Bailout Uncertainty in a Microfounded General Equilibrium Model of the Financial System*

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PRELIMINARY AND INCOMPLETE

Abstract

This paper develops a micro-founded general equilibrium model of the financial system composed of ultimate borrowers, ultimate lenders and financial intermediaries. The model is used to investigate the impact of uncertainty about the likelihood of governmental bailouts on leverage, interest rates, the volume of defaults and the real economy. The distinction between risk and uncertainty is implemented by applying the Gilboa-Schmeidler (1989) maxmin with multiple priors framework to lenders' beliefs about the probability of bailout. Events like Lehman's collapse are conceived of as "black swan" events that led lenders to put a positive mass on bailout probabilities that were previously assigned zero mass.

Results of the analysis include: (i) An unanticipated increase in bailout uncertainty raises interest rates, the volume of defaults in both the real and financial sectors and may lead to a total drying up of credit markets. (ii) Lower exante bailout uncertainty is conducive to higher leverage - which raises moral hazard and makes the economy more vulnerable to expost increases in bailout uncertainty. (iii) Bailout uncertainty raises the likelihood of bubbles, the amplitude of booms and busts and the banking spread. (iv) Bailout uncertainty is associated with higher returns' variability in diversified portfolios and systemic risks, (v) Expansionary monetary policy reinforces those effects by inducing higher aggregate leverage levels.

Keywords and Phrases: Risk, Uncertainty, Lehman's default, Leverage, Financial intermediaries, Bailouts, Duration mismatches.

JEL Classification Codes: G01, G11, G2, G18, E3, E4, E5, E6, D81, D83.

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1 Introduction

Financial sector bailouts in the US and more recently in Europe have revived the well known dilemma between restoration of confidence in the face of a panic and the costs of moral hazard. On one hand, when a panic engulfs financial markets, bailouts appear indispensable in order to restore confidence and prevent further collapses in the financial system. On the other, by subsidizing opportunistic behavior at the expense of taxpayers, bailouts encourage excessive risk taking on the part of financial institutions, borrowers and lenders, and plant the seeds of the next bubble.

Different experts in both policymaking circles as well as in academia often find themselves at odds regarding the ways to handle this problem. In spite of currently ongoing reforms in regulation this dilemma is, therefore, likely to be a central issue during the upcoming decade. Whether, and how exactly will bailout policies be deployed in the future is largely an open issue. However, due to the lack of consensus about the precise ways to deal with the (exante and expost) trade-offs induced by bailouts, it is extremely likely that bailout uncertainty is likely to be non negligible in the foreseeable future. The 2008 bailout zigzags in the US (Bear-Stern versus Lehman) and current uncertainties about the reaction of EMU governments to potential sovereign debt problems of a large country like Spain attest to that.

This paper develops a micro-founded general equilibrium model of the financial system and uses it in order to investigate the impact of an increase in bailout uncertainty on financial markets and the real economy. It also investigates the exante, leverage expanding, moral hazard problems created by perceived generous governmental bailout policies.

As is well known since Knight's (1921) work risk and uncertainty are distinct concepts. Modern formulations of this distinction in the context of pecuniary returns conceptualize risk as some measure of spread for a **known distribution** of the stochastic return. Uncertainty, on the other hand, is a situation in which individuals are **unsure about the probability distribution** of returns and entertain the possibility that several alternative probability distributions have positive measure. An increase in uncertainty is then viewed as a an enlargement of the set of plausible probability distributions with positive measure. Ellsberg

(1961) and others have demonstrated by means of experiments that individuals are averse to ambiguity in the sense that, other things the same, they prefer a lottery with a known probability distribution to a lottery in which several distributions are believed to be possible.

Gilboa and Schmeidler (1989) (GS in the sequel) conceptualize an investor's uncertainty by postulating that he possesses a subjective set of probability measures, or multiple priors, over outcomes. Under several axioms they show that, if the investor is averse to ambiguity his action is determined by the Gilboa-Schmeidler max-min ambiguity aversion criterion. That is, for each possible action the investor assumes that the worst (by the expected utility criterion) possible distribution will realize and chooses his action so as to attain maximum expected utility over this set of worst outcomes.

This paper utilizes the GS notion of uncertainty and the associated max-min behavioral criterion to analyze the impact of an increase in uncertainty about governmental bailout policy on financial markets, the aggregate level of credit and, through them, on the real economy. The riskiness of bailouts at the level of an individual creditor is captured by a binomial distribution in which conditional on default by a borrower there is a bailout with probability, p, or there is no bailout with probability $1 - p$. Bailout uncertainty then means that individuals entertain the view that several alternative binomial distributions, each characterized by a different value of p possess positive mass. In this context an increase in uncertainty means that there is an enlargement in the set of possible bailout distributions.

Prior to Lehman's collapse financial market's beliefs about the probability of bailout have been relatively optimistic due to Bear-Stern's bailout in March 2008 as well as to the implicit US government guarantees of Fannie Mae and Freddie Mac's liabilities (Meltzer (2009)). In terms of the GS framework this means that the family of binomial bailout distributions with positive mass was concentrated in the relatively high range of p's.

Taleb (2007) has popularized the notion of a "black swan" event. Such an event is perceived to have zero mass before it realizes for the first time. However, once it realizes, individuals assign to it (a possibly small) but positive mass. We view Lehman's collapse in mid September 2008 as such a "black swan" event. That event, deemed unthinkable, prior to this collapse had realized after all and this reduced the lowest perceived probability of bailout with positive mass.

The behavior of credit default swap (CDS) spreads during the two weeks following Lehman's collapse provides a dramatic illustration of the sensitivity of bailout expectations to public signals. In the aftermath of this collapse credit markets experienced substantial waves of deleveraging, totally drying up in some cases, and both the level and variability of CDS spreads went through the roof. Table 1[1] shows the behavior of Citibank's CDS spread index during the period just preceding Lehman's default and the final approval of the TARP bailout package at the beginning of October 2008.

Date	**Event**	**CDS Spread**
13-14/9		150
15/9	Lehman files for chapter 11	
16-17/9	Paulson suggests TARP to Congress	250
18-19/9		150
22-23/9	Paulson & Bernanke address Congress	450
24-25/9		350
29/9	Congress rejects TARP proposal	Almost 450
3/10	Amended TARP approved by Congress	
5-10/10	Aftermath of approval	150

Table 1: **Chronology of CDS spread around Lehman's collapse**

The table demonstrates the sensitivity of the CDS spread to ongoing public signals. In particular, following rejection of the proposed TARP bailout package by Congress in September 2008 the CDS spread goes up and following its approval in early October it goes down supporting the view that financial markets participants are quite sensitive to news about the likelihood of bailout.[2] Our view is that, following Lehman's collapse and the ensuing public debate among policymakers about the wisdom of governmental bailouts, the lower bound on the set of binomial distributions with perceived positive mass went down,

[1]Source: Cochrane and Zingales (2009) .

[2]Following Keynes, Akerlof and Shiller (2009) attribute changes in expectations to exogenous animal spirits. By contrast this paper takes the view that changes in expectations can be traced back to new information in noisy but relevant public signals.

say, from π_0 to π_1 (here π_t is the lower bound of distributions with perceived positive mass in period t).

The analysis in the paper shows that lenders' expected utility is lower the lower is p. In conjunction with the GS max-min criterion this increase in bailout uncertainty implies that, once a "black swan" event like Lehman's collapse materializes, lenders become more reluctant to lend, sending shock waves through both financial and real markets. One objective of the paper is to trace some of the mechanisms through which the consequent changes in perceptions affect short term credit within the financial system, as well as credit to the real sector. Another related objective is to analyze the impact of expansionary monetary policy on leverage and risk appetite. The paper's framework makes it possible to trace out both the exante and the expost consequences of (perceived) generous bailout policies. Exante, a more generous bailout policy increases moral hazard in all segments of the financial system and induces an overall expansion of credit.[3] But expost the maintenance of a generous bailout policy may become necessary just to avoid a crisis even if government no longer desires to maintain high bailout levels.

Important features of the model include:

(i) An individual trade-off between return seeking through higher levels of leverage and higher probability of total loss at the **individual level**.

(ii) Exante and expost relations between the worst probability of bailout and leverage at the **aggregate level**.

(iii) **Duration mismatches:** Borrowers need financing for two periods but get only one period loans from financial intermediaries in each period.

(iv) The model's focus is on the segment of **the shadow banking system** (like SIV and hedge funds) in which funds are secured only for short periods. Accordingly, financial intermediaries are assumed to borrow for only one period.

The rest of the paper is organized as follows. Section 2 presents a general outlook of the model. Sections 3, 4 and 5 introduce a typical borrower, a typical financial intermediary

[3] Borio C. and M. Drehmann (2009) convincingly argue that such a credit buildup raises the likelihood of a financial crisis.

and a typical lender and characterize the optimal microeconomic behavior of each type of agent. Government's bailout policy is specified in Section 5. General equilibrium of the financial system and the determination of market rates are discussed in section 6. Section 7 analyzes the impact of an exogenous decrease in perceptions about the likelihood of bailout on financial markets and utilizes it to explain some of the events observed following Lehman's collapse. Section 8 discusses the exante choice of leverage by borrowers in general equilibrium including, in particular, the impact of perceived bailout policy and the associated moral hazard problem. This is followed by concluding remarks in Section 9. A central result of the paper (implied by the discussion in sections 7 and 8 and elaborated in the conclusion) is that higher bailout uncertainty raises the amplitude of booms and busts. Most proofs are in the Appendix.

2 Framework

There is a large number of each of the following risk averse (identical within each group) 3 types of agents: Borrowers (B), Financial intermediaries (F) and Lenders (L) each possessing one unit of equity capital.[4] The initial masses of each type of agent are M_B, M_L and M_F for borrowers, financial intermediaries and lenders, respectively.[5]

There are 3 time periods labeled 0, 1 and 2. Only borrowers-investors have access to real investment decisions. All such decisions are made by them in period 0 and are long term in the sense, that once chosen, the project's size cannot be adjusted. The selected project size is $(1 + L_B)$, where 1 is the borrower's initial equity capital and L_B is the leverage he selects to take. Only short term loans are available to borrowers with interest rates r_{B1} and r_{B2} for loans assumed in the first and in the second period, respectively. Interest rates on loans and project's yields are all specified in terms of net returns.

[4]We use the following notational conventions: the subscript $j = \{B, F, L\}$ to a variable x_{jt} indicates the agent type, and subscript $t = \{0, 1, 2\}$ indicates time. When the time index is omitted the variable refers to any of the time periods between 0 and 2. Random variables are identified by a tilde on top of the variable (e.g. $\widetilde{X}$).

