Mixed-Form Indices: A Study of Their Properties

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Abstract

This paper reviews the main properties of mixed-form (price) indices, that is, chained indices which in real time behave as direct (fixed-base) indices. The European Union HICP is used as demonstration material.

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1 Introduction

Apart from his many teaching activities, as a result of which we got his 2007 book *Index Theory and Price Statistics*[1], Peter von der Lippe will be remembered for his life-long struggle against chained indices, as summarized in his 2001 book (described by himself as "a sort of pamphlet") *Chain Indices;*

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[1]Peter Lang GmbH, Frankfurt am Main.

A Study in Price Index Theory.[2] As is well known, the “battle between chainers and non-chainers” has by and large be concluded in favour of the party of “chainers”. Their two paradigms have been extensively compared in my 2010 (originally 2004) review article; see also Balk (2008, Section 3.9), to which not much can be added in terms of novel insights.

A hidden presumption in much of this discussion has been that data are annual (or, more abstractly formulated, the time periods considered are of equal length and price and quantity data of the aggregate studied are available for all the periods). However, most officially compiled indices, such as CPIs and PPIs, are monthly, and appear to exhibit a functional form that is a mix of direct and chained elements. A good example is the structure prescribed for the Harmonized Index of Consumer Prices (HICP) of the European Union member states.

In memory of the lasting contributions of Peter von der Lippe, in the present paper I will study the main properties of such mixed-form indices, thereby using the HICP as demonstration material.

The paper is structured as follows. Section 2 provides the various definitions. Section 3 considers properties, notably those of derived rates of change. Section 4 concludes.

2 Definitions

The Framework Regulation concerning the HICP, the HICP-CT, the OOH price index and the HPI[3] states in Article 3.2 that “The harmonised indices shall be annually chain-linked Laspeyres-type indices.” The latter term is defined in Article 2(14) as follows:

> Laspeyres-type index means a price index that measures the average change in prices from the price reference period to a comparison period using expenditure shares from some period prior to the price reference period, and where the expenditure shares are adjusted to reflect the prices of the price reference period.

[2]Volume 16 of the Publication Series Spectrum of Federal Statistics, Statistisches Bundesamt, Wiesbaden.

[3]Regulation (EU) 2016/792 of 11 May 2016. Official Journal of the European Union L135/11, 24 May 2016.

My reading[4] of this text and concomitant explanations is that 'Laspeyres-type index' means Lowe price index[5] What does this mean when a monthly index must be compiled?

Let the comparison period be month $m = 1, ..., 12$ of year t, and let the price reference period be month 12 (= December) of the preceding year $t-1$. In the sequel such a price reference period will be denoted as month 0 of year t. It is assumed that during the year t the scope of the price index is determined by a set N^t of commodities. For any time period τ considered, be it a month or a year, (positive) quantities of commodities will be denoted by x_n^τ and (positive) prices by p_n^τ.

The Lowe price index for the comparison period relative to the price reference period is then compiled as

$$P(mt, 0t; b) \equiv \frac{\sum_n p_n^{mt} x_n^b}{\sum_n p_n^{0t} x_n^b}, \tag{1}$$

where the summations in numerator and denominator run over all the commodities $n \in N^t$, and b is some weight reference period. It is thereby assumed that for all the commodities $n \in N^t$ the quantities x_n^b exist. Notice that $P(0t, 0t; b) = 1$.

It is important to realize that in this construct, the month of December plays a double role: once every year this month acts as comparison period, but it always acts as price reference period. To distinguish clearly between these two roles, and to avoid complications, the notation is deliberately chosen as in expression (1). Thus, being in year t and occurring in the numerator of expression (1), December is labeled as $m = 12$, whereas being in year $t-1$ but occurring in the denominator of expression (1) December is labelled as month $m = 0$ of year t. Put otherwise, each year t is considered as consisting of 13 months, running from December of year $t-1$ to December of year t.

