Gross Migration, Housing and Urban Population Dynamics*

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May 27, 2014

Abstract

Cities experience significant, near random walk productivity shocks, yet population is slow to adjust. In practise local population changes are dominated by variation in net migration, and we argue that understanding gross migration is essential to quantify how net migration may slow population adjustments. Housing is also a natural candidate for slowing population adjustments because it is difficult to move, costly to build quickly, and a large durable stock makes a city attractive to potential migrants. We quantify the influence of migration and housing on urban population dynamics using a dynamic general equilibrium model of cities which incorporates a new theory of gross migration motivated by patterns we uncover in a panel of US cities. After assigning values to the model's parameters with an exactly identified procedure, we demonstrate that its implied dynamic responses to productivity shocks of population, gross migration, employment, wages, home construction and house prices strongly resemble those we estimate with our panel data. The empirically validated model implies that costs of attracting workers to cities drive slow population adjustments. Housing plays a very limited role.

JEL Classification Numbers: E0, O4, R0
Keywords: Housing, house prices, labor reallocation

*An earlier draft of this paper circulated under the title "The Role of Housing in Labor Reallocation." We thank Marco Bassetto, John Kennan, Erzo Luttmer, Robert Shimer and seminar participants at various institutions and conferences for helpful comments. The views expressed herein are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

1 Introduction

As we document in this paper, cities experience significant, random-walk-like productivity shocks, yet population is slow to adjust. In the light of Blanchard and Katz (1992)'s empirical evidence that internal migration is integral to equilibrating the US labor market, explaining population's slow adjustment should inform our understanding of macroeconomic labor reallocation. Ultimately migration to and from cities is the main driver of a city's population adjustments. Migration frictions associated with leaving and attracting workers to a city naturally impede population adjustments. Housing is another natural candidate because it is difficult to move, requires time to build, and a large durable housing stock makes a city attractive to potential migrants.

To understand the quantitative importance of migration and housing in urban population dynamics we develop a dynamic general equilibrium model of cities with endogenous migration and local housing and compare it to panel data on 365 US cities over the period 1985-2009. Our model is a version of the Lucas and Prescott (1974) islands economy in which islands are interpreted as cities. We propose a new theory of migration between cities interpreted as population movements between the islands. Population adjustments involve net migration, but we argue that it is essential to model the underlying gross flows. Our argument builds on new evidence from our panel data. We find that gross in- and out-migration are strikingly linear in net migration, evidence that both the decisions to leave and move to a city drive changes in net migration, and that migration clearly involves directed search.

In the model workers face idiosyncratic shocks to their taste for where they currently live and this influences the decision to leave a city. After this decision has been made a worker chooses between directed and undirected search for a new city. Workers understand the distribution of city characteristics, but must use costly directed search to find a city with specific labor and housing markets. Undirected search leaves a worker randomly assigned to a city. Including both directed and undirected migration coincides with evidence that moves involve decisions about where to work and enjoy amenities like housing but also intangible factors such as to be near family members. Increases in employment of the existing population are a obvious alternative to net migration for accommodating local fluctuations in labor demand and so labor supply is endogenous in our model as well.

We introduce this theory of migration and labor supply into an otherwise familiar generalization of the neoclassical growth model. The employed population in each city produces intermediate goods that are imperfectly substitutable in the production of the tradeable final goods equipment and consumption. Local production combines employment with freely mobile, durable capital, augmentable by equipment investment, and subject to local total factor productivity (TFP) shocks. Individuals have preferences for consumption and housing services, but only enjoy housing in the same city they work or rest. Housing services are derived from locally produced, immobile and durable residential structures and local residential land.

The model is calibrated to aggregate statistics familiar from other studies that work with the neoclassical growth model and features of the data that are specific to our model's environment. For the latter, we use our new evidence on the relationship between gross and net migration and microeconomic estimates of migration costs to obtain the key migration parameters. In addition, we estimate the idiosyncratic TFP process using our panel data thereby pinning down the model's exogenous source of persistence and variability. Our estimation of the TFP process facilitates estimation of the dynamic responses of key variables to TFP shocks. We use the estimated elasticity of the employment to population ratio with respect to wages from the impact period of a TFP shock to identify the model's labor supply elasticity. Finally, we calibrate the substitutability of city-specific intermediate goods so that our model matches the empirical cross-section distribution of population. In so doing we confirm that our model is consistent with Zipf's law, that in its upper tail city population is distributed exponentially with an exponent close to unity. In turn the idiosyncratic process that we estimate and introduce into the model is able to generate a similar law for TFP that we uncover also in our panel data.

We validate the model by studying several of its over-identifying restrictions, observations not used to calibrate its parameters. Specifically, we compare the model's dynamic responses to TFP shocks of population, gross in- and out-migration, employment, wages, home construction and house prices to those we estimate from our panel data. The model does surprisingly well along this dimension and importantly it is consistent with the slow response of population to TFP shocks that motivates this study even though this evidence is not directly targeted in our calibration. With only TFP shocks driving within-city dynamics we also find that the model is broadly consistent with the unconditional volatility, persistence, and contemporaneous co-movement of the key variables, although there are some interesting shortcomings.

Having established the empirical relevance of our model, we use it to examine how migration and housing influence population adjustments. We find that the process of attracting workers to cities through costly directed search is sufficient to explain slow population adjustments to TFP shocks. Housing plays a surprisingly limited role. In the absence of migration frictions introducing immobile housing does lower the amplitude of population's response to a TFP shock but has very little influence over its persistence. However, if migration frictions are already present making housing immobile does little to influence population dynamics.

We also investigate our model's implications for the persistence of urban decline. Glaeser and Gyourko (2005) explore this phenomenon emphasizing housing's immobility and slow depreciation and argue that these features are a significant source of persistent urban decline. There are many cities in our data with declining populations throughout the sample period, evidence of persistent urban decline. These cities also typically experience declining TFP suggesting that our model might account for the persistence of urban decline in addition to short run population dynamics. We find that the combination of previous declines in TFP and the slow response of population to them can explain persistent urban decline. Apart from indicating an important role for TFP in urban decline this finding strongly suggests that costly migration is a major factor determining persistent urban decline. Glaeser and Gyourko (2005) do not consider a role for costly migration.

Our model builds on an extensive empirical and theoretical microeconomic literature on migration, surveyed by Greenwood (1997) and Lucas (1997). Kennan and Walker (2011) (hereafter, KW) is an important recent contribution to this literature. They analyze individual migration decisions in the face of wage shocks and moving costs with directed search, but without explicit housing or equilibrium interactions. They use their estimated model to calculate the speed of adjustment of state's population to permanent wage changes. Despite using very different different methodologies we find similarly slow population adjustments. One of our contributions is to show how their microeconomic estimates of moving costs based on inter-state migration can be used to calibrate a general equilibrium model of inter-city migration.

The classic references for systems-of-cities models like ours are Roback (1982) and Rosen (1979). These authors consider static environments in which individuals allocate themselves across cities so that they are indifferent to where they live. Recent contributions using this approach include Albouy (2009) and Diamond (2012). Because it is static, the Roback-Rosen model does not inform our understanding of migration and local population adjustments. Van Nieuwerburgh and Weil (2010) introduce dynamics to this framework and therefore their model speaks to migration. It has implications for net population flows, but not for gross flows. Coen-Pirani (2010) also constructs a dynamic Roback-Rosen model. He studies gross population flows among US states in an environment similar to that used by Davis, Faberman, and Haltiwanger (2011) and others to model gross worker flows among firms. Our empirical work demonstrates that gross population flows in a city are very different from gross worker flows in a firm so we introduce a new theory.

Our model also contributes to the literature by introducing a city's dynamic response to an identified TFP shock as a model validation tool and by estimating the underlying stochastic process for TFP.[1] Model validation in the existing literature emphasizes unconditional cross-sectional and time-series patterns. Even so, the papers that focus on cities abstract from Zipf's law, perhaps the most notable feature of the cross-section of cities.[2] While the literature relies on idiosyncratic TFP shocks to drive variation, it does not provide evidence on the nature of these shocks as we do.[3]

The recent housing boom and bust has prompted a growing literature that seeks to quantify how frictions in housing may impede migration and labor reallocation and possibly give rise to persistent high unemployment. Karahan and Rhee (2012), Lloyd-Ellis and Head (2012) and Nenov (2012) study how the recent collapse in house prices may have limited labor reallocation through disincentives to migrate arising from home ownership and within-location search frictions in housing and labor markets. We abstract from these within-location labor and housing market frictions and instead study between-location migration frictions and focus on housing's basic technological characteristics.[4] However our finding that migration frictions alone can account for slow population adjustments and that also including the basic technological characteristics of housing does not slow down population adjustments very much suggests it is unlikely that once the costs of migration are taken into account that adding more frictions in the housing market will have much of an impact on labor reallocation.

The rest of the paper is organized as follows. Section 2 describes new empirical evidence on migration and population's response to TFP shocks based on our panel of cities. After this we use two stripped down versions of our quantitative model to describe our approach to modeling migration and the possible role for housing in slowing population adjustments. Sec-

[1] Lloyd-Ellis, Head, and Sun (2014) study the within-city responses of population, residential construction and house prices to personal income shocks identified using a panel VAR and a Choleski decomposition of the variance-covariance matrix of the residuals. These authors abstract from migration decisions and equilibrium interactions among cities.

[2] See for example Gabaix (1999) and Eeckhout (2004).

[3] Karahan and Rhee (2012) estimate an auto-regressive process in the level of GDP per worker using a short panel of cities.

[4] There is also an empirical literature that investigates the effects of housing related financial frictions on mobility. See for example Ferreira, Gyourko, and Tracy (2011), Modestino and Dennett (2012) and Schulhofer-Wohl (2012). We abstract from financial friction in this paper.

tion 5 introduces the complete dynamic quantitative model economy and Section 6 describes how we calibrate its parameters. Section 7 validates the quantitative model by comparing its predictions for within city dynamics we estimate from our panel and quantifies the roles of housing and migration in labor reallocation. The last section concludes.

2 Empirical Evidence

In this section we introduce the empirical evidence that motivates our analysis and guides our modeling of migration. We work with an annual panel data set covering 1985 to 2009 that includes population, net and gross migration, employment, wages, residential construction, and house prices for 365 Metropolitan Statistical Areas (MSAs) comprising 83% of the aggregate population.[5] An MSA is a geographical region with a relatively high population density at its core and close economic ties throughout the area measured by commuting patterns. Such regions are not legally incorporated as a city or town would be, nor are they legal administrative divisions like counties or sovereign entities like states. A typical MSA is centered around a single large city that wields substantial influence over the region, *e.g.* Chicago. However, some metropolitan areas contain more than one large city with no single municipality holding a substantially dominant position, *e.g.* the Dallas–Fort Worth metroplex or Minneapolis–Saint Paul. With these caveats, for convenience we refer to our MSAs as cities. The section begins with our evidence on gross and net migration and then we describe the dynamic response of TFP, population and gross migration to a shock to a city's TFP.

2.1 Gross Versus Net Migration

We use IRS data to calculate city-level net and gross migration rates. These data have wide coverage of US cities which are the natural unit of analysis for studying migration between geographically distinct labor markets. Due to limited sample sizes gross migration rates can only be calculated for a small number of cities using the other main data sources, the Current Population Survey and the American Community Survey. State-level migration rates can be calculated using these surveys. In our context, these data yield very similar results to those

[5] See Davis, Fisher, and Veracierto (2011) for a detailed description of these data.

we obtain with city-level and state-level migration rates calculated using the IRS data.[6]

Let a_{it} and l_{it} denote the number of people flowing into and out of city i in year t and p_{it} the population of that city at the end of the same year. For an individual city the *arrival* (in-migration) rate is $a_{it}/\tilde{p}_{it}$ and the *leaving* (out-migration) rate is $l_{it}/\tilde{p}_{it}$, where $\tilde{p}_{it} = (p_{it-1} + p_{it})/2$. These measures of gross migration mirror the measures of gross job flows defined in Davis, Haltiwanger, and Schuh (1998).[7] The difference between the arrival and leaving rates is the net migration rate. Gross migration rates fluctuate over the business cycle and have been falling over our sample period.[8] To abstract from these dynamics we subtract from each city's gross rate in a year the corresponding cross section average in that year. The net migration rate calculated from the difference between these gross rates is equivalent to subtracting from each city's raw net migration rate the corresponding cross-section average net migration rate in each year.

Figure 1 contains plots of gross and net migration rates by population decile with only time effects removed. Net migration is essentially unrelated to city size. This finding reflects Gibrat's law for cities, that population growth is independent of city size. However, the arrival and leaving rates are clearly diminishing in city size. While we think this is an interesting finding worthy of further study, its presence confounds across-city variation with the within-city dynamics we are interested in. Therefore, after removing time fixed effects, for every city we subtract from each year's arrival and leaving rate the time series average of the sum of the arrival and leaving rates for that city. This removes city fixed effects in gross migration without affecting net migration rates.

Figure 2 displays mean arrival and leaving rates against mean net migration for each net migration decile, after removing both time and city fixed effects and adding back the corresponding unconditional mean to the gross migration rates. Notice first that gross migration is far in excess of the amount necessary to account for net migration. For example, when net migration is zero an average of 11% of the population either moves in or out of a city in any given year.

Second, the arrival rate is monotonically increasing (and the leaving rate is monotonically

[6]As emphasized by Kaplan and Schulfofer-Wohl (2012) there are three drawbacks to using the IRS data: tax filings under-represent the poor and elderly; addresses on tax forms are not necessarily home addresses; and tax returns may be filed late. The ultimate affects of these features on measurement is unclear.

[7]In practice we approximate $\tilde{p}_{it}$ as the average of the beginning of year t and end of year t IRS-based population. For additional details see the appendix.

[8]See Molloy, Smith, and Wozniak (2011) and Kaplan and Schulfofer-Wohl (2012) for studies of the trend in gross migration rates.

