UNSW Business School
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# Substitution Bias in Multilateral Methods for CPI Construction using Scanner Data

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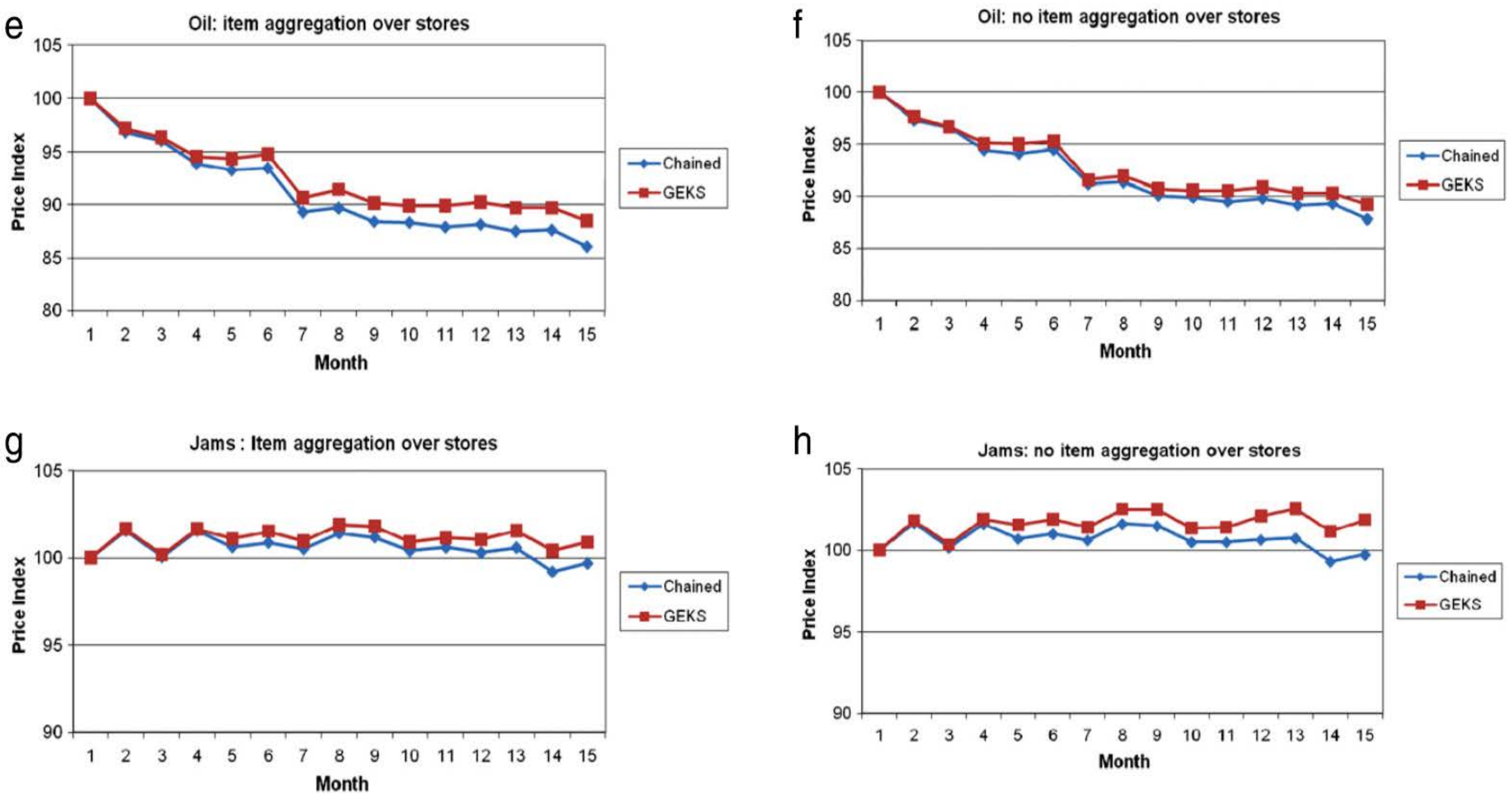
# Chain Drift Bias

