Do Sticky Prices Increase Real Exchange Rate Volatility at the Sector Level?*

Mario J. Crucini[†], Mototsugu Shintani[‡] and Takayuki Tsuruga[§]

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Abstract

We introduce the real exchange rate volatility curve as a useful device to understand the relationship between price stickiness and the fluctuations in Law of One Price deviations. In the presence of both nominal and real shocks, the theory predicts that the real exchange rate volatility curve is a U-shaped function of the degree of price stickiness. Using sector-level US-European real exchange rate data and frequency of price changes, we estimate the volatility curve and find the predominance of real effects over nominal effects. Good-by-good variance decompositions show that the relative contribution of nominal shocks is smaller at the sector level than what previous studies have found at the aggregate level. We conjecture that this is due to significant averaging out of good-specific real microeconomic shocks in the process of aggregation.

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[†]Department of Economics, Vanderbilt University and NBER; e-mail: `mario.crucini@vanderbilt.edu`.

[‡]Department of Economics, Vanderbilt University; e-mail: `mototsugu.shintani@vanderbilt.edu`.

[§]Graduate School of Economics, Kyoto University; e-mail: `tsuruga@econ.kyoto-u.ac.jp`.

1 Introduction

Among international macroeconomists, it is widely believed that the variability of real exchange rates is increasing in the degree of price rigidity. A reasoning is found in a prominent textbook by Dornbusch, Fischer, and Startz (2004):

> "Exchange rate overshooting results from the rapid response of exchange rates to monetary policy and the sluggish adjustment of prices. A monetary expansion will lead to an immediate depreciation but only a gradual increase in prices. Exchange rate overshooting implies that real exchange rates are highly volatile (p. 534)."

The basic idea is as follows. The nominal exchange rate is an asset price (since currencies are actively traded in the foreign exchange market) and thus it adjusts instantaneously in response to nominal shocks. However, if prices of many goods and services adjust sluggishly, the real exchange rate will be highly volatile because it comoves with the nominal exchange rate. The expectation, then, is a *positive* correlation between the volatility of real exchange rates and the degree of price stickiness if *nominal* shocks dominate the landscape, as they do in much theorizing on the topic. Quantitative investigations of this prediction have been undertaken, by Chari, Kehoe, and McGrattan (2002) who focus on the aggregate real exchange rates and by Kehoe and Midrigan (2007) who focus on Law of One Price (LOP) deviations.

An early advocate for the role of real shocks in the equilibrium determination of real exchange rates is Stockman (1980). Stockman casts his model in a flexible price setting, so that nominal shocks make no contribution to real exchange rate volatility. Crucini, Shintani, and Tsuruga (2010), on the other hand, neutralize the effect of nominal shocks by focusing on intranational trade and investigate the role of real shocks on good-level real exchange rate volatility across cities in the presence of price rigidity. Unlike models emphasizing the role of the nominal shocks, their model predicts a *negative* correlation between price stickiness and real exchange rate variability because only *real* shocks affect real exchange rates across locations within a country.

The current paper puts these two views of real exchange rate determination on the same playing field by combining the model of Kehoe and Midrigan (2007) which emphasizes nominal shocks with the model of Crucini, Shintani, and Tsuruga (2010) which emphasizes real shocks. These models rely on the time dependent pricing assumption, but allow the frequencies of price changes to vary across goods, as measured in the micro-data. Under the synthesized framework, we theoretically explore the cross-sectional relationship between price stickiness and real exchange rate volatility at the level of individual goods. We refer to this relationship as the real exchange rate volatility curve: the functional relationship between the forecast error variance of the real exchange rate and the infrequency of price changes at the level of a good. When real shocks are absent, the volatility curve is upward-sloping: an increasing function of the price stickiness parameter and the good with the stickiest price should exhibit the greatest amount of real exchange rate variability. When nominal shocks are turned off, the volatility curve is downward-sloping: a decreasing function of the price stickiness parameter and the good with the stickiest price has the least amount of real exchange rate variability. When both real and nominal shocks are present, the real exchange rate volatility curve becomes U-shaped.

We estimate the volatility curve using sector-level real exchanges of Austria, Belgium, France, and Spain vis à vis the US, constructed by Kehoe and Midrigan (2007). We find the estimated U-shaped curve is monotonically decreasing over the majority of the range of price stickiness. Our main findings regarding the shape of the curve are confirmed by both parametric and nonparametric estimation methods. The negative correlation together with the theoretical prediction of our model suggests the predominance of real shocks over nominal shocks in explaining the volatility of real exchange rates at the sector level.

At the aggregate level, the relative contribution of real and nominal shocks to real exchange rate variability has been typically evaluated in terms of forecast error variance decompositions (e.g., Clarida and Galí, 1994, Eichenbaum and Evans, 1995, and Rogers, 1999).[1] Following this literature, we further conduct variance decompositions of real exchange rates at the sector level, and evaluate the relative contribution of nominal and real shocks. For the majority of goods, the contribution of nominal shocks is smaller than that of real shocks, and real shocks rise in dominance as the forecast horizon lengthens. To reconcile our microeconomic evidence with the macroeconomic evidence, it seems necessary to allow for large idiosyncratic real shocks at the sector level such that these microeconomic sources of variation average out in the move to the CPI-based real exchange rate (see, Crucini and Telmer, 2007, Broda and Weinstein, 2008, and Bergin, Glick and Wu, 2012).

2 The Model

2.1 The real exchange rate volatility curve

The theory combines the key features of Kehoe and Midrigan (2007) and Crucini, Shintani and Tsuruga (2010). Both of these models assume heterogeneous price stickiness across goods, but the former relies on nominal exchange rate variations whereas the latter focuses on the labor productivity variations along with trade costs in explaining the volatility of good-level real exchange rates.

In what follows, we present a sketch of our model to discuss its main implication for good-level real exchange rate volatility.[2] The (log) real exchange rate for a bilateral pair of

[1] Some exceptions, such as Steinsson (2008), focus on the shape of impulse response function to evaluate the relative importance of nominal and real shocks.

[2] The full model is presented in the technical appendix of this paper, which is available from the authors upon request.

countries is defined as:

$$q_{it} = s_t + p^*_{it} - p_{it}, \tag{1}$$

where p_{it} (p^*_{it}) denotes the (log) price index for good i in the home (foreign) country and s_t is the (log) nominal exchange rate, at period t. Throughout the paper, variables marked with an asterisk denote foreign analogs of home variables.

To introduce the real exchange rate volatility curve, some simplifying assumptions are made on the sources of real exchange rate variation. The first assumptions concern nominal shocks and exchange rates. The nominal shocks in the model are the home and foreign money growth rate, μ_t and μ^*_t, which are independent and identically distributed (i.i.d.). Semi-log household preferences over consumption and leisure, combined with a local-currency cash-in-advance constraint, lead to the equality of the money growth differential and the nominal exchange rate growth (i.e., $\mu_t - \mu^*_t = \Delta s_t$).[3] These assumptions are taken from Kehoe and Midrigan (2007) and conveniently lead the nominal exchange rate s_t to follow a random walk, a characteristic similar to the data.[4]

The second assumptions concern real shocks and trade costs. Monopolistically competitive firms set prices of their goods, which are produced using a technology that is linear in labor and subject to productivity shocks. Productivity shocks in two countries (a_{it} and a^*_{it}) are given by:

$$a_{it} = z_t + \eta_t + \varepsilon_{it}, \qquad a^*_{it} = z_t + \eta^*_t + \varepsilon^*_{it}. \tag{2}$$

Due to our microeconomic focus, the productivity shock for each good consists of three components: a possibly nonstationary global trend component (z_t), an i.i.d. nation-specific

[3]To be specific, semi-log period utility is given by $\ln C_t - \chi L_t$, where C_t, L_t and $\chi(> 0)$, denote aggregate consumption, hours worked, and marginal disutility of labor supply, respectively.

