Dynamics or diversity? An empirical appraisal of distinct means to measure inflation uncertainty

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June 2nd, 2012

Bundesbank/Ifo Workshop
Uncertainty and Forecasting in Macroeconomics

Introduction

- (Inflation-) expectations play a key role in many economic models
- Examples: New Keynesian Phillips curve, consumption smoothing, firms' investment, price setting,...

$\Rightarrow$ Under risk aversion, considering inflation uncertainty makes sense whenever inflation expectations are part of the model

  $\rightarrow$ Inflation uncertainty (IU) is unobservable

  $\rightarrow$ Distinct ways to measure IU have been proposed

  - Any empirical study involving inflation risk has to motivate choice of particular uncertainty measure

Objective

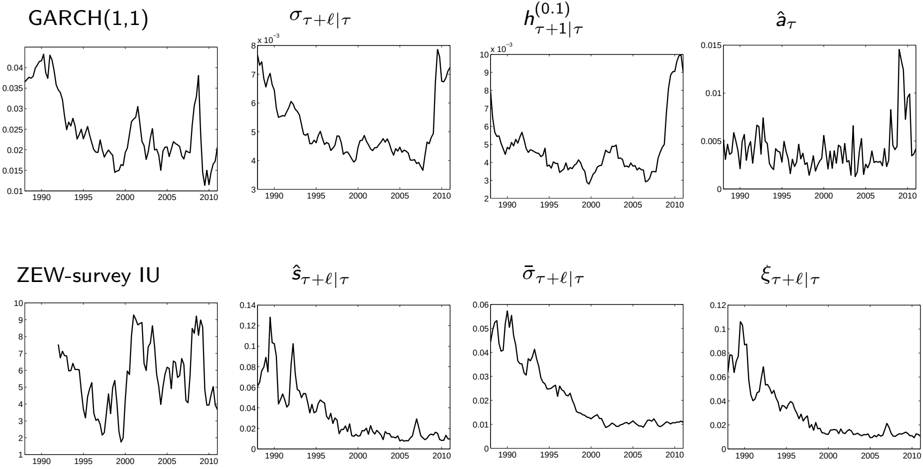
- This study: Pseudo out-of-sample forecasting 'horse race' with alternative IU measures as predictors for interest rates
- Objective: Empirical ranking of distinct approaches to measure inflation uncertainty (IU)

Distinguish two families of IU measurement:

→ Dynamic approaches (e.g. (G)ARCH)

→ Disparity (or Dispersion) of expectations, typically based on surveys of expert forecasts, e.g. *ASA-NBER Quarterly Economic Outlook Survey, ZEW survey*

Median IU trajectories - 4×Dynamic (above), 4×Dispersion (below)



The figure shows the median over 18 economies. GARCH(1,1) and ZEW-survey IU are benchmark measures from the related literature

Measuring IU by means of inflation forecasting

- We consider forecast-based measures of IU
- Autoregressive (AR) scheme is among most successful models to predict inflation $\pi_t = \ln(CPI_t / CPI_{t-4})$

$$\pi_{t+\ell} = \alpha_0 + \alpha_1 t + \alpha_2 \pi_t + \epsilon_{t+\ell}, \quad t = \tau - B + 1, ..., \tau, \quad \epsilon_{t+\ell} \overset{iid}{\sim} (0, \sigma_\epsilon^2) \quad (1)$$

- Predictions $\hat{\pi}_{\tau+\ell|\tau}$ obtained at forecast horizons $\ell \in \{1, 2, 3, 4\}$
- $\tau = T_0 - \ell, ..., T - \ell :=$ rolling forecast origin, B is estimation window size
- time instances T_0 and T delimit period for which IU measures are obtained (1988Q1 to 2011Q1)
- Cross section comprises 18 developed economies (Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, UK, US)

Distinct ways to measure IU - 1. Dynamic measures

- *1.1 Predictive standard deviation*

$$\hat{\sigma}_{\tau+\ell|\tau} = \tilde{\sigma}_\epsilon \sqrt{(1 + \mathbf{z}'_\tau (Z'_\tau Z_\tau)^{-1} \mathbf{z}_\tau)}, \quad (2)$$

  with Z_τ := design matrix of linear (AR) inflation forecasting model, $\mathbf{z}_\tau$:= most recent observations for out-of-sample forecasting.

