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"Bank Regulation and Risk Management: An Assessment of the Basel Market Risk Framework"

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An Examination of the Basel Market Risk Framework*

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Abstract

In attempting to promote bank stability, the Basel Committee on Banking Supervision (2006) provides a framework that seeks to control the amount of tail risk that large banks take in their trading books. However, banks around the world suffered sizeable trading losses during the recent crisis. Due to the size and prevalence of losses, a formal examination of whether the Basel framework allows banks to take substantive tail risk in their trading books without a capital requirement penalty is of particular interest. In this paper, we provide such an examination and show that the Basel framework indeed allows banks to do so. Hence, our paper supports the view that the Basel framework did not promote bank stability.

JEL classification: G11; G21; G28: D81

Keywords: bank regulation; bank stability; Basel framework; crisis; tail risk

1. Introduction

In attempting to promote bank stability, the Basel Committee on Banking Supervision (2006) provides a framework that seeks to control the amount of tail risk that large banks take in their trading books.[1] However, in contrast with the Basel framework's intent, banks around the world suffered sizeable trading losses during the recent crisis.[2] The size and prevalence of these losses suggest that the Basel framework is ineffective in controlling tail risk within trading books. Due to the prominence of the losses, a formal examination of whether the Basel framework allows banks to take substantive tail risk in their trading books without a capital requirement penalty is of particular interest. In this paper, we provide such an examination and show that the Basel framework indeed allows banks to do so. Hence, our paper supports the view that the Basel framework did not promote bank stability.[3]

Importantly, the Basel framework requires large banks to use Value-at-Risk (VaR) to measure tail risk in their trading books and to determine the corresponding minimum capital requirements. This framework also requires such banks to use Stress Testing (ST) in supplementing VaR. Not surprisingly, banks utilize VaR and ST to set risk exposure limits (see Basel Committee on Banking Supervision (2005, p. 12) and Committee on the Global Financial System (2005, pp. 1, 15)).

It is worth emphasizing that VaR does not capture the size of losses beyond VaR. Hence, portfolios with relatively small VaRs might have substantive tail risk. An important advantage of Conditional Value-at-Risk (CVaR) over VaR is that it captures the size of losses beyond VaR. Due

[1] While the Basel framework is often considered in the context of commercial banks, regulators have also endorsed its use for investment banks. In 2004, the U.S. Securities and Exchange Commission (SEC) adopted this framework for the capital requirements of certain broker-dealers whose holding companies voluntarily elect to be supervised by the SEC; see Financial Crisis Inquiry Commission (2011, p. 152). Subsequently, Bear Stearns, Lehman Brothers, Merrill Lynch, Goldman Sachs, and Morgan Stanley elected to be supervised by the SEC. However, recognizing that voluntary supervision did not work, the SEC ended it in 2008; see Financial Crisis Inquiry Commission (2011, p. 154).

[2] For example, in 2008, the trading losses of Bank of America, Royal Bank of Scotland, and UBS reached, respectively, USD 5.9 billion, GBP 8.5 billion, and CHF 25.8 billion. For a discussion of the causes of the crisis, see Kane (2009), Caprio, Demirgüç-Kunt, and Kane (2010), Dewatripont, Rochet, and Tirole (2010, Ch. 2), and Levine (2010a).

[3] Caprio, Demirgüç-Kunt, and Kane (2010), Dewatripont, Rochet, and Tirole (2010, Ch. 2-4), and Levine (2010b) provide recommendations on how to restructure bank regulation in light of the crisis.

to this advantage, our paper uses CVaR in measuring tail risk.[4]

We examine the effectiveness of three sets of constraints in controlling tail risk: (1) a VaR constraint; (2) ST constraints; and (3) VaR and ST constraints. Within the context of the Basel framework, our results are pertinent in two cases of interest.[5] The first one occurs when both types of constraints (i.e., VaR and ST) are used in seeking to control tail risk but just one type binds. In this case, the results for one of the two sets involving only one type of constraint are applicable. The second one occurs when both types of constraints are used in seeking to control tail risk and bind. In this case, our results for the set of constraints involving both types are applicable.

The Basel Committee on Banking Supervision (2006, p. 157) notes that market risk exposures subject to VaR-based minimum capital requirements include "the risks pertaining to interest rate related instruments and equities in the trading book." Accordingly, we examine the effectiveness of each set of constraints by solving a plausible problem of wealth allocation among Treasury bills, government bonds, corporate bonds, and the six size/book-to-market Fama-French equity portfolios. In doing so, we utilize historical simulation to estimate VaR, CVaR, and losses in ST events given its popularity among banks (see, e.g., Pérignon and Smith (2010)). Also, we consider two ST events: (1) the crash in the U.S. stock market of 1987; and (2) the terrorist attacks in the U.S. of September 2001. Our motivation for using these events is two-fold. First, they are the two most frequently used historical ST events according to a survey of banks and securities firms conducted by the Committee on the Global Financial System (2005, Table 12). Second, the Committee on the Global Financial System (2005, p. 10) notes that participants in this survey focus

[4] Also, VaR fails to possess the subadditivity property (i.e., two assets in combination can have a VaR greater than the sum of their individual VaRs), whereas CVaR does possess it; see Artzner, Delbaen, Eber, and Heath (1999). However, Garcia, Renault, and Tsafack (2007) argue that the cases when VaR is not subadditive are rare.

[5] Our analysis is also pertinent in contexts beyond the Basel framework. For example, it is applicable in a risk management context where just one type of constraint (i.e., VaR or ST) is used in seeking to control tail risk and binds. It is also applicable to any organization (such as hedge funds) that manages the inherent tail risk present in a trading book with either VaR or ST constraints. Liang and Park (2007, 2010) provide empirical support to the use of CVaR as a measure of tail risk for hedge funds.

on major historical events (the average number of reported historical ST events across participants is 3.3). For robustness checks, we consider the 1997 Asian and 1998 Russian crises as additional ST events and the ten size Fama-French equity portfolios.

First, we examine the effectiveness of a VaR constraint in controlling tail risk. We find that when using a VaR bound that does not depend on the required expected return ('fixed bound'), there is no value for the bound such that the constraint precludes all portfolios with substantive tail risk from being selected while allowing the selection of portfolios with a wide range of expected returns. We find next that the use of an appropriately chosen bound that depends on the required expected return ('variable bound') has advantages over the use of one that does not in that: (i) it is more effective in precluding the selection of portfolios with substantive tail risk; and (ii) it allows the selection of portfolios with a notably wider range of expected returns. However, the constraint still allows the selection of portfolios with substantive tail risk.

Second, we examine the effectiveness of ST constraints in controlling tail risk. Like a VaR constraint, we find that the ST constraints allow the selection of portfolios with substantive tail risk regardless of whether fixed or variable bounds are used. Unlike a VaR constraint, however, the benefits of using variable-bound ST constraints (relative to fixed-bound ST constraints) are small. Moreover, the ST constraints are even less effective in controlling tail risk than the VaR constraint.

Third, we examine the joint effectiveness of VaR and ST constraints in controlling tail risk. As in the case where only a VaR constraint is imposed, there are notable benefits to using variable bounds (relative to using fixed bounds). However, while the joint use of VaR and ST constraints is beneficial (relative to using just one type of constraint), they continue to allow the selection of portfolios with substantive tail risk.

We take the view that an unqualified positive assessment of the Basel framework requires that the joint use of VaR and ST constraints is fully effective in controlling tail risk within simple settings that are consistent with common practice among banks. Therefore, our results on the ineffectiveness of the constraints in controlling tail risk within these settings raise doubts about the adequacy of the Basel framework in preventing banks from taking substantive tail risk in their trading books.[6] Our robustness checks indicate that these doubts remain even if we increase the complexity of our setting by considering either a larger number of assets, or more ST events, or both. Recognizing that bank trading activities have experienced tremendous growth before the recent crisis, our paper supports the view that the Basel framework did not promote bank stability.[7]

There is an extensive literature on the impact of bank capital regulation on stability (see Barth, Caprio, and Levine (2008) and Freixas and Rochet (2008, Ch. 9) for a review). For example, a number of papers theoretically show that bank capital regulation might increase the risk-taking activities of banks (see, e.g., Koehn and Santomero (1980), Kim and Santomero (1988), and Rochet (1992)). In a related paper, Barth, Caprio, Levine (2004) do not find robust empirical evidence that stringent capital requirements reduce these activities. Lucas (2001) and Kane (2006) explicitly criticize the Basel framework by showing that it provides incentives for banks to underreport their VaR-based estimates of capital requirements.[8] Our work contributes to this literature by showing that the joint use of VaR and ST in accordance to the Basel framework and common practice among banks allows them to take substantive tail risk in their trading books.

The paper proceeds as follows. Section 2 describes the model. Section 3 examines the effectiveness of VaR and ST constraints in controlling tail risk, and Section 4 concludes.

[6] Since there are alternative approaches to implement VaR and ST, our results should not be interpreted as a general criticism to the joint use of VaR and ST for controlling tail risk. Our main point is that there are important shortcomings in doing so if VaR and ST are implemented in accordance to the Basel framework and common practice among banks.

[7] For example, the assets in the trading books of U.S. commercial banks increased almost thirty-fold between 1988 and 2008, while in comparison their overall total assets increased about three-fold; see Federal Reserve Statistical Release (2008).

