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"Capital Regulation, Liquidity Requirements and Taxation in a Dynamic Model of Banking"

Capital Regulation, Liquidity Requirements and Taxation in a Dynamic Model of Banking[*]

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ABSTRACT

This paper formulates a dynamic model of a bank exposed to both credit and liquidity risk, which can resolve financial distress in three costly forms: fire sales, bond issuance and equity issuance. We use the model to analyze the impact of capital regulation, liquidity requirements and taxation on banks' optimal policies and metrics of bank efficiency and welfare. We obtain three main results. First, mild capital requirements increase bank lending, bank efficiency and welfare relative to an unregulated bank, but these benefits turn into costs if capital requirements are too stringent. Second, liquidity requirements reduce bank lending, efficiency and welfare significantly, they nullify the benefits of mild capital requirements, and their efficiency and welfare costs increase monotonically with their stringency. Third, increases in corporate income and bank liabilities taxes reduce bank lending, bank efficiency and welfare, with tax receipts increasing with corporate taxation, but not changing significantly with liability taxation. Moreover, bank probability of default increases with liability taxation, contrary to the conjecture that these taxes may be a tool to control bank risk.

I. Introduction

The 2007–2008 financial crisis has been a catalyst for significant bank regulation reforms. The regulatory framework embedded in the Basel II capital accord has been judged inadequate to cope with large financial shocks. As a result, the proposed new Basel III framework envisions a significant raise in bank capital requirements and the introduction of new liquidity requirements.[1] At the same time, several proposals have been advanced to use forms of taxation with the twin objectives of raising funding to pay for resolution costs in stressed times, as well as a way to control risk behavior at large and complex financial institutions.[2]

Assessing the *joint* impact of these regulations and tax proposals on bank behavior is a difficult task. Recent central banks' efforts to quantify the impact of capital and liquidity requirements on both banks' behavior and the economy at large have produced mixed results.[3] One reason for the difficulty is that the relatively large literature on bank regulation offers few formal analyses where a *joint* assessment of these policies can be made in a *dynamic* context. To our knowledge, none of the existing work considers a bank exposed to both credit and liquidity risk, and evaluates the implications of capital regulation, liquidity requirements, and taxation on bank optimal policies, and metrics of bank efficiency and welfare. The formulation of a dynamic model with all these features is the main contribution of this paper.

Our model is novel in three important dimensions. First, we analyze a bank which dynamically transforms short term liabilities into longer-term illiquid assets whose returns are

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[1] According to the Basel Committee on Banking Supervision (*An Assessment of the Long–Term Economic Impact of Stronger Capital and Liquidity Requirements*, Bank for International Settlements, Basel, August 2010), the reform would increase the minimum common equity requirement from 2% to 4.5%. The Tier 1 capital requirement will increase from 4% to 6%. In addition, banks will be required to hold a "capital conservation buffer" of 2.5% to withstand future periods of stress bringing the total common equity requirements to 7%. Two new liquidity requirements are planned to be introduced: a short term liquidity coverage ratio, meant to ensure the survival of a bank for one month under stressed funding conditions, and a long-term so-called net stable funding ratio, designed to limit asset and liabilities mismatches.

[2] See e.g. Acharya, Pedersen, Philippon, and Richardson (2010) and *Financial Sector Taxation*, International Monetary Fund, Washington D.C., September 2010.

[3] Basel Committee on Banking Supervision (2010) evaluates the long–term economic impact of the proposed capital and liquidity reforms using a variety of models, including dynamic stochastic general equilibrium models. It finds that the net economic benefits of these reforms, as measured as a reduction in the expected yearly output losses associated with a lower frequency of banking crises, are positive for a broad range of capital ratios, but become negative beyond a certain range. However, the impact of higher capital levels and higher liquidity requirements on the probability of crises remains highly uncertain. Moreover, measurement errors in the computation of net economic benefits also arise from the use of banking crises classifications which record government responses to crises rather than adverse shocks to the banking system (see Boyd, De Nicolò, and Loukoianova (2010)).

uncertain. This feature is consistent with banks' special role in liquidity transformation emphasized in the literature (see e.g. Diamond and Dybvig (1983) and Allen and Gale (2007)). This allows us to capture important dynamic trade–offs, including banks' choices on whether, when, and how to continue operations in the face of financial distress.

Second, we model bank's financial distress explicitly. The bank in our model invests in risky loans and risk–less bonds financed by (random) government-insured deposits and short term fully collateralized debt. Financial distress occurs when the bank is unable to honor part or all of its debt and tax obligations for given realizations of credit and liquidity shocks. The bank has the option to resolve distress in three costly forms: by liquidating assets at a cost, by issuing fully collateralized bonds, or by issuing equity. The liquidation costs of assets are interpreted as fire sale costs, and modeled introducing asymmetric costs of adjustment of the bank's risky asset portfolio. The importance of fire sale costs in amplifying systemic banking distress has been brought to the fore in the recent crisis (see e.g. Acharya, Shin, and Yorulmazer (2010) and Hanson, Kashyap, and Stein (2011)).

Third, we evaluate the impact of bank regulations and taxation on bank optimal policies as well as in terms of metrics of bank efficiency and welfare. The first metric is the enterprise value of the bank, which can be interpreted as the efficiency with which the bank carries out its maturity transformation function. The second one, called "social value", proxies welfare in our risk-neutral world: it is given by shareholders' value less net bond holdings, plus the value of (insured) deposits, plus the value of government tax receipts net of default costs. To our knowledge, and with the exception of Van den Heuvel (2008) who focuses only on capital regulation, this is the first study that evaluates changes in welfare associated with capital regulation, liquidity requirements and taxation.

Our benchmark bank is unregulated, but its deposits are fully insured. We consider the unregulated bank as the appropriate benchmark, since one of the asserted key roles of capital regulation and liquidity requirements is the abatement of the excessive bank risk–taking arising from moral hazard under partial or total insurance of its liabilities. We use a standard calibration of the parameters of the model, with regulatory and tax parameters mimicking current capital regulation, liquidity requirement and tax proposals, to solve for the optimal policies and the metrics of interest for given state realizations, and present statistics of the steady state distribution of these policies and metrics by numerical simulation.

We obtain three sets of results. First, if capital requirements are mild, a bank subject only to capital regulation invests more in lending and its probability of default is lower than its unregulated counterpart. This additional lending is financed by higher levels of retained earnings or equity issuance, and its leverage is correspondingly lower than its unregulated peer. Our metrics of bank efficiency and social value indicate higher values of the bank subject to mild capital regulation relative to the unregulated bank. However, if capital requirements become too stringent, then the efficiency and welfare benefits of capital regulation disappear and turn into costs, even though default risk remains subdued: lending declines, and our metrics of bank efficiency and social value drop below those of the unregulated bank. These findings suggest the existence of an optimal level of bank-specific regulatory capital under deposit insurance.

Second, the impact of liquidity requirements on bank lending, efficiency and social value is significantly negative. When liquidity requirements are added to capital requirements, they eliminate the benefits of mild capital requirements, since bank lending, efficiency and social values are reduced relative to the bank subject to capital regulation only. In addition, the costs of these liquidity requirements, in terms of reductions in lending, efficiency and social values, increase monotonically with their stringency.

Lastly, an increase in corporate income taxes reduces lending, bank efficiency and social values due to standard negative income effects. However, tax receipts increase, generating higher government revenues. By contrast, the introduction of taxes on liabilities, while decreasing bank lending, efficiency and social values, do not generate an increase in government tax receipts owing to substitution effects. Interestingly, under liability taxation, bank's probability of default increases, contrary to the view that these taxes may be a tool to control bank risk.

The remainder of this paper is composed of six sections. Section II presents a brief review of the literature. Section III describes the benchmark model of an unregulated bank subject to standard corporate taxation. Section IV introduces capital regulation and liquidity requirements. Section V details the impact of bank regulation, while Section VI evaluates the impact of taxation. Section VII concludes with a brief discussion of the policy implications of the results. The Appendix describes some properties of the bank dynamic program and the computational procedures used for the simulation of the model.

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II. A brief literature review

The literature on bank regulation is large, but it offers few formal analyses of the impact of regulatory constraints on bank optimal policies in a dynamic framework.

The great majority of studies have focused on capital regulation. Capital requirements has been typically justified by their role in curbing excessive risk–taking (risk–shifting or asset substitution) induced by moral hazard of banks whose deposit are insured (for a review, see Freixas and Rochet (2008)). Based on the experience of the 2007-2008 financial crisis, Brunnermeier, Crockett, Goodhart, Persaud, and Shin (2009) have recently advocated the use of capital requirements designed to reduce the pro–cyclicality of financial institutions' leverage. Such pro–cyclicality has been identified as an important factor underlying the amplification of credit cycles, and likely to increase the probability of so–called illiquidity spirals, in which the entire banking system liquidates its assets at fire sale prices in a down–turn with adverse systemic consequences.[4]

However, whether an increase in capital requirements unambiguously reduces banks' incentives to take on more risk appears an unsettled issue even in the context of static models of banking. While several studies using partial equilibrium set–ups show that an increase in capital results in less risk (see, e.g. Besanko and Kanatas (1996), Hellmann, Murdock, and Stiglitz (2000), and Repullo (2004)), in other models of this type this conclusion can be reversed (see e.g. Blum (1999), and Calem and Rob (1999)), and such reversal can also occur in general equilibrium set–ups (see e.g. Gale and Özgür (2005), and De Nicolò and Lucchetta (2009) and Gale (2010)).

The relatively sparse literature of dynamic models of banking has exclusively focused on the pro-cyclicality aspects of bank capital regulation: capital requirements may increase in a recession and become less stringent in an expansion, thus amplifying fluctuations in lending and real activity. Even in this dimension, results are mixed depending on the details of the modeling set-ups. Estrella (2004) and Repullo and Suarez (2008) find that capital requirements are indeed pro-cyclical, while Peura and Keppo (2006) and Zhu (2008) find that this is not necessarily the case.

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[4]For a review of the literature on pro–cyclicality as related to capital regulation and some empirical evidence, see Zhu (2008) and Panetta and Angelini (2009)).

The papers presenting a modeling approach closest to ours are Van den Heuvel (2009) and Zhu (2008). Van den Heuvel (2009) focuses on bank responses to monetary shocks. He presents a dynamic model of a bank which invests in risky loans and risk–free securities, its deposits are government-insured, and it is subject to capital requirements, and finds such requirements are pro-cyclical. Extending the model by Cooley and Quadrini (2001), Zhu (2008) considers a bank that invests in a risky decreasing return to scale technology, its sole source of financing are uninsured (and fairly priced) deposits, faces linear equity issuance costs, and it is subject to minimum capital requirements. In sum, none of the papers we have reviewed consider a bank subject to both credit and liquidity risk, where financial distress can be resolved in three costly forms (fire sales, bond issuance and equity issuance).

III. The model

Time is discrete and the horizon is infinite. We consider a bank that receives a random stream of short term deposits, can issue risk–free short term debt, and invests in longer-term assets and short term bonds. The bank manager maximizes shareholders' value, so there are no managerial agency conflicts, and bank's shareholders are risk–neutral.

A. Bank's balance sheet

On the asset side, the bank can invest in a liquid, one–period bond (a T-bill), which yields a risk–free rate r, and in a portfolio of risky assets, called loans. We denote with B_t the face value of the risk–free bond, and with $L_t \geq 0$ the nominal value of the stock of loans outstanding in period t (i.e., in the time interval $(t-1, t]$). Similarly to Zhu (2008), we make the following

Assumption 1 (Revenue function). The total revenue from loan investment is given by $Z_t \pi(L_t)$, where $\pi(L_t)$ satisfies conditions $\pi(0) = 0$, $\pi > 0$, $\pi' > 0$, and $\pi'' < 0$.

This assumption is empirically supported, as there is evidence of decreasing return to scale of bank investments.[5] Loans may be viewed as including traditional loans as well as risky securities. Z_t is a random *credit shock* realized on loans in the same time period, which can

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[5]See for instance, Berger, Miller, Petersen, Rajan, and Stein (2005), Carter and McNulty (2005), Cole, Goldberg, and White (2004).

be viewed as capturing variations in banks' total revenues as determined, for example, by business cycle conditions. Note that the choice variables B_t and L_t are set at the beginning of the period, while Z_t is realized only at the end of the period.

The maturity of deposits is set to one period. Bank maturity transformation is introduced with the following

Assumption 2. (Loan reimbursement) A constant proportion $\delta \in (0, 1/2)$ of the existing stock of loans at t, L_t, becomes due at $t+1$.