Figure 1: Gross and Net Migration Rates by Population Decile

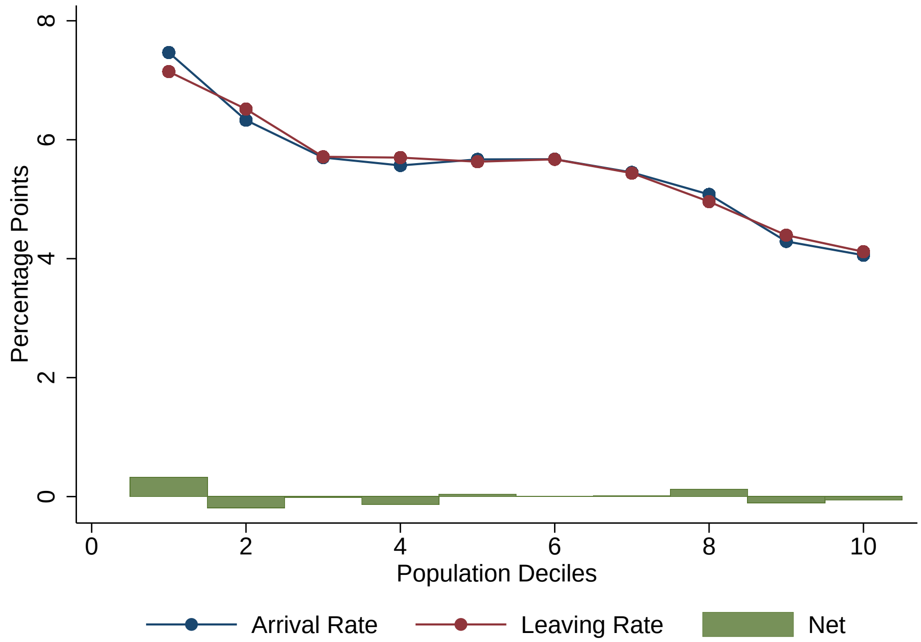
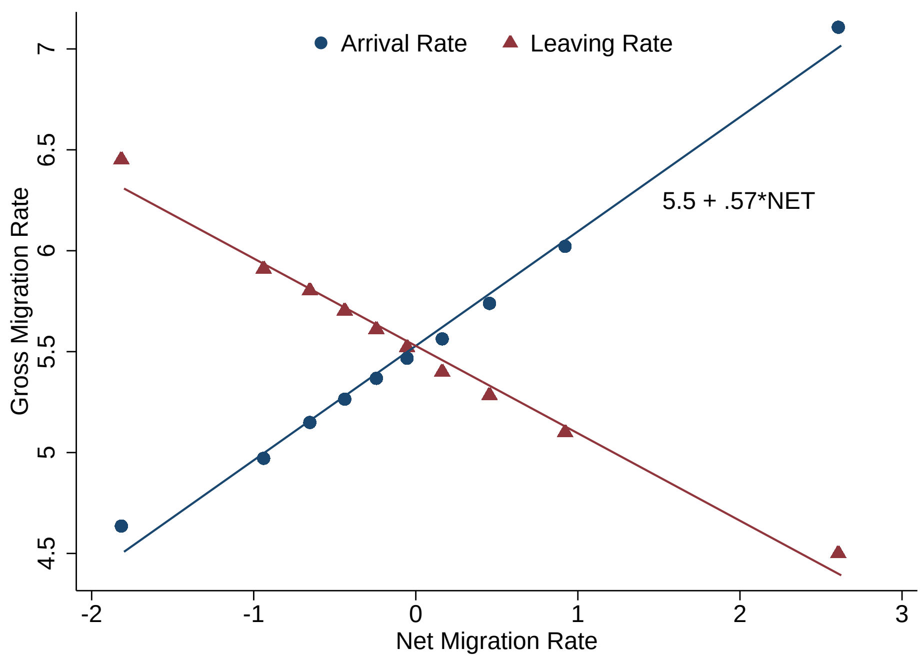


Figure 2: Gross Migration Rates by Net Migration



decreasing) in net migration. The rising arrival rate suggests that migration involves directed search. Otherwise gross arrivals would be independent of net migration. The fact that the arrival rate rises and the departure rate falls with increases in net migration suggests both margins are important when a city's population adjusts to shocks.

Third, and most striking, the gross migration rates all fall almost exactly on the corresponding regression lines.[9] This evidence sharply contrasts with the non-linear relationships for worker flows at firms described by Davis, Faberman, and Haltiwanger (2006). They find a kink at zero for hires and separations as functions of net worker flows. For negative net flows hires are flat and close to zero while for positive net flows they are linearly increasing; separations as a function of net flows are essentially the mirror image. The linear relationships between gross and net migration displayed in Figure 2 motivate how we specify migration decisions in our model.

The clear negative relationship between the arrival and leaving rates evident in Figure 2 may be surprising given Coen-Pirani (2010)'s focus on a positive correlation between the two gross migration rates at the state level. This difference does not arise because we consider cities rather than the states considered by Coen-Pirani (2010). It arises from our removal of city-specific fixed effects from the gross migration rates. As suggested by Figure 1, when we do not remove these effects the gross migration rates are strongly positively correlated.[10]

2.2 Responses of Population and Gross Migration to TFP Shocks

We now describe how we estimate dynamic responses of city-level variables to local TFP shocks and report estimates for population and the gross migration rates. To proceed we exploit the first order conditions of final good producers and intermediate good in the quantitative model described in Section 5. These conditions can be used to derive an equation involving TFP, employment and wages. Using this equation and data on employment and wages we obtain a measure of TFP from which we estimate a stochastic process for its growth. We estimate the dynamic response of a variable to TFP shocks by regressing it on current and lagged values of the TFP innovations derived from the estimated TFP growth process. Later we compare these estimated responses to ones calculated using the same

[9]We obtain virtually identical regression lines when we use all the data rather than first taking averages of deciles and when we estimate using data from the first 5 years of the sample and the last five years of the sample. We also find qualitatively similar results when we regress gross on net migration separately for each city in our sample.

[10]Coen-Pirani (2010) removes cross-sectional variation in the occupational characteristics of states prior to his analysis, but not state fixed effects.

procedure from data simulated from our model.

There are N cities that each produce a distinct intermediate good used as an input into the production of final goods. The production function for a representative firm producing intermediate goods in city i at date t is

$$y_{it} = s_{it} n_{y,it}^{\theta} k_{y,it}^{\gamma}, \tag{1}$$

where s_{it} is exogenous TFP for the city, $n_{y,it}$ is employment, $k_{y,it}$ is capital, hereafter referred to as equipment, $\theta > 0$, $\gamma > 0$, and $\theta + \gamma \leq 1$.[11] The output of the final good at date t, Y_t is produced using inputs of city-specific intermediate goods according to

$$Y_t = \left[\sum_{i=1}^{N} y_{it}^{\chi} \right]^{\frac{1}{\chi}}, \tag{2}$$

where $\chi \leq 1$.

Our measurement of city-specific TFP relies on the following definition. For any variable x_{it}:

$$\hat{x}_{it} \equiv \ln x_{it} - \frac{1}{N} \sum_{j=1}^{N} \ln x_{jt}. \tag{3}$$

Subtracting the mean value of $\ln x_{jt}$ in each period eliminates variation due to aggregate shocks, allowing us to focus on within-city dynamics. Under the assumption of perfectly mobile equipment the rental rate of equipment is common to all cities. It then follows from the first order conditions of competitively behaving final good and intermediate good producers that

$$\Delta \hat{s}_{it} = \frac{1 - \gamma\chi}{\chi} \Delta \hat{w}_{it} + \frac{1 - \theta\chi - \gamma\chi}{\chi} \Delta \hat{n}_{it}, \tag{4}$$

where Δ is the first difference operator and w_i denotes the wage in city i.[12] Applying the first difference operator eliminates permanent differences in TFP among the cities. Assuming values for χ, θ and γ, and substituting data on wages and employment for $\Delta \hat{w}_{it}$ and $\Delta \hat{n}_{it}$, we use this equation to measure $\Delta \hat{s}_{it}$, the growth rate of city-specific TFP.

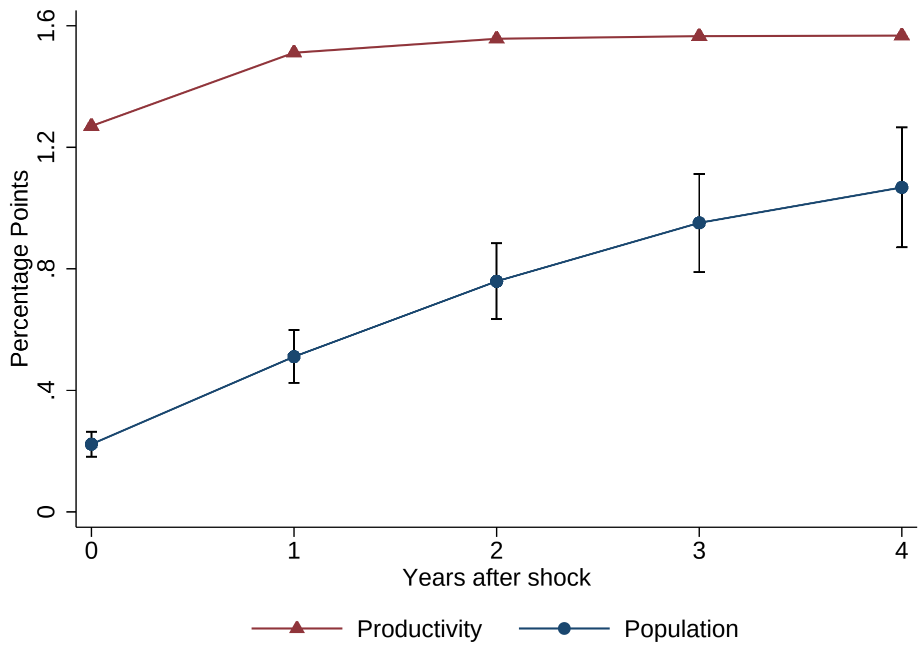
Below we calibrate θ and γ using traditional methods and find a value for χ to match the model to Zipf's law. With calibrated values $\chi = 0.9$, $\theta = 0.66$ and $\gamma = .235$ we estimate

[11] The additional subscripts on employment and equipment are used later to distinguish between employment and equipment used in the production of intermediate goods and residential construction.

[12] See the technical appendix downloadable at http:
xxx for more details.

a first order auto-regression in $\Delta\hat{s}_{it}$ with an auto-correlation coefficient equal to 0.24 and the standard deviation of the error term equal to 0.015. Wooldridge (2002)'s test of the null of no first order serial correlation in the residuals is not rejected, suggesting that this specification is a good fit for the data.

Figure 3: Responses of TFP and Population

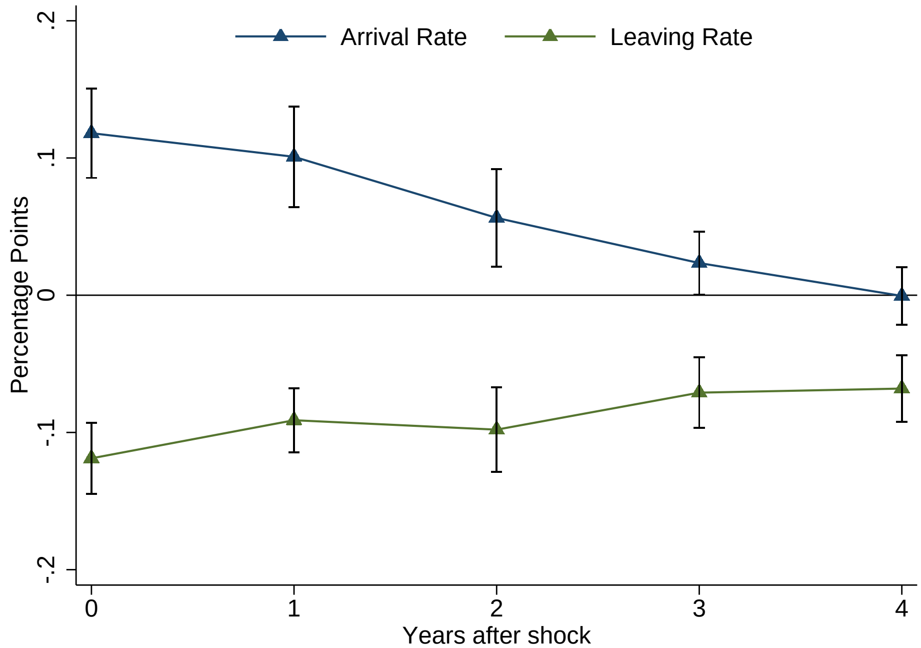


Note: Point estimates along with 2 standard error bands.

A natural concern about measuring TFP with (4) is that it ignores agglomeration. Davis, Fisher, and Whited (2013) find statistically significant agglomeration effects in model where agglomeration affects TFP endogenously through an externality in output per acre of land following Ciccone and Hall (1996). It is straightforward to modify equation (4) to include agglomeration modeled in this way and it leads to the same measurement equation for the exogenous component of TFP except that the coefficients on wage and output growth include the parameter governing the magnitude of the externality. When we re-estimate the TFP process using the estimate of the externality parameter in Davis et al. (2013) we find the serial correlation coefficient and the innovation standard deviation are a little different, falling to 0.20 and 0.013. While we do not include agglomeration in our model, we expect that doing so would reconcile the two sets of estimates but have little impact on our other results.[13]

[13]Verifying this conjecture is beyond the scope of this paper. However, in the model considered by Davis et al. (2013) the externality amplifies the response of TFP to an exogenous TFP shock and makes it more

Figure 4: Responses of Arrival and Leaving Rates



Note: Point estimates along with 2 standard error bands.

We now show how to use the estimated TFP process (without agglomeration) to identify the dynamic responses of variables to exogenous local TFP shocks. Let e_{it} denote the residual from the estimated TFP growth auto-regression. Then, we estimate the dynamic response to a TFP shock of variable $\Delta\hat{x}_{it}$ as the coefficients b_0, b_1, …, b_4 from the following panel regression:

$$\Delta\hat{x}_{it} = \sum_{l=0}^{4} b_l e_{it-l} + u_{it} \tag{5}$$

where u_{it} is an error term which is orthogonal to the other right-hand-side variables under the maintained hypothesis that the process for TFP growth is correctly specified. The dynamic response of $\hat{x}_{it}$ is obtained by summing the estimated coefficients appropriately. For the gross migration rates we replace $\Delta\hat{x}_{it}$ with the rates themselves (transformed as described above) in (5) and identify the dynamic responses with the estimated coefficients directly.

Figure 3 displays the percentage point deviation responses of TFP and population to a 1 standard deviation impulse to measured TFP. This plot establishes the claim made in the introduction that productivity, that is TFP, responds much like a random walk, rising quickly to its new long run level, and that population responds far more slowly. Figure 4 shows that

persistent.

the adjustment of population occurs along both the arrival and leaving margins, as suggested by our earlier discussion of Figure 2. On impact the arrival rate jumps up and the leaving rate jumps down and then both slowly returns to their long run levels. The indicated sampling uncertainty suggests that the arrival and leaving margins are about equally important in the adjustment of population to a TFP shock. In particular it is the improvement in local prospects encouraging workers not to move as much as the affect those prospects have on attracting workers to the city through which population adjusts to persistent improvements in local TFP.