[5]The financial markets model in the paper can be thought of as a microfounded version of general equilibrium approaches to monetary theory and policy (Brunner and Meltzer (1997) , Tobin (1969)).

Each borrower can get loans only from financial intermediaries. The amount of leverage, L_B, demanded by a borrower is determined as a function of r_{B1} and of expected r_{B2}, by means of individual optimization. Each financial intermediary can obtain short-term funds, L_F, from lenders. The intermediaries generally splits his total funds $(1 + L_F)$ between a fraction z_F allocated to a partially diversified portfolio of loans to borrowers and a fraction $(1 - z_F)$ allocated to a risk free asset that pays a fixed interest rate r_f.[6] The return on the risk free asset, r_f, is determined by the monetary authority.

Financial intermediaries pay to lenders short term interest rates r_{L1} and r_{L2} in periods 1 and 2 (for loans taken in periods 0 and 1) respectively. A typical lender splits his initial wealth of 1 between a fraction z_L of funds allocated to loans to financial intermediaries and a remaining fraction, $(1 - z_L)$, that is invested in the risk free asset. In contrast to a typical financial intermediary, whose portfolio of loans to borrowers is only partially diversified, a typical lender holds his selected portion of loans to financial intermediaries in a fully diversified portfolio of loans.

The supply of loans to borrowers by an individual financial intermediary and his demand for loans from lenders, L_F, are determined through the intermediary's individual optimization as a function of the interest rates r_{B1}, r_{B2} , r_{L1} and r_{L2}. Those interest rates are determined through general equilibrium competitive clearing in periods 0 and 1 respectively in two markets within each period: The market for loans from intermediaries to borrowers and the market for loans from lenders to financial intermediaries.[7]

As elaborated in the next section, returns on real projects are stochastic and therefore risky. They realize in period 2. A real-project yield, $\widetilde{Y}_P$, depends on two independent random variables: an aggregate economy-wide shock, $\widetilde{Y}_A$, and a specific (idiosyncratic) shock, $\widetilde{Y}_I$. The realization of the aggregate shock is not revealed prior to period 2. Although all idiosyncratic shocks realize only in period 2, the value of this future realization becomes publicly known for some borrowers already in period 1. Depending on the information available in period 1

[6] An intermediary's portfolio is partially diversified in the sense that only part of the idiosyncratic risk is eliminated. By contrast in a fully diversified portfolio all idiosyncratic risk is eliminated and only the systematic risk remains.

[7] A loan carrying interest rate r_t $(t = 1, 2)$ is contracted in period $t - 1$ and settled in period t.

borrowers can be classified to one of the following three groups: Lucky borrowers for whom it becomes known they will get a high $\widetilde{Y}_I$, unlucky borrowers for whom it becomes known they will get a low $\widetilde{Y}_I$, and regular borrowers for whom no advance return information is available in period 1. The availability of such information is important because it affects the borrower's ability to get refinancing in period 1. Since project yield is random and borrowers have some leverage obligations they generally may default in either of periods 1 or 2. A borrower defaults in period 1 if he does not succeed in securing credit to carry over his project on to period 2. He defaults in period 2 if the total final project return does no suffice to service the debt incurred in the previous period.

A financial intermediary can also default in period 1 or 2 if the principal and the interest rate paid to him by borrowers cannot cover his obligations to lenders. When a financial intermediary defaults lenders lose their entire investment in this intermediary including the principal and the interest rate. Government can possibly and selectively pay the debt of defaulting financial intermediaries to lenders. But governmental bailout policy is uncertain in the Knigthian sense. More precisely, individuals entertain multiple priors about the probability of bailout, or in the language of modern decision theory – government's bailout policy is ambiguous.

Figure 1 presents a bird's eye view of the model's financial system. In the figure z_F and z_L represent the fractions of funds Fs and Ls allocate to risky loans, and r_B and r_L are the rates paid by Bs and received by Ls respectively. $\widetilde{Y}_A$ and $\widetilde{Y}_I$ are aggregate and individual components of the total net return to a typical borrower.

3 The Typical Investor-Borrower (B)

This section presents the borrower's problem. First it specifies B's real investments opportunities and his financial requirements in each period. It then derives conditions for his solvency and utilizes them to characterize the optimal project's size and B's optimal leverage conditional on the project's characteristics (its outcomes and their probabilities) and the cost of capital faced by him in periods 0 and period 1.

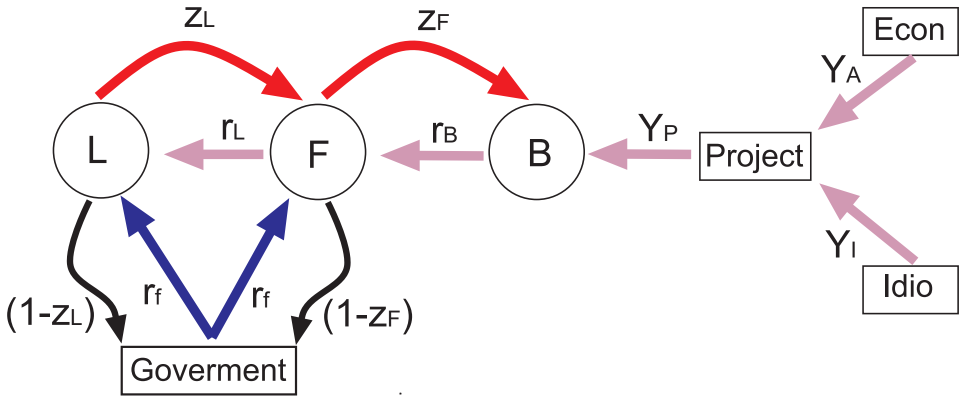

Figure 1: **The financial flows**

3.1 Real investment projects

All real investment made in period 0 are long term in the sense, that once chosen, the project's size cannot be adjusted, until returns are realized in period 2. The typical investment project yields a stochastic (net) return, $\widetilde{Y}_P$, which may be either positive or negative.[8] All real projects have the same distribution of returns, and the yields of any two different projects are correlated due to presence of the aggregate common component in $\widetilde{Y}_P$.

A project's net yield is the sum of an aggregate shock $\widetilde{Y}_A$ and of an individual idiosyncratic shock $\widetilde{Y}_I$, that is

$$\widetilde{Y}_P = \widetilde{Y}_A + \widetilde{Y}_I. \tag{1}$$

We assume, for tractability, that both the aggregate and the idiosyncratic shocks, are binomially and identically distributed. That is

$$\widetilde{Y} = \widetilde{Y}_A = \widetilde{Y}_I = \begin{cases} y, & \Pr(y) = q \\ -y, & \Pr(-y) = 1-q \end{cases}, \tag{2}$$

where $0 \leq y \leq 1/2$. The random variables $\widetilde{Y}_A$ and $\widetilde{Y}_I$ are statistically independent and the

[8]Returns, whether positive or negative, are cash flows.

idiosyncratic shock, $\widetilde{Y}_I$, is independent across projects.[9] Equations (1)-(2) imply that the distribution of $\widetilde{Y}_P$ is

$$\widetilde{Y}_P = \begin{cases} 2y, & \Pr(2y) = q^2 \\ 0, & \Pr(0) = 2q(1-q) \\ -2y, & \Pr(-2y) = (1-q)^2 \end{cases}.$$

Notice that the risk of a project is a function of y: Given two projects with identical q a higher y implies a riskier project. By equations (1)-(2) the expected return of each of the component shocks $\widetilde{Y}_i$, $i = \{A, I\}$ is

$$E\widetilde{Y}_i = \overline{Y}_i = y(2q-1), \quad i = A, I.$$

Since the project net return is the sum of $\widetilde{Y}_A$ and $\widetilde{Y}_I$, which are equal in distribution, its expected return is

$$E\widetilde{Y}_P = \overline{Y}_P = 2y(2q-1).$$

Projects must have a positive expected return to be considered; i.e. $\overline{Y}_P > 0$, which implies that $q > \frac{1}{2}$.

Assumption 1: *As of period 0 the expected return on a project is higher than the expected cost of leverage needed to carry the project to completion in period 2. That is, the distribution of the return, $\widetilde{Y}_P$, on a typical project satisfies*

$$1 \leq (1 + r_{B1})(1 + r_{B2}^e) < \left(1 + \overline{Y}_P\right),$$

where r_{B1} is the interest rate paid by the borrower in period 1 and r_{B2}^e is the interest rate he expects to pay in period 2 for refinancing in period 1 given the information set of period 0. *Since the minimal project size is* 1 *and the lowest return on a project is* $-2y$ *we impose the constraint*

$$0 \leq y \leq \frac{1}{2}.$$

[9]The cases $\widetilde{Y}_A = y$ and $\widetilde{Y}_A = -y$ are referred to as expansion and contraction respectively.

This consraint enforces limited liability by ruling out negative realizations of wealth when there is no leverage .

3.2 Borrower′s financial requirements

Projects are financed by a combination of equity and of leverage supplied by financial intermediaries to borrowers. In period 0, each borrower-entrepreneur owns one unit of equity capital. The initial financing structure (equity-1 versus leverage-L_B) is chosen by each B in period 0 along with the project's size, denoted by x. Since B's initial equity capital is 1, $x = 1 + L_B$. In each period loans by Fs to Bs are one period loans. Consequently, a B's project has to be financed by two consecutive one period loans.

In the presence of positive leverage and since projects' yields are obtained only in period 2 a B must seek refinancing in period 1. Therefore he depends on the availability and the cost of credit in period 1. If excluded from the credit market in that period he defaults and loses the entire investment project including his equity. A borrower's financial requirements in period 1 are equal to the amount needed to repay the principal, L_B, and period's 1 interest charges, $r_{B1}L_B$. Hence, *B′s total financial requirements in period 1 are*

$$FR_{B1} = \underbrace{(1 + r_{B1})\, L_B}_{Debt\ service}.$$

When he gets credit in period 1, B's ultimate debt service in period 2 is

$$FR_{B2} = (1 + r_{B2})\, FR_{B1} = (1 + r_{B1})\,(1 + r_{B2})\, L_B.$$

The borrower's cost of capital for the entire project's life (from period 0 till period 2) is therefore

$$r_B \equiv (1 + r_{B1})\,(1 + r_{B2}) - 1.$$

We assume that when the borrower cannot obtain refinancing in period 1 or cannot repay the debt in period 2, he defaults, the project is lost and neither the borrower nor the financial intermediary receives any payoff. Thus, due to limited liability, the amount needed to cover losses (if any) is 0. The next section explores the borrower's solvency condition.