The parameter $\delta < 1/2$ gauges the average maturity of the existing stock of loans, which is $(1-\delta)/\delta > 1$.[6] Thus, the bank is engaging in maturity transformation of short term liabilities into longer-term investments. Under Assumption 2, the law of motion of L_t is

$$L_t = L_{t-1}(1-\delta) + I_t, \tag{1}$$

where I_t is the investment in new loans if it is positive, or the amount of cash obtained by liquidating loans if it is negative.

To capture bank's monitoring and liquidation costs, we introduce convex asymmetric adjustment costs as in the Q-theory of investment (see e.g. Abel and Eberly (1994)) with the following

Assumption 3 (Loan Adjustment Costs). The adjustment costs function for loans is quadratic:

$$m(I_t) = |I_t|^2 \left(\chi_{\{I_t > 0\}} \cdot m^+ + \chi_{\{I_t < 0\}} \cdot m^- \right), \tag{2}$$

where $\chi_{\{\mathcal{A}\}}$ is the indicator of event $\mathcal{A}$, and $m^+ > m^- > 0$ are the unit cost parameters.

In increasing its investment in loans, the bank incurs monitoring costs, whereas in decreasing them the bank pays liquidation costs. Adjustment costs are deducted from profit. The asymmetry in the adjustment costs $(m^- > m^+)$ captures costly reversibility: the bank faces

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[6]The (weighted) average maturity of existing loans at date t, assuming the bank does not default nor it makes any adjustments on current the investment in loans, is

$$M_t = \sum_{s=0}^{\infty} s \frac{\delta L_{t+s}}{L_t} = \sum_{s=0}^{\infty} s\delta(1-\delta)^s = \frac{1-\delta}{\delta},$$

as the residual loans outstanding at date $t+s$, $s \geq 0$, is $L_{t+s} = L_t(1-\delta)^s$.

higher costs to liquidate investments rather then expanding them. The higher costs of reducing the stock of loans can be interpreted as capturing fire sales costs incurred in financial distress.

On the liability side, the bank receives a random amount of one-period deposits D_t at the beginning of period t, and this amount remains outstanding during the period. The stochastic process followed by D_t is detailed below. Deposits are insured according to the following

Assumption 4 (Deposit insurance). The deposit insurance agency insures all deposits. In the event the bank defaults on deposits and on the related interest payments, depositors are paid interest and principal by the deposit insurance agency, which absorbs the relevant loss.

Under this assumption, with no change in the model, the depositor can be viewed as the deposit insurance agency itself, and its claims are risky, while deposits are effectively risk–free from depositors' standpoint.[7] As in Zhu (2008), depositors' supply of funds is assumed to be perfectly elastic, and depositors' reservation rate is the risk–free rate, r. The difference between the ex–ante yield on deposits and the risk–free rate is a subsidy that the agency provides to the bank, as the cost of this insurance is not charged to either banks or depositors.

To fund operations, the bank can issue a one–period bond. The bank is constrained to issue fully collateralized bonds, so that their return is the risk–free rate. We denote $B_t < 0$ the notional amount of the bond issued at $t-1$ and outstanding until t. The collateral constraint is detailed below.

To summarize, at $t-1$ (or at the beginning of period t), after the investment and financing decisions have been made, the balance sheet equation is

$$L_t + B_t = D_t + K_t, \tag{3}$$

where K denotes the ex–ante book value of equity, or *bank capital*. In this equation, B denotes the face value of a risk–free investment when $B > 0$, and the face value of issued bond when $B < 0$.

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[7]This assumption is similar to Van den Heuvel (2009), but differs from Zhu (2008), who assumes that the bank will reward uninsured depositors with a risk premium.

B. Bank's cash flow

Once Z_t and D_{t+1} are realized at t, the current state (before a decision is made) is summarized by the vector $x_t = (L_t, B_t, D_t, Z_t, D_{t+1})$ as the bank enters date t with loans, bonds and deposits in amounts L_t, B_t, and D_t, respectively. *Prior* to its investment, financing and cash distribution decisions, the total internal cash available to the bank is

$$w_t = w(x_t) = y_t - \tau(y_t) + B_t + \delta L_t + (D_{t+1} - D_t). \tag{4}$$

Equation (4) says that total internal cash w_t equals bank's earnings before taxes (EBT),

$$y_t = y(x_t) = \pi(L_t)Z_t + r(B_t - D_t), \tag{5}$$

minus corporate taxes $\tau(y_t)$, plus the principal of one–year investment in bond maturing at t, $B_t > 0$ (or alternatively the amount of maturing one–year debt, $B_t < 0$) and from loans that are repaid, δL_t, plus the net change in deposits, $D_{t+1} - D_t$.

Consistently with current dynamic models of a non–financial firm (see e.g. Hennessy and Whited (2007)), corporate taxation is introduced with the following

Assumption 5 (Corporate Taxation). Corporate taxes are paid according to the following convex function of EBT:

$$\tau(y) = \tau^+ \max\{y, 0\} + \tau^- \min\{y, 0\}, \tag{6}$$

where τ^- and τ^+, $0 \leq \tau^- \leq \tau^+ < 1$, are the marginal corporate tax rates in case of negative and positive EBT, respectively.

The assumption $\tau^- \leq \tau^+$ is standard in the literature, as it captures a reduced tax benefit from loss carryforward or carrybacks. Note that convexity of the corporate tax function creates an incentive to manage cash flow risk, as noted by Stulz (1984).

Given the available cash w_t as defined in Equation (4) and the residual loans, $L_t(1 - \delta)$, bank's managers choose the new level of investment in loans, L_{t+1} and the amount of risk–free bonds B_{t+1} (purchased if positive, issued if negative). As a result, Equation (3) applies to B_{t+1}, L_{t+1}, D_{t+1}, and both L_{t+1} and B_{t+1} remain constant until the next decision date,

$t + 1$. However, these choices may differ according to whether the bank is or is not in financial distress. If total internal cash w_t is *positive*, it can be retained to change the investment in loans, it can be invested in one period risk–free bonds, or paid out to shareholders. On the other hand, if w_t is *negative*, the bank is in *financial distress*, since absent any action, it would be unable to honor part, or all, of its obligations towards either the tax authority, or depositors, or bondholders. When in financial distress, the bank can finance the shortfall w_t either by selling loans at fire sale prices, or by issuing bonds ($B_{t+1} < 0$), or by injecting equity capital. However, overcoming this shortage of liquidity is expensive, because all these transactions generate (either explicit of implicit) costs. In a fire sale, the bank incurs the downward adjustment cost defined in Equation (2), bond issuance is subject to a collateral restriction, and floatation costs are paid when seasoned equity are offered. We now present these latter two restrictions on the banks' financing channels.

Bank's issuance of bonds is constrained as described by the following[8]

Assumption 6 (Collateral constraint). If $B_t < 0$, the amount of bond issued by the bank must be fully collateralized. In particular, the constraint is

$$L_t - m(-L_t(1 - \delta)) + \pi(L_t)Z_d - \tau(y_{t+1}^{\min}) + (B_t - D_t)(1 + r) + D_d \geq 0, \tag{7}$$

where Z_d is the worst possible credit shock (i.e., the lower bound of the support of Z), D_d is the worst case scenario flow of deposits, and $y_{t+1}^{\min} = \pi(L_t)Z_d + (B_t - D_t)r$ is the EBT in the worst case end–of–period scenario for current L_t, B_t and D_t.

The constraint in (7) reads as follows: the end–of–period amount $B_t(1+r) < 0$ that the bank has to repay must not be higher than the after–tax operating income, $\pi(L_t)Z_d - rD_t - \tau(y_{t+1}^{\min})$, in the worst case scenario, plus the total available cash obtained by liquidating the loans, $L_t - m(-L_t(1 - \delta))$, plus the flow of new deposits in the worst case, D_d, net of the claim of current depositors, D_t. The proceeds from loans liquidation are the sum of the loans that will become due, $L_t\delta$, plus the amount that can be obtained by a forced liquidation of the loans, $L_t(1 - \delta)$ net of the adjustment cost $m(-L_t(1 - \delta))$, as from Equation (2).

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[8]As we will clarify later, in Assumption 9, the support for deposits and credit shock processes is compact. Therefore, the collateral constraint is well defined.

We denote with $\Gamma(D_t)$ the feasible set for the bank when the current deposit is D_t; i.e., the set of (L_t, B_t) such that condition (7) is satisfied, if $B_t < 0$, no restrictions being imposed when $B_t \geq 0$:

$$\Gamma(D_t) =$$
$$\left\{ (L_t, B_t) \mid \frac{L_t - m(-L_t(1-\delta)) + D_d + \pi(L_t)Z_d(1 - \tau(y_{t+1}^{\min}))}{1 + r(1 - \tau(y_{t+1}^{\min}))} + B_t \geq D_t, B_t < 0 \right\} \cup \{ B_t \geq 0 \}. \tag{8}$$

In the plane (L_t, B_t), the lower boundary of $\Gamma(D_t)$, when $B_t < 0$, is convex due to concavity of the revenue function. This means that a bank can fund more investment in risky loans by issuing more risk–free one period bonds. However, this investment has decreasing return to scale, and at some point the net return of a dollar raised by issuing bonds and invested in loans becomes negative. Figure 3 shows this for a given set of parameters.

Bank's costs on equity issuance are due to information asymmetry and underwriting fees, and are modeled in a standard fashion (see e.g. Cooley and Quadrini (2001)) with the following

Assumption 7 (Equity floatation costs). The bank raises capital by issuing seasoned shares incurring a proportional floatation cost $\lambda > 0$ on new equity issued.

As a result of the choice of (L_{t+1}, B_{t+1}), the residual cash flow to shareholders at date t is

$$u_t = u(x_t, L_{t+1}, B_{t+1}) = w_t - B_{t+1} - L_{t+1} + L_t(1 - \delta) - m(I_{t+1}). \tag{9}$$

When u_t is positive, it is distributed to shareholders (either as dividends or stock repurchases). If u_t is negative, it is the amount of newly issued equity. Hence, the actual cash flow to equity holders is

$$e_t = e(x_t, L_{t+1}, B_{t+1}) = \max\{u_t, 0\} + \min\{u_t, 0\}(1 + \lambda). \tag{10}$$

A description of the evolution of the state variables and of the related bank's decisions, when the bank is solvent, is in Figure 1.

Lastly, bank's insolvency is described by the following

Assumption 8 (Insolvency). In the case of bank's default, shareholders exercise the limited liability option (i.e., equity value is zero), and bank assets are transferred to the deposit insurance agency, net of verification and bankruptcy costs in proportion $\gamma > 0$ of the face value of deposits, γD_t. Right after default, the bank is restructured as a new entity endowed with capital $K_{t+1} = D_u - D_d > 0$ and deposits D_d, where D_u is the upper bound of deposit process. The restructured bank invests initially only in risk–free bonds, $B_{t+1} = D_u$, so that $L_{t+1} = 0$.

The probabilistic assumptions of our model are as follows. There are two exogenous sources of uncertainty: the credit shock on the loan portfolio, Z, and the funding available from deposits, D. Denote with $s = (Z, D)$ the pair of state variables, and with $\mathcal{S}$ the state space.

Assumption 9. The state space $\mathcal{S}$, is compact. The random vector s evolves according to a stationary and monotone (risk–neutral) Markov transition function $Q(s_{t+1} \mid s_t)$ defined as

$$Z_t - Z_{t-1} = (1 - \kappa_Z)\left(\overline{Z} - Z_{t-1}\right) + \sigma_Z \varepsilon_t^Z \tag{11}$$

$$\log D_t - \log D_{t-1} = (1 - \kappa_D)\left(\log \overline{D} - \log D_{t-1}\right) + \sigma_D \varepsilon_t^D. \tag{12}$$

The error terms ε_t^Z and ε_t^D are i.i.d and have jointly normal truncated distribution with correlation coefficient ρ.[9]

In the above equations, κ_Z is the persistence parameter, σ_Z is the conditional volatility, and $\overline{Z}$ is the long–term average of the credit shock; κ_D is the persistence parameter for the deposit process, σ_D is the conditional volatility, and $\overline{D}$ is the long–term level of deposits.

C. The unregulated bank program and the valuation of securities

Let E denote the market value of bank's equity. Given the state, $x_t = (L_t, B_t, D_t, Z_t, D_{t+1})$, bank's equity value is the result of the following program

$$E_t = E(x_t) = \max_{\{(L_{i+1}, B_{i+1}) \in \Gamma(D_{i+1}), i=t,\dots,T\}} \mathbb{E}_t\left[\sum_{i=t}^{T} \beta^{i-t} e(x_i, L_{i+1}, B_{i+1})\right],$$

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[9]As we clarify in the Appendix, the support of each state variable is within three times the unconditional standard deviation of each marginal distribution around the longterm average.

where $\mathbb{E}_t[\cdot]$ is the expectation operator conditional on D_t, on the state variables at t, (Z_t, D_{t+1}), and on the decision (L_{t+1}, B_{t+1}); β is a discount factor (assumed constant for simplicity); (L_{i+1}, B_{i+1}) is the decision at date i, for $i = t, \ldots$. In the case of bank's insolvency (Assumption 8), the bank is valued as a going concern at the current state but assuming it is unlevered (i.e., with $D = 0$). We denote T the default date.