3 Modeling Migration

The previous section documents evidence confirming a role for both gross migration margins in population adjustments. We now introduce our theory of migration that is motivated by this evidence. To do so so we employ a simple, static model which abstracts from housing, equipment, and labor supply. We use this simplified approach to develop intuition about migration choices, to describe how and why we can reproduce the relationships depicted in Figure 2, and to establish that modeling gross migration is essential for understanding population adjustments. All of the results in this section extend to our more general quantitative model.

3.1 A Static Model of Migration

The economy consists of a large number of geographically distinct cities with initial population x. In each city there are firms which produce identical, freely tradeable consumption goods with the technology sn^{θ}, where s is a city-wide TFP shock, n is labor and $0 < \theta < 1$. There is a representative household with a unit continuum of members that are distributed across city types $z = (s, x)$ according to the measure μ. Each household member enjoys consumption, C, and supplies a unit of labor inelastically. After the TFP shocks have been realized, but before production takes place, the household decides how many of its members leave each city and how many of those chosen to leave move to each city. Once these migration decisions have been made, production and consumption take place.

The leaving decision is based on each household member receiving a *location-taste* shock ψ, with measure μ_l, that subtracts from their utility of staying in the city in which they are initially located. This kind of shock is used by KW in their measurement of migration costs. To help us match the empirical evidence on the relationship between gross and net migration we make a parametric assumption for the distribution of individual location-taste shocks in a city of type z:

$$\int_{-\infty}^{\bar{\psi}(l(z)/x)} \psi d\mu_l = -\psi_1 \frac{l(z)}{x} + \frac{\psi_2}{2} \left(\frac{l(z)}{x} \right)^2$$

where the parameters ψ_1 and ψ_2 are both non-negative and $\bar{\psi}\left(l(z)/x\right)$ is defined by

$$\frac{l(z)}{x} = \int_{-\infty}^{\bar{\psi}(l(z)/x)} d\mu_l.$$

This parameterization is U-shaped starting at the origin. Initially benefits accrue to increasing the number of leavers from a city, and eventually individuals find it very costly to leave. These features are consistent with evidence in KW that individuals who move receive substantial non-pecuniary benefits and that non-movers would find it extremely costly if they were forced to move. For example, many individuals move to be near family members or find it very costly to move because they are already near family members. As more people leave a city the remaining inhabitants are those who have a strong preference for living in that city. Subject to these shocks, the household determines how many of its members from each city must find new cities in which to work. Household members chosen to find new cities are called *leavers*.

When deciding where to send its leavers the household understands the distribution of city types μ but does not know the location of any specific type z. However, it can find a particular type of city by obtaining a *guided trip*, a form of *directed search*. To match the evidence on gross and net migration, we adopt a particular functional form for producing guided trips as well. Specifically, by giving up u units of utility each individual household member can produce $\sqrt{2}A^{-1/2}u^{1/2}$ guided trips to the city in which they are initially located, where the parameter A is non-negative. Therefore, to attract $a(z)$ workers to a city of the indicated type the household must incur a total utility cost of $(A/2)\left(a(z)/x\right)^2 x$.

The production of guided trips encompasses the many ways in which workers are attracted to specific cities, including via informal contacts between friends and family, professional networks, specialized firms like head-hunters, advertising that promotes cities as desirable places to live and work, firms' human resource departments, and via recruiting by workers whose primary responsibility is some other productive activity.[14] Clearly some of these activities are part of recruiting workers within a local labor market and as such would be included in any measurement of the vacancy costs typically assumed in models of labor market search and matching. Our approach can be thought of as capturing the portion of these activities devoted to attracting workers to a local labor market from other locations.

If a household member does not obtain a guided trip it can migrate to another city using *undirected search*. Specifically, by incurring a utility cost τ a leaver is randomly allocated to another city in proportion to its initial population. Including undirected search captures the idea that choosing to move to a particular city is often the outcome of idiosyncratic factors other than wages or housing costs that are difficult to model explicitly, such as attractiveness of amenities and proximity to family members.[15] Furthermore, it is natural to let people move to a location without forcing them to find someone to guide them.

We characterize allocations in this economy by solving the following planning problem:

$$\max_{\substack{\{C,\Lambda,a(z),\\ l(z),p(z)\}}} \left\{ \ln C - \int \left[\frac{A}{2}\left(\frac{a(z)}{x}\right)^2 x + \left(-\psi_1 \frac{l(z)}{x} + \frac{\psi_2}{2}\left(\frac{l(z)}{x}\right)^2\right) x \right] d\mu - \tau\Lambda \right\} \tag{6}$$

subject to

$$p(z) \leq x + a(z) + \Lambda x - l(z), \forall\, z \tag{7}$$

$$\int [a(z) + \Lambda x]\, d\mu \leq \int l(z)\, d\mu \tag{8}$$

$$C \leq \int s p(z)^{\theta}\, d\mu \tag{9}$$

and non-negativity constraints on the choice variables. The variable Λ is the fraction of the household that engages in undirected search. Since these workers are allocated to cities in proportion to their initial populations, Λ also corresponds to the share of a city's initial population that migrates to that city within the period. Constraint (7) states that population in a city is no greater than the initial population plus arrivals through guided trips and undirected search minus the number of workers who migrate out of the city. Constraint (8) says that total arrivals can be no greater than the total number of workers who migrate out

[14] For convenience we have modeled the cost of attracting workers to a city as a direct loss of utility. Our results do not rest on this assumption.

[15] In KW migration is a combination of undirected and directed search. It is undirected because to learn a location's permanent component of wages workers have to migrate there. It is directed because workers retain information about locations to which they have previously migrated and include this information in their current migration decision along with expectations about locations they have not visited already.

of cities and (9) restricts consumption to be no greater than total production, taking into account that each individual supplies a unit of labor inelastically, $n(z) = p(z)$, $\forall z$.

3.2 Why Both Gross Migration Frictions are Necessary

We now explain why it is necessary to include frictions on both gross migration margins in order to match the evidence depicted in Figure 2. Suppose $A = 0$ so that guided trips can be produced at no cost, but that household members continue to be subject to location-taste shocks, $\psi_1 > 0$ and $\psi_2 > 0$. Then it is straightforward to show

$$\begin{aligned} \frac{a(z)}{x} &= \max\left\{\frac{p(z)-x}{x} + \frac{\psi_1}{\psi_2}, 0\right\}; \\ \frac{l(z)}{x} &= \max\left\{\frac{\psi_1}{\psi_2}, -\left(\frac{p(z)-x}{x}\right)\right\}. \end{aligned}$$

Observe that as long as the net population growth rate, $(p(z) - x)/x$, is not too negative, the planner sets the leaving rate, $l(z)/x$ at the point of maximum benefits, ψ_1/ψ_2, and adjusts population using the arrival rate, $a(z)/x$, only. In this situation the leaving rate is independent of net population adjustments, contradicting the evidence presented in Figure 2.

Now suppose that there are no location-taste shocks, $\psi_1 = \psi_2 = 0$, but it is costly to create guided trips, $A > 0$. In this case we find

$$\begin{aligned} \frac{l(z)}{x} &= \max\left\{-\left(\frac{p(z)-x}{x} - \Lambda\right), 0\right\}; \\ \frac{a(z)}{x} &= \max\left\{\frac{p(z)-x}{x} - \Lambda, 0\right\}. \end{aligned}$$

Without taste shocks the planner always goes to a corner: when net population growth is positive the leaving rate is set to zero, and when net population growth is negative the arrival rate is set to zero. Clearly the relationship between gross and net migration in this situation also contradicts the evidence depicted in Figure 2. We conclude that to be consistent with the relationship between gross and net migration, it is necessary to include frictions on both gross migration margins.

3.3 Migration Trade-offs

For the model to be consistent with the gross flows data as depicted in Figure 2, it also must be true (almost everywhere) that the number of workers leaving a city and the number arriving to the same city using guided trips are both strictly positive, $l(z) > 0$ and $a(z) > 0$. The reason we require $l(z) > 0$ is that gross out-migration is always positive in Figure 2. The reason we require $a(z) > 0$ is that otherwise there would be intervals of net migration in which arrival rates are constant, equal to Λ, which is also inconsistent with Figure 2. Therefore, unless otherwise noted, from now on we assume that $a(z) > 0$ and $l(z) > 0$.

The planner's first order conditions for $\Lambda, a(z), l(z)$ and $p(z)$ are

$$\tau = \int \lambda \xi(z) x d\mu - \lambda \eta \tag{10}$$

$$\lambda \xi(z) - A \frac{a(z)}{x} = \lambda \eta \tag{11}$$

$$\lambda \xi(z) = \psi_1 - \psi_2 \frac{l(z)}{x} + \lambda \eta \tag{12}$$

$$\xi(z) = s\theta p(z)^{\theta - 1} \tag{13}$$

where λ is the marginal utility of consumption and $\lambda \xi(z)$ and $\lambda \eta$ are the Lagrange multipliers corresponding to (7) and (8). The multipliers measure the value of an additional worker in a particular city and the cost of pulling an additional worker from the pool of available migrants. We use (10)–(13) to illustrate the trade-offs involved in allocating workers across cities.

Combining (10) with (11) we find

$$\tau = \int A a(z) d\mu.$$

This equation describes the trade-off between using guided trips and undirected search. The marginal cost of raising the fraction of household members engaged in undirected search is equated to the average marginal cost of allocating those household members using guided trips. The averaging reflects the fact that undirected search allocates workers in proportion to each city's initial population.

Equations (11) and (12) imply that

$$A \frac{a(z)}{x} = \psi_1 - \psi_2 \frac{l(z)}{x}.$$

Intuitively, migration out of a city increases to the point where the marginal benefits of doing so (recall that the location-taste shocks initially imply benefits to leaving a city) are equated with the marginal cost of attracting workers into the city.

Finally, notice from (13) that the shadow value of bringing an extra worker to a city equals the marginal product of labor in that city. It follows from (11) and (12) that absent migration frictions, $A = \psi_1 = \psi_2 = 0$, the efficient allocation of workers across cities involves equating cities' marginal products of labor. This contrasts with the classic Roback (1982) and Rosen (1979) static model of a system of cities with free mobility in which equilibrium allocations are obtained by equating the level of utility across workers in different cities. This difference arises from the fact that we have assumed perfect consumption insurance. Still, our model shares the property of the classic model that individuals are indifferent to the city they choose to locate (the quantitative model developed below has this property as well.) Equations (11)–(13) indicate that migration frictions drive a wedge between marginal products of labor because heterogeneous initial populations imply differential costs of moving workers around. In this case workers remain indifferent to the city they choose to locate in.

3.4 Connecting Figure 2 to Population Adjustments

The planner's first order conditions reveal how gross migration relates to net migration. From the first order conditions for $a(z)$ and $l(z)$, (11) and (12), and the population constraint, (7), it is straightforward to show that

$$\frac{a\left(z\right)}{x} + \Lambda = \frac{\psi_1}{A + \psi_2} + \frac{A}{A + \psi_2}\Lambda + \frac{\psi_2}{A + \psi_2}\left(\frac{p\left(z\right) - x}{x}\right). \tag{14}$$

The arrival rate is a linear function of the net migration rate $\left(p(z) - x\right)/x$ with the linear coefficient satisfying $0 < \psi_2/(\psi_2 + A) < 1$. Similarly the leaving rate is given by:

$$\frac{l\left(z\right)}{x} = \frac{\psi_1}{A + \psi_2} + \frac{A}{A + \psi_2}\Lambda - \frac{A}{A + \psi_2}\left(\frac{p\left(z\right) - x}{x}\right). \tag{15}$$

The leaving rate is also is a linear function of the net migration rate with the linear coefficient satisfying $-1 < -A/(\psi_2 + A) < 0$. Equations (14) and (15) establish that gross migration in the model can be made consistent with the linear relationships depicted in Figure 2. This result is the underlying reason for our specifications of the location-taste shocks and the production of guided trips. Clearly, the relationship between gross and net migration depicted in Figure 2 places strong restrictions on the nature of migration frictions. Moreover, since the coefficients on net migration in (14) and (15) depend on the migration parameters A and ψ_2 Figure 2 is valuable for quantifying those frictions.

Modeling both gross migration margins is important for replicating Figure 2, but it also plays a crucial role in determining the speed of population adjustments. This can be seen by substituting for $a(z)$ and $l(z)$ in the original planning problem using (14) and (15), which simplifies it to

$$\max_{\{p(z),\Lambda\}} \left\{ \ln \int sp(z)^{\theta} \, d\mu - \int \left[\Phi(\Lambda) + \frac{1}{2} \frac{A\psi_2}{A+\psi_2} \left(\frac{p(z)-x}{x} \right)^2 x \right] d\mu - \tau\Lambda \right\}$$

subject to:

$$\int p(z) \, d\mu = \int x d\mu \tag{16}$$

with non-negativity constraints on the choice variables and where $\Phi(\Lambda)$ is a quadratic function in Λ involving the underlying structural parameters ψ_1, ψ_2 and A. In deriving this simplified planning problem we have used the fact that (7) and (8) reduce to (16) and that this constraint holds with equality at the optimum. Similarly we have used (9) to substitute for consumption in the planner's objective function.

When the planning problem is written in this way we see that population adjustments do not involve the gross migration decisions $a(z)$ and $l(z)$. Nevertheless modeling these decisions matters for understanding population adjustments because the coefficient that determines the speed of adjustment of population, $A\psi_2/(A+\psi_2)$, involves parameters governing the arrival and departure decisions. The arrival decision matters through the parameter A and departure decision matters through ψ_2. Also notice that the reduced form costs of adjusting population are quadratic. This is a direct consequence of specifying the location-taste shocks and guided trip production function to reproduce Figure 2. That is, the relationship between gross and net migration displayed in Figure 2 implies quadratic adjustment costs in net population adjustments.

Finally, notice that as long as $a(z) > 0$ and $l(z) > 0$, assumed in the statement of the simplified planning problem, population adjustments are independent of the undirected search decision. Undirected search is determined by the solution to

$$\tau = \Phi'(\Lambda).$$

An implication of this property is that as long as arrivals are always positive undirected search plays no role in net population adjustments. In the more general quantitative model arrivals will be set to zero in especially undesirable cities. Still, for most cases arrivals are strictly positive so that the amount of equilibrium undirected search is essentially irrelevant for our results. This is a useful property given that there is little evidence on the share of in-migration that is a result of undirected versus directed search. Nevertheless we include undirected search in the model because, as emphasized above, otherwise workers would have no way to move other than to obtain a guided trip and we think this is implausible.

