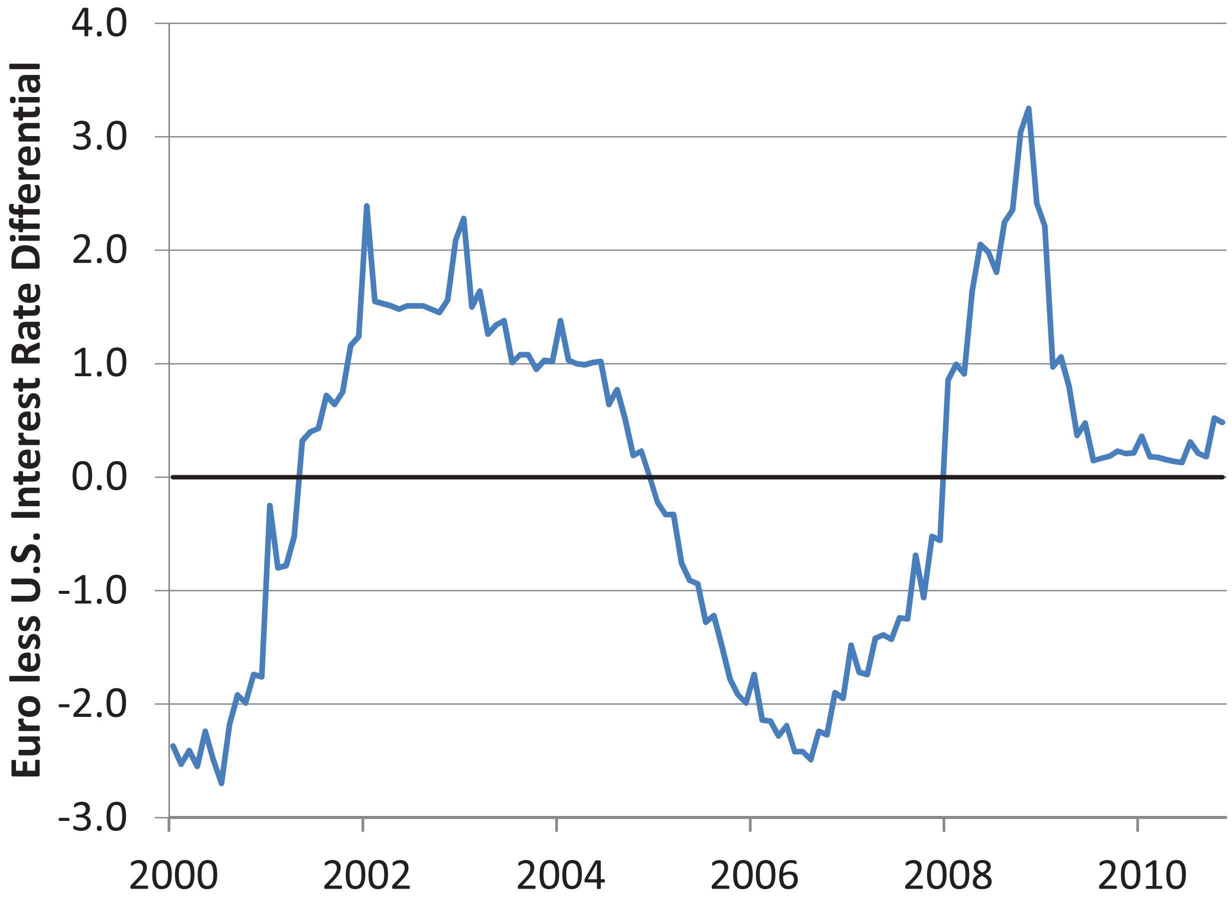
Monetary Policy and the Uncovered Interest Rate Parity Puzzle

Dave Backus, Federico Gavazzoni, Chris Telmer and Stan Zin

USD/EUR Interest Rate Differential

Eonia Less Fed Funds Interest Rate Spread



Why do countries with high-interest-rate *policies* have currencies that tend to appreciate?

- When the Fed decides to tighten vis-a-vis the ECB, why does USD get anointed as the risky currency?

... More Specifically

Domestic and foreign Taylor Rules:

$$i_t = \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t$$

$$i_t^* = \bar{\tau}^* + \tau_\pi^* \pi_t^* + \tau_x^* x_t^*$$

- ☐ How are these policies reflected in exchange rates?
- ☐ Does the answer have anything to do with currency risk?

Findings

- A relatively tight *domestic* monetary policy, $\tau_\pi > \tau_\pi^*$, makes the *foreign* currency risk premium *larger*.

- Empirical application based on U.S. - Australia
  - Qualitative predications of model confirmed
  - Quantitatively, risk premiums too small

Background and Overview

Two Points

1. Currency risk = *difference* in volatility.

2. Overview of what we do:
   - □ Take Lucas (1982).
   - □ Replace money with Taylor rules

Currency Risk in Log-Normal Models

High volatility implies low currency risk:

$$E_t(s_{t+1} - f_t) = (Var_t m_{t+1} - Var_t m^*_{t+1})/2$$

where,

- m = nominal MRS of U.S. representative agent
- m^* = nominal MRS of European representative agent
- s_t = log spot rate (price of EUR)
- f_t = log forward rate
- $E_t(s_{t+1} - f_t)$ = expected excess return on EUR

Implications:

- Time-varying volatility is necessary
- For monetary policy to matter, it must either generate volatility or respond to it.
- Our model: volatility arises from real shocks ... Taylor rule responds:

$$i_t = \bar{\tau} + \tau_\pi \pi_t(x_t, \sigma_t^2) + \tau_x x_t$$

Lucas Equation

- Lucas (1982) equation:

$$\frac{S_{t+1}}{S_t} = \frac{\frac{u'(c^*_{t+1})}{u'(c^*_t)} \frac{P^*_t}{P^*_{t+1}}}{\frac{u'(c_{t+1})}{u'(c_t)} \frac{P_t}{P_{t+1}}}$$

Lucas Equation

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Method

- Previous work on monetary policy and the UIP puzzle:

  Alvarez, Atkeson, and Kehoe (2007), Backus, Gregory, and Telmer (1993), Bekaert (1994), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006), Canova and Marrinan (1993), Dutton (1993), Grilli and Roubini (1992), Lucas (1982), Macklem (1991), Marshall (1992), McCallum (1994) and Schlagenhauf and Wrase (1995)

- Most feature explicit models of *money*.