## Ivancic, Diewert and Fox (2011), J. of Econometrics



# Our Paper

- **Ivancic, Diewert and Fox (2009)(2011) proposed using multilateral index numbers with transaction level data in order to avoid chain drift bias.**

- **Multilateral indexes were developed for use in cross-country comparisons (e.g. ICP, Penn World Table).**

- **Multilateral methods now used (in a limited fashion) in the CPIs of The Netherlands and New Zealand. Plans also for implementation in Australia. Many countries are experimenting.**

- **Consensus on two key issues has yet to be achieved:**
  1. **The best multilateral method to use.**
  2. **The best way of extending the resulting series when new observations become available.**

# Our paper

- Present theoretical and simulation evidence on the extent of substitution biases from using alternative multilateral methods.
- Examine GEKS, CCDI, Geary-Khamis, Weighted Time Product Dummy multilateral methods.
- Examine alternative extension methods: movement splice, full window splice, half splice, similarity linking.
- Also propose a new method, the “mean splice”.
- Results suggest the use of the CCDI multilateral index used in combination with the mean splice.

# The Chain Drift Problem

***Circularity Test*: $P(p^0,p^1,q^0,q^1)\ P(p^1,p^2,q^1,q^2) = P(p^0,p^2,q^0,q^2)$.**

**Multilateral indexes (for e.g. comparisons across countries) satisfy this test, but standard bilateral indexes do not.**

**Consider this related test:**

***Multiperiod Identity Test*: $P(p^0,p^1,q^0,q^1)P(p^1,p^2,q^1,q^2)P(p^2,p^0,q^2,q^0) = 1$**

**That is, if the prices in the third period revert back to period 0 prices, the product of all price changes should equal unity.**

***Chain drift* occurs when an index does not return to unity when prices in the current period return to their levels in the base period.**

# The Chain Drift Problem

**There are at least three possible solutions:**

1. **Stick to the usual Lowe index that uses annual expenditure weights from a past year → substitution bias.**

2. **Pick a base month and use fixed base superlative indexes relative to the chosen month → too much weight to the chosen base. Also, new and disappearing goods problem.**

3. **Use a Rolling Window multilateral index number approach adapted to the time series context, as suggested by IDF.**

**Focus on solution 3.**

# Multilateral Methods: GEKS

- **Method for making international index number comparisons between countries (Gini 1931).**

- **Suppose we have price and quantity information for a component of the CPI on a monthly basis for a sequence of 13 consecutive months.**

- **Pick one month (say k) in this augmented year as the base, construct Fisher price indexes for all 13 months relative to this base month.**

- **Denote the resulting sequence of Fisher indexes as $P_F(1/k)$, $P_F(2/k)$, ..., $P_F(13/k)$.**

- **The final set of GEKS indexes for the 13 months is the geometric mean of all 13 of the specific month indexes.**

# Multilateral Methods: CCDI

- **Caves, Christensen and Diewert (1982, EJ), Inklaar and Diewert (2016, J. Econometrics)**

- **Same idea as GEKS, but replaces the Fisher bilateral index in GEKS with the Törnqvist bilateral index.**

- **Turns out to have a nice interpretation.**
  - **Same as if the period t prices are compared to any base period's prices through an artificial "average" period.**

- **The algebra for this alternative form of the index is much simpler and can be analyzed more simply.**

# Multilateral Methods: WTPD

- **Suppose that prices vary in an *approximately proportional manner* from period to period:**

$$p_{tn} = a_t b_n e_{tn} \; ; \qquad t = 1,\ldots,T;\; n = 1,\ldots,N.$$

- **The parameter $a_t$ can be interpreted as the price level for period t, $b_n$ can be interpreted as a commodity n quality adjustment factor and $e_{tn}$ is a stochastic error term with mean 1.**

- **Taking logarithms leads to the following linear regression model:**

$$y_{tn} = \alpha_t + \beta_n + \varepsilon_{tn} \; ; \qquad t = 1,\ldots,T;\; n = 1,\ldots,N.$$

- **The $\alpha_t$ and $\beta_n$ can be estimated by solving a least squares minimization problem.**