3.5 One Possible Decentralization

The challenge for decentralizing the planning problem is how to treat guided trips. One valid approach is to have guided trips allocated entirely within the household through home production without any market interactions. We view guided trips in the model as an amalgam of both market and non-market activities and so we think a more natural decentralization is one that involves both market transactions and home production. We now consider such a decentralization.

Markets are competitive. Firms in a city of type z hire labor at wage $w(z)$ and produce consumption goods to maximize profits. Household members initially located in a type-z city produce $a_m(z)$ guided trips to that city which they sell to prospective migrants at price $q(z)$. The household also home produces guided trips for use by its own members and we denote these by $a_h(z)$. Let $m(z)$ denote the total number of guided trips to z-type cities purchased by household members in the market.

The representative household solves the following optimization problem

$$\max_{\substack{\{C,\Lambda,m(z),\\ a_m(z),a_h(z),\\ l(z),p(z)\}}} \left\{ \ln C - \int \left[\frac{A}{2} \left(\frac{a_m(z) + a_h(z)}{x} \right)^2 x + \left(-\psi_1 \frac{l(z)}{x} + \frac{\psi_2}{2} \left(\frac{l(z)}{x} \right)^2 \right) x \right] d\mu - \tau\Lambda \right\} \tag{17}$$

subject to:

$$C + \int q(z)\, m(z) d\mu = \int q(z)\, a_m(z) d\mu + \int w(z)\, p(z) d\mu + \int \Pi(z) d\mu \tag{18}$$

$$p(z) = x + m(z) + a_h(z) + \Lambda x - l(z),\ \forall z \tag{19}$$

$$\int \left[m(z) + a_h(z) + \Lambda x \right] d\mu = \int l(z)\, d\mu \tag{20}$$

along with non-negativity constraints on the choice variables. Equation (18) is the household's budget constraint where Π denotes profits from owning the firms. Equation (19) states that the population of a city after migration equals the initial population plus migrants from guided trips and undirected search less the initial population that migrates out of the city. Finally, equation (20) states that the household members that migrate to cities must equal the number of household members that migrate out of cities.

The unique competitive equilibrium is defined in the usual way with the market clearing conditions for guided trips, labor and consumption given by

$$\begin{aligned} m(z) &= a_m(z), \forall z; \\ n(z) &= p(z), \forall z; \\ C &= \int s n(z)^{\theta} \, d\mu. \end{aligned}$$

Using $m(z) = a_m(z)$ and the first order conditions of the household's problem we verify that a competitive equilibrium only determines the total number of guided trips into a city $a_m(z) + a_h(z)$; the composition of these guided trips between market and non-market activities is left undetermined.[16]

This particular decentralization makes it possible to calculate the total value of guided trips. In particular, as long as there are some guided trips purchased in the open market the total value of these trips is $q(z)a(z)$, with $a(z) = a_m(z) + a_h(z)$ and $q(z) = CAa(z)/x$. We use the total value of guided trips to help calibrate our model to the estimate of average moving costs in KW. Since the split of guided trips between market and non-market activities is unknowable it is ambiguous how to include them when measuring employment, wages and aggregate output in the quantitative model. Therefore another advantage of this decentralization is that we can use it to bound the impact of guided trips on our calibration.

4 Urban Population Dynamics with Housing

We expect housing to influence urban population dynamics for the reasons discussed in the introduction: it is costly to build quickly, durable and immobile. This section studies a simple model to explain why these factors may be important. The model borrows the geography and production structure from the previous section. There are three differences

[16] For details see the technical appendix.

with that model: individuals have a preference for housing services; to emphasize the role of housing, the model excludes migration frictions; to study dynamics the model introduces infinitely lived households.

To analyze this simple model it is convenient to exploit the fact, discussed further below in the context of the quantitative model, that the unique stationary competitive equilibrium can be obtained as the solution to a representative city planner's problem that maximizes local net surplus taking economy-wide variables as given, where the economy-wide variables are constrained to satisfy particular side conditions. Since here we are only interested in the qualitative implications of housing for urban population dynamics, we ignore the side conditions and study the city planner's problem assuming the economy-wide variables are exogenous. When we analyze our quantitative model below we take into account the relevant side conditions so that the economy-wide variables are determined endogenously.

Gross surplus in the representative city is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ s_t p_t^{\theta} + H \ln(\frac{h_t}{p_t}) p_t \right\}$$

where E_0 denotes the date $t = 0$ conditional expectations operator and $0 < \beta < 1$ is the household's time discount factor. Housing services are perfectly divisible so that each individual in the city enjoys h_t/p_t units of housing services where h_t denotes the local housing stock. The total utility individuals in the city derive from housing is given by $H \ln(h_t/p_t)p_t$, where $H > 0$ determines the relative weight of housing in preferences. We assume logarithmic preferences for housing here and below in the model we use for our quantitative analysis because they imply a constant share of housing in household expenditures. A constant housing expenditure share is consistent with evidence reported by Davis and Ortalo-Magné (2011). The planner must give up $\eta > 0$ units of surplus for each individual it chooses to bring to the city and employ in the production of consumption goods.

Within this framework we consider the speed of adjustment of population to a one time permanent change to TFP. To be concrete, suppose the city is in a steady state at $t = 0$ with $s = s_0$ and then at date $t = 1$ it faces a one-time unanticipated permanent change in TFP to $s = s^*$. We consider the adjustment of population to this unanticipated change in TFP under three scenarios for housing.

In the first scenario the planner can rent housing services from other cities at the exogenous price r_h. This assumption is equivalent to assuming that housing is perfectly mobile across cities. Equilibrium in this scenario is characterized by the first order conditions for population and housing:

$$H \ln(\frac{h_t}{p_t}) - H + s_t \theta p_t^{\theta-1} = \eta; \quad (21)$$

$$H \frac{p_t}{h_t} = r_h. \quad (22)$$

Condition (21) states that population is chosen to equate the marginal benefit of an additional individual working in the city to the cost of bringing that individual to the city, where the former is the sum of the marginal product of the individual plus the housing services she enjoys. The second condition equates the marginal utility of an extra unit of housing services with its cost. Replacing housing per individual in (21) using (22) yields

$$H \ln(\frac{H}{r_h}) - H + s_t \theta p_t^{\theta-1} = \eta. \quad (23)$$

The key feature of (23) is that it does not include housing services, h_t. This means that after the unanticipated change in TFP population jumps immediately to its new permanent level $p = p^*$ that equals the solution to (23) with $s_t = s^*$. So, when housing is perfectly mobile it is irrelevant for population dynamics.

Now assume the city is endowed with h_0 units of housing at $t = 0$ and that housing is immobile, meaning that it cannot be rented from or to any other city. In addition, suppose the change in TFP at $t = 1$ coincides with the onset of a potentially different exogenous path of the local housing stock satisfying

$$\ln h_t - \ln h^* = \rho_h^{t-1} (\ln h_0 - \ln h^*) \quad (24)$$

for $t \geq 1$ and $0 \leq \rho_h < 1$. Equilibrium housing is determined by (21) conditional on (24). We consider two cases for this scenerio.

First suppose that the local, immobile housing stock does not change with TFP, that is $h^* = h_0$. From (21) after the change in TFP p jumps immediately to its new level given by the value p^* that solves this equation. The new long run level of population depends on h_0 but the speed of adjustment to p^* is the same as when housing is perfectly mobile. In other words the presence of local, immobile housing is not sufficient for housing to affect population dynamics.

The second case is the new long run level of housing changes with TFP, $h^* \neq h_0$. We approximate the transition of population to its new steady state in this case by log linearizing (21) around $\ln p^*$ and $\ln h^*$. This yields

$$\ln p_t - \ln p^* = \frac{H}{H + s^* \theta (1-\theta) p^{*\theta - 1}} \rho_h^{t-1} (\ln h_0 - \ln h^*) .$$

In this case the speed of convergence of population to its new steady state p^* is directly related to the speed of convergence of housing to its new steady state through ρ_h. If the adjustment of housing is immediate, $\rho_h = 0$, then population's adjustment is instantaneous as in the case when $h^* = h_0$. If $0 < \rho_h < 1$ then population adjusts in proportion to the adjustment of housing.

We conclude that housing must be immobile and adjust slowly to changes in local productivity for it to affect population dynamics. It follows that a plausible quantitative analysis of urban population dynamics in response to TFP shocks must include endogenous housing and include the possibility of its slow adjustment. Natural candidates for influencing the speed of adjustment of housing to TFP shocks are construction depending on local resources and durability.

5 The Quantitative Model

Building on the foregoing analysis this section describes the model we use to quantify gross migration and housing's influence on urban population dynamics. The quantitative model introduces dynamics to the gross migration decision and endogenous housing. It also includes a labor supply decision. Changes in individual labor supply are a natural alternative to migration as a way for a city to adjust to labor demand shocks. The section begins with a description of the model environment and then characterizes the model's unique stationary competitive equilibrium as the solution to a representative city planning problem with side conditions.

5.1 The Environment

As before the economy consists of a continuum of geographically distinct locations called cities that are subject to idiosyncratic TFP shocks. Cities are distinguished by their stock of housing, h, initial population, x, and the current and lagged TFP, s and s_{-1}. The measure over these state variables is given by μ.[17]

Within cities there are three production sectors corresponding to intermediate goods, housing services and construction. The representative firm of each sector maximizes profits taking prices as given. Intermediate goods are distinct to a city and imperfectly substitutable in the production of the freely tradeable final goods non-durable consumption and durable equipment. The technologies for producing intermediate and final goods are identical to those underlying our estimates of TFP, described in equations (1) and (2).[18] Housing services are produced by combining residential structures with land, b_r, according to $h^{1-\zeta} b_r^{\zeta}$, $0 < \zeta < 1$. Following the convention that the prime symbol denotes next period's value of a variable, residential structures evolve as

$$h' = (1 - \delta_h)\, h + n_h^{\alpha} k_h^{\vartheta} b_h^{1-\alpha-\vartheta}, \tag{25}$$

where the factor shares are restricted to $\alpha > 0$, $\vartheta > 0$ and $\alpha + \vartheta < 1$, and $0 < \delta_h < 1$ denotes housing's depreciation rate. The last term in (25) represents housing construction. Local TFP s does not impact residential construction, reflecting our view that residential construction productivity is not a major source of cross-city variation in TFP. Equation (25) embodies our assumptions that residential structures are immobile, durable and costly to build quickly. The latter follows because residential construction requires local labor and land which have alternative uses in intermediate goods production and housing services. We assume that equipment used in production and construction is homogenous.

There is an infinitely lived representative household that allocates its unit continuum of members across the cities. The household faces the same migration choices described in Section 3, but being infinitely lived it takes into account the affects of current migration decisions on its members' allocation across cities in future periods. In particular, it is now bound by the constraint

$$x' = p \tag{26}$$

in each city where p continues to denote the post-migration population of a city. The household's members have logarithmic preferences for consumption and housing services in the city in which they are located. They also face a non-trivial labor supply decision. We assume that each period, after the migration decisions have been made, but before production

[17] Current and lagged TFP both appear in this list to accommodate the estimated TFP process described in Section 2.2. This is discussed further below.

[18] Equations (1) and (2) are written in terms of the location of a city, indexed by i, but here it is convenient to index them by the type of the city as represented by its state vector (h, x, s, s_{-1}).

and construction take place, individual household members receive a labor disutility shock φ with measure μ_n. Similar to our treatment of migration costs we make a parametric assumption for the average disutility of working. Specifically, if the household decides n of its members in a city will work for a year these costs are specified as

$$\int_{-\infty}^{\bar{\varphi}(n/p)} \varphi d\mu_n = \phi \left(\frac{n}{p}\right)^{\pi},$$

where $\phi > 0$, $\pi \geq 1$ and $\bar{\varphi}(n/p)$ satisfies

$$\frac{n}{p} = \int_0^{\bar{\varphi}(n/p)} d\mu_n.$$

The parameter π governs the elasticity of a city's labor supply with respect to the local wage.

5.2 Stationary Competitive Equilibrium

We consider the unique stationary competitive equilibrium. Since the model is a convex economy with no distortions, the welfare theorems apply. As a consequence the equilibrium allocation can be obtained by solving the problem of a social planner that maximizes the expected utility of the representative household subject to resource feasibility constraints. However, it is more useful to characterize the equilibrium allocation as the solution to a representative city social planner's problem with side conditions. This approach to studying the equilibrium allocation follows Alvarez and Shimer (2011) and Alvarez and Veracierto (2012).

The city planner enters a period with the state vector $z = (h, x, s, s_{-1})$. Taking as given aggregate output of final goods, Y, the marginal utility of consumption, λ, the shadow value of adding one individual to the city's population exclusive of the arrival and leaving costs, $\lambda\eta$, the shadow value of equipment, λr_k, the arrival rate of workers through undirected search Λ, and the transition function for TFP, $Q(s'; s, s_{-1})$, the representative city planner solves

$$V(z) = \max_{\substack{\{n_y,n_h,k_y,k_h,\\ h',b_r,b_h,p,a,l\}}} \left\{ \lambda \frac{1}{\chi} Y^{1-\chi} \left[s n_y^{\theta} k_y^{\gamma} \right]^{\chi} + H \ln\left(\frac{h^{1-\zeta} b_r^{\zeta}}{p}\right) p - \phi \left(n_y + n_h\right)^{\pi} p^{1-\pi} \right.$$

$$-\lambda r_k \left(k_y + k_h\right) - \lambda\eta \left(a + \Lambda x - l\right)$$

$$-\frac{A}{2}\left(\frac{a}{x}\right)^2 x - \left[-\psi_1 \frac{l}{x} + \frac{\psi_2}{2}\left(\frac{l}{x}\right)^2\right] x + \beta \int_{s'} V\left(z'\right) dQ\left(s'; s, s_{-1}\right)\Bigg\}$$

subject to

$$\begin{aligned} p &= x + a + \Lambda x - l & (27) \\ n_y + n_h &\leq p \\ b_r + b_h &= 1 \end{aligned}$$

plus (25), (26), and non-negativity constraints on the choice variables.

