	Average cash balances to assets	0.000	0.000	0.000	0.000
7	Frequency of collateral constraint binding	0.499	0.535	0.485	0.450
8	Average percent of debt capacity used	0.975	0.986	0.984	0.983
9	Average equity distribution ( $e/a$ )	0.044	0.044	0.043	0.042
10	Standard deviation of equity distribution ( $e/a$ )	0.039	0.037	0.040	0.041
11	Equity issuance frequency	0.060	0.084	0.091	0.088
12	Average of non-zero equity issuance/assets	0.011	0.012	0.013	0.014
	<i>Average fraction of expenses</i>				
	<i>(incl. lending) funded from:</i>				
13	Current cash flow	0.258	0.258	0.259	0.261
14	Cash balances	0.000	0.000	0.000	0.000
15	Debt issuance	0.742	0.742	0.739	0.736
16	Equity issuance	0.000	0.000	0.000	0.000
17	Swap payoffs	0.001	0.001	0.001	0.002
18	Frequency of swaps use	0.174	0.700	0.644	0.670
19	Frequency of pay-fixed (cond.)	0.032	0.762	0.704	0.619
20	Frequency of pay-float (cond.)	0.968	0.238	0.296	0.381

Table 5: Financial constraints and derivatives hedging

This table contains regression results relating banks' financial constraints to their hedging policy. These regressions are run on data simulated from the model. In Panel A, a dummy variable equal to one if the bank takes a pay-fixed position is regressed on a measure of financial constraints. The measure of financial constraints is the percentage of debt capacity used, computed as the ratio of debt taken to the maximum amount of debt possible, given by the collateral constraint. The regression is run both unconditionally (whether the bank uses derivatives or not), and conditional on using non-zero derivatives. In Panel B, the decision to hedge (a binary variable equal to one if the bank takes non-zero derivatives) and the extent of hedging (the absolute value of the swap position  $d'$ ) are regressed on the same measure of financial constraints. Data is simulated from the baseline model with interest rate swaps, with calibrated parameter values given in Table 1. In Panel B,  $\gamma = 0$ . In each panel, the model is simulated for 10,200 periods in which the bank receives stochastic productivity and interest rate shocks  $\{z, r\}$ . The first 200 periods are dropped before the regression coefficients are estimated.  $p$ -values are in parentheses. Standard errors are heteroskedasticity-consistent. \*, \*\* and \*\*\* denote respectively statistical significance at the 10%, 5% and 1% level.

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*Panel A: Regression of pay-fixed exposure on the percentage of debt capacity used*

	Unconditional	Conditional on using swaps	
	OLS	OLS	Probit
% of debt capacity used	5.153***	8.799***	30.023***
$p$ -value	(0.000)	(0.000)	(0.000)
$R^2$	0.285	0.398	0.344
N. obs.	10,000	5,761	5,761

*Panel B: Regression of the decision to hedge on the percentage of debt capacity used ( $\gamma = 0$ )*

	Decision to hedge		Extent of hedging
	OLS	Probit	OLS
% of debt capacity used	-.797***	-2.916***	-255.011***
$p$ -value	(0.000)	(0.000)	(0.000)
$R^2$	0.009	0.009	0.016
N. obs.	10,000	10,000	10,000

---

Table 6: Bank lending regressions.

This table presents regression results comparing the response of bank lending for derivatives users and non-users to both interest rate and productivity shocks. These regressions are run on data simulated from the model. In Panel A, the response of bank lending to (contemporaneous and lagged) interest rate and productivity shocks is compared between users and non-users of derivatives, using the whole sample of simulated data. The first two columns present estimates by groups, as in equation (21). The last columns contains estimates of the differences in lending between groups, i.e. of the model in equation (22). In Panel B, asymmetric responses in bank lending to “good” and “bad” shocks is investigated. The model of equation (22) is estimated on subsamples of the data in which  $\Delta r$  and  $\Delta z$  are positive or negative. Data for derivatives users is simulated from the baseline model with interest rate swaps. Each panel contains estimates obtained with  $\gamma = 0$  and  $\gamma = 3$ .  $\gamma$  captures the sensitivity of the bank’s cash flows to the short rate. Data for non-users is simulated once the policy function is solved for under the restriction that  $d' = 0$  in all periods. Each model is simulated for 10,200 periods in which the bank receives stochastic productivity and interest rate shocks  $\{z, r\}$  each period. The first 200 periods are dropped before the regression coefficients are estimated. All calibrated parameters are the same for the two models, and are given in Table 1. The simulated sequence of shocks is also the same for the two models.  $p$ -values are in parentheses. Standard errors are heteroskedasticity-consistent. \*, \*\* and \*\*\* denote respectively statistical significance at the 10%, 5% and 1% level.

