The Geary Khamis index and the Lehr index: how much do they differ?

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Abstract: *A variant of the Geary Khamis (GK) index labelled as a "Quality Adjusted Unit Value Index" has been recently proposed (e.g. Chessa (2016)) as a generic way for compiling price indices from scanner data. In this paper, we formally link the bilateral GK index to the Lehr index which can be seen as another example of a generalized unit value index. This leads us to a multilateral extension of the Lehr index which is less complex to compile than the GK index. However, both approaches are likely to give similar results. We empirically compare these multilateral indices to a monthly chained Jevons index which is the standard approach currently adopted by STATEC for working with scanner data.*

1. Introduction

Scanner data will be increasingly available to statistical agencies and consequently new methods are needed to work with this new data source. A bilateral price index with a fixed price reference period is likely not to capture well the dynamic nature of a scanner data set, with products continuously entering and leaving the market. At the same time it is known that period-to-period chaining of a matched superlative price index leads to chain drift (see de Haan and van der Grient (2011)).

Multilateral methods that are typically used in international comparisons have been found to be a solution to this problem. Initially, these methods have been developed to make comparisons in space, but they can also be used to make comparisons over time. On the one hand, these methods are transitive, hence leading to chain-drift free results. On the other hand,

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they are well adapted to a dynamic universe by taking into account price and quantity data that refer to more than just two periods.

One such example is the (Rolling Year) GEKS index (Ivancic et al (2011)). This method combines into a transitive index the bilateral price indices compiled between two periods belonging to a given time window. Another prominent example is the Time Product Dummy Method (de Haan and Krsinich (2014)). In this method, a regression model is estimated on the pooled data, assigning a dummy variable to each period and to each item.

Recently, a variant of the Geary Khamis (GK) index labelled as a "Quality Adjusted Unit Value Index" (QU-method) has been proposed (Chessa (2016)) as a new generic way for compiling price indices from scanner data. This is yet another adaptation of a method that comes from the field of international price comparisons (Geary, 1958; Khamis, 1972). This method was also assessed by the Australian Bureau of Statistics (2016) as one of the options to compile a CPI from transaction data.

The idea of a quality adjusted unit value index has already been proposed by Dalén (2001) and by de Haan (2002). A unit value price index compares the average price level change between two periods. In a quality adjusted unit value index, transformation coefficients are introduced that express how many quantities of item i are equivalent to 1 quantity of item j. A general framework was proposed by von Auer (2014) who formalized the concept of a generalized unit value index.

Although empirical results of the GK index look promising, the implications of using this method in a time series context are not entirely clear. In order to get a better understanding of this type of approach, we focus in this paper on the Lehr index (Lehr (1885)) which is another example of a generalized unit value index. The definition of the Lehr index is very similar to the one of the GK index. However, the Lehr index is easier to compute because there is no system of equations to be solved.

This paper is organized as follows. In section 2, the GK index is described in more detail whereas section 3 focuses on the bilateral GK index. In section 4, the Lehr index and its relationships with the bilateral GK index are highlighted. An augmented version of the Lehr index is introduced in section 5. The compilation of real-time indices will be discussed in section 6. Some empirical results are provided in section 7. It is shown that the augmented Lehr index and the GK index provide very similar results.

2. The Geary Khamis index

The QU-method described by Chessa (2016) foresees two stages. In a first stage, homogeneous product groups are defined. Items with possibly different GTIN codes but the same characteristics are clustered together. The unit value prices and total quantities defined per product group will enter the compilations of the GK index in a second stage.

There are practical challenges to build these groups. The definition of the groups is judgmental and in practice driven by data availability. If groups are defined too broadly, then there is a risk of a unit value bias. If groups are described too tightly, then there is the problem of not properly capturing price changes related to the "same" product. In principle, such a pre-processing step of grouping different items could also be applied in other scanner data methods and is not necessarily specific to the GK index. We focus in this paper on the second stage of the QU-method that is related to the GK index.

We assume that prices and quantities are available for the different items over a time window denoted by *T*. Using the notations introduced by Chessa (2016), the GK index can be defined as follows:

$$P_t^{GK} = \frac{\sum_{i \in N^t} p_i^t q_i^t / \sum_{i \in N^0} p_i^0 q_i^0}{\sum_{i \in N^t} v_i^{GK} q_i^t / \sum_{i \in N^0} v_i^{GK} q_i^0} \tag{1}$$

where the v_i are the transformation coefficients:

$$v_i^{GK} = \sum_{z \in T} \varphi_i^z \frac{p_i^z}{P_z^{GK}} \tag{2}$$

$$\varphi_i^z = \frac{q_i^z}{\sum_{t \in T} q_i^t} \tag{3}$$

This index can be understood as an implicit price index. In fact, a value index is divided by a quantity index[2]. The key parameter in the quality-adjusted unit values are thus the transformation coefficients. In the GK method, the transformation coefficients are implicitly defined by all the other prices data that span over a time horizon *T*. Moreover, they are obtained by solving a system of equations as the overall index is included both in equations 1 and 2.

In order to better grasp how the GK index performs, it is of utmost importance to understand how the transformation coefficients react to different data situations. Technically, multiplying all the transformation coefficients with a constant will not change the results of the quality-adjusted unit value price index. What matters is the ratio of the adjustment coefficient $\frac{v_i}{v_j}$ of one item compared to another item. This ratio of the transformation coefficients between the two items *i* and *j* indicates how many quantities of item *j* are equivalent to 1 quantity of item *i*.

