# THE FISCAL MULTIPLIER MORASS: A BAYESIAN PERSPECTIVE

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# FISCAL MULTIPLIER(S): DEFINITION

1. Present Value Multiplier:

$$\text{Present Value Multiplier(Q)} = \frac{\sum_{t=0}^{Q} E_t \left( \prod_{i=0}^{Q} R_{t+i}^{-1} \right) \Delta Y_{t+Q}}{\sum_{t=0}^{Q} E_t \left( \prod_{i=0}^{Q} R_{t+i}^{-1} \right) \Delta G_{t+Q}}$$

2. Impact Multiplier: $Q = 0$

# How Big/Small Are Fiscal Multipliers?

IMF Working Paper 10/73 March 2010

1. 17 coauthors: model builders for policy institutions

2. Seven Structural Models: QUEST, GIMF, FRB-US, SIGMA BoC-GEM, OECD Fiscal, NAWM.

3. Conclude: “Robust finding across all models that fiscal policy can have sizeable output multipliers.”

# Representative IMF Multiplier

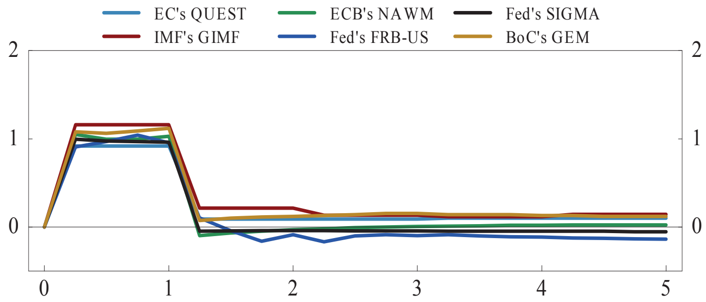


Figure 1: Estimated Impact on GDP of Increase in Government Purchases of 1 Percent of GDP

# Robust Finding?

- Cogan, Cwik, Taylor and Wieland (2010), Cwik and Wieland (2010)
    - Multipliers less than 1
- Uhlig (2010)
    - Long-run multipliers negative

# Uhlig (2010) Impulse Response

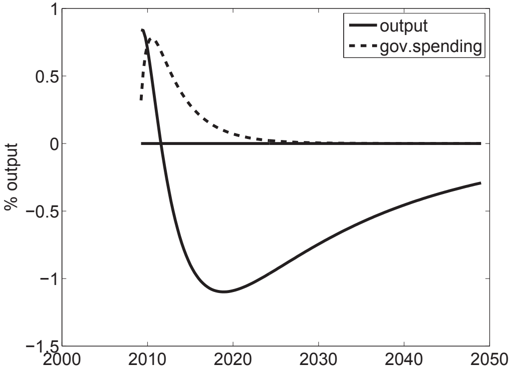


Figure 5. Output and Government Spending: 40 years.

# Motivation

Why do policy models yield very different conclusions for multipliers even when conditioning on same data set?

Answer: Multipliers are *conditional* statistics, so different specifications $\rightarrow$ different multipliers

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IMF WP10/73's Response to Uhlig (2010) and Cogan et al. (2010):

- include hand-to-mouth agents
- focus on short-run & temporary stimulus
- model different types of fiscal-monetary interactions (Davig-Leeper (2009))

# THIS PAPER

Open Question: To what extent does a DSGE model *force* a particular multiplier on the data?

- "black box" problem of DSGE models
- use Bayesian methodology to address issue

# Our Contribution

- Build suite of nested models to determine important elements for multipliers.
- Use modified prior predictive analysis (PPA) to understand *a priori* what restrictions are generated by DSGE model
- More general message: What does it mean for a prior to be “flat”?
- Distribution of object of interest should be “flat” relative to economic question at hand

# FINDINGS

- Model restrictions impose tight ranges on multipliers
- Rigidities and hand-to-mouth agents key for long run multipliers $> 0$
- Most important features for multiplier variation:
  - gov. spending process
  - hand-to-mouth agents
  - monetary-fiscal interactions

# Review of PPA

- Standard Exercise [Lancaster (2004), Geweke (2010)]: used to evaluate model's adequacy for given feature of data *before* estimation stage (model evaluation)