3.3 Borrower′s solvency conditions

3.3.1 Period 0

A borrower is able to get a loan in period 0 only if the total expected payoff from his project is higher than the total debt service liability expected for period 2, that is

$$L_B\left(1+r_B^e\right) \leq \left(1+L_B\right)\left(1+\overline{Y}_P\right), \tag{3}$$

where $r_B^e \equiv \left(1+r_{B1}\right)\left(1+r_{B2}^e\right)-1$ is the expected (as of period 0) cumulated interest rate factor over the lifetime of the project. Assumption 1 implies that this condition is satisfied for all non-negative leverage levels.

3.3.2 Period 1

Although all borrowers are identical exante (in period 0), they split into three groups in period 1. Those groups differ in terms of the information that becomes available to markets in that period about the realizations of their idiosyncratic shocks in period 2. In particular, it becomes known in period 1 that a fraction, $\theta_{LB} < q$, of borrowers will have $\widetilde{Y}_I = y$, a fraction $\theta_{UB} < 1-q$ will get $\widetilde{Y}_I = -y$, and no new information is revealed in period 1 about the remaining borrowers. We refer to those three types of borrowers as Lucky borrowers (LB), Unlucky borrowers (UB) and Regular Borrowers (RB) respectively.

A borrower who decides to leverage his project in period 0 is solvent in period 1 if and only if he is able to obtain the refinancing required to maintain his project alive till period 2. Financial intermediaries will offer the required credit in period 1 if and only if the expected cash flow of the project in period 2 suffices to cover period's 1 debt service. Obviously this expected cash flow differs across borrowers' types implying that borrower of type $i = \{LB, UB, RB\}$ obtains refinancing in period 1 if and only if

$$L_B (1 + r_{B1}) (1 + r_{B2}) \leq (1 + L_B) \left(1 + E\left[\widetilde{Y}_P | I_1 \cap B_i\right]\right), \; i = LB, UB, RB, \tag{4}$$

where I_1 is the information set of period 1. Given period's 1 information

$$E\left[\widetilde{Y}_P | I_1 \cap B_i\right] = \begin{cases} 2yq, & i = LB \\ E\left[\widetilde{Y}_P \mid I_1 \cap B_{RB}\right] = 2y(q + q_1 - 1), & i = RB \\ -2y(1-q), & i = UB \end{cases}, \tag{5}$$

and

$$q_1 \equiv \frac{q - \theta_{LB}}{1 - (\theta_{LB} + \theta_{UB})} \equiv \frac{q - \theta_{LB}}{1 - \theta} \tag{6}$$

is the probability that a regular borrower will get a good draw on the idiosyncratic shock, $\widetilde{Y}_I$, given the information available in period 1.[10]

Assumption 2: *The expected net return of a RB on a real project conditional on the information in period 1 satisfies* $E\left[\widetilde{Y}_P \mid I_1 \cap B_{RB}\right] > r_B^e$.

Assumption 2 is basically an extension of Assumption 1 from period 0 to period 1. Together those assumptions requires that, given r_B^e, the expected net return perceived by a regular borrower is larger than the expected cost of leverage given the information of both periods 0 and 1. The following Lemma identifies solvency conditions in period 1 for the three types of borrowers

Lemma 1:

(i) Regular borrowers are solvent in period 1 at any level of leverage, L_B, *if* $r_{B2} = r_{B2}^e$.

(ii) Lucky borrowers are solvent in period 1 at any level of leverage if $2yq > E\left[\widetilde{Y}_P \mid I_1\right]$.[11]

(iii) Unlucky borrowers are solvent in period 1 if and only if $L_B \leq \frac{1-2y(1-q)}{2y(1-q)+r_B^e} \equiv L_B^1$

Note that since $\frac{1}{2} < q \leq 1$ the critical level of leverage, L_B^1, is positive. The lower this critical level the wider the range of period's 0 debt for which there is a non-zero probability

[10]The reason this probability differs between periods 0 and 1 is that the realizations of $\widetilde{Y}_I$ become known with certainty for a fraction, θ, of borrowers in period 1.

[11]Overly strong jointly sufficient conditions for this requirement are $\theta_{LB} = \theta_{UB}$ and $q > 1 - q$..

that the unlucky borrower defaults in period 1. When the expected cost of capital, r_B^e, increases the critical level, L_B^1, decreases, implying that, the higher the cost of capital, the wider is the range of leverages at which a borrower might default in period 1. Increasing risk (measured in terms of returns' variance) has a similar effect since it decreases L_B^1.[12] On the other hand, when the probability of good returns increases (i.e q increases), the critical leverage, L_B^1, increases widening the range of leverages for which the probability of default in period 1 is zero.

3.3.3 Period 2

A borrower is solvent in period 2 if the payoff from his project suffices to cover his debt obligation, that is

$$L_B\left(1+r_{B1}\right)\left(1+r_{B2}\right) \leq \left(1+L_B\right)\left(1+\widetilde{Y}_P\right). \tag{7}$$

Straightforward algebra shows that this is equivalent to the requirement that ultimate wealth, $W_B(.)$, is non negative

$$W_B(L_B, \widetilde{Y}_P) = 1 + \widetilde{Y}_P + \left(\widetilde{Y}_P - r_B\right) L_B \geq 0. \tag{8}$$

When the final project's payoff does no suffice to cover the principal and interest rate payments the borrower defaults and loses the entire project including his initial equity. Due to limited liability the net return on the project in this case is $\widetilde{Y}_P = -1$ and the financial intermediary who own the debt receives nothing.

Lemma 1 has shown that when period's 0 leverage is higher than some critical value the borrower is exposed to default risk in period 1. In addition positive leverage also exposes borrowers to default risk in period 2. The following Lemma identifies regular borrower's solvency conditions in period 2 for various realizations of total returns.

Lemma 2: *If a regular borrower's ultimate return is:*
(i) $\widetilde{Y}_P = -2y$ *he is solvent in period 2 if and only if* $L_B \leq \frac{1-2y}{r_B+2y} \equiv L_B^L$,

[12]It is easily checked that increasing y, while keeping the probabilities of good and bad outcomes (q and $(1-q)$) unchanged, increases the project's variance without changing its expected return.

(ii) $\widetilde{Y}_P = 0$ *he is solvent in period 2 if and only if* $L_B \leq \frac{1}{r_B} \equiv L_B^H$,

(iii) $\widetilde{Y}_P = 2y$ *he is solvent in period 2 for any level of leverage.*

Lemma 3: Given $r_{B2} = r_{B2}^e$,

(i) If an unlucky borrower (UB) has chosen L_B^1 *in period 0 he is solvent in period 1.*

(ii) An UB that has chosen L_B^1 *is also solvent in period 2 if and only if there is an aggregate expansion*

(iii) A lucky borrower is always solvent in period 2.

Next, we analyze the probabilities of default in periods 1 and 2 as functions of the leverage chosen in period 0. Since payoffs are discrete those relations take the form of step functions.

Proposition 1: *Provided* $r_{B2} = r_{B2}^e$, *the ex-ante probabilities of default in period 1 and in period 2 (as viewed from he vantage point of period 0) are step functions of the leverage chosen in period 0. The precise probabilities of defaults are:*

L_B	$\Pr(D_1)$	$\Pr(D_2)$	$\Pr(D) = \Pr(D_1) + [1 - \Pr(D_1)]\Pr(D_2)$
$L_B \leq L_B^L$	0	0	0
$L_B^L < L_B \leq L_B^1$	0	$(1-q)^2$	$(1-q)^2$
$L_B^1 < L_B \leq L_B^H$	θ_{UB}	$(1-q)^2$	$\theta_{UB} + (1-\theta)(1-q)^2$
$L_B^H < L_B$	θ_{UB}	$1-q^2$	$\theta_{UB} + (1-\theta_{UB})(1-q^2) + ((1-\theta)2q + \theta_L)(1-q)$

,

where $\Pr(D_1)$ *and* $\Pr(D_2)$ *stand for default probabilities in period 1 and 2 respectively.*

3.4 Borrower's optimization

Not surprisingly the individual borrower faces a trade-off between expected payoff and default probability. In the large, by raising leverage, he raises the expected value of terminal equity but also the chances of default. By Proposition 1 the exante probability of default is a step function of leverage. This implies that the optimal level of leverage (and by implication also the optimal project's size) must coincide with one of the four leverage levels at the jump points of the probability of default function. The reason is that, once leverage is extended beyond a given jump point the probability of default remains constant as long as leverage is not pushed beyond the next jump point. Within such an interval, raising leverage raises the expected payoff without raising the probability of default.

Hence, once leverage is raised beyond a given jump point, it is individually optimal to push it (at least) all the way till just a tiny bit before the probability function's next jump point. It follows that, from the vantage point of period 0, the optimal level of leverage is either zero or one of the following three leverage levels:

$$\begin{aligned} L_B^L &= \frac{1-2y}{r_B^e+2y}; \\ L_B^1 &= \frac{1-2y\,(1-q)}{2y\,(1-q)+r_B^e}; \\ L_B^H &= \frac{1}{r_B^e}. \end{aligned} \tag{9}$$

The borrower's utility function is piecewise linear with a penalty in the event of default. In particular utility is linear in wealth as long as the borrower is solvent. When insolvent the borrower is subject to a penalty that increases with the magnitude of leverage he defaults on. Formally

$$u(W_B, L_B) = \begin{cases} W_B \geq 0, & \text{Solvency} \\ P_B L_B, & \text{Insolvency} \end{cases}, \tag{10}$$

where W_B is his period's 2 terminal wealth after servicing all debts and P_B is a fixed default penalty per unpaid leverage dollar in states of insolvency. Hence, the borrower's expected utility is

$$\begin{aligned} V(L_B) &\equiv Eu(W_B(\cdot), L_B) \\ &= [1-\Pr(D \mid L_B)]\, E\,[W_B(\cdot) \mid W_B(\cdot) \geq 0] - \Pr(D \mid L_B) P_B L_B. \end{aligned} \tag{11}$$

Using Proposition 1 and the definition of $W_B(\cdot)$ in equations (8) and (9) establishes that $B's$ expected utilities at each of the five candidates for optimal leverage (four discussed above plus any level of leverage $L_B^m > L_B^H$) are given by

$$\begin{aligned}
V(L_B = 0) &= q^2(1+2y) + 2q(1-q) + (1-q)^2(1-2y); \\
V(L_B^L) &= q^2\left[1+2y+(2y-r_B^e)L_B^L\right] + 2q(1-q)\left[1-r_B^e L_B^L\right]; \\
V(L_B^1) &= q^2\left[1+2y+(2y-r_B^e)L_B^1\right] + 2q(1-q)\left[1-r_B^e L_B^1\right] - (1-q)^2 P_B L_B^1; \qquad (12) \\
V(L_B^H) &= \left(\theta_{LB}q + (1-\theta)q^2\right)\left[1+2y+(2y-r_B^e)L_B^H\right] - \Pr\left[D \mid L_B^H\right] P_B L_B^H; \\
V(L_B^m) &= \left(\theta_{LB}q + (1-\theta)q^2\right)\left[1+2y+(2y-r_B^e)L_B^m\right] - \Pr\left[D \mid L_B^m\right] P_B L_B^m,
\end{aligned}$$

where

$$\begin{aligned}
\Pr\left[D \mid L_B^H\right] &\equiv \left[\theta_{UB} + (1-\theta)(1-q^2)\right], \\
\Pr\left[D \mid L_B^m\right] &\equiv \Pr\left[D \mid L_B^H\right] + \left((1-\theta)2q + \theta_{LB}\right)(1-q).
\end{aligned}$$

Let L_B^* be the optimal level of leverage. The following proposition presents (overly restrictive) sufficient condition for $L_B^* = L_B^H$.