If period b would coincide with month 0 of year t then expression (1) would turn into a (genuine) Laspeyres price index. However, it is common practice to chose as weight reference period b a period of 12 consecutive months of consumption or expenditure for some period prior to December of $t-1$. Thus, b is a function of t, denoted as $b = b(t)$.

[4]The alternative reading, see below, hinges on the interpretation of the word 'adjusted'.

[5]For generic definitions the reader is referred to CPI Manual (2004, 270) or Balk (2008, 68).

If prices are strictly positive, which is usually the case, then the Lowe price index (1) can be rewritten as a weighted mean of price relatives of individual commodities,

$$P(mt, 0t; b) = \sum_n w_n^{0tb} \frac{p_n^{mt}}{p_n^{0t}}, \tag{2}$$

with weights, adding up to 1, defined as

$$w_n^{0tb} \equiv \frac{p_n^{0t} x_n^b}{\sum_n p_n^{0t} x_n^b} \; (n \in N^t). \tag{3}$$

The weights, defined by expression (3), do not correspond to observable expenditure shares, as they depend on prices and quantities from different time periods. They are called mixed-period weights. However, they are conveniently obtained from the observed annual expenditure shares

$$w_n^b \equiv \frac{p_n^b x_n^b}{\sum_n p_n^b x_n^b} \; (n \in N^t)$$

by a procedure known as price-updating, which is carried out as follows:

$$w_n^{0tb} = \frac{\frac{p_n^{0t}}{p_n^b} w_n^b}{\sum_n \frac{p_n^{0t}}{p_n^b} w_n^b} = \frac{\frac{p_n^{0t}}{p_n^b} p_n^b x_n^b}{\sum_n \frac{p_n^{0t}}{p_n^b} p_n^b x_n^b} \; (n \in N^t). \tag{4}$$

To make this possible, it is necessary to have available the individual price relatives comparing the price reference period $0t$ to the weight reference period b, p_n^{0t}/p_n^b $(n \in N^t)$. It is interesting to note that price-updating of the expenditure shares – as in the second part of expression (4) – is the same as price-updating of the expenditures themselves – as in the third part of expression (4).

Ideally, the weight reference period $b(t)$, used for the computation of the price indices for all the months of year t, would be year $t-1$ as this is the most recent calendar year. However, these expenditure shares are generally not yet available with sufficient accuracy early in year t when they are required for the first index number computation. Therefore, the usual strategy is to set $b(t) \equiv t-2$ or $b(t) \equiv t-3$, and execute price-updating according to expression (4).

Price-updating as defined here reflects my interpretation of the word 'adjusted' in the above quote from the Framework Regulation. The alternative interpretation emerges when the price relatives p_n^{0t}/p_n^b are decomposed as

$$\frac{p_n^{0t}}{p_n^b} = \frac{p_n^{t-1}}{p_n^b}\frac{p_n^{0t}}{p_n^{t-1}} \; (n \in N^t), \tag{5}$$

and it is assumed that all $p_n^{t-1}/p_n^b = 1$. Alternatively, it is assumed that the period b weights are equal to the period $t-1$ weights, $w_n^b = w_n^{t-1}$ $(n \in N^t)$, so that only price-updating from year $t-1$ to month $0t$ is required.

Notice, however, that in both cases a specific assumption is invoked which may or may not be true. If untrue then the interpretation of the weighted mean of price relatives – that is, the right-hand side of expression (2) – as a Lowe price index gets lost.

The Lowe price index as defined by expression (1) is a direct index, comparing each month m of year t to December of the preceding year, $t-1$. When t moves through time, there results for each year t a series of 13 index numbers, running from December of year $t-1$ (its index number being equal to 1) to December of year t. Notice that the set of commodities N^t may vary through time as some goods and services disappear from the market and other goods and services enter the market.