- We replace money with Taylor rules

Basic Approach

- Usual set-up (private sector behavior):

$$i_t = -\log E_t\, n_{t+1} e^{-\pi_{t+1}}$$

- Monetary policy is a Taylor rule:

$$i_t \quad = \quad \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t$$

- Endogenous inflation (Gallmeyer, Hollifield, Palomino, and Zin (2007))

$$\pi_t = -\frac{1}{\tau_\pi}\left(\bar{\tau} + \tau_x x_t + \log E_t\, n_{t+1}\, e^{-\pi_{t+1}}\right)$$

- Do the same for foreign country, use Lucas equation to solve for exchange rate:

$$\frac{S_{t+1}}{S_t}(\tau, \tau^*) = \frac{n^*_{t+1} e^{-\pi^*_{t+1}(\tau)}}{n_{t+1} e^{-\pi_{t+1}(\tau^*)}}$$

Different Taylor Rules

- Can evaluate different Taylor rules:

  - Baseline, with/without shocks/asymmetries:

$$\begin{aligned} i_t &= \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t + z_t \\ i_t^* &= \tau^* + \tau_\pi^* \pi_t^* + \tau_x^* x_t^* + z_t^* \end{aligned}$$

  - Asymmetric w.r.t. exchange rate:

$$\begin{aligned} i_t &= \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t + z_t \\ i_t^* &= \tau^* + \tau_\pi^* \pi_t^* + \tau_x^* x_t^* + \tau_3^* \log(S_{t+1}/S_t) + z_t^* \end{aligned}$$

  - Interest rate smoothing (McCallum (1994)):

$$\begin{aligned} i_t &= \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t + \tau_4 i_{t-1} + z_t \\ i_t^* &= \tau^* + \tau_\pi^* \pi_t^* + \tau_x^* x_t^* + \tau_4^* i_{t-1}^* + z_t^* \end{aligned}$$

- Important identification issues (Cochrane (2007))

Model

Setting

$$\frac{S_{t+1}}{S_t}(\tau) = \underbrace{\frac{U'(c^*_{t+1})/U'(c^*_t)}{U'(c_{t+1})/U'(c_t)}}_{\text{Real FX Rate}} \frac{P_t}{P_{t+1}}(\tau)$$

- Complete markets
- Recursive preferences
- *Exogenous* domestic and foreign consumption (c^*_t, c_t)
  - No feedback from policy to allocations
- Taylor rules (τ, τ^*)

Preferences

- Recursive preferences for representative agent:

$$U_t = [(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}$$

$$\mu_t(U_{t+1}) \equiv E_t[U_{t+1}^\alpha]^{1/\alpha}$$

- Real pricing kernel:

$$n_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{\rho-1} \left(\frac{U_{t+1}}{\mu_t(U_{t+1})}\right)^{\alpha-\rho}.$$

- Hansen, Heaton, and Li (2005) linearization

Consumption

Consumption growth:

$$x_{t+1} = (1-\varphi_x)\theta_x + \varphi_x x_t + \sqrt{u_t}\epsilon^x_{t+1}$$

Volatility:

$$u_{t+1} = (1-\varphi_u)\theta_u + \varphi_u u_t + \sigma_u \epsilon^u_{t+1}$$

Taylor Rule

$$i_t = \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t$$

Solution: Domestic Inflation

$$\pi_t = -\frac{1}{\tau_\pi}\left(\bar{\tau} + \tau_x x_t + \log E_t\, n_{t+1}\, e^{-\pi_{t+1}}\right)$$

- Solution:

$$\pi_t \quad = \quad a + a_x x_t + a_u u_t$$

- Coefficients

$$a_x \quad = \quad \frac{(1-\rho)\varphi_x - \tau_x}{\tau_\pi - \varphi_x}$$

$$a_u \quad = \quad \frac{\frac{\alpha}{2}(\alpha-\rho)(\omega_x+1)^2 - \frac{1}{2}\Big((1-\alpha)-(\alpha-\rho)\omega_x + a_x\Big)^2}{\tau_\pi - \varphi_u}$$

Pricing Kernel

$$-\log m_{t+1} \quad = \quad \delta + \gamma_x x_t + \gamma_u u_t + \lambda_x \sqrt{u_t}\epsilon^x_{t+1} + \lambda_u \sigma_u \epsilon^u_{t+1}$$

where

$$\gamma_x = (1-\rho)\varphi_x + a_x\varphi_x \quad ; \quad \gamma_u = \frac{\alpha}{2}(\alpha-\rho)(\omega_x+1)^2 + a_u\varphi_u$$

$$\lambda_x = (1-\alpha) - (\alpha-\rho)\omega_x + a_x \quad ; \quad \lambda_u = -(\alpha-\rho)\omega_u + a_u$$

Foreign Economy

- Add asterisks to everything above
  - Cross-country consumption correlation important
- Characterize foreign pricing kernel, m^*_{t+1}
- Compute nominal depreciation rate rate:

$$\log\left(S_{t+1}/S_t\right) \;=\; \log m^*_{t+1} - \log m_{t+1}$$

Bilson-Fama Regression

Bilson-Fama Regression

Regress nominal log depreciation rate on interest rate differential:

$$s_{t+1} - s_t \quad = \quad a + b\left(i_t - i_t^*\right) + \text{residuals}$$

- Common finding: $b < 0$
- Basis of carry-trade expected returns

Theoretical Bilson-Fama Coefficient

Symmetric Taylor rules ($\tau_\pi = \tau_\pi^*$, $\tau_x = \tau_x^*$) and $\varphi_x = 0$.

- Absent real exchange rate variation ($n_{t+1} = n_{t+1}^* = n$):

$$b = \frac{\varphi_u}{\tau_\pi}$$

Theoretical Bilson-Fama Coefficient

Symmetric Taylor rules ($\tau_\pi = \tau_\pi^*$, $\tau_x = \tau_x^*$) and $\varphi_x = 0$.

- Absent real exchange rate variation ($n_{t+1} = n_{t+1}^* = n$):

$$b = \frac{\varphi_u}{\tau_\pi}$$

- Asymmetric Taylor rules can't make $b < 0$. But more complex Taylor rules can:

$$i_t = \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t + \tau_i i_{t-1} + \tau_s \log(S_{t+1}/S_t)$$

... With Real Exchange Rate Variation

- Turn on real exchange rate channel.

$$b = \frac{\gamma_u}{\gamma_u - \frac{1}{2}\lambda_x^2}$$

where,

$$\gamma_u = \frac{\alpha}{2}(\alpha - \rho)(\omega_x + 1)^2 + a_u \varphi_u$$

$$\lambda_x = (1 - \alpha) - (\alpha - \rho)\omega_x + a_x$$

$$a_u = \frac{\frac{\alpha}{2}(\alpha - \rho)(\omega_x + 1)^2 - \frac{1}{2}\Big((1 - \alpha) - (\alpha - \rho)\omega_x + a_x\Big)^2}{\tau_\pi - \varphi_u}$$

$$a_x = \frac{(1 - \rho)\varphi_x - \tau_x}{\tau_\pi - \varphi_x}$$

- Conditions for $b < 0$ include $\alpha < 0$ and $\rho > \alpha$. Point: real exchange rates play major role.

Main Result

Result

$$i_t = \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t$$
$$i_t^* = \bar{\tau} + \tau_\pi^* \pi_t^* + \tau_x x_t^*$$

If everything is symmetric, except $\tau_\pi > \tau_\pi^*$, then

1. $E(i_t) < E(i_t^*)$ and $E(\pi_t) < E(\pi_t^*)$

2. If τ_x is large enough,

$$E\big(\mathit{Var}_t\, m_{t+1}\big) > E\big(\mathit{Var}_t\, m_{t+1}^*\big)$$

   Positive expected return on *foreign* currency

3. Bilson-Fama regression coefficient *smaller*.

In Words

Relative to a world with *symmetric* monetary policies, tighter domestic policy makes

- □ Domestic interest rates and inflation unconditionally lower
- □ *Foreign* currency denominated assets unconditionally riskier
- □ The *conditional* foreign currency risk premium more variable

Intuition

Effect of τ_x

$$i_t = \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t$$

$$m_{t+1} = n_{t+1} - \pi_{t+1}$$

- $Cov(\pi_{t+1}, x_{t+1}) < 0$
  - "Inflation risk"
- $Var(m_{t+1}) < Var(n_{t+1})$
  - "Nominal risk less than real risk"

Effect of τ_π

$$i_t = \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t$$

$$m_{t+1} = n_{t+1} - \pi_{t+1}$$

$$\underline{\text{Foreign}}\text{ FX Risk} = \frac{1}{2}\left(\mathit{Var}_t m_{t+1} - \mathit{Var}_t m^*_{t+1}\right)$$

- ☐ Higher τ_π ("tighter policy") *increases* $\mathit{Var}_t m_{t+1}$
- ☐ Makes foreign currency riskier

Punchline

- A procyclical interest rate rule makes the nominal economy "less risky" than the real economy.

