preamble

Bank Regulation and Risk Management: An Assessment of the Basel Market Risk Framework

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Motivation

Basel Committe on Banking Supervision (2006) requires stress testing and the use of VaR.

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Comparison of the following risk management systems:

- VaR constraints
- Stress testing (ST) constraints
- VaR+ST constraints

Portfolio

Allocation among following asset classes (distributions are discrete):

- T-bills
- government bonds
- corporate bonds
- six size/book-to-market Fama-French equity portfolios

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Notation

- $\mathbf{w}$ portfolio weights
- $\mathbf{R}$ vector of asset returns
- $\mathbf{R}$(Crash of 87) asset returns during the crash 1987
- $\mathbf{R}$(9/11) asset returns just after 9/11/2001

VaR constraints

Mean-CVaR boundary

$E \longrightarrow CVaR_{pmin,E} := \min\{CVaR_{0.99}(-\mathbf{wR}') : \text{expected return of } \mathbf{w} \text{ is } E)\}$

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i.e. there is no portfolio with expected return E and a smaller $CVaR_{0.99}$.

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- Variable *VaR* constraint: $VaR_{0.99}(-\mathbf{w}'\mathbf{R}) \leq VaR(-\mathbf{w}^*_{E(\mathbf{w})}\mathbf{R})$ (if E increases the bound increases too)

VaR constraints

ST constraints:

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- Variable ST constraint:

$$\begin{aligned} -\mathbf{w}'\mathbf{R}(\text{Crash of 87}) &\leq -\mathbf{w}^*_{\mathbf{E}(\mathbf{w})}\mathbf{R}(\text{Crash of 87}) \text{ and} \\ -\mathbf{w}'\mathbf{R}(9/11) &\leq -\mathbf{w}^*_{\mathbf{E}(\mathbf{w})}\mathbf{R}(9/11) \end{aligned}$$

Methodology

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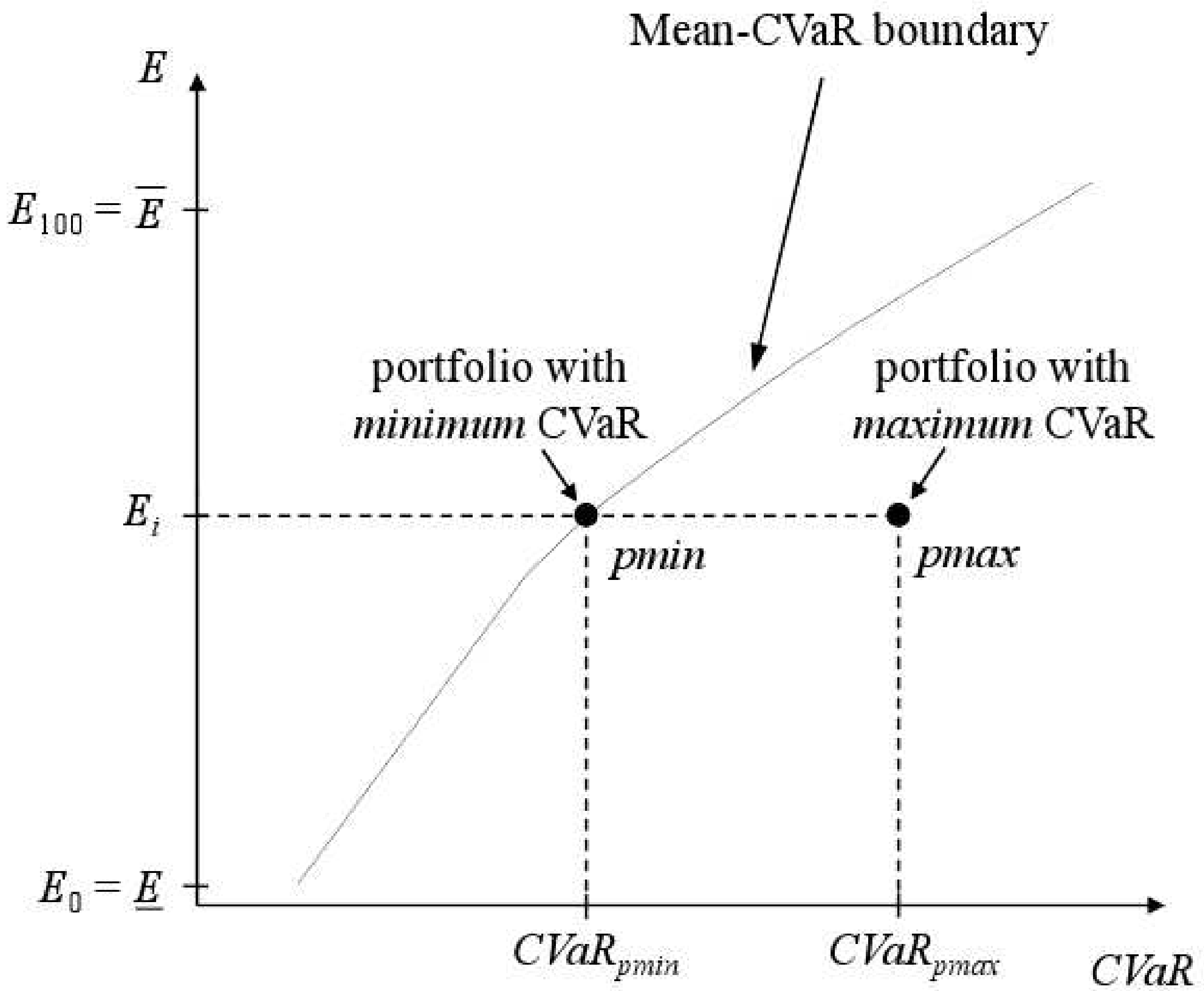
- $\delta := \frac{\overline{E} - \underline{E}}{100}$.
- $E_i := \underline{E} + (i - 1)\delta$ for $i = 1, \ldots, 100$.