Because the model is stationary and the Bellman equation involves only two dates (the current, t, and the next one, $t+1$), we can drop the time index t and use the notation without a prime for the current value of the variables, and with a prime to denote end–of–period value of the variables. The value of equity satisfies the following Bellman equation

$$E(x) = \max \left\{ 0, \max_{(L', B') \in \Gamma(D')} \left\{ e(x, L', B') + \beta \mathbb{E}\left[E(x') \right] \right\} \right\}. \tag{13}$$

Compactness of the feasible set of the bank and standard properties of the value function are described in the Appendix.

When the bank is solvent, the Bellman equation for optimality of the solution is

$$E(x) = \max_{(L', B') \in \Gamma(D')} \left\{ e(x, L', B') + \beta \mathbb{E}\left[E(x') \right] \right\}. \tag{14}$$

We denote with $(L^*(x), B^*(x))$ the optimal policy when the bank is solvent. When it is insolvent, shareholders exercise the limited liability option, which puts a lower bound on E at zero. The default indicator function is denoted $\Delta(x)$.

We solve equation (13) to determine the value of equity, the optimal policy including the optimal default policy, Δ, as a function of the current state, x. We denote φ, the state transition function based on the optimal policy:

$$\varphi(x) = \begin{pmatrix} L^* \\ B^* \\ D' \end{pmatrix} (1 - \Delta) + \begin{pmatrix} 0 \\ D_d \\ D_u \end{pmatrix} \Delta, \tag{15}$$

meaning that the new state is (L^*, B^*, D') if the bank is solvent, and $(0, D_u, D_d)$ if the bank defaults and a new bank is started endowed with seed capital $D_u - D_d$ and deposits, D_d,

and a cash balance D_u, and no loans.[10] The restructured bank is endowed with cash, which is momentarily invested in bonds. This bank will revise its investment (together with the financing) policy in the following decision dates.

The end–of–year cash flow from current deposits, D_{t+1}, for a given realization of the exogenous state variables, (Z_{t+1}, D_{t+2}), and on the related optimal policy, is

$$f(x_{t+1} \mid \varphi(x_{t+1})) = D_{t+1}(1 + r)(1 - \gamma\Delta(x_{t+1})).$$

Hence, the ex–ante fair value of newly issued deposits at t, from the viewpoint of the deposit insurance agency (i.e, incorporating the risk of bank's default), is

$$F(x_t) = \beta\mathbb{E}_t\left[f(x_{t+1} \mid \varphi(x_{t+1}))\right] = \beta D_{t+1}(1+r)\left(1 - \gamma\mathbb{E}_t\left[\Delta(x_{t+1})\right]\right) = \beta D_{t+1}(1+r)\left(1 - \gamma P(x_t)\right),$$

where $P(x_t) = \mathbb{E}_t\left[\Delta(x_{t+1})\right]$ is the conditional default probability. Dropping the dependence on the calendar date,

$$F(x) = \beta D'(1 + r)\left(1 - \gamma P(x)\right).$$

D. Efficiency and welfare metrics

A standard valuation concept is the market value of bank assets $E(x) + F(x)$, which includes current cash holdings, B. Yet, the market value of bank's assets does not necessarily capture the role of banks as maturity transformers of liquid liabilities into longer term *productive* assets (loans). One of the key economic contributions of banks identified in the literature is their role in efficiently intermediating funds toward their best productive use (see e.g. Diamond (1984) and Boyd and Prescott (1986)). But banks play no such role if they just raise funds to acquire risk–less (cash–equivalent) bonds. While the investment in risk–free bonds helps reducing the costs triggered by high cash flows volatility, it is not providing necessarily efficient intermediation. Thus, the *enterprise value* of the bank, defined as $V(x) = E(x) + F(x) - B$, is a more appropriate metric of bank efficiency in terms of its ability to create "productive" intermediation.[11]

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[10]Computationally, since default is irreversible, we must allow a new bank to re-enter otherwise there would eventually be no banks in the simulated sample.

[11]For the use of enterprise value as a metric of efficiency in the context of dynamic models of non–financial firms, see e.g. Gamba and Triantis (2008) and Bolton, Chen, and Wang (2009).

A metric proxying welfare in our risk neutral world is the "social" value of the bank, which is constructed as follows. The value of default costs on bank's current deposits is

$$DC(x) = \beta D'(1+r)\gamma P(x).$$

This is a measure of the expected losses suffered by the deposit insurance agency, which is a linear function of the loss–given–default parameter, γ, and of the default probability, $P(x)$. The value of the tax payoff to the Government is defined by the recursive equation

$$G(x) = \tau(y')\left(1 - \Delta(x)\right) + \beta\mathbb{E}\left[G(x')\right].$$

Hence, the social value of the bank is the sum of values to all the stakeholders in the model: enterprise value plus the value to (insured) depositors, plus the value to the Government, net of the value of default costs

$$SV(x) = E(x) - B + \beta D'(1+r) + G(x) - DC(x).$$

Given the above definition and the fact that $F(x) = \beta D'(1+r) - DC(x)$, we get $SV(x) = V(x) + G(x)$: the social value of the bank turns out to be the enterprise value plus the value of taxes. In essence, $SV(x)$ captures the net impact on welfare of stricter constraints on bank policies, which in general reduce the value of the bank's loans and the flow of corporate taxes, but also abate expected bailout cost.

IV. Bank regulation

Bank regulations are typically associated with specific bank closure rules that differ from those that would arise in an unfettered setting, since many of them are based on accounting norms. Specifically, capital regulations are universally based on measures of *accounting* capital, rather than economic net worth (see e.g. Saunders and Cornett (2003)). As a result, bank insolvency differs from that of an unregulated bank. In our model, bank's insolvency under regulation is defined by the following

Assumption 10 (Bank Closure Rule). The bank is closed at time t if the ex–post net asset value (i.e., ex–post bank capital, as opposed to K_t, the ex–ante bank capital) is negative:

$$v_t = L_t + B_t - D_t + y_t - \tau(y_t) = K_t + y_t - \tau(y_t) < 0. \tag{16}$$

After the closure, the market value of equity is set to zero (absolute priority rule), the assets are transferred to the deposit insurance agency, net of verification and bankruptcy costs, and the bank is restructured as per Assumption 8.

This mechanism corresponds to a condition of ex–post negative book equity, as in Zhu (2008). However, differently from Zhu, we assume that the bank, after restructuring, continues to operate as a new entity. In this case, the Bellman equation for optimality of the solution of the bank's program, when the bank is solvent (i.e., $v_t \geq 0$ as in Assumption 10), is as in equation (14).

A. Capital requirement

The first pillar of Basel II regulation establishes a lower bound K_d on the book value of equity K, set by the regulator as a function on bank's risk exposure at the beginning of the period. In particular, this requirement is a weighted average of banks risks.[12] Since our model has just one composite risky asset, we set the weight applied to loans equal to 100%. Thus, in our setting the *required* capital K_d is at least a proportion k of the principal of the loans at the beginning of the period, L, or $K_d = kL$. This requirement is equivalent to constraining net worth to be positive ex–ante. Given the definition of bank capital in (3), under the capital requirement the bank's feasible choice set is

$$\Theta(D) = \{(L, B) \mid (1 - k)L + B \geq D\}. \tag{17}$$

Relating the feasible choice set under the collateral constraint in Equation (8) to the feasible set under the capital requirement, in general neither $\Gamma(D) \subset \Theta(D)$ nor $\Theta(D) \subset \Gamma(D)$ in a

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[12]The specific norm is in *International Convergence of Capital Measurement and Capital Standards*, Bank for International Settlements, Basel, June 2006, Part 2: The First Pillar - Minimum Capital Requirements, Ia The Constituents of Capital A. Core Capital (Basic Equity or Tier 1).

proper sense. Hence, the capital requirement may (or may not) restrict the bank's feasible policies, depending on the values of the parameters.

If the bank is short term borrowing, $B < 0$, for a given D the capital constraint results in a restriction of the bank's choice set if $\Theta(D) \subset \Gamma(D)$. This is equivalent to

$$\frac{L - m(-L(1-\delta)) + D_d + \pi(L)Z_d(1 - \tau(y^{\min}))}{1 + r(1 - \tau(y^{\min}))} \geq (1 - k)L.$$

Since the inequality is independent of D, what follows holds for any current D. If we assume a constant corporate tax rate (in place of two tax rates: τ^+ and τ^-) for the sake of simplicity, for a large range of values of the model parameters and of L, the above inequality is satisfied. This means that the capital requirement restricts the bank's policy. Alternatively, if the bank is short term lending, $B \geq 0$, then the capital requirement restricts the choice set if $L < D/(1-k)$, because it forces the bank to have a fairly large cash balance B, while the constraint is not binding if $L \geq D/(1-k)$. Figure 2 shows how the capital requirement is related to the collateral constraint for a specific set of parameters.

The Bellman equation for the equity value of a currently solvent bank under a capital requirement is given by Equation (14), the only difference being a feasible set $\Gamma(D') \cap \Theta(D')$ in place of $\Gamma(D')$. Hence, the bank is forced to comply ex–ante with the capital requirement. However, at the end of the period, when the credit shock on existing loans, Z', and the new deposit, D', are realized, the bank may still face default risk if the innovations of the state variables are particularly unfavorable, and in particular, if the shock on loans is significantly negative.

B. Liquidity requirement

The current Basel III regulatory proposals include the introduction of a mandatory liquidity coverage ratio: banks would be prescribed to hold a stock of high quality liquid assets such that the ratio of this stock over what is defined as a net cash outflows over a 30-day time period is not lower than 100%. In turn, the net cash outflow is expected to be determined by what would be required to face an acute short term stress scenario specified by supervisors. Banks

would need to meet this requirement continuously as a defense against the potential onset of severe liquidity stress.[13]

In our model, the stock of high quality liquid assets over the net cash outflows over a period is given by the total cash available at the end of the period over the total net cash flow in the worst case scenario for both credit shocks and deposit flows. Formally, this liquidity coverage ratio should be not lower than a level ℓ defined by the regulator, or

$$\frac{\delta L + Z_d \pi(L) - \tau(y^{\min}) + B(1+r)}{D(1+r) - D_d} \geq \ell. \tag{18}$$

Hence, the feasible set for a bank complying with the liquidity requirement is

$$\Lambda(D) = \left\{ (L, B) \mid \frac{\delta L + \ell D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{\ell(1+r) - \tau(y^{\min})r} + B \frac{(1 + r(1 - \tau(y^{\min})))}{\ell(1+r) - \tau(y^{\min})r} \geq D \right\}. \tag{19}$$

Interestingly, for the case with $\ell = 1$, this simplifies to

$$\Lambda(D) = \left\{ (L, B) \mid \frac{\delta L + D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{(1 + r(1 - \tau(y^{\min})))} + B \geq D \right\},$$

so that we can directly compare this to the collateral constraint, $\Gamma(D)$.

For a bank that is short term borrowing, $B < 0$, the liquidity constraint restricts the feasible choice set, or $\Lambda(D) \subset \Gamma(D)$, if

$$\frac{L - m(-L(1 - \delta)) + D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{1 + r(1 - \tau(y^{\min}))} \geq \frac{\delta L + D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{(1 + r(1 - \tau(y^{\min})))},$$

or equivalently, if $L(1 - \delta) \geq (L(1 - \delta))^2 m^-$. This is indeed the case for a wide range of parameters and a large set of values of L. Moreover, the liquidity constraint always restricts the feasible choice set when the bank is short term lending, $B > 0$. In sum, the liquidity constraint turns out to restrict the bank's feasible choice set relative to the collateral constraint for a wide range of parameter values. Figure 2 shows a comparison of the liquidity requirement to the collateral constraint for a specific choice of parameters.

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[13]See Basel Committee on Banking Supervision, *International Framework for Liquidity Risk Measurement, Standards and Monitoring*, Bank for International Settlements, Basel, December 2009.

Lastly, we can compare the capital requirement with the liquidity constraint for $\ell = 1$. For a given D, the capital requirement is more restrictive than the liquidity requirement if $\Theta(D) \subset \Lambda(D)$, or

$$\frac{\delta L + D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{(1 + r(1 - \tau(y^{\min})))} \geq (1 - k)L.$$

Assuming a constant tax rate (independent of the EBT), there is a threshold level $\widehat{L}$ where the above inequality holds as an equation. For L lower than $\widehat{L}$, the capital constraint is more restrictive that the liquidity constraint. Viceversa for $L > \widehat{L}$. Overall, when considered together, the two constraints create considerable restrictions on bank's feasible choices.

