Optimal Sovereign Debt Default

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- Standard view: limited commitment + weak ex-post incentives
- Default option ex-ante inefficient - too little borrowing
- Sovereign default literature (Eaton and Gersovitz (REStud, 1981)):
  - how to generate ex-post incentives for repayment?
  - how to get them strong enough?
  - how to explain that countries default in ‘bad times’

- Committed government: can *choose* to default
- Partial repayment optimal if gov. bond markets incomplete

  => share risk / complete the market
- Grossman & van Huyck (AER, 1988):
  'excusable' vs 'non-excusable' under limited commitment
- Here full commitment => strong implications for default policies
  - default option allows for more borrowing: *relaxes* the borrowing limits (marginally binding NBL)
  - default ex-ante efficient
  - default optimal following large negative shocks or small neg shock if close to borrowing limit

- Panizza, Sturzenegger, Zettlemyer (JEL, 2009):

  ‘sovereign immunity’ & ‘act of state doctrine’: not too much bite

  US Foreign Sovereign Immunities Act (FSIA) of 1976

  Famous legal cases of hold out creditors vs sovereigns

Setup and Preview of Results

- Small open production economy
- Government can
  - internationally borrow by issuing own non-contingent bonds.
  - can accumulate foreign bonds/reserves
- Determines fully optimal policy under commitment.
- Instead of *assuming* repayment, repayment is a *decision variable*

- Without default costs:

  optimal default decisions implement first best consumption allocation

  default frequent: for all but the best productivity realization

  default proportional to news about NPV of domestic value added

Setup and Preview of Results

- Introduce (dead-weight) costs of default: proportional to size of default
- Fairly low levels of default costs:

  Default never optimal following BC cycle-sized shocks, unless country close to maximally sustainable net foreign debt position.
- Introduce economic disaster risk (Barro and Jin (2011):

  default reemerges following occurrence of a disaster shock

  optimal even if far from maximal net foreign debt position

Related Literature

- Grossman and van Huyck (AER,1988): ‘excusable’ default with limited commitment
- Chari, Kehoe and Christiano (1991) and Sims (2001): nominal bonds and price level adjustments
- Angeletos (2002): exploit yield curve for insurance purposes

The Model: Households and Firms

- Representative consumer:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subsistence consumption: $c_t \geq \bar{c} \geq 0$

- Representative firm:

$$y_t = z_t k_{t-1}^{\alpha},$$

where $z_t \in Z = \{z^1, ..., z^N\}$

- Transition probabilities are given by $\pi(z'|z)$ for $z', z \in Z$.

The Model: Government

- Government maximizes utility of the representative domestic household.
- can invest in 1-period riskless international bonds (zero coupon):

  'long position': $G_t^L \geq 0$, yield capital gain $1 + r = 1/\beta$
- can issue (potentially risky) 1-period own bonds:

  'short position' $G_t^S \geq 0$
- extension to longer maturities later on

- in $t-1$ can decide to (partially) default on bonds maturing t (commitment):

$$\Delta_{t-1} = (\delta_{t-1}^{1}, ..., \delta_{t-1}^{N}) \in [0,1]^{N},$$

where $\delta_{t-1}^{n} \in [0,1]$.

- Total repayment in state z^n in t is given by

$$G_{t-1}^{S} \cdot (1-(1-\lambda)\delta_{t-1}^{I(z^n)})$$

$\lambda \geq 0$: 'dead weight costs' of default.

- Interest rate on domestic bonds:

$$1 + r = (1 + R(z_t, \Delta_t)) \sum_{n=1}^{N} (1 - \delta_t^n) \cdot \pi(z^n | z_t)$$

Optimal Policy Problem

- Ramsey allocation problem

$$\max_{\{G_t^L \geq 0, G_t^S \geq 0, \Delta_t \in [0,1]^N, k_t \geq 0, c_t \geq \bar{c}\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t. \quad : \quad c_t + k + \frac{G_t^L}{1+r} = w_t + \frac{G_t^S}{1 + R(z_t, \Delta_t)}$$

$$w_{t+1} \geq NBL(z_{t+1}) \quad \forall z_{t+1} \in Z$$

- Beginning-of-period wealth:

$$w_t \equiv z_t k_{t-1}^{\alpha} + G_{t-1}^L - G_{t-1}^S \cdot (1 - (1-\lambda)\delta_{t-1}^{I(z_t)}).$$