Proposition 2: *Provided*

$$P_B > \frac{\left(\theta_{LB}q + (1-\theta)q^2\right)\left(2y - r_B^e\right)}{\left((1-\theta)2q + \theta_{LB}\right)(1-q)} \equiv P_B^c,$$

there exists a dense set of values for the vector of parameters $(q, \theta_{LB}, \theta_{UB})$, *such that* $1-q$, $q-\theta_{LB}$ *and* θ_{UB} *are all strictly positive but small, for which the borrower's optimal level of leverage is* L_B^H.

The broad intuition underlying this proposition can be appreciated by starting with the particular case in which the unit penalty, P_B, for default is zero. In this case, when the chances of good draws at the individual level are high (q and θ_{LB} are large) and the likelihood that the borrower will be unlucky in period 1 is low (θ_{UB} is small), expected utility is monotonically increasing in leverage. As a matter of fact, given the full linearity of the utility function in the absence of a penalty, the borrower's optimal level of leverage is infinite in this case. However, in the presence of a sufficiently large default penalty extending leverage beyond L_B^H is not individually optimal because of the increase in the risk that the penalty will be triggered once leverage crosses the L_B^H threshold.

In a broad sense the conditions in the Proposition 2 are analogous to the borrower's second order condition (SOC) when the penalties from default rise continuously with leverage. In the continuous case the SOC assures that, as leverage goes up, the favorable marginal impact of higher leverage on return in good states diminishes in comparison to the unfavorable gradual increase in the default penalty. Similarly, the conditions in Proposition 2 assure that, as leverage rises, the marginal detrimental impact of the default penalty becomes more important relatively to the marginal favorable effect on likely profits.

4 Financial intermediaries (Fs)

For reasons that will become apparent later it is convenient to open this section with a forward look at the relation between various equilibrium rates of interest.

4.1 A forward look at general equilibrium

The following proposition establishes general equilibrium relations between equilibrium interest rates, r_B, r_F and r_L.

Proposition 3: *In a general equilibrium with risk aversion on the part of borrowers, financial intermediaries and lenders, and positive levels of leverage in both the real and the financial sectors, the following inequalities hold*

$$r_f \leq r_L < r_B.$$

4.2 The typical financial intermediary

There is a large number of financial intermediaries (Fs) each of which possesses one unit of core funds consisting of a combination of equity and of long term (two periods) debt. A typical F can also raise **short term** (one period) funds from lenders.[13] Since the focus of this

[13]For instance through various deposits including certificates of deposit (CDs).

analysis is on changes in the availability of short term credit in the face of new information, the amount of short term leverage assumed by a typical F is determined endogenously while the sum of equity and of long term debt is taken to be exogenous.

Total financial resources of a typical F consist of the core funds and of short term leverage, L_F. The financial intermediary diversifies his total resources between the risk free asset whose rate, r_f, is a policy instrument, and a risky, not fully diversified, portfolio of loans to borrowers.[14] For reasons of tractability, each F lends to only two borrowers. The fraction of resources invested in the risky loan portfolio to Bs is denoted z_F. Let W_F be the intermediary's terminal wealth after debt service in each period. F is solvent or insolvent in each period depending on whether terminal wealth is non negative or strictly negative. When solvent, F's utility is described by a CRRA utility function with a coefficient, δ, of risk aversion that is close to, but not quite equal to, one. This specification implies that F is almost, but not strictly, risk neutral. When insolvent, the typical intermediary experiences a (per unit of leverage) penalty, P_F. Formally[15]

$$u(W_F) = \begin{cases} \frac{[W_F]^{1-\delta}}{1-\delta}, & \text{when } W_F \geq 0 \\ -P_F L_F, & \text{when } W_F < 0. \end{cases} \tag{13}$$

4.3 Distribution of returns, solvency and optimization

Total return to a financial intermediary depends on the performance of the two borrowers to whom he lends. Since borrowers are identical exante, the optimal risky portfolio of an F consists of a fifty-fifty split between loans to his two debtors. If both borrowers are solvent both of them pay the full face value, of the gross debt service and the payoff (from one unit) to F is $1 + r_B$. If both of them default F gets a payoff of 0 on his risky portfolio. If one borrower is solvent and the other defaults F gets the payoff $\frac{1}{2}(1 + r_B)$. Obviously, the probabilities associated with each of those three payoffs depend on the probabilities of

[14] By contrast, as shown in the next section, the risky portfolio of suppliers of funds to Fs (lenders) is fully diversified.

[15] In spite of the fact that utility functions differ across the three types of agents we use the symbol $u(\cdot)$ to stand for all of them in order to economize on notation. In each case the identity of the player should be evident from the context.

defaults of borrowers and differ between periods 0 and 1. From the vantage point of period zero, the probability a single B defaults in period 1 , $\Pr(D_1)$, is given in Proposition 1. Since $L_B^* = L_B^H$, the probability that a B is insolvent in period 1 is θ_{UB}. Since the probability of being unlucky of any borrower is statistically independent of this probability for any other borrower, the distribution of payoffs faced by a typical intermediary in period 0 is given by the following table.[16]

		Period 1	**Period 2**
State	Payoff ($\widetilde{R}_B$)	**Probability**	**Probability**
Both Bs are solvent	$1 + r_B$	$(1-\theta_{UB})^2 \equiv \gamma_{11}$	$q(1-q_1^2) + q_1^2 \equiv \gamma_{12}$
Exactly one B is solvent	$\frac{1}{2}(1 + r_B)$	$2(1-\theta_{UB})\theta_{UB} \equiv \gamma_{21}$	$2(1-q)q_1(1-q_1) \equiv \gamma_{22}$
Both Bs are insolvent	0	$\theta_{UB}^2 \equiv \gamma_{31}$	$(1-q)(1-q_1)^2 \equiv \gamma_{32}$

(14)

Recall that q is the probability of a positive (aggregate or idiosyncratic) shock and q_1, defined in Equation (6), is the probability of a positive idiosyncratic shock conditional on the project type that is revealed in period 1. The last column shows the probability distribution of period's 2 payoffs from loans to regular borrowers as perceived by F's in period 1.

The wealth of a typical F at the end of each period is

$$\widetilde{W}_F(\widetilde{R}_B, L_F) = (1 + L_F)\left[z_F\widetilde{R}_B + (1 - z_F)(1 + r_f)\right] - (1 + r_L)L_F, \tag{15}$$

where the distributions of $\widetilde{R}_B$ is given in Equation (14) and r_L is the interest rate paid by a F on its short term obligations. A representative F chooses his leverage, L_F, and the fraction, z_F, of resources invested in the risky loan portfolio so as to maximize $Eu(\widetilde{W}_F)$ in each of periods 0 and 1. The following proposition presents a preliminary characterization of F's optimal policy.

Proposition 4: *Let $r_f < r_L$. Then at an optimum with positive leverage, F invests all his resources in risky loans to Bs.*

[16]The notation γ_{bt} stands for probability in time t conditional on realized borrowers' types, b.

4.3.1 F's solvency condition

Proposition 4 and Equation (15) imply that F is solvent if and only if

$$\widetilde{W}_F(\widetilde{R}_B, L_F) = (1+L_F)\widetilde{R}_B - (1+r_L)L_F = \widetilde{R}_B + (\widetilde{R}_B - r_L)L_F \geq 0. \tag{16}$$

Since $r_B > r_L$, F is solvent for any level of leverage, L_F, when both of his borrowers are solvent, so that $\widetilde{R}_B = 1 + r_B$. In the other two cases F is solvent only if L_F is sufficiently small. The precise solvency conditions are:

$$\begin{array}{llll} L_F \leq \frac{1+r_B}{1+2r_L-r_B} \equiv L_F^c & \text{when} & \widetilde{R}_B = \frac{1}{2}(1+r_B) \\ L_F = 0 & \text{when} & \widetilde{R}_B = 0 \end{array}. \tag{17}$$

Equations (14) and (17) imply that F's probability of default is an increasing step function of F's leverage and that the precise functions for periods 1 and 2 are

$$\begin{array}{cc} \mathbf{L}_F & \Pr[D_t]\,,\ \mathbf{t=1,2} \\ 0 & 0 \\ L_F \leq L_F^c & \gamma_{1t} \\ L_F > L_F^c & \gamma_{2t}+\gamma_{3t} \end{array}. \tag{18}$$

Proposition 5: *Provided:*

(i) δ *is sufficiently small,*

(ii) $\gamma_{1t}(r_{Bt} - r_{Lt}) - (1-\gamma_{1t})P_F > 0$,

(iii) $[\gamma_{1t}(r_{Bt} - r_{Lt}) - (1-\gamma_{1t})P_F]\,(L_{Ft}^* - L_F^c) > \gamma_{2t}P_F L_F$,

(iv) $(\gamma_{2t}/\gamma_{1t})$ is sufficiently small,

F's optimal leverage is

$$L_{Ft}^* = \left(\frac{\gamma_{1t}}{1-\gamma_{1t}}\right)^{\frac{1}{\delta}} (r_{Bt} - r_{Lt})^{\frac{1-\delta}{\delta}} - \frac{1+r_{Bt}}{r_{Bt} - r_{Lt}}. \tag{19}$$

Here L_{Ft}^*, $t = \{0, 1\}$ *is the financial intermediary's optimal short term leverage in periods 0* *and 1.*

The following proposition formulates the financial intermediary's solvency condition. If solvent a financial intermediary pays the full debt service to lenders. Otherwise he defaults and pays nothing.