[4]With the cost of losing computational simplicity of the real exchange rate volatility curve, we can also replace i.i.d. money growth with serially correlated money growth.

component (η_t and η_t^*) and an i.i.d. good-specific component (ε_{it} and ε_{it}^*).[5] Finally, firms in each country are required to pay an iceberg transportation cost τ to send a good across the border. This transportation cost leads to home bias in consumption because the home variety of each good is cheaper than the imported variety of the same good. We denote the elasticity of substitution among differentiated products by θ and the discount factor by β; these parameters are assumed to satisfy $\theta > 1$ and $0 < \beta < 1$. Finally, firms reset their prices under Calvo-type pricing with the good-specific degree of price stickiness given by the probability of no price change λ_i.

The focal equation of the model is the k-period-ahead forecast error variance of the sector-level real exchange rate:

$$Var_{t-k}(q_{it}) = \Lambda_{ik}[\lambda_i^2 Var(\mu_t - \mu_t^*) + (1-\lambda_i)^2(1-\lambda_i\beta)^2\psi^2 Var(a_{it} - a_{it}^*)] \tag{3}$$

where $\Lambda_{ik} = \sum_{j=1}^{k} \lambda_i^{2(j-1)}$ and $\psi = \left(1-(1+\tau)^{1-\theta}\right) / \left(1+(1+\tau)^{1-\theta}\right)$. The parameter ψ appears because a productivity shock to good i in one country asymmetrically transmits to the price indexes of the same good in two countries due to the home bias.

Equation (3) attributes the forecast error variance of the good-level real exchange rate to the variance of the money growth differential, $\mu_t - \mu_t^*$ (the nominal shocks) and the variance of the cross-country productivity differential, $a_{it} - a_{it}^*$ (the real shocks). Recall that price stickiness parameter λ_i is assumed to be common across countries but differs across goods. Viewed as a function of λ_i, this equation is called *the real exchange rate volatility curve.*

Note that the coefficient on the nominal shock $\lambda_i^2\Lambda_{ik}$ is increasing in λ_i, while the coefficient on the real shock $(1-\lambda_i)^2(1-\lambda_i\beta)^2\psi^2\Lambda_{ik}$ is decreasing in λ_i. Therefore, for any forecast

[5] Here we impose an i.i.d. assumption on nation- and good-specific component for ease of exposition. As in the case of nominal shocks, however, we can easily relax this assumption and introduce persistence without changing the qualitative implication of real exchange rate volatility curve. For the same reason, we further assume that variance of good-specific component is common across i. In the empirical part of the paper, the possiblity of heterogenous variances of real shocks across goods is considered.

horizon k, an increase in λ_i increases the contribution of the nominal effect (measured by $\Lambda_{ik}\lambda_i^2 Var(\mu_t - \mu_t^*)$) and decreases the contribution of the real effect (measured by $\Lambda_{ik}(1 - \lambda_i)^2(1 - \lambda_i\beta)^2\psi^2 Var(a_{it} - a_{it}^*)$) to the total forecast error variance of the good-level real exchange rate. These two opposing forces give rise to a real exchange rate volatility curve that is U-shaped over the support $\lambda_i \in [0, 1]$.[6]

To gain some intuition behind the mechanism, recall that q_{it} is defined as $s_t + p_{it}^* - p_{it}$. To see the impact of nominal shocks, consider a positive money growth rate shock in the home country. The model predicts an immediate depreciation of the nominal exchange rate, that is, an increase in s_t. The responses of p_{it}, however, depend on the good-specific frequencies of price adjustment. For goods with prices that change every period, an increase in p_{it} completely offsets an increase in s_t keeping q_{it} unchanged. At the other end of the continuum, goods with prices that are extremely sticky will have q_{it} that basically follows the path of s_t with negligible pass-through of the nominal shock to p_{it}. The nominal effect on real exchange rate variability is amplified by sluggish adjustment of prices. Simply put: conditional on nominal shocks, the correlation between real exchange rate volatility and the degree of price stickiness is positive.

Turning to real shocks, consider a positive shock in home productivity in good i. In our model, this productivity shock has no equilibrium consequences for s_t. What it does is reduce both p_{it} and p_{it}^*, because firms in the home country sell their goods in both countries. However, due to home bias generated by trade costs, p_{it} will decrease more than p_{it}^*. This increases $p_{it}^* - p_{it}$ which results in an increase in $q_{it} = s_t + p_{it}^* - p_{it}$. Because this channel requires prices

[6] To prove this, evaluate the first derivative of the total variance with respect to λ_i at $\lambda_i = 0$ and 1. When evaluated at $\lambda_i = 0$, the first derivative of the variance due to the nominal effect is zero but that due to the real effect is negative and finite, which implies that the first derivative of the total variance with respect to λ_i is strictly negative when $\lambda_i = 0$. Analogously, we can also show that the total variance has a strictly positive slope at $\lambda_i = 1$. Because total variance is continuous in λ_i, there exists $\lambda_i \in (0, 1)$ that minimizes total variance.

to actually change and thereby induce asymmetric price changes across countries, it is more quantitatively important when prices are flexible. Conversely, the real effect is mitigated by sticky prices. Simply put: conditional on real shocks, the correlation between real exchange rate volatility and the degree of price stickiness is negative.

2.2 Numerical Examples

Let us provide numerical examples to evaluate how different intensities of real and nominal shocks alter the shape of the real exchange rate volatility curve. We focus on the one-period-ahead forecast error variance by setting $k = 1$ in equation (3),

$$Var_{t-1}(q_{it}) = \lambda_i^2 Var(\mu_t - \mu_t^*) + (1 - \lambda_i)^2 (1 - \lambda_i \beta)^2 \psi^2 Var(a_{it} - a_{it}^*). \tag{4}$$

The structural parameters are calibrated as follows: (i) the data is monthly, so the discount factor is set to $\beta = 0.96^{1/12}$; (ii) trade costs, broadly defined at the retail level, are in the neighborhood of $\tau = 0.5$; and (iii) the elasticity of substitution is set at $\theta = 10$.[7] Figure 1 shows the shape of the curve under three distinct stochastic environments: (a) $Std(a_{it} - a_{it}^*)/Std(\mu_t - \mu_t^*) = 5$; (b) $Std(a_{it} - a_{it}^*)/Std(\mu_t - \mu_t^*) = 1$; and (c) $Std(a_{it} - a_{it}^*)/Std(\mu_t - \mu_t^*) = 1/5$.[8] The height of the volatility curve in each panel is the model's prediction for the total forecast error variance of the real exchange rate for a particular good indexed by λ_i. The blue and red segments of a vertical line drawn at each λ_i gives the contributions of real and nominal shocks, respectively. For example, real exchange rate fluctuations of goods with fully flexible price (e.g., crude petroleum) are driven solely by real shocks while goods with completely rigid prices (e.g., postage stamps) are driven solely by nominal shocks.

Each panel of the figure clearly suggests that the shape of the curve depends crucially on

[7] Here we follow Carvalho and Nechio (2011) in setting the substitution across varieties within the same sector as $\theta = 10$. However, the shape of the curve is qualitatively insensitive to the values of θ and τ.