[8] The Financial Crisis Inquiry Commission (2011) recognizes that the use of the Basel framework might have lead to lower capital requirements. For example, it notes an SEC estimate indicating that the use of VaR would reduce the average capital charges of brokers-dealers by 40%; see Financial Crisis Inquiry Commission (2011, p. 152).

2. The Model

Suppose that uncertainty is described by S states ($s = 1, ..., S$). Let $p_s > 0$ be the probability of state s. There are J risky assets ($j = 1, ..., J$) and a risk-free asset ($j = J + 1$). Asset returns are given by a $(J + 1) \times S$ matrix $\boldsymbol{R}$. The return of asset j in state s is R_{js}.

Let $\mathbf{1}$ denote the $(J + 1) \times 1$ vector $[1 \quad \cdots \quad 1]^\top$. A portfolio is a $(J + 1) \times 1$ vector $\boldsymbol{w} = [w_1 \quad \cdots \quad w_{J+1}]^\top$ with $\boldsymbol{w}^\top \mathbf{1} = 1$. Here, w_j represents the weight of asset j. Note that a positive (negative) weight represents a long (short) position. However, the weight of each asset is constrained to be between some lower bound $l < 0$ and some upper bound $u > 1$.

2.1. VaR

In defining VaR, we follow Rockafellar and Uryasev (2002, Proposition 8). Fix a confidence level $\alpha \in (1/2, 1)$. Let $\widetilde{R}_{\boldsymbol{w}}$ denote the random return of portfolio $\boldsymbol{w}$. Let $z_{1,\boldsymbol{w}} < z_{2,\boldsymbol{w}} < \cdots < z_{N_{\boldsymbol{w}},\boldsymbol{w}}$ denote the ordered values that $\widetilde{z}_{\boldsymbol{w}} \equiv -\widetilde{R}_{\boldsymbol{w}}$ can take where $N_{\boldsymbol{w}} \leq S$ is the number of these values. Define n_α as the unique index number with:

$$\sum_{n=1}^{n_\alpha} p_{n,\boldsymbol{w}} \geq \alpha > \sum_{n=1}^{n_\alpha - 1} p_{n,\boldsymbol{w}}, \tag{1}$$

where $p_{n,\boldsymbol{w}} \equiv P[\widetilde{z}_{\boldsymbol{w}} = z_{n,\boldsymbol{w}}]$. Note that while n_α depends on $\boldsymbol{w}$, we write 'n_α' instead of '$n_{\alpha,\boldsymbol{w}}$' for notational simplicity. Portfolio $\boldsymbol{w}$'s *VaR at the* $100\alpha\%$ *confidence level* is given by:

$$V_{\alpha,\boldsymbol{w}} \equiv z_{n_\alpha,\boldsymbol{w}}. \tag{2}$$

Using Eqs. (1) and (2), we have:

$$P[\widetilde{R}_{\boldsymbol{w}} \geq -V_{\alpha,\boldsymbol{w}}] = P[\widetilde{z}_{\boldsymbol{w}} \leq z_{n_\alpha,\boldsymbol{w}}] \geq \alpha, \tag{3}$$

$$P[\widetilde{R}_{\boldsymbol{w}} > -V_{\alpha,\boldsymbol{w}}] = P[\widetilde{z}_{\boldsymbol{w}} < z_{n_\alpha,\boldsymbol{w}}] < \alpha. \tag{4}$$

As Eqs. (3) and (4) show, this definition of VaR is based on the upper quantile.

Pérignon and Smith (2010) find that 73% of the banks who report their VaR estimation methodologies use historical simulation (see also Committee on the Global Financial System (2005) and Pritsker (2006)).[9] Accordingly, we use historical simulation to estimate VaR for all portfolios.

2.2. CVaR

In defining CVaR, we follow Rockafellar and Uryasev (2002, Proposition 8). Portfolio $\boldsymbol{w}$'s *CVaR at the* $100\alpha\%$ *confidence level* is given by:

$$C_{\alpha,\boldsymbol{w}} \equiv \frac{1}{1-\alpha}\left[\left(\sum_{n=1}^{n_\alpha} p_{n,\boldsymbol{w}} - \alpha\right) z_{n_\alpha,\boldsymbol{w}} + \sum_{n=n_\alpha+1}^{N_{\boldsymbol{w}}} p_{n,\boldsymbol{w}} z_{n,\boldsymbol{w}}\right]. \tag{5}$$

Eqs. (2) and (5) imply that: (a) $C_{\alpha,\boldsymbol{w}} \geq V_{\alpha,\boldsymbol{w}}$; and (b) $C_{\alpha,\boldsymbol{w}} > V_{\alpha,\boldsymbol{w}}$ if $P[\widetilde{R}_{\boldsymbol{w}} < -V_{\alpha,\boldsymbol{w}}] > 0$. As in the case of VaR, we use historical simulation to estimate CVaR for all portfolios.

2.3. Loss in an ST event

While in practice there are many tools to stress test a portfolio (see, e.g., Committee on the Global Financial System (2005)), we use scenario analysis to set risk exposure limits. In doing so, we focus on the case where the scenarios that are analyzed are based on historical events.[10] Two historical events are considered: the crash in the U.S. stock market of 1987 (hereafter, 'crash of 87') and the terrorist attacks in the U.S. of September 2001 (hereafter, '9/11'). As noted earlier, our motivation for using these events is two-fold. First, they are the two most frequently used historical ST events according to a survey of banks and securities firms conducted by the Committee on the Global Financial System (2005, Table 12). Second, the Committee on the Global Financial System

[9] While the variance-covariance approach is also used in practice to estimate VaR and CVaR, there is a one-to-one correspondence between VaR and CVaR when asset returns have a normal distribution; see Alexander and Baptista (2004). This correspondence does not occur in our paper since we use historical simulation and asset returns are assumed to have a discrete distribution with finitely many jumps.

[10] In practice, the scenarios that are analyzed can also be based on hypothetical events such as the U.S. economic outlook and oil price scenarios; see Committee on the Global Financial System (2005, Table 3a). However, there is an important reason for us to solely examine historical events. While there is a clear-cut way of capturing the manner in which historical events are used in practice, there is no general way of doing so for hypothetical events. For example, while asset returns during an historical event are precisely observed, we do not know exactly how users of hypothetical events relate them to asset returns (for a discussion of the subjectivity of such events, see, e.g., Hull (2007, pp. 212–213)). Hence, we focus on historical events.

(2005, p. 10) notes that participants in this survey focus on major historical events (since the average number of reported historical ST events across participants is 3.3, we later consider the 1997 Asian and 1998 Russian crises as additional ST events).

We refer to the crash of 87 and 9/11 as ST events 1 and 2, respectively. Let $\boldsymbol{R}_k$ be the $(J+1)\times 1$ vector of asset returns in ST event k. Portfolio $\boldsymbol{w}$'s *loss in ST event k* is given by

$$T_{k,\boldsymbol{w}} = -\boldsymbol{w}^{\top}\boldsymbol{R}_k. \tag{6}$$

Eqs. (2) and (6) imply that $T_{k,\boldsymbol{w}}$ can be smaller than, equal to, or larger than $V_{\alpha,\boldsymbol{w}}$. Similarly, Eqs. (5) and (6) imply that $T_{k,\boldsymbol{w}}$ can be smaller than, equal to, or larger than $C_{\alpha,\boldsymbol{w}}$.

The calculation of a portfolio's loss in an ST event differs from that of its VaR and CVaR in two respects. First, while ST uses asset returns during a fixed (historical) event, VaR and CVaR use asset returns in a set of states that depends on the confidence level and the portfolio. Second, while the period of time used to determine the loss in an ST event depends on the event (one day for the crash of 87, and several days for 9/11), the period of time used to compute VaR and CVaR is fixed (e.g., several years of monthly data). Due to these two differences, portfolios with similar losses in an ST event may have notably different VaRs and CVaRs.[11]

2.4. Using VaR and ST constraints to control tail risk

Given a confidence level α, we consider the following VaR constraint:

$$V_{\alpha,\boldsymbol{w}} \leq V, \tag{7}$$

where V is the VaR bound. The set of portfolios that meet a VaR constraint is typically smaller in cases where either its confidence level is higher or its bound is smaller. In these cases, the constraint

[11] As we explain shortly, the distribution of monthly returns used in our paper to estimate VaR and CVaR incorporates the effect of the crash of 87 and 9/11 in the returns of, respectively, October 1987 and September 2001. If this effect were not incorporated in such a distribution, then the difference between the VaRs of the aforementioned portfolios (i.e., portfolios with similar losses in an ST event) and the difference between their CVaRs could be even larger.

can be thought of as being 'tightened.' As noted earlier, banks often use VaR constraints to set risk exposure limits.

Similarly, we consider the following ST constraints:

$$T_{1,\boldsymbol{w}} \leq T_1, \tag{8}$$

$$T_{2,\boldsymbol{w}} \leq T_2, \tag{9}$$

where T_1 and T_2 denote the ST bounds associated with, respectively, ST events 1 and 2 (i.e., the crash of 87 and 9/11, respectively). The set of portfolios that meet an ST constraint is typically smaller in cases where its ST bound is smaller. In these cases, the constraint can be thought of as being 'tightened.' As noted earlier, banks often use ST constraints to set risk exposure limits.