- A stronger interest rate reaction to inflation *undoes* this.

  - Domestic state prices become more variable and domestic residents view currency as risky *relative to* foreign residents

- "Weak" interest rate rules make for riskier currencies

- Broadly consistent with carry trade recipients versus funders (*e.g.*, USD vs AUD)

Calibration

Approach

$$\frac{S_{t+1}}{S_t} = \underbrace{\frac{n^*_{t+1}}{n_{t+1}}}_{\text{Real FX Rate}} \frac{e^{-\pi^*_{t+1}}}{e^{-\pi_{t+1}}}$$

- Calibrate n_{t+1}, n^*_{t+1} to consumption, real FX rate
  - Assume countries are symmetric
- Choose Taylor rule parameters to match U.S.-Australia inflation data
- See what exchange rates, interest rates look like

Consumption

Moment	Sample	Theoretical	Parameter
Consumption Growth			
Mean	1.80	1.80	$\theta_x = 0.0015$
Standard Deviation	2.72	2.72	$\theta_u =$ 6.355E-05
Autocorrelation	–	0.00	$\varphi_x = 0$
Cross-Country Correlation	0.35	0.35	$\eta_{x,x^*} = 0.35$
Cross-Country Vol Correlation	–	0.99	$\eta_{u,u^*} = 0.99$
Real Interest Rate			
Mean	0.86	0.86	$\beta = 0.99988$
Standard Deviation	0.97	0.05	$\sigma_u =$ 6.500E-06
Autocorrelation	–	0.987	$\varphi_u = 0.987$
Real Depreciation Rate			
Standard Deviation	11.41	11.41	$\alpha = -2.630$
Bilson-Fama Coefficient	–	-1.66	$\rho = 0.500$