# Multilateral Methods: WTPD

- **Rao (1995) suggested a weighted-by-economic importance version.**

- **Using expenditure shares, in our context, yields the Weighted Time Product Dummy (WTPD) approach suggested by IDF (2009).**

- **The WTPD multilateral method is recommended from the viewpoint of the economic approach to index number theory if:**

  - **Purchaser preferences are well approximated by Cobb-Douglas preferences**

    → ***Elasticity of substitution equal to one.***

  - **Purchaser preferences are well approximated by linear preferences**

    → ***Perfect substitutability*.**

    **(We show that it is an approximately additive multilateral method.)**

# Multilateral Methods: Geary-Khamis

**Total consumption vector over a time period "window":**

$$q \equiv \sum_{t=1}^{T} q^t$$

**where q ≡ [$q_1$,$q_2$,…,$q_N$].**

**Equations that determine price levels and quality adjustment factors:**

$$P_t = \frac{p^t \cdot qt}{b \cdot q^t}$$

$$b_n = \sum_{t=1}^{T} \left[\frac{q_{tn}}{q_n}\right] \left[\frac{p_{tn}}{P_t}\right]$$

**(Normalization required for a unique solution.)**

# Multilateral Methods: Geary-Khamis

**Period t quantity is then:**

$$Q_t = p^t \cdot q^t / P_t = b \cdot q^t$$

- **So it is an additive method, consistent with linear preferences**
- **Can also show (surprisingly) that GK is consistent with Leontief preferences.**
- **To summarize: the GK method is recommended from the viewpoint of the economic approach to index number theory if:**
  - **Purchaser preferences are well approximated by Leontief preferences**

    → ***Elasticity of substitution equal to zero.***
  - **Purchaser preferences are well approximated by linear preferences**

    → ***Perfect substitutability*.**

# Rolling Windows and the Linking Problem

- **A headline CPI cannot be revised from month to month.**
- **What to do when another period's data becomes available? IDF(2011):**
- **Add the data for the new period (T+1) and drop the oldest period (t=1).**
- **Multilateral indexes for the new time window (t=2,…, T+1) are calculated.**
- **Choose a linking period and extend the old window. IDF used the most recent overlapping observation (T) → "*movement splice*"**
- **The resulting indexes are called Rolling Window indexes, or for a thirteen month window, Rolling Year indexes.**

# Rolling Windows and the Linking Problem

- **IDF noted that there are other potential extension methods.**

- **Krsinich (2016): Link the windows at t=2 rather than T.**

  → "***window splice***"

- **de Haan (2015): Link period should be near the middle of the first window, i.e. t = T/2, or t = (T+1)/2 if T is odd.**

  → "***half splice***"

- **DF: *Ex ante*, each choice of linking period t = 2 to t = T is equally valid. Suggest taking the geometric mean of the period T+1 price levels obtained by using each linking period in turn.**

  → "***mean splice***"

- ***NB*: Linking method may introduce chain drift bias (but in practice, it is usually small)**

# Rolling Windows and the Linking Problem

- **An alternative is to try to link through a "similar" period: leads to the Similarity Linking Method (see the work of Robert Hill)**
- **If the price and quantity data for period T+1 are exactly equal to the data for period t, then linking the windows at observation t will preserve the identity test over the two windows**
- **Also holds for the case for equal shares and proportional price vectors.**
- **This is attractive, but need measures of "similarity" of the data between T+1 and t.**
- **Diewert (2009) proposed:**
  - **Weighted Log Quadratic Index of Relative Price Dissimilarity**
  - **Asymptotically Linear Index of Relative Price Dissimilarity**

# Rolling Windows and the Linking Problem

- **Can use these measures for linking windows in a rolling window approach.**

- **Or can use these to create a new multilateral index, using the dissimilarity measures to determine a unique "path" for bilateral index comparisons that can then be linked.**

- **For example:**
  - **Period 2 most similar to period 1**
  - **Period 3 most similar to period 1**
  - **Period 4 most similar to period 3**

$$1,\ P_F(p^1,p^2,q^1,q^2),\ P_F(p^1,p^3,q^1,q^3),\ P_F(p^1,p^3,q^1,q^3)P_F(p^3,p^4,q^3,q^4),\ldots$$

