Sharing the Burden: International Policy Cooperation in a Liquidity Trap *

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April, 2011: Preliminary

Abstract

A collapse in aggregate demand concentrated in one country may have international spillover effects which lead 'natural real interest rates' to fall below zero in all countries. We explore the optimal policy response to this type of shock, when governments cooperate on both fiscal and monetary responses. The main focus of the analysis is to explore how both the transmission of shocks, and the optimal policy responses are affected by the degree of trade openness of each economy. With fully integrated trade, both countries enter a liquidity trap simultaneously, and the optimal policy response is to have identical, expansionary fiscal packages. When trade is less than fully open, the source country is worst hit by the shock. Then the optimal policy response is to have fiscal expansion in that country, a much smaller expansion in the foreign country, combined with relatively tight monetary policies in the foreign country. Strikingly, the foreign country may choose to have a positive policy interest rate, even though its 'natural real interest rate' is below zero. Thus, the optimal policy response to a global liquidity trap may differ substantially from the standard closed economy prescription.

Keywords:Liquidity Trap, Monetary Policy, Fiscal Policy, International Spillovers

JEL: E2, E5, E6

*This paper was prepared for the 23rd TRIO Conference, University of Tokyo, December 16-17, 2010. Devereux thanks SSHRC, the Bank of Canada, and the Royal Bank of Canada for financial support. The opinions in this paper are those of the authors alone and cannot be ascribed to the Bank of Canada.

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1 Introduction

This paper is concerned with global policy responses to a world liquidity trap. The macroeconomic situation of the world economy was profoundly altered by the experience of the Great Recession that began in 2008. By general consensus, the source of the shock was the US financial sector, but this subsequently led to a fall in world aggregate demand, spilling over to the economies of many other countries. How should policymakers respond when aggregate demand shocks push world 'natural real interest rates' below zero? As is well known (e.g. Eggertson and Woodford 2003), when desired real interest rates are below zero, there is a failure of the 'divine coincidence' that monetary policy can simultaneously deliver zero inflation and a zero output gap. Of course, even when policy interest rates are at a lower bound, monetary policy may still be effective through an expectations channel, but the effectiveness of announcements about future monetary policy is questionable, given the implausibility of 'committing to be irresponsible' in the future (Krugman 1998). An alternative is to use fiscal policy. In the aftermath of the crisis, many countries followed significant expansions in government deficits, reducing taxes and/or increasing government spending. At the beginning of the downturn, there was a concerted effort to coordinate these fiscal expansion across countries, through the G20 process and other venues. But the ensuing fiscal responses were far from uniform across different countries. In addition, some countries have already begun to raise policy rates, while in the US, interest rates remain effectively at their zero bound.

While there has been a significant growth in research on the economics of liquidity traps in both open and closed economy settings, to date there has been little investigation of the global dimension of optimal policy responses to large macro shocks which push one or more countries towards the zero lower bound[1]. In particular, a key question is how the 'burden of adjustment' to a global recession should be shared across countries that experience the downturn at different levels of severity.

This specific focus of the paper is to identify an optimal policy response to a world liquidity trap in which two trading partners are well integrated through financial markets but less than perfectly integrated in goods markets. We think of an aggregate demand shock as coming from one country, but spilling over into other countries by pushing down desired real interest rates below zero in all countries. A policy response in our model is a joint monetary-fiscal package, and we focus on cooperative policy optima. We emphasize that a liquidity trap is not a mechanical occurance, but a decision to reduce policy rates to zero when

[1] References are discussed below.

the natural real interest rate goes below zero. In this respect, the international dimension to macroeconomic policy at the zero lower bound introduces some intriguing complications. The particular complicating feature is the degree of trade integration. With highly open trade linkages, the optimal policy response closely mirrors that of a closed economy. Policy interest rates are set at zero, and both economies should follow similarly expansionary fiscal packages. The reason is that when international trade is highly integrated, a demand slump in one country is felt equally in all other countries, via interconnected goods and financial markets. Output and inflation in all countries will respond symmetrically to demand shocks, regardless of the source of the shock, and the optimal policy response is to have interest rates as low as possible, and an equal fiscal expansion in all countries.

However, the benchmark of fully open trade does not closely approximate the current configuration of the world economy, where large, but relatively closed economies, such as Japan and the United States, are stuck in a liquidity trap. In these countries, exports make up substantially less than 20% of GDP. With home bias in consumption baskets, which acts so as to reduce trade linkages between countries, both the propagation of demand shocks and the optimal response of policy to shocks takes on very different characteristics. Typically, a large negative demand shock in one country will push down the desired real interest rate in that country more than those of its trading partners. Moreoever, the introduction of home bias complicates the analysis of optimal policy, since the international effects of a demand shock on output and inflation are not distributed equally across countries.