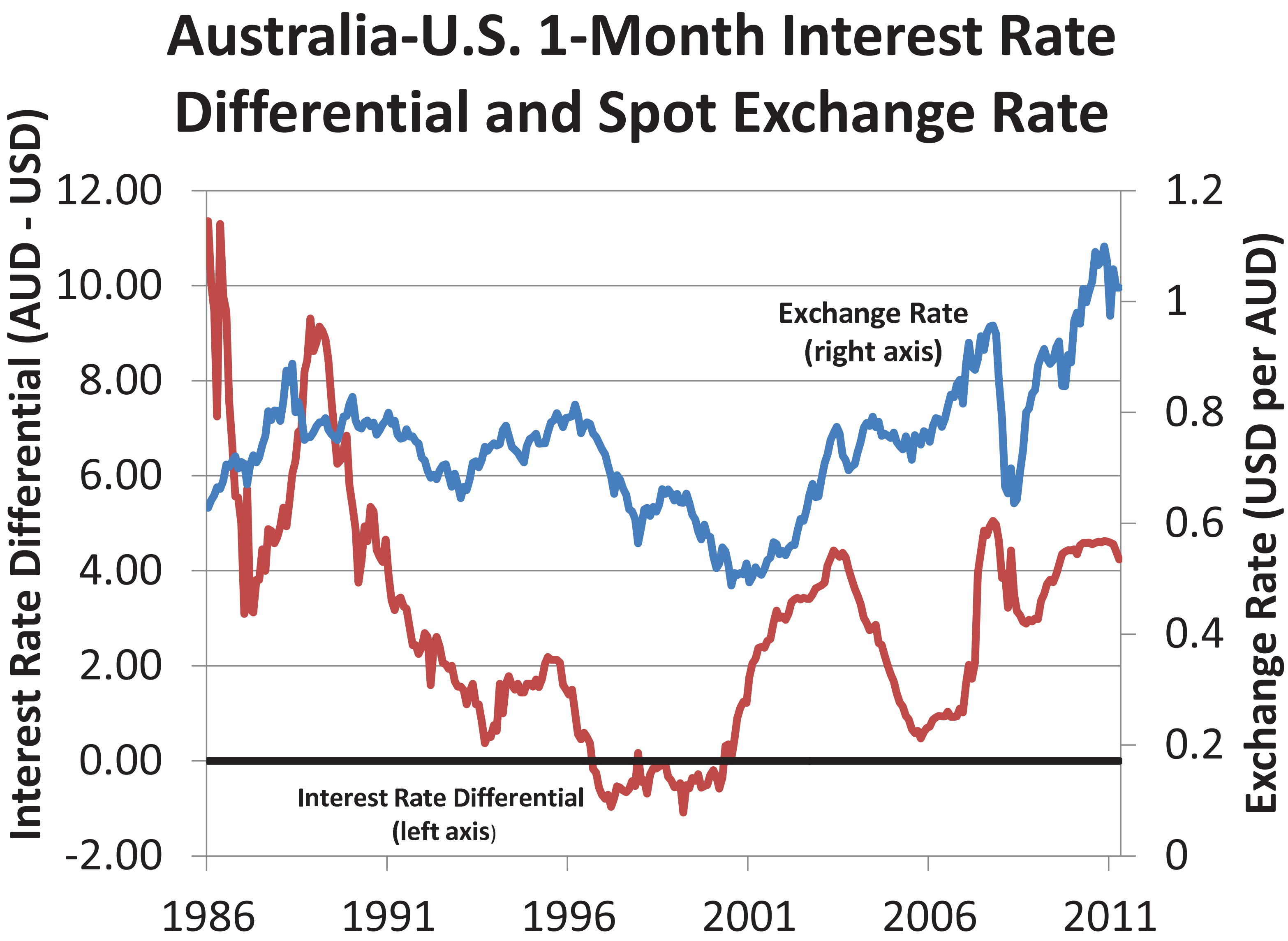
Taylor Rule Coefficients

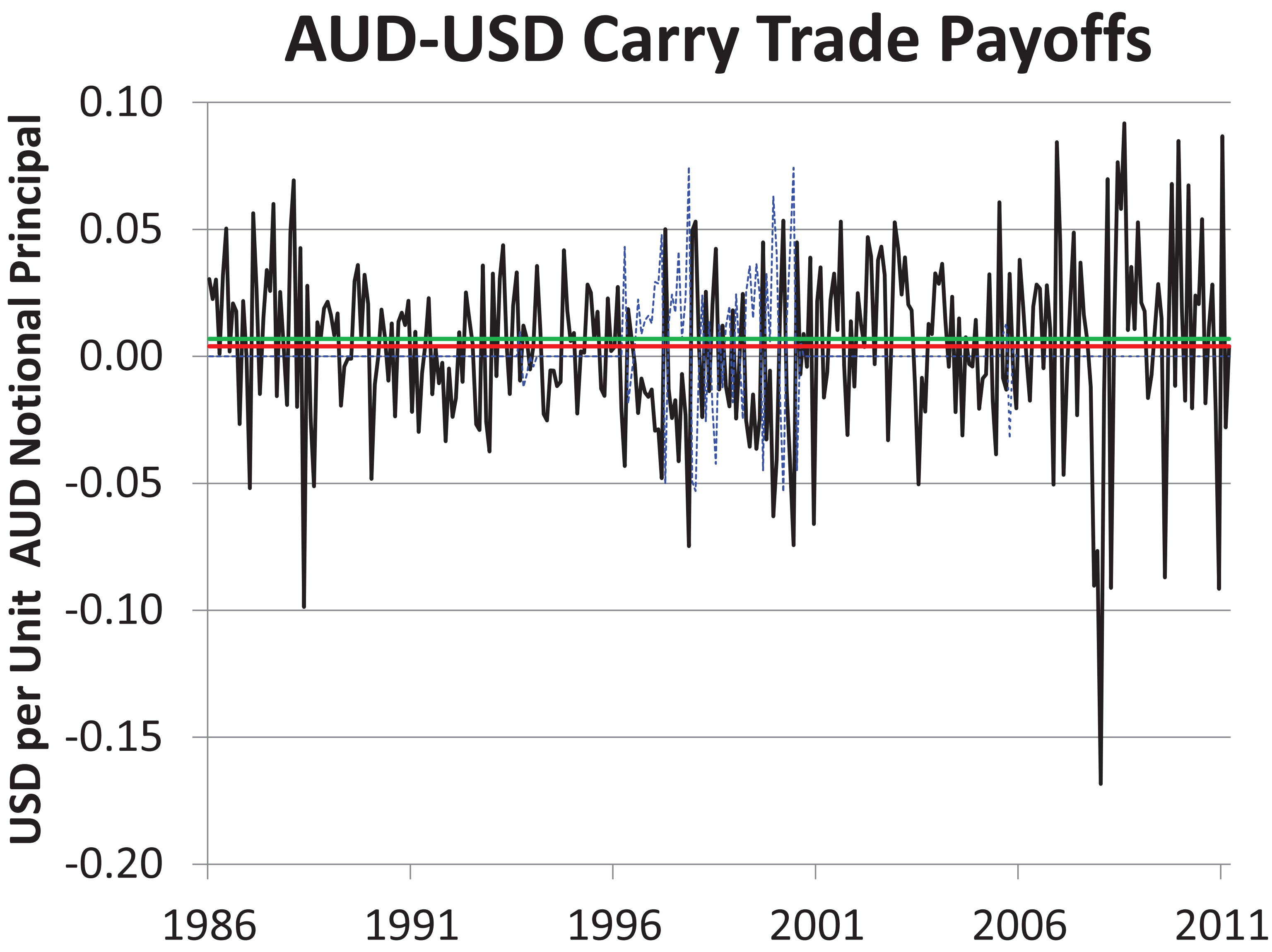
Using U.S.-Australia data:

- τ_x, τ_x^* not separately identified
- Five coefficients uniquely identified by
  - Average inflation: $E(\pi_t)$, $E(\pi_t^*)$
  - Volatility of inflation: $Var(\pi_t)$, $Var(\pi_t^*)$
  - Nominal Bilson-Fama coefficient of -1.00.

	U.S.	Australia
$\bar{\tau}$	-0.0033	-0.0004
τ_x	0.7623	0.7623
τ_π	2.2636	1.0517

U.S. – Australia Data




AUD-USD Carry Trade Payoffs
USD per Unit AUD Notional Principal
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
1986
1991
1996
2001
2006
2011

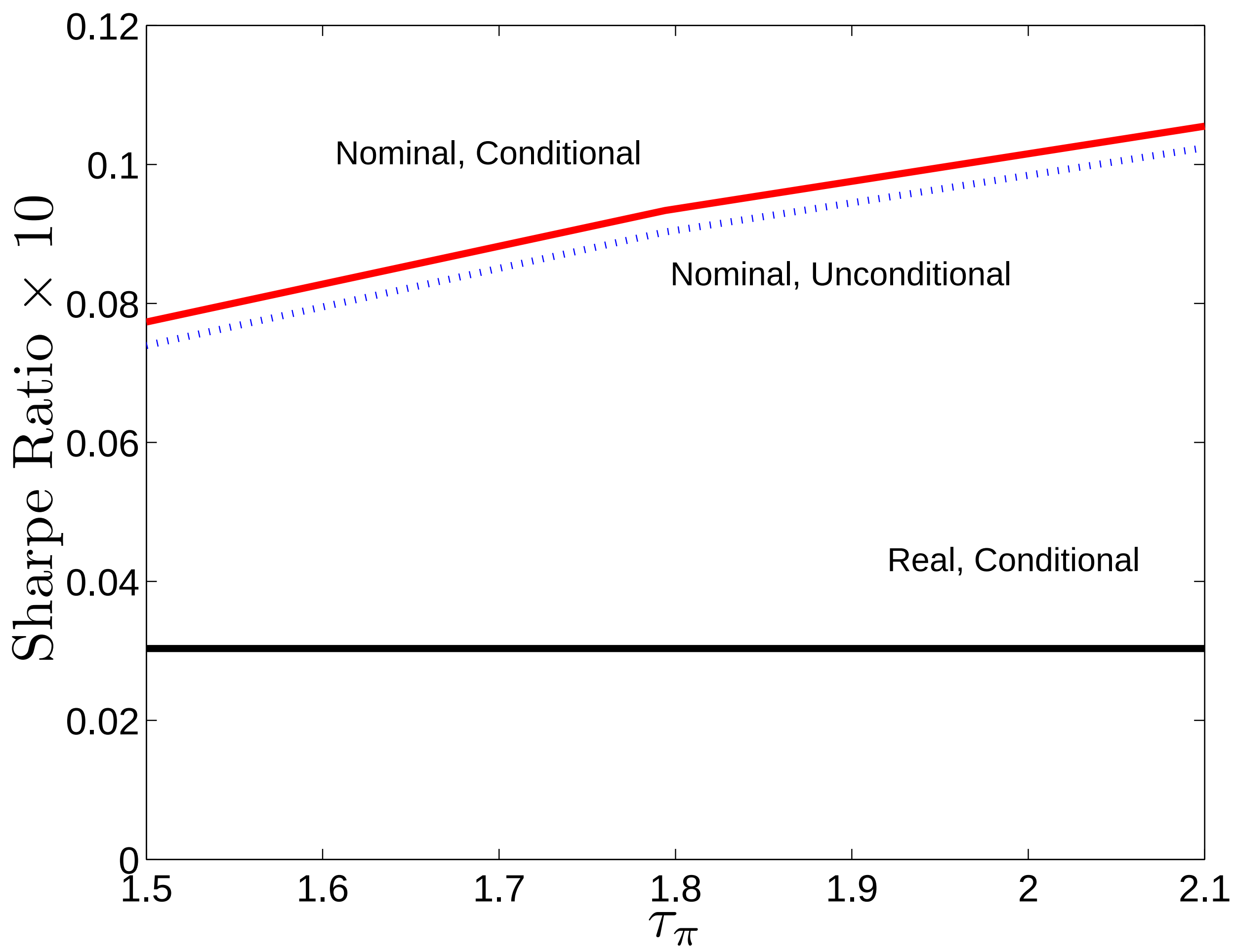
Nominal Results

Moment	Sample	Theoretical
Inflation $(\pi_t\ ,\ \pi_t^*)$		
Domestic, U.S.		
Mean	2.80	2.80
Standard Deviation	0.93	0.93
Autocorrelation	0.84	0.0002
Foreign, Australia		
Mean	3.67	3.67
Standard Deviation	2.01	2.01
Autocorrelation	0.75	0.0001
Nominal Interest Rates $(i_t,\ i_t^*)$		
Domestic, U.S.		
Mean	4.48	3.77
Standard Deviation	2.54	0.032
Autocorrelation	0.99	0.98
Foreign, Australia		
Mean	7.25	4.71
Standard Deviation	3.69	0.024
Autocorrelation	0.99	0.98