Maximum efficiency loss



Maximum efficiency loss of E_i: $M_i = CVaR_{pmax,E_i} - CVaR_{pmin,E_i}$

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  $\text{and VaR/ST/VaR+ST constraint is satisfied}\}$.

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- **Maximum efficiency loss** of E_i:

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- **Average efficiency loss**: $\frac{1}{100}\sum_{i=1}^{100} M_i$

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- **Largest efficiency loss**: $max_{i=1,...,100} M_i$

Efficiency loss

- **Relative efficiency loss** of E_i:

$$R_i = \frac{CVaR_{pmax,E_i} - CVaR_{pmin,E_i}}{CVaR_{pmin,E_i}}$$

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Efficiency loss with VaR constraints

	short sell. disallow.			short sell. allow.		
	fixed bound		var. bound	fixed. bound		var. bound
	4%	8%		4%	8%	
Efficiency loss:						
Average	4.46	8.38	1.29	8.25	14.94	3.86
Largest	6.20	11.79	2.65	11.11	20.97	9.59
Relative efficiency loss:						
Average	224.21	362.56	37.10	366.99	501.75	105.93
Largest	592.30	1056.73	89.31	934.56	1844.67	174.24
Maximal expected return	1.03	1.26	1.58	1.59	2.07	2.16