A number of papers suggest the use of a mean-CVaR model to control tail risk (see, e.g., Alexander and Baptista (2004) and Bertsimas, Lauprete, and Samarov (2004)). Hence, the question of whether the use of VaR and ST constraints lead to the selection of portfolios with small efficiency losses relative to the mean-CVaR frontier is of particular interest.[12] A portfolio belongs to the *mean-CVaR frontier* if there is no portfolio with the same expected return and a smaller CVaR. We measure a portfolio's *efficiency loss* by the difference between: (i) its CVaR and (ii) the CVaR of the portfolio on the mean-CVaR frontier with the same expected return. When tail risk is measured by CVaR, a portfolio's efficiency loss represents the increase in tail risk arising from selecting it instead of the portfolio with the same expected return that has minimum tail risk.

2.5. Methodology

Fig. 1 summarizes the methodology used to examine the effectiveness of three sets of constraints in controlling tail risk: (1) a VaR constraint; (2) ST constraints; and (3) VaR and ST constraints.

[12] Previous work examines the impact of adding either VaR or ST constraints to the mean-variance model; see Alexander and Baptista (2004, 2009). Importantly, our paper differs from previous work in two respects. First, we consider the joint use of VaR and ST constraints, whereas previous work focuses on the use of just one type of constraint. Second, while we impose no assumption on the portfolio selection model that is used in the presence of the constraints, previous work focuses on portfolio selection within the mean-variance model.

For each set of constraints, we proceed as follows. In Step 1, a confidence level α and constraint bounds are chosen. Based on the requirements of the Basel framework (see Basel Committee on Banking Supervision (2006, p. 195)), we let $\alpha = 99\%$.[13] Also, we constrain the weight of each asset to be between lower bound $l = -50\%$ and upper bound $u = 150\%$.[14] Hence, short selling is allowed.[15] Letting a portfolio's leverage ratio be defined as the sum of its positive weights, these bounds allow the selection of portfolios with a maximum leverage ratio of 400%.[16] We should emphasize that the possibility of short selling and the range of leverage ratios allowed by the asset weight constraints are realistic in the context of the trading portfolios of large banks. Consider U.S. depository institutions with total assets of $100 billion or more and positive trading assets as of December 31, 2009, which amounts to seventeen institutions (see Federal Deposit Insurance Corporation (2010)). First, the trading portfolios of sixteen out of these seventeen institutions involve short selling. Second, of the fourteen institutions for which leverage ratios are well-defined, twelve have leverage ratios less than 400%.[17] Third, the average leverage ratio across these fourteen institutions is 219%.

While the minimum required expected return $\underline{E}$ is assumed to be the risk-free rate, the maximum feasible expected return $\overline{E}$ is found in Step 2. Step 3 uses the values of $\underline{E}$ and $\overline{E}$ to calculate $\delta \equiv (\overline{E} - \underline{E})/100$. The value of δ is then used in Step 4 to create a grid of 101 expected returns $\{E_i\}_{i=0}^{100}$ that range from $E_0 = \underline{E}$ to $E_{100} = \overline{E}$ in return increments of δ.

In Step 5, the *maximum efficiency loss* M_i is determined for each E_i. Fig. 2 illustrates how M_i is determined. The curve represents the mean-CVaR frontier. Point p_{min} represents the portfolio

[13] Similar results, available upon request, are obtained when $\alpha = 95\%$.

[14] Similar results, available upon request, are obtained when $l = -100\%$ and $u = 200\%$.

[15] The results when short selling is disallowed are available upon request. They differ from those when it is allowed in that the VaR and ST constraints are less ineffective in controlling tail risk. Nevertheless, the constraints still allow the selection of portfolios with substantive tail risk when either just one type of constraint or fixed bounds are used.

[16] As we explain shortly, we consider a wealth allocation problem among nine assets. Hence, since portfolio weights sum to one, a leverage ratio of 400% is achieved by any portfolio with the following weights: (a) 150% in each of two assets; (b) 100% in one asset; and (c) -50% in each of the remaining six assets.

[17] The difference between trading assets and trading liabilities is negative for three institutions. Hence, their leverage ratios are not well-defined.

that has an expected return of E_i and minimum CVaR, denoted by C_{min} (i.e., the portfolio on this frontier with an expected return of E_i). Point p_{max} represents the portfolio that has the same expected return, satisfies the VaR and/or ST constraints, and has maximum CVaR, denoted by C_{max}. Since $M_i = C_{max} - C_{min}$, M_i measures the maximum increase in CVaR allowed by the VaR and/or ST constraints given an expected return of E_i.

In Step 6, the average of $\{M_i\}_{i=0}^{100}$, referred to as the *average efficiency loss*, is determined. The single largest maximum efficiency loss, referred to as the *largest efficiency loss*, is also determined. Lastly, we compute average and largest *relative* efficiency losses where a portfolio's *relative efficiency loss* is defined as the ratio between: (1) its efficiency loss; and (2) the CVaR of the portfolio on the mean-CVaR frontier with the same expected return.[18] Subsequent analysis is based on the values of $\{M_i\}_{i=0}^{100}$, average and largest efficiency losses, and average and largest relative efficiency losses associated with the three sets of constraints.[19]

An examination of maximum efficiency losses captures the idea of being agnostic regarding the portfolio selection model that is used in the presence of VaR and ST constraints. The motivation for this idea is two-fold. First, we are interested in exploring the effectiveness of such constraints in controlling CVaR without making any assumption on the portfolio selection model that is used in the presence of the constraints. If a set of constraints precludes all portfolios with substantive efficiency losses from being selected, then it is effective in controlling CVaR. Thus, any portfolio that meets the constraints, no matter how selected (e.g., using a mean-variance model), will have tail risk that is similar in magnitude to that of the portfolio with the same expected return that

[18] There are two difficulties in using this measure of relative efficiency loss. First, portfolios on the mean-CVaR frontier with expected returns close to the risk-free return have negative CVaRs and thus their relative efficiency losses are negative. Second, even when the CVaRs of certain portfolios on the mean-CVaR frontier are positive, they could be arbitrarily close to zero, resulting in arbitrarily large relative efficiency losses. In order to circumvent these two difficulties, we compute the average and largest relative efficiency losses by solely using levels of expected return for which the correspondent portfolios on the mean-CVaR frontier have CVaRs greater than 1%. Note that such losses are larger if we include levels of expected return for which the correspondent portfolios on the mean-CVaR frontier have smaller (but positive) CVaRs.

[19] As a robustness check, we also compute the losses when only the interquartile range of expected returns $\{E_i\}_{i=25}^{75}$ is considered. The results, available upon request, are similar to those reported when using the entire range of expected returns.

minimizes CVaR. However, if the set of constraints allows the selection of portfolios with substantive efficiency losses, then it is not effectively controlling CVaR.

Second, while the use of VaR and ST constraints by large banks is apparent, we do not know the exact models that they utilize for portfolio selection. Indeed, trading book managers might have incentives to take on as much tail risk as possible (subject to existing risk constraints); see, e.g., Lucas (2001) and Hull (2007, p. 198). For example, generous deposit insurance and compensation schemes might lead such managers to take excessive risks; see, e.g., Kane (1989) and Cai, Cherny, and Milbourn (2010).[20]

2.6. Optimization inputs

In assessing the effectiveness of VaR and ST constraints in controlling tail risk, we consider a simple yet plausible problem of wealth allocation among the following assets: (i) Treasury bills (assumed to be risk-free), (ii) government bonds, (iii) corporate bonds, and (iv) six size/book-to-market Fama-French equity portfolios.[21] Monthly returns on Treasury bills and Fama-French portfolios are obtained from Kenneth French's website. Monthly returns on bonds are obtained from Bloomberg by using the Merrill Lynch government and corporate bond master indices.

Our results use returns from the sample period 1982–2006, which precedes the recent crisis. Nevertheless, similar results are obtained when using the period 1982–2009, which includes the crisis (see Section 3.4). The first four rows of Table 1 present summary statistics expressed as percentages on the monthly asset returns during the sample period.[22] Four facts are worth noting. First, average returns for stocks are larger than those for bonds with a single exception: the average

[20] Such schemes might induce excessive risk-taking since the distribution of profits and losses among managers and others is possibly asymmetric. Whereas extreme trading profits are beneficial from the perspective of managers, extreme trading losses are costly from the perspective of, for example, taxpayers and debt holders.

[21] Similar results, available upon request, are obtained if (1) Treasury bills, or (2) bonds, or (3) both Treasury bills and bonds are removed from consideration.

[22] Note that we not consider estimation risk. For work that recognizes estimation risk in VaR and CVaR, see, for example, the November 2000 issue of the *Journal of Empirical Finance*, the July 2002 issue of the *Journal of Banking and Finance*, and Pritsker (2006).

return on the small-size/low-book-to-market-ratio Fama-French portfolio is slightly smaller than that on corporate bonds. Second, standard deviations for stocks are much larger than those for bonds.[23] Third, VaRs and CVaRs for stocks are much larger than those for bonds. Fourth, for any given risky asset, CVaRs are larger than VaRs.

The last two rows show that government bonds have positive returns in the ST events, whereas corporate bonds have negative returns.[24] Also, all Fama-French portfolios have notably negative returns in these events.

3. Results

This section presents our results.