3. The bilateral Geary Khamis index

To simplify the analysis, we now assume only two time periods. We thus go back from a multilateral to a bilateral situation. With only two periods, it is known that the GK index reduces to the so called bilateral GK index formula (see also Chessa (2016)):

$$P_t^{BGK} = \frac{\sum_{i \in N_0 \cap N_t} h(q_i^0, q_i^t) p_i^t}{\sum_{i \in N_0 \cap N_t} h(q_i^0, q_i^t) p_i^0} \tag{4}$$

where $h(q_i^0, q_i^t)$ is the harmonic mean of the quantities observed in the two comparison periods. This index is similar to the Walsh index which uses a geometric mean and to the Edgeworth index which uses an arithmetic mean instead of the harmonic mean used in the bilateral GK index. Diewert (2005) notes that: *"All three indexes will approximate each other to the second order around an equal price and quantity point. Thus while the Geary Khamis bilateral index number formula is not superlative, it will approximate a superlative index to the second order around an equal price and quantity point."*

[2] This index is part of the family of additive methods. According to Paragraph 16.62 of the ILO manual, this type of method must satisfy the additivity test which states that the implicit quantity price index has the form of a Lowe quantity index.

The bilateral GK price index is based on a matched sample. This means that an item which is available in only one of the two comparison periods has no impact on the result. The numerator and the denominator of the bilateral GK index are denoted as follows:

$\delta^t = \sum_{i \in N_0 \cap N_t} h(q_i^0, q_i^t)\, p_i^t$ and $\delta^0 = \sum_{i \in N_0 \cap N_t} h(q_i^0, q_i^t)\, p_i^0$

Applying equation 2 to the 2-period case, we then have (see also Peter von der Lippe (2007), p. 542):

$$\frac{v_i^{BGK}}{v_j^{BGK}} = \frac{\dfrac{p_i^0 \varphi_i^0}{P_0^{BGK}} + \dfrac{p_i^t \varphi_i^t}{P_t^{BGK}}}{\dfrac{p_j^0 \varphi_j^0}{P_0^{BGK}} + \dfrac{p_j^t \varphi_j^t}{P_1^{BGK}}} = \frac{\dfrac{p_i^0 \varphi_i^0}{1} + \dfrac{p_i^t \varphi_i^t}{\delta^t/\delta^0}}{\dfrac{p_j^0 \varphi_j^0}{1} + \dfrac{p_j^t \varphi_j^t}{\delta^t/\delta^0}} = \frac{p_i^0 \varphi_i^0 \delta^t + p_i^t \varphi_i^t \delta^0}{p_j^0 \varphi_j^0 \delta^t + p_j^t \varphi_j^t \delta^0} \qquad (5)$$

The ratio of the transformation coefficients between the two items *i* and *j* thus depends on the average prices of these two items, but all the other items also play a role because prices of the current period are deflated with the overall bilateral GK index.

4. The Lehr index

The bilateral GK index can be seen as a generalized unit value price index with the transformation coefficients shown by equation 5. In fact, there are many alternative options how the transformation coefficients can be defined (see Von Auer (2014)). For instance, if the coefficients v_i correspond to the prices of the base period, then the quality-adjusted unit value price index reduces to a Paasche index. If the coefficients v_i correspond to the prices of the current period, then we have a Laspeyres index.

A simplification of the BGK coefficients consists in removing the deflator part:

$$\frac{v_i^L}{v_j^L} = \frac{p_i^0 \varphi_i^0 + p_i^t \varphi_i^t}{p_j^0 \varphi_j^0 + p_j^t \varphi_j^t} \qquad (6)$$

With such a definition of the transformation coefficients, the generalized unit value index reduces to the Lehr index (Lehr (1885)):

$$P_t^L = \frac{\sum_{i \in N^t} p_i^t q_i^t / \sum_{i \in N^0} p_i^0 q_i^0}{\sum_{i \in N^t} v_i^L q_i^t / \sum_{i \in N^0} v_i^L q_i^0} \qquad (7)$$

It may be plausible under certain circumstances that quality differences are derived from price differences. The interpretation of the transformation coefficients for the bilateral GK index seems more complex because they take into account the prices of all the other items. From this perspective, the Lehr index looks more transparent than the bilateral GK index.

We now compare the Lehr index to the bilateral GK price index.

$$\frac{P_t^L}{P_t^{BGK}} = \frac{\left(\sum_{i\in N^t} p_i^t q_i^t / \sum_{i\in N^0} p_i^0 q_i^0\right) \Big/ \left(\sum_{i\in N^t} v_i^L q_i^t / \sum_{i\in N^0} v_i^L q_i^0\right)}{\left(\sum_{i\in N^t} p_i^t q_i^t / \sum_{i\in N^0} p_i^0 q_i^0\right) \Big/ \left(\sum_{i\in N^t} v_i^{BGK} q_i^t / \sum_{i\in N^0} v_i^{BGK} q_i^0\right)} \quad (8)$$