Optimal policy problem

- Solving optimal policy problem difficult:
  - Interest rate $R(z_t, \Delta_t)$ depends on default policy: unclear if problem is concave & use of FOCs justified....
  - Many occasionally binding inequality constraints $G_t^L \geq 0$, $G_t^S \geq 0$ and particular $\Delta_t \in [0, 1]^N$ that are difficult to handle computationally
  - Optimal default policies Δ_t turn out to be non-continuous, complicating numerical solutions difficult.
- Derive an equivalent problem: concave (can use FOCs), economizes on inequality constraints, continuous optimal policies...

- Equivalent optimization problem:

$$\max_{\{b_t, a_t \geq 0, k_t \geq 0, c_t \geq \bar{c}\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t. \ \forall t : c_t = \widetilde{w}_t - k_t - \frac{1}{1+r} b_t - p_t \cdot a_t$$

$$\widetilde{w}_{t+1} \geq NBL(z_{t+1}) \quad \forall z_{t+1} \in Z$$

$b \gtrless 0$: riskless bond
a : vector of Arrow securities
p_t : price vector for Arrow securities (indep. of policy)
$\widetilde{w}_0 = w_0$: initial condition

- Beginning-of-period wealth

$$\widetilde{w}_t \equiv z_t k_{t-1}^{\alpha} + b_{t-1} + (1-\lambda) a_{t-1}(z_t)$$

- Concave problem, economizes on inequality constraints

Equivalence of Problems

- Equivalence proof in paper.....
- b has an interpretation as the net foreign asset position

$$b_t = G_t^L - G_t^S,$$

- Arrow securities capture state contingent default policies on own bonds

  In a setting with 2 productivity states:

$$a_t = \begin{pmatrix} G_t^S \delta^1 \\ G_t^S \delta^2 \end{pmatrix}$$

Analytical Result

Proposition

Without default costs ($\lambda = 0$) the solution involves constant consumption equal to

$$c = (1-\beta)(\Pi(z_0) + \widetilde{w}_0)$$

where $\Pi(\cdot)$ denotes the maximized expected value added

$$\Pi(z_t) \equiv E_t \left[\sum_{j=0}^{\infty} \beta^j \left(-k^*(z_{t+j}) + \beta z_{t+j+1} \left(k^*(z_{t+j}) \right)^{\alpha} \right) \right]$$

with

$$k^*(z_t) = (\alpha \beta E(z_{t+1}|z_t))^{\frac{1}{1-\alpha}}$$

denoting the profit maximizing capital level. For any period t, the optimal default level satisfies

$$a_0(z_t) \propto -\left(\Pi(z_t) + z_t \left(k^*(z_{t-1})\right)^{\alpha}\right)$$

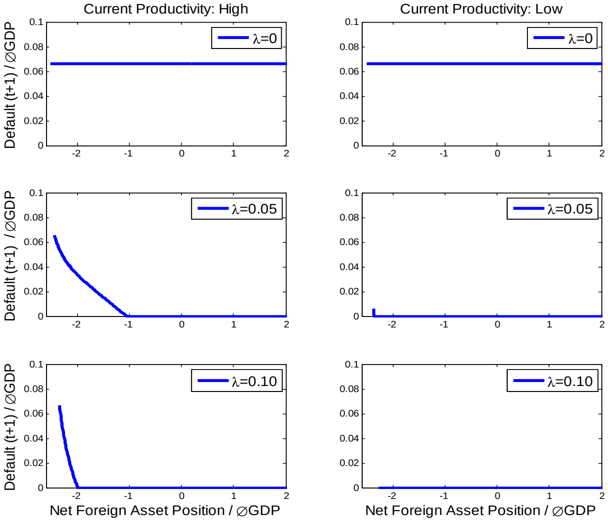
- Positive default costs: require numerical solution
- Calibrate the model at annual frequency
- Tauchen's method to generate obtain a two-state productivity process (implied quarterly persistence of technology 0.9 & std dev of 0.5)
- Utility function is given by

$$u(c) = \frac{(c - \bar{c})^{1-\sigma}}{1 - \sigma}$$

- $\overline{c}$: if bonds must be repaid always, max sustainable NFA equals -100% of GDP (Lane and Milesi-Ferretti (2007))
- Remaining parameters:

α	β	σ	$\bar{c}$	1+r
0.34	0.97	2	0.357	$1/\beta - 0.0005$

The Effect of Default Costs



- Default option:
  - relaxes borrowing limit from 100% of GDP to 220% of GDP
  - with default costs suboptimal to use if above max sustainable NFA position
  - less default in the future if current state low: persistence....

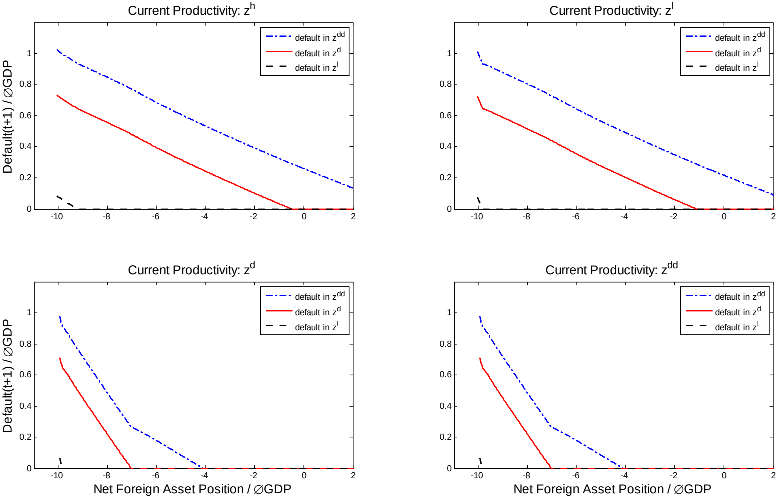
Calibrating Economic Disasters following Barro and Jin (2011):

Shock process $Z = \{z^h, z^l, z^d, z^{dd}\} = \{1.0133, 0.9868, 0.9224, 0.6696\}$ with transition matrix

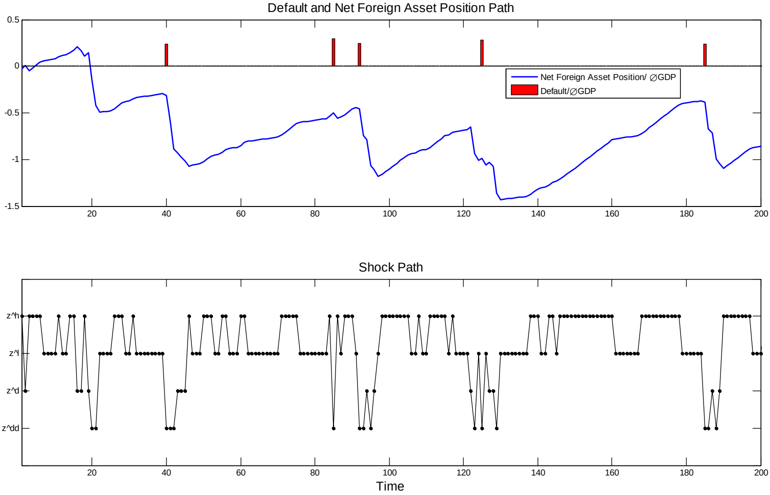
$$\pi = \begin{pmatrix} 0.7770 & 0.1850 & 0.019 & 0.019 \\ 0.1850 & 0.7770 & 0.019 & 0.019 \\ 0.1429 & 0.1429 & 0.3571 & 0.3571 \\ 0.1429 & 0.1429 & 0.3571 & 0.3571 \end{pmatrix}.$$

We recalibrate the subsistence level of consumption to $\bar{c} = 0.198$.

