Understanding the Equity-premium Puzzle and the Correlation Puzzle

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The correlation puzzle

- The covariance and correlation between stock returns and measurable fundamentals, especially consumption, is weak at the 1, 5, 10 and 15 year horizons.
- This fact underlies virtually all modern asset-pricing puzzles.
  - The equity premium puzzle, Hansen-Singleton-style rejection of asset pricing models, Shiller's excess volatility of stock prices, etc.
- Hansen and Cochrane (1992) and Cochrane and Campbell (1999) call this phenomenon the "correlation puzzle."

Asset prices and economic fundamentals

- Classic asset pricing models load all uncertainty onto the supply-side of the economy.
  - Stochastic process for the endowment in Lucas-tree models.
  - Stochastic process for productivity in production economies.
- These models abstract from shocks to the demand for assets.
- It's not surprising that one-shock models can't simultaneously account for the equity premium puzzle and the correlation puzzle.

Fundamental shocks

- What's the other shock?
- We explore the possibility that it's a shock to the demand for assets.

Shocks to the demand for assets

- We model the shock to the demand for assets in the simplest possible way: time-preference shocks.
- Macro literature on zero lower bound suggests these shocks are a useful way to model changes in household savings behavior.
  - e.g. Eggertsson and Woodford (2003).
- These shocks also capture effects of changes in the demographics of stock market participants or other institutional changes that affect savings behavior.

Key results

- The model accounts for the equity premium and the correlation puzzle (taking statistical uncertainty into account).
    - It also accounts for the level and volatility of the risk free rate.

- The model's estimated risk aversion coefficient is very low (close to one).

- Our findings are consistent with Lucas' conjecture about fruitful avenues to resolve the equity premium puzzle.

"It would be good to have the equity premium resolved, but I think we need to look beyond high estimates of risk aversion to do it."
Robert Lucas, Jr., "Macroeconomic Priorities," American Economic Review, 2003.

Key results

- Model with Epstein-Zin preferences and no time-preference shocks
  - Very large estimated risk-aversion coefficient, no equity premium and cannot account for correlation puzzle.
- CRRA preferences and time-preference shocks.
  - Can't account for the equity premium or the correlation puzzle.
- Bansal, Kiku and Yaron (2011)
  - Can account for the equity premium puzzle with a risk aversion coefficient of 10.
  - Can't account for the correlation puzzle.

Trade-offs

- On the one hand, we introduce a new source of shocks into the model.
- On the other hand, our model is simpler than many alternatives.
- We assume that consumption and dividends are a random walk with a homoskedastic error term.
- We don't need:
  - Habit formation, long-run risk, time-varying endowment volatility, model ambiguity.
  - Any of these features could be added.
- Straightforward to modify DSGE models to allow for these shocks.

The importance of Epstein-Zin preferences

- For time-preference shocks to improve the model's performance, it's critical that agents have Epstein-Zin preferences.
- Introducing time-preference shocks in a model with CRRA preferences is counterproductive.
- In the CRRA case, the equity premium is a *decreasing* function of the variance of time-preference shocks.

The correlation puzzle

- We use data for 17 OECD countries and 7 non-OECD countries, covering the period 1871-2006.
- Correlations between stock returns and consumption, as well as correlations between stock returns and output are low at all time horizons.
- The correlation between stock returns and dividend growth is substantially higher for horizons greater than 10 years, but it's similar to that of consumption at shorter horizons.

Historical data

- Sample: 1871-2006.
- Nakamura, Steinsson, Barro, and Ursúa (2011) for stock returns.
- Barro and Ursúa (2008) for consumption expenditures and real per capita GDP.
- Shiller for real S&P500 earnings and dividends.
- We use realized real stock returns and risk free rate.

The correlation puzzle

Correlation between real stock market returns and the growth rate of fundamentals
United States, 1871-2006

	Consumption	Output	Dividends	Earnings
1 year	0.090 (0.089)	0.136 (0.101)	-0.039 (0.0956)	0.126 (0.1038)
5 years	0.397 (0.177)	0.249 (0.137)	0.382 (0.148)	0.436 (0.179)
10 years	0.248 (0.184)	-0.001 (0.113)	0.642 (0.173)	0.406 (0.125)
15 years	0.241 (0.199)	-0.036 (0.148)	0.602 (0.158)	0.425 (0.111)
Episodes	0.615 (0.271)	0.308 (0.303)	0.713 (0.305)	0.708 (0.292)
Weighted Episodes	0.631 (0.147)	0.268 (0.168)	0.787 (0.131)	0.692 (0.149)

Standard errors are indicated in parenthesis.