- *1.2 Exponential smoothing* (Zangari 1996)

$$h^{(\lambda)}_{\tau+1|\tau} = \sqrt{\lambda(\Delta\pi_\tau)^2 + (1-\lambda)\overline{(\Delta\pi)^2}}. \quad (3)$$

  In (3), $\Delta\pi_t = \pi_t - \pi_{t-1}$, and $\overline{(\Delta\pi)^2} = (1/(B-1)) \sum_{t=\tau-B+1}^{\tau-1} (\Delta\pi_t)^2$, Presetting: $\lambda \in \{0.1, 0.2\} \approx$ typical estimates (e.g. Bollerslev 1986)

- *1.3 Unanticipated volatility* (Ball and Cecchetti 1990)

$$\hat{a}_{\tau+\ell} = |\hat{\pi}_{\tau+\ell|\tau} - \pi_{\tau+\ell}|, \quad (4)$$

  based on AR-implied inflation forecasts $\hat{\pi}_{\tau+\ell|\tau}$

Distinct ways to measure IU - 2. Dispersion measures

- *2.1 Disagreement of expectations*

$$\hat{s}_{\tau+\ell|\tau} = \sqrt{(1/(J-1)) \sum_{j=1}^{J} (\hat{\pi}_{j,\tau+\ell|\tau} - \overline{\pi}_{\tau+\ell|\tau})^2} \quad (5)$$

from $j = 1, ..., 5$ linear autoregressive distributed lag (ADL) forecasting models

- *2.2 Average uncertainty* (Zarnowitz and Lambros 1987)

$$\bar{\sigma}_{\tau+\ell|\tau} = (1/J) \sum_{j=1}^{J} \hat{\sigma}_{j,\tau+\ell|\tau} \quad (6)$$

- *2.3 Augmenting the disagreement measure* (cf. Lahiri and Liu 2005, Wallis 2005)

$$\xi_{\tau+\ell|\tau} = 0.5(\hat{s}_{\tau+\ell|\tau} + \bar{\sigma}_{\tau+\ell|\tau}) \quad (7)$$

- *2.4 Alternative augmentation* (cf. Lahiri and Sheng 2010)

$$\zeta_{\tau+\ell|\tau} = 0.5(\hat{s}_{\tau+\ell|\tau} + h^{(0.1)}_{\tau+1|\tau}) \quad (8)$$

Forcasting by means of the 'augmented Fisher equation'

$$
\begin{aligned}
R_{\tau+\ell} \quad = \quad & \gamma_{10} + \gamma_{11}\tau + \sum_{p=1}^{P} \gamma_{12,p}\pi_{\tau-p+1} + \sum_{p=1}^{P} \gamma_{13,p} R_{\tau-p+1} + \\
& + \sum_{p=1}^{P} \gamma_{14,p} IU_{\tau-p+\ell+1|\tau} + e_{\tau+\ell}, \quad \tau = T_0 - \ell, ..., T - \ell \qquad (9)
\end{aligned}
$$

following Levi and Makin (1979), Blejer and Eden (1979), inter alia.