(continued)

Moment	Sample	Theoretical
Nominal Depreciation $(\log(m_t^*/m_t))$		
Mean	2.05	-0.87
Standard Deviation	11.43	9.78
Autocorrelation	0.04	$\approx$ 0.0
Nominal Currency Risk Variables		
Nominal Bilson-Fama Coefficient	-1.00	-1.00
Uncond. Risk Premium on AUD, $-E(p_t)$	4.77	0.13
Uncond. Sharpe Ratio	0.41	0.01
Cond. Sharpe Ratio	0.73	0.02

Comparative Statics



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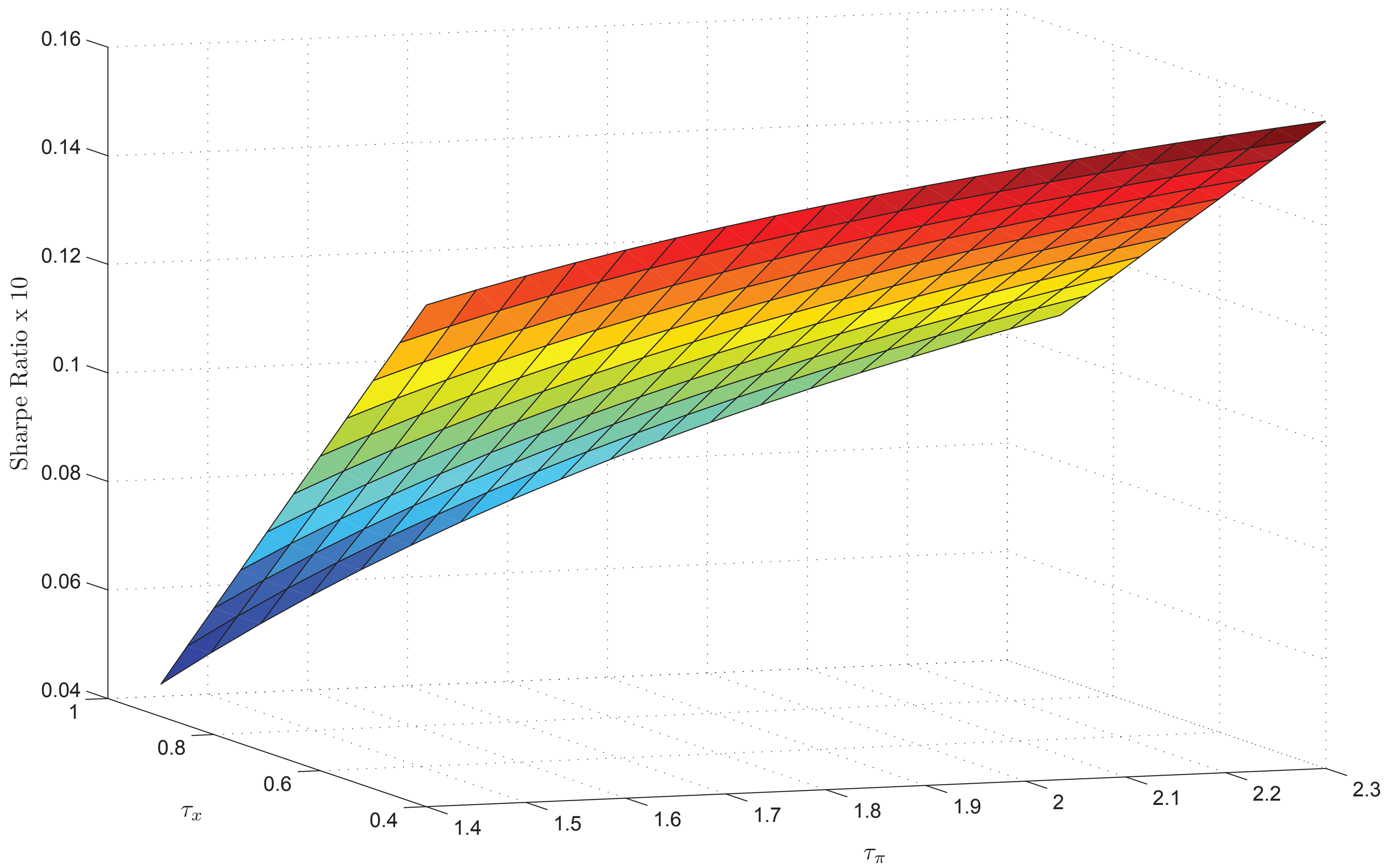

Sharpe Ratio x 10
0.16
0.14
0.12
0.1
0.08
0.06
0.04
1
0.8
0.6
0.4
τ_x
1.4
1.5
1.6
1.7
1.8
1.9
2
2.1
2.2
2.3
τ_π

Enhanced Model

- Incorporate long-run risk in consumption
  - Decouples conditional mean of x_t from other moments
  - Allows for low cross-country consumption correlations and low real exhange rate variability
  - Used previously by Bansal and Shaliastovich (2008)
- Fixes interest rate volatility, but not low FX Sharpe ratios
- Qualitative aspects of Taylor mechanism survive

Conclusions

Taylor Rules and the Carry Trade

Is there a link between monetary policy and the carry trade?

- Asymmetric Taylor rules can generate inflation processes that magnify expected carry trade profits

- Mechanism: Taylor rules affect the volatility of nominal pricing kernels through their effect on inflation.

  – Tight policy country has (i) low volatility in inflation, (ii) high volatility in nominal pricing kernel.

  – Fits some broad facts about carry-trade funding currencies (*e.g.*, U.S., Germany, Switzerland, Japan) versus recipient currencies (*e.g.*, Australia, New Zealand).

Future Work

- Nominal frictions:
  - Link between Taylor rules and real exchange rates
- Richer model of how monetary policy interacts with volatility

"Change of units risk:"

- ☐ Suppose there is a global risk factor that affects international equities, fixed-income, etc.
  - If currency-specific pricing kernels load on it symmetrically it won't matter for exchange rates
  - There must be some asymmetries

- ☐ Asymmetric monetary policy is a plausible, coherent source of asymmetries in pricing kernels
  - Cross-sectional predictions

Last Thoughts

Question: does monetary policy *cause* carry trade profits?

- Consider India in recent years:
  - RBI policy has been to accumulate USD reserves and sterilizes the effect on domestic money supply
  - Short side of the carry trade (Indian rates high, U.S. rates low)
  - Are carry trade *losses* a cost of conducting monetary policy?
  - Is this a good policy?

(continued)

Example of India is pretty explicit. Other centrals banks much less so. However, consider

- U.K. increases rates, Fed lowers rates.
- Open-market operations:
  - Bank of England sells gilts to JPM
  - Fed buys U.S. treasuries from JPM
- JPM is long the carry trade
- *Consolidated* balance sheets of Fed and Bank of England are short.