Proposition 6: *Provided δ is sufficiently small the financial intermediary is solvent if and only if the two borrowers to whom he has lent are solvent.*

Proposition 6 implies that (since $L_{Ft}^{*} > L_{F}^{c}$) overly restrictive sufficient conditions for the two requirement in Proposition 5 are that γ_{1t} is sufficiently large and/or P_F sufficiently small. Next, we characterize F's optimal leverage.

Proposition 7: *The optimal leverage of a typical financial intermediary is higher*
(i) the lower are the intermediary's risk aversion, δ, and the default penalty, P_F,
(ii) the lower the cost of borrowing, r_{Lt},
(iii) the higher the probability, γ_{1t}, that the intermediary remains solvent,
(iv) the higher the interest rate, r_{Bt}, paid by borrowers.

5 The representative lender and government's bailout policy

Through pension or mutual funds the representative lender (L) splits his equity between a fully diversified portfolio of loans to financial intermediaries and the risk free asset.[17] Since, exante, all Fs have identical distributions of returns the optimal shares of loans to different Fs are all equal. The fraction invested in the risky loan portfolio to Fs is denoted z_L. The typical lender possesses mean-variance (or Constant Absolute Risk Aversion - CARA)[18] preferences

$$u\left(W_L\right) = -\frac{1}{\alpha}e^{-\alpha W_L}, \qquad \alpha \geq 0, \tag{20}$$

[17] The lender is representative in the sense that all lenders are identical.
[18] See Sargent (1987) pages 154-155.

where W_L is his terminal wealth in each period and α characterizes the degree of constant absolute risk aversion.

5.1 Perceived government's bailout policy

Government may repay the gross debt owed to lenders by defaulting Fs. The perceived probability that the debt service of a defaulting F is paid by government (a bailout) is denoted by p. The likelihood of bailout is independent across Fs debt. In case of bailout a lender receives the full debt service, r_L. In the presence of risk but no bailout uncertainty p is unique. We use the Gilboa Schmeidler's (1989) multiple priors framework to formalize Knightian uncertainty.[19] Accordingly, in the presence of bailout uncertainty perceptions include the convex set of all possible binomial distributions characterized by p's that the lenders believe to have positive mass (i.e. are considered as plausible). The lowest value of p in the set is denoted by π. As will become clear below this is also the worst plausible prior from a typical lender's point of view.

The degree of uncertainty is determined by the "size" of the set of possible priors. That is, when circumstances become more uncertain (ambiguous) the set of possible priors expands to include priors that previously were considered implausible. Consequently, provided some of the set enlargement is toward lower p's, the worst prior, π, is revised downward.

5.2 Representative lender's returns and optimization

5.2.1 Period 0

On one period loans taken in period 0 borrowers face only idiosyncratic risk because only individually unlucky borrowers default in period 1, implying that financial intermediaries who lend to them also face only idiosyncratic risk in that period. By contrast, since they fully diversify their loans across intermediaries, lenders face no risk at all in period 0. Consequently, and since lenders know from Equation (14) that only a fraction, $(1-\theta_{UB})^2$, of Fs will be solvent in period 1, equilibrium r_{L1} includes a compensation for the average fraction

[19] Knightian uncertainty is also known as "ambiguity" in modern decision theory.

of unpaid debt but no compensation for variability of this fraction, since this variability equals zero due to full diversification. Hence,

$$1 + r_{L1} = \frac{1 + r_{f1}}{1 - (1 - \pi)\left[1 - (1 - \theta_{UB})^2\right]} \tag{21}$$

where $(1 - \pi)\left[1 - (1 - \theta_{UB})^2\right]$ is the probability that a lender loses the investment in a loan to a single intermediary. It is obtained as the product of the probabilities of the two following independent events: "the intermediary defaults" and "government does not reimburse the delinquent debt to the lender". As a consequence, in period 0, lenders are indifferent between investing in the standard risk free asset at rate r_{f1} and between investing in loans to Fs.

5.2.2 Period 1

By contrast, in period 1, lenders face risk in loans to Fs in spite of their fully diversified portfolios. The reason is that the returns to lenders from loans to different Fs are correlated due to the common shock, $\widetilde{R}_A$, in the return to real investments of borrowers. As explained in the previous section a financial intermediary either pays his debt in full to lenders or fully defaults. When F defaults on the debt service, government may or may not step in and pay the delinquent debt service to a lender. Consequently the lender faces a binomial distribution of returns from lending to an individual F – he either gets the full debt service, $1 + r_L$, (from F or from government) or 0. Although the bailout policy of government does not affect the binomial nature of the payoffs from a single F, it **does alter their distribution**. Since the risky portfolio of L's contains a large number of such binomially distributed loans the risky portfolio of Ls is normally distributed with a mean and a variance that depend on both economic (q) and political (p) uncertainties. Details appear in the following proposition.

Proposition 8: *For a given p the period's 2 payoff to a lender on his fully diversified portfolio of loans, $\{\widetilde{R}_{L2}\}$, is normally distributed with mean*

$$E\left(\{\widetilde{R}_{L2}\}\right) = \left[p + \gamma_{12}(1 - p)\right](1 + r_{L2}) = \left[p + \left[q(1 - q_1^2) + q_1^2\right](1 - p)\right](1 + r_{L2}) \tag{22}$$

and variance[20]

$$Var\left(\{\widetilde{R}_{L2}\}\right) = (1-p)^2 (1-q) (1-q_1)^2 \left(q + 2qq_1 + q_1^2\right) (1+r_{L2})^2,$$

where

$$q_1 \equiv \frac{q - \theta_{LB}}{1 - (\theta_{LB} + \theta_{UB})}, \tag{23}$$

and $\{\widetilde{R}_{L2}\}$ *stands for the set of return from loans.*

A representative L chooses the fraction, z_L, of resources invested in the risky loan portfolio to Fs so as to maximize[21]

$$E\left(\widetilde{W}_L(z_L)\right) = -\frac{1}{\alpha} E\left(e^{-\alpha \widetilde{W}_L(z_L)}\right) \tag{24}$$

in each period, where

$$\widetilde{W}_L(z_L) = z_L \widetilde{r}_L + (1 - z_L) r_f. \tag{25}$$

Proposition 9: *At an individual optimum, a lender allocates the fraction*

$$z_L^* (\pi, R_L, q, q_1) \cong \frac{E\left(\{\widetilde{R}_L\}\right) - (1 + r_f)}{\alpha Var\left(\{\widetilde{R}_L\}\right)} \tag{26}$$

of each single \$ to the diversified risky portfolio of loans to Fs.[22,23]

5.3 Partial equilibrium comparative statics

We now investigate the impact of less generous bailouts on the size of lenders' risky portfolios and the impact of ambiguity aversion in partial equilibrium. In the absence of bailout uncertainty government's bailout policy is characterized by a unique perceived probability,

[20] The variance is scaled by the term $(1 + r_{L2})^2$ for reasons of space.

[21] The right hand side of Equation (24) is obtained by using a typical lender's utility function in equation (20).

[22] This approximation is accurate for a small risk premium, $E(\{\widetilde{r}_L\}) - r_f$.

[23] An identical allocation, $z_L^* (\pi, R_L, q, q_1)$, is obtained for constant relative risk aversion (CRRA) preferences (see Merton 1971, 1973).

p, that government will pay the debt of delinquent F's to L's. A more generous (towards L's) bailout policy is characterized by a higher p and a less generous bailout policy by a lower p. By changing the distribution of $\widetilde{r}_L$ the value of p affects both the mean and the variance of lenders' risky portfolios.

Proposition 10: *Holding r_{L2} constant a less generous bailout policy (lower p)*
(i) reduces the mean return on the portfolio of loans from lenders to financial intermediaries,
(ii) raises the covariance between any two loans in the (fully diversified) portfolio, and therefore, the portfolio's variance,
(iii) Both changes reinforce each other in inducing a "flight to safety" by lenders.

In the presence of uncertainty about p, and since they are averse to ambiguity in the Gilboa-Schmeidler (1989) sense, lenders behave as if the probability of bailout is the lowest within the set of p's with positive mass (denoted π).[24,25] Stated differently, they choose the fraction of their portfolio invested in risky loans to Fs so as to maximize expected utility under the assumption that bailout probability is π. The operational consequence of such behavior is that p should be replaced with π in propositions 8 and 10.

Propositions 10 implies that higher bailout uncertainty has two effects: Not surprisingly it lowers the expected return from the risky loan portfolio of lenders. More surprisingly, but not less importantly, it raises the correlation between loans in the portfolio which implies in turn higher variances in lenders portfolios. This result appears surprising at first blush since intuition may lead one to conclude that an increase in bailout probability, by decreasing the likelihood of default, will increase the correlation between loans' returns in the portfolio. But this intuition is mistaken. The reason is that the correlation originates uniquely from the aggregate shock whose impact operates only through the fraction of loans in the portfolio that are no bailed out. Since the impact of this fraction on the overall correlation diminishes as more intermediaries are bailed out the variance goes down. Consequently, in the limit, when bailouts are almost certain, this variance tends to zero.

[24] For simplicity we use the Gilboa-Schmeidler (1989) model to capture uncertainty, however other model of uncertainty can be incorporated into our model; Klibanoff, Marinacci and Mukerji (2005) or Izhakian and Izhakian (2009a, 2009b), for example.

[25] Lowest in the sense that under this probability distribution expected utility attains its minimal value.

Proposition 10 implies that, when due to an increase in bailout uncertainty π decreases, lenders reduce the share of funds supplied to financial intermediaries. This conclusion plays an important role in the following general equilibrium sections.

Proposition 11: *When $1 + 2r_f \geq r_L$, L's optimal allocation to risky loans, $z_L^* (\pi, r_L, q, q_1)$ is increasing in r_L.*

6 General equilibrium of the financial system

Given expectations about the future, general equilibrium of the financial system is characterized by two market clearing conditions. One for credit from Ls to Fs and the other for credit from Fs to Bs. These two conditions simultaneously determine r_B and r_L in each period. In period 1, expectations about the future only involve realizations of period's 2 returns to borrowers. As a consequence, the formulation of this equilibrium is relatively simple. But in period 0 they also involve expectations about period's 1 market clearing values of r_B and r_L in period 1 (r_{B2}^e and r_{L2}^e). Those expectations are assumed to be model consistent in the sense that, in period 0, financial market participants use the information available in that period along with their knowledge of the fact that period's 1 rates will be determined by market clearing to derive r_{B2}^e and r_{L2}^e.