- $\theta$ parameters, $y$ data, $\omega$ vector of interest

$$
\begin{aligned}
\theta^{(m)} &\sim p(\theta) \\
y^{(m)} &\sim p(y|\theta^{(m)}) \\
\omega^{(m)} &\sim p(\omega|y^{(m)}, \theta^{(m)})
\end{aligned}
$$

- Compare distribution of $\omega$ to data

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- Compare distribution of $\omega$ to data

- compuationally inexpensive

# Modified PPA

- Issue: What is multiplier in data? Requires model and identification

- $A_j$ DSGE model, $\theta$ parameters of DSGE, $\omega =$ multipliers

$$\begin{aligned}&\text{Draw } \theta^{(m)} \sim p(\theta)\\&\text{Solve DSGE Model}\\&\text{Calculate } \omega^m|\theta^{(m)}\\&\text{Form } p(\omega|A_j)\end{aligned}$$

# Modified PPA

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- $A_j$ DSGE model, $\theta$ parameters of DSGE, $\omega =$ multipliers

$$\text{Draw } \theta^{(m)} \sim p(\theta)$$
$$\text{Solve DSGE Model}$$
$$\text{Calculate } \omega^m | \theta^{(m)}$$
$$\text{Form } p(\omega | A_j)$$

- PPA gives entire range of *possible* multipliers

# Our Model

1. forward-looking, optimizing agents
2. utility from consumption and leisure
3. capital and labor inputs in production
4. monopolistic competition
5. nominal & real frictions
6. fiscal and monetary policy
7. open economy features

# Nested Specifications

- Model 1: Basic RBC

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- Model 5: NK model with open economy features

# Model 1: Basic RBC

- CRRA, time-separable utility

$$E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\xi}}{1+\xi} \right]$$

- Cobb-Douglas production

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

- Law of motion for capital:

$$K_t = I_t + (1-\delta) K_{t-1}$$

# Model 1: Basic RBC

- GBC:

$$B_t + \tau_t^K R_t^K K_{t-1} + \tau_t^L W_t L_t + \tau_t^C C_t = R_{t-1} B_{t-1} + G_t + Z_t$$

- capital tax, labor tax, government consumption, transfers follow

$$\hat{X}_t = \rho_x \hat{X}_{t-1} + (1 - \rho_x)\gamma_x \hat{s}_{t-1}^b + \epsilon_t^x$$

where $s_{t-1}^b = B_{t-1}/Y_{t-1}$

# MODEL 1: BASIC RBC

- 5,000 draws from priors: $\gamma \sim N^+(2, 0.6)$, $\xi \sim N^+(2, 0.6)$, $\rho_x \sim B(0.5, 0.2)$, $\gamma_x \sim N^+(0.2, 0.05)$
- Priors similar to Smets and Wouters (2003) and others
- Other parameters fixed at well known values (e.g., $\beta = 0.99$)

# Model 1: Basic RBC

| Variable | Impact | 4 quart. | 10 quart. | 25 quart. | $\infty$ |
|---|---|---|---|---|---|
| Prob$\left(PV\frac{\Delta Y}{\Delta G} > 1\right)$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Prob$\left(PV\frac{\Delta C}{\Delta G} > 0\right)$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Prob$\left(PV\frac{\Delta I}{\Delta G} > 0\right)$ | $<$0.01 | $<$0.01 | $<$0.01 | $<$0.01 | 0.00 |

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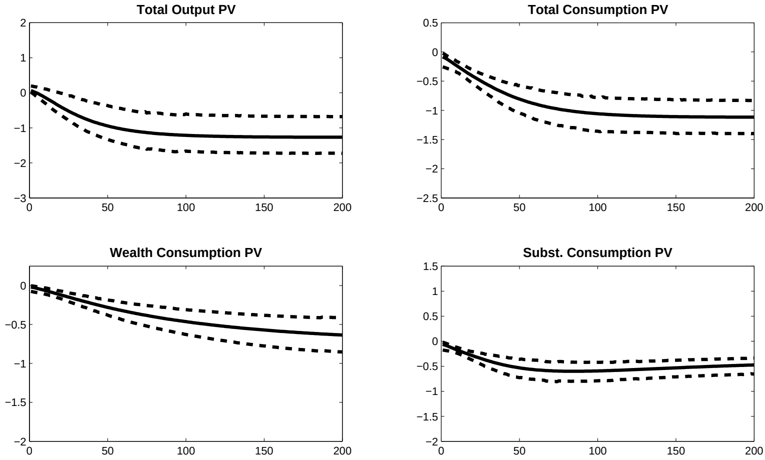