Initially, we show analytically that, after a negative demand shock that is large enough, the *average* world output gap declines, the average level of the *world* interest rate is set to zero, and the average *world* optimal fiscal policy response should be expansionary. In fact the decline in the average world output gap and the optimal world fiscal policy response are invariant to home bias, and precisely equal to that observed in a closed economy.

However, the distribution of the effects of the shock among countries is less clear. With substantially home bias, the decline in global interest rates that occur during the slump may actually be expansionary for global trading partners. The reason is that, when interest rates are at a zero bound, aggregate demand shocks tend to drive the terms of trade in the 'wrong direction'. The intuition is that a fall in aggregate demand will generate relatively higher disinflation in the source country. When nominal interest rates cannot respond to inflation, a persistent fall in inflation will raise relatively real interest rates for the source country, and lead to terms of trade *appreciation,* generating an expenditure switching of world demand away from the source country. The terms of trade movement thus tends to *exacerbate* rather than mitigate country specific demand shocks in a global liquidity trap.

In such a case, the optimal policy response for the foreign country will be to combine very slight fiscal expansion with monetary *contraction*. That is, the least hit (foreign) economy should only minimally engage in a cooperative fiscal expansion, but should raise its policy rates. Strikingly, we find that the best policy (from a global cooperative perspective), is for the foreign country to tighten monetary policy, even though using the standard criterion from the closed economy logic, it should still be in a liquidity trap (where its 'natural' real interest rate is below zero). The logic behind this policy is that a foreign interest rate increase acts so as weaken the appreciation of the home terms of trade caused by the original demand shock, limiting the degree of world expenditure switching away from the home economy. Overall, it is best for both countries to have higher interest rates in the trading partner, when the source country shock requires zero home interest rates.

Our results in fact show that the response of policy interest rates in a global liquidity trap are piecewise functions of the degree of trade-openness, as measure by the parameter of 'home bias' in preferences. When preferences are identical, trade is fully open, and a global liquidity trap is associated with zero policy rates in all countries. For a shock coming from the home country, home policy rates are always set equal to zero. As preferences display more home bias, both policy rates are still zero for some interval. But at a critical threshold level of home bias, foreign interest rates are raised, even when the foreign natural real interest rate is negative. As the degree of home bias rises, foreign policy rates rise more and more, and are always set above the foreign natural real interest rate.

The message is that the open economy dimension has very substantial implications for both the occurance of a liquidity trap, in the sense that it predicts that policy is not restricted by the zero lower bound even when traditional indicators (which look at the value of the 'natural real interest rate') say that it should be, and for the way in which policy is designed when the world economy 'on average' is in a liquidity trap. More generally, the model predicts that the 'burden of adjustment' to a global liquidity trap may be spread quite unequally across countries, and implies some apparently counterintuitive policy responses.

The paper builds on a substantial recent literature on monetary and fiscal policy in a liquidity trap. In particular, with the experience of Japan in mind Krugman (1999), Eggertson and Woodford (2003, 2005), Jung et al. (2005), Svensson (2003), Auerbach and Obstfeld (2004) and many other writers explored how monetary and fiscal policy could be usefully employed even when the authorities have no further room to reduce short term nominal interest rates. Recently, a number of authors have revived this literature in light of the very similar problems now encountered by the economies of Western Europe and North America. Papers by Christiano et al (2009), Devereux (2010), Eggertson (2009), Taylor et al. (2008) have explored the possibility for using government spending expansions, tax cuts, and monetary policy when the economy is in a liquidity trap. For the most part, these papers did not focus on the international dimension of liquidity traps. Some recent expections are Fujiwara et al. (2009, 2010), Erceg et al. (2009) and Jeanne (2009). Jeanne (2009) examines a 'global liquidity trap' in a model of one-period ahead pricing similar to that of Krugman (2009). Erceg. et al (2009) use a fully specific two country DSGE model to examine the international transmission of shocks when one country is in a liquidity trap, but do not focus on optimal monetary policy or fiscal policy choices. Fujiwara et al. (2009) examine the optimal monetary problem with commitment in a multi country situation, but do not examine the determination of fiscal policy, or the transmission of demand shocks across countries. Fujiwara et al. (2010) look at the impact of the international effects of fiscal policy in a liquidity trap, examing the sign and size of domestic and international fiscal multipliers. Our paper may be seen as complementary to theirs in that we extend the analysis to incorporate trade frictions, but more importantaly, investigate the determination of optimal policy[2].