6.1 General equilibrium in period 0

Proposition 1 implies that a borrower is insolvent in period 1 only if he is unlucky, implying (from the discussion in Section 4) that a financial intermediary defaults in period 1 if and only if at least one of his borrowers is unlucky. Being unlucky is related to B's and Fs individual fortunes rather than to the aggregate shock. Lenders are exposed to aggregate shocks and idiosyncratic shocks; aggregate shocks are relevant only in period 2 while idiosyncratic shocks are fully diversified. Hence, since lenders are fully diversified and lend to Fs lend for only one period, they do not face any risk in lending to F's in period 0. In particular, they know for sure (from Equation (14)) that a fraction $(1 - (1 - \theta_{UB})^2)$ of intermediaries will default in period 1. Hence, they demand a compensation only for this perfectly anticipated fraction of defaults. Consequently, in period 0, r_{L1} is determined exogenously by the condition

$$1 + r_{L1} = \frac{1 + r_{f1}}{1 - (1 - p)\left[1 - (1 - \theta_{UB})^2\right]}. \tag{27}$$

Actual period's 0 equilibrium conditions in the markets for loans from Ls to Fs and from Fs to Bs are given respectively by

$$M_F L_F^*(r_{B1}, r_{L1}) = M_L z_{L1}, \tag{28}$$

$$L_B^*(r_{B1}, r_{B2}^e) = \frac{M_B}{(1 + r_{B1})(1 + r_{B2}^e) - 1} = M_F \left(1 + L_F^*(r_{B1}, r_{L1})\right). \tag{29}$$

They determine z_{L1} and r_{B1} as functions of r_{L1} and r_{B2}^e. Since $L_B^*(r_{B1}, r_{B2}^e)$ also depends on period's 0 expectation of the cost of funds to borrowers, r_{B2}^e, in the subsequent period a full characterization of period's 0 equilibrium requires additional conditions for the determination of r_{B2}^e. Those conditions are provided by the hypothesis that, in period 0, agents form their perceptions about r_{B2}^e and r_{L2}^e by utilizing period's 1 expected market clearing conditions given their period's 0 information. Expected period's 1 equilibrium conditions in the markets for loans from Ls to Fs and from Fs to Bs are given respectively by

$$(1 - \theta_{UB})^2 M_F L_F^*(r_{B2}^e, r_{L2}^e) = (1 + r_{f1}) M_L z_L^*(\pi_0, r_{L2}^e, q, q_1), \tag{30}$$

$$\frac{M_B}{(1 + r_{B1})(1 + r_{B2}^e) - 1} = (1 - \theta_{UB}) \begin{bmatrix} 1 + r_{B1} + (r_{B1} - r_{L1}) L_F^*(r_{B1}, r_{L1}) \\ + L_F^*(r_{B2}^e, r_{L2}^e) \end{bmatrix}. \tag{31}$$

The two period's 0 market clearing conditions and the two clearing conditions expected for period 1 jointly determine r_{B1}, r_{B2}^e, r_{L2}^e and z_{L1}. This system is recursive since the last three equations simultaneously determine the first three variables leaving the first equation for the determination of z_{L1}.

6.2 General equilibrium in period 1

The actual period's 1 market clearing conditions in the markets for loans from Ls to Fs and from Fs to Bs are given respectively by

$$(1-\theta_{UB})^2 M_F L_F^*(r_{B2}, r_{L2}) = (1+r_{f1}) M_L z_L^*(\pi_1, r_{L2}, q, q_1), \tag{32}$$

$$\frac{M_B}{(1+r_{B1})(1+r_{B2}^e)-1} = (1-\theta_{UB}) \begin{bmatrix} 1 + r_{B1} + (r_{B1} - r_{L1}) L_F^*(r_{B1}, r_{L1}) \\ + L_F^*(r_{B2}, r_{L2}) \end{bmatrix}. \tag{33}$$

Those equilibrium conditions determine the *actual* interest rates, r_{B2} and r_{L2} in period 1 for predetermined, values of r_{B1}, r_{B2}^e, r_{L1} and of π_1. Comparing $z_L^*(\pi_0, r_{L2}^e, q, q_1)$ from Equation (30) with $z_L^*(\pi_1, r_{L2}, q, q_1)$ from Equation (32) we note that the first two arguments of those functions differ. Not surprisingly the first, which refers to the expected equilibrium, features r_{L2}^e, while the second, that refers to the actual equilibrium, features r_{L2}. Importantly, the effective bailout probabilities, π_0 and π_1, differ across the expected and the actual period's 1 equilibria. This (at this stage) notational difference is introduced in anticipation of the discussion in the next section that introduces an unanticipated increase in bailout uncertainty.

7 The impact of bailout uncertainty and Lehman's collapse

This section considers the impact of an unanticipated increase in bailout uncertainty on financial markets in period 1. Recall first that π_0 is the lowest perceived bailout probability as of period 0 for **both** periods 0 and 1.[26] The fact that, as of period 0, financial market participants do not expect this probability to change in period 1 is reflected in the formulation of the expected period's 1 general equilibrium conditions for period 0 (equations (28) through (31)).

[26] More generally π_t stands for the lowest belief in period t about the probability of governmental bailout in all relevant future periods.

Suppose now that, following a major indication of a shift in government's bailout policy – like not rescuing Lehman — bailout uncertainty increases. In particular, the lowest perceived bailout probability with positive mass decrease from π_0 to π_1. Proposition 10 implies that, holding r_{L2} constant, this change reduces the supply of funds to Fs by Ls. Application of comparative statics methods to period's 1 equilibrium conditions shows that this change triggers a general equilibrium increases in both r_{L2} and r_{B2} above their expected counterparts – r^e_{L2} and r^e_{B2}. Those increases raise the fraction of defaulting borrowers in period 2 and may, under some circumstances, lead to a total drying up of credit in period 1. The string of propositions in this section provides a precise formulation of these and related results

Let

$$\frac{\partial L_F^*}{\partial r_B}, \quad \frac{\partial L_F^*}{\partial r_L} \tag{34}$$

be respectively the responses of $F's$ optimal leverage to changes in r_{B2} and in r_{L2} and let

$$\frac{\partial z_{L2}^*}{\partial r_L}, \quad \frac{\partial z_{L2}^*}{\partial \pi} \tag{35}$$

be the responses of $L's$ optimal share of investments in the risky portfolio to changes in r_{L2} and in π.

Proposition 12: *Given propositions 7, 10, and 11 an increase in uncertainty about governmental bailout policy, and thus a decrease in the effective probability of bailout from* π_0 *to* π_1 *leads to:*

(i) An increases in r_{L2} *and* r_{B2} above r^e_{L2} *and* r^e_{B2} respectively;

(ii) When the increase in r_{B2} *is such that* $r_{B2} > r^e_{B2} + 2y(q + q_1 - 1)(1 + r^e_{B2})$, *period's 1 credit is denied to* ***both regular*** *and unlucky borrowers, so both types of borrowers default in period 1;*

(iii) A sufficiently large increase in r_{B2} *beyond* $r^e_{B2} + 2y(q + q_1 - 1)(1 + r^e_{B2})$, *triggers a total "financial arrest" in period's 1 credit to Bs in the sense that credit is denied to* ***all*** *borrowers.*[27]

The comparative statics impacts in Proposition 12 accord well with actual developments

[27] This term has been recently been suggested by Ricardo Caballero.

following the downfall of Lehman Brothers. They are consistent with the view that much of the financial market panic, and the associated arrest of financial markets, in the aftermath of this collapse was due to an increase in uncertainty about the willingness of the US government to use public funds to compensate creditors' for losses due to defaulting financial intermediaries.

We turn next to the impact of an increase in bailout uncertainty on the banking spread.

Proposition 13:

(i) Generally, depending on whether $\frac{\partial L_F^*}{\partial r_B}$ *is larger or smaller than* $\mid \frac{\partial L_F^*}{\partial r_L} \mid$*, an increase in bailout uncertainty raises or reduces the banking spread,* $r_{B2} - r_{L2}$*.*

(ii) For the specification of intermediaries' utility used in the model an increase in bailout uncertainty raises the banking spread, $r_{B2} - r_{L2}$*.*

8 Exante bailout perceptions and moral hazard

8.1 The impact of higher bailout perceptions

Unlike period 1 in which the demand for credit by borrowers is already predetermined, period's 0 leverage depends on the borrowing rate in that period as well as on the borrowing rate expected to prevail in period 1. In general equilibrium both of those rates, as well as the rates at which financial intermediaries borrow from lenders, depend on financial markets participants' perceptions about the likelihood of bailout. Hence, by affecting equilibrium interest rates, perceptions about the likelihood of bailout in period 0 affect the volume of leverage in financial markets. This section investigates the impact of period's 0 permanent beliefs about governmental bailout policy as summarized by the parameter π_0 on expected future rates, on actual equilibrium interest rates and on the volume of leverage. The main results are summarized in the following two propositions

Proposition 14: *For model consistent expectations higher permanent values of* π_0 *(lower* *bailout uncertainty) are associated with lower values of* r^e_{B2}*. Formally*

$$\frac{dr^e_{B2}}{d\pi_0} < 0.$$

Proposition 15: *Provided the direct impact of* π_0 *on* z^*_{L1} *is sufficiently large in comparison to the absolute value of the (negative) impact of* π_0 *on* r^e_{B2}*, a higher* π_0 *is associated with*
(i) Lower level of r^e_{B2} *and of* r_{l2}*,*
(ii) Overall larger levels of credit by lenders to financial intermediaries and by intermixtures to borrowers.

The results of Proposition 15 arise through several interconnected channels. By part (i) of Proposition 8 perceptions of more generous bailout policy directly raises the fraction of their portfolio that lenders desire to invest in risky loans to financial intermediaries. This effect exerts a **downward** pressure on r_{L1} and, via the reaction of Fs, also on r_{B1}. But the higher level of π_0 also reduces the (model consistent) borrowing rate expected to prevail in period 1 (r^e_{B2}) and this raises period's 0 demand for credit by borrowers. This effect creates **upward** pressures on both r_{L1} and r_{B1}.