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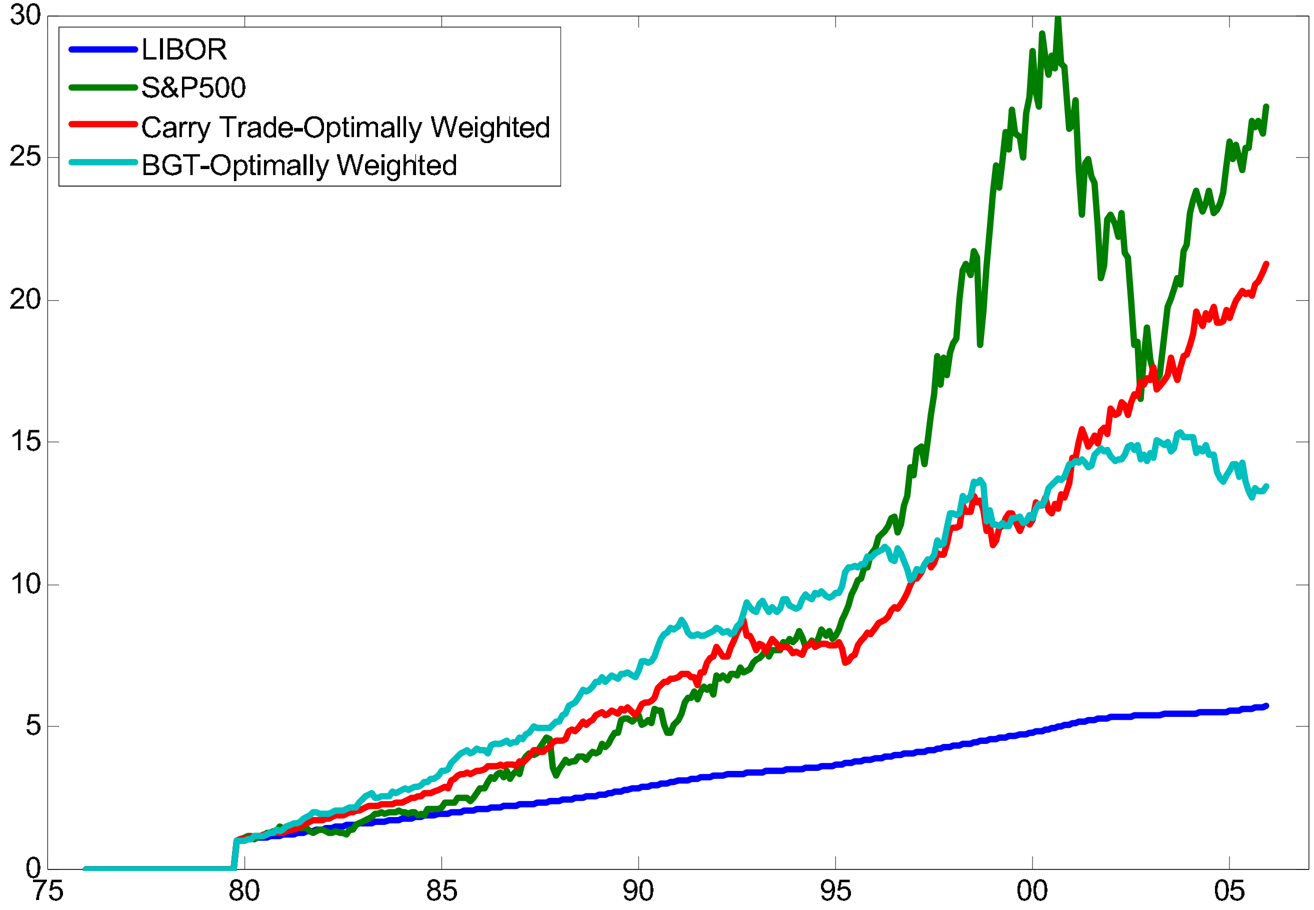
Extra Slides

Carry Trade Profits (Again)

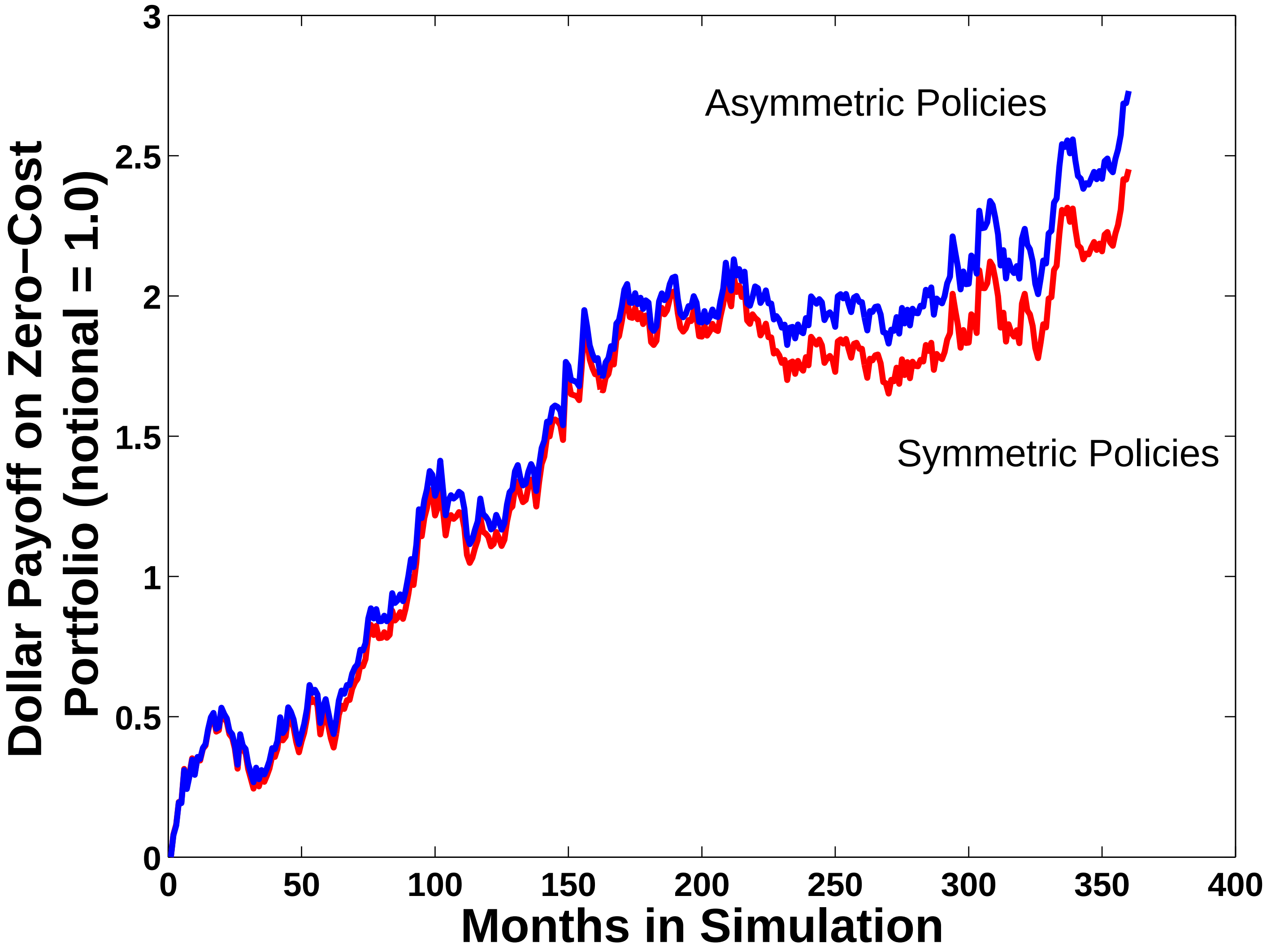
FIGURE 3

Cumulative Realized Nominal (USD) Returns to Currency Speculation (Nov. 1979=1)



From "The Returns to Currency Speculation," by Burnside, Eichenbaum, Kleshchelski and Rebelo, August 2006.