[8] For all three cases, $Std(\mu_t - \mu_t^*)$ is normalized to 1 percent.

the relative magnitude of the two shocks. The U-shaped curves are minimized at $\lambda_i = 0.75$ in panel (a), 0.40 in panel (b), and 0.06 in panel (c). While shock structures in terms of variance ratios are symmetric between panels (a) and (c), the shapes of their volatility curves are asymmetric because of asymmetric roles played by nominal and real shock volatility.[9] This feature also explains that the curve in panel (b) is minimized at $\lambda_i = 0.40$ rather than 0.50, the midpoint of the unit interval.

When the real shock dominates, the model predicts that the volatility curve is downward sloping over almost its entire range as in panel (a). The middle panel (b), with equal variances of real and nominal shocks, displays an obvious U-shape. When the nominal shock dominates, the curve is upward sloping over almost its entire range as in panel (c). In practice the sign of the correlation of price stickiness and real exchange rate volatility will depend on the distribution of goods in the sample as well as the relative importance of real and nominal shocks. In addition, since the real exchange rate volatility curve is U-shaped in general, introducing a nonlinear functional form in the regression, rather than relying a simple linear regression, is essential for detecting the underlying structure.

When the degree of price stickiness is uniformly distributed across goods, blue and red areas in Figure 1 can be interpreted as the cross-sectional average of the variance decomposition for each individual good. In panel (a), the red area (nominal effects) accounts for only 7% and the blue area (real effects) contributes the remaining 93%. In contrast, the nominal effects account for 65% in panel (b) and 98% in panel (c). While the average variance decomposition is a convenient measure under the uniform distribution assumption, it should be noted that variance decompositions can still greatly vary across the individual goods. For

[9] Since β and ψ are close to unity in our setting, the curve can be approximated by $\lambda_i^2 Var(\mu_t - \mu_t^*) + (1 - \lambda_i)^4 Var(a_{it} - a_{it}^*)$. This shows that the relative role of nominal and real shock volatilities is asymmetric since the weight on the former approaches 0 at a slower rate as $\lambda_i \to 0$, compared to the coefficient on the latter which approaches 0 at a faster rate as $\lambda_i \to 1$.

example, consider two goods, one with $\lambda_1 = 0.25$ and the other with $\lambda_2 = 0.56$ in panel (b). These two real exchange rates turn out to have the same total forecast error variance. However, the relative contribution of nominal shocks is only 18% for the first good but 90% for the second good.

3 Empirical Analysis

3.1 Data

The empirical analysis focuses on (i) examining the relationship between total variance of real exchange rates and the degree of price stickiness based on the theoretical volatility curve; and (ii) assessing the relative importance of the real and nominal effects at the disaggregated level. The data is from Kehoe and Midrigan (2007) and consists of 66 sectoral real exchange rates for four European countries (Austria, Belgium, France, and Spain) relative to the US from January 1996 to December 2006. The series are constructed by matching monthly local currency micro-price data from Eurostat and the Bureau of Labor Statistics and converting to a common-currency using spot nominal exchange rates. In addition, Kehoe and Midrigan (2007) take the cross-country average monthly infrequencies of price changes within each sector. The country-specific frequencies for the US are from Bils and Klenow (2004) and those for each of the European countries are taken from the individual country studies by: Baumgartner, Glatzer, Rumler, and Stiglbauer (2005) for Austria; Aucremanne and Dhyne (2004) for Belgium; Baudry, Le Bihan, Sevestre, and Tarrieu (2007) for France; Alvarez and Hernando (2006) for Spain. The details of the data construction are found in the appendix of Kehoe and Midrigan (2007).

The Euro was officially introduced part-way through our sample. Even before the introduction of the Euro, the volatility of the nominal exchange rate against the US dollar was quite similar across these European countries.[10] This rationalizes a pooled regression of the four country-pairs as a benchmark in the analysis below. However, the productivity differentials may differ across bilateral pairs, so we estimate the relationship for each country separately as a robustness check.

3.2 Regression analysis

Let V_{ij} be the one-period-ahead forecast error variance of the real exchange rate for good i for country j, vis à vis the United States. The technical appendix of the paper proves that q_{ijt} follows an AR(1) process with an AR coefficient λ_{ij} under a set of maintained assumptions. Effectively, this means V_{ij} is equal to the sample variance of $q_{ijt} - \lambda_{ij} q_{ijt-1}$ using the observed infrequency of price changes, λ_{ij}. When either λ_{ij} or q_{ijt} is missing or when V_{ij} can be computed from only a short time sample, we exclude such goods from the sample.[11]

As a preliminary analysis, results of simple linear regressions of V_{ij} on λ_{ij} are reported in Table 1. The estimated coefficients on λ_{ij} are significantly negative and similar across all cases. The pooled regression explains 70 percent of the cross-sectional variation in the volatility of real exchange rates. The nation-specific regressions fit well especially for Austria and France. This preliminary regression estimates suggest that the real exchange rate volatility curve is downward sloping within the range of the observed λ_{ij} (denoted $\lambda_{\min}$ and $\lambda_{\max}$ in Table 1).

Recall, however, that the theory predicts a nonlinear relationship, rather than linear relationship, between the frequency of price adjustment and real exchange rate variability.[12] Therefore, following the structural form of (4), the estimation is augmented with a quadratic

[10] The standard deviations of the nominal exchange rate growth of the US dollar against Austrian Schillings, Belgian Francs, French Francs, and Spanish Pesetas are 2.36, 2.37, 2.35, and 2.36 percent, respectively.

[11] After excluding the samples, the number of sectors amounts to 57 for Austria, 46 for Belgium, 48 for France, and 31 for Spain.

[12] Using Ramsey's (1969) RESET test, the null hypothesis of linearity is rejected at the one percent significance level for the pooled case, as well as for the Austrian and French cases. Weaker evidence of nonlinearity is obtained for Belgian and Spanish cases possibly because the power of the test is lower in smaller samples.

term and a quartic term:

$$V_{ij} = b_{1j}\lambda_{ij}^2 + b_{2j}(1-\lambda_{ij})^2(1-\lambda_{ij}\beta)^2 + u_{ij}, \tag{5}$$

where the b's are regression coefficients and u_{ij} is the regression error term for good i for country j. The second regressor is constructed by setting $\beta = 0.96^{1/12}$. According to (4), regression coefficient b_{1j} should capture the nominal effects, due to $Var(\mu_t - \mu_{jt}^*)$. The regression coefficient, b_{2j}, captures the real effects $\psi_j^2 Var(a_{it} - a_{ijt}^*)$, with the restriction that the variance of productivity differentials are common across i.

Table 2 presents the estimation results of the quartic regression model (5). In all cases, the estimated coefficients, b_{1j} and b_{2j}, are significantly positive, which is consistent with the theory. The quartic regression is comparable to the linear regression in terms of the goodness of fit. The relative role of nominal and real shocks, as implied by the estimates b_{1j} and b_{2j}, indicates a much larger role for real shocks.[13] Because the home bias term satisfy $0 \leq \psi_j^2 \leq 1$, b_{2j} can be considered as a lower bound on $Var(a_{it} - a_{ijt}^*)$. The same argument establishes that $\sqrt{b_{2j}/b_{1j}}$ can be used as a lower bound for $Std(a_{it} - a_{ijt}^*)/Std(\mu_t - \mu_{jt}^*)$. The lower bounds are inferred to be: 5.28, 6.17, 4.42, 5.30 and 9.64, for the pooled case and the Austrian, Belgian, French and Spanish nation-specific cases, respectively. Thus, the quartic regression result is very comparable to the numerical example of case (a) where $Std(a_{it} - a_{it}^*)/Std(\mu_t - \mu_t^*) = 5$. This is further confirmed in the left panel of Figure 2 showing the fitted curve of the pooled quartic regression (the solid line) from Table 2, along with that of the pooled linear regression (the dashed line) from Table 1. The fitted curve of the quartic regression resembles panel (a) of Figure 1 in terms of the shape of the curve, suggesting the importance of real effects.