Clearly, the belief that government may repay the debt of some delinquent financial intermediaries creates a moral hazard problem. An important implication of Proposition 15 is that, by raising leverage in the economy, the perception of a more generous bailout policy aggravates this problem and increases the likelihood and the severity of a financial crisis in period 1.

8.2 The impact of a temporary expansionary monetary policy

Within the context of the model a temporary expansionary monetary in period 0, policy takes the form of a decrease in r_f holding the borrowing rate expected for period 1, r^e_{B2}, constant. The following proposition summarizes the impact of such a policy.

Proposition 16: *A temporary decrease in* r_f *leads to a decrease in both* r_{B1} *and* r_{L1}*, and to an increase in leverage within both the financial and the real sectors (both* L^*_F *and* L^*_B *go* *up).*

Interpretation: The subprime crisis counterpart of period 0 in the model can be thought of as the buildup phase of the crisis. During this phase market participants believed that the set of bailout probabilities with positive mass is concentrated in a range with relatively high values of p. In addition, monetary policy was loose by historical standards. Propositions 15 and 16 imply that both factors contributed to the exante expansion of credit and to a real investment boom, making the system more fragile to a sudden downward revision of perceptions about the likelihood of governmental bailouts.

9 Concluding remarks

A major result of our analysis is that the larger the change in bailout uncertainty the stronger the pre-crisis buildup and the deeper the ensuing crisis. The detailed mechanics of this result can be appreciated by thinking of period 0 as the pre-crisis phase during which the worst scenario perceived likelihood of bailout is high and monetary policy relatively loose leading to credit expansion and to an investment boom. Taylor (2009) argues that loose monetary policy caused, prolonged and worsened the financial crisis. Period 1 can be thought of as the phase in which, due to the arrival of some major public signal — like not rescuing Lehman — financial market operators adjust their worst scenario perceptions about the likelihood of bailout downward. By Proposition 9 this adjustment induces a general increase in market interest rates, a rise in the proportion of insolvent borrowers along with the destruction of real investments and, for some realizations of real returns, a complete drying up of short term credit markets.

By Proposition 10 the pre-crisis bubbly credit boom is larger the larger π_0 . By Proposition 10 the magnitudes of deleveraging and of insolvencies (real and financial) is larger the lower is π_1 . Since it measures the extent to which the set of possible bailout distributions widened between periods 0 and 1 the difference $\pi_0 - \pi_1$ is a natural proxy for the increase in bailout uncertainty. Combining this proxy with Propositions 9 and 10 yields the conclusion that higher changes in bailout uncertainty are associated with larger pre-crisis bubbles as well as with higher levels of insolvencies and destruction of real economic activity when the bubble bursts. The crucial variable through which those effects operate is leverage. It expands more during periods of optimism about the likelihood of bailouts but, by the same token, it shrinks more violently during periods of pessimism about the likelihood of bailouts. Given π_1 the deleveraging process during period 1 involves a larger volume of insolvencies the larger is π_0. The reason is that a larger π_0 raises the exante leverage buildup in comparison to what market operators would have engaged in, had they known already in period 0 that the probability of bailout in period 1 will drop to π_1. The larger this "excessive" credit buildup, the larger the expost volume of insolvencies in the real economy.

Importantly, higher bailout uncertainty, by raising the correlation between the returns on different loans, is associated with larger systemic risks. It is also associated with wider banking spreads

Somewhat analogously to Diamond and Dybvig (1983) (DD in the sequel) classic model of bank runs, a main objective of this paper was to identify circumstances that trigger a financial crises. A main result of the DD framework is that deposit insurance eliminates runs on the banks. Although there is an analogy between the role of deposit insurance in DD and bailouts in our framework, a crucial difference between these frameworks is that, up to a given limit, deposit insurance is backed by the exante certainty of a legal act while the availability (and scope) of the generalized bailouts considered here is shrouded in uncertainty and is likely to remain in this state also in the future. Besides other obvious differences two additional difference worth emphasizing are: (i) In DD liquidity shocks are exogenous while here they are related to an increase in uncertainty due to the arrival of new information about "black swan" events. (ii) Our framework is designed to make statements about the impact of monetary policy on leverage and the economy.

Reinhart and Rogoff (2009) present broad evidence supporting the view that private financial crises are often followed by substantial reductions in tax collections and defaults on sovereign debt. Motivated by this findings and some of the results in this paper we speculate in what follows on an additional channel through which higher exante leverage buildups possibly makes the economy more crisis prone when new information arrives. Higher leverage raises the probability as well as the magnitude of potential defaults, and with it the cost of potential bailouts. The more costly is a bailout to tax payers the more reluctant is government to engage in such bailouts. As a consequence, the higher is leverage, the higher bailout uncertainty making beliefs more sensitive to news.

The punch line is that the sensitivity of expectations to various news becomes larger the larger is leverage. In terms of the Gilboa-Schmeidler (1989) uncertainty framework this means that the range of bailout probability distributions entertained by individuals becomes more sensitive to news. As a consequence, the same pessimistic new information about the likelihood of bailout is more likely to puncture a bubble the higher is leverage.

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A Appendix

Proof of Lemma 1

For regular borrowers, and given $r_{B2} = r^e_{B2}$, the condition in Equation (4) is identical to the condition in Equation (3). But, by assumption 1a, the last condition is satisfied for all $L_B \geq 0$. Hence, regular borrowers are solvent at any level of leverage. Since $2yq > 2y\,(2q-1)$ this is afortiori true for lucky borrowers. The proof for unlucky borrowers follows by using Equation (5) in solvency condition (4) and by rearranging. QED

Proof of Lemma 2

The proof of parts (i) and (ii) is obtained by substituting $\widetilde{Y}_P = -2y$ and $\widetilde{Y}_P = 0$ respectively into Equation (8) and by rearranging. The proof of part (iii) follows by inserting $\widetilde{Y}_P = 2y$ into Equation (8) and by utilizing Assumption 1. QED

Proof of Lemma 3

Part (i) is a direct consequence of Lemma 1. To prove part (ii) note that, since $\frac{1}{2} < q < 1$, $L^L_B < L^1_B < L^H_B$. The ultimate payoff of an UB is either 0 in case of expansion or $-2y$ in case of contraction. By Lemma 2, and since $L^1_B < L^H_B$, this borrower is solvent in the first case. By Lemma 2, and since $L^L_B < L^1_B$, this borrower is insolvent in the second case. Part (iii) is a direct consequence of Assumption 1 in conjunction with condition (7). QED

Proof of Proposition 1

The first two default probabilities follow directly from Lemmas 1 through 3. The third probability is derived by noting that, when leverage is larger than L^1_B default may occur in period 1 if the borrower turns out to be unlucky (probability θ_{UB}) and may also occur in period 2 if he turns out to be a regular borrower and $\widetilde{Y}_P = -2y$ (probability $(1-\theta)(1-q)^2$). The last probability follows by noting that, in addition to the states in which he defaults in the previous case, the borrower defaults also in the following two cases: (i) If he is a LB and there is a contraction, (ii) If he is a regular borrower in and $\widetilde{Y}_P = 0$. QED

Proof of Proposition 2

To show that L_B^H is the optimal level of leverage it suffices to establish that

$$
\begin{aligned}
V(L_B^H) &> V(L_B^1) > V(L_B^L) > V(L_B = 0), \\
V(L_B^H) &> V(L_B^m).
\end{aligned}
$$

The proof is implemented by using Equation (12) to form explicit expressions for the differences $V(L_B^L) - V(L_B = 0)$, $V(L_B^1) - V(L_B^L)$, $V(L_B^H) - V(L_B^1)$, $V(L_B^m) - V(L_B^H)$ and by showing that they are all positive.

(i) $V(L_B^L) - V(L_B = 0) = q^2(2y - r_B^e)L_B^L - 2q(1-q)r_B^e L_B^L$.

Assumption 1 and q sufficiently close to one imply that this difference is positive.

(ii) $V(L_B^1) - V(L_B^L) = \{q^2(2y - r_B^e) - 2q(1-q)r_B^e\}(L_B^1 - L_B^L) - (1-q)^2 P_B L_B^1$.

Since $(L_B^1 - L_B^L) > 0$ this expression is positive for q sufficiently close to one.

(iii) Letting θ_{LB} approach q from below and θ_{UB} approach zero from above

$V(L_B^H) - V(L_B^1) = q^2(2y - r_B^e)(L_B^H - L_B^1) - 2q(1-q)(1 - r_B^e L_B^1) - (1-q)^2 P_B \left[(1-q)L_B^H - L_B^1\right]$.

Since $(L_B^H - L_B^1) > 0$ this expression is positive for q sufficiently close to one.

(iv) The condition $V(L_B^m) - V(L_B^H) > 0$ is equivalent to

$[[(1-\theta)2q(1-q) + \theta_{LB}(1-q)]\, P_B - [\theta_{LB} q + (1-\theta)q^2]\,(2y - r_B^e)]\left(L_B^m - L_B^H\right) > 0$.