Now these separate annual series can be chained together as a single series of index numbers, which compares month m of year t to some earlier time period, called the index reference period. In the HICP case the natural month to use as linking pin is December. Then the chained index[6]

$$P(mt, 0t; b(t)) \times P(12(t-1), 0(t-1); b(t-1)) \times P(12(t-2), 0(t-2); b(t-2)) \times ...$$
$$\times P(12(0), 00; b(0))$$

compares month m of year t with month 0 of a certain year 0. Recall that month 0 of any year τ is the same as month 12 of year $\tau - 1$. Notice that, in principle, each link of this chained index employs a different weight reference period and a different set of commodities. In the expression above month 0 of year 0 serves as index reference period.

It is more convenient, however, to use some calendar year as index reference period. This can be achieved by rescaling the chained index; that is, dividing by the unweighted (arithmetic) mean of the index numbers for the months of year 0. Thus, the final index for month m of year t relative to reference year 0 is defined by

[6]Where misunderstanding is possible, here and in the sequel, years and months, such as $t-1$ or $m-1$, are put within brackets.

$$P^c(mt, 0) \equiv \frac{P(mt, 0t; b(t)) \prod_{\tau=0}^{t-1} P(12\tau, 0\tau; b(\tau))}{(1/12) \sum_{m=1}^{12} P(m0, 00; b(0))}. \tag{6}$$

This is a typical instance of a *mixed-form index*. The first factor in the numerator compares month m of year t to month 0 of year t (= month 12 of year $t-1$); the second factor in the numerator compares month 12 of year $t-1$ to month 0 of year 0; and the denominator rescales the numerator so that the mean of the year 0 index numbers becomes equal to 1. In real time, that is, moving from January to December of a certain year t only the first factor in the numerator matters; basically $P^c(mt, 0)$ is a direct index, its fixed base being December of year $t-1$, multiplied by a constant. The constant, however, changes every year. Viewed as a whole, $P^c(mt, 0)$ is a rescaled, chained index.

3 Some properties

3.1 In a static economy

In a static economy the set of commodities does not change and the weights are constant; that is, $N^\tau = N^0$ and $b(\tau) = b(0)$ for $\tau = 0, ..., t$. Substituting expression (1) into expression (6) it turns out that

$$P^c(mt, 0) = \frac{\sum_n p_n^{mt} x_n^{b(0)}}{\sum_n p_n^0 x_n^{b(0)}}, \tag{7}$$

where, for any period τ, $p_n^\tau \equiv \frac{1}{12} \sum_{m=1}^{12} p_n^{m\tau}$ is the (arithmetic) mean price of commodity $n \in N^\tau$ in this period.

Thus, in this situation, the chained price index is again a direct Lowe price index, comparing prices of the comparison month to mean prices of the index reference year, using quantities of some earlier weight reference period.

3.2 Consistency-in-aggregation

Suppose that the set of all commodities N^t is divided into mutually disjunct subsets N_k^t $(k = 1, ..., K)$. Then the following is true:

$$P(mt, 0t; b)$$

$$= \sum_{k=1}^{K} \Big(\sum_{n \in N_k^t} w_n^{0tb} \Big) \sum_{n \in N_k^t} \frac{w_n^{0tb}}{\sum_{n \in N_k^t} w_n^{0tb}} \frac{p_n^{mt}}{p_n^{0t}}$$

$$= \sum_{k=1}^{K} w_k^{0tb} P_k(mt, 0t; b). \tag{8}$$

Thus the overall Lowe index is a weighted mean of the Lowe indices for the subsets of commodities, defined as

$$P_k(mt, 0t; b) \equiv \frac{\sum_{n \in N_k^t} p_n^{mt} x_n^b}{\sum_{n \in N_k^t} p_n^{0t} x_n^b} \; (k = 1, ..., K). \tag{9}$$

The weights

$$w_k^{0tb} \equiv \sum_{n \in N_k^t} w_n^{0tb} = \frac{\sum_{n \in N_k^t} p_n^{0t} x_n^b}{\sum_{n \in N^t} p_n^{0t} x_n^b} \; (k = 1, ..., K) \tag{10}$$

are the mixed-period expenditure shares of the subsets. Put otherwise, the overall Lowe index can be calculated in one stage from the individual commodity price relatives, as in expression (2); or in two stages, as in expression (8): from commodity price relatives to subset Lowe indices and then from these subset indices to the overall index. This is called consistency-in-aggregation (CIA); a very useful property in statistical practice.