The rest of the paper is organized as follows. The next section develops the basic model. Section 3 examines the solution under sticky prices. Then in section 4 we analyze the impact of fiscal policies at the zero lower bound, and the role of international spillovers of policies. Section 5 examines the optimal policy making problem in a global cooperative agreement, including the possibility of using both monetary and fiscal policy for the least affected countries. Some conclusions are then offered.

2 A two country model of interacting monetary and fiscal policy

We construct a model in which there are two countries in the world economy. In each country, households consume both private and government goods, and supply labour. Denote the countries as 'home' and 'foreign', with foreign variables denoted with an asterisk superscript. The population of each country is normalized to unity. Each country produces a range of differentiated goods. Complete asset markets allow full insurance of consumption risk across countries. Households also hold their own country's nominal government bonds. Firms produce private goods, while governments produce government goods which are distributed

[2] In addition, a previous paper (Cook and Devereux, 2010a) examines the linkages of natural real interest rates, the determination of fiscal multipliers and optimal fiscal policy in a simpler version of the model of the present paper, but does not allow for the endogenous response of monetary policy.

uniformly across households. Firms production and supply is constrained by sticky prices. Governments have access to lump sum taxation.

2.1 Households

Utility of a representative infinitely lived home household evaluated from date 0 is:

$$U_t = E_0 \sum_{t=0}^{\infty} (\beta)^t (U(C_t, \xi_t) - V(N_t) + J(G_t)) \tag{1}$$

where U, V, and J represent the utility of the composite home consumption bundle C_t, disutility of labour supply N_t, and utility of the government supplied public good G_t,respectively. The variable ξ_t represents a shock to preferences or 'demand' . We assume that $U_{12} > 0$..

Composite consumption is defined as

$$C_t = \Phi C_{Ht}^{v/2} C_{Ft}^{1-v/2}, \quad v \geq 1$$

where $\Phi = \left(\frac{v}{2}\right)^{\frac{v}{2}} (1 - \left(\frac{v}{2}\right))^{\frac{v}{2}}$, C_H is the consumption of the home country composite good by the home household, and C_F is consumption of the foreign composite good. If $v > 1$ then there is a home preference bias for domestic goods. The case v>1 is most realistic for thinking about policy in large open economies.

Consumption aggregates, C_H and C_F are composites, defined over a range of home and foreign differentiated goods, with elasticity of substitution θ between goods, so that:

$$C_H = \left[\int_0^1 C_H(i)^{1-\frac{1}{\theta}} di\right]^{\frac{1}{1-\frac{1}{\theta}}}, \qquad C_F = \left[\int_0^1 C_F(i)^{1-\frac{1}{\theta}} di\right]^{\frac{1}{1-\frac{1}{\theta}}}, \quad \theta > 1.$$

Price indices for home and foreign consumption are:

$$P_H = \left[\int_0^1 P_H(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}, \qquad P_F = \left[\int_0^1 P_F(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}},$$

while the aggregate (CPI) price index for the home country is $P = P_H^{v/2} P_F^{1-v/2}$ and for the foreign is $P_t^* = P_F^{*v/2} P_H^{*1-v/2}$

Demand for each differentiated good ($j = H, F$) is

$$\frac{C_j(i)}{C_j} = \left(\frac{P_j(i)}{P_j}\right)^{-\theta}$$

The law of one price holds for each good so $P_j(i) = SP_j^*(i)$.where S_t is the nominal exchange rate (home price of foreign currency). Relative demand for the composites is:

$$\frac{C_H}{C_F} = \frac{P_F}{P_H} = \frac{SP_F^*}{P_H}$$

Home government spending falls on the home composite good and foreign government spending on the foreign composite good. Thus, government spending is assumed to have full 'home bias'. I addition, we assume that government spending demand for each variety of home goods has price elasticity θ, the same as that for private spending.

The household's implicit labour supply at nominal wage W_t is:

$$U_C(C_t, \xi_t)W_t = P_t V'(N_t). \tag{2}$$

Optimal risk sharing implies

$$U_C(C_t, \xi_t) = U_C(C_t^*, \xi_t^*)\frac{S_t P_t^*}{P_t} = U_C(C_t^*, \xi_t^*)T_t^{v-1}, \tag{3}$$

.Nominal bonds pay interest, R_t. Then the Euler equation is:

$$\frac{U_C(C_t, \xi_t)}{P_t} = \beta R_t E_t \frac{U_C(C_{t+1}, \xi_{t+1})}{P_{t+1}}. \tag{4}$$

Foreign household preferences and choices can be defined exactly symmetrically. The foreign representative household has weight $v/2$, $(1-v/2)$ on the foreign (home) compositive good in preferences.

