The Geary-Khamis index and the Lehr index: how much do they differ?

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Introduction

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- The **Geary-Khamis (GK) index is a multilateral price index** that provides transitive, hence chain-drift free, results when applied to the intertemporal context.

- Based on the GK index, the QU-method ("Quality adjusted unit value") has been proposed as a generic way for compiling price indices from scanner data.

- The (bilateral) **Lehr index** is another example of a generalized unit value index which looks **similar** to the GK index, but the compilations are much **simpler**.

The Geary-Khamis index

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- The QU-method foresees a pre-processing step that aggregates individual GTIN codes into “homogeneous” product groups.

- In a second step, the GK index is compiled based on this pre-aggregated data.

$$P_t^{GK} = \frac{\sum_{i \in N^t} p_i^t q_i^t / \sum_{i \in N^0} p_i^0 q_i^0}{\sum_{i \in N^t} v_i^{GK} q_i^t / \sum_{i \in N^0} v_i^{GK} q_i^0}$$

$$v_i^{GK} = \sum_{z \in T} \varphi_i^z \frac{p_i^z}{P_z^{GK}} \qquad \varphi_i^z = \frac{q_i^z}{\sum_{t \in T} q_i^t}$$

- The transformation coefficients v_i^{GK} are the key parameters in the GK index.

$$\frac{v_i^{GK}}{v_j^{GK}}$$

How many quantities of item j are equivalent to 1 quantity of item i ?

The bilateral Geary-Khamis index and the Lehr index

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- In the 2-period case, the GK index reduces to the bilateral Geary-Khamis (BGK) index:

$$P_t^{BGK} = \frac{\sum_{i \in N_0 \cap N_t} h(q_i^0, q_i^t) p_i^t}{\sum_{i \in N_0 \cap N_t} h(q_i^0, q_i^t) p_i^0}$$

where $h(q_i^0, q_i^t)$ is the harmonic mean of the quantities observed in the two comparison periods.

- See also:
  - Walsh index (geometric average of quantities)
  - Marshall-Edgeworth index (arithmetic average of quantities)

The bilateral Geary-Khamis index and the Lehr index

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- We simplify the transformation coefficients of the BGK by removing the deflator part:

$$\frac{v_i^{BGK}}{v_j^{BGK}} = \frac{\frac{p_i^0 \varphi_i^0}{P_0^{BGK}} + \frac{p_i^t \varphi_i^t}{P_t^{BGK}}}{\frac{p_j^0 \varphi_j^0}{P_0^{BGK}} + \frac{p_j^t \varphi_j^t}{P_t^{BGK}}} \longrightarrow \frac{v_i^L}{v_j^L} = \frac{p_i^0 \varphi_i^0 + p_i^t \varphi_i^t}{p_j^0 \varphi_j^0 + p_j^t \varphi_j^t}$$

- This leads us to the **Lehr index**:

$$P_t^L = \frac{\sum_{i \in N^t} p_i^t q_i^t / \sum_{i \in N^0} p_i^0 q_i^0}{\sum_{i \in N^t} v_i^L q_i^t / \sum_{i \in N^0} v_i^L q_i^0}$$

The bilateral Geary-Khamis index and the Lehr index

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- Using a Bortkiewicz decomposition, we show how the BGK index compares to the Lehr index:

$$\frac{P_t^L}{P_t^{BGK}} = 1 + RelCov_w\left(\frac{q_i^t}{q_i^0}; \frac{v_i^{BGK}}{v_i^L}\right)$$

- Both indices give identical results if:
  1. The change in quantities is the same for all items
  2. The transformation coefficients of the BGK compared to those of the Lehr are in the same proportion for all items

⇔ *The expenditure share of an item in the base period relative to the total expenditure of that item in the base and current periods must be identical for all items.*

The bilateral Geary-Khamis index and the Lehr index

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- Unlike the BGK, the Lehr index fails the proportionality test.

If the prices of all items are increasing by the same rate λ, then we have: $P_t^L < P_t^{BGK} = \lambda$.

If the prices of all items are decreasing by the same rate λ, then we have: $P_t^L > P_t^{BGK} = \lambda$.

Example:

	Prices		Quantities	
	t0	*t1*	*t0*	*t1*
Item 1	2	4	8	1
Item 2	1	2	8	16

All prices are doubled.