3.1. VaR constraint

We begin by analyzing the effectiveness of a VaR constraint in controlling tail risk. Consider the case when the bound does not depend on the required expected return, which we refer to as a *fixed* bound. Table 2 shows the results, expressed in percentages (subsequent tables and figures also use percentages). In the first two columns, V is assumed to be either 4% or 8%.[25] The first two rows report average and largest efficiency losses. Two results can be seen. First, losses are sizeable for both values of V. Second, losses are smaller when the lower value of V is used. The next two rows indicate that average and largest relative efficiency losses are also sizeable, and again are smaller when the lower value of V is used.

The last row shows that using the lower value of the VaR bound (and thus a tighter constraint) precludes the selection of more portfolios with large expected returns. Hence, there exists no

[23]Since we assume that Treasury bills are risk-free, the standard deviation of the return on Treasury bills is reported as zero. In each state, Treasury bills are assumed to have a return equal to the average monthly return on Treasury bills during the sample period.

[24]For simplicity, we assume that the return on the risk-free security in an ST event is equal to the product of: (i) the duration of the event expressed as a fraction of a month times (ii) the average risk-free return.

[25]While these (and subsequent) fixed bound values are utilized for illustrative purposes, similar results have been obtained when other reasonable values are used.

value for the fixed bound such that a VaR constraint precludes the selection of all portfolios with substantive efficiency losses while allowing the selection of portfolios with large expected returns. Accordingly, we examine next the case of a bound that depends on the required expected return, which we refer to as a *variable* bound. This bound is set in such a way that it is typically increasing in the required expected return. Recognizing that a higher expected return is generally associated with more risk, the use of a variable bound is also motivated by the practical plausibility of senior management setting larger bounds for trading book managers who would like to select portfolios with higher expected returns since failure to do so could result in the non-existence of a feasible portfolio.

Let $\boldsymbol{w}_{\alpha,E}$ denote the portfolio on the mean-CVaR frontier at the $100\alpha\%$ confidence level with an expected return of E. Consider setting the bound equal to the VaR of this portfolio:

$$V^*_{\alpha,E} \equiv V_{\alpha,\boldsymbol{w}_{\alpha,E}}. \tag{10}$$

A VaR constraint with bound $V^*_{\alpha,E}$: (i) allows (but typically does not force) the selection of portfolio $\boldsymbol{w}_{\alpha,E}$, which has by construction an expected return of E and a zero efficiency loss; and (ii) precludes (as much as possible) portfolios with an expected return of E that have the greatest efficiency losses, thereby leading to the smallest maximum efficiency loss when using a VaR constraint. Generally, when a VaR constraint with confidence level α and a bound V other than $V^*_{\alpha,E}$ is imposed, either (i) or (ii) do not hold. First, the use of a bound $V < V^*_{\alpha,E}$ precludes the selection of portfolio $\boldsymbol{w}_{\alpha,E}$. Second, the use of a bound $V > V^*_{\alpha,E}$ generally results in a larger maximum efficiency loss. Hence, the choice of variable bound $V^*_{\alpha,E}$ is appealing.

The last column of Table 2 presents the results when this bound is used.[26] The first two rows

[26]In order to restrict our attention to a set of plausible levels of expected return, we let $\overline{E} = 2.16\%$ when variable bounds are used (when fixed bounds are used, $\overline{E}$ is still set to be equal to the maximum feasible expected return as it is less than 2.16%). Note that a portfolio with a weight of (a) 150% in small cap value stocks, (b) -50% in Treasury bills, and (c) zero in the remaining asset classes has an expected return of 2.16% [$= 1.5 \times 1.58\% - 0.5 \times 0.43\%$]. Nevertheless, the results when $\overline{E}$ is set to be equal to the maximum feasible expected return (i.e., 3.24%) are qualitatively similar to when $\overline{E} = 2.16\%$. Quantitatively,

report average and largest efficiency losses. Two results can be seen. First, losses are notably smaller than those in the first two columns, indicating that a VaR constraint with a variable bound is useful to control CVaR. The intuition for this result is as follows. While a constraint with a fixed bound is tight for only the largest feasible levels of expected return, a constraint with a variable bound is tight for all feasible levels. Hence, losses are smaller when the variable bound is used. Second, losses can still be noticeable. This result can be understood by noting that VaR does not capture the size of losses beyond VaR.

Similarly, the next two rows indicate that the average (largest) relative efficiency loss is much smaller than when a fixed bound is used. As the last row shows, a VaR constraint with a variable bound allows the selection of the portfolio with larger expected returns than when a fixed bound is used.

The first column of Fig. 3 shows a box plot of maximum efficiency losses associated with variable bounds.[27] The three horizontal lines in the box represent the lower quartile, median, and upper quartile of losses. The dashed vertical lines extending from each end of the box show the range of losses. Hence, the horizontal line at the bottom (top) of the lower (upper) dashed vertical line represents the lowest (highest) value of the loss. Note that losses vary noticeably across required levels of expected return.

Next, we illustrate the tail risk of: (i) portfolios on the mean-CVaR frontier and (ii) portfolios with maximum efficiency losses. For brevity, consider required expected returns of E_{33} and E_{67}, which are, respectively, 33% and 67% of the way between the risk-free rate ($\underline{E}$) and the portfolio on the mean-CVaR frontier with maximum expected return ($\overline{E}$). The first row of Table 3 shows the

it is worth noting that the average relative efficiency loss is smaller. This result is driven by two facts: (1) portfolios on the mean-CVaR frontier with larger expected returns have larger CVaRs; and (2) efficiency losses for levels of expected return close to 3.24% are relatively small (since the set of feasible portfolios with these levels of expected return is also relatively small).

[27] In most of the box plots presented in the paper, there are no outliers. When there are outliers, our main results are not affected by their exclusion. Here, an outlier is a value of the loss that is smaller (larger) than the lower (upper) quartile by an amount that exceeds 1.5 times the interquartile range (i.e., the difference between the upper and lower quartiles).

VaR, losses in ST events, CVaR, efficiency loss, and relative efficiency loss of the portfolio on the mean-CVaR frontier with an expected return of E_{33}.[28] The second row shows the VaR, losses in ST events, and CVaR of the portfolio with such expected return that has the maximum efficiency loss when a variable-bound VaR constraint is used. Note that the latter portfolio has an efficiency loss of 2.38% [= 3.91% − 1.53%] and a relative efficiency loss of 155.18% [= 2.38%/1.53%] even though it has the same VaR as the portfolio on the mean-CVaR frontier. A similarly substantive result can be seen in the fifth and sixth rows where an expected return of E_{67} is used, with efficiency and relative efficiency losses of 3.82% [= 8.89% − 5.07%] and 75.44% [= 3.82%/5.07%], respectively.

We examine the *distance* between any two given portfolios $\boldsymbol{w}^1$ and $\boldsymbol{w}^2$ by computing $\frac{\left|\boldsymbol{w}^1-\boldsymbol{w}^2\right|}{\sqrt{J+1}}$, where $|\cdot|$ denotes the Euclidean norm.[29] Using a variable bound, let D_i denote the distance between the portfolio with maximum efficiency loss in the case when a VaR constraint is imposed and the portfolio on the mean-CVaR frontier when the required expected return is E_i.[30] The average of $\{D_i\}_{i=0}^{100}$ is referred to as *average distance*. With a VaR constraint, this distance is 44%.

3.2. ST constraints

We now analyze the effectiveness of ST constraints in controlling tail risk. The first two columns of Table 4 examine the case of fixed bounds. The first two rows report average and largest efficiency losses when T_1 and T_2 are assumed to be either 4% or 8% (for brevity, we assume that $T_1 = T_2$). Three results can be seen. First, losses are sizeable for both values of T_1 and T_2. Second, losses are smaller when the lower value for T_1 and T_2 is used. Third, losses are larger than those in Table 2 where VaR constraints were imposed. The next two rows indicate that average and largest relative efficiency losses are also sizeable and larger than when a VaR constraint is imposed.

The last row shows that using the lower values of the ST bounds (and therefore tighter con-

[28] Note that a negative loss corresponds to a positive return; see Eq. (6).
[29] Note that $\left|\boldsymbol{w}^1 - \boldsymbol{w}^2\right| / \sqrt{J+1} = [\Sigma_{j=1}^{J}(w_j^1 - w_j^2)^2/(J+1)]^{1/2}$.
[30] In subsequent cases, we continue to assume that variable bounds are used when determining D_i.

straints) precludes the selection of more portfolios with large expected returns. Hence, there exists no value for the fixed bounds such that the constraints preclude the selection of all portfolios with substantive efficiency losses while allowing the selection of portfolios with large expected returns. Accordingly, we examine the case of variable bounds next.

For a given expected return E and confidence level α, we set the ST bounds to be equal to the ST event losses of the portfolio on the mean-CVaR frontier:

$$(T^*_{1,\alpha,E}, T^*_{2,\alpha,E}) \equiv (T_{1,\boldsymbol{w}_{\alpha,E}}, T_{2,\boldsymbol{w}_{\alpha,E}}). \tag{11}$$

Note that ST constraints with bounds $T^*_{1,\alpha,E}$ and $T^*_{2,\alpha,E}$: (i) allow (but typically do not force) the selection of portfolio $\boldsymbol{w}_{\alpha,E}$, which has by construction an expected return of E and a zero efficiency loss; and (ii) preclude (as much as possible) the portfolios with an expected return of E that have the greatest efficiency losses, thereby leading to the smallest maximum efficiency loss when using ST constraints. Hence, the choice of variable bounds $T^*_{1,\alpha,E}$ and $T^*_{2,\alpha,E}$ is appealing.