V. The impact of bank regulation

The evaluation of the impact of bank regulations on banks' optimal policies and the two metrics defined previously proceeds as follows. First, we illustrate basic bank policy trade-offs through the lenses of a simplified version of our model (Subsection A). Second, we describe a set of benchmark parameters calibrated using selected statistics from U.S. banking data, some previous studies, and current regulatory and tax parameters (Subsection B), and we carry out a simulation exercise using these parameters. We present the results of the optimal bank policies and relevant metrics under bank regulation for given realized states (Subsection C) and, finally, we report the statistics of the steady state distributions of optimal policies and our metrics (Subsection D).

A. Optimal policies in a simplified version of the model

To illustrate key trade-offs on bank optimal policies implied by regulatory restrictions in the simplest possible way, we collapse our model to two periods, where t is the decision date, $t+1$ is the final date, and the bank initial conditions are determined at $t-1$,

We make the following two sets of simplifying assumptions. First, there are no taxes, no adjustment costs, no floatation costs on equity issuance, no financial distress at t ($w_t \geq 0$), and no uncertainty about deposit flows ($D_t = D_{t+1} = D > 0$, and $D_{t+2} = 0$ since $t+1$ is the last period). Second, we set $\delta = 0$ and $\beta = (1 + r)^{-1}$, and assume a simple two point

credit shock distribution: Z^H with probability $p \in (0,1)$, and Z^L otherwise, where Z^L is such that $Z_d = \frac{Z^L L_{t+1}}{\pi(L_{t+1})}$, with $Z^H > (1-p)r > Z^L > -1$. Under these assumptions, the collateral constraint (C) for $B_{t+1} < 0$, the capital constraint (K), and the liquidity constraint (L) with $\ell = 1$ are

$$B_{t+1} \geq \frac{r}{1+r}D - \frac{1+Z^L}{1+r}L_{t+1} \tag{C}$$

$$B_{t+1} \geq D - (1-k)L_{t+1} \tag{K}$$

$$B_{t+1} \geq \frac{r}{1+r}D - \frac{Z^L}{1+r}L_{t+1} \tag{L}$$

The bank chooses (L_{t+1}, B_{t+1}) to maximize

$$e_t + \frac{1}{1+r}\mathbb{E}_t\left[e_{t+1}\right] = w_t + L_t - (1-p)B_{t+1} - L_{t+1}$$
$$+ \frac{1}{1+r}\left[p\left(Z^H\pi(L_{t+1}) - (1+r)D + L_{t+1}\right)\right.$$
$$\left. + (1-p)\max\left\{0, Z^L L_{t+1} + (1+r)(B_{t+1} - D) + L_{t+1}\right\}\right] \tag{20}$$

Since $1 - p > 0$, it is optimal to maximize debt $(B_{t+1} < 0)$, since in the good state profits are increasing in debt, while in the bad state losses are bounded to be positive by limited liability. This implies that at most one of the constraints (C), (K), and (L) will be binding.

The unregulated bank maximizes (20) subject to constraint (C). Substituting (C) into (20), the term $\max\{\cdot\}$ vanishes and the optimal loan level L_{t+1}^c satisfies

$$pZ^H\pi'(L_{t+1}^c) = r - (1-p)Z^L. \tag{21}$$

The expected return on lending is equated to the return on holding cash implied by the binding collateral constraint. Clearly, L_{t+1}^c declines with r and Z^L.

Suppose now that the capital constraint (K) is tighter than (C) at the optimal choice L_{t+1}^c, that is, (K) is binding. For a sufficiently small k representing a "mild" capital constraint, it can be verified that the term $\max\{\cdot\}$ in (20) is zero, and the the optimal loan level L_{t+1}^k satisfies

$$PZ^H\pi'(L_{t+1}^k) = r + (1-p)(1-(1-p)(1-k)). \tag{22}$$

In this case, the return on holding cash varies positively with the capital constraint, with a higher k being associated with a lower L_{t+1}^k.

Using (21) and (22), we have

$$L_{t+1}^k > L_{t+1}^c \quad \text{if} \quad Z^L < (1+r)(1-k) - 1 \tag{23}$$

If $k = 0$ (or sufficiently close to zero), then it can be easily seen that (23) is satisfied and $L_{t+1}^k > L_{t+1}^c$. Thus, when (K) is binding, lending can be higher than in the unregulated case under mild capital requirements even though borrowing is lower ($B_{t+1}^k > B_{t+1}^c$ holds when constraint (K) is more stringent than (C)). This is because the capital requirement raises the expected return on loan investment relative to holding cash.

Note that the difference between enterprise values in the two cases is

$$V^k - V^c = Z^H \frac{p}{1+r} \left(\pi(L_{t+1}^k) - \pi(L_{t+1}^c) \right) + \left(B_{t+1}^U - B_{t+1}^K \right).$$

Therefore, under mild capital requirements, $V^k - V^c$ can be strictly positive if the first term at the right–hand–side of the above expression, which is positive under a mild capital requirement, is sufficiently large to offset the second difference term. In other words, under mild capital requirements, bank efficiency can be enhanced relative to the unregulated case.

Finally, consider the addition of a liquidity requirement to the capital constraint. Inspecting (L) and (C), it can be seen that the liquidity constraint is always tighter than the collateral constraint. Suppose now that the liquidity constraint (L) is tighter than (K) at the optimal choice L_{t+1}^k, that is, (L) is binding. Putting (L) in (20), the max$\{\cdot\}$ term turns into max$\{0, -D + L_{t+1}\}$.

If at the optimal solution $L_{t+1} < D$, then L_{t+1}^ℓ satisfies

$$pZ^H \pi'(L_{t+1}^\ell) = r - (1-p)Z^L. \tag{24}$$

If, alternatively, at the optimal solution $L_{t+1} > D$, then L_{t+1}^ℓ satisfies

$$pZ^H \pi'(L^\ell t + 1) = r - Z^L. \tag{25}$$

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Comparing (23) with either (24) or (25), it is easy to verify that the right hand side of (22) is always strictly lower than that of (24) and (25) when the capital requirement is mild and $Z^L < r(1-p)$, as assumed. This implies that $L_{t+1}^{\ell} < L_{t+1}^k$: the liquidity constraint unambiguously reduces lending relative to the bank subject to a (binding) mild capital constraint. Moreover, the enterprise value of the bank subject to liquidity constraint is always lower than that of a bank subject to a (binding) mild capital constraint. Thus, the liquidity constraint imposes fairly strong restrictions on bank's optimal choices.

B. Calibration

Our calibration of the model is based on three sets of parameters, summarized in Table I. The first set comprises parameters of the two exogenous state variables. We estimated the VAR of equations (11) and (12) using U.S. yearly aggregate time series for the period 1983-2009 for the entire universe of banks included in the Federal Reserve Call Reports constructed by Corbae and D'Erasmo (2011). The shock process was proxied by the return on bank investments before taxes, given by the ratio of interest and non-interest revenues to total lagged assets. As can be seen in Table I, the shock process exhibits high persistence and the correlation with the process of (log)deposit is negative. Estimates of the autocorrelation process for (log) deposit produced estimates closed to unity, indicating the possibility that such process has a unit root. To guarantee convergence of the fixed point algorithm, we set this parameter equal to 0.95.

The second set of parameters is taken from previous research. The annual discount factor β is set to 0.95, equal to that used by Zhu's (2008) and Cooley and Quadrini (2001)). The risk–free rate is set to 2.5%. This value is consistent with the average effective cost of funds documented in Corbae and D'Erasmo (2011), and falls between the one used by Zhu (2008) and that used by Cooley and Quadrini (2001). With regard to corporate taxation, recall that the tax function is defined by the marginal tax rates, τ^+ and τ^-, for positive and negative income, respectively. Since we do not explicitly consider dividend and capital gain taxation for shareholders or interest taxation for depositors and bond holders, the two marginal rates for corporate taxes are to be considered net of the effect of personal taxes. For this reason we choose $\tau^+ = 15\%$, which is close to the values determined by Graham (2000) for the marginal tax rate. The marginal tax rate for negative income is $\tau^- = 5\%$ to allow for convexity in the corporate tax schedule.

Furthermore, the proportional bankruptcy cost is $\gamma = 0.10$, This is a value close to the (structural) estimate of 0.104 for this cost based on U.S. non–financial firms found by Hennessy and Whited (2007). Since this estimate is based on nonfinancial firms, it can be viewed as a lower bound for bankruptcy costs incurred in the financial sector. The annual percentage of reimbursed loan is 20%, so that the average maturity of outstanding loans is 4 years, in line with the assumption made by Van den Heuvel (2009). The floatation cost for seasoned equity issuance is 30%, as in Cooley and Quadrini (2001). This means the bank incurs a significant transaction cost to tap the equity capital market when in financial distress.

We specify the revenue function from loan investment as $\pi(L) = L^{\alpha}$, as in Zhu (2008), and set our base case value for α to 0.95, which is in line with the one used in other papers. Lastly, we set $m^+ = 0.03$ and $m^- = 0.04$ by matching two moments from empirical data. The first moment is the average Bank Credit over Deposit ratio, in which bank credit is loans and other financial investments. From our dataset, this is 1.271. The second moment we match is bank's book leverage, or deposits plus other financing liabilities over loans and other financial investments. In the data, the average book leverage is 0.89. The corresponding unconditional moments from a Monte Carlo simulation of the model with the selected parameters are respectively 1.1098 and 0.9031.

The third set of parameters is based on regulatory prescriptions. In our case, these are the ratio of capital to risk–weighted assets and the liquidity coverage ratio. The capital ratio k is set to 4%, as in current Basel II regulation of Tier 1 capital ratios. The liquidity ratio is $\ell = 1$, based on current Basel III proposals.

C. State-dependent analysis

In this section we present the results of the impact of bank regulations on optimal policies and our metrics of bank efficiency and welfare for given realization of the states. We consider three cases: the unregulated bank, the bank subject to capital regulation only, and that subject to both capital regulation and liquidity requirements. While many states can be possibly chosen, we set our analysis at the steady state for both deposits ($D = 2$) and credit shock ($Z = 0.0717$), while choosing $B = 0$ to avoid the impact of current liquidity, and $L = 4.1$, which is very close

to the unconditional median of L for several versions of the model. As a result, bank's capital is $K = 2.1$.

Figure 3 depicts the impact of regulatory restrictions on the bank's policy related to loan investment and to short term investment and financing. Tables II and III report average capital ratios and liquidity coverage ratios for a solvent bank under the three cases as functions of different levels of credit shocks and different levels of bank capital at a point in time.

Consider first the unregulated bank. Figure 3 shows that when D' is low (liquidity shock) or Z is low (credit shock), the bank reduces short term debt and liquidates loans. Conversely, with an expansion both in liquidity (high D') or in credit (high Z), there is an increase in loan investment funded by issuing short term bonds, although less then proportionally for a liquidity shock, when compared to a credit shock. It is important to note that the loan policy of an unregulated bank is *pro-cyclical*, since loans vary positively with credit and liquidity conditions. Thus, pro-cyclicality of lending is a feature of an optimal policy independently of capital requirements. As shown in Table II and Table III), the resulting average capital and liquidity coverage ratios are *negative* in all cases. This means that the unregulated bank takes an exposure in loans so that, in the worst case scenario for the credit and the liquidity shocks, a forced liquidation of loans is likely needed. In essence, the bank is trading off liquidation costs with the benefits of a larger investment in loans.

Consider now the bank subject to capital regulation only. Relative to the unregulated bank, this bank takes on less debt but invests more in loans than its unregulated counterpart for every realization of liquidity and credit shocks. This is the scenario identified in the simplified model under a mild capital requirement. More generally, by equation (17), the bank satisfies the capital requirement by increasing loans at a rate proportionally higher than the capital ratio coefficient. On the one hand, the capital requirement forces the bank to reduce its indebtedness relative to its loan investment, while also reducing the rate of return of holding cash and the risk of a bank closure. As a result, the expected returns on loans may increase, prompting a higher investment in loans. Importantly, loan increase with both higher levels of new deposits and more favorable credit shocks although not proportionally when there is a positive liquidity shock (see Figure 3). Thus, as in the case of the unregulated bank, loans vary positively with credit and liquidity conditions. However, under our parameterization, the

pro-cyclicality of lending relative to the unregulated case does not appear to be significantly enhanced by capital requirements, as the slope of the relevant loan policies are almost parallel.