- $IU_{\tau+\ell|\tau}$ represents a particular inflation uncertainty measure, $e_{\tau+\ell} \overset{iid}{\sim} (0, \sigma_e^2)$
- $R_{\tau+\ell}$: Interest rate on 10-year government bond

→ Each observation $R_{\tau+\ell}$ from the sample period $\tau = T_0 - \ell, ..., T - \ell$ is predicted ℓ-steps ahead by means of a respective leave-one-out cross-validation estimate

→ This yields distinct forecasts of $R_{\tau+\ell}$ based on alternative IU measures (2) to (8)

- Maximum lag order $P = 4 \;\Rightarrow 2^{12}$ distinct subset models

Subset modelling by Bayesian model averaging (BMA)

- Averaging forecasts improves predictive accuracy (Bates and Granger 1969, Timmermann 2005, Wright 2009)
- Combine forecasts from $m = 1, ..., M = 2^{12}$ reformulations of augmented Fisher equation:

$$\hat{R}_{\tau+\ell|\tau} = \sum_{m=1}^{M} w_m^* \hat{R}_{\tau+\ell|\tau}^{(m)}, \tag{10}$$

- $$w_m^* = \frac{w_m}{\sum_m w_m} \text{ and } w_m = \int L_m(\gamma^{(m)}) p_m(\gamma^{(m)}) d\gamma^{(m)}. \tag{11}$$

  $L_m(\gamma^{(m)})$:= likelihood function, $p_m(\gamma^{(m)})$:= a-priori distribution of $\gamma^{(m)}$
- Based on log-likelihood $l(\gamma^{(m)}) = \ln L(\gamma^{(m)})$, posterior probabilities w_m in (11) can be approximated as

$$\ln \hat{w}_m = l(\hat{\gamma}^{(m)}) - \frac{n_m}{2} \ln(T - T_0), \tag{12}$$

  $\hat{\gamma}^{(m)}$:= (Q)ML estimator of $\gamma^{(m)}$ and n_m stands for the number of right hand side variables in model m.
- Forecast combination weights obtain as w_m in (11) by $\exp\left(l(\hat{\gamma}^{(m)}) - \frac{n_m}{2} \ln(T - T_0)\right)$

Performance criterion

- Forecast ranking based on absolute forecast error (AE)

$$|e^{\bullet}_{\tau+\ell|\tau}| = |\hat{R}^{\bullet}_{\tau+\ell|\tau} - R_{\tau+\ell}| \tag{13}$$

'•' represents IU measures $\hat{\sigma}_{\tau+\ell|\tau}, h^{(\lambda)}_{\tau+1|\tau}, \hat{a}_{\tau}, \hat{s}_{\tau+\ell|\tau}, \bar{\sigma}_{\tau+\ell|\tau}, \xi_{\tau+\ell|\tau},$ $\zeta_{\tau+\ell|\tau}, \max(\mathrm{IU}), \min(\mathrm{IU}), \mathrm{median}(\mathrm{IU}), \mathrm{mean}(TS), \mathrm{mean}(DS).$

→ Frequency by which IU measure • produces forecasts among the 3 best (Stock and Watson 1999):

$$\mathrm{TOP3}^{\bullet} = (1/((T - T_0 + 1) \times 18)) \sum_{\tau=T_0-\ell}^{T-\ell} \sum_{i=1}^{18} \mathrm{I}(|e^{\bullet}_{i,\tau+\ell}| \leq |e^{(3)}_{i,\tau+\ell}|), \tag{14}$$

where $|e^{(3)}_{i,\tau+\ell}|$ is the 3rd smallest AE and $\mathrm{I}(\cdot)$ is the indicator function

TOP3• frequencies

	Dynamic measures					Dispersion measures			
	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$		$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$
$\hat{\sigma}_{\tau+\ell\|\tau}$	21.45	24.16	25.32	25.19	$\hat{s}_{\tau+\ell\|\tau}$	21.51	20.09	20.54	20.74
$h^{(0.1)}_{\tau+1\|\tau}$	23.32	22.03	21.77	21.77	$\bar{\sigma}_{\tau+\ell\|\tau}$	22.35	**27.65**	**28.62**	**27.97**
$h^{(0.2)}_{\tau+1\|\tau}$	23.26	21.77	17.57	18.09	$\varsigma_{\tau+\ell\|\tau}$	15.96	18.15	20.80	21.90
$\hat{a}_{\tau}$	26.94	23.06	22.93	23.13	$\zeta_{\tau+\ell\|\tau}$	19.44	20.22	22.22	20.93
$\overline{TS}$	**28.81**	24.22	21.38	20.99	$\overline{DS}$	19.06	18.28	18.99	21.12
	Further IU statistics								
max(IU)	15.70	16.86	20.80	20.09	median	20.74	23.00	22.48	23.39
min(IU)	19.51	21.83	20.16	17.12	$\circ$	22.16	19.51	18.22	19.32