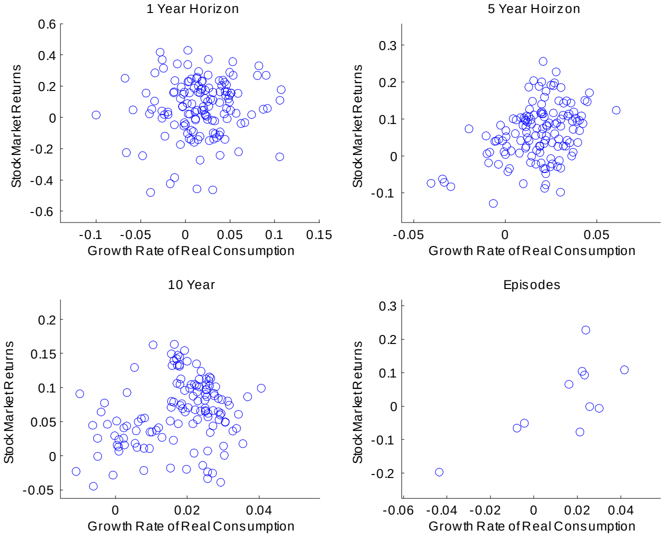
The correlation puzzle

Correlation between real stock market returns and growth rate of fundamentals
G7 and non G7 countries

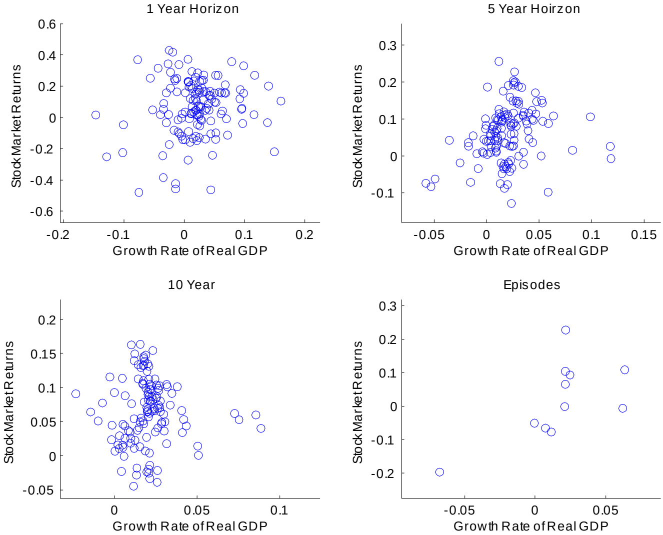
	G7 countries		Non G7 countries	
	Consumption	Output	Consumption	Output
1 year	0.008	0.182	0.050	0.089
	(0.062)	(0.081)	(0.027)	(0.031)
5 years	0.189	0.355	0.087	0.157
	(0.105)	(0.092)	(0.069)	(0.074)
10 years	0.277	0.394	0.027	0.098
	(0.132)	(0.119)	(0.122)	(0.130)
15 years	0.308	0.374	0.023	0.084
	(0.176)	(0.171)	(0.166)	(0.176)
Episodes	0.651	0.702	0.376	0.474
	(0.100)	(0.073)	(0.107)	(0.109)
Weighted Episodes	0.741	0.770	0.342	0.445
	(0.036)	(0.040)	(0.028)	(0.029)

Standard errors are indicated in parenthesis.