As shown in Table II, the capital ratio is constantly higher than the prescribed level of 0.04, due to the (shadow) cost of the capital requirement, which prompts the bank to use retained earnings as a precautionary tool to avoid hitting the relevant constraint. As shown in Table III, the average liquidity coverage ratio of the bank under capital regulation is larger than its unregulated counterpart, owing to the higher level of loan investment and a less than proportional increase in short term financing, which raise the numerator of Equation (18).

By adding a liquidity requirement to a capital requirement, Figure 3 shows that both debt and investment levels shrink substantially, and in particular, given the current state, the bank likely ends up reducing the loan investment. As we have seen with our example of a simple static version of the model, and as is apparent in Figure 2, the fact that the liquidity requirement is far more restrictive than the capital requirement for large enough L induces a sharp decline in both debt and loan investment.

The dominant tightness of the liquidity requirement is also reflected in the average capital ratios and the liquidity coverage ratios reported in Tables II and III respectively. The average capital ratio under a liquidity requirement becomes inflated relative to the previous case, and is pushed up by a relatively large net bond holding (the numerator) and a lower investment in loans (the denominator). Note that this mechanism is totally different from that induced by capital regulation, since in that case the capital ratio is ultimately pushed up by retained earnings and possibly equity issuance. Not surprisingly, the average liquidity coverage ratio is higher than the prescribed level ($\ell = 1$), since the (shadow) cost associated with the liquidity constraint forces the bank to hold precautionary extra cash to avoid hitting that constraint.

Turning to our efficiency and welfare metrics, Figure 4 shows enterprise and social values divided by the corresponding values for the unregulated case for the bank subject to capital regulation, as well as that subject to both capital regulation and liquidity requirements.

With regard to capital regulation, there is a value loss in both metrics, since the relevant ratios are all below one. The value losses associated with capital regulation are more severe the lower are the levels of new deposits and the credit shock. This is because in a downturn, during which credit quality deteriorates, the bank is forced to liquidate more loans, thereby incurring

in significant liquidation (fire sale) costs. Thus, these results suggest that the losses in values associated with capital regulation appear to be largest for adverse realizations of both credit and liquidity risk parameters. Here the efficiency and social costs of "pro-cyclicality" take the form of relative over–investment in loans in an upswing, and relative high liquidation costs in a downturn. This result is somewhat at variance with the conjecture that capital regulation may abate both the private or social values of bank distress, since the cost it imposes on the bank and its intermediation capacity are highest precisely when the bank is likely to be in financial distress.

With regard to the bank subject to capital regulation and liquidity requirements, Figure 4 shows that the losses of efficiency and social value are significantly larger than those in the previous case. But as in the previous case, the value loss appears to be largest for adverse realizations of both credit risk and liquidity risk parameters.

While the point chosen for our state-dependent analysis is representative of the unconditional distribution of the shocks and of the policies of the bank, the results presented in this part cannot be considered decisive. For this reason, in the next part, we will provide numerical results based on a Monte Carlo simulation, in order to capture the steady state behavior of the bank.

D. Dynamics

Here we report results based on the unconditional distribution of the values and policies produces by the model. Recall that a bank is represented by the set of parameters governing credit and liquidity shocks, its revenue function, the maturity of its loan portfolio, and loan adjustment costs. We subject this bank to a large number of shocks and compute statistics of its optimal choices and our metrics. Since the bank's optimal policies are path-dependent, we simulate a random sample of 10,000 paths of the exogenous shocks (or 10,000 possible scenarios this bank may face) of 100 annual periods each, and apply the optimal policy to these random paths. To better approximate the unconditional distribution and reduce the dependence on the initial state, we drop the first 50 periods. Hence, we obtain a panel of 50 "years" for the 10,000 scenarios of the bank, and summarize the distribution of the relevant policies and metrics by their mean and standard deviation. Table IV illustrates these statistics when the

bank is unregulated, when it is subject to capital requirements only, or when it is subject to both capital and liquidity requirements.

Compared to the unregulated bank, the bank operating under capital regulation invests more in loans and holds approximately the same debt on average. Since deposits are not a control variable and follow the same exogenous process of the unregulated case, the regulated bank can fund this additional investment increasing retained earnings and equity issuance. This is what we showed in the simple two-period version of our model under mild capital requirements. Specifically, from equations (9) and (10), more earnings are retained from w_t or shares (incurring floatation costs λ) are issued if w_t is negative. As a result of these optimal policies, the bank holds a higher capital ratio than that prescribed by regulation, as the positive shadow price of that constraint forces the bank to manage its earnings and investments so as to maintain a capital buffer to ensure that the constraint is not hit. This result is consistent with the empirical evidence regarding banks holding (ex–ante) capital larger than required by regulations (see Flannery and Kasturi (2008)).

Importantly, capital regulation results in a bank with a lower probability of default than in the unregulated case. Thus, capital regulation is successful in abating the probability of default under deposit insurance.

Remarkably, mild capital regulation implies an *increase* in the efficiency of intermediation, since the enterprise value is larger than the unregulated case. This result suggests that the unregulated bank, relative to the bank subject to capital regulation, is inefficiently under-investing in loans, as it prefers to distribute earnings as dividends rather than retaining them to support higher loan investment. The social value of the bank is also larger than in the unregulated case, due to both higher enterprise and government values. The higher government value stems from higher tax receipts accruing from a larger taxable profit base as well as from a lower probability of bank default, which reduces expected bailout costs. However, as we show below, for these rankings of efficiency and welfare to hold, capital regulation needs to be "mild".

Turning to the case of a bank subject to both capital regulation and liquidity requirement, results are significantly different. Relative to the bank subject only to capital requirements, lending is significantly reduced, and enterprise, government and social values are all signifi-

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cantly lower. As we have already noted, the liquidity requirement results in an over-bloated book capital ratio. Such a ratio may be viewed as an indication of a safe but very inefficient bank. These results are all consistent with the trade-offs we have described in the previous subsections.

Let's now turn to examine the impact of *changes* in bank regulations. The results are presented in Table V. We consider the impact of an increase in the required capital ratio (from 8% to 12%, column k) and an increase in the liquidity ratio (from 1 to 1.2, column ℓ).

For the bank under capital regulation, an increase in the capital ratio implies now a *reduction* in loans and a corresponding reduction in indebtedness. Previously, we saw that under a lower capital ratio, the bank was satisfying the capital requirement by increasing loans at a rate proportionally higher than the capital ratio coefficient, financing them with higher retained earnings or equity issuance. However, when the capital ratio coefficient becomes too high, such a strategy becomes too costly. Thus, the bank is compelled to reduce both loan investments and further reduce debt.

The increase in the required capital ratio impact negatively on bank's efficiency and on social value. The bank enterprise value declines significantly, and such decline accounts for the bulk of the decline in bank's social value. These results suggest that the benefits of a mild capital regulation (relative to the unregulated case) rapidly disappear as the cost of capital regulation increases more than proportionally with its stringency beyond a certain threshold. This result suggests the existence of an optimal level of regulatory capital as a function of banks' characteristics.

For the bank subject to capital and liquidity requirements, an increase in the capital ratio implies a small increase in loans matched by a small decrease in indebtedness. However, the dominance of the liquidity requirement in constraining bank's decisions discussed previously allows the bank to respond to an increase in the capital ratio only in a limited way. As loan slightly increase, all metrics also increase moderately. On the other hand, an increase in the liquidity requirement again reduces loans and reduces short term financing due to the mechanisms already discussed. Both the efficiency and welfare metrics indicate a significant reduction.

The two key results of this section can be summarized as follows. Capital requirements can achieve the twin objectives of abating banks' incentives to take on excessive risk induced by deposit insurance and limited liability, and of mitigating inefficient investment absent capital regulation. However, if these requirements are too strict, then the benefits of capital regulation disappear and the associated efficiency and social costs may be significant. By contrast, liquidity requirements appear an extremely inefficient way of attaining a balance between efficiency, social value and risk–taking in the banking system. Moreover, when an increase in liquidity requirements is superimposed on a more stringent capital regulation, then the decline in enterprise and social values is even more pronounced.

VI. The impact of taxation

In this section we examine the impact of *changes* in taxation on the optimal policies and the efficiency and welfare metrics of a bank subject to both capital regulation and liquidity requirements. We focus on higher taxation mimicking some current tax policy proposals.

A. Corporate income taxes

The results of an increase of taxation of bank's income (from $\tau^+ = 15\%$ and $\tau^- = 5\%$ to $\tau^+ = 20\%$ and $\tau^- = 7.5\%$) are in Table VI.

Consider first the bank subject to capital regulation only. As the after–tax loan return is lowered with higher taxes, the investment in loans is reduced. On the liability side, short term financing is reduced. Moreover, both the enterprise value and the social value of the bank are reduced when the tax rate is increased, although heighten tax receipts increase government value.

When we consider the bank subject to both capital regulation and liquidity requirements, the income effect is still visible, as the bank under capital requirement lowers its loan investment as well as its indebtedness. As in the previous case, both enterprise value and social values are reduced and government value increases.

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Of some interest is the case when the bank under capital requirements is subject simultaneously to a higher taxation ($\tau^+ = 20\%$ and $\tau^- = 7.5\%$), and a higher capital ratio ($k = 0.12$). The reduction in loan investment and short term financing is significant but less than proportional. As for the bank subject to both capital and liquidity restrictions, the above increase, together with an increase in the liquidity ratio to $\ell = 1.2$, produces effects similar to the ones due to an increase of the liquidity ratio only. This confirms that the liquidity requirement is the most demanding of the regulatory restrictions.

Summing up, an increase in corporate income taxes results in a reduction in lending and indebtedness due to income effects, and an attendant reduction in bank efficiency and social value. Interestingly, when the bank is subject to both tightening in capital regulation and higher income taxation, the effects of taxation are dampened, being almost nullified when the bank is also subject to an increase in liquidity requirements. Finally, in all cases considered, government value increases as a result of an increase in tax receipts.

B. Taxation of bank liabilities

Recently a variety of additional taxes on financial institutions have been proposed or enacted.[14] The justifications of these taxes are: a) to address the budgetary costs of the crisis (ex-post), b) to create resolution funds to address future distress (ex-ante), c) to better align bank managers' incentives to target levels of bank risks, and d) to control systemic risk in the banking system. Pigouvian taxation have been proposed to internalize the negative externalities arising from collective bank failures. Current proposals of systemic risk levies are designed to mimic such Pigouvian levies (see, for example, Acharya and Richardson (2009), Perotti and Suarez (2009), and for a critical evaluation see Shackelford, Shaviro, and Slemrod (2010)).

Particular emphasis has been placed on taxes on bank liabilities. Some have stressed the role of these taxes as a potential complementary tool for prudential regulation. On the one hand, Shin (2010) has argued that a well designed taxation of so–called "non-core" bank liabilities (excluding deposits and other stable sources of funding) would be easier to implement than other regulations, such as time varying capital requirements or expected loss provisioning, and would be more effective in dampening pro-cyclicality. On the other hand, the IMF advocated a

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[14]See *Financial Sector Taxation*, International Monetary Fund, Washington D.C., September 2010.

Financial Activities Tax that is a flat–rate tax imposed on total bank liabilities, on the ground that total liabilities are those that would need to be supported should a resolution of distress arise.[15]

Here we consider three simple liability taxation schemes that mimic taxes either already in place in some countries, or that are part of some current proposals. The first one imposes a flat tax rate τ_B on banks' uninsured liabilities. The tax revenue is therefore $-\tau_B \min\{0, B\}$. The second one imposes a flat-tax rate τ_D on insured deposits. In this case the tax rate can be also interpreted as a flat-rate deposit insurance premium, and the tax revenue is $\tau_D D$. The third one combines the two schemes imposing the same tax rate on both insured and uninsured liabilities, i.e. $\tau_B = \tau_D$. In all cases, we assume that these taxes are deductible from earnings for income taxation purposes.

Table VII reports the impact on bank optimal choices and relevant metrics of these taxes for a bank subject to capital regulation, and one subject to both capital regulation and liquidity requirements. We set $\tau_B = \tau_D = 0.005$, which is a value in the range of taxes currently in place or proposed.[16]

Consider first a bank subject to capital regulation. A tax on uninsured liabilities results in a decline in lending and indebtedness, owing to the combination of a negative income effect, and a substitution effect due to an increase in the cost of uninsured debt relative to deposits. Both enterprise and social values decrease significantly relative to the base case, indicating non-trivial efficiency and social costs associated with this form of taxation. Importantly, government value slightly declines, suggesting that the total tax revenue may not increase because of the reduction in the tax base implied by a relatively strong substitution effect.