[13] We also conduct the robustness analysis, using the first-order autocorrelation (ρ_{ij}) of the sector-level real exchange rates, rather than λ_{ij}. In particular, we obtain the sample variance of $q_{ijt} - \rho_{ij}q_{ijt-1}$ and run the quartic regression (5) by replacing λ_{ij} with ρ_{ij}. Even when we use ρ_{ij} for the quartic regressions, the estimated coefficients are both significantly positive in the pooled regression and country-by-country regressions except for Spain and again suggest a large role for real shocks as in the case of λ_{ij}.

As reported in the last two columns of Table 2, estimates of b_{1j} and b_{2j} can be also used to locate the degree of price stickiness which minimizes the forecast error variance. For the pooled case, the variance of the real exchange rate is minimized at $\lambda_i = 0.76$ which is remarkably close to the value of 0.75 from our numerical example with dominant real shocks. This frequency of price changes implies that the duration between price changes is 4.2 months.

The quartic regression (5) imposes a strict theoretical shape restriction on the real exchange rate volatility curve. As a robustness check, the functional form restriction is replaced with a general nonparametric regression,

$$V_{ij} = m_j(\lambda_{ij}) + u_{ij}$$

where $m_j(\cdot)$ is an unknown conditional mean function for country j. The right panel of Figure 2 shows the estimated curve using the nonparametric local linear regression estimator with pooled data.[14] The shape of the fitted curve shown as the solid line is very different from the linear regression fit shown as the dashed line. This suggests the plausibility of a nonlinear structure in the real exchange volatility curve.

Turning to a comparison between the quartic regression (5) and the nonparametric regression, both similarities and differences are evident. Both estimates imply convexity in the real exchange rate volatility curve. When the first derivative of the m function is evaluated nonparametrically, it tends to be increasing in λ_{ij}, which is consistent with the theoretical prediction. The slope of the curve is negative over the empirical range of λ_{ij} and it becomes flatter as λ_{ij} increases. The most notable difference between the two is the location of the bottom of the curve. The value of λ_{ij} which minimizes the forecast error variance in the nonparametric regression is close to unity, larger than the value predicted by the quartic

[14]In estimation, Gaussian kernel is used along with the bandwidth selected by the rule of thumb.

regression.

To formally investigate the shape of the estimated curve, a nonparametric test of monotonicity developed by Ghosal, Sen and van der Vaart (2000) is employed – a test of the null hypothesis that the m function is an increasing (or decreasing) function over a certain interval. In the present context, the shape of the curve is examined over the observed range of the data, $[\lambda_{\min}, \lambda_{\max}]$. The test is also applied to establish the monotonicity of the first derivative of the m function. Table 3 shows that the hypothesis of an increasing function in λ_{ij} is rejected, and that of decreasing function is not, based on a conventional significance level. For the first derivative, the test fails to reject a monotonically increasing function while a monotonically decreasing function is rejected. Therefore, the estimated real exchange rate volatility curve is a convex function consistent with the U-shape prediction of the theory.

3.3 Variance decomposition

Let us now turn to the relative importance of the real and nominal effects at the sector level by directly using equation (3) at various horizons. First, we compute the k-period-ahead forecast error variance, $Var_{t-k}(q_{ijt})$, from the sample variance of the quasi-difference $q_{ijt} - \lambda_{ij}^k q_{ijt-k}$, using observed sectoral infrequency of price changes, λ_{ij}. Second, because $\mu_t - \mu_{jt}^* = \Delta s_{jt}$ according to the theory, the nominal effect can be evaluated by $\lambda_{ij}^2 \Lambda_{ijk} Var(\Delta s_{jt})$. Combining the two, the relative contribution of nominal shocks to the forecast error variance of the real exchange rate is:

$$\phi(i,j,k) = \frac{\lambda_{ij}^2 \Lambda_{ijk} Var(\Delta s_{jt})}{Var_{t-k}(q_{ijt})}$$

where the indices of the variance decomposition, $\phi(i,j,k)$, reflect good, country-pair and horizon, respectively. For the limiting case of $k \to \infty$, we utilize the sample variance of q_{ijt} and $[\lambda_{ij}^2/(1-\lambda_{ij}^2)]Var(\Delta s_{jt})$ to measure the relative contribution of the nominal shocks to the unconditional variance. For the purpose of evaluating the relative role of shocks, this approach has an advantage over the direct estimation of the volatility curve in the sense that it allows for heterogeneous variance of real shocks across sectors.

Table 4 reports the summary statistics for the contributions of the nominal shocks to the forecast error variance of sectoral real exchange rates at monthly horizons of $k = 1, 3, 6, 12$ and ∞. Note that, unlike the variance decomposition of aggregate real exchange rates often reported in the literature, the decomposition is calculated for each sectoral good. The first row of the table shows the average contribution of nominal shocks, with the average taken across all goods and all four bilateral pairs. The numbers in parentheses in the second row are the standard deviations of these contributions across goods and countries. The remaining rows report corresponding results for each pair of countries.

For the one-period-ahead forecast error decomposition, nominal shocks account for about 40 percent of real exchange rate variation and range from a high of 49 percent for Austria to a low of 35 percent for Spain. The large standard deviations in the table imply that the contributions of nominal shocks differ considerably across goods. This cross-sectional dispersion is similar across countries. For the shortest horizon it seems sensible to conclude that the contribution of real shocks is at least as large as that of nominal shocks for many goods.

The role of nominal shocks becomes smaller as the horizon lengthens. At a horizon of 6 months, the relative contribution is about one half of the 1-month horizon. The long-run contribution of nominal shocks, evaluated at $k = \infty$, is lower than 10 percent for all countries except for Austria, thus leaving 90 percent to be explained by real shocks. The cross-sectional variation at the longest horizon is much smaller than that at shorter horizons, implying the dominance of real shocks in real exchange rate fluctuations for most goods.

Let us now compare the variance decompositions of sector-level real exchange rates with previous studies involving the aggregate real exchange rate. Using a structural VAR model, Clarida and Galí (1994, Table 3) find that the relative contribution of nominal shocks to one-period-ahead forecast error variance of quarterly real exchange rate is 47 percent for Germany and 36 percent for Japan. In contrast, our three-month (the counterpart to one quarter) ahead variance decomposition indicates nominal shocks account for between 19 to 31 percent, depending on the country (see Table 4). Using over 100 years of annual UK-US real exchange rate data, Rogers (1999) finds that the contribution of nominal shocks to the one-year-ahead forecast error variance ranges from 19 percent to 60 percent, with a median value of 41 percent. Our 12-month ahead forecast error variance decomposition estimates indicate nominal shocks only account for about 14 percent. The benchmark estimates of Eichenbaum and Evans (1995, Table 1a) show a nominal shock contribution at horizons of 31- to 36-months averaging 38 percent for France, while our estimates imply long-run contributions between 9 and 12 percent for France. Thus, largely independent of the horizon or countries examined, nominal shocks play a more important role in accounting for aggregate real exchange rate fluctuations than for sector-level real exchange rate fluctuations.

