Discussion of "Jensen's Inequality and the Success of Linear Prediction Pools"

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Outline

Background of the paper

- Forecast averaging for point forecasts easier to justify than density forecast averaging.
- Paper shows that we usually do not mess up too much - averaging has "insurance function" for different, commonly used, score functions.
- Furthermore
  - presents MC study to analyze performance of linear pools - even if one model in the set is the true model, the linear pool with EW is doing relatively well.
  - presents empirical results on US marco series - linear pool with EW does very well and often beats all individual models.

Concave Score Functions

If $S(\cdot)$ is concave, we have for a fixed y and with $\sum_i \omega_i = 1$, $\omega_i \geq 0$

$$S(\sum_i \omega_i f_i(y)) \geq \sum_i \omega_i S(f_i(y)).$$

Therefore, for deterministic weights ω_i:

$$E\left[S(\sum_i \omega_i f_i(y))\right] \geq \sum_i \omega_i E\left[S(f_i(y))\right] \geq \min_i E\left[S(f_i(y))\right]$$

holds for the Log Score (LS).
Quadratic Score (QS), Continuous Ranked Probability Score (CRPS) depend on more than the value $f(y)$.

Log Pool needs "Strongly" Concave Functions

We have for a fixed y and with $\sum_i \omega_i = 1$, $\omega_i \geq 0$

$$\begin{aligned}
S\left(\frac{\Pi_i f_i^{\omega_i}(y)}{\int \Pi_i f_i^{\omega_i}(y)dy}\right) &= S\left(\exp(\log \Pi_i f_i^{\omega_i}(y) - \log \int \Pi_i f_i^{\omega_i}(y)dy)\right) \\
&\geq S\left(\exp(\sum \omega_i \log f_i(y) - \log \int \sum_i \omega_i f_i(y)dy)\right) \\
&= S\left(\exp(\sum \omega_i \log f_i(y) - \log \sum_i \omega_i \cdot 1)\right) \\
&= S\left(\exp(\sum \omega_i \log f_i(y))\right)
\end{aligned}$$

If S(.) is Concave enough:

Therefore, if $S(\exp(\cdot))$ is concave, we have for fixed y:

$$S\left(\frac{\prod_i f_i^{\omega_i}(y)}{\int \prod_i f_i^{\omega_i}(y)dy}\right) \geq \sum_i \omega_i \, S\Big(\exp(\log f_i(y))\Big) = \sum_i \omega_i \, S\Big(f_i(y)\Big)$$

LS is concave enough. For QS and CRPS, similar reasoning should show that their are not concave enough.
Therefore, for deterministic weights ω_i:

$$E\left[S\left(\frac{\prod_i f_i^{\omega_i}(y)}{\int \prod_i f_i^{\omega_i}(y)dy}\right)\right] \geq \sum_i \omega_i \, E\left[S(f_i(y))\right] \geq \min_i E\left[S(f_i(y))\right]$$

Some outcomes of simulations and emp. Example:

Simulation:

- EW linear pool does do relatively well
- results qualitatively similar across scoring rules
- power of GW test sometimes low sometimes high

Empirical Example

- four-dimensional post 1985 US monthly macro sample (323 obs), h=1,3,6
- models differ by system size (K=1,2) and estimation windows (short rolling, long rolling)
- Results:
  - short better than long
  - EW is good
  - EW better with LS than with QS and CRPS (LS more sensitive to outliers)

Some related Literature:

- Hendry and Clements *(2004, 'Pooling of forecasts', Econometrics Journal, 7, 1-31*) show that deterministic combinations provide insurance against the worst forecast. *They point out the role of models being differentially misspecified.*
- Kascha and Ravazzolo (*2010, Combining Inflation Density Forecasts. Journal of Forecasting*) show that for the LS deterministic linear and logarithmic pools provide insurance because of the concavity of the LS.
- Timmermann (*2006, Forecast Combinations, Handbook of Economic Forecasting*) lists reasons for the success of equal weights

Some eventually interesting questions

- Arithmetic and geometric averaging and loss functions:
  - insurance effect vs. performance
  - loss functions and nonlinear averaging

⇒compute results also for an equally weighted logarithmic average.

- Insurance effect of deterministic weights but what about a random model set?
  - How far should one restrict the set? Does the answer depend on the way pooling is done? Does the answer depend on the score?
- Jensen's inequality, density forecast averaging and decision making:
  - Grinblatt, Linnainmaa (2011): *Jensen's Inequality, Parameter Uncertainty, and Multi-period Investment*