# Model 1: Basic RBC

Intuition Straightforward:

- Baxter-King (1993) Monacelli-Perotti (2008) + distortionary fiscal financing
- $\uparrow G \rightarrow$ negative wealth and substitution effects, crowding out
- Consumption, Investment falls
- Increase in public demand cannot offset decrease in private demand

# Model 2: RBC with Real Frictions

Add to Model 1

- Habit formation in utility

$$E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{(c_t - \theta C_{t-1})^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\xi}}{1+\xi} \right]$$

$\theta \sim B(0.5, 0.2)$

- Capacity utilization: $\psi(v_t)$ cost per unit of $K$

$v = 1$, $\psi(1) = 0$, $\frac{\psi''(1)}{\psi'(1)} = \frac{\psi}{1-\psi}$, $\psi \sim B(0.6, 0.15)$

# Model 2: RBC with Real Frictions

- Investment adjustment costs

$$K_t = (1-\delta)K_{t-1} + \left[1 - s\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

where $s(1) = s'(1) = 0$, and $s''(1) = s > 0$, $s \sim N(6, 1.5)$

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where $s(1) = s'(1) = 0$, and $s''(1) = s > 0$, $s \sim N(6, 1.5)$

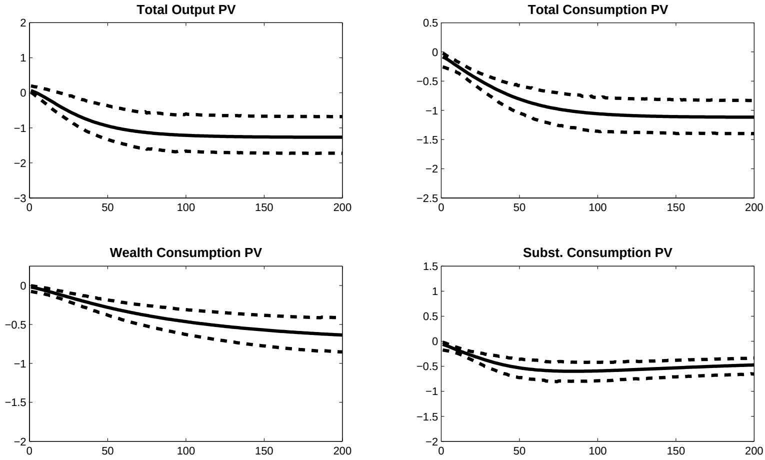
- Aggregate resource constraint:

$$Y_t = C_t + G_t + I_t + \psi(v_t)K_{t-1}$$

# Model 2: RBC with Real Frictions

| Variable | Impact | 4 quart. | 10 quart. | 25 quart. | $\infty$ |
|---|---|---|---|---|---|
| Prob$\left(PV\frac{\Delta Y}{\Delta G} > 1\right)$ | 0.01 | 0.00 | 0.00 | 0.00 | $<$0.01 |
| Prob$\left(PV\frac{\Delta C}{\Delta G} > 0\right)$ | 0.00 | 0.00 | 0.00 | 0.00 | $<$0.01 |
| Prob$\left(PV\frac{\Delta I}{\Delta G} > 0\right)$ | $<$0.01 | $<$0.01 | $<$0.01 | $<$0.01 | $<$0.01 |