Cell entries represent the frequencies in which distinct IU measures lead to forecasts which are among the 3 most accurate ones. The row labelled as '$\circ$' reports respective ranking frequencies for a forecasting model without an IU term.

Percentage of cases where $|e^{\bullet}_{\tau+\ell|\tau}| < c \times |e^{(\circ)}_{\tau+\ell|\tau}|$

	$c=1$				$c=0.8$			
	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$
$\hat{\sigma}_{\tau+\ell\|\tau}$	51.03	53.29	55.49	54.84	22.22	29.07	30.75	31.20
$h^{(0.1)}_{\tau+1\|\tau}$	**51.87**	54.20	52.71	52.00	18.09	21.25	21.25	19.77
$h^{(0.2)}_{\tau+1\|\tau}$	51.74	53.94	52.84	51.74	15.50	17.64	17.70	15.31
$\hat{a}_{\tau}$	49.55	51.16	52.78	52.78	26.94	29.13	28.94	28.10
$\hat{s}_{\tau+\ell\|\tau}$	51.42	53.55	54.97	54.97	26.49	31.65	35.79	34.82
$\bar{\sigma}_{\tau+\ell\|\tau}$	49.68	53.04	53.29	55.10	22.87	**34.04**	35.34	**36.82**
$\varsigma_{\tau+\ell\|\tau}$	50.19	53.10	56.07	55.56	25.32	31.65	35.47	35.92
$\zeta_{\tau+\ell\|\tau}$	50.45	52.71	54.91	54.72	**27.45**	33.01	**37.34**	35.21
max(IU)	50.45	53.10	**56.20**	55.62	25.26	32.11	35.47	35.59
min(IU)	49.94	53.62	52.97	50.97	18.80	22.42	22.80	22.87
median(IU)	51.55	**55.88**	52.26	55.62	21.45	30.62	30.30	34.30
$\overline{TS}$	51.16	51.36	52.39	53.23	26.94	28.42	28.55	27.78
$\overline{DS}$	50.65	53.23	56.14	**55.62**	25.97	31.91	35.85	35.79

'$\circ$' represents forecast errors for Fisher eq. WITHOUT IU term.

Comparison to benchmark measures

Percentage of cases where $|e^{\bullet}_{\tau+\ell|\tau}| < |e^{(bm)}_{\tau+\ell|\tau}|$