By contrast, a tax on insured liabilities (deposits) induces no relevant change in loans and debt, as the substitution effect works in the opposite direction of the previous case. However, both enterprise and social values slightly increase, with a slight increase in government value. By contrast, the probability of default increases, contrary to the conjecture that these type of taxation may reduce bank risk. Finally, under both taxes, the changes in loans and debt induces

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[15]Some of these proposals also mention the possibility of further designing taxation of bank liabilities by imposing different tax rates either based on banks' different risk profiles, or bank size, or both, in manners resembling risk-based deposit insurance charges. On the problems associated with implementing risk-based deposit insurance owing to asymmetric information, see Chan, Greenbaum, and Thakor (1992)

[16]See the relevant tables in *Financial Sector Taxation*, International Monetary Fund, Washington D.C., September 2010.

by the tax on uninsured liabilities seem to dominate, as loans and debt remain at levels close to those under such taxation. Moreover, enterprise and social values increase slightly relative to the cases of imposition of each of the taxes individually, but the probability of bank default increases.

When we consider a bank subject to both capital regulation and liquidity requirements, we obtain essentially the same qualitative results we have described for the case of capital regulation only with the important exception that the probability of default increases significantly. Overall, all changes in policies and metrics are smaller than those witnessed when capital regulation is in place. In other words, the strength of the liquidity constraint is such that the cumulative effects of these taxes are partly "neutralized".

In sum, our model yields three main results concerning the impact of liability taxation. First, taxes on uninsured liabilities have a significant negative impact on lending, while the impact on lending of taxes on insured liabilities (deposits) goes in the opposite direction, but is quantitatively negligible under our parameterization. Second, bank efficiency and social values decline and, importantly, and differing from the effects of increases in corporate income taxation, total tax receipts do not change significantly. Third, under taxation of bank liabilities the probability of bank default increases, contrary to the conjecture that these taxes may be a tool to control for bank risk.

VII. Conclusions

This paper has formulated a dynamic model of a bank exposed to credit risk, liquidity risk, and financial distress, and gauged the joint impact of capital regulation, liquidity requirements and taxation on banks' optimal policies and metrics of efficiency of intermediation and social value.

We have uncovered an important inverted U-shaped relationship between bank lending, bank efficiency, social value and regulatory capital ratios. This result suggests the existence of optimal levels of regulatory capital. In a policy perspective, this implies that the determination of "efficient" regulatory capital ratios is likely to be highly bank-specific, depending crucially on the configuration of risks a bank is exposed to as a function of the chosen business strategies.

Similarly, our results on the high costs of liquidity requirements point out the consequences of the repression of the key maturity transformation role of bank intermediation. We believe our results contribute to clarify the terms of the current intense debate on the desirability of increasing bank capital ratios and introducing liquidity requirements (see e.g. Admati, DeMarzo, Hellwig, and Pfleiderer (2011) and Calomiris (2011).

On taxation, we highlighted the implications of basic trade-off between income and substitution effects. For the purpose of rising tax revenues, corporate income taxation seems to be preferable to taxes on bank liabilities, since the associated substitution effects of the latter may imply at the same time higher bank efficiency and social costs and lower tax receipts. This is an under-researched area where more could be gained by the explicit modeling of the trade-offs of different tax schemes in the context of bank regulation.

Overall, our results suggest that prudence should be exercised in implementing non-trivial changes in capital and liquidity requirements, both at an individual and "systemic" level, before having gained a clear grasp of the associated private and social costs, as these costs could be significantly larger than originally thought.

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Appendix

Properties the unregulated bank program

Compactness of the feasible set of the bank can be shown as follows. Given the strict concavity of $\pi(L)$, there exists a level L_u such that $\pi(L_u)Z_u - rL_u = 0$, where r is the cost of capital of the marginal dollar raised either through deposits or short term financing.[17] Thus, any investment $L > L_u$ would be unprofitable. This establishes an upper bound on the feasible set of L, given by $[0, L_u]$ for some L_u. With an upper bound on L, and because the stochastic process D has compact support, the collateral constraint sets a lower bound B_d (i.e., an upper bound on bond issuance). Specifically, this is obtained by putting D_d in place of D_t and L_u in place of L_t in equation (7).

Lastly, an upper bound on B can be obtained assuming that the proceeds from risk–free investments made by the bank are taxed at a rate not higher than the personal tax rate. Specifically, assume that the current deposits D are all invested in short term bonds, B, with no investments in loans. To further increase the investment in bonds of one dollar, the bank must raise equity capital. A shareholder thus incurs a cost $1 + \lambda$, where λ is the floatation cost. This additional dollar is invested at the risk–free rate, so that at the end of the year, the proceeds of this investment that can be distributed are $(1 + r(1 - \tau^+))$. Alternatively, the shareholder can invest $1 + \lambda$ in a risk–free bond, obtaining $(1 + \lambda)(1 + r)$. Because $\tau^+ \geq 0$ and $\lambda \geq 0$, then $(1 + \lambda)(1 + r) \geq (1 + r(1 - \tau^+))$, there is no incentive of the bank to have a cash balance that is larger than D as long as either λ or τ^+ are strictly positive. The foregoing argument is made for simplicity. If the shareholders are taxed on their investment proceeds at a rate τ_p, they obtain $(1 + \lambda)(1 + r(1 - \tau_p))$ from their investment in the risk–free asset. If $\tau_p \leq \tau^+$, then $(1 + \lambda)(1 + r(1 - \tau_p)) > (1 + r(1 - \tau^+))$, and the bank has no incentive to increase the investment in risk–free bonds beyond D. Moreover, if floatation costs associated with equity issuance are strictly increasing in the amount issued, no assumption about differential tax rates are needed to establish an upper bound on B. In conclusion, the feasible set of the bank can be assumed to be $[0, L_u] \times [B_d, B_u]$.

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[17]Deposits and short term bonds are the cheapest form financing. If the same dollar were raised by issuing equity, the cost would be higher due to both the higher cost of equity capital and to floatation costs. In this case the upper bound would be even lower.

Furthermore, standard arguments establish the existence of a unique value function $E(x) = E(L, B, D, Z, D')$ that satisfies (14) and is continuous, increasing, and differentiable in all its arguments, and concave in L. The existence and uniqueness of the value function E follow from the Contraction Mapping Theorem (Theorem 3.2 in Stokey and Lucas (1989)). The continuity, monotonicity, and concavity of the value function E in the argument L follow from Theorem 3.2 and Lemma 9.5 in Stokey and Lucas (1989). The continuity and monotonicity of E in B, D, Z, and D' follow from the continuity and monotonicity of e in Z and D' and the monotonicity of the Markov transition function of the process (Z, D).

Algorithm

The solution of the Bellman equation in (14) is obtained numerically by a value iteration algorithm. The valuation model for bank's equity is a continuous–decision and infinite–horizon Markov Decision Processes. The solution method is based on successive approximations of the fixed point solution of the Bellman equation. Numerically, we apply this method to an approximate discrete state-space and discrete decision valuation operator.[18]

Given the dynamics of Z_t in (11) and of $\log D_t$ in (12), using a vector notation $\xi(t) = (Z(t), \log D(t))$, we have a VAR of the form

$$\xi(t) = c + \mathcal{K}\xi(t-1) + \varepsilon(t) \tag{26}$$

where $\varepsilon = (\varepsilon^Z, \varepsilon^D)$ is a bivariate Normal variate with zero mean and covariance matrix Σ,

$$c = \begin{pmatrix} (1-\kappa_1)\bar{\xi}_1 \\ (1-\kappa_2)\bar{\xi}_2 \end{pmatrix}, \qquad \mathcal{K} = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho \\ \sigma_1\sigma_2\rho & \sigma_2^2 \end{pmatrix},$$

where, differently from the approach proposed in Tauchen (1986), Σ may be non–diagonal and singular (when $|\rho| = 1$). We restrict the support of each stochastic process within three times the unconditional standard deviation of the marginal distribution around the long–term average.

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[18]See Rust (1996) or Burnside (1999) for a survey on numerical methods for continuous decision infinite horizon Markov Decision Processes.

Given this alternative assumption for the error covariance matrix, we follow an efficient approach first proposed by Knotek and Terry (2011) based on numerical integration of the multivariate Normal distribution. In particular, the continuous–state process $\xi(t)$ is approximated by a discrete–state process $\widehat{\xi}(t)$, which varies on the finite grid $\{X_1, X_2, \ldots, X_n\} \subset \mathbb{R}^2$. Define a partition of $\mathbb{R}^2$ made of n non–overlapping 2–dimensional intervals $\{\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n\}$ such that $X_i \in \mathcal{X}_i$ for all $i = 1, \ldots, n$ and $\cup_{i=1}^n \mathcal{X}_i = \mathbb{R}^2$. The $n \times n$ transition matrix Π is defined as

$$
\begin{aligned}
\Pi_{i,j} &= \text{Prob} \left\{ \widehat{\xi}(t+1) \in \mathcal{X}_j \mid \widehat{\xi}(t) = X_i \right\} \\
&= \text{Prob} \left\{ c + \mathcal{K}\xi(t) + \varepsilon(t+1) \in \mathcal{X}_j \right\} \\
&= \text{Prob} \left\{ \varepsilon(t+1) \in \mathcal{X}_j' \right\} = \int_{\mathcal{X}_j'} \phi(\xi, 0, \Sigma) d\xi
\end{aligned}
$$

where $\mathcal{X}_j' = \mathcal{X}_j - c - \mathcal{K}\xi(t)$, and $\phi(\xi, 0, \Sigma)$ is the density of the bivariate Normal with mean zero and covariance matrix Σ. The integral on the last line is computed numerically by Monte–Carlo integration. We select the grid points for each variable based on approximately equal weighting from the univariate normal cumulative distribution function.

The feasible interval for loans, $[0, L_u]$, and for the face value of bonds, $[B_d, B_u]$ (with $B_d < 0 < B_u$), is set so that they are never binding for the equity maximizing program. We discretize $[L_d, L_u]$, to obtain a grid of N_L points

$$
\widetilde{L} = \left\{ \widetilde{L}_j = L_u(1 - \delta)^j \mid j = 1, \ldots, N_L - 1 \right\} \cup \{L_{N_L} = 0\}
$$

such that, if the bank choose inaction, the loan's level is what remains after the portion δL has been repaid. The interval $[B_d, B_u]$ is discretized into N_b equally–spaced values, making up the set $\widetilde{B}$. To keep the notation simple, we denote $x = (\xi, L, B)$ the generic element of the discretized state.

We solve the problem

$$
E(x) = \max \left\{ 0, \max_{(L', B') \in \mathcal{A}(D)} \left\{ e(x, L', B') + \beta \mathbb{E} \left[E(x') \right] \right\} \right\},
$$

where function $e(x, L, B)$, is defined in equation (10), and $\mathcal{A}(D)$ is the case specific feasible set defined differently for the unregulated and the regulated case, in all points of the discrete

state space. The solution is found by successive approximations, starting from a guess function $E_0(\cdot)$, putting it on the right–hand–side of the Bellman equation obtaining $E_1(\cdot)$ and than by iterating on the same procedure obtaining the sequence $\{E_n(\cdot), n = 0, 1, \ldots\}$. The procedure is terminated when the error $\|E_{j+1} - E_j\|$ is lower than the desired tolerance.

For the set of parameters in Table I, we use $L_u = 10$, $B_d = -5$ and $B_u = 5$. Given the properties of the quadrature scheme, we solve the model using only 9 points for Z, 9 points for D_t. However, we need to allow for many more points when discretizing the control variables, so we choose $N_L = 27$, and $N_B = 27$. The tolerance for termination of the iterative procedure is set at 10^{-5}.