Optimal Default with Disaster Risk



NFA and Default under Optimal Policy

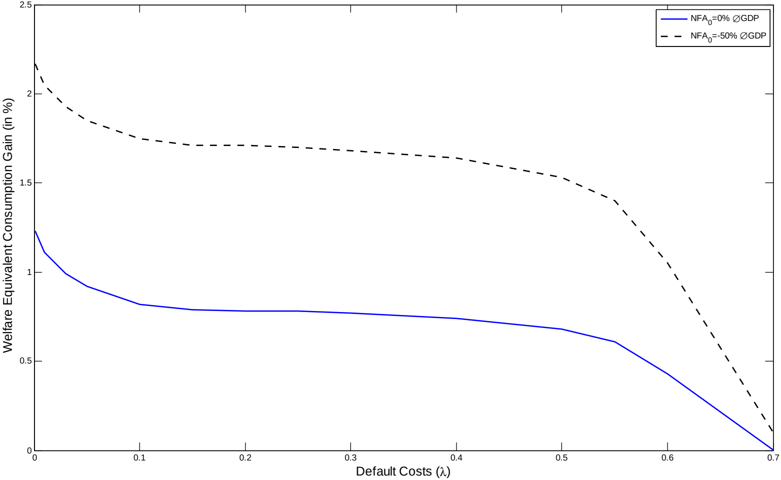


- Welfare equivalent consumption gain from default (first 500 years)
- Compute consumption change ω solving

$$E_0\left[\sum_{t=0}^{500}\beta^t\frac{((c_t^1(1+\omega)-\bar{c}))^{1-\gamma}}{1-\gamma}\right]=E_0\left[\sum_{t=0}^{500}\beta^t\frac{(c_t^2-\bar{c})^{1-\gamma}}{1-\gamma}\right]$$

- c_t^1 : optimal consumption path in the no-default economy (repayment assumed)
- c_t^2 : the corresponding consumption path with optimal (costly) default.

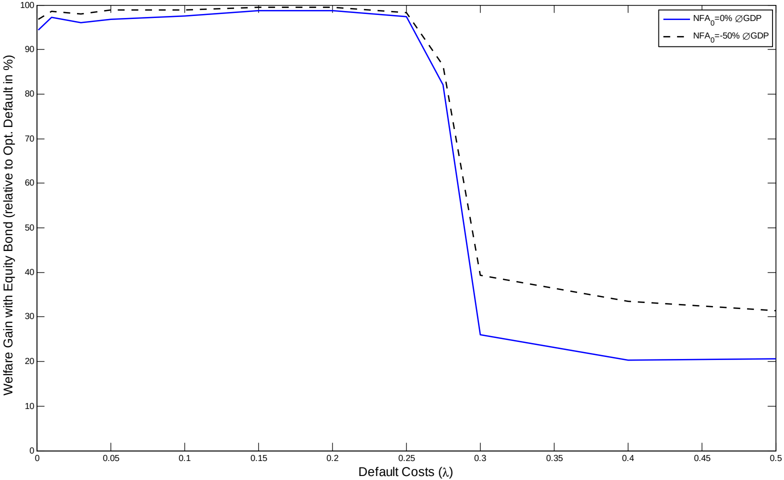
NFA and Default under Optimal Policy



Approximate Implementation

- The government now issues two kinds of financial instruments:
- Simple non-contingent bond with payoff $(1, 1, 1, 1)$
- Equity-like bond with payoff $(1, 1, 0, 0)$ plus default cost λ in disaster

Approximate Implementation: Welfare Gain Relative to Optimal Implementation



Longer Maturities

- No difference from introducing long foreign bonds: no value for insurance
- No difference from long domestic bonds if repayment is assumed (unlike in Angeletos(2001))
- Long domestic bonds with default option:

  (partial) default *in the future* after bad event *today* => bonds fall in value

  repurchase at depreciated value & realize a capital gain
- Improvements possible: if repurchase has lower dead weight costs than outright default....

Conclusion

- Default can be optimal under commitment if bond markets incomplete
- Relaxes borrowing limits, increases welfare & optimal after bad output realizations
  - following large disasters
  - if NFA position close to borrowing limit
- Welfare gains large (1-2% of cons.) & not very sensitive to default costs
- Simple equity bonds can approx. implement optimal default policies (for moderate default costs)
- Buyback program may be even more efficient