The last column of Table 4 presents the results when these bounds are used. The first two rows report average and largest efficiency losses. Two results can be seen. First, average losses are smaller than those in the first two columns, indicating that ST constraints with variable bounds are useful to control CVaR. Second, the largest efficiency loss is larger than that in the first column but smaller than that in the second column. The next two rows show similar results for the average and largest relative efficiency losses. Importantly, the benefits of using variable-bound constraints relative to fixed-bound constraints are small in comparison to those obtained with a VaR constraint as shown in Table 2. As the last row shows, ST constraints with a variable bound allow the selection of portfolios with larger expected returns than when a fixed bound is used.

The second column of Fig. 3 shows a box plot of maximum efficiency losses associated with variable bounds. The losses differ from those when a VaR constraint is imposed in that the ST box plots are much larger (compare the first two columns of this figure). Specifically, the highest value, upper quartile, median, and lower quartile are all larger.[31] The intuition for why the ST constraints are less effective than the VaR constraint is as follows. Because stocks have notable losses in ST events, a portfolio with noticeable short positions in stocks might satisfy the ST constraints. However, since the state used to compute a portfolio's VaR depends on the portfolio's return distribution, the aforementioned portfolio might not satisfy the VaR constraint (if it was imposed as in the previous section). Since the states used to compute a portfolio's CVaR also depend on the portfolio's return distribution, such a portfolio might have a large CVaR. Because there are portfolios that: (1) satisfy the ST constraints, (2) do not satisfy the VaR constraint, and (3) have relatively large CVaRs, efficiency losses with ST constraints are larger than with the VaR constraint. In results not presented in the tables (but available upon request), we indeed find that portfolios with maximum efficiency losses when ST constraints are imposed generally involve more significant short positions than when a VaR constraint is imposed.

In assessing the statistical significance of the difference between the distributions of losses with variable-bound VaR and ST constraints, we utilize: (i) the two-sample Kolmogorov-Smirnov test and (ii) the Wilcoxon rank sum test. Using (i), we test the null hypothesis that the cumulative distribution function (cdf) of losses when a VaR constraint is imposed coincides with the cdf when ST constraints are imposed against the alternative hypothesis that the two cdfs differ. Similarly, using (ii), we test the null hypothesis that the median of the distribution of losses when a VaR constraint is imposed equals the median when ST constraints are imposed against the alternative hypothesis that the two medians differ. In results available upon request, we find that both null

[31] Note that our weight restrictions allow the selection of portfolios where the sum of the weights of the asset classes involving stocks is negative (hereafter, 'short equity portfolios'). These portfolios may actually have positive returns in the ST events considered in our paper but may suffer large losses when the stock market has large returns. Hence, our results on the ineffectiveness of ST constraints in controlling tail risk could be driven by short equity portfolios. However, upon further analysis, we find that this does not occur. Indeed, we find that ST constraints are still ineffective when short sales are allowed but short equity portfolios are disallowed. The fact that short equity portfolios do not drive our results when ST constraints are imposed is also true when both VaR and ST constraints are imposed.

hypotheses are rejected at the 1% level.[32] Hence, there is statistical evidence that ST constraints are less effective in controlling tail risk than a VaR constraint.

The third row of Table 3 shows the VaR, losses in ST events, CVaR, efficiency loss, and relative efficiency loss of the portfolio with an expected return of E_{33} that has the maximum efficiency loss when variable-bound ST constraints are used. Note that such a portfolio has an efficiency loss of 21.06% [= 22.59% − 1.53%] and a relative efficiency loss of 1,372.18% [= 21.06%/1.53%] even though its returns in ST events are equal to or higher than those of the portfolio on the mean-CVaR frontier. Hence, these losses are notably larger than when a VaR constraint is used (i.e., 2.38% and 155.18%). Similar results can be seen in the seventh row where an expected return of E_{67} is used in that the efficiency and relative efficiency losses with ST constraints are still substantial, being equal to 9.86% [= 14.93% − 5.07%] and 194.48% [= 9.86%/5.07%], respectively.

We now examine the distance between: (1) portfolios with maximum efficiency losses when ST constraints are used; and (2) portfolios on the mean-CVaR frontier. The average distance is 83% and thus about two times larger than when a VaR constraint is imposed.

3.3. VaR and ST constraints

Next, we analyze the joint effectiveness of VaR and ST constraints in controlling tail risk. The first four columns of Table 5 present the results with fixed bounds. The first two rows report average and largest efficiency losses for various values of V, T_1, and T_2. Three main results can be seen. First, losses are sizeable for all of these values. Second, losses are smaller than those in Table 2 where just VaR constraints were imposed. Third, losses are notably smaller than those in Table 4 where just ST constraints were imposed. The next two rows indicate that average and largest relative efficiency losses are also sizeable but smaller than when either just VaR or ST constraints

[32] Here, we continue to assume that the confidence level used to compute VaR and CVaR is $\alpha = 99\%$. The results are statistically significant at the 1% level when this value of α is used.

are imposed.

The last row shows that using the lower values of the VaR and ST bounds precludes the selection of portfolios with large expected returns. Hence, there exist no values for the fixed bounds such that the constraints preclude the selection of all portfolios with substantive efficiency losses while allowing the selection of portfolios with large expected returns. Next, we examine the case of variable bounds given by Eqs. (10) and (11).

The last column of Table 5 presents the results when these bounds are used. The first two rows report average and largest efficiency losses. Four results can be seen. First, losses are not close to zero, indicating that the joint use VaR and ST constraints with variable bounds is ineffective in controlling tail risk. Second, losses are notably smaller than those in the first four columns, indicating that the use of variable bounds is beneficial relative to the use of fixed bounds. Third, losses are smaller than those in the last column of Table 2 where only variable-bound VaR constraints are imposed. Fourth, losses are notably smaller than those in the last column of Table 4 where only variable-bound ST constraints are imposed.[33] The next two rows show similar results for the average and largest relative efficiency losses. As the last row shows, VaR and ST constraints with variable bounds allow the selection of portfolios with larger expected returns than when fixed bounds are used.

The third column of Fig. 3 shows a box plot of maximum efficiency losses associated with variable bounds. Note that the upper quartile, median, and lower quartile with both VaR and ST constraints are: (i) smaller than with only the VaR constraint (compare the first and third columns); and (ii) much smaller than with only ST constraints (compare the second and third columns). Hence, the joint use of the constraints is clearly an improvement over using just one

[33] By construction, for any given feasible level of expected return, the maximum efficiency loss when these VaR and ST constraints are both used is smaller than or equal to that when solely using either constraint. Accordingly, we do not perform statistical tests comparing losses when both constraints are imposed with those when just one type of constraint is imposed. The main point of our paper is to show that while the loss when both types of constraints are imposed is typically smaller when just one type of constraints is imposed, it is sizeable.

of type of the constraints in that efficiency losses are notably reduced. However, losses are still substantial.

The fourth row of Table 3 shows the VaR, losses in ST events, CVaR, efficiency loss, and relative efficiency loss of the portfolio with an expected return of E_{33} that has the maximum efficiency loss when variable-bound VaR and ST constraints are used. Note that such a portfolio has an efficiency loss of 0.91% [= 2.44%−1.53%] and a relative efficiency loss of 58.78% [= 0.91%/1.53%] even though it has the same VaR and losses in ST events as the portfolio on the mean-CVaR frontier. Hence, these losses are notably smaller than when either only: (a) a VaR constraint is used (i.e., 2.38% and 155.18%); or (b) ST constraints are used (i.e., 21.06% and 1,372.18%). Similar results can be seen in the last row where an expected return of E_{67} is used, but the efficiency and relative efficiency losses with VaR and ST constraints are 2.13% [= 7.20% − 5.07%] and 42.09% [= 2.13%/5.07%], respectively.

We now examine the distance between: (1) portfolios with maximum efficiency losses when both VaR and ST constraints are used; and (2) portfolios on the mean-CVaR frontier. The average distance is 40% and thus smaller than that when either just VaR (44%) or ST (83%) constraints are imposed.

3.4. Additional robustness checks

Next, we further assess the robustness of the result that variable-bound VaR and ST constraints are ineffective in controlling tail risk by examining five additional cases. The first three cases consider a larger number of: (1) ST events; (2) assets; and (3) both ST events and assets. The fourth case uses the period 1982–2009 (instead of 1982–2006). The fifth case uses daily data (instead of monthly data). We focus on variable-bound constraints since they are more effective in controlling tail risk than fixed-bound constraints.

First, consider a larger number of ST events along with the original nine assets. Suppose that four ST events are used: (a) crash of 87; (b) 9/11; (c) the 1997 Asian crisis; and (d) the 1998 Russian crisis.[34] The second column of Fig. 4 provides a box plot of maximum efficiency losses with four ST events. For easier comparison, the box plot of maximum efficiency losses with two ST events from the third column of Fig. 3 now re-appears in the first column of Fig. 4. Note that the two box plots are quite similar. Thus, when a larger number of ST events is used, VaR and ST constraints are similarly ineffective in controlling tail risk.