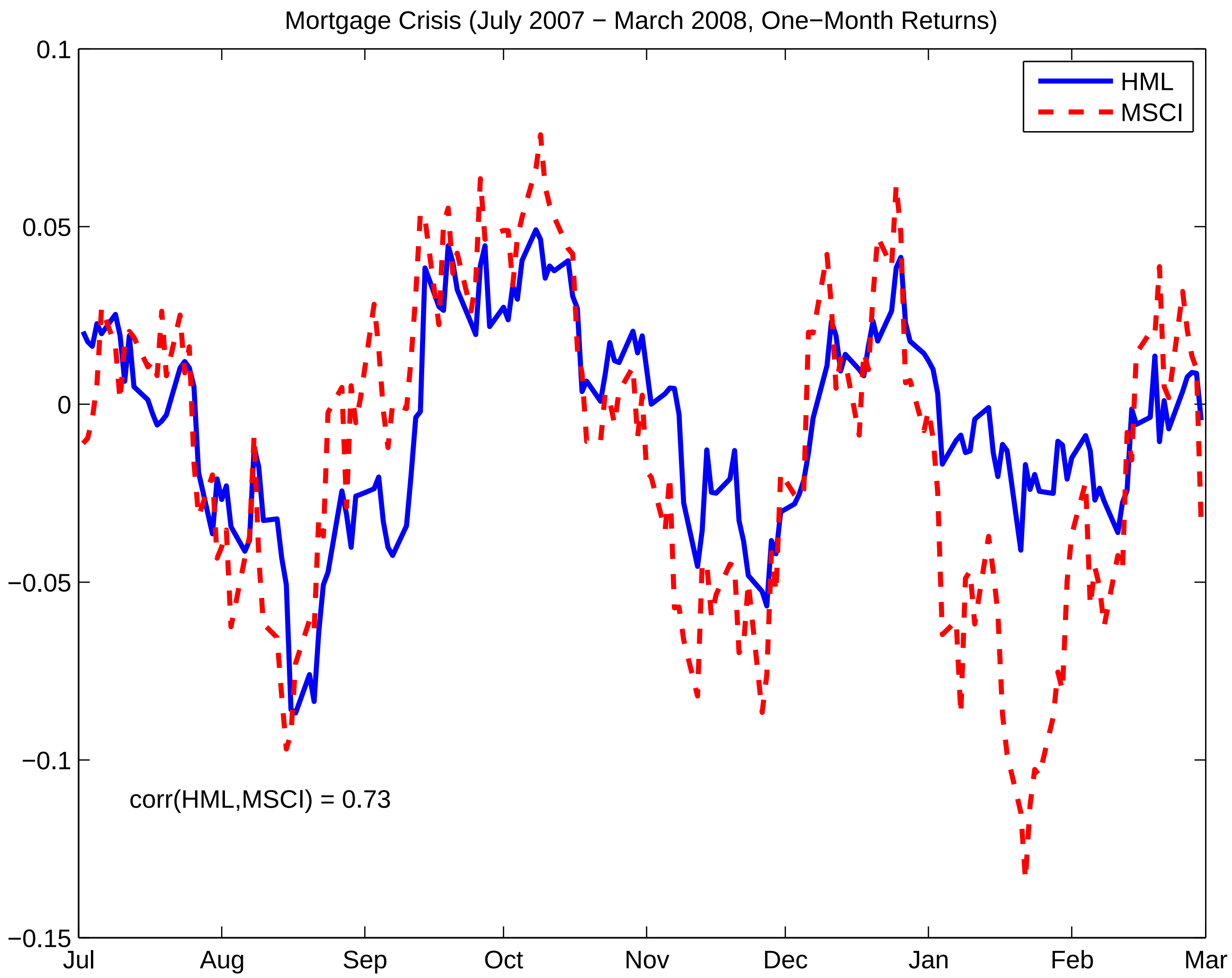
Effect of Relatively Tight Domestic Monetary Policy



Volatility, Skewness

- Recent evidence: volatility is bad news for carry-trade returns
- Lustig-Roussanov-Verdelhan (2010)
  - Correlation of FX returns and equity returns increasing in market volatility
- Brunnermeier-Nagel-Pedersen (2008)
  - FX returns negatively correlated with market volatility
  - Negative skewness of FX returns increasing in $i_t - i_t^*$

FX and Equity During the Crisis



Source: Adrien Verdelhan

Volatility Difference

- These are statements about how FX returns are related to:

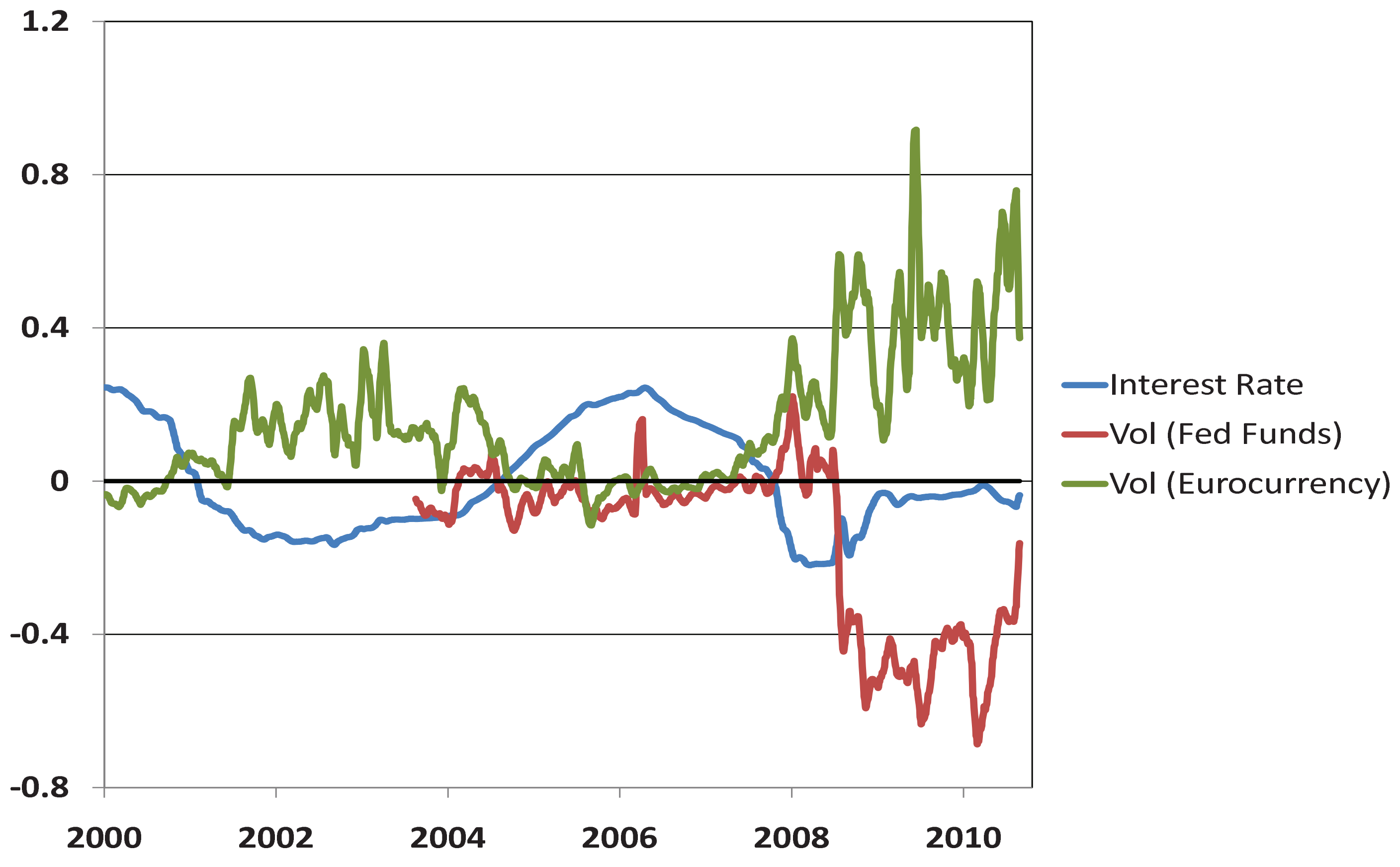
$$\mathit{Var}_t\left(S_{t+1}/S_t\right) = \mathit{Var}_t\left(\log m^*_{t+1} - \log m_{t+1}\right)$$