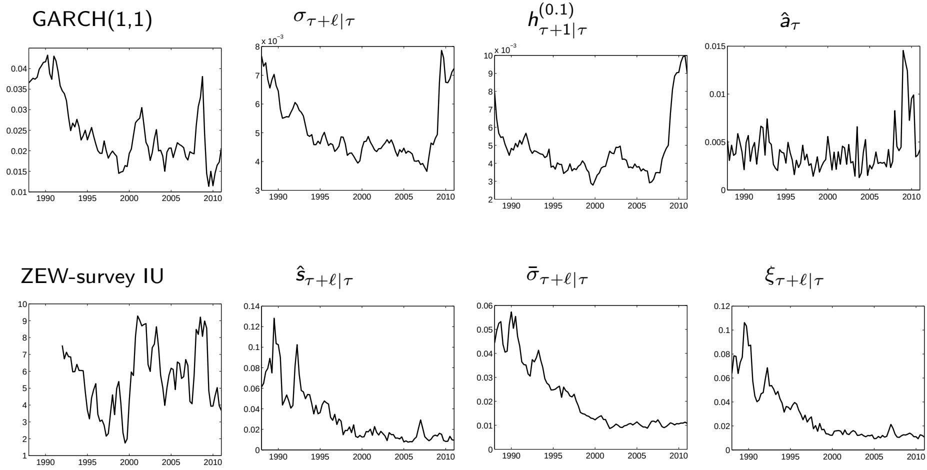
$\hat{\sigma}_{\tau+1\|\tau}$	$h^{(0.1)}_{\tau+1\|\tau}$	$h^{(0.2)}_{\tau+1\|\tau}$	$\hat{a}_{\tau}$	$\hat{s}_{\tau+1\|\tau}$	$\bar{\sigma}_{\tau+1\|\tau}$	$\varsigma_{\tau+1\|\tau}$	$\zeta_{\tau+1\|\tau}$
52.97	52.58	**54.13**	50.06	52.00	52.07	52.45	51.94
$\hat{\sigma}_{\tau+4\|\tau}$	$h^{(0.1)}_{\tau+1\|\tau}$	$h^{(0.2)}_{\tau+1\|\tau}$	$\hat{a}_{\tau}$	$\hat{s}_{\tau+4\|\tau}$	$\bar{\sigma}_{\tau+4\|\tau}$	$\varsigma_{\tau+4\|\tau}$	$\zeta_{\tau+4\|\tau}$
52.65	52.78	47.62	48.94	52.91	52.53	**56.61**	53.70

Upper panel: bm = GARCH(1,1)
Lower panel: bm = IU based on ZEW survey

TOP3• for subsamples

	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$
	Turbulent periods				Calm periods			
$\hat{\sigma}_{\tau+\ell\|\tau}$	19.64	23.39	22.61	24.55	23.26	24.94	28.04	25.8
$\hat{a}_{\tau}$	**26.23**	23.00	21.19	20.93	**27.24**	23.13	24.68	25.32
$\bar{\sigma}_{\tau+\ell\|\tau}$	22.61	**27.91**	**28.29**	**27.00**	22.09	**27.39**	**28.94**	**28.94**
	Sample period 1988Q1-1998Q3				Sample period 1998Q4-2011Q1			
$\hat{\sigma}_{\tau+\ell\|\tau}$	21.71	24.68	27.00	26.61	21.19	23.64	23.64	23.77
$\hat{a}_{\tau}$	**29.36**	23.51	23.64	21.83	**25.19**	22.61	22.22	24.42
$\bar{\sigma}_{\tau+\ell\|\tau}$	20.67	**26.61**	**27.91**	**27.26**	24.03	**28.37**	**29.33**	**28.68**
	Higher-inflation economies				Lower-inflation economies			
$\hat{\sigma}_{\tau+\ell\|\tau}$	19.38	22.48	24.94	22.74	23.51	25.84	25.71	27.43
$\hat{a}_{\tau}$	**25.19**	21.06	19.12	19.64	**28.68**	25.06	26.74	26.61
$\bar{\sigma}_{\tau+\ell\|\tau}$	20.93	**27.15**	**29.33**	**28.29**	23.77	**27.11**	**27.91**	**27.65**

Median IU trajectories - 4×Dynamic (above), 4×Dispersion (below)



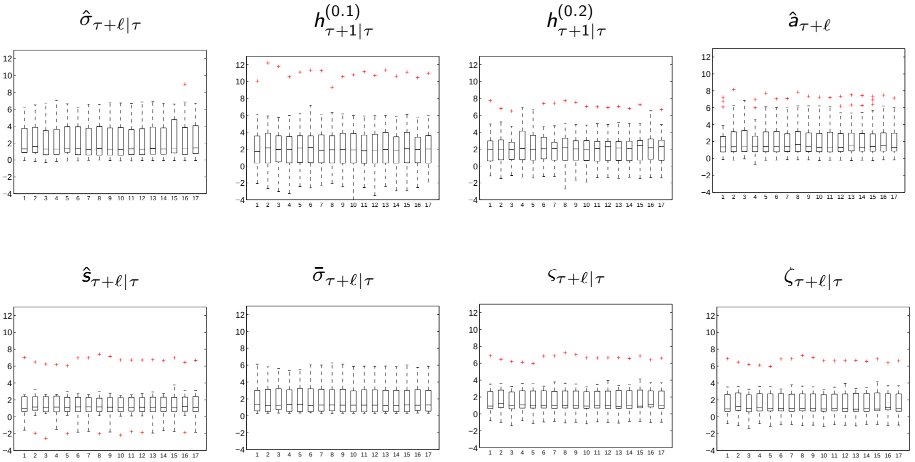
The figure shows the median over 18 economies. GARCH(1,1) and ZEW-survey IU are benchmark measures from the related literature