2.2 Firms

Each firm i employs labor to produce a differentiated good.

$$Y_t(i) = N_t(i),$$

Profits are $\Pi_t(i) = P_{Ht}(i)Y_t(i) - W_t H_t(i)\frac{\theta-1}{\theta}$ indicating a subsidy financed by lump-sum taxation to eliminate steady state first order inefficiencies. Each firm re-sets its price according to Calvo pricing with probability of adjusting prices equal to $1-\kappa$. Firms that adjust their price set new price given by $\widetilde{P}_{Ht}(i)$:

$$\widetilde{P}_{Ht}(i) = \frac{E_t \sum_{j=0} m_{t+j}\kappa^j \frac{W_{t+j}}{A_{t+j}} Y_{t+j}(i)}{E_t \sum_{j=0} m_{t+j}\kappa^j Y_{t+j}(i)}. \tag{5}$$

where stochastic discount factor $m_{t+j} = \frac{P_t}{U_C(C_t,\varepsilon_t)} \frac{U_C(C_{t+j},\xi_{t+j})}{P_{t+j}}$. In the aggregate, the price index for the home good then follows the process given by:

$$P_{Ht} = [(1-\kappa)\widetilde{P}_{Ht}^{1-\theta} + \kappa P_{Ht-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{6}$$

The behaviour of foreign firms and the foreign good price index may be described analogously.

2.3 Market Clearing

Equilibrium in the market for good i as

$$Y_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} \left[\frac{v}{2}\frac{P_t}{P_{Ht}}C_t + (1-\frac{v}{2})\frac{S_t P_t^*}{P_{Ht}}C_t^* + G_t\right],$$

where G_t represents total home government spending. Aggregate market clearing in the home good is:

$$Y_{Ht} = \frac{v}{2}\frac{P_t}{P_{Ht}}C_t + (1-\frac{v}{2})\frac{S_t P_t^*}{P_{Ht}}C_t^* + G_t. \tag{7}$$

Here $Y_{Ht} = V_t^{-1}\int_0^1 Y_{Ht}(i)di$ is aggregate home country output, where we have defined $V_t = \int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} di$. It follows that home country employment (employment for the representative home household) is given by $N_t = \int_0^1 N(i)di = Y_{Ht}V_t$.

The aggregate market clearing condition for the foreign good is

$$Y_{Ft} = \frac{v}{2}\frac{P_t^*}{P_{Ft}^*}C_t^* + (1-\frac{v}{2})\frac{P_t}{S_t P_{Ft}^*}C_t^* + G_t^*, \tag{8}$$

where: $N_t^* = \int_0^1 N_t^*(i)di = Y_{Ft}V_t^*$, where $V_t^* = \int_0^1 \left(\frac{P_{Ft}^*(i)}{P_{Ft}^*}\right)^{-\theta} di$.

An equilibrium in the world economy with positive nominal interest rates may be described by the equations (3), and (2), (4), (5) (6) and (**??**) for the home and foreign economy, as well as (7) and (8). For given values of V_t and V_t^*, and given government spending policies, these equations determine an equilibrium sequence for the variables $C_t, C_t^*, W_t, W_t^*, S_t, P_{Ht}$, $P_{Ft}^*, \widetilde{P}_{Ht}, \widetilde{P}_{Ft}^*$, R_t, R_t^*, and N_t, N_t^*.

3 New Keynesian Open Economy Model

3.1 Demand Shocks and Natural Interest Rates

Define $\sigma \equiv -\frac{U_{CC}\overline{C}}{U_C}$ as the inverse of the elasticity of intertemporal substitution in consumption, $\phi \equiv -\frac{V''\overline{H}}{V'}$ as the elasticity of the marginal disutility of hours worked and $\sigma_g \equiv -\frac{J''\overline{G}}{J'}$ as the elasticity of marginal utility of public goods. In addition, we assume that $\sigma_g = \sigma > 1$. Finally, $\varepsilon_t = \frac{U_{C\xi}}{U_C}\ln(\xi_t)$ is the measure of a positive demand shock in the home country, with an equivalent definition for the foreign country. Define $c_y = \frac{C}{Y}$ is the steady state share of consumption in output.We examine the effects of once and for all demand shocks which have a probability $1-\mu$ of reverting to mean.