The objective of the optimization problem is to maximize the expected present discounted value of net local surplus. To see this note that the first two terms are the value of intermediate good production and the housing services consumed in the city. The next five terms comprise the contemporaneous costs to the planner of obtaining this surplus: the disutility of sending the indicated number of people to work, the shadow cost of equipment used in the city, and the disutility of net migration inclusive of guided trip production and taste-for-location shocks. The last term is the discounted continuation value given the updated state vector. Constraining the achievement of the city planner's objective are the local resource constraints, the housing and population transition equations and the non-negativity constraints on the choice variables. Note that in the statement of the land constraint we have normalized the local endowment of residential land to unity and used the fact that land used for current housing services cannot be built on in the same period.[19]

Let $\lambda\xi(z)$ denote the Lagrange multiplier corresponding to constraint (27) in the city planner's problem. This function represents the shadow value of bringing an additional individual to a type-z city. From the first order conditions of the city planner's problem it is easy to show that

$$\lambda\xi\left(z\right) = \begin{cases} A\left[\frac{a(z)}{x}\right] + \lambda\eta, \text{ if } a\left(z\right) > 0, \\ \left[\psi_1 - \psi_2\left(\frac{l(z)}{x}\right)\right] + \lambda\eta, \text{ if } l\left(z\right) > 0. \end{cases} \tag{28}$$

which takes into account the fact that $a(z) = l(z) = 0$ will never occur in equilibrium. Comparing equation (28) to equations (11) and (12) we see that if gross migration rates are

[19] In our calibration $0 < \theta + \gamma < 1$ which implies the presence of a fixed factor in the production of the city's intermediate good. As written the production function assumes that the supply of this fixed factor is constant (equal to one) across cities. One interpretation of this fixed factor is that it represents commercial land. Under this interpretation commercial land cannot be converted into residential land and vice versa.

positive then the shadow value of a migrant is related to migration costs in the same way as in the simple migration model of Section 3.

The unique stationary allocation is the solution to the city planner's problem that satisfies particular side conditions we now describe. To begin, let $\{n_y, n_h, k_y, k_h, h', b_r, b_h, p, a, l\}$ denote the optimal decision rules (which are functions of the state z) for the city planner's problem that takes $\{Y, \lambda, \eta, r_k, \Lambda\}$ as given and μ be the invariant distribution generated by the optimal decision rules $\{h', p\}$ and the transition function Q. In addition define that aggregate stock of equipment and per capita consumption:

$$\begin{aligned} K &= \int (k_y + k_h)\, d\mu; \\ C &= Y - \delta_k K, \end{aligned}$$

where $0 < \delta_k < 1$ denotes equipment's depreciation rate. Now suppose the following equations are satisfied

$$Y = \left\{ \int \left[s n_y (z)^{\theta} k_y (z)^{\gamma} \right]^{\chi} d\mu \right\}^{\frac{1}{\chi}} \tag{29}$$

$$\lambda = \frac{1}{C} \tag{30}$$

$$\int a(z)\, d\mu + \Lambda = \int l(z)\, d\mu \tag{31}$$

$$r_k = \frac{1}{\beta} - 1 + \delta_k \tag{32}$$

$$\lambda \int [\xi(z) - \eta]\, x d\mu - \tau \leq 0, (= 0 \text{ if } \Lambda > 0) \tag{33}$$

Then $\{C, K, n_y, n_h, k_y, k_h, h', b_r, b_h, p, \Lambda, a, l\}$ is a steady state allocation.[20]

In the steady state the variables taken as given in the city planner's problem solve the side conditions given by (29)–(33). Equation (29) expresses aggregate output in terms of intermediate good production in each city. This equation is the theoretical counterpart to equation (2) used to estimate city-specific TFP. The marginal utility of consumption is given by equation (30). Equation (31) states that total in-migration equals total out-migration.

[20] We prove this result in the technical appendix. We take a traditional dynamic programming approach to solving the city planner's problem. This is complicated substantially by the fact that there are four state variables in the city planner's problem, two of them endogenous. Furthermore the TFP process has a large domain. We overcome the computational challenges of a large dimensional and high variance state space in two main ways. First we exploit a parsimonious spline method to approximate the planner's value function and one-period return function. Second we take advantage of the large number of processors contained in graphics cards. For details see the appendix below.

Equation (32) defines the rental rate for equipment. The last side condition (33) is equivalent to (10) in the static model and similarly determines steady state undirected search.

The function $\xi(z)$ in (33) is central to the determination of migration in the model. It can be shown to satisfy

$$\begin{aligned}\xi(z) \quad = \quad & C\phi\left[n_y(z)+n_h(z)\right]^{\pi}(\pi-1)\,p(z)^{-\pi}+CH\ln\left(\frac{h(z)^{\varsigma}\,b_r(z)^{1-\varsigma}}{p(z)}\right)-CH \\ & +\beta\int\left(CA\left[\frac{a(z')}{p(z)}\right]^2+C\psi_2\left[\frac{l(z')}{p(z)}\right]^2+\Lambda\left[\xi(z')-\eta\right]\right)dQ(s';s,s_{-1}) \\ & +\beta\int\xi(z')dQ(s';s,s_{-1}).\end{aligned} \tag{34}$$

The value of bringing an additional individual to a city is the expected discounted value of four terms: the benefits of obtaining a better selection of worker disutilities given the same amount of total employment n_y+n_h; the benefits of the local housing services that the additional person will enjoy; the costs of reducing the amount of housing services that everybody else in the city will enjoy when an additional person is brought in; and the expected discounted value of starting the following period with an additional person. This last term includes the benefits of having an additional person producing guided trips to the city, the benefits of obtaining a better selection of location-taste shocks (given the same number of individuals leaving the city), and the benefits of attracting additional people to the city through the undirected search technology.

When there are no migration frictions, $A=\psi_1=\psi_2=0$, equation (28) implies that the marginal value of bringing an additional individual to a city is equated across cities as in the static case, $\xi(z)=\eta$, $\forall z$. However, unlike the static case this does not imply that wages are equated across cities. Instead, equation (34) says that the marginal savings in worker disutility plus the marginal impact on the utility of housing services is equated. When in addition to $A=\psi_1=\psi_2=0$ housing structures are made perfectly mobile across cities, the same condition is obtained because land remains immobile. However, when land is also made mobile, then the marginal savings in work disutility and the marginal utility of housing services are each equated across cities.

6 Calibration

We now calibrate the steady state competitive equilibrium to U.S. data.[21] Our calibration has two important characteristics. First, the city-specific TFP process is chosen to match our estimates presented in Section 2.2 thereby pinning down the model's exogenous source of persistence and volatility. Second, the calibration targets for the remaining parameters involve features of the data that are not primary to our study. So, for instance, we do not choose parameters to fit our estimated response of population to a TFP shock. The model's response of population to a TFP shock is the consequence of the estimated TFP process and the remaining parameters that are chosen to fit other features of the data.

In addition to specifying the stochastic process for TFP we need to find values for 16 parameters:

$$\theta, \gamma, \alpha, \vartheta, \delta_k, \delta_h, \beta, H, \zeta, \pi, \phi, \psi_1, \psi_2, A, \tau, \chi.$$

These include the factor shares in production and construction, depreciation rates for equipment and structures, the discount factor, the housing coefficient in preferences, land's share in housing services, and the parameters governing labor supply, migration, and intermediate goods' substitutability in final goods production.

We calibrate these parameters conditional on a given quantity of undirected search Λ determined by τ. For larger values of τ undirected search is relatively small so that $a(z) > 0,\ \forall z$. In these cases the behavior of the model is invariant to the specific value of τ. For smaller values of τ undirected search is large and $a(z) = 0$ for some z. In these cases the behavior of the model is affected. It turns out that even for seemingly large steady state Λ corner solutions for $a(z)$ are either non-existent or extremely rare. We set our baseline so that undirected arrival rate is 3.8%, roughly 70% of all moves.[22]

The baseline calibration for the assumed value of τ is summarized in Table 1. There we indicate for each parameter the proximate calibration target, the actual value for the target we obtain in the baseline calibration, and the resulting parameter value. In the remainder of this section we discuss the calculations underlying Table 1. We begin with the novel aspects of our calibration which involve the parameters governing migration, the city-level TFP process, the elasticity of substitution of city-specific intermediate goods in final good production, and labor supply.

[21] Except where noted the aggregate data used to calibrate our model is obtained from Haver Analytics.

[22] The specific value is $\tau = 1$. For this value the baseline calibration has 0.3% of city-year observations involving zero arrivals.

Table 1: Baseline Calibration

Parameter	Parameter Description	Calibration Target	Target Value	Actual Value	Parameter Value
θ	Labor's share in intermediate goods	$\int w \left[n_y + n_h\right] d\mu / GDP$	0.64	0.64	0.66
α	Labor's share in construction	$\int n_h d\mu / \int \left[n_y + n_h\right] d\mu$	0.042	0.042	0.41
β	Discount factor	Real interest rate	0.04	0.04	0.9615
γ	Intermediate goods' equipment share	K_y/GDP	1.63	1.63	0.235
δ_k	Depreciation rate of equipment	$\delta_k K/GDP$	0.16	0.16	0.104
ϑ	Equipment's share in construction	K_h/GDP	0.022	0.022	0.05
δ_h	Depreciation rate of structures	I/GDP	0.064	0.064	0.045
ζ	Land's share in housing services	$\int q^b b_r d\mu / \left[\int q^h h d\mu + \int q^b b_r d\mu\right]$	0.37	0.36	0.215
H	Housing coefficient in preferences	$\int q^h h d\mu / GDP$	1.55	1.50	0.205
ϕ	Labor disutility	$\int \left[n_y + n_h\right] d\mu / \int p d\mu$	0.63	0.63	1.61
π	Labor supply elasticity	$\partial \ln \left[n_y + n_h/p\right] / \partial \ln w$	0.24	0.25	5.0
ψ_1	Taste shock slope	Mean arrivals	5.5	5.5	6.07
ψ_2	Taste shock curvature	Slope of arrivals versus net	0.55	0.55	43.9
A	Guided trip cost	Average moving costs/average wages	-1.9	-1.9	35.6
χ	Intermediate goods' complimentarity	Zipf's law for population	-1.0	-1.3	0.9
g	Drift in technology	Zipf's law for TFP	-3.5	-3.4	-0.0017
ρ	TFP lag coefficient	Serial corr. of TFP growth	0.24	0.24	0.35
σ	TFP innovation std. err.	TFP growth innovation std. err.	0.015	0.015	0.019

Note: The calibration is based on $\tau = 1$. See the text for details. The actual values for the serial correlation and innovation of TFP growth are based on simulations. The underlying parameter values for the TFP process are somewhat different due because of the discrete approximation we use.

6.1 Migration Parameters

Section 3.4 establishes that the migration parameters A, ψ_1 and ψ_2 are central to determining the model's implications for the speed of population adjustment to TFP shocks. Fortunately there is evidence at hand that makes assigning values to these parameters straightforward. First, conditional on a value for A reproducing reproducing Figure 2 pins down ψ_1 and ψ_2. To reproduce Figure 2 we set the constant and slope coefficients in equation (14) to their empirical counterparts displayed in Figure 2.[23] In particular

$$\begin{aligned} \frac{\psi_1}{A+\psi_2} + \frac{A}{A+\psi_2}\Lambda &= 5.5; \\ \frac{\psi_2}{A+\psi_2} &= 0.55. \end{aligned}$$

To identify A we take advantage of KW's estimate of the average net cost of migration for those who move. Specifically, we match the statistic defined as the average net cost of migration of those who move divided by average wages where we take the latter from KW as well.[24] It is straightforward to replicate their concept of moving costs in our model. In KW, net moving costs sum two components of the utility flow of an individual in the period of a move. One component called "deterministic moving costs" is a function of the distance of the move, whether the move is to a location previously visited or not, the age of the mover, and the size of the destination location. The second component is the difference between idiosyncratic benefits in the current and destination location. We interpret guided trips in our model as representing the first component and the location-taste-shocks the second one. Consequently we measure average moving costs of individuals who move as

$$\frac{\int q(z)a(z)\ d\mu + C\tau\Lambda}{\int a(z)\ d\mu + \Lambda} + \frac{C\int\left(-\psi_1\frac{l(z)}{x} + \psi_2\left(\frac{l(z)}{x}\right)^2\right)xd\mu}{\int l(z)d\mu}.$$

[23]The constant term for arrivals in Figure 2 is the sample average gross migration rate, but gross migration is declining over our sample. Our measure of migration costs depends on this choice and so in principle our findings do as well. We examined the implications of calibrating to the average gross migration rate at the start and end of our sample and found that our results are substantially the same.

[24]Using KW's estimates, average net moving costs of those who move divided by average wages equal -1.9. This value equals the ratio -\$80,768/\$42,850. The numerator is the entry in the row and columns titled 'Total' in Table V and the denominator is the wage income of the median AFQT scorer aged 30 in 1989 reported in Table III. The negative value of the estimate indicates that individuals receive benefits to induce them to move.

Average wages are simply

$$\frac{\int w(z)\,[n_y(z)+n_h(z)]\,d\mu}{\int [n_y(z)+n_h(z)]\,d\mu},$$

where wages in a type-z city, $w(z)$, equal the marginal product of labor.

There are two potential drawbacks to using KW's estimate of moving costs. Firstly, KW identify moving costs using individual-level data. Individual behavior is not observable in our model so we cannot replicate their estimation strategy. Still, our model implies a value for the moving cost statistic so it is natural to take advantage of available estimates. More concerning is the fact that KW's estimate moving costs using data on frequency of inter-state moves, while our quantitative model describes inter-city moves. Inter-city moves are more frequent than moves between states. Consequently it is possible that KW would have estimated a different value for moving costs had they had data on all inter-city moves, in which case we would be calibrating our model to the wrong value. This suggests it is important to quantify any bias in KW's estimate arising from their focus on inter-state moves only.

We do this using a calibrated variant of their model which suggests any bias is likely to be small. Details of how we arrive at this conclusion are in the appendix; here we summarize our argument.[25] The model includes essentially the same individual discrete choice problem in KW inserted within an equilibrium setting. We posit 365 cities allocated across 50 states as in our data. There are a continuum of infinitely lived risk neutral individuals who begin each period identical except for the city they currently reside in and a random vector of city-specific location-taste shocks distributed as in KW. Living in a city for a period entitles an individual to the exogenous city-specific, and non-storable endowment called the wage, and individuals know the wage in every city.[26] An individual's only choice is to decide which city to live in the following period. They choose the city with the highest expected benefits, where the benefits of a given city equal the difference between the individual's location-taste shock for that city and the one for their current city, moving costs if applicable, and the discounted continuation value of living in the city next period. Costs of moving between cities may differ according to whether the move is within a state or across state lines. We study the unique stationary equilibrium in which individual choices are consistent with a

[25] The appendix is not ready yet.