# Simulations: CES Preferences

- **A problem with existing comparisons of methods is that it is not known which method is closest to the “truth”.**
- **We constructing an artificial data set that is exactly consistent with purchasers having CES preferences over a group of related items.**

***CES unit cost function* has the following functional form:**

$$c(p_1,\ldots,p_N) \equiv [\textstyle\sum_{n=1} \alpha_n p_n{}^{1-\sigma}]^{1/(1-\sigma)} \quad \text{if } \sigma \geq 0 \text{ and } \sigma \neq 1$$

$$\equiv \textstyle\prod_{n=1} p_n^{\alpha_n} \quad \text{if } \sigma = 1$$

**where $\sigma$ and the $\alpha_n$ are positive parameters, with $\sum_{n=1} \alpha_n = 1$.**

**The parameter $\sigma$ is the *elasticity of substitution.***

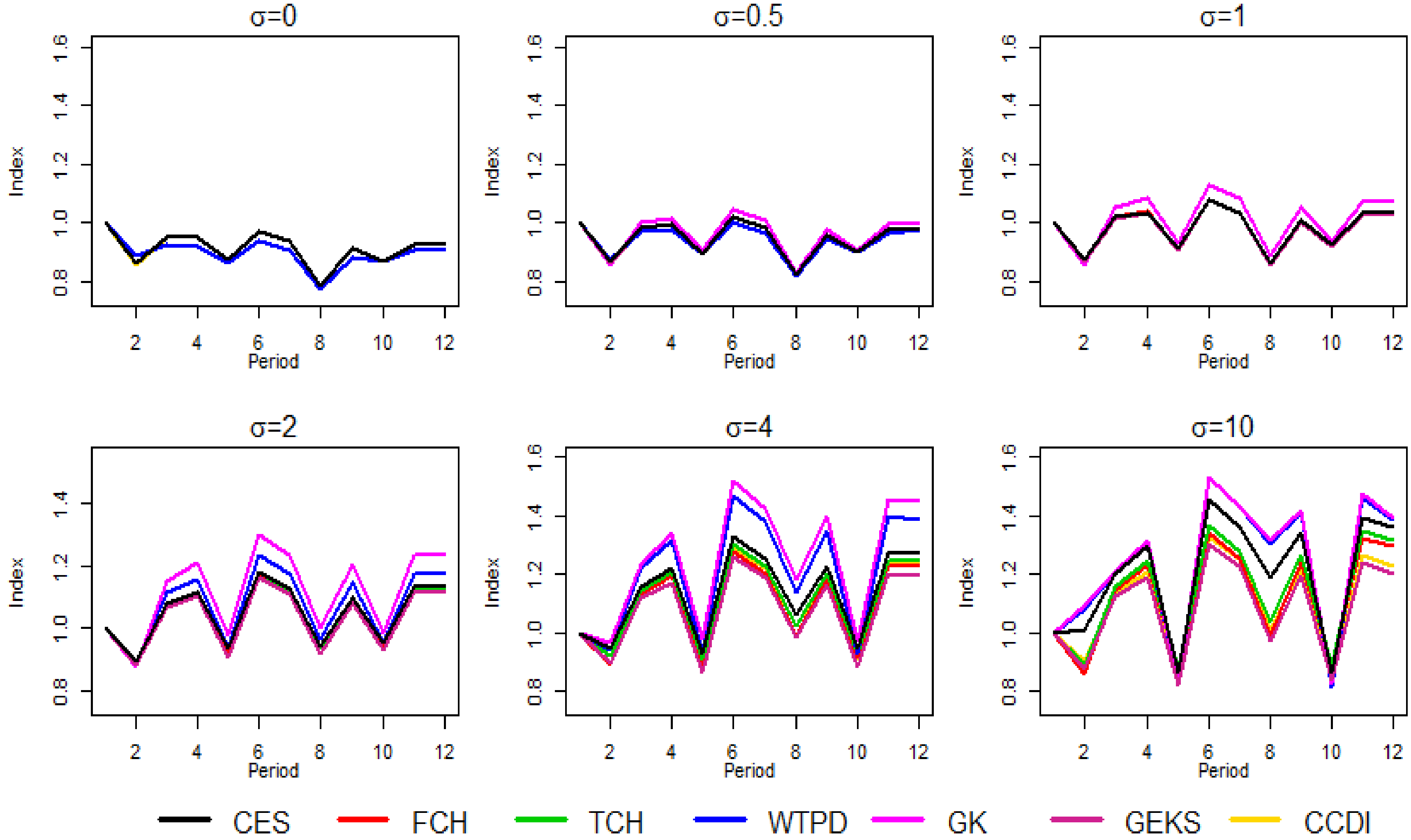
# Simulations: CES Preferences

- **T = 12 and N = 4**
- **$\alpha \equiv [\alpha_1,\alpha_2,\alpha_3,\alpha_4] \equiv [0.2, 0.2, 0.2, 0.4]$**
- **$\sigma$ will take on the values 0, 0.5, 1, 2, 4, 10 and 20**
- **In the scanner data context, it is likely that $\sigma$ is between 1 and 5.**
- **Set up:**
  - **Prices of commodities 1 and 3 trend downward while the prices of commodities 2 and 4 trend upward.**
  - **The trends in commodities 1 and 4 are very smooth but the trends in commodities 2 and 3 are interrupted by sales: item 2 goes on sale in periods 2 and 8 and item 3 goes on sale in periods 5 and 10.**
  - **Total expenditures trend upwards except in the four periods after a sale when aggregate expenditures fall a bit.**

# Table 3: Price and Expenditure Data for the Artificial Data Set

| $t$ | $p_{t1}$ | $p_{t2}$ | $p_{t3}$ | $p_{t4}$ | $e_t$ |