# MODEL 2: RBC WITH REAL FRICTIONS



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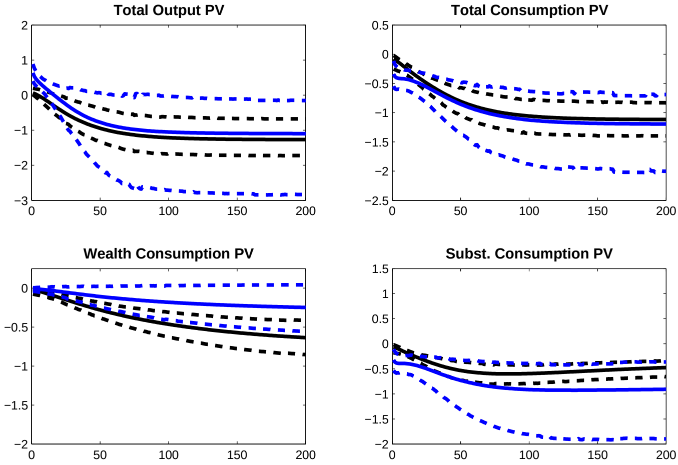

# Model 2: RBC with Real Frictions

- More dispersed range of multipliers
- Agents and firms want to smooth consumption and investmtent
- Smaller wealth effects (agents care about $c_t, c_{t-1}$), larger substitution effects (more sensitive to price changes)
- Same policy implications

# Model 3: Sticky Price & Wage

Add to Model 2

- Monopolistically competitive intermediate goods & labor services

$$Y_t = \left[ \int_0^1 y_t(i)^{\frac{1}{1+\eta_p}} di \right]^{1+\eta_p}$$

- Price & wage stickiness via Calvo (1983)

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$$Y_t = \left[ \int_0^1 y_t(i)^{\frac{1}{1+\eta_p}} di \right]^{1+\eta_p}$$

- Price & wage stickiness via Calvo (1983)
  - prob. $1 - \omega_p$ re-optimize
  - prob. $\omega_p$ partial indexation: $p_t = \pi_{t-1}^{\chi_p} p_{t-1}$

# Model 3: Sticky Price & Wage

- Monetary policy via Taylor rule

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) \left[ \phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t \right] + \epsilon_t^r$$

# Model 3: Sticky Price & Wage

| Variable | Impact | 4 quart. | 10 quart. | 25 quart. | $\infty$ |
|---|---|---|---|---|---|
| Prob$\left(PV\frac{\Delta Y}{\Delta G} > 1\right)$ | 0.35 | 0.01 | $<$0.01 | 0.00 | 0.00 |
| Prob$\left(PV\frac{\Delta C}{\Delta G} > 0\right)$ | $<$0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| Prob$\left(PV\frac{\Delta I}{\Delta G} > 0\right)$ | $<$0.01 | $<$0.01 | $<$0.01 | $<$0.01 | 0.00 |

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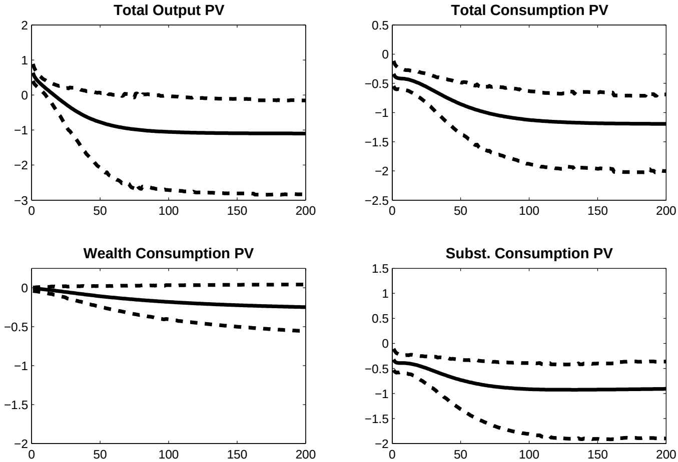

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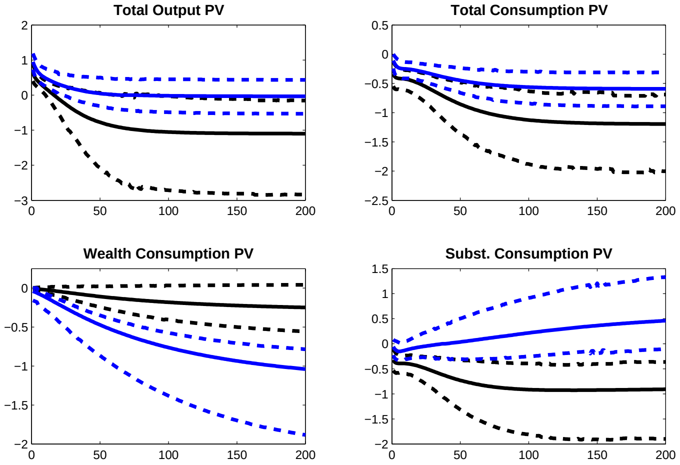