Second, consider a larger number of assets along with the original two ST events. Instead of the six size/book-to-market Fama-French equity portfolios, we now use the ten size Fama-French equity portfolios. As before, suppose that Treasury bills, government bonds, and corporate bonds are available. Hence, the number of assets increases from nine to thirteen. The third column of Fig. 4 shows that the box plot of maximum efficiency losses with thirteen assets and two ST events is much larger than that with nine assets and two ST events. Thus, when a larger number of assets is available, VaR and ST constraints are even less effective in controlling tail risk.

Third, consider larger numbers of both ST events and assets. The fourth column of Fig. 4 shows a box plot of maximum efficiency losses with four ST events (crash of 87, 9/11, Asian crisis, and Russian crisis) and thirteen assets (Treasury bills, government bonds, corporate bonds, and the ten size Fama-French portfolios). Note that the box plot is larger (but the lower quartile is slightly smaller) than that with two ST events and nine assets. Thus, when larger numbers of both ST events and assets are used, VaR and ST constraints are still ineffective in controlling tail risk.

Fourth, consider the use of the period 1982–2009 along with the original nine assets and two ST events. The last column of Fig. 4 shows that the box plot of maximum efficiency losses with this

[34] In determining the time periods for the ST events involving the Asian and Russian crises, we follow RiskMetrics (a leading provider of risk management products); see <www.riskgrades.com/retail/events/events.cgi>. However, our starting date for the Russian crisis event is one day earlier than that used by RiskMetrics so that the event includes the day when the Russian government decided to default on its debt.

period is larger than that with the period 1982–2006. Thus, when the period 1982–2009 is used, VaR and ST constraints are even less effective in controlling tail risk.

Fifth, consider the use of a one-day investment period along with the original nine assets and two ST events. We use the period January 13, 2003–December 29, 2006 so that we have 1000 daily observations; the second column of Fig. 5 shows the corresponding box plot of maximum efficiency losses.[35] These losses are scaled to a period of one month by multiplying daily efficiency losses by $(250/12)^{0.5}$ so that the results can be compared with those when using monthly data.[36] For easier comparison, the box plot of maximum efficiency losses with monthly data from the third column of Fig. 3 re-appears in the first column of Fig. 5. While the box plots with daily data are smaller than those with monthly data, efficiency losses are still substantial. For example, the median monthly loss exceeds 1%, whereas the largest loss exceeds 2% (see the second column of Fig. 5). Thus, when daily data are used, VaR and ST constraints are still ineffective in controlling tail risk.

4. Conclusion

In attempting to promote bank stability, the Basel Committee on Banking Supervision (2006) provides a framework that seeks to control the amount of tail risk that large banks take in their trading books. However, the size and prevalence of bank trading losses during the recent crisis raises the question of whether this framework indeed promotes bank stability. In this paper, we shed light on this question by showing that the Basel framework allows banks to take substantive tail risk in their trading books without a capital requirement penalty. Hence, our paper supports the view that the Basel framework did not promote bank stability.

Broadly speaking, our results are in line with Lucas (2001) and Kane (2006). However, we

[35] A caveat is in order here in that the daily data extends back to 2003 while the monthly data extends back to 1982. Hence, the two sample periods differ.

[36] Our scaling convention is consistent with the fact that the Basel framework allows the use of an estimate of VaR for a period of ten trading days by taking an estimate of VaR for a period of one trading day and multiplying it by the square root of ten; see Basel Committee on Banking Supervision (2006, p. 195). This convention is often used in practice; see Hull (2007, p. 203).

should emphasize that our arguments differ from theirs. They show that banks have incentives to underreport their VaR-based estimates of capital requirements. Our paper shows that banks can take substantive tail risk in their trading books without a capital requirement penalty even if their VaR-based estimates of capital requirements are not underreported.

Perhaps recognizing that the Basel framework did not prevent banks from taking substantive tail risk in their trading books, the Basel Committee on Banking Supervision (2010) proposes a new regulatory framework for trading books.[37] An examination of the effectiveness of the new Basel framework in promoting bank stability is left for further research.

[37]In the new Basel framework, a trading book's minimum capital requirement depends on both its VaR and stressed VaR while still requiring banks to use ST. The estimation of stressed VaR is based on the use of a sample period where significant losses occurred (e.g., the recent crisis).

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Table 1: Summary statistics on asset returns

This table presents summary statistics on the monthly returns of nine assets during the period 1982–2006: (i) Treasury bills, (ii) government bonds as measured by the Merrill Lynch government bond master index, (iii) corporate bonds as measured by the Merrill Lynch corporate bond master index, and (iv) the six size/book-to-market Fama-French portfolios. Returns on Treasury bills and Fama-French portfolios are obtained from Kenneth French's website. Returns on bonds are obtained from Bloomberg. Since Treasury bills are assumed to be risk-free, the standard deviation of the return on Treasury bills is reported as zero. A confidence level of 99% is used to compute VaR and CVaR. Also presented are the asset returns in two ST events: (i) the crash in the U.S. stock market of 1987 (October 19, 1987) and (ii) the terrorist attacks in the U.S. in September 2001 (September 11–21, 2001). For simplicity, we assume that the return on the risk-free security in an ST event is equal to the product of: (i) the duration of the event expressed as a fraction of a month times (ii) the average risk-free return. All numbers are reported in percentage points per month except for losses in ST events, which refer to the periods of time capturing the events as defined earlier.

				Fama-French portfolios					
	Treas.	Govt.	Corp.	Small			Big		
	bills	bonds	bonds	Low	Inter.	High	Low	Inter.	High
Mean	0.43	0.73	0.83	0.82	1.44	1.58	1.10	1.22	1.27
Std. dev.	0.00	1.47	1.65	6.82	4.81	4.65	4.74	4.19	4.12
VaR	−0.43	2.44	3.11	16.23	12.93	14.38	10.91	9.37	10.21
CVaR	−0.43	3.34	3.75	24.48	20.20	19.91	16.36	16.11	14.50
	Returns in ST events								
Crash of 87	0.04	0.34	−0.55	−13.03	−11.12	−10.97	−17.94	−18.60	−17.92
9/11	0.16	0.69	−0.60	−15.06	−13.14	−15.34	−11.72	−11.36	−11.72

Table 2: Efficiency losses and maximum feasible expected returns when a VaR constraint is imposed

The first and second rows of each panel report, respectively, average and largest efficiency losses with a VaR constraint. The third and fourth rows report, respectively, average and largest relative efficiency losses with this constraint. The last row reports the maximum feasible expected return with the constraint. The first two columns use a fixed bound V of either 4% or 8%. The third column uses variable bound $V^*_{\alpha,E}$ as defined by Eq. (8), which depends on required expected return E. A confidence level of 99% is used to compute VaR and efficiency losses. Losses and expected returns are reported in percentage points per month. Relative losses are measured in percentage points.

	V		
	4%	8%	$V^*_{\alpha,E}$
Efficiency loss:			
Average	8.25	14.94	3.86
Largest	11.11	20.97	9.56
Relative efficiency loss:			
Average	366.99	501.75	105.93
Largest	934.56	1,844.67	174.24
Maximum feasible expected return	1.59	2.07	2.16

Table 3: VaRs, losses in ST events, CVaRs, efficiency losses, and relative efficiency losses of portfolios on the mean-CVaR frontier and of portfolios with maximum efficiency losses when various constraints are imposed

Consider the two levels of expected return, E_{33} and E_{67}, that are, respectively, 33% and 67% of the way between the risk-free rate ($\underline{E}$) and the expected return of the portfolio on the mean-CVaR frontier with maximum expected return ($\overline{E}$). The table presents the VaR, losses in ST events, CVaR, efficiency loss, and relative efficiency loss of the portfolio on the mean-CVaR frontier with an expected return of E_{33} (E_{67}). It also presents the VaRs, losses in ST events, CVaRs, efficiency losses, and relative efficiency losses of portfolios with this level of expected return that have maximum efficiency losses when variable-bound VaR and/or ST constraints are used. A confidence level of 99% is used to compute VaR and CVaR. VaRs, CVaRs, and efficiency losses are reported in percentage points per month. Losses in ST events are reported in percentage points but refer to the periods of time capturing the events as defined in Table 1. Relative efficiency losses are reported in percentage points.

		Losses in ST events				Relative
	VaR	Crash of 87	9/11	CVaR	Efficiency loss	efficiency loss
E_{33}:						
Mean-CVaR frontier	1.38	−0.26	0.84	1.53	0.00	0.00
VaR	1.38	2.94	3.46	3.91	2.38	155.18
ST	16.59	−0.26	−6.85	22.59	21.06	1,372.18
VaR + ST	1.38	−0.26	0.84	2.44	0.91	58.78
E_{67}:						
Mean-CVaR frontier	4.38	3.90	4.83	5.07	0.00	0.00
VaR	4.38	5.59	7.59	8.89	3.82	75.44
ST	10.68	3.90	3.07	14.93	9.86	194.48
VaR + ST	4.38	−1.43	4.83	7.20	2.13	42.09

Table 4: Efficiency losses and maximum feasible expected returns when ST constraints are imposed

The first and second rows report, respectively, average and largest efficiency losses with ST constraints that use the crash of 87 and 9/11 as the ST events. The third and fourth rows report, respectively, average and largest relative efficiency losses with these constraints. The last row reports the maximum feasible expected return with the constraints. The first two columns use fixed bounds T_1 and T_2, where $T_1 = T_2 = 4\%$ or 8%. The third column uses variable bounds $T^*_{1,\alpha,E}$ and $T^*_{2,\alpha,E}$ as defined by Eq. (11), which depend on required expected return E. A confidence level of 99% is used to compute efficiency losses. Losses and expected returns are reported in percentage points per month. Relative losses are measured in percentage points.