What accounts for this difference in the microeconomic and macroeconomic evidence? Our suspicion is that the real shocks tend to average out across sectors. Recall productivity shocks in equation (2) are expected to embody idiosyncratic sector-specific shocks ε_{it} and ε_{it}^*, at least in part. In our model, the aggregation of goods prices eliminates this idiosyncratic component but has no effect on the nominal shocks, μ_t and μ_t^*, which are common across sectors. It is therefore not surprising that nominal and real shocks are more on par as contributors to real exchange rate variation at disaggregated level. This averaging-out argument is consistent with recent micro studies including Crucini and Telmer (2007) who show that only a small fraction of LOP changes are common to all goods, Broda and Weinstein (2008) who find that enormous volatility in the barcode prices is eliminated at the aggregate price level, and Bergin, Glick and Wu (2012) who claim that idiosyncratic industry price shocks account for about 80% of variation in LOP deviations. Thus, explaining the LOP volatility only by nominal exchange rates would leave most of the variation unaccounted for. The decomposition performed here fills this gap with common and sector-specific real shocks.

4 Conclusion

We use a Calvo-type pricing model with real and nominal shocks to develop the concept of a real exchange rate volatility curve. The curve has a U-shape as a function of the degree of price stickiness, implying an ambiguous correlation between the forecast error variance of real exchange rates and the degree of price stickiness. Using US-European real exchange rate data, the correlation was found to be negative over the wide range of observed degree of price stickiness. The downward-sloping profile suggested that for this micro-sample of goods and countries, real shocks account for most of the volatility of sectoral real exchange rates, though nominal shocks are important as well. The good with minimal real exchange rate volatility was estimated to have a duration between the price changes of about 4.2 months based on the benchmark quartic regression, and longer based on a nonparametric regression. However, to validate the generality of the theory, it is important to explore other samples of goods, cross-sections of countries and historical periods.

Our results also point to the value of examining cross-sectional differences in real exchange rate variability in order to flesh out the rich quantitative predictions of models of micro-price adjustment currently under development. Differences across goods help us to disentangle heterogeneous responses to common shocks due to differences in economic propagation mechanisms such as costs of price adjustment and trade costs from heterogeneity in the underlying shocks themselves. Averaging across goods, as is inevitable in the move to an aggregate real exchange rate, is not innocuous in terms of the weight given to real and nominal shocks. The same averaging may also lead to an under-appreciation of the sources of the risks that individuals and firms face. We hope to explore these possibilities in future work. Much remains to be done.

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Table 1: Linear regressions

	Constant	λ_i	Adj. R^2	Obs.	$[\lambda_{\min}, \lambda_{\max}]$
Pooled	0.013	-0.014	0.700	182	[0.223, 0.979]
	(0.001)	(0.001)			
Austria	0.016	-0.016	0.886	57	[0.223, 0.979]
	(0.002)	(0.002)			
Belgium	0.011	-0.011	0.454	46	[0.296,0.956]
	(0.001)	(0.001)			
France	0.013	-0.014	0.833	48	[0.254, 0.958]
	(0.002)	(0.002)			
Spain	0.014	-0.015	0.589	31	[0.524, 0.964]
	(0.003)	(0.003)			

Notes: The heteroskedasticity-consistent standard errors are in parentheses. The "Adj. R^2" denotes the adjusted R^2. The "Obs." denotes the number of observations. The last column shows the empirical range of infrequencies of price changes $[\lambda_{\min}\ , \lambda_{\max}]$ in our dataset.

Table 2: Structural regressions

	λ_i^2	$(1-\lambda_i)^2(1-\lambda_i\beta)^2$	Adj. R_{uc}^2	$\underline{\lambda}$
Pooled	0.0016	0.0456	0.751	0.762
	(0.0001)	(0.0039)		(0.008)
Austria	0.0012	0.0475	0.895	0.784
	(0.0001)	(0.0066)		(0.011)
Belgium	0.0021	0.0419	0.623	0.735
	(0.0004)	(0.0056)		(0.017)
France	0.0015	0.0411	0.905	0.763
	(0.0001)	(0.0040)		(0.009)
Spain	0.0017	0.1567	0.716	0.836
	(0.0002)	(0.0292)		(0.011)

Notes: The heteroskedasticity-consistent standard errors are in parentheses. The "Adj. R_{uc}^2" denotes the adjusted uncentered R^2. $\underline{\lambda}$ presents the estimates of λ which minimize the total variance.

Table 3: Tests of monotonicity

	$m(\lambda_i)$ Null hypothesis		$m'(\lambda_i)$ Null hypothesis		Critical values		
	Increasing	Decreasing	Increasing	Decreasing	1%	5%	10%
Pooled	10.329***	-1.831	-2.324	17.014***	5.342	4.386	3.964
Austria	4.927**	-1.546	1.883	9.362***	5.530	4.488	4.027
Belgium	4.608*	-0.647	1.866	9.378***	5.737	4.609	4.111
France	6.369***	-1.212	1.397	6.675***	5.645	4.554	4.072
Spain	5.098*	-1.281	-0.025	7.731***	6.924	5.382	4.701

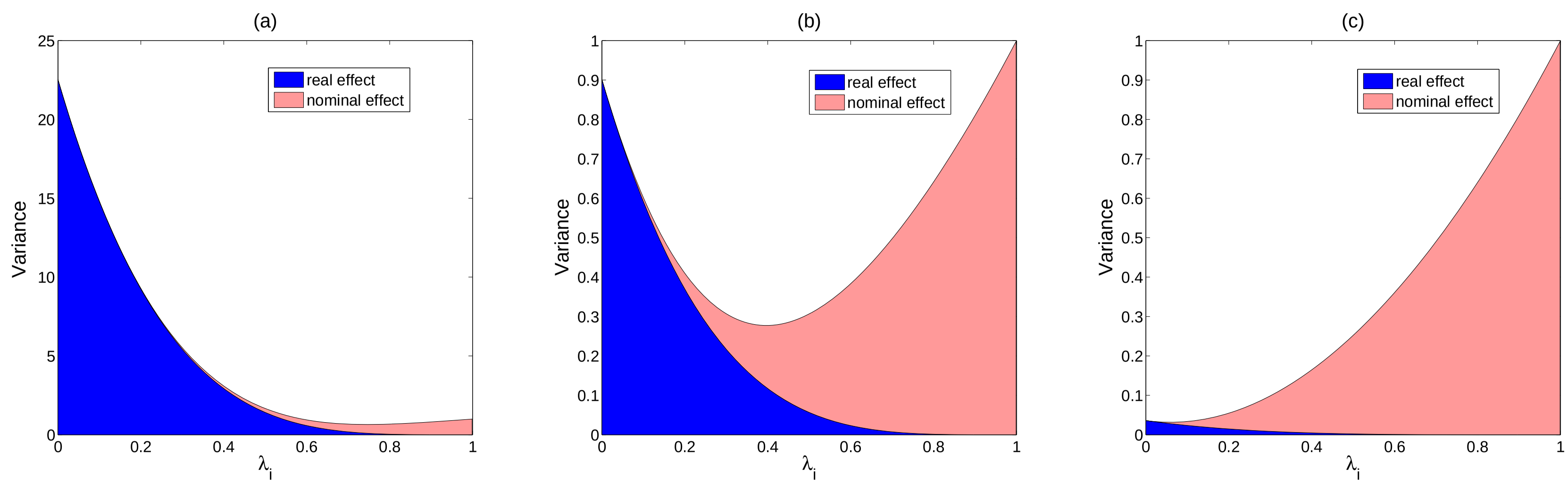
Notes: The first two columns correspond to the hypothesis testing for $m(\lambda_i)$ and the second two columns correspond to the test for the first derivative of $m(\lambda_i)$ with respect to λ_i. Critical values shown in the last three columns are computed from the method by Ghosal, Sen, and van der Vaart (2000).