# Model 3: Sticky Price & Wage

- Much larger multipliers
- sticky prices $\rightarrow$ firms respond to a government spending increase by increasing production rather than their price
- Sub Effect: sticky wages $\rightarrow$ wage substitution effect is now often positive (increasing real wages)
- CB doesn't raise nominal rate enough initially to keep real rate from falling
- Wealth Effect: initial real value of debt higher (than flex price case), requires larger fiscal adjustment

# Model 4: Non-Savers

Add to Model 3

- Non-savers consume entire per period disposable income

$$c_t^N = (1 - \tau_t^L) w_t L_t^N + Z_t^N$$

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- Set wage to average of savers

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$$c_t^N = (1 - \tau_t^L) w_t L_t^N + Z_t^N$$

- Set wage to average of savers
- Crucial parameter: percentage of non-savers
  $\mu \sim B(0.3, 0.1)$

# Model 4: Non-Savers

| Variable | Impact | 4 quart. | 10 quart. | 25 quart. | $\infty$ |
|---|---|---|---|---|---|
| Prob$\left(PV\frac{\Delta Y}{\Delta G} > 1\right)$ | 0.88 | 0.32 | 0.07 | 0.02 | 0.01 |
| Prob$\left(PV\frac{\Delta C}{\Delta G} > 0\right)$ | 0.84 | 0.46 | 0.18 | 0.02 | 0.01 |
| Prob$\left(PV\frac{\Delta I}{\Delta G} > 0\right)$ | $<$0.01 | $<$0.01 | $<$0.01 | $<$0.01 | 0.01 |

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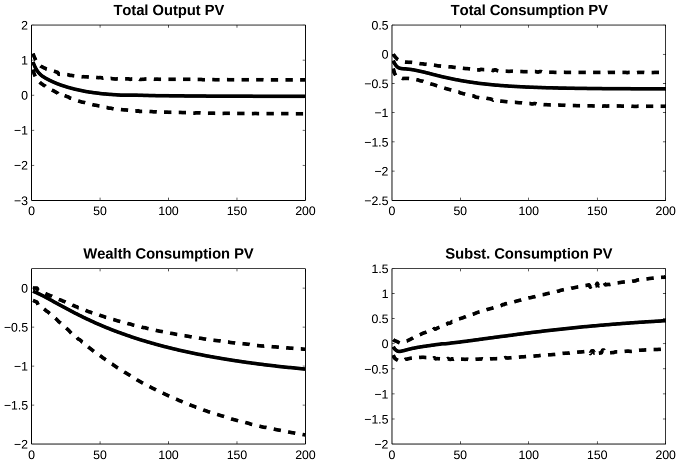
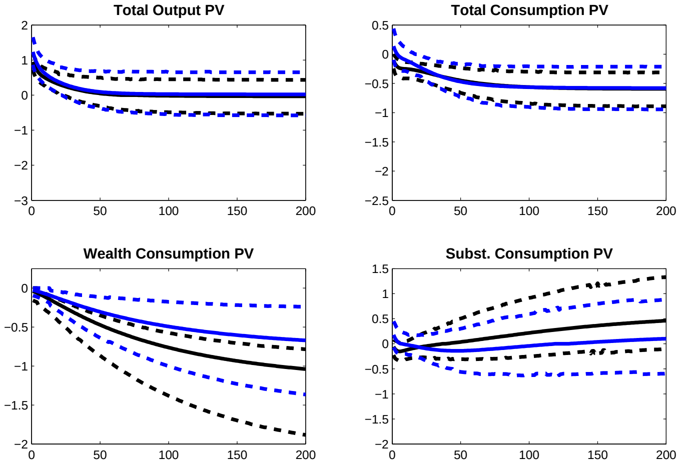