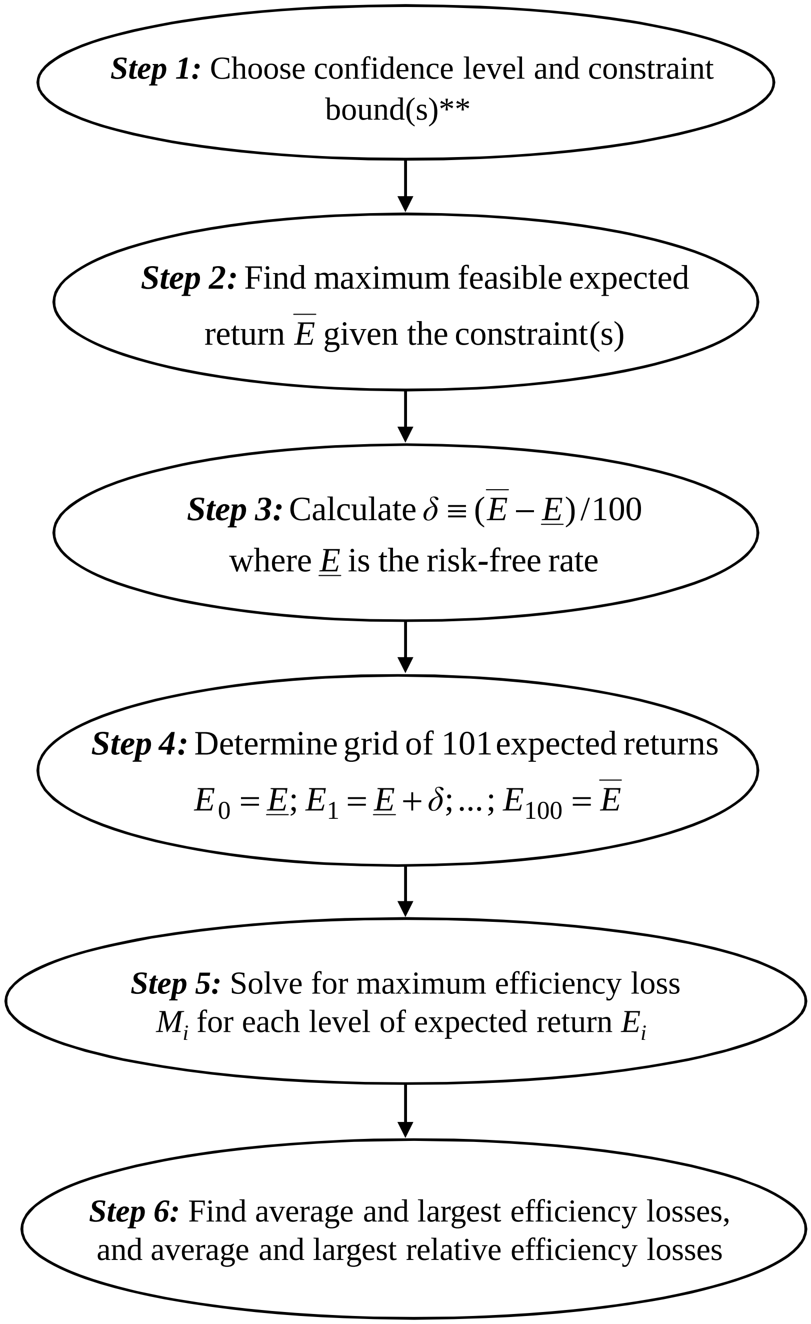
	$T_1 = T_2$		
	4%	8%	$T^*_{1,\alpha,E}$, $T^*_{2,\alpha,E}$
Efficiency loss:			
Average	16.52	17.80	15.54
Largest	24.18	28.19	26.97
Relative efficiency loss:			
Average	636.80	626.66	505.27
Largest	1,810.70	2,329.70	2,280.65
Maximum feasible expected return	1.84	2.04	2.16

Table 5: Efficiency losses and maximum feasible expected returns when VaR and ST constraints are imposed

The first and second rows report, respectively, average and largest efficiency losses with a VaR constraint and ST constraints that use the crash of 87 and 9/11 as the ST events. The third and fourth rows report, respectively, average and largest relative efficiency losses with these constraints. The last row reports the maximum feasible expected return with the constraints. The first four columns use a fixed bound V of either 4% or 8%, and fixed bounds T_1 and T_2 where $T_1 = T_2 = 4\%$ or 8%. The last column uses variable bounds $V^*_{\alpha,E}$, $T^*_{1,\alpha,E}$, and $T^*_{2,\alpha,E}$ as defined by Eqs. (8) and (11), which depend on required expected return E. A confidence level of 99% is used to compute VaR and efficiency losses. Losses and expected returns are reported in percentage points per month. Relative losses are measured in percentage points.

	V				$V^*_{\alpha,E}$
	4%		8%		
	$T_1 = T_2$				$T^*_{1,\alpha,E}$, $T^*_{2,\alpha,E}$
	4%	8%	4%	8%	
Efficiency loss:					
Average	6.59	7.91	12.34	12.22	1.96
Largest	10.35	10.89	17.54	17.54	4.03
Relative efficiency loss:					
Average	288.29	349.27	484.05	437.12	56.56
Largest	831.20	976.37	1,552.29	1,511.05	138.53
Maximum feasible expected return	1.59	1.59	1.78	1.95	2.16

Figure 1: Methodology*



* There are three sets of constraints: (1) a VaR constraint; (2) ST constraints; and (3) VaR and ST constraints. Historical simulation is used to compute VaR, CVaR, and losses in ST events for all portfolios.

** The bounds for the VaR and ST constraints are either: (a) fixed (do not depend on required expected return E_i); or (b) variable (depend on E_i). The variable bound for the VaR constraint is given by Eq. (10), whereas the variable bounds for the ST constraints are given by Eq. (11).

Figure 2: Determining maximum efficiency losses

The curve represents portfolios on the mean-CVaR frontier for various levels of expected return. The minimum required expected return $E_0 = \underline{E}$ is assumed to be the risk-free rate. The maximum required expected return $E_{100} = \overline{E}$ depends on whether short selling is disallowed or allowed. Point p_{min} represents the portfolio that has an expected return of E_i and minimum CVaR, C_{min}. Point p_{max} represents the portfolio that has the same expected return, meets the VaR and stress testing constraints, and has maximum CVaR, C_{max}. Note that maximum efficiency loss M_i is given by $C_{max} - C_{min}$.