Table 4: Percentage of forecast error variance accounted for by nominal shocks

k	1	3	6	12	∞
Pooled	40.6	23.6	18.7	14.2	11.4
	(24.1)	(16.5)	(15.7)	(13.5)	(11.8)
Austria	48.6	30.5	25.7	20.3	17.1
	(24.4)	(16.9)	(17.1)	(15.7)	(16.3)
Belgium	34.9	19.9	15.3	11.4	8.9
	(23.0)	(14.9)	(13.6)	(11.2)	(8.2)
France	40.2	21.8	16.2	11.7	9.2
	(22.7)	(15.2)	(13.5)	(10.2)	(7.6)
Spain	35.2	18.9	14.6	11.1	7.9
	(24.5)	(16.4)	(15.6)	(13.3)	(8.0)

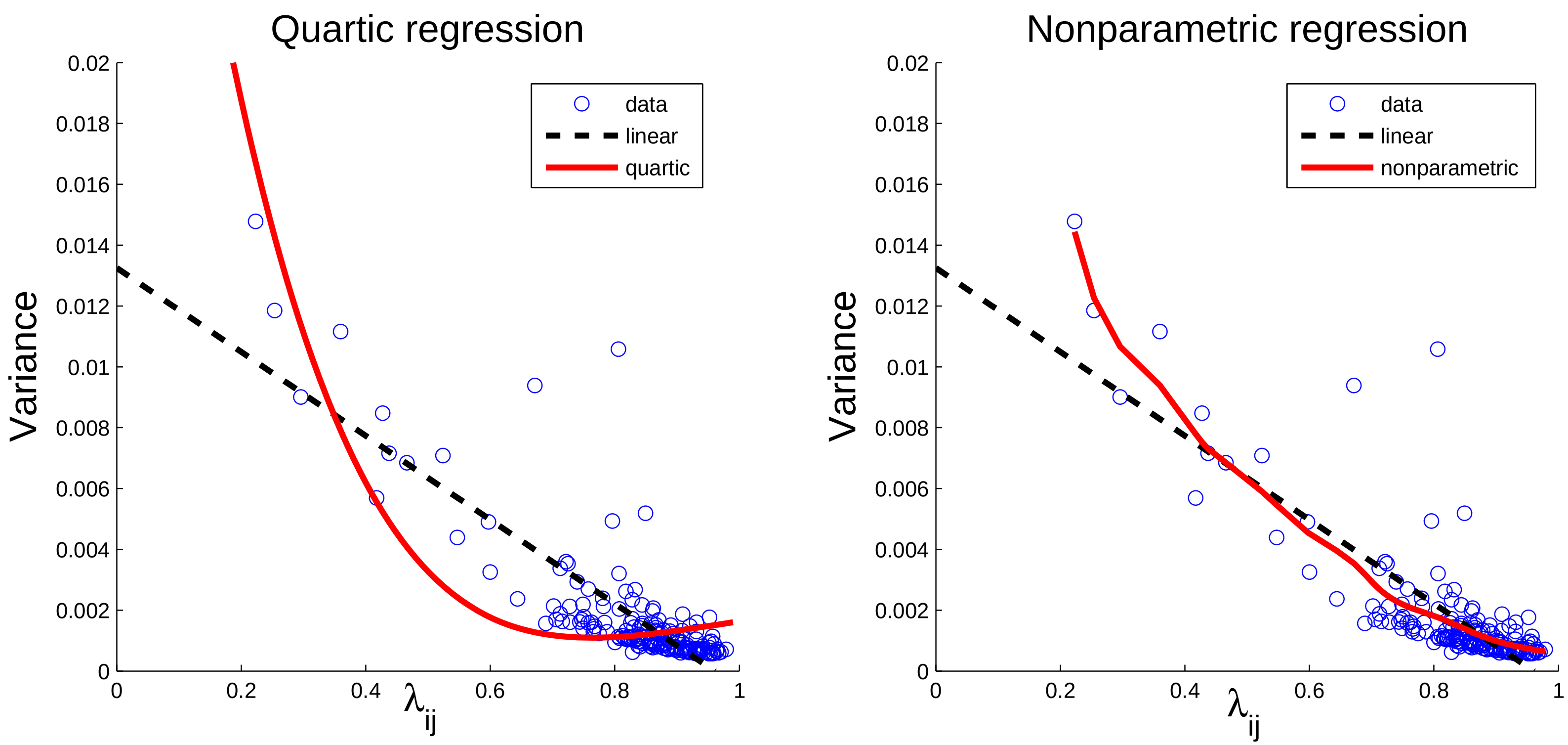
Notes: Numbers are in percent. Each column corresponds to the cross-sectional average of the k-period-ahead forecast error variance of sector-level real exchange rates accounted for by nominal shocks. Numbers in parentheses are standard deviations.

Figure 1: Simulated real exchange rate volatility curves



Notes: Each panel of the figure shows contributions of the nominal and real shocks on the good-level real exchange rate volatility over the range of the degree of price stickiness. The volatility is measured by the one-period-ahead forecast error variance. For each good, the variances due to real and nominal shocks are expressed by the heights of blue and red areas. The combined height of the two areas corresponds to the total variance. Panels (a), (b), and (c) show the cases of $Std(a_{it} - a_{it}^{*})/Std(\mu_t - \mu_t^{*}) = 5$, 1, and 1/5, respectively. Standard deviation of nominal shocks is set to one.

Figure 2: Estimated real exchange rate volatility curves



Notes: The both panels show the scatter plot of the one-period-ahead forecast error variance of the sector-level real exchange rates against the degree of price stickiness. The solid line in the left panel shows the fitted curve based on the quartic regression. The solid line in the right panel represents the fitted curve based on the nonparametric regression. The dashed line in both panels shows the linear regression fit for comparisons.

Appendix to: Do Sticky Prices Increase Real Exchange Rate Volatility at the Sector Level?*

Mario J. Crucini†, Mototsugu Shintani‡ and Takayuki Tsuruga§

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The model is a variant of the New Open Economy Macroeconomics with heterogeneous price stickiness. The world economy consists of symmetric home and foreign countries. Households choose consumption and labor supply over infinite horizon subject to a cash-in-advance (CIA) constraint. They also hold complete state-contingent money claims denominated in the home currency. Household's consumption consists of domestically produced goods and imported goods. Firms are monopolistic competitors who set prices in local currency to satisfy the demand for their goods in each country. In what follows, the unit of time is one month. Due to the symmetry of the model, we mostly focus on the equations of the home country and use an "$*$" superscript to denote foreign variables, if necessary.

Each country has a continuum of goods $i \in [0,1]$, each of which consists of a continuum of brand $v \in [0,1]$. Brands of good i in the home country are indexed by $v \in [0,1/2]$ while those in the foreign country are indexed by $v \in (1/2,1]$. Integrating over brand, we have the constant elasticity of substitution (CES) index for consumption of good i in the home country:

$$C_{it} = \left[\int C_{it}(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where $C_{it}(v)$ denotes the consumption of brand v of good i in the home country and $\theta \in (1, \infty)$ is the elasticity of substitution across brands. CES aggregation across goods gives aggregate consumption C_t:

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†Department of Economics, Vanderbilt University and NBER; e-mail: mario.crucini@vanderbilt.edu.