# MODEL 4: NON-SAVERS



# MODEL 4: NON-SAVERS

- Much, much larger impact multipliers, similar long-run multipliers
- intuition straightfoward: nonsavers are nonsavers
- *the* most crucial parameter value

# Model 5: Open Economy

Add to Model 4

- Two large symmetric countries ($H$ & $F$)
- Complete financial markets
- $C$ and $I$ consist of domestic and imported goods

$$Q_t^C = \left[ (1-\nu_c)^{\frac{1}{\mu_c}} (C_t^H)^{\frac{\mu_C - 1}{\mu_C}} + \nu_C^{\frac{1}{\mu_C}} (C_t^F)^{\frac{\mu_C - 1}{\mu_C}} \right]^{\frac{\mu_C}{\mu_C - 1}}$$

- $G$ non-traded

# Model 5: Open Economy

- Home market domestic demand:

$$y_t^H(i) = Y_t^H \left( \frac{p_t^H(i)}{P_t^H} \right)^{-\frac{1+\eta^p}{\eta^P}}$$

- Home market foreign demand:

$$m_t(i) = M_t^* \left( \frac{p_t^{H*}(i)}{P_t^{H*}} \right)^{-\frac{1+\eta^p}{\eta^P}}$$

- local currency pricing

# Model 5: Open Economy

| Variable | Impact | 4 quart. | 10 quart. | 25 quart. | $\infty$ |
|---|---|---|---|---|---|
| Prob$\left(PV\frac{\Delta Y}{\Delta G} > 1\right)$ | 0.81 | 0.27 | 0.05 | 0.01 | 0.01 |
| Prob$\left(PV\frac{\Delta C}{\Delta G} > 0\right)$ | 0.82 | 0.48 | 0.23 | 0.02 | $<$0.01 |
| Prob$\left(PV\frac{\Delta I}{\Delta G} > 0\right)$ | $<$0.01 | $<$0.01 | $<$0.01 | $<$0.01 | 0.01 |

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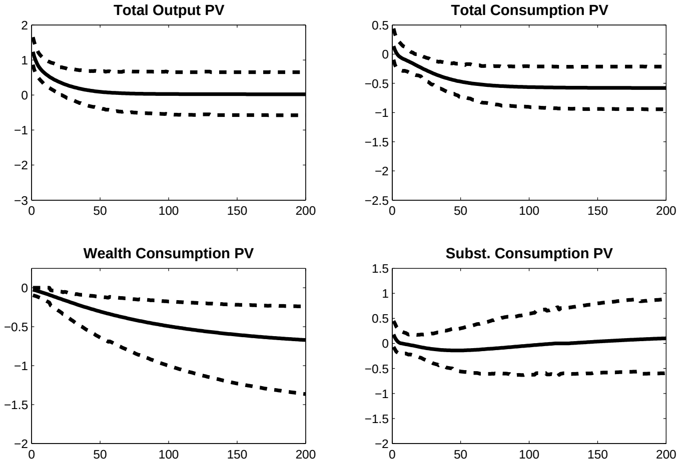

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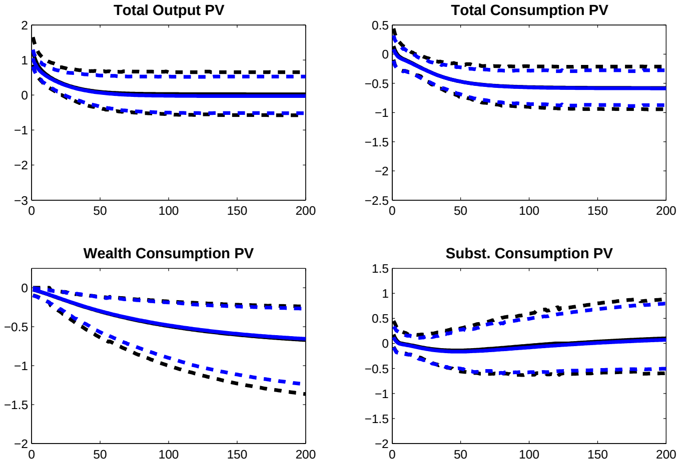

# Model 5: Open Economy

- smaller multipliers
- import-substitution effect: increases in government expenditures induce a substitution away from domestically produced goods towards imported goods.
- Multipliers are smaller still when government spending is a traded good as part of the increase in government spending is “leaked” to the foreign country

# Root Mean Square Deviations

How much do multipliers vary on average due to particular parameter?