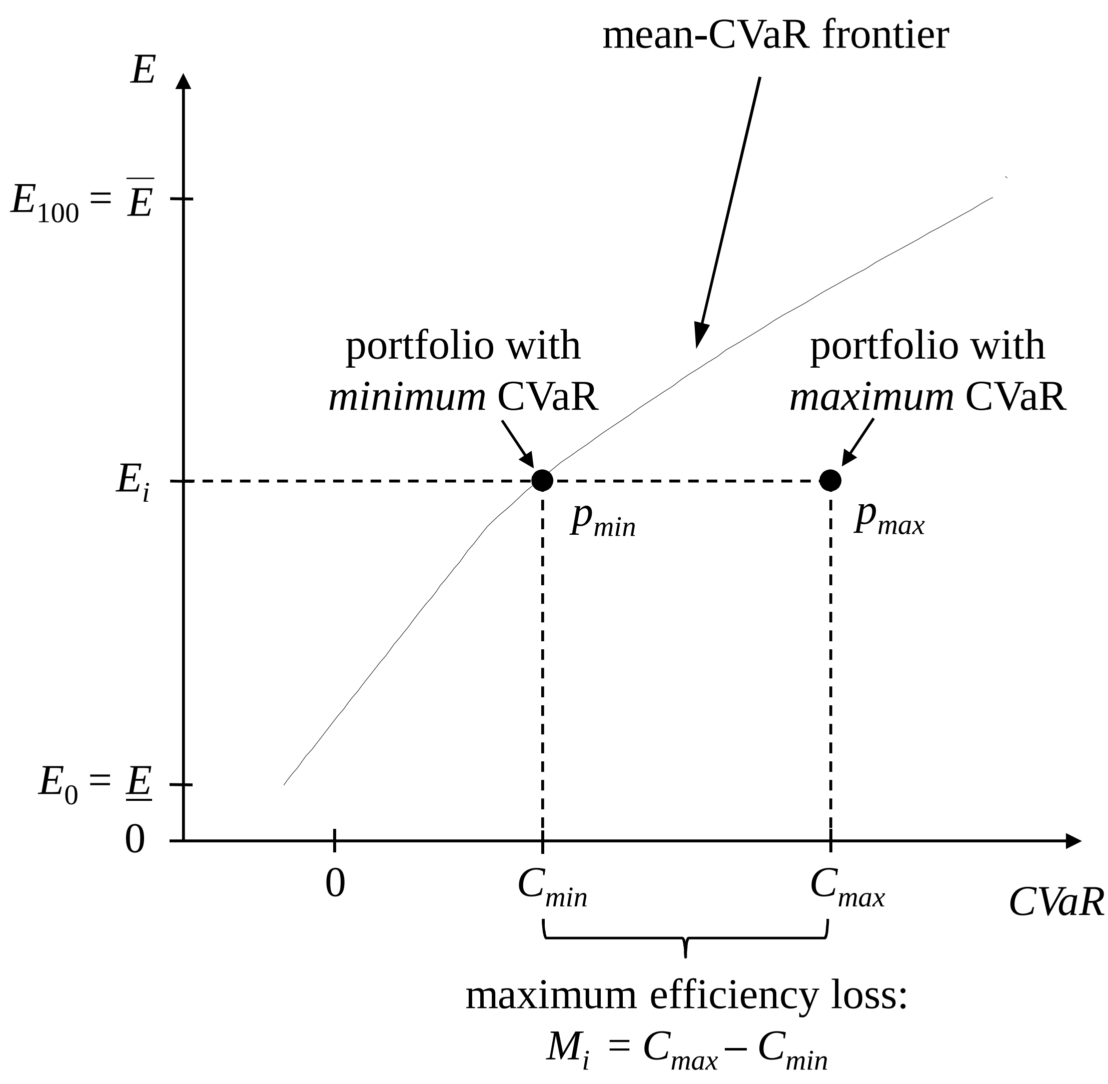


Figure 3: Box plots of maximum efficiency losses

This figure shows box plots of maximum efficiency losses in the range of feasible expected returns with variable-bound constraints. In the first column, we solely impose a VaR constraint with bound $V^*_{\alpha,E}$, which depends on the required expected return E as Eq. (8) shows. In the second column, we solely impose ST constraints that use the crash of 87 and 9/11 as the ST events and bounds $T^*_{1,\alpha,E}$ and $T^*_{2,\alpha,E}$, which depend on required expected return E as Eq. (11) shows. In the third column, we jointly impose both VaR and ST constraints. The three horizontal lines in the box represent the lower quartile, median, and upper quartile of losses. The dashed vertical lines extending from each end of the box show the range of losses. Hence, the horizontal line at the bottom (top) of the lower (upper) dashed vertical line represents the lowest (highest) value for the loss. A confidence level of 99% is used to compute VaR and efficiency losses. Losses are reported in percentage points per month.

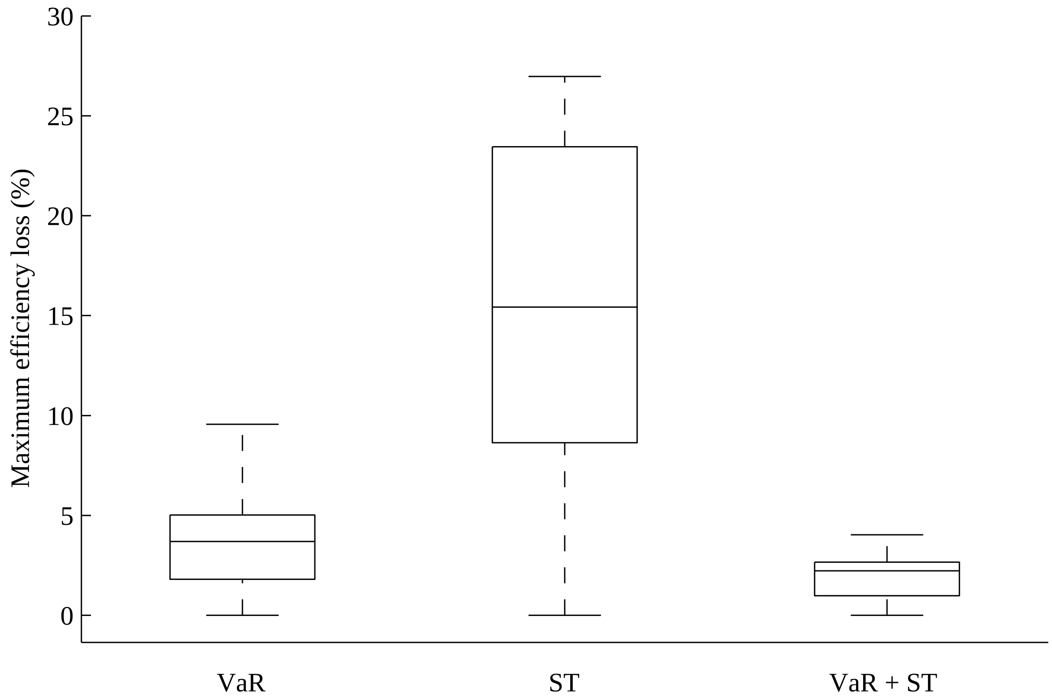


Figure 4: Box plots of maximum efficiency losses with VaR and ST constraints when different ST events, assets, and periods are used

This figure shows box plots of maximum efficiency losses in the range of feasible expected returns when both VaR and ST constraints are imposed. As in Fig. 3, the VaR and ST constraints use variable bounds. While the first four columns use the period 1982–2006, the last column uses the period 1982–2009. The first, second, and fifth columns consider nine assets: Treasury bills, government bonds, corporate bonds, and the six size/book-to-market Fama-French portfolios. The third and fourth columns consider thirteen assets: Treasury bills, government bonds, corporate bonds, and the ten size Fama-French portfolios. The first, third, and fifth columns use two ST events: crash of 87 and 9/11. The second and fourth columns use four ST events: crash of 87, 9/11, Asian crisis, and Russian crisis. The three horizontal lines in the box represent the lower quartile, median, and upper quartile of losses. The dashed vertical lines extending from each end of the box show the range of losses. Hence, the horizontal line at the bottom (top) of the lower (upper) dashed vertical line represents the lowest (highest) loss. A confidence level of 99% is used to compute VaR and efficiency losses. Losses are reported in percentage points per month.

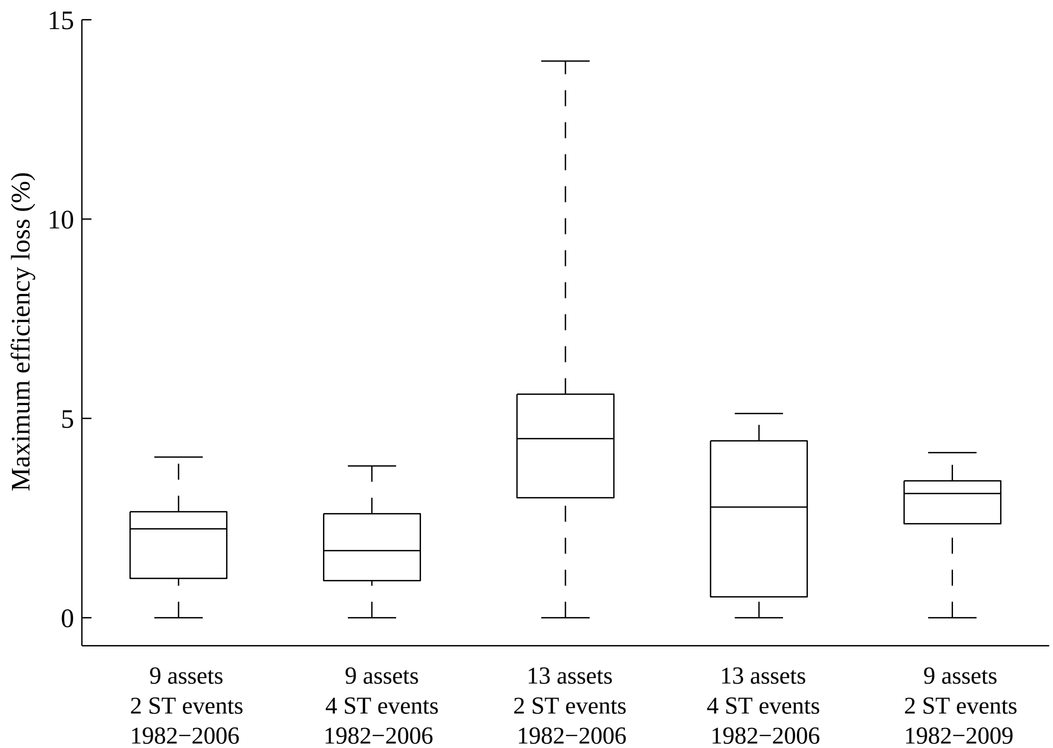


Figure 5: Box plots of maximum efficiency losses when daily data are used

This figure shows box plots of maximum efficiency losses in the range of feasible expected returns when both VaR and ST constraints are imposed. As in Fig. 3, the VaR and ST constraints use variable bounds. There are nine assets (Treasury bills, government bonds, corporate bonds, and the ten size Fama-French portfolios) and two ST events (crash of 87 and 9/11). In the first column, the investment horizon is one month and monthly data are used in estimating monthly VaR, CVaR, and efficiency losses as before. In the second column, the investment horizon is one day and daily data are used in estimating daily VaR, CVaR, and efficiency losses. These efficiency losses are scaled to a period of one month by multiplying them by $(250/12)^{0.5}$ so that the results with monthly and daily data can be compared. The three horizontal lines in the box represent the lower quartile, median, and upper quartile of losses. The dashed vertical lines extending from each end of the box show the range of losses. Hence, the horizontal line at the bottom (top) of the lower (upper) dashed vertical line represents the lowest (highest) loss. A confidence level of 99% is used to compute VaR and efficiency losses.