Chained indices, however, are not CIA. For them there does *not* exist a relation like expression (8); put ottherwise, there does *not* exist a set of weights (adding up to 1) such that

$$P^c(mt, 0) = \sum_{k=1}^{K} w_k P_k^c(mt, 0),$$

where $P_k^c(mt, 0)$ is the chained price index for subset $k = 1, ..., K$. Only the assumption that the weights are constant over the entire time span implies that the chained index exhibits CIA, which doesn't come as a surprise since then any chained Lowe index reduces to a direct Lowe index.

3.3 Derived measures

3.3.1 Monthly change

The fixed-base nature of a mixed-form index is revealed most clearly when we consider the price change between consecutive months. Thus, the price change between month $m-1$ and month m of current year t is obtained from the series of chained index numbers $P^c(m\tau, 0)$ $(\tau = 0, ..., t; m = 1, ..., 12)$ as[7]

$$\frac{P^c(mt,0)}{P^c((m-1)t,0)} = \frac{P(mt,0t;b(t))}{P((m-1)t,0t;b(t))} = \frac{\sum_n p_n^{mt} x_n^{b(t)}}{\sum_n p_n^{(m-1)t} x_n^{b(t)}}. \tag{11}$$

These equalities are obtained respectively by using expression (6) and substituting expression (1). Expression (11) means that the price change between months $m-1$ and m of the same year is measured by a Lowe price index based on quantities of a prior period $b(t)$. An important consequence is that if between these two months all the prices change by the same factor, that is, $p_n^{mt} = \lambda p_n^{(m-1)t}$ $(\lambda > 0, n \in N^t)$, then the overall price change equals λ.[8]

Another consequence is that the overall price change can be written as a weighted arithmetic mean of individual price relatives $p_n^{mt}/p_n^{(m-1)t}$ $(n \in N^t)$.

3.3.2 Annual change

We now look at the price change between corresponding months of consecutive years; that is, $P^c(m\tau, 0)/P^c(m(\tau-1), 0)$ $(\tau = 1, ..., t; m = 1, ..., 12)$. Employing expression (6), the price change between months m of years $t-1$ and t is measured as

$$\frac{P^c(mt,0)}{P^c(m(t-1),0)} = \frac{P(mt,0t;b(t))P(12(t-1),0(t-1);b(t-1))}{P(m(t-1),0(t-1);b(t-1))}. \tag{12}$$

By substituting expression (1) we obtain

$$\frac{P^c(mt,0)}{P^c(m(t-1),0)} = \frac{\sum_n p_n^{mt} x_n^{b(t)}}{\sum_n p_n^{0t} x_n^{b(t)}} \times \frac{\sum_n p_n^{12(t-1)} x_n^{b(t-1)}}{\sum_n p_n^{m(t-1)} x_n^{b(t-1)}}. \tag{13}$$

The annual price change appears to be the product of two Lowe price indices, each having their own set of quantity weights: the first index compares prices of month m of year t to December of year $t-1$, and the second index compares prices of December of year $t-1$ to month m of year $t-1$. The first index is based on the commodity set N^t and the second on commodity set N^{t-1}.

[7]This is usually presented as a percentage, or a rate of change; that is, $(P^c(mt,0)/P^c((m-1)t,0) - 1) \times 100\%$.

[8]Put otherwise, $P^c(mt,0)/P^c((m-1)t,0)$ satisfies the Proportionality Test.