Comments to the table

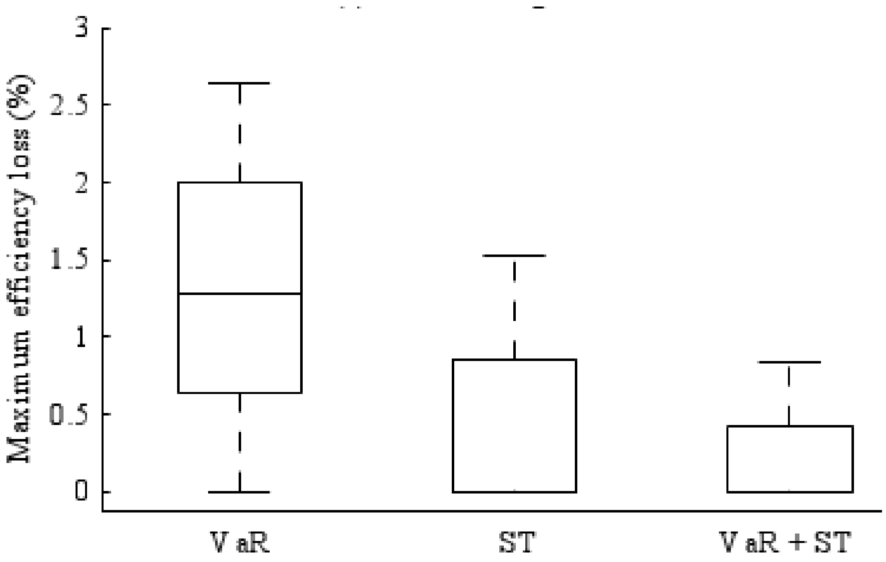
Table with VaR, ST and VaR+ST constraints have in common:

- Efficiency losses are sizeable.
- Variable constraints have lower eff. losses than fixed constraints.
- The eff. losses for a fixed bound of 4% are lower than of 8%, but the expected returns are also lower.
- If short selling is allowed then the losses and the expected returns are higher than if short selling is disallowed (number of portfolios increase).

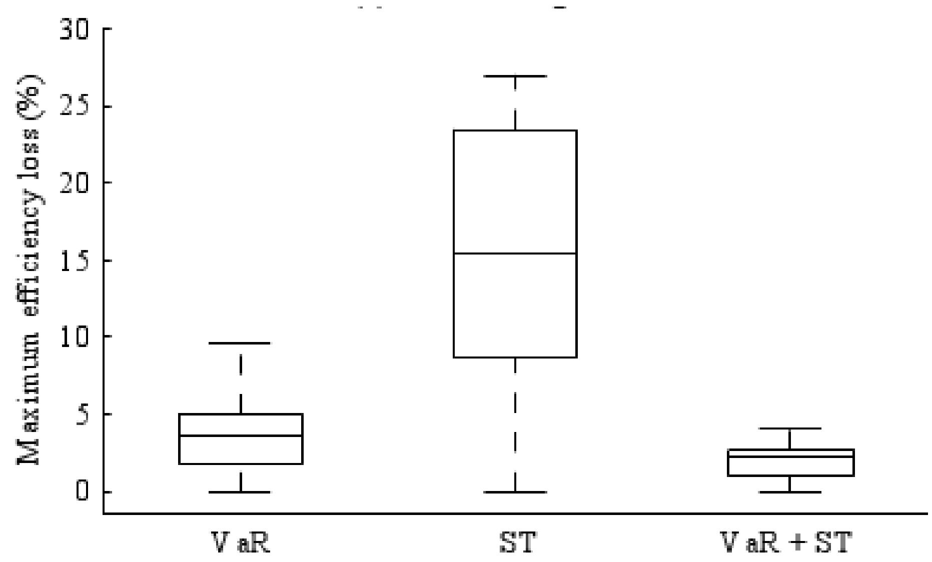
Boxplot of maximum efficiency loss

Variable constraints:

Short selling disallowed



Short selling allowed



Comments to the plots

- Note the different scaling of the left and the right plot.
- Losses are much higher when short selling is allowed than when short selling is disallowed.
- If short selling is disallowed then ST constraints perform better than VaR constraints.
- If short selling is allowed then VaR constraints perform better than ST constraints.
- Tests on the difference of the distribution of losses (on one plot, or comparing short selling allowed and disallowed) as Kolmogorov Smirnov and Wilcoxon rank sum test reject the equality on the 1% confidence level.