Consequently, it follows that:

$$\frac{P_t^L}{P_t^{BGK}} = \frac{\dfrac{\sum_{i\in N^t} v_i^{BGK} q_i^t}{\sum_{i\in N^t} v_i^L q_i^t}}{\dfrac{\sum_{i\in N^0} v_i^{BGK} q_i^0}{\sum_{i\in N^0} v_i^L q_i^0}} \quad (9)$$

Using a Bortkiewicz decomposition[3], this is equal to a weighted relative covariance[4], with weights corresponding to $w_i = \frac{v_i^L q_i^0}{\sum_k v_k^L q_k^0}$:

$$\frac{P_t^L}{P_t^{BGK}} = 1 + RelCov_w\left(\frac{v_i^{BGK}}{v_i^L}\ ;\ \frac{q_i^t}{q_i^0}\right) \quad (10)$$

This decomposition helps to understand what drives the difference between these two solutions. Recall that:

$$\frac{v_i^{BGK}}{v_i^L} = \frac{p_i^0 \varphi_i^0 + \dfrac{p_i^t \varphi_i^t}{P_t^{BGK}}}{p_i^0 \varphi_i^0 + p_i^t \varphi_i^t} = \frac{p_i^0 q_i^0 + \dfrac{p_i^t q_i^t}{P_t^{BGK}}}{p_i^0 q_i^0 + p_i^t q_i^t} \quad (11)$$

In the right-hand side of equation 11, the overall price change P_t^{BGK} is identical for all items. The ratio of the BGK and the Lehr coefficient differs by item because of different item

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[3] To apply this decomposition, we must assume that the set of items remains constants: $N_t = N_0$.

[4] The weighted relative covariance is defined here as follows : $RelCov_w(a,b) = \frac{\sum_i w_i (a_i - \sum_j w_j a_j)(b_i - \sum_j w_j b_j)}{(\sum_i w_i a_i)(\sum_i w_i b_i)}$

expenditures. It follows that the Lehr index and the bilateral GK index both lead to the same results if the relative covariance is equal to zero. This happens if one of the two variables has a zero variance. This means that at least one of the following two conditions is satisfied:

- $\frac{p_i^0 q_i^0}{p_i^0 q_i^0 + p_i^t q_i^t} = constant \quad \forall\, i$
- $\frac{q_i^t}{q_i^t} = constant \quad \forall\, i$

The first condition states that the expenditure share of an item in the base period relative to the total expenditure of that item in the base and current periods must be identical for all items. The second condition covers the trivial situation where all quantities change at the same rate. Apart from these degenerated cases, it would be interesting to say something about the sign of this covariance.

Let us for instance assume that all prices are changing by the same rate $\frac{p_i^t}{p_i^0} = \lambda$. It is known that the bilateral GK index then also changes by the same rate. The first variable of the covariance (see equation 11) can thus be rewritten as follows:

$$\frac{v_i^{BGK}}{v_i^L} = \frac{p_i^0 q_i^0 + \frac{p_i^t q_i^t}{P_t^{BGK}}}{p_i^0 q_i^0 + p_i^t q_i^t} = \frac{1 + \frac{p_i^t}{p_i^0}\frac{1}{P_t^{BGK}}\frac{q_i^t}{q_i^0}}{1 + \frac{p_i^t}{p_i^0}\frac{q_i^t}{q_i^0}} = \frac{1 + \frac{q_i^t}{q_i^0}}{1 + \lambda\frac{q_i^t}{q_i^0}} \qquad (12)$$

If $\lambda > 1$, then this is a decreasing function of $\frac{q_i^t}{q_i^0}$. This means the covariance between this term and $\frac{q_i^t}{q_i^0}$ will be negative[5]. In other words, if prices are increasing all at the same rate, then we must have that $P_t^L < P_t^{BGK}$. Similarly, if prices are decreasing ($\lambda < 1$), then the covariance will be positive and consequently $P_t^L > P_t^{BGK}$. This indicates that the absolute value of the price increase or decrease will be lower in the Lehr index than in the bilateral GK index.

This discussion also highlights that the Lehr index violates the proportionality axiom (see Von Auer 2014). This axiom states that if all individual prices change by the same factor λ, then the price index must also change by that same rate. It is known that the bilateral GK index satisfies this axiom. However, the Lehr index will either understate (if prices are increasing)

[5] Except for the trivial case in which the change in quantities is identical across all items. In such a situation, the covariance collapses to zero.

or overstate (if prices are decreasing) this factor λ . Empirically, the difference between the Lehr index and the bilateral GK index may not be so large if the price change remains “moderate” (see section 7).

Related to the proportionality axiom is the proportionality in current prices[6] test which is referred to as T5 in the system of 20 tests that characterizes the Fisher price index. Just as the unit value price index, the Lehr index fails this test[7]. The ILO manual states that *“most index number theorists regard this property as a very fundamental one that the index number formula should satisfy.”* Compilers that use the Lehr index must thus be aware of the theoretical properties of this index.

In a dynamic context, the items available in the base period are not necessarily the same as the ones available in the current period. The bilateral GK is a matched index in the sense that only items that are available in both periods impact the result (see equation 4). In the Lehr index however, the impact of items that are only available in one of the two comparison periods is not completely neutral.

