The multiplex structure of interbank networks

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The issue

- The financial crisis stressed the importance of interconnectedness among financial institutions
- Network analysis contributed to explain the map of linkages and to assess the systemic risk in the financial system
- Interbank market, i.e., has been seen as a single layer
- ... but credit relationships turn out to be more complex

Goal of this paper

- We extend the analysis to different kind of contracts
- The interbank market is studied as a multiplex or multilayer network
- Main questions:
  - are the layers of the multiplex topologically different?
  - is there a specific layer driving the properties of the total network
  - is the occurrence of a link in a layer predictive of link in another layer?

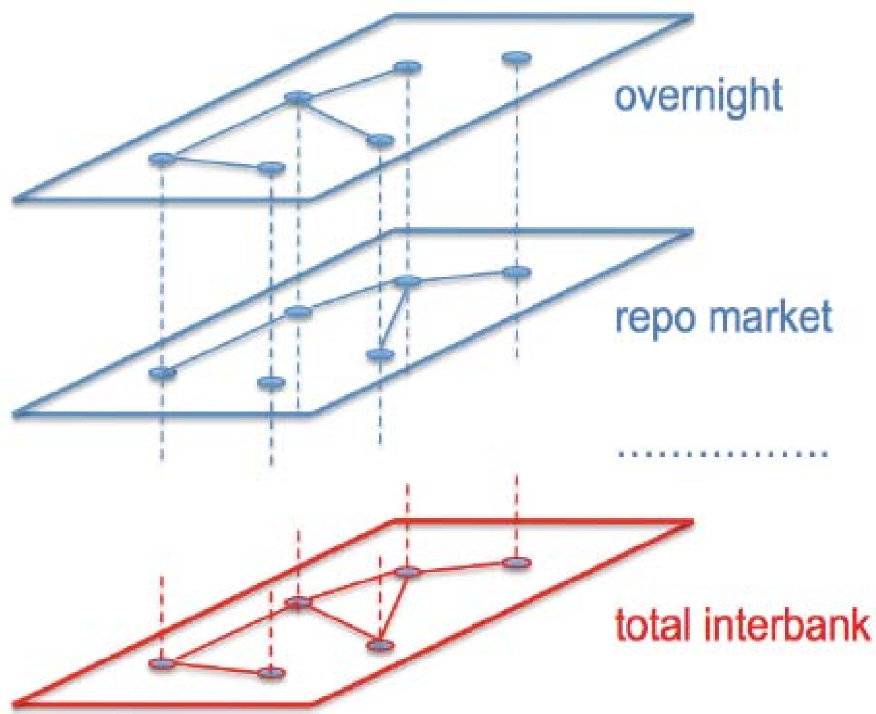

Figure 1: Stylized representation of the multiplex structure of the interbank market. Each node is a bank, and links represent credit relations. A layer (e.g. overnight, repo market,...) is the set of all credit relations of the same type. The network in red is the total interbank market, obtained by aggregating all the layers.

The methodology

- Comparison of the topological and metric properties of different layers and of the total layer
- Similarity analysis
- Does Random models fit the layers of the Multiplex?

A quick tour on the Literature

- Based on Italian data, Mistrulli (2007) finds that banks default hardly triggers a systemic risk
- Montagna and Kok (2013) develop an agent-based model exploiting a multi-layered network representation of interbank market
- Abbassi et al. (2013) study the different reaction of Euro interbank markets using econometric technique and network covariates
- Among non-network papers, Afonso et al. (2012) analyse the counterparty risk and liquidity hoarding taking into account different segments of the market
- Kuo et al. (2013) study US term market exploiting price and quantity information

Data description

- Interbank transactions based on the supervisory reports transmitted to Bank of Italy
- End of year data for the period 2008-2012
- We distinguish between Unsecured and Secured transactions
- Data are reclassified w.r.t. maturity:
    - overnight
    - short term (less than 12 months)
    - long term
- Consolidation at Group Level (self-loops)
- In this analysis we focus only on domestic data

Layer	2008	2009	2010	2011	2012
Unsecured overnight	185	147	71	68	79
Unsecured ST	157	192	97	97	81
Unsecured LT	68	110	95	102	103
Secured ST	74	39	43	65	36
Secured LT	0.1	8.0	0.8	2.5	4.9
Total	485	497	308	336	306

(a) Non consolidated data (intragroup lending is included)

Layer	2008	2009	2010	2011	2012
Unsecured overnight	22	19	16	17	19
Unsecured ST	26	27	13	14	12
Unsecured LT	6	3	7	17	28
Secured ST	15	5	17	11	6
Secured LT	0.03	0.3	0.7	0.6	1,4
Total	70	55	54	61	68

(b) Consolidated data (intragroup lending is excluded)

Table 1: Domestic credit exposures in the Italian interbank market. Billions of euros. End-of-period outstanding amounts.