‡Department of Economics, Vanderbilt University; e-mail: mototsugu.shintani@vanderbilt.edu

§Graduate School of Economics, Kyoto University; e-mail: tsuruga@econ.kyoto-u.ac.jp

$$C_t = \left[\int C_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{2}$$

Here we assume that the elasticity of substitution across goods are the same as that across brands for simplicity.

1 Households

The home consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \chi L_t \right), \tag{3}$$

subject to the intertemporal budget and CIA constraints:

$$M_t + E_t(\Upsilon_{t,t+1} D_{t+1}) = R_{t-1} W_{t-1} L_{t-1} + D_t + (M_{t-1} - P_{t-1} C_{t-1}) + T_t + \Pi_t \tag{4}$$

$$M_t \geq P_t C_t, \tag{5}$$

where β is the discount factor of the household satisfying $0 < \beta < 1$ and $E_t(\cdot)$ is the expectations operator. On the intertemporal budget constraint (4), M_t, D_{t+1}, and $\Upsilon_{t,t+1}$ denote cash holding, bond holding, and the nominal stochastic discount factor, respectively. Thus, the left hand side of the equation represents the nominal total value of wealth brought into the beginning of the period $t+1$. Also, R_t, W_t, and L_t are the (gross) nominal interest rate, nominal wages, and hours of work. Thus, the household receives labor income in the end of period $t-1$ and earns nominal interest per unit of labor income until period t in terms of the home currency. Next, the household carries the nominal bonds in amount of D_t and the cash remaining after consumption in amount of $M_{t-1} - P_{t-1} C_{t-1}$, where P_t is the aggregate price index defined below. Finally, the household receives nominal transfers from the government, T_t and the nominal profits from firms, Π_t. The other constraint (5) is the CIA constraint. Here P_t is given by $P_t = \left[\int P_{it}{}^{1-\theta} di \right]^{1/(1-\theta)}$ and the price index for good i is given by $P_{it} = \left[\int P_{it}(\upsilon)^{1-\theta} d\upsilon \right]^{1/(1-\theta)}$.

The foreign consumer has the same preference and the same constraints except that we express the intertemporal budget constraint in the foreign currency with the nominal exchange rate S_t. In particular, the budget constraint for the foreign consumer is given by

$$M_t^* + E_t \left(\frac{\Upsilon_{t,t+1} D_{t+1}^*}{S_t} \right) = \frac{S_{t-1} R_{t-1}}{S_t} W_{t-1}^* L_{t-1}^* + D_t^* + (M_{t-1}^* - P_{t-1}^* C_{t-1}^*) + T_t^* + \Pi_t^*. \tag{6}$$

The first-order conditions are standard:

$$
\begin{aligned}
\frac{W_t}{P_t} &= \chi C_t, && (7)\\
\Upsilon_{t,t+1} &= \beta\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1}\frac{P_t}{P_{t+1}}\right] = \beta\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-1}\frac{S_tP^*_t}{S_{t+1}P^*_{t+1}}\right] && (8)\\
M_t &= P_tC_t. && (9)
\end{aligned}
$$

Combining the first-order conditions (7) and (9) yields

$$W_t = \chi M_t. \tag{10}$$

Using (8), the aggregate real exchange rate, Q_t, is proportional to the consumption ratio:

$$Q_t = \frac{S_tP^*_t}{P_t} = \kappa\frac{C_t}{C^*_t}, \tag{11}$$

where $\kappa = Q_0C^*_0/C_0$. In what follows, we assume that $\kappa = 1$. Furthermore, using the CIA constraint, (11) gives an expression for the nominal exchange rate:

$$S_t = \frac{M_t}{M^*_t}. \tag{12}$$

2 Firms

The technology to produce a brand of good is

$$Y_{it}(\upsilon) = A_{it}L_{it}(\upsilon) \tag{13}$$

where $Y_{it}(\upsilon)$ and $L_{it}(\upsilon)$ is output and labor demand for firms producing brand υ of good i and A_{it} is the labor productivity which varies by good i but is common across brands. Let a_{it} and a^*_{it} be the log labor productivity in the home and foreign countries, respectively. The stochastic processes are is given by

$$
\begin{aligned}
a_{it} &= z_t + \eta_t + \varepsilon_{it} && (14)\\
a^*_{it} &= z_t + \eta^*_t + \varepsilon^*_{it} && (15)\\
z_t &= z_{t-1} + \varepsilon_{zt}, && (16)
\end{aligned}
$$

where ε_{zt} ~i.i.d.$(0,\sigma_z^2)$, is a common global stochastic trend, η_t ~i.i.d.$(0,\sigma_\eta^2)$ and η_t^* ~i.i.d.$(0,\sigma_\eta^{*2})$ are country-specific productivity shock, and ε_{it} ~i.i.d.$(0,\sigma_\varepsilon^2)$ and ε_{it}^* ~i.i.d.$(0,\sigma_\varepsilon^{*2})$ are idiosyncratic shocks to the production of each good in each country.

Firms in the home country sell their goods to home and foreign consumers. To send one unit of goods into the foreign country, the firm must pay trade cost, τ. Thus, the resource constraint for the home market is

$$C_{it}(\upsilon) + (1+\tau)C_{it}^*(\upsilon) = Y_{it}(\upsilon) \text{ for } \upsilon \in [0, 1/2] \tag{17}$$

The resource constraint for the foreign market is similar to (17):

$$(1+\tau)C_{it}(\upsilon) + C_{it}^*(\upsilon) = Y_{it}^*(\upsilon) \text{ for } \upsilon \in (1/2, 1]. \tag{18}$$

Next, we introduce the Calvo-type nominal price rigidities. Every month, a fraction λ_i of firms are randomly drawn from the unit interval and are not allowed to change prices; otherwise, the remaining fraction of firms reset prices. We allow for heterogeneity of this Calvo parameter across goods whereas we assume that it is common across countries. The optimal price for firms in home country to sell good i in the home country, $P_{H,it}$, solves

$$\underset{P_{H,it}}{\text{Max}} E_t \sum_{h=0}^{\infty} \lambda_i^h \Upsilon_{t,t+h} \left[P_{H,it} - \frac{W_{t+h}}{A_{it+h}} \right] \left(\frac{P_{H,it}}{P_{it+h}} \right)^{-\theta} C_{it+h}.$$

Prices are set in the local currency of the country of consumption (local currency pricing). The optimal price for firms in the foreign country to sell good i in the home country, $P_{F,it}$, is the solution to the following:

$$\underset{P_{F,it}}{\text{Max}} E_t \sum_{h=0}^{\infty} \lambda_i^h \Upsilon_{t,t+h} \left[P_{F,it} - (1+\tau) \frac{S_{t+h} W_{t+h}^*}{A_{it+h}^*} \right] \left(\frac{P_{F,it}}{P_{it+h}} \right)^{-\theta} C_{it+h}.$$

The first-order conditions for $P_{H,it}$ and $P_{F,it}$ are

$$\begin{aligned} E_t \sum_{h=0}^{\infty} \lambda_i^h \Upsilon_{t,t+h} \left[P_{H,it} - \frac{\theta}{\theta-1} \frac{W_{t+h}}{A_{it+h}} \right] \left(\frac{P_{H,it}}{P_{it+h}} \right)^{-\theta} C_{it+h} &= 0 && (19) \\ E_t \sum_{h=0}^{\infty} \lambda_i^h \Upsilon_{t,t+h} \left[P_{F,it} - (1+\tau) \frac{\theta}{\theta-1} \frac{S_{t+h} W_{t+h}^*}{A_{it+h}^*} \right] \left(\frac{P_{F,it}}{P_{it+h}} \right)^{-\theta} C_{it+h} &= 0, && (20) \end{aligned}$$

respectively.