An important consequence is that if these commodity sets are the same and in the corresponding months m all the later prices happen to be the same multiple of the former prices, that is, $p_n^{mt} = \lambda p_n^{m(t-1)}$ ($\lambda > 0, n \in N^{t-1} = N^t$), then the annual price change not necessarily equals λ. Put otherwise, the annual price change cannot be written as a weighted mean of the individual price relatives $p_n^{mt}/p_n^{m(t-1)}$.

Nevertheless, it is possible to decompose the annual price change into contributions of the individual commodities and/or contributions of the current and the previous year. The solution suggested by Ribe (1999) amounts to

$$\frac{P^c(mt,0)}{P^c(m(t-1),0)} - 1 = \left(\frac{\sum_n p_n^{mt} x_n^{b(t)}}{\sum_n p_n^{0t} x_n^{b(t)}} - 1\right) \times \frac{\sum_n p_n^{12(t-1)} x_n^{b(t-1)}}{\sum_n p_n^{m(t-1)} x_n^{b(t-1)}} + \left(\frac{\sum_n p_n^{12(t-1)} x_n^{b(t-1)}}{\sum_n p_n^{m(t-1)} x_n^{b(t-1)}} - 1\right). \tag{14}$$

However, by looking at the structure of the right-hand side it becomes clear that this decomposition is not completely satisfactory. Though the second factor between brackets can be interpreted as previous year's contribution, and the first factor between brackets likewise as current year's contribution (and both factors can be decomposed commodity-wise), this first factor is multiplied by previous year's price change. Thus there seems to be a whiff of double-counting here.

A different solution, involving logarithmic means[9], was proposed by Balk (2006). Consider the first factor at the right-hand side of expression (13) and use the logaritmic mean and its linear homogeneity (that is, property (3)) to obtain the following decomposition:

$$\ln\left(\frac{\sum_n p_n^{mt} x_n^{b(t)}}{\sum_n p_n^{0t} x_n^{b(t)}}\right) = \sum_{n \in N^t} w_n^{mt} \frac{p_n^{mt} - p_n^{0t}}{p_n^{0t}}, \tag{15}$$

where the weights are defined as

[9]For any two positive real numbers a and b, their logarithmic mean is defined by $LM(a,b) \equiv (a-b)/\ln(a/b)$ when $a \neq b$, and $LM(a,a) \equiv a$. It has the following properties: (1) $\min(a,b) \leq LM(a,b) \leq \max(a,b)$; (2) $LM(a,b)$ is continuous; (3) $LM(\lambda a, \lambda b) = \lambda LM(a,b)$ ($\lambda > 0$); (4) $LM(a,b) = LM(b,a)$; (5) $(ab)^{1/2} \leq LM(a,b) \leq (a+b)/2$; (6) $LM(a,1)$ is concave. More details in Balk (2008, 134-136).

$$w_n^{mt} \equiv \frac{1}{LM(P(mt,0t;b(t)),1)} \frac{p_n^{0t} x_n^{b(t)}}{\sum_n p_n^{0t} x_n^{b(t)}} \ (n \in N^t).$$

Similarly, decomposing the second factor delivers

$$\ln\left(\frac{\sum_n p_n^{12(t-1)} x_n^{b(t-1)}}{\sum_n p_n^{m(t-1)} x_n^{b(t-1)}}\right) = \sum_{n \in N^{t-1}} w_n^{m(t-1)} \frac{p_n^{12(t-1)} - p_n^{m(t-1)}}{p_n^{m(t-1)}}, \tag{16}$$

where the weights are defined as

$$w_n^{m(t-1)} \equiv \frac{1}{LM(P(12(t-1),m(t-1);b(t-1)),1)} \frac{p_n^{m(t-1)} x_n^{b(t-1)}}{\sum_n p_n^{m(t-1)} x_n^{b(t-1)}} (n \in N^{t-1}).$$