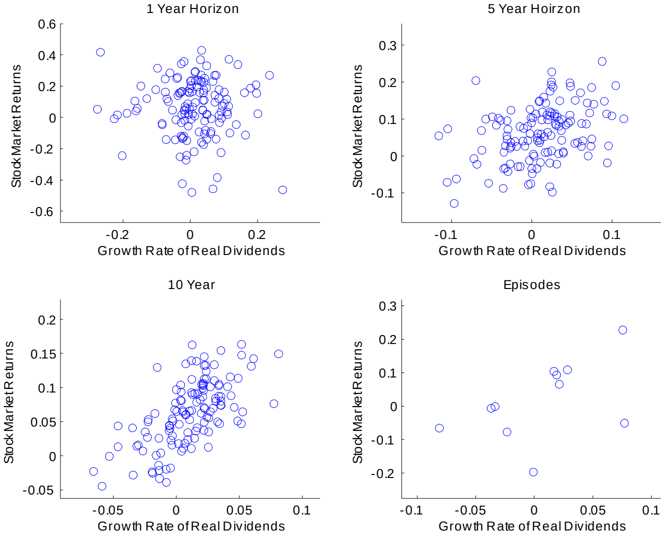
U.S. stock returns and consumption growth



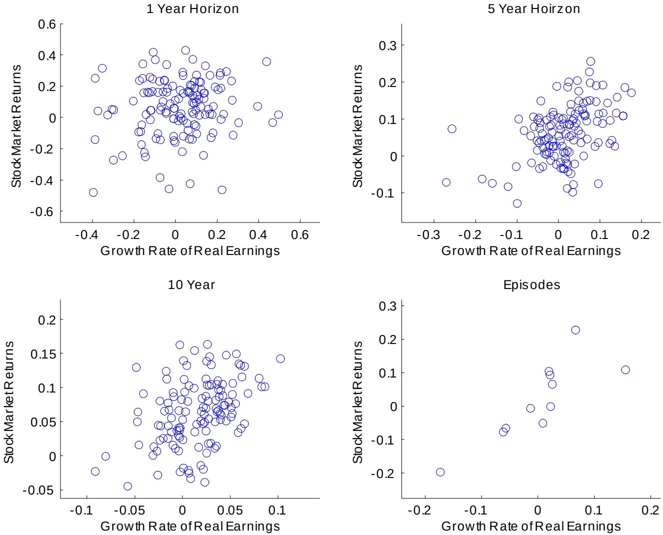
U.S. stock returns and output growth



U.S. stock returns and dividend growth



U.S. stock returns and earnings growth



A model with time-preference shocks

- Epstein-Zin preferences
  - Life-time utility is a CES of utility today and the certainty equivalent of future utility, U^*_{t+1}.

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U^*_{t+1} \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}$$

- λ_t determines how agents trade off current versus future utility, isomorphic to a time-preference shock.

- ψ is the elasticity of intertemporal substitution.

A model with time-preference shocks

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U_{t+1}^* \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}$$

- The certainty equivalent of future utility is the sure value of $t+1$ lifetime utility, U_{t+1}^* such that:

$$\left(U_{t+1}^* \right)^{1-\gamma} = E_t \left(U_{t+1}^{1-\gamma} \right)$$

$$U_{t+1}^* = \left[E_t \left(U_{t+1}^{1-\gamma} \right) \right]^{1/(1-\gamma)}$$

- γ is the coefficient of relative risk aversion.

Special case: CRRA

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U_{t+1}^* \right)^{1-1/\psi} \right]^{1-1/\psi}$$

- When $\gamma = 1/\psi$, preferences reduce to CRRA with a time-varying rate of time preference.

$$V_t = \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} C_{t+i}^{1-\gamma},$$

where $V_t = U_t^{1-\gamma}$.

- Case considered by Garber and King (1983) and Campbell (1986).

Stochastic processes

- Consumption follows a random walk

$$\begin{aligned} \log(C_{t+1}) &= \log(C_t) + \mu + \eta^c_{t+1} \\ \eta^c_{t+1} &\sim N(0, \sigma^2_c) \end{aligned}$$

- Process for dividends:

$$\begin{aligned} \log(D_{t+1}) &= \log(D_t) + \mu + \pi\eta^c_{t+1} + \eta^d_{t+1} \\ \eta^d_{t+1} &\sim N(0, \sigma^2_d) \end{aligned}$$

Stochastic processes

- Time-preference shock:

$$\log\left(\lambda_{t+1}/\lambda_t\right) = \rho \log\left(\lambda_t/\lambda_{t-1}\right) + \varepsilon_{t+1}$$
$$\varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$$

- It's convenient to assume that agents know λ_{t+1} at time t.

- What matters for agents' decisions is the growth rate of λ_t, which we assume is highly persistent but stationary (ρ is very close to one).