3 Equilibrium

The money supply in each country follows a random walk:

$$\ln M_t = \ln M_{t-1} + \xi_{Mt},$$

where ξ_{Mt} and the foreign analog ξ^*_{Mt} are i.i.d with finite variance. Due to this assumption and (12), the nominal exchange rate S_t also follows a random walk:

$$\ln S_t = \ln M_t - \ln M^*_t = \ln S_{t-1} + \xi_t,$$

where $\xi_t = \xi_{Mt} - \xi^*_{Mt}$. Total transfers from the governments satisfy $T_t = M_t - M_{t-1} - (R_{t-1} - 1)W_{t-1}L_{t-1}$ and $T^*_t = M^*_t - M^*_{t-1} - (R_{t-1}S_{t-1}/S_t - 1)W^*_{t-1}L^*_{t-1}$. Thus, the total transfers in each country equal domestic money injections minus the lump sum tax from the government paying interest. The profits of firms accrue exclusively to consumers in the same country: $\Pi_t = \int\int_{\upsilon=0}^{1/2} \Pi_{it}(\upsilon)d\upsilon di$ and $\Pi^*_t = \int\int_{\upsilon=1/2}^{1} \Pi^*_{it}(\upsilon)d\upsilon di$. The labor market clearing conditions are $L_t = \int\int_{\upsilon=0}^{1/2} L_{it}(\upsilon)d\upsilon di$ and $L^*_t = \int\int_{\upsilon=1/2}^{1} L^*_{it}(\upsilon)d\upsilon di$. The state-contingent bond market is $D_t + D^*_t = 0$ at each date and state. Finally, good market clearing conditions are given by (17) and (18).

4 Sectoral real exchange rates

To consider the implications for sectoral real exchange rates, we approximate the first-order conditions and resource constraints by the log deviation from the steady state and derive the reduced form solution for the real exchange rates. In what follows, unless otherwise indicated, the log deviation of a generic variable X_t from the steady state is expressed by $x_t = \ln X_t - \ln X$, where a variable without a time subscript is the steady state value of X_t. Also, we normalize all nominal prices by dividing by the nominal money stock in the place of consumption and by multiplying the labor productivity in the place of production to assure the stationarity. For example, we normalize the nominal home price index for good i by dividing by the money stock of the home country and by multiplying the labor productivity of the home country to have $\bar{P}_{it} = P_{it}A_{it}/M_t$. We also normalize the optimal prices set for the home market $P_{H,it}$ and $P_{F,it}$ to have $\bar{P}_{H,it} = P_{H,it}A_{it}/M_t$ and $\bar{P}_{F,it} = P_{F,it}A^*_{it}/M_t$.

Log-linearizing (19) and (20) gives the optimal normalized price for domestic and foreign firms

$$\bar{p}_{H,it} = \sum_{h=1}^{\infty}(\lambda_i\beta)^h E_t\left[\mu_{t+h} - g_{t+h}^{A_i}\right] \tag{21}$$

$$\bar{p}_{F,it} = \sum_{h=1}^{\infty}(\lambda_i\beta)^h E_t\left[\mu_{t+h} - g_{t+h}^{A_i^*}\right]. \tag{22}$$

Here $\mu_{t+1} = \ln M_{t+1} - \ln M_t$ and $g_{t+1}^{A_i} = a_{it+1} - a_{it}$. To derive (22), we used (10) and (12) implied by the first-order conditions from household's maximization problem. The equations (21) and (22) mean that the optimal reset prices are determined in a forward-looking manner, based on future money growth and labor productivity growth. Our assumptions on the stochastic processes of money supply and good- and location-specific labor productivity allow us to express the optimal prices as

$$\bar{p}_{H,it} = \lambda_i\beta\left(\eta_t + \varepsilon_{it}\right) \tag{23}$$

$$\bar{p}_{F,it} = \lambda_i\beta\left(\eta_t^* + \varepsilon_{it}^*\right). \tag{24}$$

We log-linearize the price index for good i. Due to Calvo pricing, we have

$$\bar{p}_{it} = \lambda_i\bar{p}_{it-1} - \lambda_i\mu_t + \lambda_i g_t^{A_i} + (1-\lambda_i)\left[\omega\bar{p}_{H,it} + (1-\omega)(\bar{p}_{F,it} + \eta_t + \varepsilon_{it} - \eta_t^* - \varepsilon_{it}^*)\right], \tag{25}$$

where ω is the expenditure share of the household in the home country which is given by $1/[1 + (1+\tau)^{1-\theta}]$.

Given the stochastic processes of the money growth rate and productivity, combining (21) and (22) with (25) gives

$$\bar{p}_{it} = \lambda_i\bar{p}_{it-1} - \lambda_i\mu_t + \lambda_i g_t^{A_i} + (1-\lambda_i)\lambda_i\beta\left[\omega(\eta_t + \varepsilon_{it}) + (1-\omega)(\eta_t^* + \varepsilon_{it}^*)\right] + (1-\lambda_i)(1-\omega)(\eta_t + \varepsilon_{it} - \eta_t^* - \varepsilon_{it}^*). \tag{26}$$

The foreign analog of the normalized price index is

$$\bar{p}_{it}^* = \lambda_i\bar{p}_{it-1}^* - \lambda_i\mu_t^* + \lambda_i g_t^{A_i^*} + (1-\lambda_i)\lambda_i\beta\left[\omega(\eta_t^* + \varepsilon_{it}^*) + (1-\omega)(\eta_t + \varepsilon_{it})\right] - (1-\lambda_i)(1-\omega)(\eta_t + \varepsilon_{it} - \eta_t^* - \varepsilon_{it}^*). \tag{27}$$

Let the sectoral real exchange rate be $Q_{it} = S_t P_{it}^*/P_{it}$. We have the log real exchange rate at sector level:

$$q_{it} = \lambda_i q_{it-1} + \lambda_i(\mu_t - \mu_t^*) + (1-\lambda_i)(1-\lambda_i\beta)\psi\left[a_{it} - a_{it}^*\right], \tag{28}$$

where $\psi = \left[\left(1-(1+\tau)^{1-\theta}\right) / \left(1+(1+\tau)^{1-\theta}\right)\right]$. To derive (28), we used the fact that $\eta_t + \varepsilon_{it} - \eta_t^* - \varepsilon_{it}^*$ can be replaced with $a_{it} - a_{it}^*$ because a common stochastic trend z_t is be eliminated in technological differential.

Equation (28) describes equilibrium dynamics of real exchange rates at the sector level. Notice that $\mu_t - \mu_t^*$ and $a_{it} - a_{it}^*$ follow an iid process. Hence, Equation (28) is the reduced form solution for real exchange rates and the real exchange rate follows an AR(1) process with two iid shocks. By using (28), we can obtain the k-period-ahead forecast error variance:

$$Var_{t-k}(q_{it}) \;=\; \left(\sum_{j=1}^{k} \lambda_i^{2(j-1)}\right) \left[\lambda_i^2 Var\left(\mu_t - \mu_t^*\right) + (1-\lambda_i)^2(1-\lambda_i\beta)^2\psi^2 Var\left(a_{it} - a_{it}^*\right)\right]. \qquad (29)$$

This is Equation (2.3) in the paper.