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*Panel A: Response of bank lending to shocks  
for derivatives users vs. non-users.*

With  $\gamma = 0$ .

	Derivatives non-users	Derivatives users	Non-users minus users
$\sum_{k=0}^{k=3} \Delta r_{t-k}$	-40.867*** (0.000)	-40.878*** (0.000)	-.272 (0.431)
$\sum_{k=0}^{k=3} \Delta z_{t-k}$	9.387*** (0.000)	9.459*** (0.000)	.031 (0.208)
N. Obs.	9,996	9,996	9,996
Adj.- $R^2$	0.939	0.937	0.400

With  $\gamma = 3$ .

$\sum_{k=0}^{k=3} \Delta r_{t-k}$	-73.782*** (0.000)	-75.118*** (0.000)	1.617*** (0.000)
$\sum_{k=0}^{k=3} \Delta z_{t-k}$	9.337*** (0.000)	9.392*** (0.000)	-.061** (0.022)
N. Obs.	9,996	9,996	9,996
Adj.- $R^2$	0.925	0.938	0.399

*Panel B: Asymmetric response of bank lending to shocks  
for derivatives users vs. non-users.*

With  $\gamma = 0$ .

Subsample	Coefficient	Non-users minus users	$p$ -value	N. Obs.	Adj.- $R^2$
$\Delta r > 0$	$\sum_{k=0}^{k=3} \Delta r_{t-k}$	.418	(0.404)	4,806	0.519
$\Delta r < 0$	$\sum_{k=0}^{k=3} \Delta r_{t-k}$	-.391	(0.542)	4,844	0.281
$\Delta z > 0$	$\sum_{k=0}^{k=3} \Delta z_{t-k}$	.013	(0.769)	4,860	0.372
$\Delta z < 0$	$\sum_{k=0}^{k=3} \Delta z_{t-k}$	.025	(0.502)	4,856	0.439

With  $\gamma = 3$ .

Subsample	Coefficient	Non-users minus users	$p$ -value	N. Obs.	Adj.- $R^2$
$\Delta r > 0$	$\sum_{k=0}^{k=3} \Delta r_{t-k}$	1.182**	(0.034)	4,806	0.482
$\Delta r < 0$	$\sum_{k=0}^{k=3} \Delta r_{t-k}$	1.834***	(0.005)	4,844	0.312
$\Delta z > 0$	$\sum_{k=0}^{k=3} \Delta z_{t-k}$	-.079*	(0.091)	4,860	0.378
$\Delta z < 0$	$\sum_{k=0}^{k=3} \Delta z_{t-k}$	-.037	(0.358)	4,856	0.430

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Figure 1: The growth of derivatives markets.

This figure represents the gross notional amount of all over-the-counter (OTC) derivative contracts outstanding worldwide (blue line). Its most sizeable component is the interest rate derivatives market (solid green line). Most of the exposures in the interest rate derivatives market are held by financial institutions (dashed green line). These derivatives market are compared with the global nominal GDP (red line) over the period from 1998 to 2014. Data sources: BIS (for derivatives data) and World Bank (for GDP data).