[26] The wage distribution is calibrated to its 1990 empirical counterpart. For simplicity we abstract from idiosyncratic wage shocks included by KW. KW assume that individuals are finitely lived and only know the wages of their current city and any city they have lived in previously. In an infinite horizon context individuals eventually live in every city and therefore have knowledge of the complete wage distribution.

time invariant population distribution.

The model has three key parameters: the two moving costs and a scale factor for the location-taste shocks. We calibrate the moving costs to reproduce moving rates for all city-to-city moves and for state-to-state moves only in 1990. The scale factor is identified by forcing the model's net moving costs relative to average wages for all inter-city moves to match KW's -1.9 estimate. The model is able to match the calibration targets and we find the gross cost of moving across state lines is larger than moving within a state, consistent with KW's finding of deterministic moving costs being increasing in the distance of a move.

Since all moves are observable in this model we can also calculate the moving cost statistic using data on just inter-state moves. We find that the net costs of moving across state lines are 4% lower than when they are calculated using all moves. In other words within this simplified version of the KW framework the net costs of a move are approximately the same whether the move is across state lines or not. That the net cost of moving across state lines is lower than a move within a state reflects that the higher gross moving cost required to induce fewer inter-state moves is more than offset by larger differences in location preferences. We conclude that using KW's estimate is valid in our context.

6.2 TFP and Substitutability of City-specific Goods

The calibration of the substitution parameter χ and the stochastic process for TFP are interconnected because χ is used to measure TFP. When we measure TFP using the procedure described in Section 2.2 its growth rate is well-represented as a stationary AR(1) process which is non-stationary in levels and therefore inconsistent with a steady state. To overcome this we assume a reflecting barrier process for TFP:

$$\ln s_{t+1} = \max\left\{g + (1+\rho)\ln s_t - \rho \ln s_{t-1} + \varepsilon_{t+1}, \ln s_{\min}\right\}. \tag{35}$$

where $\varepsilon_{t+1} \sim N(0, \sigma^2)$, $g < 0$ and $\rho > 0$. With this process TFP growth is approximately AR(1), while its level is stationary due to having a negative drift and being reflected at the barrier $\ln s_{\min}$ (which we normalize to zero).[27]

The case $\rho = 0$ was used by Gabaix (1999) to explain the cross section distribution of cities by population. In this case the invariant distribution has an exponential upper tail

[27] Coen-Pirani (2010) considers a stationary AR(2) process for the level of TFP, calibrating it to match serial correlation in net state-to-state worker flows.

given by

$$\Pr[s_t > b] = \frac{d}{b^{\omega}}$$

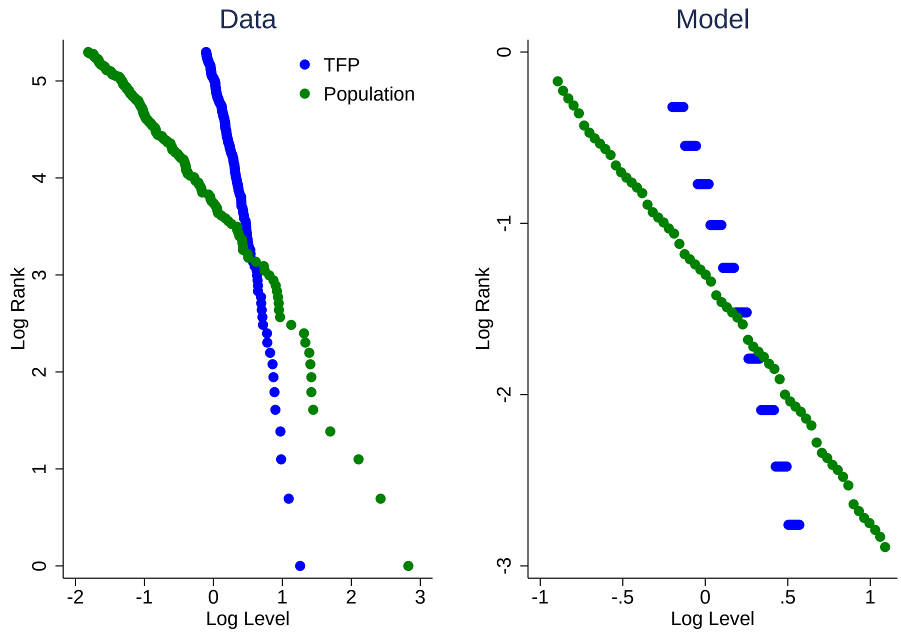
for scalars d and b. A striking characteristic of cities is that when s measures a city's population one typically finds that $\omega \simeq 1$. Equivalently a regression of log rank on log level of city populations yields a coefficient close to -1. This property is called Zipf's law and so we refer to ω as the Zipf coefficient. The case $\rho > 0$, which applies when TFP growth is serially correlated, has not been studied before. Simulations suggest this case behaves similarly to the $\rho = 0$ case in that it has an invariant distribution with an exponential-like upper tail. We verify below that a version of Zipf's law holds for TFP and so using the reflecting barrier process with $\rho > 0$ seems justified.

Our calibration of χ and (35) proceeds as follows. For a given χ (and θ and γ which are calibrated independently as discussed below) we measure TFP in the data following the procedure in Section 2.2, obtain its Zipf coefficient, and estimate an AR(1) in its growth rate. We then find the g, ρ and σ to match the Zipf coefficient and the serial correlation and innovation variance of the estimated AR(1) using data simulated from our model and based on these parameters calculate the model's population Zipf coefficient. The calibrated value of χ is the one that generates a population Zipf coefficient that is as close as possible to the one we find in the data, 1.0. The best fit is at $\chi = 0.9$ with a population Zipf coefficient equal to 1.3. The corresponding values of g, ρ and σ are in Table 1.

To demonstrate how well our model replicates the Zipf's laws for population and TFP, Figure 5 displays plots of log rank versus log level for population and TFP from the data and our calibrated model.[28] Notice how in the data the Zipf coefficient is larger for TFP than population. This arises naturally in the model because population tends to be allocated away from lower toward higher TFP cities. Luttmer (2007) finds a similar relationship between employment and TFP in an equilibrium model of firm size.

[28]The scales for the data and model differ because we use the cumulative distribution functions to measure rank in the model and TFP's domain is narrower in the model because of the extreme cost of matching the data. The narrower domain does not matter for our quantitative analysis. For example, the migration parameters are based on Figure 2 and migration costs. The former does not depend on the level of TFP and the latter depends on the distribution of TFP growth which is essentially independent of the domain. Our quantitative analysis relies on growth rates and is similarly independent of the underlying domain.

Figure 5: Zipf's Laws for Population and TFP



6.3 Labor Supply

The labor disutility parameters are calibrated to match statistics involving employment to population ratios. The multiplicative parameter ϕ is chosen to match the ratio of aggregate civilian employment to population obtained from Census Bureau data. The curvature parameter π is chosen using the first order condition for labor supply in a city. In the model's decentralization the representative household chooses labor supply to equate the disutility of putting an additional household member to work in a city with that city's wage. This implies:

$$(1-\pi)\left(\Delta\hat{n}_{it}-\Delta\hat{p}_{it}\right)+\Delta\hat{w}_{it}=0,$$

where n is the sum n_y and n_h and the "delta" and "hat" notation is from Section 2.2. Using the methods described in Section 2.2, we estimate the dynamic responses of $\Delta\hat{n}_{it}$, $\Delta\hat{p}_{it}$ and $\Delta\hat{w}_{it}$ to a local TFP shock and calibrate π so that this equation holds in the period of shock. Note that this procedure does not force the model to match these variables' individual impulse responses even in the period of a shock. We use these responses later to validate the model.

6.4 Remaining Parameters

Our strategy for calibrating the remaining parameters borrows from studies based on the neo-classical growth model. Several calibration targets involve GDP and we measure this in the model as

$$GDP = Y + I, \tag{36}$$

where Y is output of non-construction final goods and I is residential investment. Residential investment is measured as the value in contemporaneous consumption units of the total additions to local housing in a year. Specifically,

$$I = \int \left[\beta \int q^h(z')dQ(s'; s, s_{-1})\right] n_h(z)^{\alpha} k_h(z)^{\vartheta} b_h(z)^{1-\alpha-\vartheta} d\mu$$

where q^h denotes the price of residential structures. This price is obtained as the solution to the following no arbitrage condition

$$q^h(z) = r_h(z) + (1 - \delta_h)\beta \int q^h(z')\, dQ(s'; s, s_{-1})$$

where the rental price of residential structures, r_h, equals the marginal product of structures in the provision of housing services. The National Income and Product Accounts (NIPA) measure of private residential investment is the empirical counterpart to I. Our empirical measure of Y is the sum of personal consumption expenditures less housing services, non-residential fixed investment and private business inventory investment. Because our model does not include government expenditures and net exports we exclude these from our empirical concept of GDP.

Our measurement of model GDP and wages excludes the value of guided trip services, which might be problematic. For example, workers produce guided trips and in principle they should be compensated for this. Using the decentralization discussed in Section 3.5, we calculate the total value of guided trips in our baseline calibration to be 1.8% of model GDP. Recall that we interpret guided trips as encompassing many market and non-market activities. Some of these activities appear in the national accounts as business services and therefore count as intermediate inputs that do not end up directly in measured GDP. Others do not appear anywhere in the national accounts because they are essentially home production or are impossible to measure. Fortunately, given its small size including the total value of guided trips in our model-based measures of GDP and wages does not change our baseline calibration.

Measuring employment also is complicated by the fact that all household members participate in generating guided trips. We count those agents engaged in intermediate good production, n_y, and residential construction, n_h, as employed and measure their wages by their marginal products excluding the value of guided trips. The non-employed who also produce guided trips are assumed to be engaged in home production and so are not included in our accounting of employment. In Table 1 the labor share parameters are chosen to match total labor compensation as a share of GDP (the target is borrowed from traditional real business cycle studies) and our estimate of the share of construction employment in total private non farm employment.

We fix the discount rate so the model's real interest rate is 4%. Combined with this target the equipment-output ratio in the non-construction sector, K_y/Y, identifies equipments's share in that sector's production. Our empirical measure of equipment for this calculation is the Bureau of Economic Analysis' (BEA) measure of the stock of non-residential fixed capital. Equipment's depreciation rate is identified using the investment to GDP ratio, where we measure investment using the NIPA estimate of non-residential fixed investment. Equipment's share in residential construction is identified by the ratio of capital employed in the residential construction sector, K_h, to GDP where the empirical counterpart to capital in this ratio is the BEA measure of non-residential fixed capital employed in residential construction. The depreciation rate of residential structures is identified using the residential investment to GDP ratio.[29]

We identify the housing service parameters as follows. First the housing coefficient H is chosen to match the residential capital to GDP ratio, where the measurement of residential capital is consistent with our measure of residential investment described above. Land's share in housing services, ζ, is chosen to match the estimate of land's share of the total value of housing in Davis and Heathcote (2007). To measure this object in the model we need the price of land, q_b. We obtain this variable as the solution to the arbitrage condition

$$q^b(z) = r_b(z) + \beta \int q^b(z')\, dQ(s'; s, s_{-1}),$$

where r_b denotes the rental price of land which equals the marginal product of land in the

[29]The depreciation rate for residential structures obtained this way is close to the mean value of the (current cost) depreciation-stock ratio for residential structures obtained from the BEA publication "Fixed Assets and Consumer Durable Goods," once output and population growth are taken into account. Calibrating to this alternative depreciation rate has virtually no impact on our quantitative findings.

provision of housing services. Land's share of the economy-wide value of housing is then given by $\int q^b b_r d\mu / \left[\int q^h h d\mu + \int q^b b_r d\mu\right]$.

7 Quantitative Analysis

We now consider the model's empirical predictions. First we examine how well the model is able to reproduce our finding that a city populations are slow to adjust to TFP shocks. We confirm that the model is able to account for population's slow adjustment and that this success comes with generally accurate predictions for gross migration and the behavior of local labor and housing markets. We also study the model's predictions for unconditional dynamics and find the model is similarly successful at accounting for the data even though TFP shocks are the only source of city level fluctuations in the model. So, despite choosing parameters to match evidence not directly related to the dynamics of interest our model nonetheless excels in replicating them.

After establishing the empirical credibility of our framework, we investigate how migration and housing influence slow population adjustments. We find that costly directed search through the model's guided trip technology is the principle source of slow population adjustments. We interpret this finding as demonstrating that the myriad ways individuals get informed about desirable locations to live and work represent significant barriers to rapid population and worker reallocation. The fact that we identify the model's migration parameters without consideration of within-city dynamic responses to TFP shocks lends substantial credibility to this interpretation. Interestingly, we find that housing plays only a small role slowing population adjustments.

Finally, we investigate the implications of our model's successful accounting of slow short run population adjustments for persistent urban decline. There are many cities in our data that experience declining population throughout the sample period. These cities also typically experience declining TFP as well suggesting our model might account for the persistence of urban decline. We study the average experience of the 15 cities with the largest population declines. Simulating our model using the empirical path of TFP for these cities shows that our model accounts for essentially all of the average population decline. Apart from indicating an important role for TFP in urban decline this finding strongly suggests that costly migration, in particular the costs of finding desirable alternative locations to live and work, is a major factor determining the persistence of urban decline.

7.1 Model Validation with Conditional Correlations

We begin by comparing the model's dynamic responses to TFP shocks of population, gross migration, employment, wages, home construction and house prices to those we estimate from our panel data. We estimate the responses for both the model and the data using the identical procedure described in Section 2.2, basing our model responses on the simulation of a large panel of cities over a long time period. Comparisons of model and empirical impulse response functions is a model validation tool common in macroeconomics, see for example Christiano, Eichenbaum, and Evans (2005). Its key advantage over studying unconditional statistics, is that it is in principle robust to the presence of other shocks.