For any variable x_t, define the world average and world relative level, $x_t^W = \frac{x_t + x_t^*}{2}$ and $x_t^R = \frac{x_t - x_t^*}{2}$. Solving a first order approximation of the flexible price, zero inflation version of the above economy, we can derive the Wicksellian (or 'natural') real interest rate of the home and foreign economy, $\widetilde{r}_t$ as a function of the demand shocks as follows[3].

$$\widetilde{r}_t = \overline{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W + \frac{\phi c_y (v-1)}{\Delta} \varepsilon_t^R \right)(1-\mu) \tag{9}$$

The foreign efficient nominal interest rate is:

$$\widetilde{r}_t^* = \overline{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W - \frac{\phi c_y (v-1)}{\Delta} \varepsilon_t^R \right)(1-\mu) \tag{10}$$

where $\Delta \equiv \phi c_y D + \phi(1-c_y) + \sigma$ and $\sigma > D \equiv (\sigma v(2-v) + (1-v)^2) > 1$. The way to interpret (9) and (10) is as the Fisherian, consumption based real interest rates that would obtain in an economy with fully flexible prices and no other distortions. These are critical variables for our analysis, since they govern the degree to which monetary policy can be efficiently employed to stabilize the economy. In particular, our model has the characteristic that when (9) and (10) are both positive, then monetary policy can perfectly achieve the joint target of zero inflation and zero output gaps, since home and foreign policy rates can simply be set to equal (9) and (10), respectively.

Note that in the no home bias case, when $v = 1$, then the natural interest rate for both economies should be the same. $\widetilde{r}_t = \widetilde{r}_t^* = \overline{r} + \left(\frac{\phi c_y}{\phi+\sigma}\varepsilon_t^W\right)(1-\mu)$. This is an example where the real exchange rate is constant, and given integrated financial markets, consumption-based real interest rates are equated across countries. But in fact, the case $v = 1$ is not particularly realistic. For most economies, and particlarly for large open economies, the lion's share of

[3]Note that this is defined as the value of $r_t - E_t\pi_{Ht+1}$ in a flexible price economy, or in other words, the PPI based real interest that would hold with flexible prices.

demand will come from the domestic economy, making the home bias case most relevant. We will therefore focus on the more general case where $v > 1$.

For concreteness, we also look at the case where the home country is the source of the shocks. In particular, we will assume that home consumers are affected by preference shocks which affect their propensity to save, whereas consumers in the foreign economy is not directly affected by these shocks. Of course foreign consumers will be indirectly affected by the shock, since integrated financial markets lead to linkages between interest rates. Thus, a saving shock with its source in the home economy, pushing the monetary authority into a liquidity trap, may have similar effects on the foreign economy, even though the foreign consumers are not directly affected by the shock.

Making this assumption, we have in this case, $\varepsilon_t^* = 0$ and $\varepsilon_t^W = \varepsilon_t^R = \frac{\varepsilon_t}{2}$ and we can write the home country natural rate as

$$\widetilde{r}_t = \overline{r} + \left(\frac{\Delta + (\phi + \sigma)(v - 1)}{(\phi + \sigma)\,\Delta} \right) (1 - \mu)\phi c_y \frac{\varepsilon_t}{2} \tag{11}$$

Then the foreign natural rate is:

$$\widetilde{r}_t^* = \overline{r} + \left(\frac{\Delta - (\phi + \sigma)(v - 1)}{(\phi + \sigma)\,\Delta} \right) (1 - \mu)\phi c_y \frac{\varepsilon_t}{2} \tag{12}$$

It is clear that, when there is home bias, $(v > 1)$, and then $\Delta + (\phi + \sigma)(v - 1) > \Delta - (\phi + \sigma)(v - 1) > 0$, so that the home economy natural interest rate falls by a greater amount with a negative ε_t shock.[4] But a home preference shock that is sufficiently large can still push the natural interest rate of both countries below zero, regardless of the degree of home bias.[5]

3.2 First Order Approximation

We derive a sticky price log-linear approximation of the model in terms of inflation and output gaps in a similar manner to Clarida et al. (2002) and Engel (2010). Let $\widehat{x}_t$ be the percentage deviation of a given variable x_t from the efficient zero flexible price equilibrium. Thus, $\widehat{x}_t$ is interpreted as a 'gap' variable. As defined before, $D \equiv \sigma v(2 - v) + (1 - v)^2 > 1$. In addition, let $s \equiv \frac{\sigma}{c_y}$, and $s > s_D \equiv \frac{s}{D} > 1$. In order to explore the implications of the zero

[4]Note that $\Delta - (\phi + \sigma)(v - 1) = \phi c_y(D - 1) + (\phi + \sigma)(2 - \nu)$

[5]However, it is also possible, in the case of strong home bias or in the case of a sufficiently small preference shock, that the natural interest rate of the home economy will be pushed below zero, while the foreign economy will retain a natural interest rate that will be positive.

lower bound constraint, we begin with the standard forward looking inflation equations and open economy IS relationships for the home and foreign economy.