- BGK = 2.0000
- Lehr = 1.6154
- Unit value index = 1.4118

The augmented Lehr index

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- The augmented Lehr index is a generalized unit value index where the transformation coefficients are based on the average price over a time window T:

$$\frac{v_i^{AL}}{v_j^{AL}} = \frac{\sum_{z \in T} p_i^z \varphi_i^z}{\sum_{z \in T} p_j^z \varphi_j^z} \qquad P_t^{AL} = \frac{\sum_{i \in N^t} p_i^t q_i^t / \sum_{i \in N^0} p_i^0 q_i^0}{\sum_{i \in N^t} v_i^{AL} q_i^t / \sum_{i \in N^0} v_i^{AL} q_i^0} \qquad \forall\, t \in T$$

- This definition is a **generalization of the bilateral case**.
- The augmented Lehr index satisfies **transitivity**.
- It is based on the assumption that the **quality difference** can be (implicitly) explained by the **difference in the average price** over the time window.
- The augmented Lehr transformation coefficients are **easier to compile** than those of the GK index.

The augmented Lehr index

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- The result obtained in the bilateral case extend to the multilateral case:
  - If prices are “increasing”, then the augmented Lehr index will understate the GK index.
  - If prices are “decreasing”, then the augmented Lehr index will overstate the GK index.

Simulations

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- The data: Luxembourg retail chain, December 2014-December 2015, for a selection of food products

- Results are compared to a monthly chained Jevons price index.
  - Approach currently adopted by STATEC
  - Items are resampled every month using a cut-off sampling technique
  - Use of filters and imputation rules

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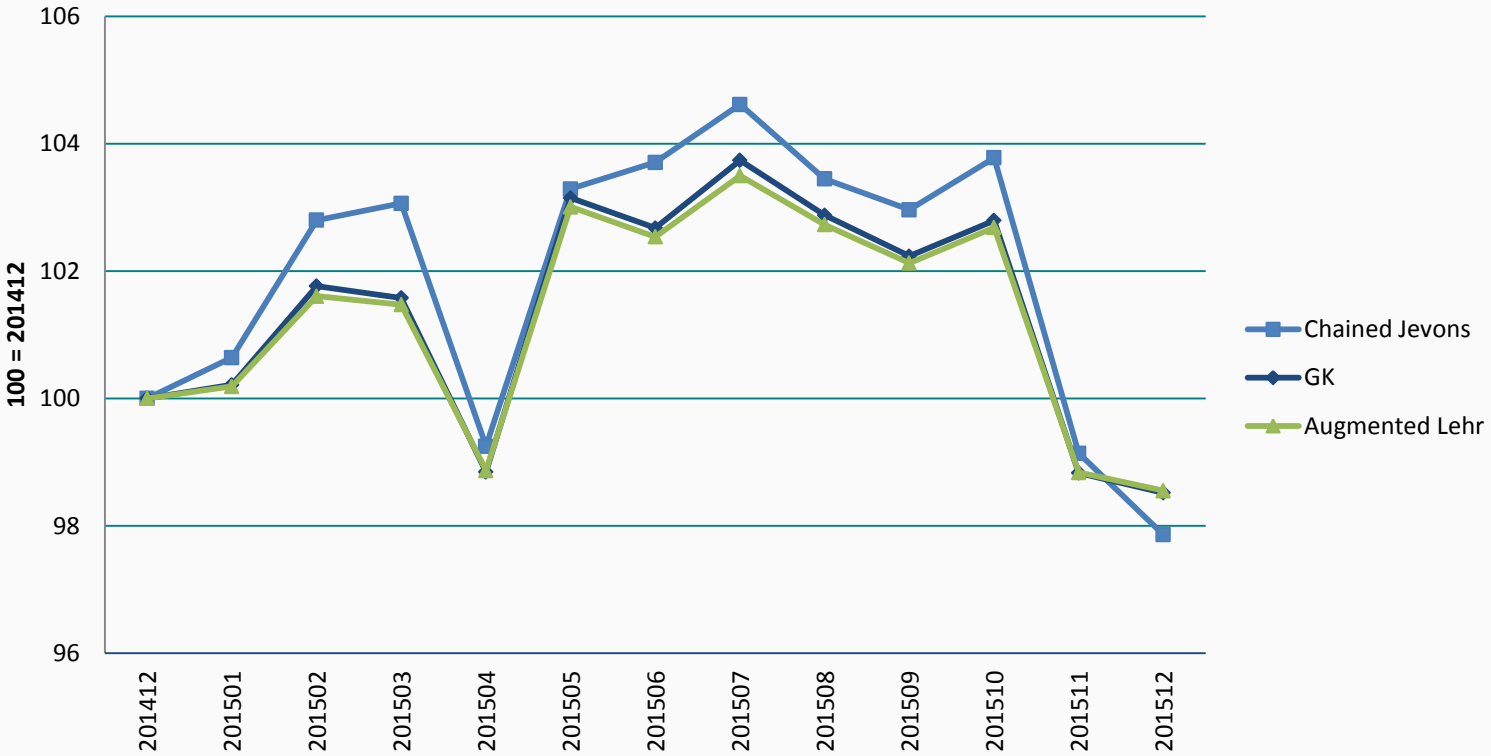
Coffee