- The idea is to capture, in a parsimonious way, persistent changes in agents' attitudes towards savings.

Solving the model

- Returns to the stock market are defined as returns to claim on dividend process:
  - Standard assumption in asset-pricing literature (Abel (1999)).
- Realized gross stock-market return:

$$R^d_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}.$$

- Define:

$$\begin{aligned} r_{d,t+1} &= \log(R^d_{t+1}), \\ z_{dt} &= \log(P_t / D_t). \end{aligned}$$

Solving the model

- Realized gross return to a claim on the endowment process:

$$R^c_{t+1} = \frac{P^c_{t+1} + C_{t+1}}{P^c_t}.$$

- Define:

$$\begin{aligned} r_{c,t+1} &= \log(R^c_{t+1}), \\ z_{ct} &= \log(P^c_t / C_t). \end{aligned}$$

Solving the model

- Using a log-linear Taylor expansion:

$$
\begin{aligned}
r_{d,t+1} &= \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}, \\
r_{c,t+1} &= \kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct} + \Delta c_{t+1},
\end{aligned}
$$

$$
\begin{aligned}
\kappa_{d0} &= \log\left[1 + \exp(z_d)\right] - \kappa_{1d} z_d, \\
\kappa_{c0} &= \log\left[1 + \exp(z_c)\right] - \kappa_{1c} z_c,
\end{aligned}
$$

$$
\kappa_{d1} = \frac{\exp(z_d)}{1 + \exp(z_d)}, \quad \kappa_{c1} = \frac{\exp(z_c)}{1 + \exp(z_c)}.
$$

- z_d and z_c are the values of z_{dt} and z_{ct} in the non-stochastic steady state.

Solving the model

- The log-SDF is:

$$m_{t+1} = \theta \log(\delta) + \theta \log(\lambda_{t+1}/\lambda_t) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1},$$

$$\theta = \frac{1-\gamma}{1-1/\psi}.$$

- $r_{c,t+1}$ is the log return to a claim on the endowment,

$$r_{c,t+1} = \log(R_{t+1}) = \frac{P_{t+1} + C_{t+1}}{P_t}$$

- Euler equation:

$$E_t \left[\exp\left(m_{t+1} + r_{d,t+1}\right)\right] = 1$$

Solving the model

- Use Euler equation:

$$E_t \left[\exp\left(m_{t+1} + r_{d,t+1}\right)\right] = 1$$

- Replace m_{t+1} and $r_{d,t+1}$ using equations:

$$m_{t+1} = \theta \log\left(\delta\right) + \theta \log\left(\lambda_{t+1}/\lambda_t\right) - \frac{\theta}{\psi}\Delta c_{t+1} + \left(\theta - 1\right) r_{c,t+1},$$

$$r_{d,t+1} = \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}.$$

- Replace $r_{c,t+1}$ with:

$$r_{c,t+1} = \kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct} + \Delta c_{t+1}.$$

Solving the model

- Guess and verify that the equilibrium solution for z_{dt} and z_{ct} take the form:

$$
\begin{aligned}
z_{dt} &= A_{d0} + A_{d1} \log\left(\lambda_{t+1}/\lambda_t\right), \\
z_{ct} &= A_{c0} + A_{c1} \log\left(\lambda_{t+1}/\lambda_t\right).
\end{aligned}
$$

- Since consumption is a martingale, price - dividend ratios are constant absent movements in λ_t.
- In calculating conditional expectations use properties of lognormal distribution.
- Use method of indeterminate coefficients to compute A_{d0}, A_{d1}, A_{c0}, and A_{c1}.

The risk-free rate

$$
\begin{aligned}
r^f_{t+1} &= -\log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \mu/\psi - (1-\theta)\,\kappa^2_{c1} A^2_{c1} \sigma^2_{\varepsilon}/2 \\
&\quad + \left[\frac{(1-\theta)}{\theta}(1-\gamma)^2 - \gamma^2\right]\sigma^2_c/2,
\end{aligned}
$$

$$
\theta = \frac{1-\gamma}{1-1/\psi}.
$$

- $\theta = 1$ when preferences are CRRA.
- The risk-free rate is a decreasing function of $\log(\lambda_{t+1}/\lambda_t)$.
  - If agents value the future more, relative to the present, they want to save more. Since aggregate savings cannot increase, the risk-free rate has to fall.