Now we can write the home and foreign forward looking inflation equations in terms of gaps as:

$$\pi_{Ht} = k(\phi\widehat{n}_t + \frac{s_D}{2}\left[\widehat{ng}_t(1+D) + \widehat{ng}_t^*(D-1)\right]) + \beta E_t \pi_{Ht+1} \tag{13}$$

.

$$\pi_{Ft}^* = k(\phi\widehat{n}_t^* + \frac{s_D}{2}\left[\widehat{ng}_t^*(1+D) + \widehat{ng}_t(D-1)\right]) + \beta E_t \pi_{Ft+1}^* \tag{14}$$

Likewise, the home and foreign 'dynamic IS equations' are

$$\begin{aligned} & s_D E_t(\widehat{ng}_{t+1} - \widehat{ng}_t)(D+1) + s_D E_t(\widehat{ng}_{t+1}^* - \widehat{ng}_t^*)(D-1) \\ = \; & 2E_t\left(r_t - \widetilde{r}_t - \pi_{Ht+1}\right) \end{aligned} \tag{15}$$

$$\begin{aligned} & s_D E_t(\widehat{ng}_{t+1}^* - \widehat{ng}_t^*)(D+1) + s_D E_t(\widehat{ng}_{t+1} - \widehat{ng}_t)(D-1) \\ = \; & 2E_t\left(r_t^* - \widetilde{r}_t^* - \pi_{Ft+1}\right) \end{aligned} \tag{16}$$

where π_{Ht} and π_{Ft}^* are the inflation rates of the domestic and foreign composite goods, $\widehat{ng}_t = (\widehat{n}_t - (1-c_y)\widehat{g}_t)$ and $\widehat{ng}_t^* = (\widehat{n}_t^* - (1-c_y)\widehat{g}_t^*)$, and the coefficient k depends on the degree of price rigidity. Note that, approximated around the steady state, $\widehat{n}_t \approx \widehat{y}_t$, $\widehat{n}_t^* \approx \widehat{y}_t^*$, so the labor gap will stand in for the output gap.

If the natural interest rates of both economies are always above zero, then the monetary and fiscal authorities can achieve perfect price and output stability by setting the nominal interest rate equal to the natural real interest rate and keeping the fiscal gaps, $\widehat{g}_t$ and $\widehat{g}_t^*$ equal to zero. However, if one or both countries have a natural real interest rate below zero, this outcome is not possible. As we will see below, then one or both countries will be in a liquidity trap, where nominal rates are set at zero. As a result, there is a role for fiscal policy in stabilizing inflation and output gaps.

3.3 The World and Relative Economy

We can simplify the equations (13)-(16) by writing them in terms of world average and relative levels. The world average for global inflation is written as:

$$\pi_t^W = k(\phi + s)\widehat{n}_t^W - ks \cdot \widehat{cg}_t^W + \beta E_t \pi_{t+1}^W \tag{17}$$

$$sE_t(\widehat{n}_{t+1}^W - \widehat{n}_t^W) - sE_t(\widehat{cg}_{t+1}^W - \widehat{cg}_t^W) = E_t\left(r_t^W - \widetilde{r}_t^W - \pi_{t+1}^W\right) \tag{18}$$

The world 'relative' variables are written as:

$$\pi_t^R = k(\phi + s_D)\widehat{n}_t^R - ks_D\widehat{cg}_t^R + \beta E_t \pi_{t+1}^R \tag{19}$$

$$s_D E_t(\widehat{n}_{t+1}^R - \widehat{n}_t^R) - s_D E_t(\widehat{cg}_{t+1}^R - \widehat{cg}_t^R) = E_t\left(r_t^R - \widetilde{r}_t^R - \pi_{t+1}^R\right) \tag{20}$$

where $\widehat{cg}_t^W \equiv (1 - c_y)\widehat{g}_t^W$ and $\widehat{cg}_t^R \equiv (1 - c_y)\widehat{g}_t^R$.