- Average index : 101.89 (CJev); 101.32 (GK); 101.24 (AL)



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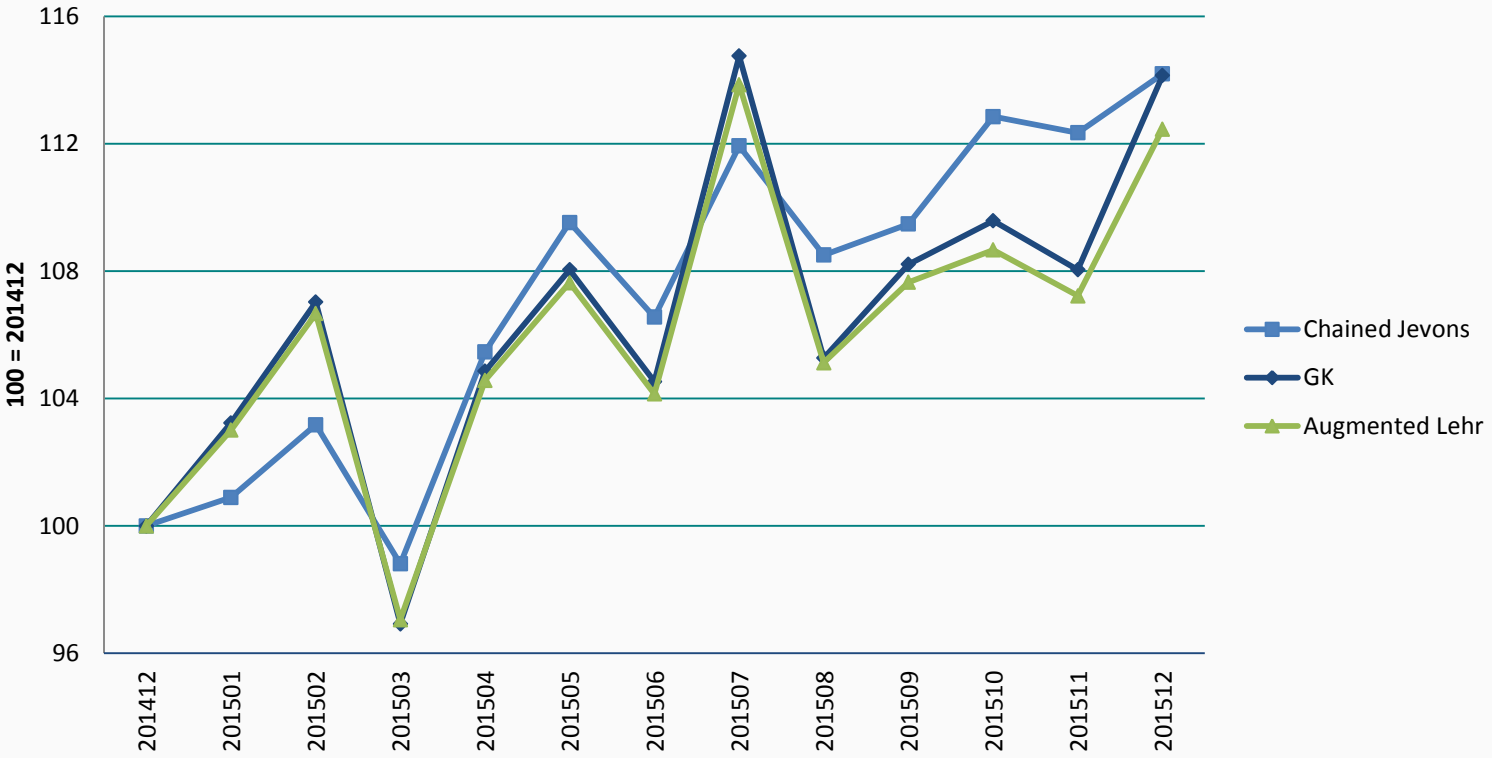
Olive oil