|---|---|---|---|---|---|
| 1 | 2.00 | 1.00 | 1.00 | 0.50 | 10 |
| 2 | 1.75 | 0.50 | 0.95 | 0.55 | 13 |
| 3 | 1.60 | 1.05 | 0.90 | 0.60 | 11 |
| 4 | 1.50 | 1.10 | 0.85 | 0.65 | 12 |
| 5 | 1.45 | 1.12 | 0.40 | 0.70 | 15 |
| 6 | 1.40 | 1.15 | 0.80 | 0.75 | 13 |
| 7 | 1.35 | 1.18 | 0.75 | 0.70 | 14 |
| 8 | 1.30 | 0.60 | 0.72 | 0.65 | 17 |
| 9 | 1.25 | 1.20 | 0.70 | 0.70 | 15 |
| 10 | 1.20 | 1.25 | 0.40 | 0.75 | 18 |
| 11 | 1.15 | 1.28 | 0.70 | 0.75 | 16 |
| 12 | 1.10 | 1.30 | 0.65 | 0.80 | 17 |

# Figure 2: Alternative Price Levels for Different Methods and Elasticities of Substitution



# Simulations: CES Preferences

**For elasticities of substitution in the most likely range of 1 to 4:**

- **The four methods based on the use of bilateral superlative indexes approximate CES preferences reasonably well with the chained Törnqvist generally doing the best. (Consistent with the 2004 CPI Manual advice).**
- **But the GK indexes have substantial upward biases in all cases.**
- **WTPD indexes also have substantial upward biases when $\sigma$ equals 2 or 4, but they are unbiased when $\sigma = 1$.**
- **The above results were derived when we knew the "truth" and the data were consistent with cost minimizing CES consumers.**
- **In the following slides, consumers will move away from their CES preferences in periods following a sale of some products.**

# Simulations: CES Preferences

- **The economic approach to index number theory assumes that the consumption of goods takes place within the period of purchase.**

- **But consumers tend to stock up during sales to partially satisfy their needs for the subsequent period.**