By combining the two factors it appears that the annual price change can be decomposed as follows:

$$\ln\left(\frac{P^c(mt,0)}{P^c(m(t-1),0)}\right) = \sum_{n \in N^t} w_n^{mt} \frac{p_n^{mt} - p_n^{0t}}{p_n^{0t}} + \sum_{n \in N^{t-1}} w_n^{m(t-1)} \frac{p_n^{12(t-1)} - p_n^{m(t-1)}}{p_n^{m(t-1)}}. \tag{17}$$

Since for any positive real number $a \neq 1$, $\ln a = (a-1)/LM(a,1)$, the left-hand side of expression (17) is a simple transformation of the inflation rate. In fact, for $a \approx 1$, the usual approximation is $\ln a \approx a - 1$. The right-hand side of the expression then provides a decomposition of the inflation rate into contributions of the individual commodities, divided with respect to current year t and previous year $t-1$. The commodities can be grouped into those available in both years, those available in the current year but not in the previous year, and those available in the previous year but not in the current year.

3.3.3 Mean annual change

The annual price change, obtained by comparing the same month of two adjacent years, is in principle able to remove any regularly occuring seasonal variation. To provide a statistic that is robust to both regular and irregular (seasonal) variations, a 12-months moving average of annual price changes is frequently used.

We will consider here in particular the moving average as obtained in December of any year; that is, the mean of ratios

$$\frac{1}{12}\sum_{m=1}^{12}\frac{P^c(mt,0)}{P^c(m(t-1),0)}, \tag{18}$$

and compare this with the ratio of mean indices

$$\frac{\frac{1}{12}\sum_{m=1}^{12}P^c(mt,0)}{\frac{1}{12}\sum_{m=1}^{12}P^c(m(t-1),0)}. \tag{19}$$

The last ratio can be given a solid interpretation. To see this, we substitute expression (6) into expression (19). Cancelling common factors in numerator and denominator, we obtain

$$\frac{\frac{1}{12}\sum_{m=1}^{12}P^c(mt,0)}{\frac{1}{12}\sum_{m=1}^{12}P^c(m(t-1),0)} =$$

$$\frac{\frac{1}{12}\sum_{m=1}^{12}P(mt,0t;b(t))}{\frac{1}{12}\sum_{m=1}^{12}P(m(t-1),0(t-1);b(t-1))}P(12(t-1),0(t-1);b(t-1)), \tag{20}$$

which after substituting expression (1) thrice becomes

$$\frac{\frac{1}{12}\sum_{m=1}^{12}P^c(mt,0)}{\frac{1}{12}\sum_{m=1}^{12}P^c(m(t-1),0)} =$$

$$\frac{\frac{1}{12}\sum_{m=1}^{12}\frac{\sum_n p_n^{mt}x_n^{b(t)}}{\sum_n p_n^{0t}x_n^{b(t)}}}{\frac{1}{12}\sum_{m=1}^{12}\frac{\sum_n p_n^{m(t-1)}x_n^{b(t-1)}}{\sum_n p_n^{0(t-1)}x_n^{b(t-1)}}}\frac{\sum_n p_n^{12(t-1)}x_n^{b(t-1)}}{\sum_n p_n^{0(t-1)}x_n^{b(t-1)}} =$$

$$\frac{\sum_n p_n^{t}x_n^{b(t)}}{\sum_n p_n^{t-1}x_n^{b(t-1)}}\frac{\sum_n p_n^{0t}x_n^{b(t-1)}}{\sum_n p_n^{0t}x_n^{b(t)}}, \tag{21}$$

where $p_n^\tau \equiv \frac{1}{12}\sum_{m=1}^{12}p_n^{m\tau}$ $(n \in N^\tau)$ are annual mean prices and it is useful to recall that month 12 of year $t-1$ is the same as month 0 of year t. The last line of expression (21) contains two components. The left factor is a (counterfactual) value index comparing weight reference period $b(t)$ quantities at mean period t prices to weight reference period $b(t-1)$ quantities at mean period $t-1$ prices. The right factor is the reciprocal of a (Lowe) quantity index comparing weight reference period $b(t)$ quantities to those of weight reference period $b(t-1)$. If $b(t) = b(t-1)$ then this factor vanishes.