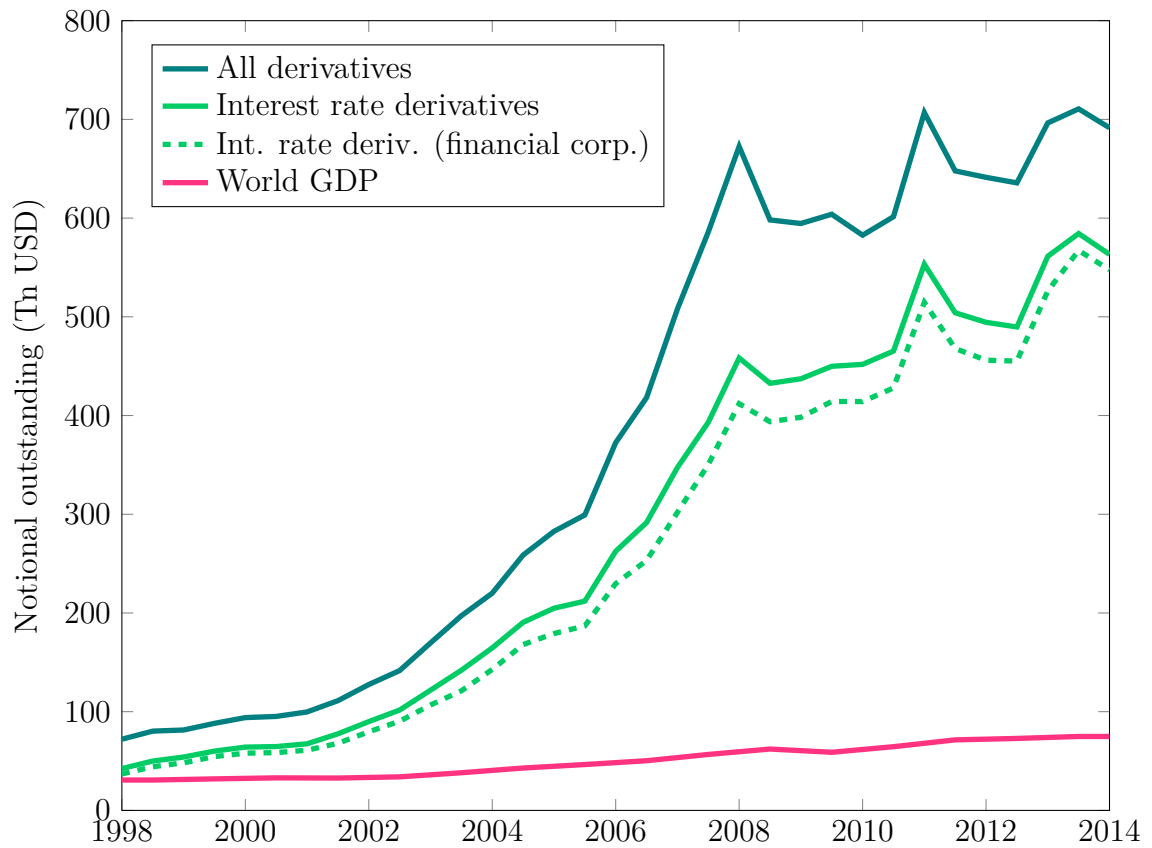
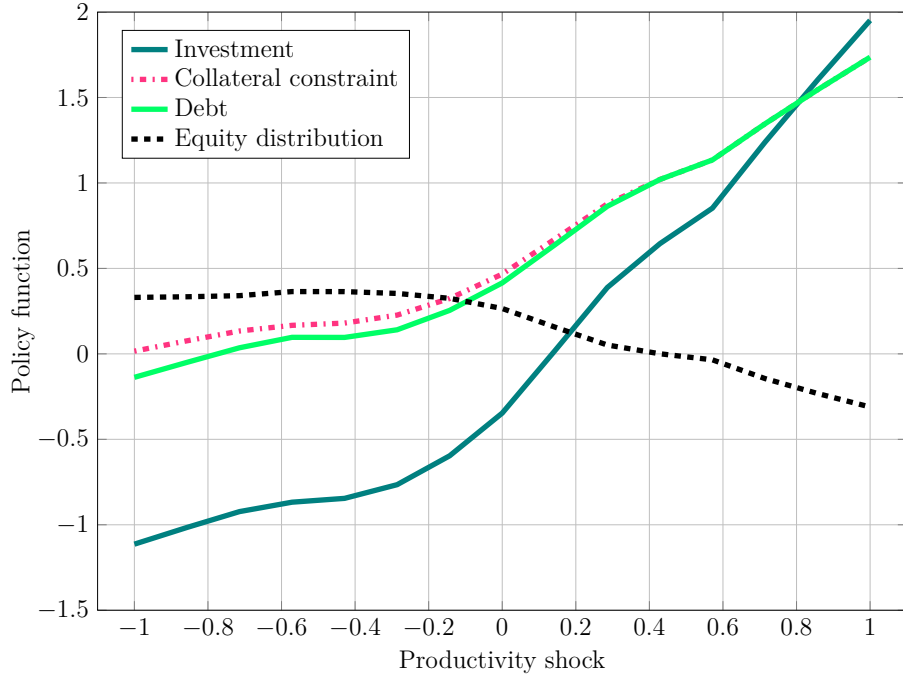