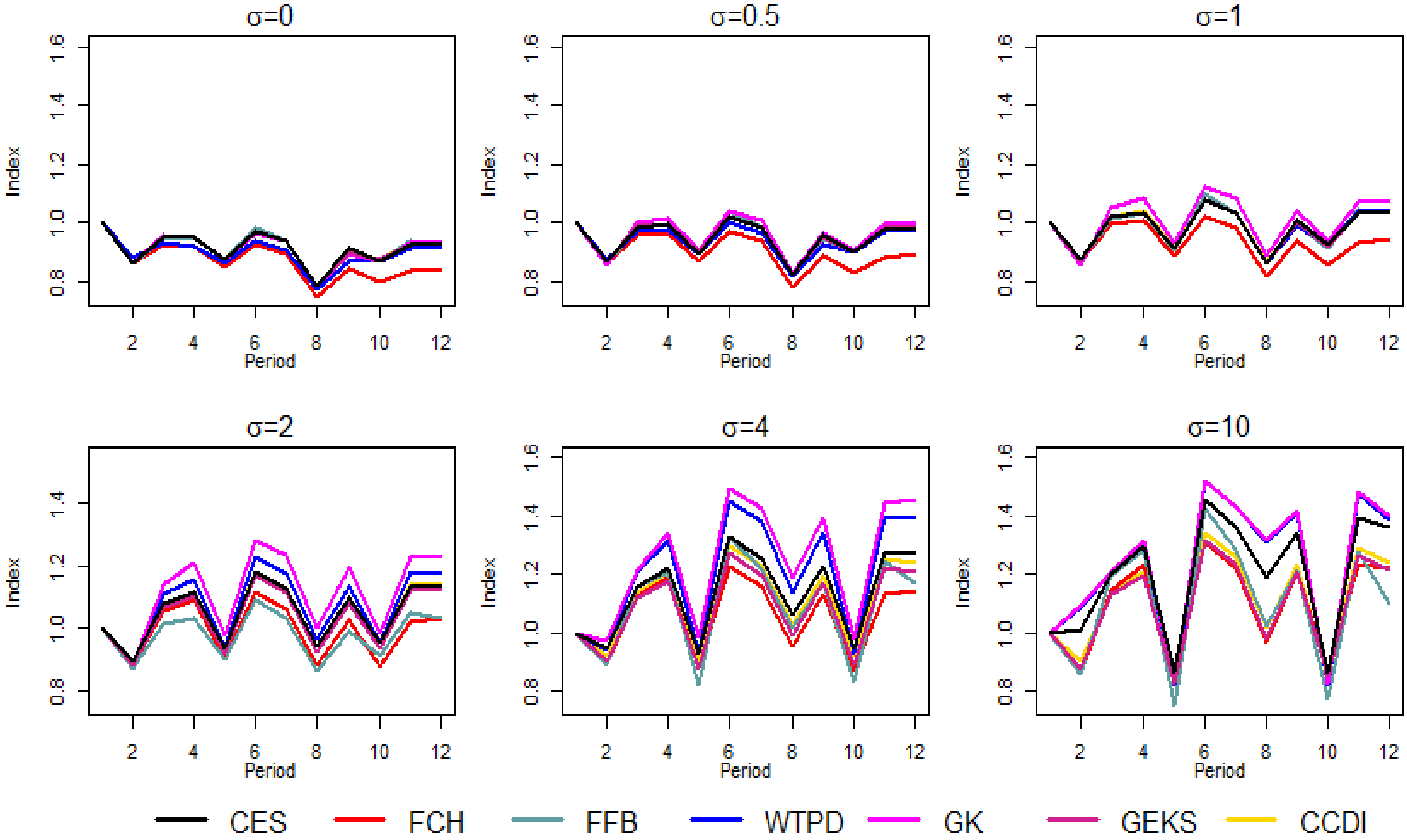
- **We adjust the data so that quantities in the periods following sales are half of the predicted levels generated by the CES model.**

- **Hence, for these periods, have new total expenditures, new quantity vectors and new expenditure shares for each elasticity.**

- **For periods 1,2,4,5,7,8,10 and 12, the $P_t$ and $\pi_{CES}^t \equiv P_t/P_1$ for the new data set are the same as before.**