The multiplex Italian interbank network: some properties

Statistics (2008)	U OVN	U ST	U LT	S ST	S LT	TOT
# of nodes	573	550	238	72	8	573
# of edges	2936	1457	354	125	7	3534
Density	0.8%	0.5%	0.6%	2.4%	12.5%	1.0%
Largest weak compon.	573	549	230	48	6	573
Largest strong compon.	498	333	27	14	1	528
Avg undir. path length	2.3	2.5	3.1	2.3	1.8	2.2
Avg dir. path length	2.4	2.7	2.4	1.9	-	2.3
Statistics (2012)	**U OVN**	**U ST**	**U LT**	**S ST**	**S LT**	**TOT**
# of nodes	532	521	450	45	18	533
# of edges	2560	1254	887	67	25	3235
Density	0.8%	0.4%	0.4%	3.3%	7.9%	1.0%
Largest weak compon.	532	520	447	35	11	533
Largest strong compon.	456	375	165	16	3	513
Avg undir. path length	2.3	2.6	2.6	2.7	1.7	2.2
Avg dir. path length	2.4	2.8	2.8	2.5	1.3	2.4

- The network is very sparse and connected for all the layer
- The Unsec. Overn. shares similar properties to the Total
- The secured layers show smaller size

Spearman correlation coefficient between degree and strength

Layer	2008	2009	2010	2011	2012
U OVN	0.4433	0.4755	0.4680	0.4933	0.5190
U ST	0.5974	0.6313	0.6300	0.5596	0.5906
U LT	0.9372	0.9172	0.9430	0.9530	0.9391
S ST	0.9119	0.9579	0.9972	0.9913	0.9339
Total	0.5564	0.5641	0.5355	0.5115	0.5192

(a) out-degree vs out-strength

Layer	2008	2009	2010	2011	2012
U OVN	0.7332	0.7756	0.7050	0.6423	0.7081
U ST	0.9018	0.9009	0.9008	0.8053	0.7545
U LT	0.9134	0.8776	0.7373	0.4695	0.4939
S ST	0.9508	0.9115	0.6128	0.7816	0.8084
Tot	0.7562	0.7414	0.6972	0.5612	0.5066

(b) in-degree vs in-strength

- Lower correlation for the Unsecured Overn.
- The high correlation for the secured segment may be driven by the fixed costs of establishing bilateral lending agreements

Assortativity and Cluster coefficient

Date: 2008	U OVN	U ST	U LT	S ST	S LT	TOT
Out-degree assort.	-0.26**	-0.40**	-0.52**	-0.43**	0.00	-0.27**
In-degree assort	-0.34**	-0.32**	-0.35**	-0.32**	0.15	-0.33**
Out-weight assort.	-0.02	-0.01	-0.06	-0.18*	-0.21	-0.05**
In-weight assort.	-0.03*	-0.01	-0.06	-0.14	0.38	-0.06**
Degree reciprocity	0.43*	0.45*	0.10*	0.18*	0.14	0.47*
Weight reciprocity	0.43*	0.16*	0.05*	0.13*	0.04	0.29*
Date: 2012	**U OVN**	**U ST**	**U LT**	**S ST**	**S LT**	**TOT**
Out-degree assort.	-0.27**	-0.40**	-0.51 **	-0.17	0.06	-0.31**
In-degree assort.	-0.42**	-0.39**	-0.38**	-0.31*	0.12	-0.37**
Out-weight assort.	-0.03	-0.05	-0.32**	-0.16	-0.29	-0.11**
In-weight assort.	-0.18**	-0.04	-0.03	-0.15	-0.05	-0.07**
Degree reciprocity	0.40**	0.56**	0.31**	0.31**	0.05	0.45**
Weight reciprocity	0.20**	0.00**	0.01**	0.05*	-0.00	0.07**

Date: 2008	U OVN	U ST	U LT	S ST	S LT	TOT
Avg dir. clustering	0.393	0.112	0.056	0.161	0.135	0.463
Avg undir. clustering	0.527	0.170	0.083	0.180	0.270	0.571
Date: 2012	**UOVN**	**U ST**	**U LT**	**S ST**	**S LT**	**TOT**
Avg dir. clustering	0.402	0.131	0.156	0.118	0.184	0.448
Avg undir. clustering	0.547	0.209	0.303	0.169	0.311	0.577

Similarity Analysis of Layers: measures

- We use the following functions:
  - Jaccard similarity for binary data:

$$J(p,q) = \frac{|p \wedge q|}{|p \vee q|}$$

  - Cosine similarity for valued data:

$$\cos(\theta) = \frac{pq}{\|p\|\|q\|}$$

p and q stand for the network

Θ is the angle formed by p and q

Jaccard and Cosine measures: the similarity over time

	2008	2009	2010	2011
2009	61%*			
2010	35%*	42%*		
2011	18%*	21%*	42%*	
2012	15%*	17%*	32%*	70%*