Figure 2: Policy function.

This figure depicts the policy function of the model, using the baseline calibration. Each function maps a current shock to the optimal investment (lending), debt and dividend distribution/issuance to equity holders. Panel A depicts the policy function with respect to the log productivity shock and Panel B with respect to the short rate. The policy functions are computed at the steady state asset size, defined as the asset size to which the bank would converge if it were to receive a long series of zeros shocks, i.e.  $\epsilon_z = \epsilon_r = 0$ . Each variable is normalized by this steady state asset size. The collateral constraint is also depicted. The collateral constraint is that from the model with no swaps,  $d' = 0$ .

Panel A: Policy function with respect to  $z$



Panel B: Policy function with respect to  $r$

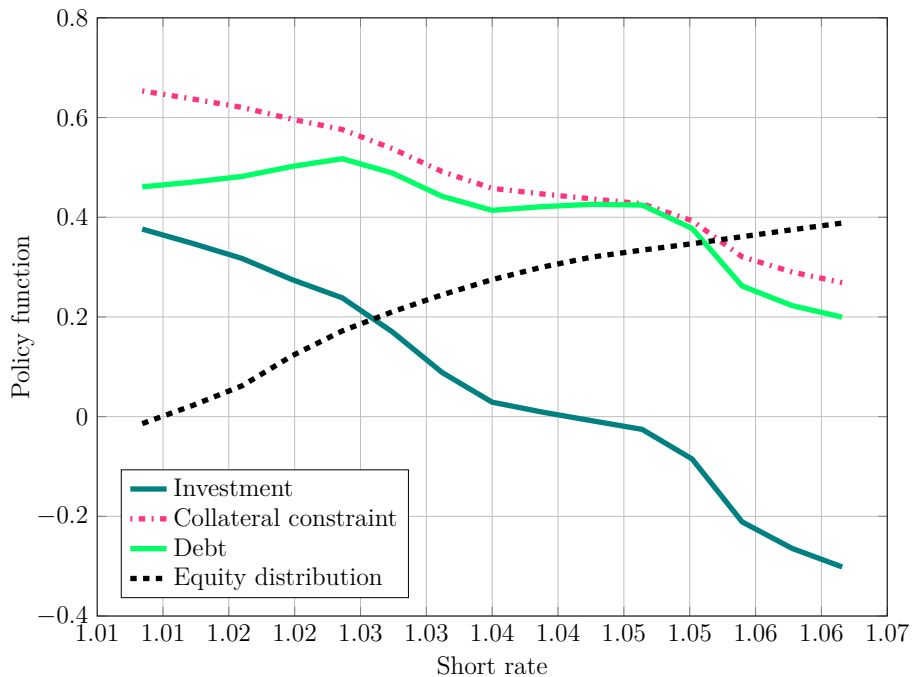




Figure 3: “Worst-case” interest rate realization  $\hat{r}$ .

This figure plots the “worst-case” interest rate realization for the bank at date  $t + 1$ , i.e.  $\hat{r}$ , given its choice of its choice of interest rate swaps  $d'$  at date  $t$ . The value of  $\hat{r}$  is given by equation (7). The value of  $\hat{r}$  is evaluated at the steady state asset stock  $a^*$ , as a function of  $d'$  and for three values of  $\gamma$ . The parameter  $\gamma$  captures the sensitivity of the bank’s cash flow to the short rate. When  $\gamma = 0$  (black line), cash flows do not depend on the realization of  $r$ . When  $d' < 0$ , it is always the case that  $\hat{r} = \bar{r}$ , because states in which cash flows from assets in place and states in which the bank is a net swap payer coincide. When  $d' > 0$ , swaps provide hedging on low cash flow realizations, and whether  $\hat{r}$  equals  $\bar{r}$  or not depends on the relative magnitude of  $\gamma$  and of the swap position  $d'$ . In this calibration,  $\bar{r} = 0.065$  and  $\bar{r} = 0.005$ .