In fact, the transformation coefficient for an item that is only available in one of the two comparison periods is simply equal to the price of that item:

$$v_i^L = p_i^t \quad if\ i \in N_t \backslash N_0 \qquad (13)$$

$$v_i^L = p_i^0 \quad if\ i \in N_0 \backslash N_t \qquad (14)$$

Consequently, the Lehr index can be defined as follows in a dynamic context:

$$P_t^L = \frac{\left(\sum_{i \in N^t \cap N^0} p_i^t q_i^t + \sum_{i \in N^t \backslash N^0} p_i^t q_i^t\right) / \left(\sum_{i \in N^t \cap N^0} p_i^0 q_i^0 + \sum_{i \in N^0 \backslash N^t} p_i^0 q_i^0\right)}{\left(\sum_{i \in N^t \cap N^0} v_i^L q_i^t + \sum_{i \in N^t \backslash N^0} p_i^t q_i^t\right) / \left(\sum_{i \in N^t \cap N^0} v_i^L q_i^0 + \sum_{i \in N^0 \backslash N^t} p_i^0 q_i^0\right)} \qquad (15)$$

This can be decomposed into a “matched” Lehr index and other factors:

[6] Symmetrically, there is also the test T6 on inverse proportionality in base period prices.
[7] It is straightforward to check that “Proportionality in current prices” (T5) plus “Identity test” (T3) implies the “Proportionality axiom” (PA). Because the Lehr index satisfies (T3) but not (PA), it must also fail (T5).

$$P_t^L = \frac{\sum_{i \in N^t \cap N^0} p_i^t q_i^t / \sum_{i \in N^t \cap N^0} p_i^0 q_i^0}{\sum_{i \in N^t \cap N^0} v_i^L q_i^t / \sum_{i \in N^t \cap N^0} v_i^L q_i^0} \frac{\left(1 + \frac{\sum_{i \in N^t \setminus N^0} p_i^t q_i^t}{\sum_{i \in N^t \cap N^0} p_i^t q_i^t}\right)}{\left(1 + \frac{\sum_{i \in N^t \setminus N^0} p_i^t q_i^t}{\sum_{i \in N^t \cap N^0} v_i^L q_i^t}\right)} \frac{\left(1 + \frac{\sum_{i \in N^0 \setminus N^t} p_i^0 q_i^0}{\sum_{i \in N^t \cap N^0} v_i^L q_i^0}\right)}{\left(1 + \frac{\sum_{i \in N^0 \setminus N^t} p_i^0 q_i^0}{\sum_{i \in N^t \cap N^0} p_i^0 q_i^0}\right)} \quad (16)$$

This formally shows how the Lehr index is affected by new or disappearing items. In general, the factors in brackets do not cancel out. Consequently, the Lehr index differs from a "matched" Lehr index.

5. The augmented Lehr index

The bilateral GK index is a special case of the GK index where the time window is restricted to two periods only. In section 4, the bilateral Lehr index has been introduced. We are now going to expand the definition of this index in the context of a larger time window. The augmented Lehr index is a generalized unit value index where the adjustment coefficients are based on the average price over a time window *T* (see Von Auer (2016)):

$$\frac{v_i^{AL}}{v_j^{AL}} = \frac{\sum_{z \in T} p_i^z \varphi_i^z}{\sum_{z \in T} p_j^z \varphi_j^z} \quad (17)$$

The augmented Lehr index is then defined as follows:

$$P_t^{AL} = \frac{\sum_{i \in N^t} p_i^t q_i^t / \sum_{i \in N^0} p_i^0 q_i^0}{\sum_{i \in N^t} v_i^{AL} q_i^t / \sum_{i \in N^0} v_i^{AL} q_i^0} \qquad \forall\, t \in T \quad (18)$$

The underlying idea is that transforming units of item *i* into units of item *j* is based on the difference in the average price over the time window of item *i* and *j*. This definition is clearly a generalization of the bilateral case. With a 2-period time window, this boils down to the standard Lehr index. Compared to the GK index, it does not rely on a deflating factor measuring the price change across all items. Because the transformation coefficients for each item are fixed over the entire time window *T*, the augmented Lehr index satisfies transitivity over that same time window.

From an operational point of view, the augmented Lehr transformation coefficients are thus more transparent and easier to compute. It is of importance for compilers to be able to explain the complexity of these methods in order to gain their acceptability by users.

In fact, it can be argued that the augmented Lehr index is not a “true” multilateral price index. It is merely a “bilateral” unit value index, adjusted by a factor that is based on the average price level of each item. In fact, the item factor only depends on the prices and quantities of that same item whereas in the GK index, the item factor depends on the prices and quantities of all available items.

Similarly to equations 10 and 11, it is possible to compare the augmented Lehr index and the GK index using a Bortkiewicz decomposition. The difference between both indices can be written as a weighted relative covariance, with weights corresponding to $w_i = \frac{v_i^{AL} q_i^0}{\sum_k v_k^{AL} q_k^0}$:

$$\frac{P_t^{AL}}{P_t^{GK}} = 1 + RelCov_w\left(\frac{v_i^{GK}}{v_i^{AL}}\ ;\ \frac{q_i^t}{q_i^0}\right) \tag{19}$$