Portfolios on the mean-CVaR boundary

Short selling disallowed (short selling allowed):

	VaR	Losses in ST events		CVaR
		Crash of 87	9/11	
E_{33}				
Mean-CVaR boundary	2.38 (1.38)	1.91 (-0.26)	2.52 (0.84)	2.42 (1.53)
VaR	2.38 (1.38)	1.91 (-0.26)	2.52 (0.84)	4.22 (3.91)
ST	2.91 (16.59)	1.91 (-0.26)	2.52 (0.84)	3.60 (22.59)
VaR+ST	2.38 (1.38)	1.91 (-0.26)	2.52 (0.84)	3.15 (2.44)
E_{67}				
Mean-CVaR boundary	6.69 (4.38)	5.50 (3.90)	7.61 (4.83)	8.91 (5.07)
VaR	6.69 (4.38)	5.50 (3.90)	7.61 (4.83)	9.58 (8.89)
ST	6.69 (10.68)	5.50 (3.90)	7.61 (4.83)	8.91 (14.93)
VaR+ST	6.69 (4.38)	5.50 (3.90)	7.61 (4.83)	8.91 (7.20)

Comments to the table

- For example in the second row we see the VaR, the losses in the Crash of 87 and 9/11 and the CVaR of a portfolio with expected return E_{33} satisfying the variable VaR constraint with maximum efficiency loss.

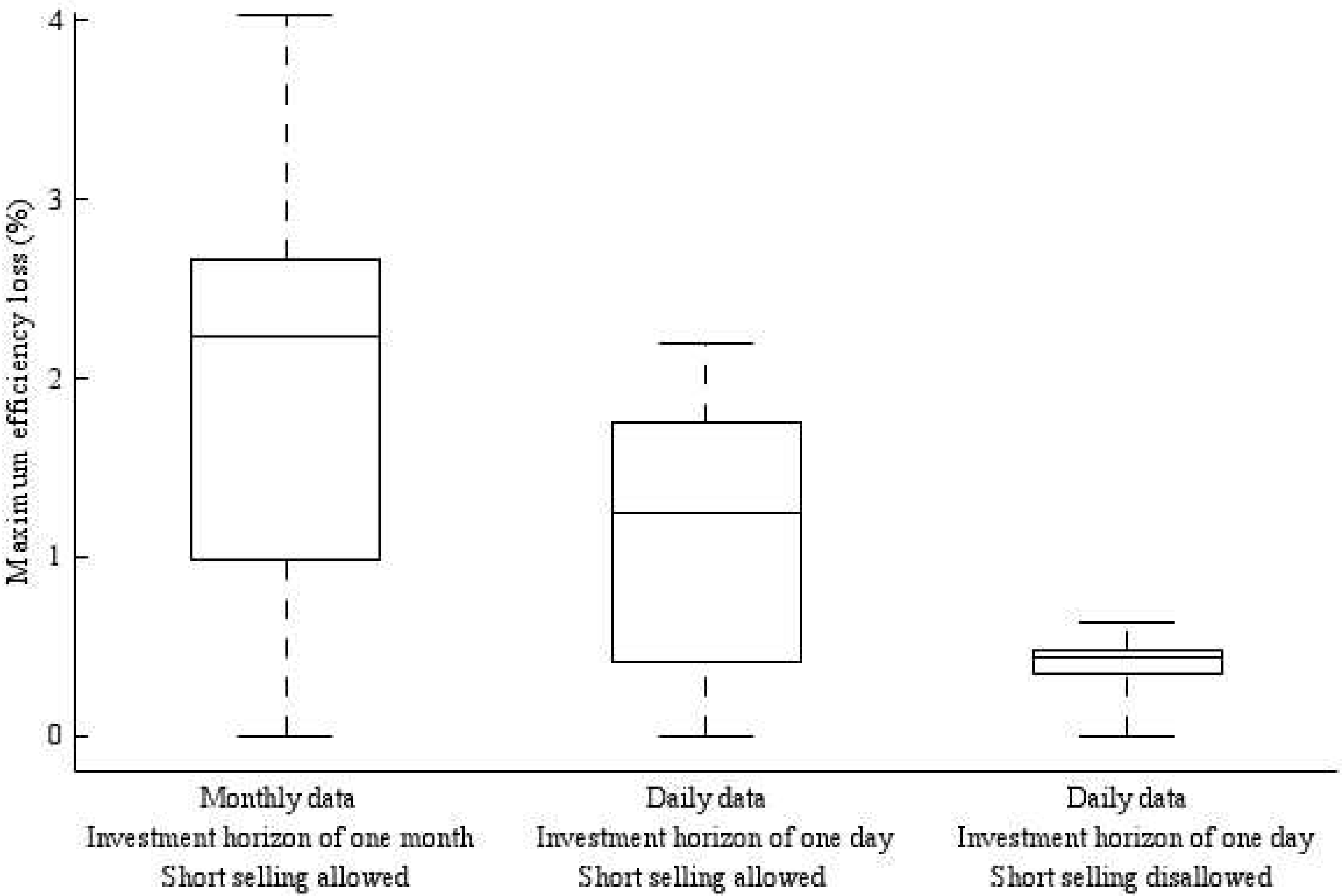
- Short selling disallowed: CVaR of the VaR constraint is the highest. CVaR of the VaR+ST constraint is closest to the CVaR on the mean-CVaR boundary.