If we abstract from fiscal gaps, (i.e. $\widehat{cg}_t^W = \widehat{cg}_t^R = 0$) we can rewrite these equations as follows:

$$\pi_t^W = k(\phi + s)\widehat{n}_t^W + \beta E_t \pi_{t+1}^W \tag{21}$$

$$sE_t(\widehat{n}_{t+1}^W - \widehat{n}_t^W) = E_t\left(r_t^W - \widetilde{r}_t^W - \pi_{t+1}^W\right) \tag{22}$$

$$\pi_t^R = k(\phi + s_D)\widehat{n}_t^R + \beta E_t \pi_{t+1}^R \tag{23}$$

$$s_D E_t(\widehat{n}_{t+1}^R - \widehat{n}_t^R) = E_t\left(r_t^R - \widetilde{r}_t^R - \pi_{t+1}^R\right) \tag{24}$$

We can clearly see that both systems of equations (for the world average and the world relative economies) are in the canonical form of the New Keynesian closed economy equations. The only difference comes in the parameterization of the inverse elasticity of consumption: s , in the case of the average economy; and, s_D, in the case of the relative world economy.

Note that $s_D < s$, so the world average level of demand is less sensitive to the average interest rate than the relative level of demand is sensitive to the relative interest rate. This reflects the expenditure switching effect of terms of trade changes. When worldwide interest rates are relatively low, then for intertemporal substitution reasons, world demand will be relatively high. But when there is a large gap in interest rates across country, again, through intertemporal substitution, demand will be relatively high in the low interest rate country. On top of that however, in order to satisfy interest rate parity, a relatively low real interest rate country must have an anticipated terms of trade appreciation. This implies a current terms of trade deterioration, leading world aggregate demand to move towards the low interest rate country through the expenditure switching channel.

4 Global Liquidity Traps

We now examine the impact of savings shocks in the home economy, which lead to changes in the home and foreign natural interest rates. Savings shocks only pose a policy problem when they push natural interest rates below the zero lower bound. When policy rates are free to adjust, they can perfectly offset these shocks. Despite this, we first explore the impact of savings shocks that do not lead interest rates to hit their zero bound, but when the policy makers in each country do not automatically offset the shocks, and instead follow a Taylor type interest rate rule. This comparison is revealing to the extent that it provides a contrast to the effect of shocks when interest rates are constrained by the zero lower bound.

We assume that a preference shock is unanticipated, and reverts back to zero with probability $1-\mu$ in each period. Because there are no predetermined state variables in the model, this implies that all variables in the world economy will inherit the same persistence as the shock itself, in expectation. Thus, for any variable x_t, we may write $E_t(x_{t+1}) = \mu x_t$. After the shock expires, all variables will then revert to their zero initial equilibrium.

4.1 Demand Shocks under a Taylor rule

Assume that the source of the demand shock is in the home country, and the movement of natural real interest rates is as in (9) and (10). But assume that, instead of offsetting the movement in natural real interest rates, policy interest rates follow a Taylor rule, such that:

$$r_t = r + \gamma\pi_{Ht}, \quad r_t^* = r + \gamma\pi_{Ft}$$

This discussion implicitly assumes that demand shocks are small enough to leave nominal interest rates above zero. Using () in the solutions for world and relative output gaps, gives us:

$$\begin{aligned} \Delta_1 \widehat{n}_t^W &= (1-\beta\mu)(\overline{r} - \widetilde{r}_t^W) \\ \Delta_1^D \widehat{n}_t^R &= (1-\beta\mu)\widetilde{r}_t^R \end{aligned}$$

where $\Delta_1 \equiv s(1-\beta\mu)(1-\mu) + (\gamma-\mu)k\,(\phi+s) > 0$, and $\Delta_1^D = s_D(1-\beta\mu)(1-\mu) + (\gamma-\mu)k\,(\phi+s_D) > 0$, with $\Delta_1 > \Delta_1^D$.