Relation between IU and $R_{\tau+\ell}$

$$R_{\tau+\ell} = \gamma_{10} + \gamma_{11}\tau + \sum_{p=1}^{P} \gamma_{12,p}\pi_{\tau-p+1} + \sum_{p=1}^{P} \gamma_{13,p} R_{\tau-p+1} + + \sum_{p=1}^{P} \gamma_{14,p} IU_{\tau-p+\ell+1|\tau} + e_{\tau+\ell} \quad (15)$$

- Overall IU effect for $\tau = T_0 + 1, ..., T$ (i.e. 1988Q1 to 2011Q1) in economies $i = 1, ..., 18$ is denoted $\bar{\bar{\gamma}}_{i\tau}^{(IU)} = \sum_{p=1}^{P} \gamma_{14,p}$.
- First theory: *Inflation Risk Premium*
  Investors require compensation for holding non-indexed bonds (Barnea et al. 1979, Brenner and Landskroner 1983)
- Second theory: *Investment Barrier*
  IU reduces demand for loanable funds since returns to real investments are more uncertain (Blejer and Eden 1979)

IU effect on $R_{\tau+\ell}$



Summary and Conclusions

We distinguish 2 families of IU measures.

→ Both groups indicate IU decrease during *Great Moderation* period

→ Distinct IU indication after 2008, post-Lehman

Forecast ranking shows: Dispersion outperforms Dynamic measures.

→ Average over individual models' uncertainty is most informative predictor

Across time instances and economies,

impact of IU on interest is uniformly positive.

→ Call IU influence on bond yields a risk premium

Measuring IU - Inflation forecasting models

Alternative ways to predict inflation:

$$\pi_{t+\ell} = \alpha_{10} + \alpha_{11} t + \alpha_{12}\pi_{t-1} + \alpha_{13}\tilde{y}_{t-1} + \epsilon_{t+\ell}, \quad t = \tau - B + 1, ..., \tau. \tag{16}$$

$$\pi_{t+\ell} = \alpha_{20} + \alpha_{21} t + \alpha_{22}\pi_{t-1} + \alpha_{23}\tilde{y}_{t-1} + \alpha_{24}\bar{m}_{t-1} + \epsilon_{t+\ell}. \tag{17}$$

$$\pi_{t+\ell} = \alpha_{30} + \alpha_{31} t + \alpha_{32}\pi_{t-1} + \alpha_{33}\tilde{y}_{t-1} + \alpha_{34}\bar{m}_{t-1} + \alpha_{35}\Delta^2 oil_{t-1} + \epsilon_{t+\ell}. \tag{18}$$

$$\pi_{t+\ell} = \alpha_{40} + \alpha_{41}\tilde{\pi}_{t-1} + \epsilon_{t+\ell}. \tag{19}$$

- $\tilde{y}_t = y_t - \bar{y}_t$: output gap, with potential output $\bar{y}_t$ estimated by means of HP-filter
- $\bar{m}_t$: core money growth
- $\Delta^2 oil_{t-1}$ oil price dynamics (WTI crude)
- $\tilde{\pi}_{t-1} = \pi_t - \bar{\pi}_t$: inflation gap $\rightarrow$ $\tilde{\pi}_{t-1}$ in (19) resembles error-correction term