In a stylized situation, let us assume that all prices for each item are changing by the same rate $\frac{p_i^t}{p_i^0} = \lambda^t$. It is known that if prices in all periods are proportional, then the GK index only depends on these proportions (see ABS(2016)). The first variable of the relative covariance can thus be rewritten as follows:

$$\frac{v_i^{GK}}{v_i^{AL}} = \frac{\sum_{z\in T} \frac{p_i^z q_i^z}{P_z^{GK}}}{\sum_{z\in T} p_i^z q_i^z} = \frac{\sum_{z\in T} \frac{p_i^0 \lambda^z q_i^z}{\lambda^z}}{\sum_{z\in T} p_i^0 \lambda^z q_i^z} = \frac{\sum_{z\in T} q_i^z}{\sum_{z\in T} \lambda^z q_i^z} = \frac{\sum_{z\in T} \frac{q_i^z}{q_i^0}}{\sum_{z\in T} \lambda^z \frac{q_i^z}{q_i^0}} \tag{20}$$

This is a generalization of the 2 period case that was described in equation 12. Now the comparison between the GK index and the augmented Lehr index depends on all the periods. Equation 20 can be rewritten as follows:

$$\frac{\sum_{z\in T} \frac{q_i^z}{q_i^0}}{\sum_{z\in T} \lambda^z \frac{q_i^z}{q_i^0}} = \frac{\sum_{\substack{z\in T\\ z\neq t}} \frac{q_i^z}{q_i^0} + \frac{q_i^t}{q_i^0}}{\sum_{\substack{z\in T\\ z\neq t}} \lambda^z \frac{q_i^z}{q_i^0} + \lambda^t \frac{q_i^t}{q_i^0}} \tag{21}$$

The derivative of the right hand side expressed as a function of $\frac{q_i^t}{q_i^0}$ amounts to:

$$\frac{\sum_{\substack{z \in T \\ z \neq t}} \left(\lambda^z \frac{q_i^z}{q_i^0} - \lambda^t \frac{q_i^z}{q_i^0} \right)}{\left(\sum_{\substack{z \in T \\ z \neq t}} \lambda^z \frac{q_i^z}{q_i^0} + \lambda^t \frac{q_i^t}{q_i^0} \right)^2} \tag{22}$$

In a stylized situation, we assume that period *t* prices exceed the prices from the other periods[8]:

$$\lambda^z \leq \lambda^t \qquad \forall z \in T \tag{23}$$

Then equation 22 is always negative whatever $\frac{q_i^t}{q_i^0}$. This means the relative covariance of equation 19 will be negative and consequently, the Lehr index will understate the GK index. If, on the contrary, we assume the opposite[9]:

$$\lambda^z \geq \lambda^t \qquad \forall z \in T \tag{24}$$

Then equation 22 is always positive whatever $\frac{q_i^t}{q_i^0}$. This means the relative covariance of equation 19 will be positive and consequently, the Lehr index will overstate the GK index.

6. Real-time indices

When applying a multilateral method developed for spatial comparisons to a time series context, there is always the issue of revisions. As a new time period is added to the data, all the previous time periods will be revised. In a CPI context, revisions are typically to be avoided. There are practical workarounds to compile indices that are not revised but they imply giving up transitivity at some point.

In the QU-method proposed by Chessa (2016), the time window is enlarged every month by one month, starting with the December month of the previous year as the first month. The resulting price index compares current period prices to the prices of the previous December. Such an approach seems to indicate some kind of consistency with the price reference period of the HICP which corresponds to the December month of the previous year[10]. However,

[8] Note that $\lambda^0 = 1$ and consequently $1 \leq \lambda^t$.
[9] Note that $\lambda^0 = 1$ and consequently $1 \geq \lambda^t$.

[10] See Article 2(16) of Regulation (EU) 2016/792 on the harmonised indices of consumer prices and the house price index.

there is an imbalance during the year as indices at the beginning of the year rely on a shorter time window than indices at the end of the year.

An alternative strategy has been proposed in the context of the Rolling Year GEKS (Ivancic et al (2011)). According to this approach, a fixed time window length is consistently used. In practice, one may use a window of 13 months but the length of the optimal time window length remains an open issue. Once a new period t+1 becomes available and is added to the time window, the first period of the previous time window is removed. The price change between t and t+1 is then linked onto the long term price index.

Finally, a mix between both strategies could also be considered. A sequence of multilateral indices is compiled on a rolling time window always consisting of 13 months. For each of these indices, the last month of the time period is compared to the previous December month. This is in line with option 1 where the December month plays a specific role. For instance, the first time window spans from January t-1 to January t. A price change between January t and December t-1 can be derived from this. The following time window spans from February t-1 to February t. This allows the compilation of a price change between February t and December t-1.

In this third approach, the price change in January compared to the previous December is identical to the price change obtained with the second approach. At the same time, the price change in December compared to the December of the previous year is identical to the one obtained with the first approach. In the context of the HICP, option 3 would be the preferred choice. Each monthly compilation is based on a 13-month time window, ensuring that seasonal products are included at least in two months[11]. At the same time it explicitly recognizes the use of the December month as the price reference period.

In order to compile real-time indices, both the GK index and the augmented Lehr index can be extended to any of these three options.

[11] Strictly speaking, an even longer period should be considered (e.g. a 14-month period) in order to cope with changing seasonal patterns.

7. Simulations

The approach adopted by STATEC for working with scanner data is to compile a monthly chained Jevons price index. Each month, the items are resampled within each retailer and product category. A cutoff sampling based on average turnover shares is run over the matched sample of two consecutive months. In addition, different filters and imputation rules are implemented. This should for instance prevent results from being biased because of items leaving the sample at reduced prices. This is the standard method (Van der Grient (2010)) currently used by some European countries that include scanner data in the production of their CPI.