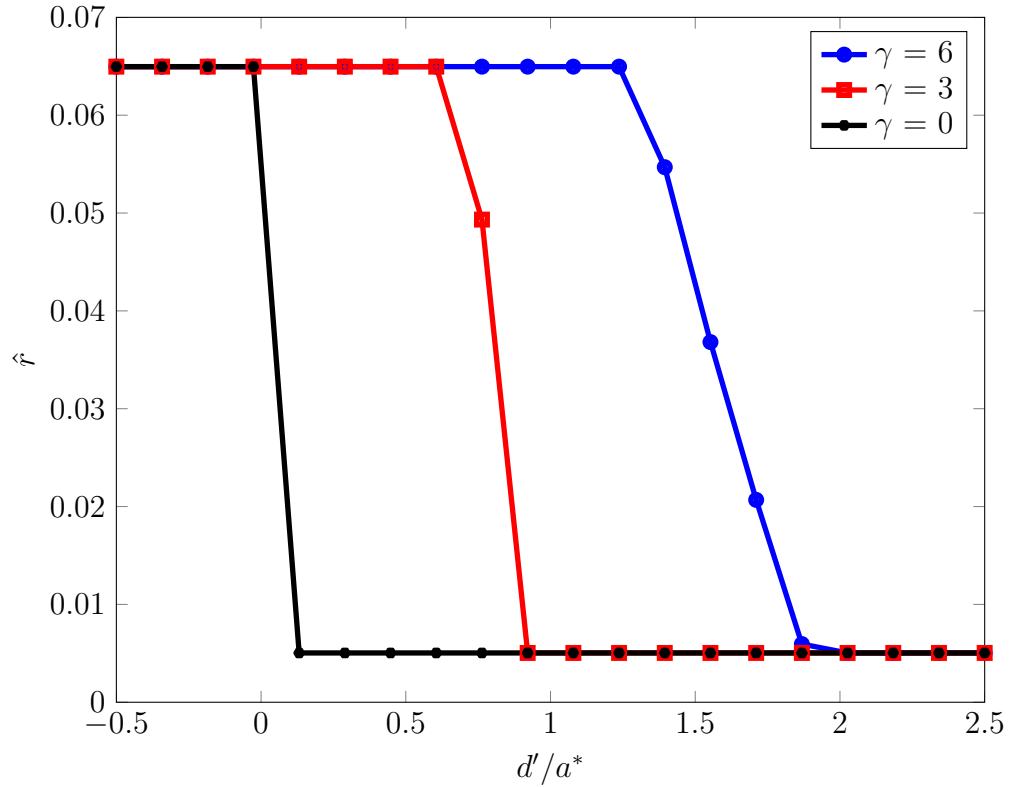


Figure 4: Hedging policy given  $\{z, r\}$ .

This figure plots the optimal hedging policy of the bank, given shock realizations  $\{z = 0, r = r^*\}$ , as a function of its net worth  $w$ . The policy function is obtained by solving the model with the baseline calibration using value function iteration. A positive value of the swap position  $d'$  denotes a pay-fixed position, while negative values denote pay-float positions. The fact that banks use pay-fixed positions when their net worth is lower, given investment opportunities (determined by  $\{z, r\}$ ), reflects the fact that pay-fixed positions are used by constrained banks to increase their present debt capacity by transferring funds to future states in which their cash flows would otherwise be low. In doing so, they increase the pledgeable value in this “worst-case” state, which determines the bank’s debt capacity, given by the collateral constraint in equation (8).