Equity premium

$$
\begin{aligned}
r^f_{t+1} &= -\log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \mu/\psi - (1-\theta)\kappa^2_{c1}A^2_{c1}\sigma^2_{\varepsilon}/2 \\
&\quad + \left[\frac{(1-\theta)}{\theta}(1-\gamma)^2 - \gamma^2\right]\sigma^2_c/2.
\end{aligned}
$$

$$
\begin{aligned}
E_t(r_{d,t+1}) - r^f_t &= \pi\sigma^2_c(2\gamma-\pi)/2 - \sigma^2_d/2 + \\
&\quad \kappa_{d1}A_{d1}\left[2(1-\theta)A_{c1}\kappa_{c1} - \kappa_{d1}A_{d1}\right]\sigma^2_{\varepsilon}/2.
\end{aligned}
$$

- It's cumbersome to do comparative statics exercises because κ_{c1} and κ_{d1} are functions of the parameters of the model.

Equity premium: CRRA case

- Suppose that $\theta = 1$:

$$r^f_{t+1} = -\log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \mu/\psi - \gamma^2\sigma_c^2/2.$$

$$E_t(r_{d,t+1}) - r^f_t = \pi\sigma_c^2(2\gamma - \pi)/2 - \sigma_d^2/2 - \kappa_{d1}^2 A_{d1}^2 \sigma_\varepsilon^2/2.$$

- Interestingly, the equity premium in this special case depends *negatively* on σ_ε^2.

Equity premium: CRRA case

- To get some intuition consider the case where the stock market is a claim to consumption ($\pi = 1$, $\sigma_d^2 = 0$).
- Replacing expectations of future price-consumption ratio we obtain:

$$
\begin{aligned}
\frac{P_t}{C_t} &= \alpha \exp(\sigma_\varepsilon \varepsilon_{t+1}) \left[E_t \left(\frac{P_{t+1}}{C_{t+1}} \right) + 1 \right] \\
\alpha &= \delta \exp \left[(1-\gamma)\, \mu + (1-\gamma)^2\, \sigma_c^2 / 2 \right]
\end{aligned}
$$

- ε_{t+1} is known at time t.
- Recursing on P_t / C_t:

$$
\frac{P_t}{C_t} = \alpha \exp(\sigma_\varepsilon \varepsilon_{t+1}) E_t \left[\begin{array}{c} 1 + \alpha \exp(\sigma_\varepsilon \varepsilon_{t+2}) \\ + \alpha^2 \exp(\sigma_\varepsilon \varepsilon_{t+2}) \exp(\sigma_\varepsilon \varepsilon_{t+3}) + \ldots \end{array} \right]
$$

Equity premium: CRRA case

$$\frac{P_t}{C_t} = \alpha \exp(\sigma_\varepsilon \varepsilon_{t+1}) E_t \left[\begin{array}{c} 1 + \alpha \exp(\sigma_\varepsilon \varepsilon_{t+2}) + \alpha^2 \exp(\sigma_\varepsilon \varepsilon_{t+2}) \exp(\sigma_\varepsilon \varepsilon_{t+3}) \\ +... \end{array} \right]$$

- Computing expectations:

$$\frac{P_t}{C_t} = \alpha \exp(\sigma_\varepsilon \varepsilon_{t+1}) \left[1 + \alpha \exp(\sigma_\varepsilon^2/2) + \alpha^2 \left[\exp(\sigma_\varepsilon^2/2)\right]^2 + ...\right]$$

- Assume that $\alpha \exp(\sigma_\varepsilon^2/2) < 1$ so price is finite.
- The price-consumption ratio is an increasing function of σ_ε^2.
  - This variance enters because the mean of a lognormal variable is increasing in the variance.

Equity premium: CRRA case

- The unconditional expected return is:

$$ER^c_{t+1} = \exp(\mu + \sigma_c^2/2)\left[1 + E\left(C_t/P_t\right)\right].$$

$$E\left(C_t/P_t\right) = \frac{1 - \delta \exp\left[(1-\gamma)\,\mu + (1-\gamma)^2\,\sigma_c^2/2\right]\left[\exp(\sigma_\varepsilon^2/2)\right]^2}{\delta \exp\left[(1-\gamma)\,\mu + (1-\gamma)^2\,\sigma_c^2/2\right]}$$

- ER^c_{t+1} is a decreasing function of σ_ε^2.
- Including time-preference shocks in a model with CRRA utility lowers the equity premium!