The advantage of such an approach is that it is consistent with the Jevons price index that is also used elsewhere in the CPI. The method is thus easy to explain to users. Unlike a chained Törnqvist index, it does not suffer from chain drift (Johansen (2011)). Its main limitation is that quantities at the lower levels are only used in the cutoff procedure but they do not play an explicit role in the index number formula.

We are going to compare the augmented Lehr index and the GK index to the chained Jevons index. The scanner data set covers 13 months (December 2014 to December 2015) for a selection of product categories of a retail chain of Luxembourg.

First, we do not compile "real-time" indices but use the full time window to compile the augmented Lehr index and the GK index. Because the multilateral indices are transitive, their results are independent of the choice of the price reference period. For comparison purposes, they are both expressed using December 2014 as the starting point. In addition, we also compile real-time indices for both the GK index and the augmented Lehr index using the three strategies outlined in section 6.

The first option consists in successively increasing every month the window length by one month. The January to December real-time index is based only on these two months. The February to December real-time index is based on a 3 month window (December 2014 – February 2015). Finally, the December to December index is based on a 13 month time window (December 2014 – December 2015).

The second option implements a rolling year approach. The short-term indices of the last two periods of the time window are spliced together. For instance, the January 2015 to December 2014 index is based on a time window that spans from January 2014 to January 2015. The

February 2015 to January 2015 index is based on a time window that spans from February 2014 to February 2015. The final real-time index is obtained by multiplying these two short-term indices, taking December 2014 as a starting point.

Finally, in option 3, the same time window is used than in option 2. However, the final result is not obtained by linking the month-on-month indices together. Instead, the December 2014 month is always used as the price reference period. So, for instance, the January 2015 to December 2014 real-time index is based on a time window that spans from January 2014 to January 2015. The February 2015 to December 2014 real-time index is based on data ranging from February 2014 to February 2015.

The results can be found in Table 1. Overall, the different indices tested here are broadly consistent. However, there are some lessons that can be learned from this preliminary empirical analysis. First, the chained Jevons price index overstates all the other indices while at the same time being less volatile. The Jevons price index is the only "equally weighted" price index. By ignoring quantities, possible substitution effects may not be properly captured which then has an upward impact on results.

The augmented Lehr index and the GK index provide very a similar result if the overall price change is moderate. As theory has already shown, we can even say something about the difference, if any, between both indices. If there is an increasing price trend, such as for coffee, then the GK index lies above the augmented Lehr index. If, on the contrary, prices globally go down, then the GK index lies below the Lehr index. This is for instance the case for mineral water. If there prices are more or less stable, such as for soft drinks, then both indices almost coincide. In all these circumstances, it may be acceptable to apply a simplified formula and to remove the deflator part from the estimation of the transformation coefficients.

A larger difference between the Lehr index and the GK index can be seen for olive oil. The December 2015 to December 2014 comparison consists in a 14.14% increase for the GK index but only in a 12.45% increase for the Lehr index. This means that more significant differences can occur between both indices if there are larger price changes.

Coffee	201412	201501	201502	201503	201504	201505	201506	201507	201508	201509	201510	201511	201512	Average
Chained Jevons	100,00	100,64	102,80	103,06	99,25	103,29	103,71	104,61	103,45	102,96	103,78	99,14	97,86	101,89
GK	100,00	100,21	101,76	101,58	98,85	103,15	102,68	103,74	102,88	102,23	102,79	98,83	98,52	101,32
Real time option 1	100,00	100,24	101,96	102,03	98,55	103,28	102,82	103,71	102,97	102,23	102,74	98,85	98,52	101,38
Real time option 2	100,00	100,76	101,68	102,10	99,03	104,01	103,63	104,62	103,92	103,20	103,83	99,83	99,52	102,01
Real time option 3	100,00	100,76	101,63	101,95	98,59	103,29	102,85	103,72	103,08	102,22	102,80	98,85	98,52	101,41
Lehr	100,00	100,19	101,61	101,47	98,87	103,01	102,54	103,50	102,73	102,13	102,68	98,84	98,55	101,24
Real time option 1	100,00	100,22	101,66	101,82	98,73	103,12	102,65	103,43	102,74	102,06	102,55	98,83	98,55	101,26
Real time option 2	100,00	100,71	101,24	101,67	98,78	103,60	103,22	104,02	103,46	102,78	103,40	99,53	99,24	101,66
Real time option 3	100,00	100,71	101,21	101,57	98,50	103,04	102,58	103,23	102,76	101,96	102,55	98,79	98,55	101,19