Summarizing, the ratio of annual mean indices in expression (19) may be interpreted as an implicit price index for year t relative to year $t-1$. If $b(t) = b(t-1)$ then this implicit price index reduces to a Lowe price index.

To relate now the mean of annual ratios, expression (18), to the ratio of annual means, expression (19), we proceed as follows. Employing the logarithmic mean, the logarithm of the ratio of annual means can be decomposed as

$$\ln\left(\frac{\frac{1}{12}\sum_{m=1}^{12} P^c(mt,0)}{\frac{1}{12}\sum_{m=1}^{12} P^c(m(t-1),0)}\right) = \sum_{m=1}^{12} \Psi^m \ln\left(\frac{P^c(mt,0)}{P^c(m(t-1),0)}\right), \tag{22}$$

with

$$\Psi^m \equiv \frac{LM\left(\frac{P^c(mt,0)}{\frac{1}{12}\sum_{m=1}^{12} P^c(mt,0)}, \frac{P^c(m(t-1),0)}{\frac{1}{12}\sum_{m=1}^{12} P^c(m(t-1),0)}\right)}{\sum_{m=1}^{12} LM\left(\frac{P^c(mt,0)}{\frac{1}{12}\sum_{m=1}^{12} P^c(mt,0)}, \frac{P^c(m(t-1),0)}{\frac{1}{12}\sum_{m=1}^{12} P^c(m(t-1),0)}\right)} \; (m = 1, ..., 12).$$

The weights Ψ^m are the normalized, mean-over-two-years ratios of monthly price index to annual mean price index, and as such reflect the aggregate seasonal price level pattern.

Now return to the mean of annual ratios in expression (18). Being an arithmetic mean it is greater than or equal to a geometric mean. Thus,

$$\begin{aligned}
&\ln\left(\frac{1}{12}\sum_{m=1}^{12} \frac{P^c(mt,0)}{P^c(m(t-1),0)}\right) \geq \sum_{m=1}^{12} \frac{1}{12} \ln\left(\frac{P^c(mt,0)}{P^c(m(t-1),0)}\right) = \\
&\sum_{m=1}^{12} \Psi^m \ln\left(\frac{P^c(mt,0)}{P^c(m(t-1),0)}\right) - \sum_{m=1}^{12}\left(\Psi^m - \frac{1}{12}\right) \ln\left(\frac{P^c(mt,0)}{P^c(m(t-1),0)}\right) = \\
&\ln\left(\frac{\frac{1}{12}\sum_{m=1}^{12} P^c(mt,0)}{\frac{1}{12}\sum_{m=1}^{12} P^c(m(t-1),0)}\right) - \sum_{m=1}^{12}\left(\Psi^m - \frac{1}{12}\right) \ln\left(\frac{P^c(mt,0)}{P^c(m(t-1),0)}\right),
\end{aligned} \tag{23}$$

where at the last step expression (22) has been used. The term after the minus sign in expression (23) measures the covariance between seasonal price level and seasonal price change (between consecutive years). If this covariance is zero or negative then the mean of ratios is greater than the ratio of means and, in view of the interpretation of the latter as provided by expression (21), can be said to overstate mean annual price change. If, however, this covariance is positive then nothing can be said with certainty.

4 Conclusion

The traditional distinction runs between direct and chained indices. Mixed-form indices share features of the two kinds: in the short run they behave as direct indices, and in the long run as chained indices.

Mixed-form indices materialize usually where monthly indices must be compiled, typical examples being the HICP and CPIs compiled by national statistical agencies. Their mixed form is revealed most clearly when derived measures such as monthly or annual rates of change are considered. The interpretation of these requires some care.

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