Mixed-Form Indices: A Study of Their Properties

Bert M. Balk
Rotterdam School of Management, Erasmus University

15th Meeting of the Ottawa Group
10-12 May 2017, Eltville am Rhein

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- Attended OG meetings once: Neuchâtel 2009 → Special issue *Index Number Theory and Price Statistics*, edited by Peter von der Lippe and Erwin Diewert, Jahrbücher für Nationalökonomie und Statistik 230(6), 2010.
- Had (joint) papers at Wellington 2011 and Copenhagen 2013 meetings.

Why pay attention?

- Large influence via his courses, organized by Eurostat in Western- and Eastern-European countries, over a large number of years → *Index Theory and Price Statistics* (2007).
- His life-long struggle against chained indices.

Direct and chained indices (1)

- Classic paper "Der Unsinn von Kettenindices", Allgemeines Statistisches Archiv 84(1), 67-82, 2000.
- Classic book *Chain Indices; A Study in Price Index Theory* (2001).
- Many other publications … (see website www.von-der-lippe.org; still available).
- Chained indices were first suggested by Julius Lehr in 1885. See Peter von der Lippe, "Recurrent Price Index Problems and Some Early German Papers on Index Numbers", Jahrbücher für Nationalökonomie und Statistik 233(3), 336-366, 2013.

The battle has by and large been concluded in favour of the party of "chainers", and, as Peter observed in the preface of his 2007 book, "I am pretty sure that it is most unlikely that I will outlive the chain index era."

Direct and chained indices (2)

For an extensive comparison of the two paradigms I might refer to

- Balk, B. M., 2010, “Direct and Chained Indices: A Review of Two Paradigms”, in *Price and Productivity Measurement: Volume 6 -- Index Number Theory*, edited by W. E. Diewert, B. M. Balk, D. Fixler, K. J. Fox and A. O. Nakamura (Trafford Press, www.vancouvervolumes.com and www.indexmeasures.com).
- Balk, B. M., 2008, *Price and Quantity Index Numbers: Models for Measuring Aggregate Change and Difference* (Cambridge University Press, New York), Section 3.9.

Direct and chained indices (3)

A hidden presumption in much of this discussion has been that data are annual (or, more abstractly formulated, the time periods considered are of equal length and price and quantity data of the aggregate studied are available for all the periods). However, most officially compiled indices, such as CPIs and PPIs, are *monthly*, and appear to exhibit a functional form that is a mix of direct and chained elements.

Comparing month *m* of year *t* to reference year 0

Here is a characteristic example:

$$P^c(mt, 0) \equiv \frac{P(mt, 0t; b(t)) \prod_{\tau=0}^{t-1} P(12\tau, 0\tau; b(\tau))}{(\frac{1}{12}) \sum_{m=1}^{12} P(m0,00; b(0))}$$

with

$$P(mt, 0t; b(t)) \equiv \frac{\sum_{n \in N^t} p_n^{mt} x_n^{b(t)}}{\sum_{n \in N^t} p_n^{0t} x_n^{b(t)}}$$

Explanation

- $P(mt, 0t; b(t))$ is a Lowe index, comparing prices of month *m* of comparison year *t* to prices of month 0 of the same year (= December of previous year), conditional on quantities of some (earlier) weight reference period $b(t)$; N^t is the set of commodities employed during year *t*. This is a *direct* index.
- $\prod_{\tau=0}^{t-1} P(12\tau, 0\tau; b(\tau))$ is a *chained* Lowe index, comparing December of year $t-1$ to December of the year preceding the reference year 0.
- $(\frac{1}{12})\sum_{m=1}^{12} P(m0,00; b(0))$ rescales such that the mean of reference year index numbers becomes equal to 1.

Warning

$P^c(mt, 0)$ is *path-dependent*; that is, selecting instead of December another linking month (or period) results in a different index.

Some properties

- In a static economy $P^c(mt, 0)$ reduces to a direct Lowe index, comparing prices of month *m* of year *t* to mean prices of year 0.
- $P^c(mt, 0)$ is *not* consistent-in-aggregation.
- Monthly change, $P^c(mt, 0)/P^c((m-1)t, 0)$, is a Lowe index; hence, satisfies the Proportionality Test.
- Annual change, $P^c(mt, 0)/P^c(m(t-1), 0)$, is a product of two Lowe indices; hence, does not satisfy the Proportionality Test.
- Moving average of annual change, in particular $\frac{1}{12}\sum_{m=1}^{12} P^c(mt, 0)/P^c(m(t-1), 0)$, is difficult to interpret.