Tea	201412	201501	201502	201503	201504	201505	201506	201507	201508	201509	201510	201511	201512	Average
Chained Jevons	100,00	99,84	98,86	100,04	100,53	100,74	100,64	100,46	100,00	100,42	98,73	99,97	97,45	99,82
GK	100,00	99,85	98,80	100,03	100,70	100,50	100,46	99,95	99,72	100,33	97,46	99,95	97,04	99,60
Real time option 1	100,00	99,93	98,72	100,03	100,53	100,34	100,37	99,87	99,71	99,95	97,29	99,85	97,04	99,51
Real time option 2	100,00	100,59	98,78	100,22	100,86	100,08	100,05	99,83	99,88	100,16	97,35	99,97	97,05	99,60
Real time option 3	100,00	100,59	98,72	100,16	100,79	99,99	100,02	99,80	99,88	100,09	97,40	99,85	97,04	99,56
Lehr	100,00	99,85	98,82	100,03	100,70	100,49	100,45	99,95	99,70	100,35	97,55	99,99	97,14	99,62
Real time option 1	100,00	99,94	98,80	100,03	100,51	100,34	100,37	99,88	99,72	99,96	97,40	99,88	97,14	99,54
Real time option 2	100,00	100,56	98,76	100,18	100,84	100,06	99,99	99,82	99,83	100,12	97,38	99,93	97,07	99,58
Real time option 3	100,00	100,56	98,73	100,16	100,80	99,99	99,98	99,84	99,87	100,12	97,54	99,91	97,14	99,59

Mineral Water	201412	201501	201502	201503	201504	201505	201506	201507	201508	201509	201510	201511	201512	Average
Chained Jevons	100,00	99,20	99,35	97,74	102,01	93,96	97,93	98,36	96,33	100,44	99,12	97,71	99,71	98,60
GK	100,00	98,55	96,40	92,21	100,94	90,72	98,68	94,17	92,86	97,18	98,41	95,06	99,42	96,51
Real time option 1	100,00	99,00	97,78	93,43	101,26	91,86	98,99	94,59	92,96	97,22	98,64	95,19	99,42	96,95
Real time option 2	100,00	99,64	99,04	92,48	103,76	93,79	101,47	96,77	95,52	100,04	101,47	97,99	102,48	98,80
Real time option 3	100,00	99,64	98,31	91,09	101,92	92,08	99,68	94,39	93,32	97,68	98,94	95,35	99,42	97,06
Lehr	100,00	98,67	96,49	92,60	101,00	91,01	98,79	94,43	93,15	97,28	98,61	95,16	99,50	96,67
Real time option 1	100,00	99,08	97,97	93,99	101,26	92,28	99,05	94,83	93,28	97,33	98,83	95,30	99,50	97,13
Real time option 2	100,00	99,76	99,02	92,95	103,65	93,99	101,43	96,91	95,67	99,96	101,49	97,93	102,40	98,86
Real time option 3	100,00	99,76	98,27	91,52	101,84	92,32	99,67	94,61	93,55	97,70	99,07	95,41	99,50	97,17

Soft drinks	201412	201501	201502	201503	201504	201505	201506	201507	201508	201509	201510	201511	201512	Average
Chained Jevons	100,00	100,99	101,20	101,94	102,74	100,16	101,70	99,45	100,05	100,27	101,73	101,58	100,71	100,96
GK	100,00	100,39	99,41	99,70	101,70	98,86	101,06	97,17	99,49	98,45	100,89	101,08	98,97	99,78
Real time option 1	100,00	100,77	99,94	99,91	101,98	99,29	101,29	97,60	99,62	98,59	100,91	101,11	98,97	100,00
Real time option 2	100,00	100,79	99,66	100,16	102,66	99,58	101,72	98,18	100,17	99,21	101,61	101,81	99,68	100,40
Real time option 3	100,00	100,79	99,44	99,89	102,30	99,23	101,33	97,77	99,67	98,70	100,98	101,19	98,97	100,02
Lehr	100,00	100,30	99,42	99,73	101,54	98,87	101,01	97,26	99,52	98,51	100,89	101,07	99,00	99,78
Real time option 1	100,00	100,67	99,96	99,92	101,74	99,29	101,20	97,71	99,66	98,66	100,91	101,09	99,00	99,99
Real time option 2	100,00	100,73	99,58	100,12	102,48	99,50	101,63	98,18	100,11	99,19	101,53	101,71	99,62	100,34
Real time option 3	100,00	100,73	99,39	99,88	102,12	99,16	101,25	97,82	99,67	98,76	100,99	101,16	99,00	99,99

Olive Oil	201412	201501	201502	201503	201504	201505	201506	201507	201508	201509	201510	201511	201512	Average
Chained Jevons	100,00	100,89	103,17	98,81	105,46	109,52	106,56	111,93	108,50	109,48	112,85	112,34	114,19	107,21
GK	100,00	103,23	107,03	96,93	104,86	108,04	104,53	114,75	105,27	108,21	109,58	108,04	114,14	106,51
Real time option 1	100,00	101,71	104,92	96,85	105,22	108,54	104,90	114,39	105,26	109,96	109,37	108,35	114,14	106,43
Real time option 2	100,00	100,41	103,25	94,29	102,03	106,82	103,12	112,72	103,36	108,57	108,80	107,85	113,94	105,01
Real time option 3	100,00	100,41	103,49	94,97	101,56	106,48	102,91	111,95	105,15	110,24	109,30	108,08	114,14	105,28
Lehr	100,00	103,01	106,66	97,05	104,57	107,63	104,14	113,85	105,12	107,65	108,67	107,22	112,45	106,00
Real time option 1	100,00	101,33	104,08	97,61	104,59	107,85	104,20	112,93	104,59	109,14	108,47	107,46	112,45	105,75
Real time option 2	100,00	100,28	102,58	94,04	101,28	106,01	102,09	111,40	102,83	107,33	107,61	106,54	111,74	104,13
Real time option 3	100,00	100,28	102,79	94,98	101,24	106,02	101,93	110,49	104,50	109,19	108,40	107,08	112,45	104,57

Table 1: Price indices for coffee, tea, mineral water, soft drinks and olive oil (100=December 2014).