Figure 6: Responses of Population

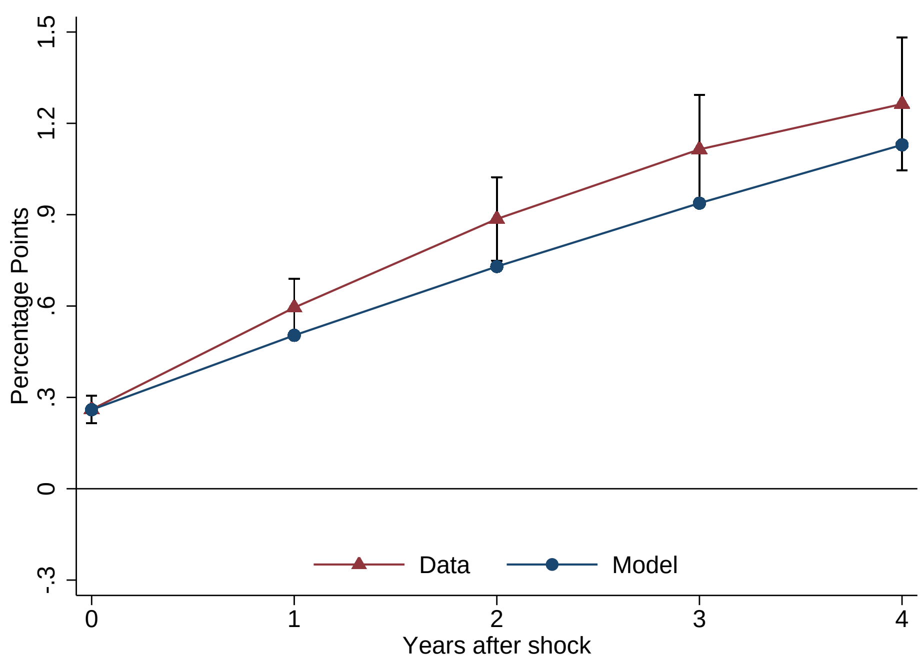
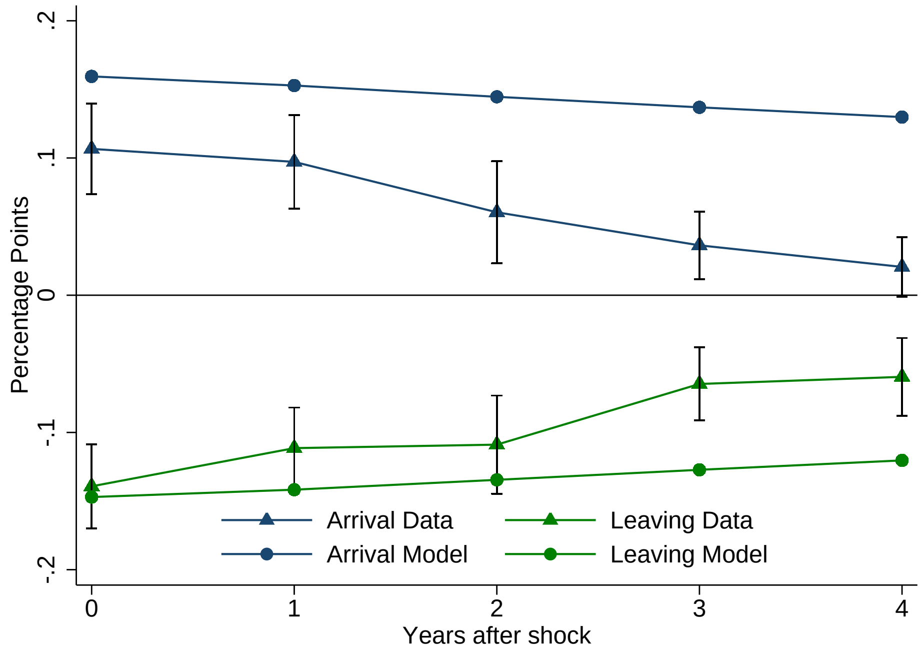


Figure 6 displays model and estimated responses of population to a one standard deviation positive innovation to TFP. Here and for similar figures below the vertical lines with hash marks indicate plus and minus 2 standard error bands for the estimates.[30] Figure 6 demonstrates that the model's population response is statistically and economically close to the one for the data; our model accounts for the slow response of city populations to local TFP shocks. It may appear that the model's slow population response is inconsistent with

[30] These standard errors do not take into account the sampling uncertainty in our estimates of the underlying TFP process.

Figure 7: Responses of Arrival and Leaving Rates



population's variance exceeding that for TFP as indicated by Figure 5. The model is able to account for the unconditional population distribution with a slow conditional response of population to a TFP shock because ultimately the long run response of population exceeds that for TFP.

Figure 7 shows this accounting for slow population adjustments involves replicating quite closely the dynamic responses of the arrival and leaving rates.[31] Crucially the model is consistent with the negative conditional correlation between the gross migration rates. The intuition for this finding is simple. Having multiple margins to respond to the increase in productivity, the city planner takes advantage of all of them. It can raise employment per person and bring more workers to the city. For the latter it can cut back on the fraction of the initial population that leaves for other cities, that is reduce the leaving rate, and attract more workers to the city taking advantage of more guided trips. The goodness-of-fit is weaker for gross migration than it is for population, for example both responses are more persistent than in the data and the arrival rate's initial response is a little too strong. Nevertheless given its simplicity the model does surprisingly well.

[31] The difference between the model's migration rates in the the first period do not correspond exactly to the response of population which in principle it should according to equation (27). The discrepancy is due to using logarithmic first differences to approximate the net rate of population growth.

Figure 8: Responses of Labor and Housing Markets

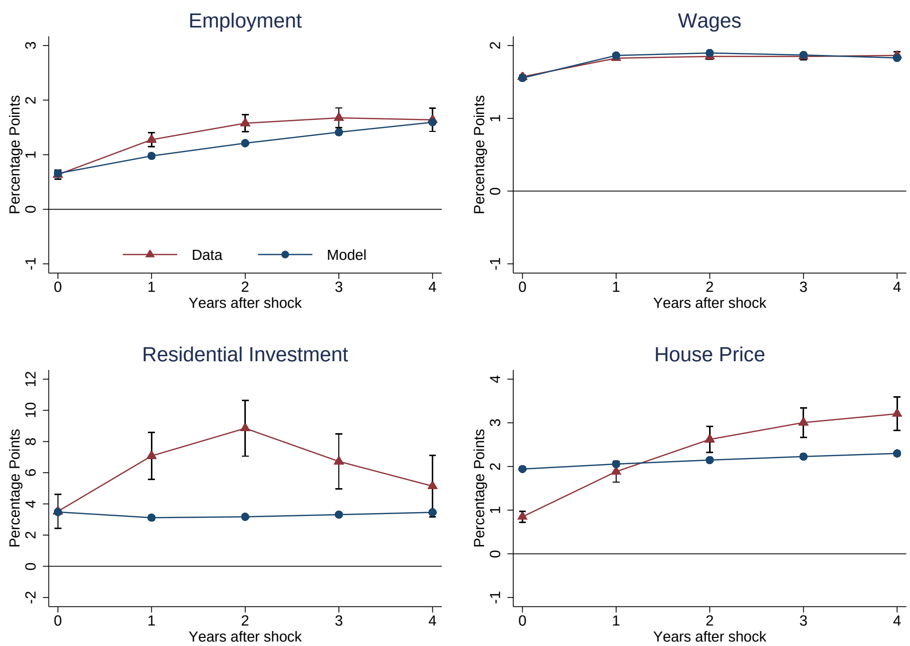


Figure 8 shows the dynamic responses of employment, wages, residential investment and house prices. We define house prices, q^{sf}, as the total value of structures and land used to produce housing services per unit of housing services provided:

$$q^{sf}(z) = \frac{q^{h}(z)\, h(z) + q^{b}(z)}{h(z)^{1-\varsigma} b_r(z)^{\varsigma}}.$$

The price q^{sf} corresponds to the price of housing per square foot under the assumption that every square foot of built housing yields the same quantity of housing services.

The labor market responses are a very good fit. Observe that the employment response in the model, as in the data, is stronger than the population response. That is, the employment to population ratio rises after a positive TFP shock indicating that the labor supply margin is indeed exploited in both the data and the model. The qualitative responses of construction and housing also are consistent with the data. These findings derive from a higher population desiring additional housing and that local factor inputs with alternative uses are used in construction thereby creating an imperfectly elastic supply of new housing. The model is less successful accounting for the quantitative responses of housing. Residential investment misses the hump shape in the data and the house price response is too fast. However in both cases the order of magnitude of the responses are about right. One explanation for housing's discrepancy with the data is that our model does not include search frictions in the local housing market. Lloyd-Ellis et al. (2014) demonstrate that search frictions show promise in generating serially correlated responses of construction and house price growth to productivity shocks.

7.2 Model Validation with Unconditional Statistics

The response of a city to TFP shocks is in principle robust to the presence of other shocks and is therefore informative about the validity of the model even if there are other shocks. However it is likely that there are other shocks to cities, for example to local taxes, amenities and demand for the locally produced intermediate good, and so it is worth knowing the extent to which TFP shocks alone account for the totality of variation, that is unconditional moments of the data. Tables 2 and 3 display unconditional standard deviations, contemporaneous correlations, and serial correlations of the same variables discussed above in the model and in our data for this purpose. Except for population, the standard deviations are expressed relative to the standard deviation of population and the contemporaneous correlations are all with population. The statistics are based on the levels of the gross migration rates and on the growth rates of the other variables. The variables have been transformed as described in Section 2 prior to the analysis.

The first thing to notice from Table 2 is that TFP shocks generate about two thirds of the overall variation in population – they are a quantitatively important source of local variation. The model is striking successful replicating the qualitative pattern of relative volatilities and only somewhat less successful quantitatively. Gross migration is less volatile than population and the labor and housing market variables are all more volatile than population, just as in the data. The relative volatility among the variables other than population also mostly match the data. Only the labor market variables miss, with wages a little too volatile compared to employment. The model is consistent with residential construction being the most volatile variable, but it fluctuates much less in the model than in the data. House prices in the model are more than twice as volatile as population, but not quite as volatile as in the data. The high volatility of house prices is a direct consequence of local land and labor that have alternative uses being factor inputs in construction.

The model is qualitatively consistent with all the correlations with population growth. The largest discrepancies with the data involve the arrival and leaving rates being perfectly

Table 2: Volatility and Co-movement Within Cities

Variable	Standard Deviation Data	Standard Deviation Model	Correlations Data	Correlations Model
Population	1.33	0.87	–	–
Arrival Rate	0.65	0.53	0.59	1.00
Leaving Rate	0.58	0.48	-0.42	-1.00
Employment	1.58	1.23	0.56	0.93
Wages	1.23	1.81	0.16	0.32
Construction	19.7	4.27	0.14	0.40
House Prices	3.76	2.32	0.29	0.47

Note: The statistics are based levels of the gross migration rates and on the growth rates of the other variables. The latter variables have been transformed as described in Section 2.2 prior to calculating growth rates. Standard deviations of all variables except population are expressed relative to the standard deviation for population. Correlations are with population.

positively and negatively correlated with population growth. Perhaps the mechanism inducing a positive correlation between the gross migration rates described in Coen-Pirani (2010), absent from our model, could overcome this deficiency.

From Table 3 we see that population, gross migration, employment and wages all display similar persistence to that in the data, although the model's variables are more persistent. Construction in the model and data are similarly random-walk like, although this feature of the unconditional moments clearly is due to the affects of other shocks given the serially correlated growth rate of construction in response to TFP shocks we find in the data. House prices display the greatest differences with house price growth displaying substantial serial correlation in the data while in the model house prices are more like a random-walk.

7.3 The Source of Slow Population Adjustments

We now address the sources of slow population adjustments in our model. Figure 9 displays impulse responses to TFP shocks implied by several different versions of the model for this purpose. The different versions consist of perturbations relative to the baseline, calibrated version of the model, holding parameters not involved in the perturbation fixed at their baseline values. These perturbations are as follows:

Table 3: Serial Correlation Within Cities

	Lag			
Variable	1	2	3	4
Population				
Data	0.81	0.74	0.67	0.63
Model	0.93	0.87	0.81	0.75
Arrival Rate				
Data	0.82	0.70	0.58	0.47
Model	0.93	0.87	0.81	0.75
Leaving Rate				
Data	0.77	0.74	0.67	0.60
Model	0.93	0.87	0.81	0.75
Employment				
Data	0.52	0.29	0.21	0.15
Model	0.73	0.63	0.58	0.54
Wages				
Data	0.15	0.04	0.05	0.07
Model	0.20	0.02	-0.02	-0.02
Construction				
Data	0.12	0.10	-0.02	-0.11
Model	-0.09	0.02	0.04	0.04
House Prices				
Data	0.73	0.31	-0.06	-0.25
Model	0.07	0.06	0.05	0.05

Note: The variables are have been transformed as described in Section 2.2 prior to calculating the statistics. The gross migration rates are levels and all other variables are growth rates.

- The "Free Guided Trips" case sets $A = 0$. This case has the same implications as assuming all the migration parameters are set to zero, because when guided trips are free the city-planner sets the leaving rate in each city to the constant value that minimizes leaving costs and adjusts population by changing the arrival rate at zero cost.

- "No Location-Taste Shocks" is the case where $\psi_1 = \psi_2 = 0$ so that costly guided trips are the only migration friction.

- "Mobile Housing" corresponds to the case discussed in Section 4 in which housing can be rented at a fixed price from any city; housing is perfectly mobile. In this case a city's dynamics are not influenced by the durability or the size of the local housing stock nor the city's ability to produce houses to accommodate new workers.

- "Full Flexibility" combines all the perturbations so there are no mobility costs and housing is perfectly mobile.