- Average index : 107.21 (CJev); 106.51 (GK); 106.00(AL)



Real-time indices

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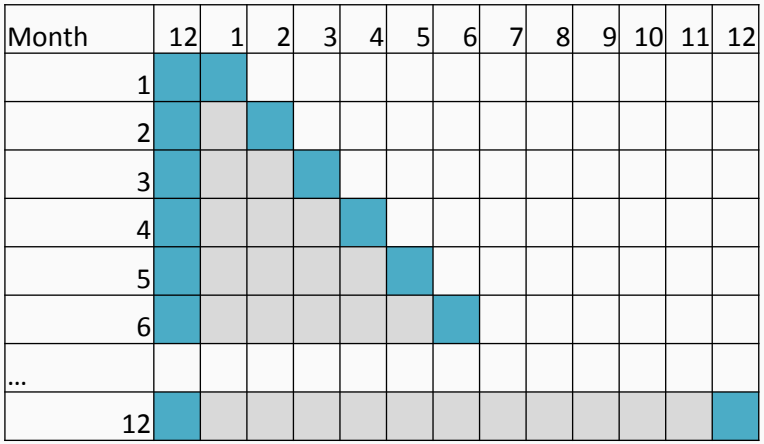
- Option 1 : “Fixed Base Enlarging Window”
  - The time window is enlarged every month by one month, starting with the December month.
  - The index compares the current month to the December month of the previous year.



Real-time indices

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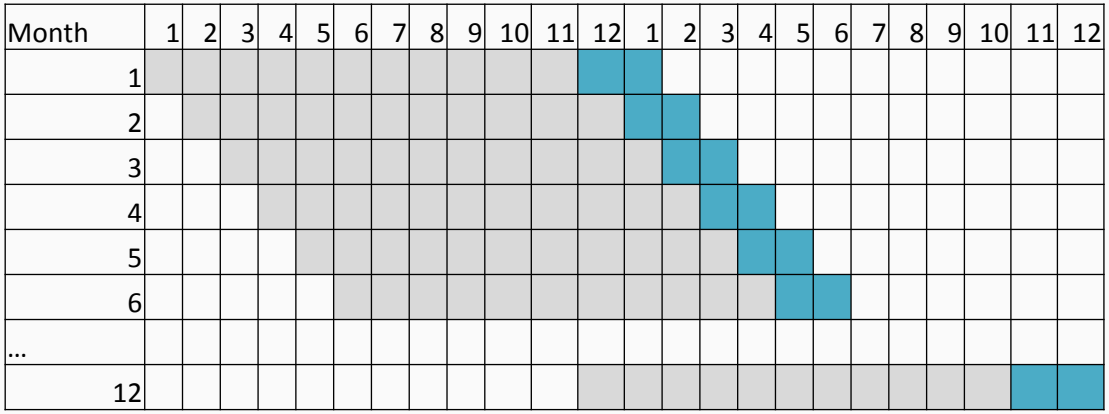
- Option 2: “Movement Splicing”
  - A moving time window of 13 months is used.
  - The price change between the last two periods of the time window is spliced onto the long term index.



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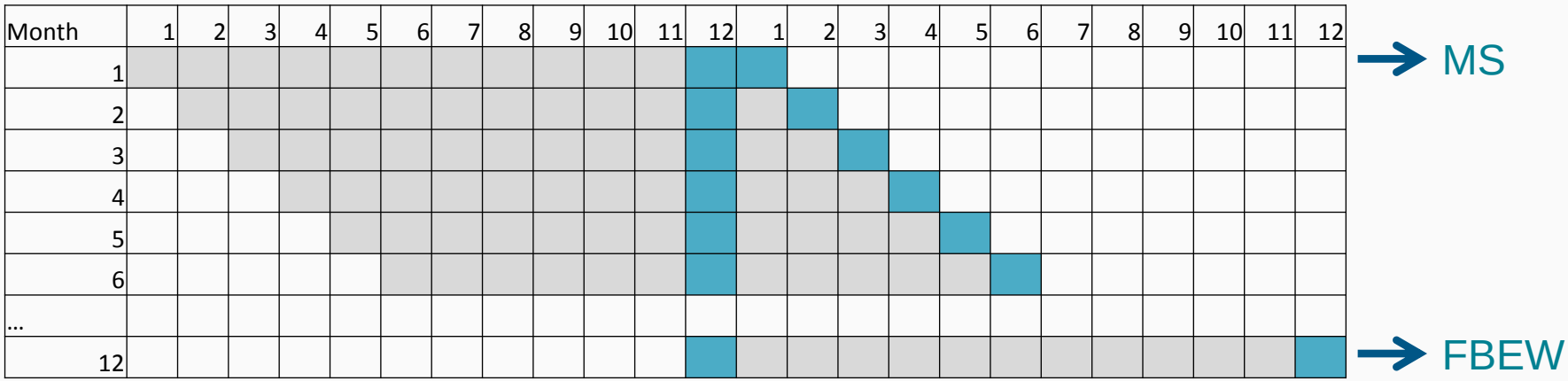
- Option 3: “Fixed Base Moving Window”
  - A moving time window of 13 month is used.
  - The index compares the current month to the December month of the previous year.