Except for tea, option 1 (fixed base and extending window) and option 3 (fixed base and rolling window) are closer to the transitive benchmark than option 2 (rolling window and splicing month-on month movements). This conclusion seems to hold for both the GK index and the Lehr index. For instance, the average GK index for coffee stands at 101.32. This compares with an average real-time index of 101.38 (option 1), 102.01 (option 2) and 101.41 (option 3). This comparison is by construction unfavorable to option 2 as both options 1 and 3 are designed to coincide with the transitive benchmark index in December 2015.

These simulations also show that the sign of the difference between the transitive indices and their real-time counterparts is an empirical matter. For instance, for olive oil, the real-time indices are lower than the transitive benchmark indices. For coffee, the opposite conclusion holds. At a more aggregate level, it may happen that the positive and negative differences compensate each other to a certain extent.

The differences between the real-time indices and their transitive benchmark are in most cases larger than the difference between the Lehr index and the GK index. In other words, the choice of the method for compiling real-time indices matters as much if not more than the choice between the Lehr index and the GK index at a detailed (product) level.

In all these simulations the individual items are linked at the GTIN level. There is a risk that prices changes between "similar" items with different GTIN codes are not properly captured. This relaunch problem can be solved by defining a homogenous product that consists of items with different GTIN codes. While solving the relaunch issues, this in turn can then lead to some unit value bias. It cannot be excluded that any preliminary grouping of GTIN codes can have a significant impact on results.

8. Conclusions

The GK index is one of the different multilateral methods that are seriously considered for working with scanner data. In this paper, we have investigated a simplification of this approach which can be readily used in practice. The Lehr index and its multilateral counterpart are more transparent and easier to compute. We have formally shown that under an increasing (decreasing) price trend the Lehr index understates (overstates) the GK index. However, from an empirical point of view, results are very similar. Our simulations indicate that at least at product level, the strategy adopted for compiling real-time indices can matter more than the choice between GK and Lehr. These conclusions will be further investigated on alternative scanner data sets.

References

Australian Bureau of Statistics (2016), “Making Greater Use of Transactions Data to Compile the Consumer Price Index”, Information paper No. 6401.0.60.003, Australia.

Auer, L. von (2014), “The Generalized Unit Value Index”, *Review of Income and Wealth*, 60(4), 843-861.

Auer, L. von (2016), “Processing Scanner Data by an Augmented GUV Index”, unpublished report.

Bortkiewicz, L. (1923). Zweck und Struktur einer Preisindexzahl, *Nordisk Statistik Tidsskrift*.

Chessa, A. (2016), “A new methodology for processing scanner data in the Dutch CPI”, *EURONA N°1/2016.*

Dalén, J. (2001), “Statistical targets for Price Indexes in Dynamic Universes”, Proceedings of the 6th Meeting of the International Working Group on Price Indices, Canberra, Australia.

De Haan, J. (2015), “A Framework for Large Scale Use of Scanner Data in the Dutch CPI”, Paper presented at the 14th meeting of the Ottawa Group, 20-22 May 2015, Tokyo, Japan.

De Jaan, J. (2004), Generalised Fisher Price Indexes and the Use of Scanner Data in the Consumer Price Index (CPI), *Journal of Official Statistics*, 18(1), 61-85.

De Haan, J. and van der Grient H, (2011), “Eliminating chain drift in price indexes based on scanner data”, *Journal of Econometrics*, Vol. 161.

De Haan, J. and Krsinich, F. (2014), “Time Dummy Hedonic and Quality-Adjusted Unit Value Indexes: Do They Really Differ?”, Paper presented at the Society for Economic Measurement Conference, 18-20 August 2014, Chicago, U.S.

Diewert, E. (2005), “Weighted Country Dummy Variable Regressions and Index Number Formulae”, *Review of Income and Wealth*, 51(4), 561-570

Geary, R.C. (1958), “A Note on the Comparison of Exchange Rates and Purchasing Power between Countries”, *Journal of the Royal Statistical Society, Series A*, 121(1), 97-99.

Ivancic L., Diewert WE. and Fox KJ (2011), “Scanner data, time aggregation and the construction of price indexes”, *Journal of Econometrics*, Vol. 161.

ILO/IMF/OECD/UNECE/Eurostat/The World Bank (2004). “Consumer Price Index Manual: Theory and Practice”, International Labour Office.

Johansen, I. and Nygaard R., (2011), "Dealing with bias in the Norwegian superlative price index of food and non-alcoholic beverages". Paper presented at the 12th meeting of the Ottawa Group, May 2011, Wellington, New Zealand.

Khamis, H. (1972), "A New System of Index Numbers for National and International Purposes", *Journal of the Royal Statistical Society, Series A*, 135(1), 61-85.

Lehr, J. (1885), "Beiträge zur Statistik der Preise, insbesondere des Geldes und des Holzes". J. D. Sauerländer, Frankfurt am Main.

Lippe, P. von der (2007), "Index Theory and Price Statistics", Peter Lang Verlag.

Van der Grient, H and de Haan, J. (2010), "The use of supermarket scanner data in the Dutch CPI", Technical paper, CBS.