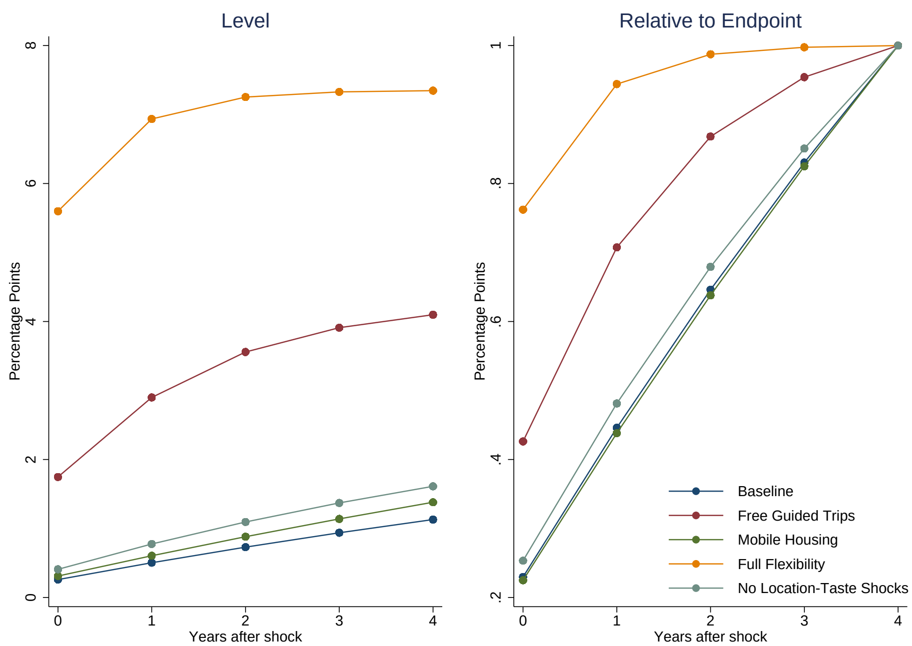
The left plot in Figure 9 displays the levels of the responses and the right one shows the responses after first dividing them by the value attained in the last (fifth) period of the response to more clearly show the speed of adjustment. Figure 9 shows that in the Full Flexibility case the population dynamics essentially follow the path of TFP with roughly 90% of the long run (five year) adjustment occurring after 2 years compared to 85% for TFP (see Figure 3) – absent migration and housing frictions the model has essentially no internal mechanism to propagate TFP shocks.

The No Location-Taste Shocks and Mobile Housing cases are very close to the baseline. In other words removing from the model costly out-migration or immobile housing, leaving costly guided trips as the only model friction, leaves the population response essentially as slow as it is in the baseline economy. In the Free Guided Trip case the population response is closer to the full-flexibility case, but does not take the model all the way there. Recall that making guided trips free leads to the same model responses as when migration is completely costless. Therefore in the Free Guided Trip case the only friction is that housing is immobile, suggesting some role for housing in slowing population adjustments.

Despite this last result, we still conclude that costly guided trips are the main source of slow population adjustments. The discrepancy with Full Flexibility arises from a property of adjustment costs highlighted by Abel and Eberly (1994). The first adjustment cost introduced to an otherwise frictionless model always has a relatively large impact on dynamics. So, introducing immobile housing into an otherwise frictionless model has large effects. However immobile housing on its own is not sufficient to deliver the amplitude and persistence of the population response in the data. Yet, the population dynamics with migration costs and mobile housing, the Mobile Housing case, are essentially the same as the baseline. This suggests that the prime driver of slow population adjustment in the model is the costly guided trip technology.

The finding of slow population adjustment driven mostly by costly migration confirms and reinforces results in KW. Using the parameters of a migration choice problem estimated with data on the frequency of inter-state moves taken from the National Longitudinal Survey of Youth, KW calculate optimizing responses of individuals in all states to a one-time permanent change in wages of one particular state (they consider changes in California, Illinois

Figure 9: Impact of Model Features on Population Adjustment



and New York). From these choices they obtain a matrix of transition probabilities which they simulate to trace out the response of population in the state with the permanent change in wages. Strikingly we find roughly the same five year elasticity of population with respect to the wage, about .5.[32] KW find that about 30% of the five period response occurs in the first period (see their Figure 1), whereas we find a response closer to 20%.

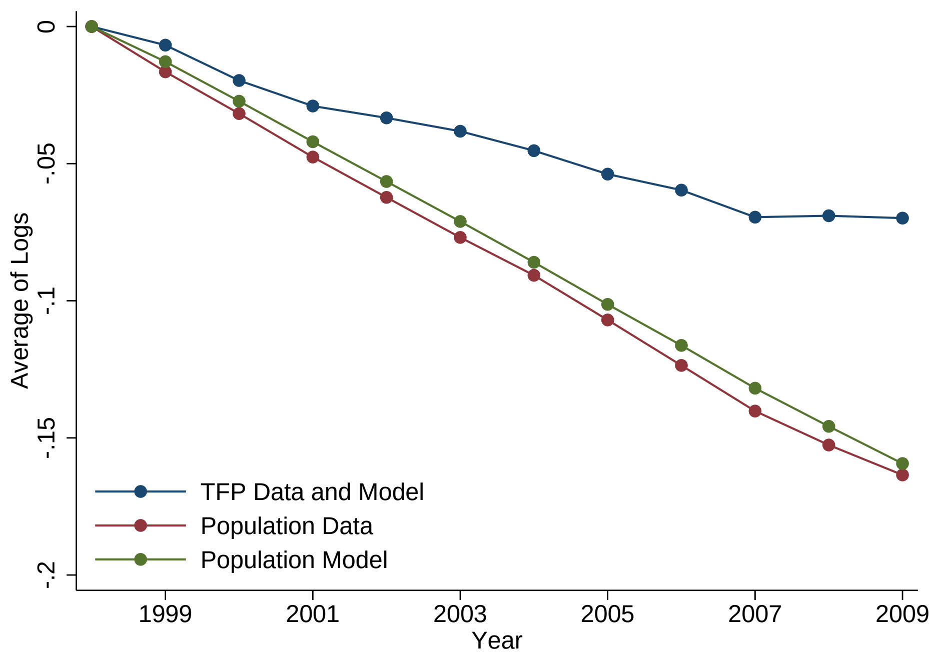
These similarities are quite striking given the very different methodologies used to generate the responses. The slower initial response we obtain is consistent with the fact that in our analysis wages take a few periods to reach their long run level due to the nature of the TFP process we estimate and our identification takes into account feedback from lower wages induced by greater net in-migration to future migration. The fact that our model includes housing does not appear to be an important source of the difference. Overall our results establish that Kennan and Walker (2011)'s findings are robust to the presence of housing and equilibrium interactions.

[32]KW consider a 10% increase in wages and find that population is 5% higher after five years. We find a 1.1% response of population to a 2% (roughly) permanent increase in the wage.

7.4 Migration and Urban Decline

There are many cities which experience declining populations (relative to the aggregate) over the sample period 1985-2009. This is evidence of the persistent urban decline studied by Glaeser and Gyourko (2005). Interestingly, the cities with declining populations also have TFP declining for most of the sample. Our model's ability to reproduce the *short run* response of population to TFP shocks then suggests it might account for population dynamics over the *long run* and in particular persistent urban decline. The success of the model accounting for the short run responses and Zipf's law suggests that it does. We now describe the outcome of a simple experiment that verifies this conjecture.

Figure 10: Persistent Urban Decline



We focus on the 15 cities of the 365 total that experience the greatest population declines in our sample. The corresponding TFP paths are fed into the model from the common initial condition that takes the mid-point of our TFP grid and assumes TFP stays at that level for a long time. We use the first 12 years of our sample, 1985 to 1997, to simulate unique initial conditions for each city based on each city's empirical TFP path. This procedure builds in the possibility that past declines in TFP show through into future population declines. For each city we calculate the predicted path for log population starting in 1998, average over these paths and compare the result to the same object constructed using the data.

Figure 10 shows the average log paths for TFP and population for the data and the model. TFP falls by 5 log points from 1998-2009 and population falls by three times as much.[33] Strikingly, the model's predicted path for population lies very close to its empirical counterpart. Obviously the fit is not as perfect for the individual cities, but the general impression is similar. There are two key factors driving the model's success: persistent declines in TFP taken from the data and the slow response of population to past declines in TFP predicted by the model. The impact of past TFP declines on current population growth is demonstrated clearly in the figure by the gradual slowdown in TFP's rate of decline alongside the almost constant decline of population.

Since the dominant source of slow population adjustment in the model is the cost of attracting workers to a city, we conclude from this experiment that these costs are integral to our model's explanation of persistent urban decline. Housing is not very important at all in our model in the sense that migration frictions alone account for slow population adjustments. This contrasts with Glaeser and Gyourko (2005) who argue that durable and immobile housing underly persistent urban decline.[34] These authors do not consider the costs of attracting workers to a city in their analysis. We consider both housing and migration, but housing turns out to be relatively unimportant.

8 Conclusion

This paper documents that population adjusts slowly to near random-walk TFP shocks and proposes an explanation for why. The explanation is that the incentive to reallocate population after a TFP shock is limited by the costs of attracting workers to desirable cities, that is adjustment costs to increasing population through in-migration are the dominant source of slow population adjustments. Our model of migration that delivers this result is not arbitrary, but is dictated by the nature of the relationship between gross and net population flows in cities that we uncover in our panel of 365 cities from 1985 to 2007.

Our model has left out other interesting model features that are undoubtedly important

[33] The much larger drop in population is a reflection of the forces driving our model's reproduction of Zipf's law discussed above.

[34] They show that irreversible housing in cities with declining populations has several empirical predictions which they verify in the data. Our model does not share these predictions since the irreversibility constraint is never binding. It is never binding because of the relatively small variance of TFP innovations compared to the depreciation rate for housing. Incidently, this constraint appears not to bind in the data as well as new building permits are always strictly positive in our panel of cities. The constraint presumably binds for neighborhoods within a city and this may have a role in explaining Glaeser and Gyourko (2005)'s findings.

for understanding the full range of adjustments to shocks within cities. Chief among these omissions are search frictions in local labor and housing markets. We think it would be interesting to add these features to our framework. Doing so would help disentangle the contributions to labor reallocation of traditional search frictions from the migration frictions we have introduced in this paper.

Taken together our findings point to a heretofore ignored mechanism in the determination of macroeconomic adjustment. While we have not done this experiment, our results strongly suggest that a mis-allocated housing stock due to overbuilding in the recent housing boom would have little impact on macroeconomic labor adjustment and the sluggish economy since the housing bust is unlikely to have been driven by such a mis-allocation. This conclusion of course derives from a model without any frictions in the financing of housing. If housing is to be important for macroeconomic labor adjustment it must be through these or other frictions.

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Appendix: Computations

While representing the solution of the economy-wide social planner as the solution to a city planner problem plus side conditions was a huge simplification, computing the city planner problem remains a nontrivial task.

The first difficulty is that the value function of the city planner's problem has two endogenous variables and two exogenous state variables. Each exogenous state variable takes values in a finite grid but this grid cannot be too coarse if the resulting discrete process is to represent the original AR(2) in a satisfactory way. To make the task of computing the value function manageable we resorted to spline approximations.

Cubic spline interpolation is usually used in these cases. A difficulty with those method is that it does not necessarily preserve the shape of the original function, or if it is does (as with Schumacher shape-preserving interpolation) it is somewhat difficult to compute. For these reasons, we use a local method that does not interpolate the original function but that approximates it while preserving shape (monotonicity and concavity). An additional benefit is that it is extremely simple to compute (there is no need to solve a system of equations). The method is known as the Shoenberg's variation diminishing spline approximation. It was first introduced by Shoenberg (1967) and is described in a variety of sources (e.g. Lyche and Morken (2011)). In what follows we provide its definition.

For a given continuous function f on an interval $[a, b]$, let p be a given positive integer, and let $\tau = (\tau_1, ..., \tau_{n+p+1})$ be a knot vector with $n \geq p+1$, $a \leq \tau_i \leq b$, $\tau_i \leq \tau_{i+1}$, $\tau_{p+1} = a$ and $\tau_{n+1} = b$. The *variation diminishing spline approximation of degree p to f* is then defined as

$$S_p(x) = \sum_{j=1}^{n} f\left(\tau_j^*\right) B_{jp}(x)$$

where $\tau_j^* = (\tau_{j+1} + ... + \tau_{j+p})/p$ and $B_{jp}(x)$ is the *jth* B-spline of degree p evaluated at x. The B-splines are defined recursively as follows

$$B_{jp}(x) = \frac{x - \tau_j}{\tau_{j+p} - \tau_j} B_{j,p-1}(x) + \frac{\tau_{j+1+p} - x}{\tau_{j+1+p} - \tau_{j+1}} B_{j+1,p-1}(x)$$

with

$$B_{j0}(x) = \begin{cases} 1, \text{ if } \tau_j \leq x < \tau_{j+1} \\ 0, \text{ otherwise} \end{cases}$$

As already mentioned, this spline approximation preserves monotonicity and concavity of the original function f (e.g. Lyche and Morken (2011), Section 5.2). The definition of variation diminishing splines is easily generalized to functions of more than one variable using tensor products (e.g. Lyche and Morken (2011), Section 7.2.1). These properties greatly simplify the value function iterations of the city planner problem and they should prove useful in a variety of other settings. In actual computations we decided to work with an approximation of degree $p = 3$.

An additional complication involves the return function of the city planner's problem.

Conditional on the current states (h, x, s, s_{-1}) and future states (h', x'), evaluating the one period return function of the city planner requires solving a nonlinear system of equations in $(n_y, n_h, k_y, k_h, b_r, b_h)$ allowing for the possibility that the constraint $n_y + n_h \leq x'$ may bind. This is not a hard task. However, doing this for every combination of (h, x, s, s_{-1}) and (h', x') considered in solving the maximization problem at each value function iteration would slow down computations quite considerably. For this reason, we chose to construct a cubic variation-diminishing spline approximation to the return function $R(h, x, s, , s_{-1}, h', x')$ once, before starting the value function iterations, and use this approximation instead. In practice, for each value of (h, x, s, s_{-1}) we used a different knot vector for h' and x' to gain accuracy of the return function over the relevant range.

Performing the maximization over (h', x') for each value of (h, x, s, s_{-1}) at each value function iteration is a well behaved problem given the concavity of the spline approximations to the return function and the next period value function. There are different ways of climbing such a nice hill in an efficient way. In our case, given that we could offload computations into two Tesla C2075 graphic cards (with a total of 896 cores), we used the massively parallel capabilities of the system to implement a very simple generalized bisection method. Essentially for each value of (h, x, s, s_{-1}) we used a block of 16×16 threads to simultaneously evaluate 16×16 combinations of (h', x') over a predefined square. We then zoom to the smallest square area surrounding the highest value and repeat. In practice, a maximum would be found after only three or four passes.

Statistics under the invariant distribution were computed using Monte Carlo simulations. This part of the computations was also offloaded to the graphic cards to exploit their massively parallel capabilities. To avoid costly computations similar to those encountered in the evaluation of the return function, cubic spline approximations were used for all decision rules.

Speeding up the solution to the city planner's problem and Monte Carlo simulations was crucial since finding solutions (Y, C, Λ, η) to the side conditions requires solving the city planner's problem and simulating its solution several times.

The source code, which is written in CUDA Fortran, is available upon request. Compiling it requires the PGI Fortran compiler. Running it requires at least one NVIDIA graphic card with compute capability higher than 2.0.