- Option 3 is a compromise between options 1 and 2:
  - In January, option 3 is equivalent to option 2 (MS).
  - In December, option 3 is equivalent to option 1 (FBEW).

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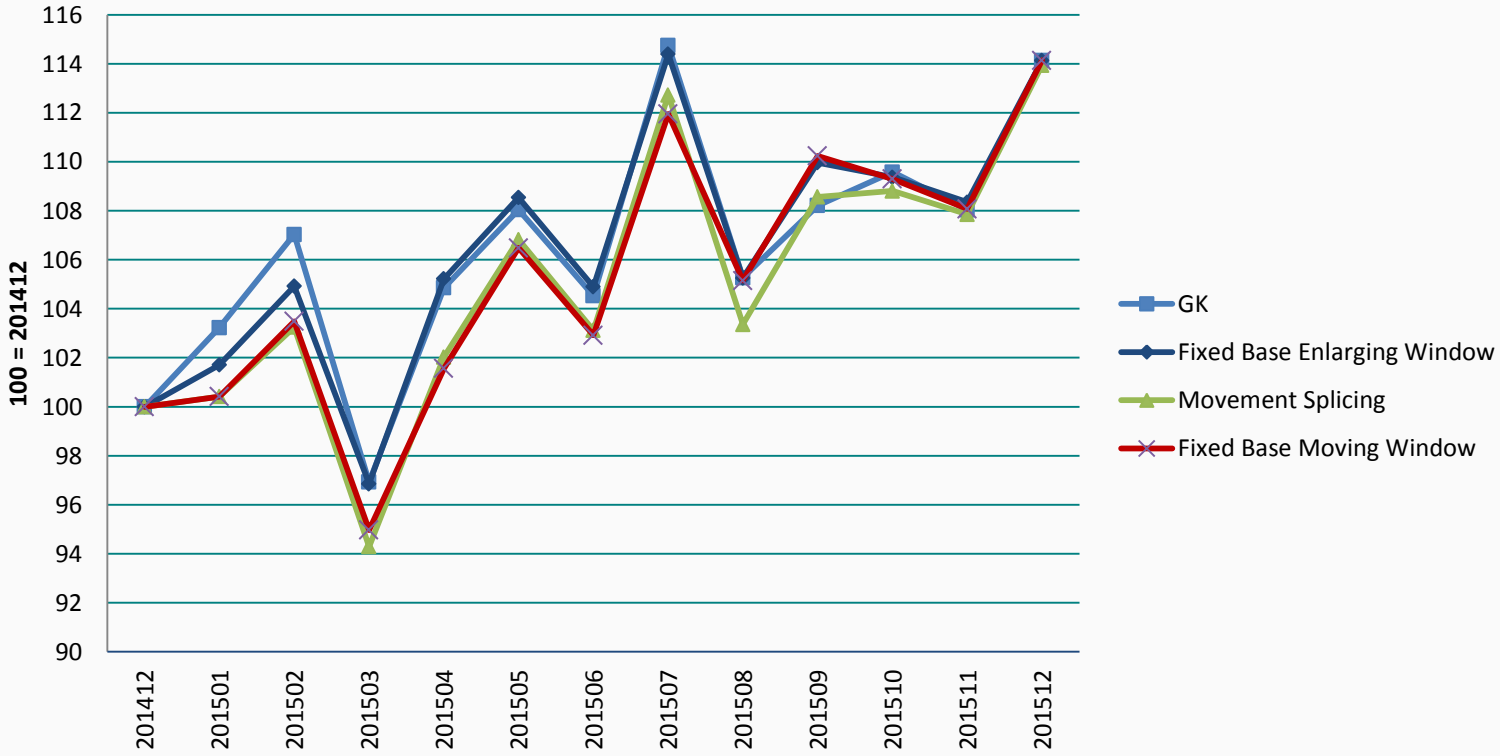
Olive Oil