- **For periods 3, 6, 9 and 11, there are no CES price levels but for convenience, in Figure 3 we simply use the old $\pi_{CES}^t$ .**

## Figure 3: Alternative Price Levels for Sales Adjusted Data



# Simulations: CES Preferences

- **Chained superlative indexes are not useful target indexes for a CPI when dealing with aggregating scanner data where discounted prices are prevalent. They have substantial downward chain drift biases.**

- **The CCDI multilateral method worked best overall for our numerical example for elasticities of substitution in the range $0 \leq \sigma \leq 4$.**

- **Similarity Linking also worked well.**

- **GK indexes had substantial upward biases relative to the corresponding CES true cost of living price levels for elasticities of substitution in the range $1 \leq \sigma \leq 4$.**

- **Weighted Time Product Dummy indexes will work well if $\sigma = 1$ or if $\sigma \geq 10$ but for our example, they had substantial upward biases for elasticities of substitution in the range $2 \leq \sigma \leq 4$.**

# Simulations: Linking the Windows

**In what follows, three tables of simulation results are presented:**

1. **Differences at Period 12 between the single window CCDI price levels and the linked CCDI price levels as functions of the linking period and the elasticity of substitution.**
   - **If these differences are large in magnitude, then this indicates a chain drift problem with the use of successive CCDI linked windows.**

2. **Biases at Period 12 as a function of the linking period and the elasticity of substitution.**
   - **The bias in the various two window CCDI period 12 price levels compared to the corresponding period 12 true (CES) cost of living indexes.**

3. **The mean absolute differences between our ten approximating indexes to the corresponding true CES cost of living indexes.**
   - **Exclude periods 3, 6, 9 and 11 from this comparison because the true cost of living is not defined for these observations.**

# Table 4: Differences at Period 12, D(t,σ), between the Single Window CCDI Price Levels and the Linked CCDI Price Levels as Functions of the Linking Period t and the Elasticity of Substitution σ

| t | D(t,0) | D(t,0.5) | D(t,1) | D(t,2) | D(t,4) | D(t,10) |
|---|---|---|---|---|---|---|
| 2 | 0.00030 | 0.00014 | 0.00021 | 0.00067 | 0.00212 | 0.01004 |
| 3 | -0.00197 | -0.00149 | -0.00098 | 0.00050 | 0.00567 | 0.02197 |
| 4 | -0.00001 | 0.00011 | 0.00021 | 0.00098 | 0.00603 | 0.02640 |
| 5 | -0.00029 | 0.00000 | 0.00021 | 0.00006 | -0.00222 | 0.01370 |
| 6 | 0.00154 | 0.00206 | 0.00265 | 0.00442 | 0.01035 | 0.03114 |
| 7 | 0.00002 | 0.00011 | 0.00021 | 0.00092 | 0.00537 | 0.02639 |
| 8 | 0.00041 | 0.00022 | 0.00021 | 0.00125 | 0.00645 | 0.02581 |
| 9 | -0.00177 | -0.00137 | -0.00098 | 0.00014 | 0.00451 | 0.02312 |
| 10 | -0.00015 | 0.00003 | 0.00021 | 0.00010 | -0.00202 | 0.01647 |
| 11 | 0.00151 | 0.00204 | 0.00265 | 0.00432 | 0.00946 | 0.02675 |
| Mean | -0.00004 | 0.00019 | 0.00046 | 0.00133 | 0.00457 | 0.02216 |