Given the optimal solution, we can determine the optimal policy and the transition function $\varphi(x)$ in (15) based on the arg-max of equity value at the discrete states x. We use Monte Carlo simulation to generate a sample of Ω possible future paths (or scenarios) for the bank. In particular, we obtain the simulated dynamics of the state variable $\xi = (Z, D)$ by application of the recursive formula in (26), starting from $Z(0) = \overline{Z}$ and $D(0) = D_d$. Then, setting a feasible initial choice $L(0) = 0$ and $B(0) = D_u$ (so that the initial bank capital is $D_u - D_d$), we apply the transition function φ along each simulated path recursively. If a bank defaults at a given step, then the current depositors receive the full value of their claim, while the deposit insurance agency pays the bankruptcy cost. Afterwards, a seed capital $D_u - D_d$ is injected in the bank. Together with deposit D_d, the total amount D_u is momentarily invested in bonds, $B = D_u$, while $L = 0$. Then the "new" bank follows on the same path by applying the optimal policy. In our numerical experiments, we generate simulated samples with $\Omega = 10,000$ paths and $T = 100$ years (steps). To limit the dependence of our results on the initial conditions, we drop the first 50 steps.
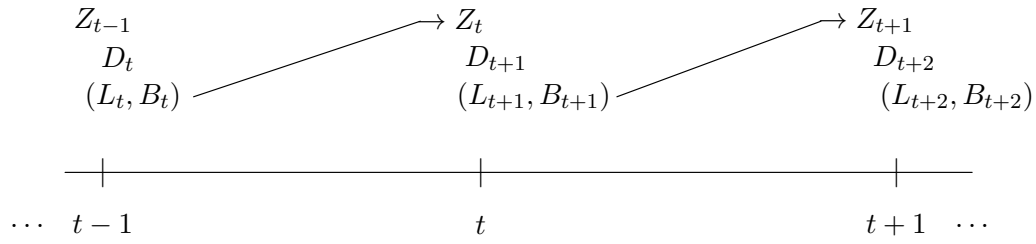
Figure 1: **Bank's dynamic**. Evolution of the state variables (credit shock, Z, and deposits, D) and of the bank's control variables (cash and liquid investments, B, and loans, L) assuming the bank is solvent at each date.

|		
κ_Z	annual persistence of the credit shock	0.88
σ_Z	annual conditional std. dev. of the credit shock	0.0139
$\overline{Z}$	unconditional average of the credit shock	0.0717
κ_D	annual persistence of the log of deposits	0.95
σ_D	annual conditional std. dev. of the log of deposits	0.0209
$\overline{D}$	unconditional average of deposits	\$2
ρ	correlation between log–deposit and credit shock	-0.85
β	annual discount factor	0.95
r	annual risk–free borrowing and investment rate	2.5%
τ^+	corporate tax rate for positive earnings	15%
τ^-	corporate tax rate for negative earnings	5%
δ	annual percentage of reimbursed loan	20%
γ	bankruptcy costs	0.1
λ	flotation cost for equity	0.3
α	return to scale for loan investment	0.95
m^+	unit price for loan investment	0.03
m^-	unit price for loan fire sales	0.04
k	percentage of loans for capital regulation	4%
ℓ	liquidity coverage ratio	100%
τ_B	tax rate on uninsured liabilities	0.005
τ_D	tax rate on insured liabilities	0.005

Table I: **Model parameters**
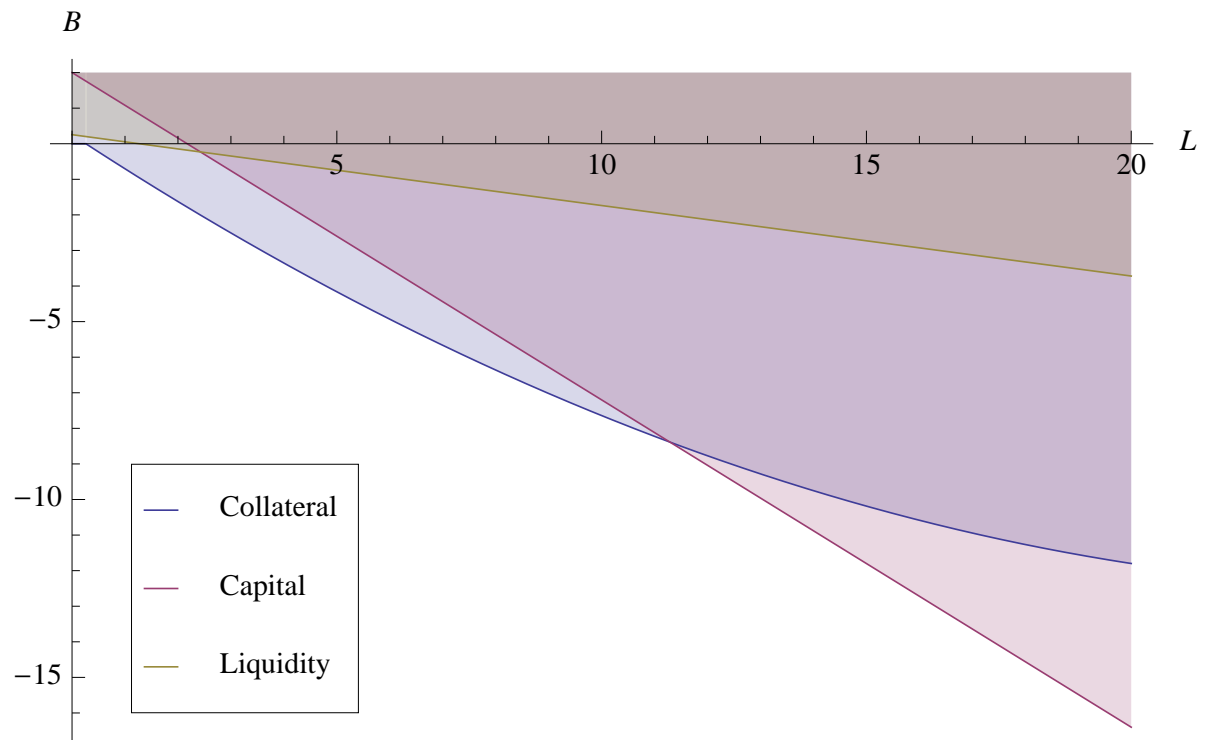
Figure 2: **Comparison of constraints**. This figures presents the three feasible region of (L, B) defined by the collateral constraint, $\Gamma(D)$ in Equation (8), the capital requirement, $\Theta(D)$ from (17), and by the liquidity requirement, $\Lambda(D)$ from Equation (19). The plot is based on the parameter values in Table I, for a current $D = 2$.
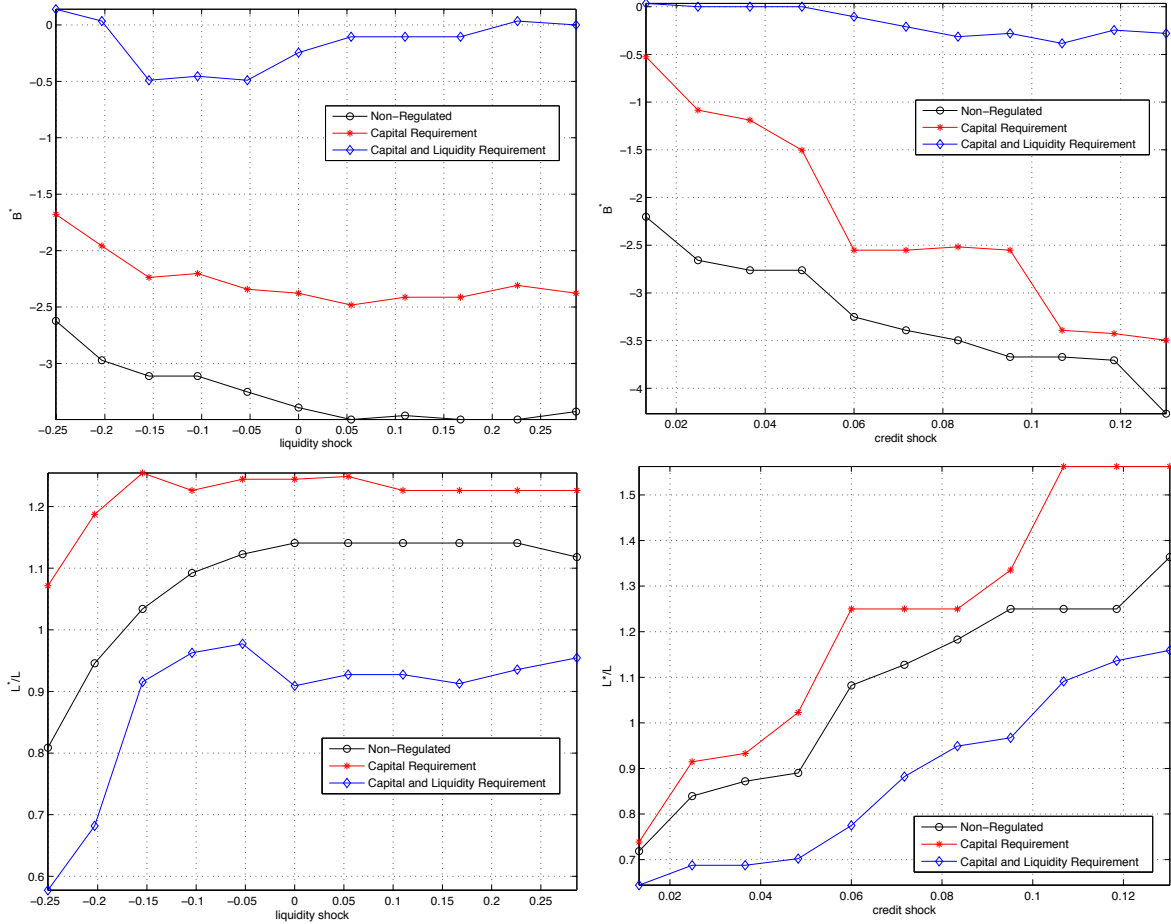
Figure 3: **Bank's policy**. This figure illustrates the impact of regulatory restrictions on the bank's policy related to loan investment and to short term investment and financing, for the non–regulated case, and for the cases with capital constraint, and with both capital and liquidity constraints. The short term investment/financing policy is given by the optimal B^* given the current state, averaged across all possible Z in the left panel and averaged across all possible D' in the right panel. The loan investment policy is represented by the ratio L^*/L given the current state and averaged across Z in the left panel and across D' in the right panel. These values are plotted against the liquidity shock, $D' - D$, in the left panels and the credit shock, Z, on loans in the right panels, and are obtained assuming that the bank is currently at the steady–term state (so that the credit shock is 0.0717, and the deposits from the previous date are $D = 2$, respectively), while $B = 0$, and $L = 4.1$ so that bank capital is $K = 2.1$. The values are from the numerical solution of the model using 9 points for Z, 9 points for D, 27 points for L, and 36 points for C, based on the parameter values in Table I.

Z	0.037	0.048	0.060	0.072	0.083	0.095	0.107
	Unregulated						
$K = 2.22$	-0.280	-0.240	-0.219	-0.153	-0.142	-0.142	-0.142
$K = 2.18$	-0.240	-0.240	-0.190	-0.121	-0.121	-0.132	-0.142
$K = 2.14$	-0.227	-0.240	-0.153	-0.110	-0.110	-0.121	-0.121
$K = 2.10$	-0.305	-0.268	-0.110	-0.099	-0.099	-0.110	-0.110
$K = 2.05$	-0.428	-0.391	-0.276	-0.251	-0.167	-0.167	-0.099
	Capital						
$K = 2.22$	0.095	0.089	0.083	0.083	0.083	0.083	0.127
$K = 2.18$	0.135	0.089	0.083	0.083	0.083	0.104	0.136
$K = 2.14$	0.135	0.098	0.083	0.083	0.104	0.137	0.146
$K = 2.10$	0.162	0.142	0.083	0.104	0.115	0.153	0.155
$K = 2.05$	0.159	0.166	0.148	0.137	0.190	0.170	0.163
	Capital & liquidity						
$K = 2.22$	0.322	0.322	0.337	0.417	0.417	0.433	0.433
$K = 2.18$	0.322	0.322	0.337	0.417	0.417	0.433	0.446
$K = 2.14$	0.322	0.322	0.337	0.417	0.417	0.441	0.446
$K = 2.10$	0.297	0.322	0.337	0.398	0.417	0.417	0.446
$K = 2.05$	0.294	0.293	0.312	0.333	0.353	0.398	0.398

Table II: **Capital ratio**. Average capital ratio (i.e., bank capital over loans, or $(L^* + B^* - D')/L^*$, where (L^*, B^*) is the optimal solution for a solvent bank and D' is the new level of deposits) as a function of the credit shock, Z, at different levels of current bank capital, $K = L + B - D$. The average is computed over the different possible levels of D', for $B = 0$. The current level of L is 4.1. The different levels of K are obtained by changing D. These results are based on the numerical solution of the valuation problem in (14) with the parameters in Table I.