Equity premium: Epstein-Zin

$$E_t\left(r_{d,t+1}\right) - r_t^f = \pi\sigma_c^2(2\gamma - \pi)/2 - \sigma_d^2/2 + \kappa_{d1}A_{d1}\left[2\left(1-\theta\right)A_{c1}\kappa_{c1} - \kappa_{d1}A_{d1}\right]\sigma_\varepsilon^2/2.$$

- Recall that:

$$r_{d,t+1} = \kappa_{d0} + \kappa_{d1}z_{dt+1} - z_{dt} + \Delta d_{t+1}, \kappa_{d1} = \frac{\exp(z_d)}{1+\exp(z_d)}$$

$$r_{c,t+1} = \kappa_{c0} + \kappa_{c1}z_{ct+1} - z_{ct} + \Delta c_{t+1}, \quad \kappa_{c1} = \frac{\exp(z_c)}{1+\exp(z_c)}$$

- Necessary condition for time-preference shocks to help explain equity premium: $\theta < 1$ $(\gamma > 1/\varphi)$.

- This condition is more likely to be satisfied for higher risk aversion, higher IES.

Estimating the parameters of the model

- We estimate the model using GMM.
- We find the parameter vector $\hat{\Phi}$ that minimizes the distance between the empirical, Ψ_D, and model population moments, $\Psi(\hat{\Phi})$,

$$L(\hat{\Phi}) = \min_{\Phi} \left[\Psi(\Phi) - \Psi_D\right]' \Omega_D^{-1} \left[\Psi(\Phi) - \Psi_D\right].$$

- Ω_D is an estimate of the variance-covariance matrix of the empirical moments.

Estimated parameters

- Agents make decisions on a monthly basis. We compute moments at an annual frequency.
- The parameter vector, Φ, includes the 9 parameters:
  - γ: coefficient of relative risk aversion;
  - ψ: elasticity of intertemporal substitution;
  - δ: rate of time preference;
  - μ: drift in random walk for the log of consumption and dividends;
  - σ_c: volatility of innovation to consumption growth;
  - π: parameter that controls correlation between consumption and dividend shocks;
  - σ_d: volatility of dividend shocks;
  - ρ: persistence of time-preference shocks;
  - σ_λ: volatility of innovation to time-preference shocks.

Moments used in estimation

- The vector Ψ_D includes the following 14 moments:
  - Consumption growth: mean and standard deviation;
  - Dividend growth: mean, standard deviation, and 1st order serial correlation;
  - Correlation between growth rate of dividends and growth rate of consumption;
  - Real stock returns: mean and standard deviation;
  - Risk free rate: mean and standard deviation;
  - Correlation between stock returns and consumption growth (1 and 10 years);
  - Correlation between stock returns and dividend growth (1 and 10 years).

Parameter estimates, benchmark model

Parameter	Estimates	Parameter	Estimates
γ	0.95	σ_d	0.0158
ψ	0.90	π	0.73
δ	0.9993	σ_λ	0.00011
σ_c	0.0058	ρ	0.9992
μ	0.00135		

Moments (annual), data and model

Moments	Data	Model
Std (Δd_t)	9.16 (1.82)	5.66
Std (Δc_t)	3.50 (0.62)	2.00
Corr$(\Delta c_t, \Delta d_t)$	0.20 (0.13)	0.26

Moments (annual), data and model

Moments	Data	Model	Moments	Data	Model
$E(R_t^d)$	6.24 (1.47)	3.12	$\mathrm{Std}(R_t^d - R_t^f)$	18.20 (2.77)	19.02
$E(R_t^f)$	1.74 (0.58)	0.45	$\mathrm{Std}(R_t^d)$	18.18 (2.65)	19.0
$E(R_t^d) - E(R_t^f)$	4.50 (1.50)	2.67	$\mathrm{Std}(R_t^f)$	4.68 (1.11)	3.22