- Average index : 106.51 (GK); 106.43 (FBEW); 105.01(MS); 105.28 (FBMW)



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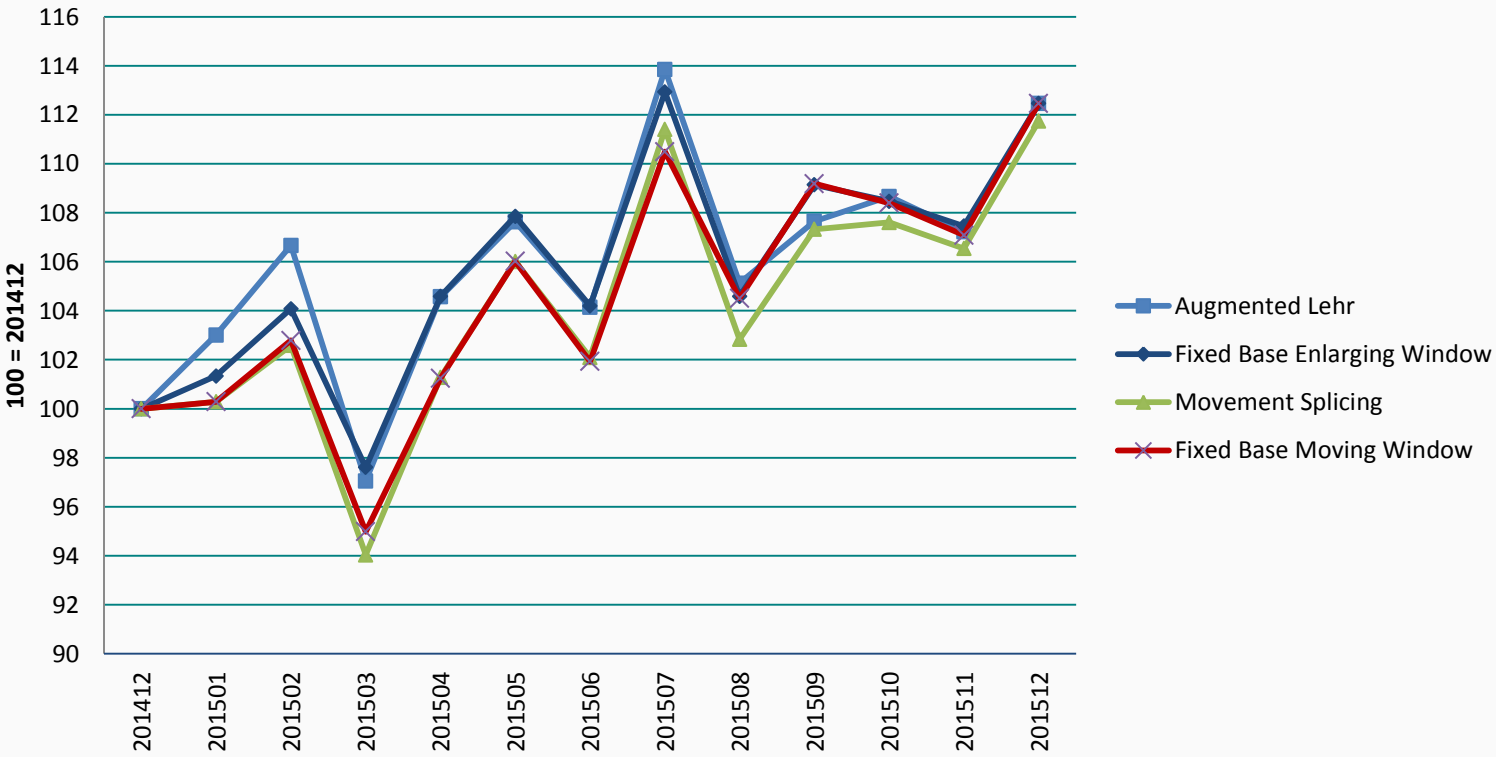
Olive Oil

- Average index : 106.00 (AL); 105.75 (FBEW); 104.13(MS); 104.57 (FBMW)



Conclusions

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- The Lehr index and its multilateral counterpart are more transparent and easier to compile than the GK index.
- Under an increasing (decreasing) price trend the Lehr index understates (overstates) the GK index.
- In practice, results are very similar.
- The strategy adopted for compiling real-time indices can matter more than the choice between GK and Lehr.

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THANK YOU !