## Table 5: Biases at Period 12, B(t, σ), as Functions of the Linking Period t and the Elasticity of Substitution σ

| t | B(t,0) | B(t,0.5) | B(t,1) | B(t,2) | B(t,4) | B(t,10) |
|---|---|---|---|---|---|---|
| 2 | -0.00002 | 0.00068 | 0.00249 | 0.00187 | -0.03189 | -0.11420 |
| 3 | -0.00230 | -0.00094 | 0.00130 | 0.00169 | -0.02833 | -0.10227 |
| 4 | -0.00034 | 0.00065 | 0.00249 | 0.00218 | -0.02797 | -0.09784 |
| 5 | -0.00061 | 0.00054 | 0.00249 | 0.00126 | -0.03622 | -0.11054 |
| 6 | 0.00122 | 0.00260 | 0.00494 | 0.00562 | -0.02365 | -0.09310 |
| 7 | -0.00031 | 0.00065 | 0.00249 | 0.00212 | -0.02864 | -0.09785 |
| 8 | 0.00008 | 0.00076 | 0.00249 | 0.00244 | -0.02756 | -0.09843 |
| 9 | -0.00210 | -0.00083 | 0.00130 | 0.00134 | -0.02949 | -0.10112 |
| 10 | -0.00048 | 0.00057 | 0.00249 | 0.00130 | -0.03603 | -0.10777 |
| 11 | 0.00118 | 0.00258 | 0.00494 | 0.00551 | -0.02455 | -0.09749 |
| Mean | -0.00037 | 0.00073 | 0.00274 | 0.00253 | -0.02944 | -0.10208 |

## Table 8: Mean Absolute Differences in Percentage Points between $\pi_{CES}^{t}(\sigma)$ and Ten Approximating Indexes as Functions of the Elasticity of Substitution σ

| σ | $B_{FCH}$ | $B_{TCH}$ | $B_{FFB}$ | $B_{TFB}$ | $B_{WTPD}$ | $B_{GK}$ | $B_{GEKS}$ | $B_{CCDI}$ | $B_{AL}$ | $B_{LQ}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 3.78 | 4.77 | 0.00 | 0.11 | 1.66 | 0.00 | 0.12 | 0.08 | 0.55 | 0.63 |
| 0.5 | 3.89 | 4.84 | 0.12 | 0.04 | 0.89 | 1.19 | 0.05 | 0.17 | 0.47 | 0.43 |
| 1 | 4.06 | 4.81 | 0.46 | 0.00 | 0.00 | 2.58 | 0.17 | 0.29 | 0.53 | 0.27 |
| 2 | 4.81 | 4.75 | 1.91 | 0.37 | 2.19 | 5.68 | 1.40 | 0.10 | 1.07 | 0.47 |
| 4 | 6.84 | 5.65 | 5.96 | 3.41 | 5.37 | 8.90 | 4.98 | 2.47 | 2.69 | 1.61 |
| 10 | 9.08 | 7.68 | 10.83 | 9.19 | 4.91 | 5.01 | 9.57 | 7.83 | 6.05 | 5.08 |

# Summary

- **The Chained Fisher and Chained Törnqvist indexes performed poorly for all elasticities of substitution.**

- **The Weighted Time Product Dummy indexes worked well for our numerical example when the elasticity of substitution $\sigma$ was equal to 1 or 10 but they did not work well when $\sigma$ was equal to 2 or 4.**

- **The Geary-Khamis indexes worked well when $\sigma$ = 0 or 10 but poorly when $\sigma$ = 1, 2 or 4.**

- **For $0 \leq \sigma \leq 0.5$, the Fixed Base Fisher, Fixed Base Törnqvist, GEKS and CCDI indexes all worked well. However the cases where $\sigma \geq 1$ are the cases of interest.**

- **For $1 \leq \sigma \leq 2$, the CCDI indexes performed well.**

- **The LQ price similarity linked indexes performed the best for $\sigma$ = 4 and the LQ generally performed well for $1 \leq \sigma \leq 10$.**

# Caveats and Conclusions

- **The conclusions of this study are based on only a single artificial data set example. More research into how the different multilateral methods perform under different conditions is needed.**

- **We have assumed that all prices and quantities are positive over all periods, hence ignoring the problem of new and disappearing goods.**

- **An important result in our study is that linking the price and quantity data for a new period to the data of previous periods by using a price dissimilarity measure is the only multilateral method that is consistent with Walsh's powerful *multiperiod identity test*.**

- **But similarity linking requires agreement on how to measure the degree of price and share dissimilarity. More research needed.**

- **In the meantime, for elasticities of substitution in reasonable ranges that are expected to be found empirically, overall our results suggest the use of the CCDI index, combined with a new method, the mean splice, for updating.**