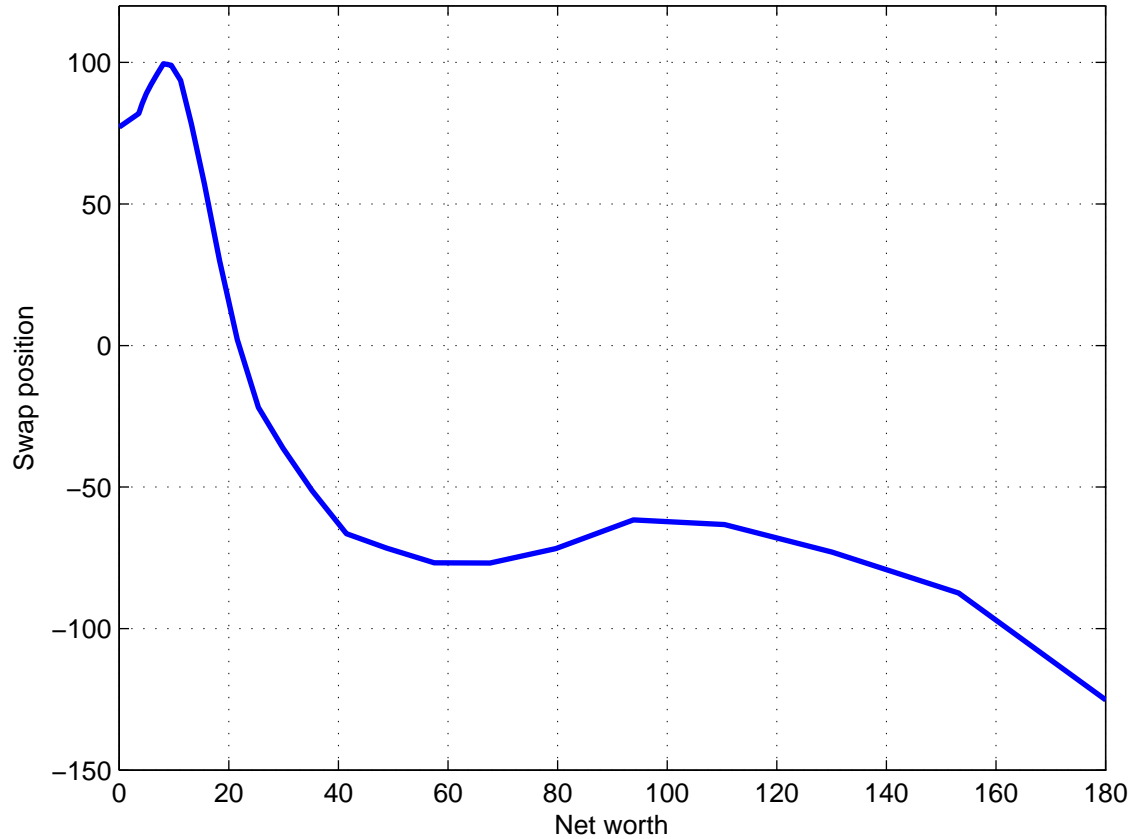


Figure 5: Net swap position of U.S. commercial banks.

This figure plots the distribution of the net swap position for U.S. commercial banks. There is one cross-sectional box plot for each quarter from 1995Q1 to 2013Q4. In each of them, the horizontal dash is the median and the diamond is the mean. The whiskers represent the 5th and 95th percentiles. The grey rectangle represents the 25th and 75th percentiles. The net swap position is computed as the difference between gross notional pay-fixed and gross notional pay-float swap positions, normalized by total assets (equation 24). A positive (resp. negative) value of the net swap position indicates a net pay-fixed (resp. pay-float) position at the bank level. The sample is restricted to a subset of commercial banks that hedge interest rate risk using interest rate swaps only, as the net swap position cannot be computed for other banks. There are between 120 and 207 banks in the sample, depending on the quarter. Data is from the call reports, obtained from the Federal Reserve Bank of Chicago. Total assets is variable  $rcon2170$  in the call reports. Swaps on which the bank pays a pay-fixed position is variable  $rcon589$  and total derivatives for purposes other than trading is  $rcon8725$  (plus  $rcon8729$  between 1995 and 2000). For banks that use only swaps for hedging, pay-float swaps are thus equal to  $rcon8725 - rcon589$ . Derivatives used for trading purposes are not included in the calculation of the net swap position.