- Draw $\tilde{\theta} = [\tilde{\theta}_1 \quad ... \quad \tilde{\theta}_n]'$ from $p(\theta)$. Calculate $\tilde{\omega}|\tilde{\theta}_n$
- Let $\tilde{\theta}^i = [\tilde{\theta}_1 \quad ... \quad E[\theta_i] \quad ... \quad \tilde{\theta}_n]'$. Calculate $\tilde{\omega}^i|\tilde{\theta}^i$
- Calculate $\sqrt{\frac{\sum_{j=1}^{M}(\tilde{\omega}_j - \tilde{\omega}_j^i)^2}{M}}$

# RMSDs for NK open economy model.

| Impact $\frac{\Delta C}{\Delta G}$ | |
|---|---|
| $\mu$, fraction of non-savers | 0.115 |
| $\rho_G$, lagged govt cons resp. | 0.065 |
| $\theta_c$, habit formation | 0.048 |
| $\rho_r$, lagged interest rate resp. | 0.047 |
| $\gamma$, risk aversion | 0.035 |

| $PV_\infty \frac{\Delta C}{\Delta G}$ | |
|---|---|
| $\rho_G$, lagged govt cons resp. | 0.202 |
| $\gamma$, risk aversion | 0.055 |
| $\rho_r$, lagged interest rate resp. | 0.047 |
| $\omega_w$, wage stickiness | 0.044 |
| $\xi$, inverse Frisch labor elast. | 0.042 |

# RMSDs for NK open economy model.

| Impact $\frac{\Delta Y}{\Delta G}$ | |
|---|---|
| $\mu$, fraction of non-savers | 0.123 |
| $\rho_G$, lagged govt cons resp. | 0.120 |
| $\psi$, capital utilization | 0.095 |
| $\rho_r$, lagged interest rate resp. | 0.065 |
| $\theta_c$, habit formation | 0.052 |

| $PV_\infty \frac{\Delta Y}{\Delta G}$ | |
|---|---|
| $\rho_G$, lagged govt cons resp. | 0.427 |
| $\rho_r$, lagged interest rate resp. | 0.096 |
| $\omega_w$, wage stickiness | 0.086 |
| $\xi$, inverse Frisch labor elast. | 0.086 |
| $\phi_\pi$, interest rate resp. to inflation | 0.068 |

# Alternative MP-FP Interaction

- Multipliers depend on MP-FP interaction
  - Davig & Leeper (2009), Christiano, Eichenbaum, Rebelo (2009)

- Calculate multipliers for passive monetary and active fiscal policy regime
  - FP unconstrained: doesn't control $B$ growth
  - MP satisfies equilibrium conditions: $R$ adjusts less than 1-1 with $\pi$

# Model 5: Open Economy PMAF

| Variable | Impact | 4 quart. | 10 quart. | 25 quart. | $\infty$ |
|---|---|---|---|---|---|
| Prob$\left(PV\frac{\Delta Y}{\Delta G} > 1\right)$ | 1.00 | 1.00 | 0.97 | 0.93 | 0.91 |
| Prob$\left(PV\frac{\Delta C}{\Delta G} > 0\right)$ | 1.00 | 1.00 | 1.00 | 0.99 | 0.93 |
| Prob$\left(PV\frac{\Delta I}{\Delta G} > 0\right)$ | 0.73 | 0.53 | 0.45 | 0.44 | 0.47 |

# Conclusion

- DSGE specification matters! If not careful, results can be imposed on data
- Most important features for multiplier variation:
  - gov. spending process
  - hand-to-mouth agents
  - monetary-fiscal interactions

# CONCLUSION

- DSGE specification matters! If not careful, results can be imposed on data
- Most important features for multiplier variation:
  - gov. spending process
  - hand-to-mouth agents
  - monetary-fiscal interactions
- Broader message: use PPA to shine light on DSGE black box