(a) Unsecured long-term, J

	2008	2009	2010	2011
2009	67%*			
2010	53%*	61%*		
2011	50%*	56%*	71%*	
2012	44%*	48%*	60%*	69%*

(b) Unsecured overnight, J

	2008	2009	2010	2011
2009	29%*			
2010	15%*	16%*		
2011	2%*	7%*	78%*	
2012	1%*	3%*	62%*	89%*

(c) Unsecured long-term, cosine similarity

	2008	2009	2010	2011
2009	30%*			
2010	13%*	39%*		
2011	15%*	47%*	52%*	
2012	19%*	41%*	48%*	76%*

(d) Unsecured overnight, cosine similarity

- The overnight layer displays more stability
- Similarity is lower when weights are taken into accounts
- There is a trend toward a greater stabilization and shift toward longer maturities

Jaccard and Cosine measures: the similarity across layers

	S LT	S ST	U OVN	U LT
S ST	18% (3%)			
U OVN	12% (0%)	15%* (3%*)		
U LT	5% (0%)	13%* (5%*)	12%* (6%*)	
U ST	13% (0%)	16%* (4%*)	29%* (29%*)	19%* (10%*)

(a) 2008. Intersection, Union in parenthesis

	S LT	S ST	U OVN	U LT
S ST	26%* (15%*)			
U OVN	11%* (0%*)	11%* (1%*)		
U LT	0% (0%)	9%* (0%*)	22%* (17%*)	
U ST	13%* (0%*)	11%* (1%*)	32%* (31%*)	31%* (28%*)

(b) 2012. Intersection, Union in parenthesis

- The probability that links in a network, i.e., overnight, are found also in another network is quite low
- In the unsecured term layers in 2012 there's an increase of probability (wrt to overnight) that we read as an evidence of a shift on longer maturity

Looking for a Null model

- Moving from single topological properties toward a network model able to replicate the main measures
- What would be the value of a metric if we allowed each bank to retain the number of lenders and borrowers with a random assignment of the counterparties?
- Maximum Entropy Principle subject to a set of constraints, imposed by observations (Park and Newmann, 2004)
- Hierarchy of observables in a network
- First order properties (connectivity, degree distrib.) vs Higher order properties

Three Models

- Directed Binary Configuration model (DBCM)
  - Where the in- and out-degree distributions are preserved
- Reciprocal Configuration Model (RCM)
  - where also the number of reciprocated relations of each node is preserved
- Directed Weighted Configuration Model (DWCM)
  - where the in- and out-degree distributions, along with in- and out-strenght are preserve
- The checked properties are:
  - The number of reciprocated links (not for the RCM)
  - The assortativity
  - The number of triangles
  - Weakly and strong connected component (high order prop.)
  - Number of distinct triads (high order prop.)

Directed Binary Configuration Model: some results

	2008	2009	2010	2011	2012
Largest weak component	573	565	556	551	532
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	556	547	544	538	515
Largest strong component	498	486	511	501	456
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	374	359	385	384	335
Reciprocal links	1,265	1,231	1,271	1,189	1,033
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	855	814	843	820	677
Und. triangles	14,114	11,747	11,645	10,704	10,098
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	18,418	16,252	15,953	14,871	13,755

(a) Unsecured overnight

- The selected high order properties are highly unlikely for realizations of the model
- The size of the largest weak and strong components, i.e., are much larger than those expected under the null model
- In the secured short-term the results appear noisier and less stable

Directed Weighted Configuration Model: some results

	2008	2009	2010	2011	2012
Out-strength assortativity	-0.0204	-0.0221	-0.0334	-0.0367	-0.0316
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	-0.1458	-0.269	-0.3016	-0.3328	-0.2711
In-strength assortativity	-0.0378	-0.0528	-0.1551	-0.197	-0.1827
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	-0.2739	-0.2527	-0.2817	-0.2623	-0.2543
Strength reciprocity	0.4325	0.1407	0.1157	0.0714	0.2070
(p-values)	(0.000)	(0.124)	(0.498)	(0.149)	(0.004)
Simulation average	0.1929	0.1078	0.1158	0.0912	0.1334

(a) Unsecured overnight

- The strength reciprocity is often explained by the null model
- The values of the other layers are in line with the null models.
- This results imply net exposures between couples of banks is mostly determined by out- and in-streghts
- Layers tend to be less disassortative than the null model, the model potentially could reflect more stability than real data

Conclusions

- This work provides a broad analysis of the different layers in the Italian interbank market
- The market reacted in several ways:
  - Significant shift from short term to longer maturities
  - Domestic overnight money market displayed a strong resilience
- The topological properties differ significantly across layers
- The heterogeneity may be a good news for financial stability, since it is likely to slow contagion
- Unsecured overnight, the focus of monetary policy operations, mirrors the features of the overall total network: that is a good news!
- But…in case policy makers were to target another segment they should avoid adopting tools based on overall features of the network