Z	0.037	0.048	0.060	0.072	0.083	0.095	0.107
				Unregulated			
$K = 2.22$	-9.683	-8.769	-11.281	-11.362	-10.967	-10.967	-10.967
$K = 2.18$	-8.769	-8.769	-11.707	-10.281	-10.281	-10.571	-10.967
$K = 2.14$	-8.374	-8.769	-11.090	-10.053	-10.053	-10.281	-10.281
$K = 2.10$	-7.838	-7.865	-9.292	-9.866	-9.866	-10.053	-10.053
$K = 2.05$	-6.481	-6.518	-7.968	-8.071	-9.052	-9.052	-9.866
				Capital			
$K = 2.22$	-3.627	-6.132	-6.424	-6.424	-6.424	-6.424	-8.079
$K = 2.18$	-2.713	-6.132	-6.424	-6.424	-6.424	-5.739	-8.258
$K = 2.14$	-2.713	-5.164	-6.424	-6.424	-5.906	-4.928	-8.415
$K = 2.10$	-2.028	-2.705	-6.424	-6.009	-5.719	-5.395	-8.019
$K = 2.05$	-1.322	-1.155	-4.197	-5.166	-3.721	-5.702	-7.729
				Capital & liquidity			
$K = 2.22$	1.577	1.577	1.345	1.829	1.829	1.417	1.417
$K = 2.18$	1.577	1.577	1.345	1.829	1.829	1.417	1.604
$K = 2.14$	1.577	1.577	1.345	1.829	1.829	1.990	1.604
$K = 2.10$	1.438	1.577	1.345	1.655	1.829	1.829	1.604
$K = 2.05$	1.835	1.626	1.206	1.289	1.389	1.655	1.655

Table III: **Liquidity coverage ratio**. Average liquidity coverage ratio (i.e., end–of–period total cash available in the worst case scenario over the end–of–period net cash outflows due to a variation in deposits, or $(\delta L^* + \pi(L^*)Z_d - \tau(y^{\min}) + B^*(1+r))/(D'(1+r) - D_d)$, where (L^*, B^*) is the optimal solution for a solvent bank and D' is the new level of deposits) as a function of the credit shock, Z, at different levels of current bank capital, $K = L + B - D$. The average is computed over the different possible levels of D', for $B = 0$. The current level of L is 4.1. The different levels of K are obtained by changing D. These results are based on the numerical solution of the valuation problem in (14) with the parameters in Table I.
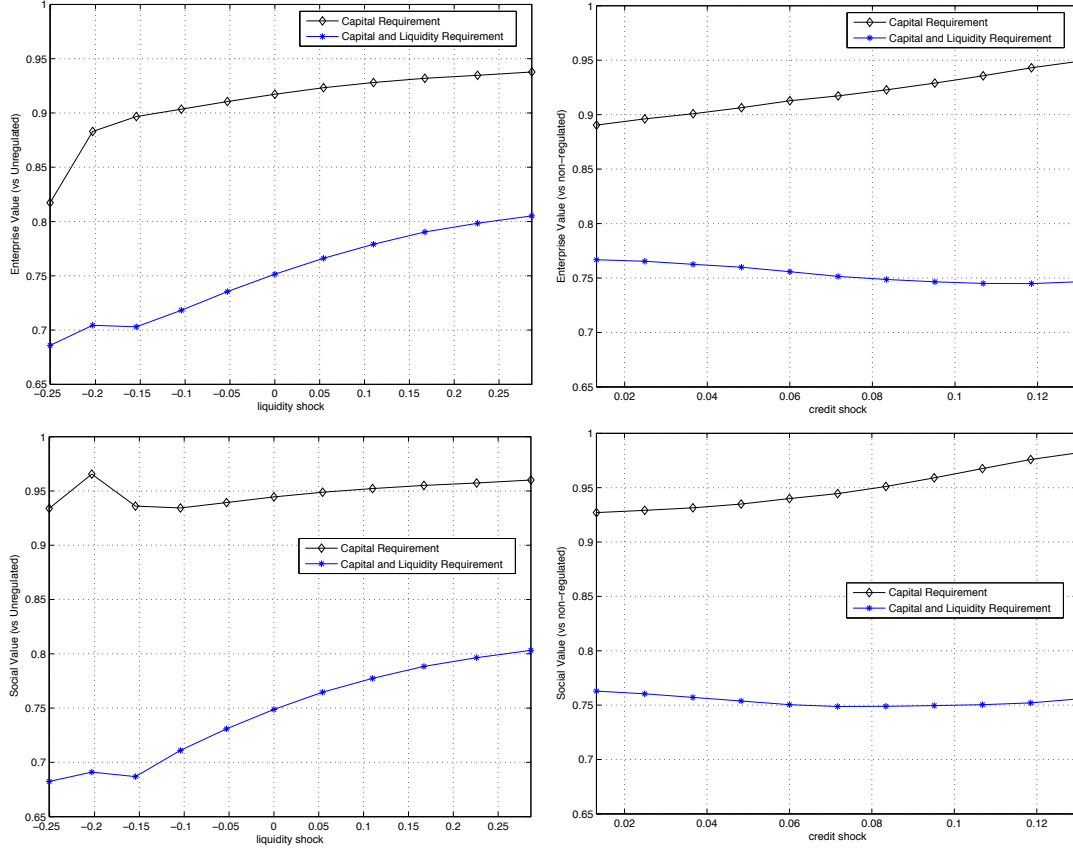
Figure 4: **Value loss associated with regulatory restrictions**. This figure illustrates the impact of regulatory restrictions by comparing the enterprise value (i.e., market value of deposits plus market value of equity net of cash balance, plus short term debt) and the social value (i.e., enterprise value of the bank plus the value of taxes) of the bank, for the case with a capital constraint, and with both capital and liquidity constraints as a proportion of the relevant values from the non–regulated case. These values are plotted against the shock on deposits, $D' - D$, in the left panels and the credit shock, Z, on loans in the right panels, and are obtained assuming that the bank is currently at the steady–term state (so that the credit shock is 0.0717, and the deposits from the previous date are $D = 2$, respectively), while $B = 0$, and $L = 4.1$ so that bank capital is $K = 2.1$. The values are from the numerical solution of the model using 9 points for Z, 9 points for D, 27 points for L, and 36 points for C, based on the parameter values in Table I.

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	Unregulated		Capital		Cap. & Liq.	
	ave.	s.d.	ave.	s.d.	ave.	s.d.
Loan (book)	4.39	2.36	5.87	2.36	2.51	1.63
Net Bond Holdings (book)	-3.23	2.28	-3.22	2.13	0.32	0.97
Bank Capital (book)	-0.79	0.61	0.65	0.40	0.82	0.92
Equity (mkt)	3.97	2.75	4.00	2.74	2.79	2.48
Deposits (mkt)	1.90	0.15	1.95	0.15	1.95	0.15
Enterprise Value (mkt)	9.10	3.42	9.17	4.40	4.43	3.14
Government Value (mkt)	0.54	0.29	0.76	0.35	0.35	0.27
Social value (mkt)	9.64	3.68	9.93	4.72	4.78	3.39
Annual Default Rate (pct)	7.84	0.24	0.00	0.00	0.00	0.00
Leverage (book)	–	–	0.90	0.04	0.78	0.17
Leverage (mkt)	0.62	0.19	0.64	0.17	0.55	0.25

Table IV: **The impact of bank regulation**. These are obtained by applying the optimal policy from the solution of the bank valuation problem (using the parameters in Table I) for 10,000 random paths of 50 years length of the liquidity and credit shocks. Capital Ratio is the Bank's Capital over the principal of Loans (K/L). Book Leverage is the book value of Deposits plus the short term financing over the book value of Loans plus the short term investment in bonds $((D - \min\{B,0\})/(L + \max\{B,0\}))$. Market Leverage is the book value of Deposits plus the short term financing over the quasi–market value of the assets $((D - \min\{B,0\})/(D - \min\{B,0\} + E))$. The results are presented for the unregulated case, the case with capital ratio restrictions, and the case with both capital and liquidity restrictions.

	Capital		Capital & Liquidity		
	base	k	base	k	ℓ
Loan (book)	5.87	5.76	2.51	2.57	2.44
Net Bond Holdings (book)	-3.22	-2.90	0.32	0.34	0.37
Bank Capital (book)	0.65	0.85	0.82	0.90	0.81
Equity (mkt)	4.00	4.11	2.79	2.87	2.76
Deposits (mkt)	1.95	1.95	1.95	1.95	1.95
Enterprise Value (mkt)	9.17	8.96	4.43	4.49	4.34
Government Value (mkt)	0.76	0.77	0.35	0.36	0.34
Social value (mkt)	9.93	9.74	4.78	4.85	4.68
Annual Default Rate (pct)	0.00	0.00	0.00	0.00	0.00
Leverage (book)	0.90	0.87	0.78	0.76	0.79
Leverage (mkt)	0.64	0.62	0.55	0.55	0.56

Table V: **Increases in capital and liquidity requirements**. These are obtained by applying the optimal policy from the solution of the bank valuation problem for 10,000 random paths of 50 years length of the liquidity and credit shocks. The table shows, respectively, from left to right, the cases of the bank with capital requirement, and the bank subject to both capital and liquidity restrictions. The table presents the average values (computed on non–defaulted banks) of the different metrics. The columns represent different choices of parameters: the column denoted "base", is the base case, with the parameters in Table I. The others are obtained by changing only the parameter(s) we use to denominate the column: in "k" we set the capital ratio to 0.12; in "ℓ" is with $\ell = 1.2$.

	Capital				*Capital & liquidity*				
	base	k	τ	all	base	k	ℓ	τ	all
Loan (book)	5.87	5.76	5.57	5.45	2.51	2.57	2.44	2.03	2.14
Net Bond Holdings (book)	-3.22	-2.90	-2.97	-2.64	0.32	0.34	0.37	0.45	0.48
Bank Capital (book)	0.65	0.85	0.60	0.81	0.82	0.90	0.81	0.48	0.62
Equity (mkt)	4.00	4.11	3.65	3.76	2.79	2.87	2.76	2.26	2.40
Deposits (mkt)	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Enterprise Value (mkt)	9.17	8.96	8.57	8.35	4.43	4.49	4.34	3.76	3.87
Government Value (mkt)	0.76	0.77	0.95	0.98	0.35	0.36	0.34	0.38	0.40
Social value (mkt)	9.93	9.74	9.52	9.33	4.78	4.85	4.68	4.14	4.27
Default Probability (pct)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Leverage (book)	0.90	0.87	0.91	0.87	0.78	0.76	0.79	0.84	0.80
Leverage (mkt)	0.64	0.62	0.65	0.63	0.55	0.55	0.56	0.58	0.57

Table VI: **Increases in corporate income taxes**. These are obtained by applying the optimal policy from the solution of the bank valuation problem for 10,000 random paths of 50 years length of the liquidity and credit shocks. The table shows, respectively, from left to right the bank with capital requirement, and the bank subject to both capital and liquidity restrictions. The table presents the average values (computed on non–defaulted banks) of the different metrics. The columns represent different choices of parameters: the column denoted "base", is the base case, with the parameters in Table I. The others are obtained by changing only the parameter(s) we use to denominate the column: "τ" is with $\tau^+ = 20\%$ and $\tau^- = 7.5\%$; in "k" we set the capital ratio to 0.12; "ℓ" is with $\ell = 1.2$; and in "all" we set $\tau^+ = 20\%$, $\tau^- = 7.5\%$, $k = 0.12$, and $\ell = 1.2$.

	Capital				*Capital & liquidity*			
	base	τ_B	τ_D	$\tau_B = \tau_D$	base	τ_B	τ_D	$\tau_B = \tau_D$
Loan (book)	5.87	5.40	5.87	5.47	2.51	2.46	2.62	2.54
Net Bond Holdings (book)	-3.22	-2.80	-3.21	-2.87	0.32	0.33	0.25	0.27
Bank Capital (book)	0.65	0.59	0.67	0.62	0.82	0.79	0.89	0.84
Equity (mkt)	4.00	3.64	4.09	3.79	2.79	2.74	3.17	3.09
Deposits (mkt)	1.95	1.95	1.94	1.94	1.95	1.95	1.92	1.92
Enterprise Value (mkt)	9.17	8.40	9.24	8.60	4.43	4.36	4.84	4.74
Government Value (mkt)	0.76	0.70	0.76	0.72	0.35	0.34	0.37	0.36
Social value (mkt)	9.93	9.10	10.01	9.31	4.78	4.70	5.21	5.10
Default Probability (pct)	0.00	0.00	2.11	2.78	0.00	0.00	5.27	5.37
Leverage (book)	0.90	0.91	0.90	0.90	0.78	0.79	0.76	0.77
Leverage (mkt)	0.64	0.65	0.63	0.63	0.55	0.56	0.52	0.52

Table VII: **The impact of taxation of liabilities**. These are obtained by applying the optimal policy from the solution of the bank valuation problem for 10,000 random paths of 50 years length of the liquidity and credit shocks. The table presents the average values (computed on non–defaulted banks) of the different metrics. The table shows, respectively, from left to right the bank with capital requirement, and the bank subject to both capital and liquidity restrictions. The table presents three different choices of parameters for taxation of liabilities. The first has $\tau_B = 0.005$ and $\tau_D = 0$; the second has $\tau_B = 0$ and $\tau_D = 0.005$; the third has $\tau_B = 0.005 = \tau_D$. These cases are compared with the base.