- Short selling allowed: CVaR of the ST constraint is the highest (extraordinary high). CVaR of the VaR+ST constraint is closest to the CVaR on the mean-CVaR boundary.

Boxplot of maximum efficiency loss

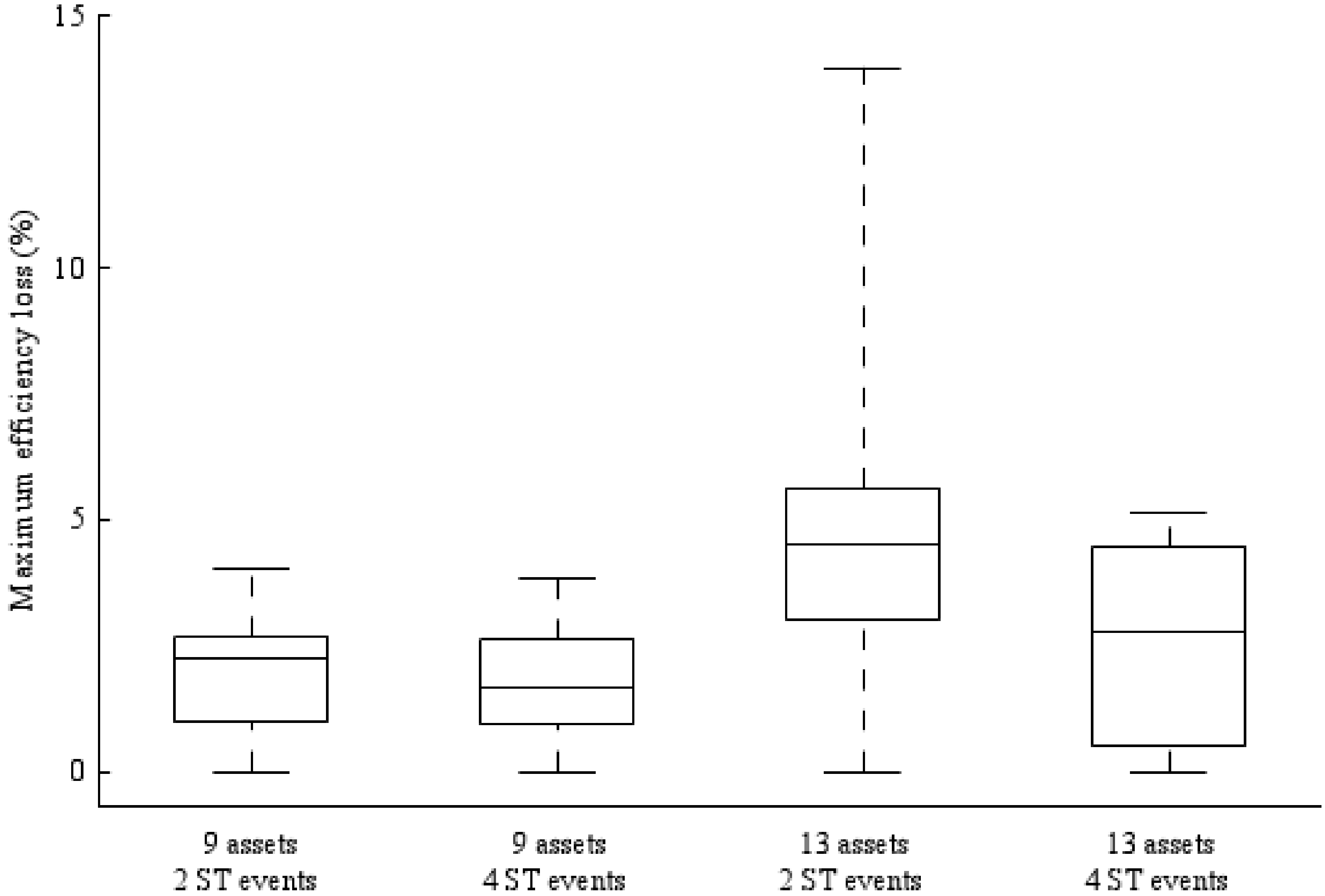
VaR+ST constraints with variable bounds

(losses are scaled to a period of one month):



Boxplot of maximum efficiency loss

VaR+ST constraints with variable bounds where short selling is allowed:



Comments to the plot

- 4ST: includes in addition to Crash of 1987 and 9/11 the 1997 Asian crises and the 1998 Russian crises.
- If the number of assets increase then the losses are getting higher (cf. first and third boxplot).
- If the number of ST events are increasing, the losses are still large (cf. first and second boxplot look quite similar).

Conclusion

- The effectiveness of VaR+ST constraints depends if short selling is allowed or disallowed.
- There are shortcomings in using VaR+ST constraints to control tail risk.
- There exists also other possibilities for implementation of this question.

Remarks

(R1) Reference on (non)-effectiveness of Basel II framework:

- J. Danielsson, P. Embrechts, C. Goodhart, C. Keating, F. Muennich, O. Renault and H. S. Shin (2001) An Academic Response to Basel II.

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(R2) Reference on VaR and CVaR:

- C. Acerbi and D. Tasche (2002) On the coherence of expected shortfall. *J. Bank. Finan.* 26: 1487-1403.
- A. McNeil, R. Frey and P. Embrechts (2005) *Quantitative Risk Management: Concepts, Techniques, Tools*. Princeton University Press

Remarks

(R3) Cases where VaR is not subadditive are rare. Superadditivity typically occurs when returns are:

(i) very heavy tailed (infinite variance)

(ii) very skew (credit?)

(iii) special dependence (copula) with normal returns

A combination of (i) and (ii) (a bit of (iii)) occurs in the analysis of operational risk.

Remarks

(R4) Have you tried other methods (beyond historical simulation): i.e. Monte Carlo, or analytic solutions for specific models like:

- multivariate normal
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- elliptical models and skew versions

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(R5) Going from VaR to Risk Capital:

- Risk Capital formula
- Specific VaR
- CoVaR
- Incremental VaR

Remarks

(R6) Reference on Stress Testing:

- R. Rebonato (2010) *Coherent Stress Testing: A Bayesian Approach to the Analysis of Financial Risk*. John Wiley & Sons.

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(R7) Concerning Basel II and III, some general issues, e.g.:

- Accounting
- REPO 105
- Solvency II
- Pillar 2