Annual correlations between fundamentals and real stock returns

Consumption	Data	Model
1 year	0.100 (0.089)	0.077
5 year	0.397 (0.177)	0.073
10 year	0.248 (0.184)	0.074
15 year	0.241 (0.199)	0.074

Dividends	Data	Model
1 year	−0.039 (0.0956)	0.297
5 year	0.382 (0.148)	0.281
10 year	0.642 (0.173)	0.288
15 year	0.602 (0.158)	0.288

The importance of the correlation puzzle

- Since $\text{corr}(\Delta d_t, R_t^d)$ and $\text{corr}(\Delta c_t, R_t^d)$ are estimated with more precision than average rates of returns, the estimation criterion gives them more weight.

- If we drop the correlations from the criterion, the parameters move to a region where the equity premium is larger.

- The value of $\theta = (1-\gamma)/(1-1/\psi)$ goes from -0.45 to -1.23, which is why the equity premium implied by the model rises.

Model comparison

	Data	Benchmark	Benchmark No corr. in criterion
γ	-	0.95	0.80
ψ	-	0.90	0.86
$E(R_t^d)$	6.24 (1.47)	3.12	5.39
$E(R_f)$	1.74 (0.58)	0.45	1.78
$E(R_t^d) - R_f$	4.50 (1.50)	2.67	3.60
$\text{corr}(\Delta d_t, R_t^d)$	-0.039 (0.0956)	0.30	0.49
$\text{corr}(\Delta c_t, R_t^d)$	0.100 (0.089)	0.08	0.08

Model without time preference shocks

- Without time-preference shocks, the estimation criterion settles on a very high risk aversion coefficient ($\gamma = 18$).
- Even then, the model cannot generate an equity premium.
- It also cannot account for the correlation puzzle
  - $\text{corr}(\Delta d_t, R_t^d) = 1$, $\text{corr}(\Delta c_t, R_t^d) = 0.40$.

Model comparison

	Data	Benchmark	Benchmark No time pref.shocks
γ	-	0.95	18.27
ψ	-	0.90	0.17
$E(R_t^d)$	6.24 (1.47)	3.12	4.52
$E(R_f)$	1.74 (0.58)	0.45	4.33
$E(R_t^d) - R_f$	4.50 (1.50)	2.67	0.19
$\text{corr}(\Delta d_t, R_t^d)$	-0.039 (0.0956)	0.30	1.00
$\text{corr}(\Delta c_t, R_t^d)$	0.100 (0.089)	0.08	0.40

Model comparison

	Data	Benchmark	CRRA	CRRA No time pref. shocks
γ	-	0.95	1.62	0.21
ψ	-	0.90	1/1.62	1/0.21
$E(R_t^d)$	6.24 (1.47)	3.12	1.66	4.95
$E(R_f)$	1.74 (0.58)	0.45	3.20	4.95
$E(R_t^d) - R_f$	4.50 (1.50)	2.67	-1.54	0.00
$\text{corr}(\Delta d_t, R_t^d)$	-0.039 (0.0956)	0.30	0.56	1.0
$\text{corr}(\Delta c_t, R_t^d)$	0.100 (0.089)	0.08	0.13	0.45

Imposing an EIS > 1

- When $\psi < 1$, good news about the future drives down stock prices.
- Suppose agents learn that they will receive a higher future dividend from the tree.
- On the one hand, the tree is worth more, so agents want to buy stock shares (substitution effect).
- On the other hand, agents want to consume more today, so they want to sell stock shares (income effect).