Since $(L_B^H - L_B^1) > 0$ this expression is positive if and only if $P_B > P_B^c$. QED

Proof of Proposition 3

The proof is a direct consequence of the fact that all three types of agents are risk averse and that leverage levels are positive. Consequently, financial intermediaries require a markup over their leverage costs as compensation for investing in risky loans to investors. As a consequence $r_B > r_L$. Similarly, lenders demand a risk premium when they invest in risky loans to financial intermediaries rather than in risk free asset. Hence, $r_L \geq r_f$.[28] . QED

Proof of Proposition 4

By construction, since $r_f < r_L$, an F with positive short term leverage and some fraction of the portfolio invested in risk free assets can increase his profits by reducing both short term

[28]As elaborated later in the general equilibrium section, the equality sign is allowed to accommodate a special case in which, due to full diversification, lenders face no risk in period 0.

leverage and, in parallel, the investment in risk free assets. Consequently, a configuration with both positive leverage and some investment in risk free assets cannot be a financial intermediary's optimum. Hence $z_f = 0$. QED

Proof of Proposition 5

Equation (19) is obtained by maximizing the expected utility of F with respect to L_F for $z_f = 0$ under the assumption that F's optimal leverage is above L_F^c. Conditions (ii) and (iii) are needed to rule out the possibility that optimal leverage is at L_F^c or at zero. To derive those conditions let L_F^m be any leverage level above L_F^c and let.

$$EV_F\left[L_F\right] \equiv Eu\left[W_F(L_F)\right]. \tag{36}$$

Then necessary conditions for the optimal level of leverage to be above L_F^c are

$$\begin{aligned} EV_F\left[L_F^m\right] &> EV_F\left[L_F = 0\right] \\ EV_F\left[L_F^m\right] &> EV_F\left[L_F^c\right] \end{aligned} \tag{37}$$

Conditions (i) and (ii) are obtained by using equations (13) and (16) to express F's utility in terms of the appropriate levels of leverage, substituting the resulting expression into Equation (37) and by rearranging. To complete the proof it remains to show that, when $(\gamma_{2t}/\gamma_{1t})$ is sufficiently small, $EV_F\left[L_F^c\right] > EV_F\left[L_F^s\right]$ for any $0 < L_F^s < L_F^c$ establishing (since $EV_F\left[L_F^m\right] > EV_F\left[L_F^c\right]$) that $EV_F\left[L_F^m\right]$ is also larger than $EV_F\left[L_F^s\right]$. When $(\gamma_{2t}/\gamma_{1t})$ is small the only two terms that could potentially make $EV_F\left[L_F^s\right]$ larger than $EV_F\left[L_F^c\right]$ involve γ_{2t}, while the terms that operate to reverse this inequality involve γ_{1t}. In the extreme case, $(\gamma_{2t}/\gamma_{1t}) = 0$, it is unambiguously the case that $EV_F\left[L_F^c\right] > EV_F\left[L_F^s\right]$. By continuity this is also the case for $(\gamma_{2t}/\gamma_{1t})$ positive but sufficiently small. QED

Proof of Proposition 6

The condition in Equation (17) implies that, when $L_{Ft}^* > L_F^c$, F is solvent if and only if both of his debtors are solvent. Recalling that $r_{Bt} - r_{Lt} > 0$ and inspecting Equation (19) reveals that L_{Ft}^* is a monotonically increasing function of δ and that it tends to infinity when δ tends to zero. It follows that, for sufficiently small but positive values of δ, $L^*_{Ft} > L^c_F$ implying that F is solvent if and only if both of his borrowers are solvent. QED

Proof of Proposition 7

The first three parts follow directly from inspection of Equation (19). Part (iv) is established by differentiating this equation with respect to r_{Bt}. QED

Proof of Proposition 8

Calculation of the expected value is relatively straightforward. Derivation of the variance utilizes the fact that the variance of a fully diversified risky portfolio composed of (equally weighted) infinitely many identically distributed assets is equal to the covariance between any two assets within the portfolio. Calculation of this covariance simplifies the derivation of an explicit expression for $Var\left(\{\widetilde{R}_{L2}\}\right)$ but still involves some messy intermediate algebra.

Details to be completed.

The expression for L's portfolio variance in Proposition 8 is obtained by using the joint distribution of $(\widetilde{r}_{Li}, \widetilde{r}_{Lj})$ in the definition of the covariance between $\widetilde{r}_{Li}$ and $\widetilde{r}_{Lj}$ and by simplifying using the Mathematica software. QED

Proof of Proposition 9

Recall that, to keep notation simple, we use the symbol $\widetilde{R}_L$ to denote the gross (one plus) return on a portfolio that consist of an infinite number of loans, $\{\widetilde{r}_L\}$, and $\widetilde{R}_f = 1 + r_f$ is the gross risk free rate. A typical lender's maximization problem is given by

$$\max_{z_L} E\left[u\left(z_L(\widetilde{R}_L - R_f) + R_f\right)\right],$$

where $u(\cdot)$ stands for the utility function. The first order condition implies

$$E\left[u'\left(z^*_L(\widetilde{R}_L - R_f) + R_f\right)(\widetilde{R}_L - R_f)\right] = 0.$$

Taking a Taylor approximation of the marginal utility with respect to $\widetilde{R}_L$ around R_f yields

$$E\left[u'\left(R_f\right)\left(\widetilde{R}_L - R_f\right)\right] + E\left[u''\left(R_f\right)\left(\widetilde{R}_L - R_f\right)\left(\widetilde{R}_L - R_f\right)z_L^*\right] \cong 0.$$

For a sufficiently small risk premium

$$z_L^* \cong -\frac{u'\left(R_f\right)\left(E\widetilde{R}_L - R_f\right)}{u''\left(R_f\right)E\left[\left(\widetilde{R}_L - R_f\right)^2\right]} = \frac{\left(E\widetilde{R}_L - R_f\right)}{-\frac{u''\left(R_f\right)}{u'\left(R_f\right)}Var\left(\widetilde{R}_L\right)},$$

but for constant absolute risk aversion, $u\left(x\right) = -e^{-\alpha x}$, the coefficient of absolute risk aversion is $-\frac{u''\left(R_f\right)}{u'\left(R_f\right)} = \alpha$, and thus $z_L^* \cong \frac{E(\{1+\widetilde{r}_L\}) - \left(1+r_f\right)}{\alpha Var(\{\widetilde{r}_L\})}$. QED

Proof of Proposition 10

Part (i) is follow immediately form Equation (22).

Part (ii) The first derivative of $Var\left(\{\widetilde{R}_{L2}\}\right)$ with respect to p is

$$\frac{\partial Var\left(\{\widetilde{R}_{L2}\}\right)}{\partial q} = -2\left(1-p\right)\left(1-q\right)\left(1-q_1\right)^2\left(q + 2qq_1 + q_1^2\right),$$

which is negative since $p, q \in [0,1]$ are probabilities.

Part (iii) is follow immediately form Equation (22). QED

Proof of Proposition 11

Writing Equation (26) as

$$z_L^*\left(\pi, r_L, q, q_1\right) \cong \frac{E\left(\{\widetilde{R}_L\}\right) - \left(1+r_f\right)}{\alpha Var\left(\{\widetilde{R}_L\}\right)} = \frac{\lambda_1\left(1+\widetilde{r}_L\right) - \left(1+r_f\right)}{\alpha\lambda_2\left(1+\widetilde{r}_L\right)^2} = \frac{1}{\alpha\lambda_2}y,$$

where λ_1 and λ_2 are determined by equations (22) and (**??**), respectively. Differentiating y with respect to $\widetilde{r}_L$ yields

$$\frac{\partial y}{\partial \widetilde{r}_L} = \frac{-\lambda_1\left(1+\widetilde{r}_L\right) + 2\left(1+r_f\right)}{\left(1+\widetilde{r}_L\right)^3}.$$

Since it is a probability $\lambda_1 \in [0,,1]$. Hence, the derivative is positive for $\frac{2(1+r_f)-\lambda_1}{\lambda_1} \geq 1+2r_f \geq \widetilde{r}_L$. QED

Proof of Proposition 12

Part (i): Differentiating equations (32) and (33) totally with respect to π yields

$$0 = (1-\theta_{UB})M_F \left[\frac{\partial L_F^*}{\partial r_B}\frac{dr_{B2}}{d\pi} + \frac{\partial L_F^*}{\partial r_L}\frac{dr_{L2}}{d\pi}\right], \tag{38}$$

$$(1-\theta_{UB})^2 M_F \left[\frac{\partial L_{F2}^*}{\partial r_B}\frac{dr_{B2}}{d\pi} + \frac{\partial L_{F2}^*}{\partial r_L}\frac{dr_{L2}}{d\pi}\right] = (1+r_{L1})M_L \left[\frac{\partial z_{L2}^*}{\partial r_L}\frac{dr_{B2}}{d\pi} + \frac{\partial z_{L2}^*}{\partial \pi}\right]. \tag{39}$$

Solving this two equations system for $\frac{dr_{B2}}{d\pi}$ and $\frac{dr_{L2}}{d\pi}$

$$\frac{dr_{B2}}{d\pi} = -\frac{\frac{\partial L_{F2}^*}{\partial r_L}\frac{\partial z_{L2}^*}{\partial \pi}}{\frac{\partial L_{F2}^*}{\partial r_B}\frac{\partial z_{L2}^*}{\partial r_L}}, \tag{40}$$

$$\frac{dr_{L2}}{d\pi} = -\frac{\frac{\partial z_{L2}^*}{\partial \pi}}{\frac{\partial z_{L2}^*}{\partial r_L}}. \tag{41}$$

By Proposition 7, $\frac{\partial L_{L2}^*}{\partial r_B} > 0$ and $\frac{\partial L_{L2}^*}{\partial r_L} < 0$. By Proposition 10 and 11, $\frac{\partial z_{L2}^*}{\partial \pi}$ and $\frac{\partial z_{L2}^*}{\partial r_L}$ are both positive. Utilization of those sign restrictions in equations (40) and (41) implies that the general equilibrium effects of a surprise decrease in π is to raise both r_{B2} and r_{L2} above what those rates had been expected to be in period 0 (r_{B2}^e and r_{L2}^e).

Part (ii): Although Assumption 2 requires that $E\left[\widetilde{Y}_P \mid I_1 \cap RB\right] = 2y([q+q_1-1] > r_B^e \equiv (1+r_{B1})(1+r_{B2}^e)-1$ the condition in part (ii) of this proposition implies that it is violated when r_B^e is replaced with r_B, or in explicit notation

$$E\left[\widetilde{Y}_P \mid I_1 \cap RB\right] = 2y([q+q_1-1] < r_B \equiv (1+r_{B1})(1+r_{B2})-1. \tag{42}$$

By equation (4) adapted to **actual** period's 1 information a RB is solvent in period 1 if and only if

$$L_B(1+r_B^e) \leq (1+L_B)\left[1+E\left[\widetilde{Y}_P \mid I_1 \cap RB\right]\right],$$

which is equivalent to

$$L_B \leq \frac{1 + 2y([q + q_1 - 1]}{r_B - 2y([q + q_1 - 1]} \equiv L_B^1(r_{B1}), \tag{43}$$

where the denominator is positive by condition (42). It follows that regular borrowers do not get refinancing in period 1 if

$$L_B^* = L_B^H = \frac{1}{r_B^e} > \frac{1 + 2y([q + q_1 - 1]}{r_B - 2y([q + q_1 - 1]} \equiv L_B^1(r_{B1}). \tag{44}$$

Rearrangement of this inequality reveals that it is equivalent to the condition in part (ii) of the proposition establishing that RB default. Given that RB default under r_{B2} UB default afortiori.

(iii) When r_{B2} increases sufficiently beyond the bound in part (ii) even LB are denied access to credit inducing a total drying up of refinancing to borrowers. QED

Proof of Proposition 13

To be completed. QED

Proof of Proposition 14

To be completed. QED

Proof of Proposition 15

To be completed. QED

Proof of Proposition 16

To be completed. QED