- But the expected FX return is:

$$E_t\left(f_t - s_{t+1}\right) \quad = \quad \mathit{Var}_t\left(\log m^*_{t+1}\right)/2 - \mathit{Var}_t\left(\log m^*_{t+1}\right)/2$$

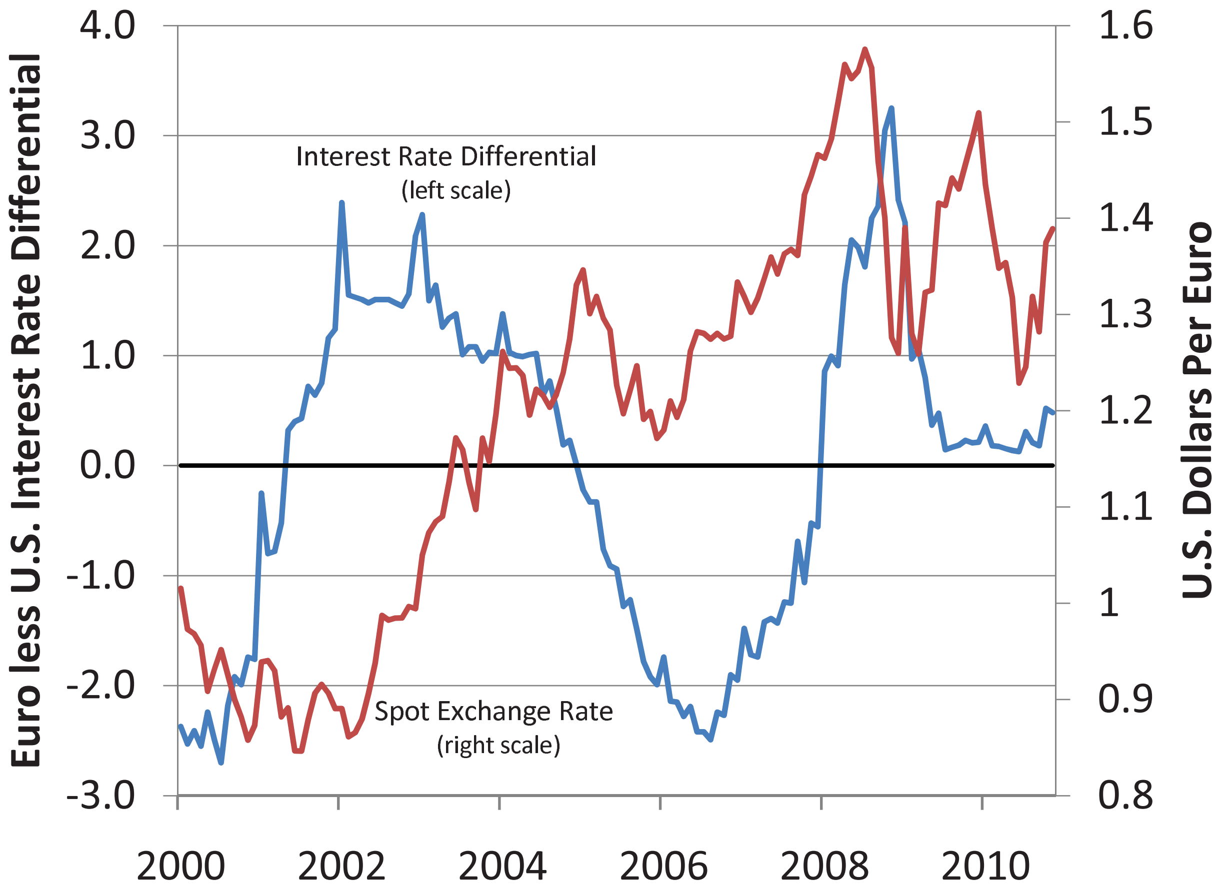
Difference in Interest Rates and Difference in Implied Volatility from Interest-Rate Options

(USD less EUR, Jan 2000 – Nov 2010)



USD/EUR Graph

Eonia Less Fed Funds Interest Rate Spread and USD/EUR Spot Exchange Rate



Changing Units in the Euler Equation

- Pricing kernel (marginal rate of substitution) for *real* units:

$$E_t \, n_{t+1} \left(1 + r_{t+1}^{goods} \right) \quad = \quad 1$$

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- *Nominal* units:

$$E_t \, \underbrace{n_{t+1} \frac{P_t}{P_{t+1}}}_{m_{t+1}} \left(1 + r_{t+1}^{USD}\right) \quad = \quad 1$$

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$$E_t\, \underbrace{n_{t+1}\frac{P_t}{P_{t+1}}}_{m_{t+1}}\left(1 + r_{t+1}^{USD}\right) \quad = \quad 1$$

- *Foreign currency* units:

$$E_t\, \underbrace{n_{t+1}\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}}_{m^*_{t+1}}\left(1 + r_{t+1}^{FX}\right) \quad = \quad 1$$

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$$E_t\, \underbrace{n_{t+1}\frac{P_t}{P_{t+1}}}_{m_{t+1}}\left(1 + r^{USD}_{t+1}\right) \quad = \quad 1$$

- *Foreign currency* units:

$$E_t\, \underbrace{n_{t+1}\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}}_{m^*_{t+1}}\left(1 + r^{FX}_{t+1}\right) \quad = \quad 1$$

- Complete markets implies *pointwise* equality

$$m^*_{t+1} \quad = \quad m_{t+1}\frac{S_{t+1}}{S_t}$$