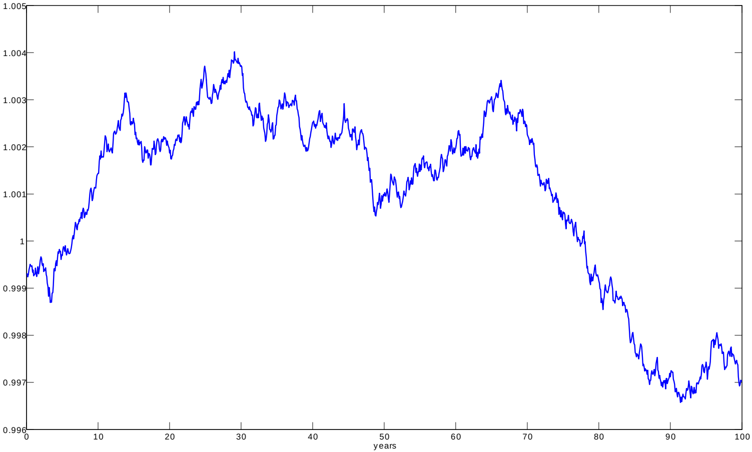
Imposing an EIS > 1

- When $\psi < 1$, income effect dominates and agents try to sell stock shares. But they can't in the aggregate.
- So, the price of the tree must fall and expected returns rise, thus inducing the representative agent to hold the tree.
- Imposing $\psi > 1$ has a modest impact on our results.
  - The equity premium rises.
  - But, $\text{corr}(\Delta d_t, R_t^d)$ and $\text{corr}(\Delta c_t, R_t^d)$ also rise.

Model comparison

	Data	Benchmark	Benchmark Impose $\psi>1$
γ	-	0.95	1.4
ψ	-	0.90	5.02
$E(R_t^d)$	6.24 (1.47)	3.12	3.68
$E(R_f)$	1.74 (0.58)	0.45	0.84
$E(R_t^d) - R_f$	4.50 (1.50)	2.67	2.84
$\text{corr}(\Delta d_t, R_t^d)$	-0.039 (0.0956)	0.30	0.41
$\text{corr}(\Delta c_t, R_t^d)$	0.100 (0.089)	0.08	0.19

A century of time-preference shocks, (a sample path)



Bansal, Kiku and Yaron (2011)

- Originally, they emphasized importance of long run risk.
- More recently they emphasized the importance of movements in volatility.

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U_{t+1}^* \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}$$

$$U_{t+1}^* = \left[E_t \left(U_{t+1}^{1-\gamma} \right) \right]^{1/(1-\gamma)}$$

$$\begin{aligned} g_t &= \mu + x_{t-1} + \sigma_{t-1} \eta_t, \\ x_t &= \rho_x x_{t-1} + \phi_e \sigma_{t-1} e_t, \\ \sigma_t^2 &= \sigma^2 (1 - \nu) + \nu \sigma_{t-1}^2 + \sigma_w^2 w_t. \end{aligned}$$

BKY parameters

Parameter	BKY	Parameter	BKY
γ	10	σ	0.0072
ψ	1.5	ν	0.999
δ	0.9989	σ_w	0.28×10^{-5}
μ	0.0015	ϕ	2.5
ρ_x	0.975	π	2.6
ϕ_e	0.038	φ	5.96

BKY

- We re-estimated our model for the period 1930-2006 for comparability with BKY

1930-2006	Data	Benchmark	BKY
$E(R_t^d)$	6.23 (2.07)	6.53	8.75
$\text{std}(R_t^d)$	19.26 (3.63)	10.25	23.37
$E(R_f)$	0.57 (0.64)	2.75	1.05
$\text{std}(R_t^f)$	3.95 (1.29)	3.29	1.22
$E(R_t^d) - R_f$	5.66 (2.15)	3.78	7.70

Correlation between stock returns and consumption growth

1930-2006	Data	Bench.	BKY
1 year	0.04 (0.15)	0.03	0.66
5 year	0.05 (0.15)	0.03	0.88
10 year	-0.30 (0.18)	0.03	0.92
15 year	-0.32 (0.15)	0.03	0.93

Correlation between stock returns and dividend growth

1930-2006	Data	Bench.	BKY
1 year	-0.10 (0.13)	0.12	0.66
5 year	0.32 (0.12)	0.12	0.90
10 year	0.73 (0.20)	0.12	0.93
15 year	0.69 (0.16)	0.12	0.94

Bansal, Kiku and Yaron (2011)

- The BKY model does a very good job at accounting for the equity premium and the average risk free rate.
- Problem: correlations between stock market returns and fundamentals (consumption or dividend growth) are close to one.

Conclusion

- We propose a simple model that accounts for the level and volatility of the equity premium and of the risk free rate.
- The model is broadly consistent with the correlations between stock market returns and fundamentals, consumption and dividend growth.
- Key features of the model
  - Consumption and dividends follow a random walk;
  - Epstein-Zin utility;
  - Stochastic rate of time preference.
- The model accounts for the equity premium with low levels of risk aversion.