A demand shock in the home country ensures that $\overline{r} - \widetilde{r}_t^W > 0$ and $\widetilde{r}_t^R < 0$. Thus, both $\widehat{n}_t^W$ and $\widehat{n}_t^R$ fall. The home and foreign output gaps are written respectively as $\widehat{n}_t = \widehat{n}_t^W + \widehat{n}_t^R$, and $\widehat{n}_t^* = \widehat{n}_t^W - \widehat{n}_t^R$. Thus:

$$\widehat{n}_t = (1-\beta\mu)\left[\frac{-(\overline{r} - \widetilde{r}_t^W)}{\Delta_1} + \frac{\widetilde{r}_t^R}{\Delta_1^D}\right]$$

$$\widehat{n}_t^* = (1-\beta\mu)\left[\frac{-(\overline{r} - \widetilde{r}_t^W)}{\Delta_1} - \frac{\widetilde{r}_t^R}{\Delta_1^D}\right]$$

The home output gap falls. The response of the foreign output gap is ambiguous, however, and depends upon both the strength of the shock as well as the openness of total trade. When $v = 1$, $\widetilde{r}_t^R = 0$, and home and foreign output gaps fall by equal amounts. Note that the first term inside the square brackets in each equation is independent of v. Then as v rises above unity, $\widetilde{r}_t^R$ falls, Δ_1^D rises, so that the foreign output gap responds by less, and the home output gap by more.

The negative demand shock always reduces home country inflation. Foreign inflation is defined as $\pi_t^W - \pi_t^R$, which may be written as:

$$\pi_t^* = k\,(\phi+s)\,\widehat{n}_t^* + k\,(s - s_D)\,\widehat{n}_t^R$$

A sufficient condition for foreign inflation to fall is that the foreign output gap falls. But even if the foreign output gap rises, foreign inflation may still fall as a result of the reduction in the home output gap reducing demand and marginal cost in the foreign economy.

Finally, we may compute the impact of the demand shock on the terms of trade for the home economy. We may derive the terms of trade response in the following way. From interest rate parity, it must be that (up to a first order), we have:

$$r_t - E_t\pi_{Ht+1} = r_t^* - E_t\pi_{Ft+1} + E_t(\widehat{\tau}_{t+1} - \widehat{\tau}_t) \tag{25}$$

Now, using the assumption on persistence of all variables, and the Taylor rule, we may write the response of the current terms of trade as:

$$\tau_t = -2\frac{\gamma - \mu}{1 - \mu}\pi_t^R \tag{26}$$

Since π_t^R is negative, the terms of trade must depreciate. Hence, when interest rates are above their zero lower bound, and policymakers follow a Taylor rule, a negative demand shock in one country is associated with a depreciation in that country's terms of trade, which cushions the impact of the shock on inflation and the output gap.

4.2 Demand Shocks in a liquidity trap

We now focus on the impact of demand shocks which push one or both countries natural real interest rates below zero. We first define some notation. Let $\widetilde{r}(\varepsilon_t, v)$ and $\widetilde{r}^*(\varepsilon_t, v)$ be defined as the functional relationships between the home country demand shock, and the home bias parameter v, and the home and foreign natural real interest rates, respectively. In addition, for given v , define $\varepsilon_H(v) < 0$ as the value of the home demand shock for which $\widetilde{r}(\varepsilon_H, v) = 0$, and likewise $\varepsilon_F(v) < 0$ as the value of the shock such that $\widetilde{r}^*(\varepsilon_F, v) = 0$. From () and (), it is clear that $\varepsilon_H(v) \geq \varepsilon_F(v)$, with a strict inequality when $v > 1$. Since the home country is hit harder by the demand shock, it reaches a zero natural real interest rate for a smaller (negative) value of the shock.

As long as the demand shock satisfies $\varepsilon < \varepsilon_H(v)$, it is not possible for policy interest rates to fully offset the shocks in both countries. Then either one or both countries will be constrained by the zero lower bound on nominal interest rates. The next section examines the optimal policy response to a demand shock. Here, we explore the impact of the shock under that assumption that both the home and foreign monetary authorities follow the rule that policy interest rates are set to equal the natural real interest rates, or zero, whichever is greatest. Thus, assume that:

$$r_t = \max(0, \widetilde{r}_t), \qquad r_t^* = \max(0, \widetilde{r}_t^*) \tag{27}$$

This is a natural extension of the optimal monetary rule in the closed economy literature on the 'zero bound' (e.g. Eggertson and Woodford 2003, Jung et al. 2005). Interestingly, we see below that this is *not* generally the optimal monetary rule in a multi-country setting.

Assume that the shock satisfies $\varepsilon < \varepsilon_H(1)$. This implies that if countries were fully open, the shock would be enough to drive the world natural real interest rate below zero. The impact of the shock on home and foreign output gaps depends, for a given shock, on the actual value of v. We focus on two cases. In both cases, the home policy interest rate is zero, but the foreign policy rate is only zero for $v \leq v_F$. If $v > v_F$, then by rule (**??**), the foreign monetary authority will set $r_t^* = \widetilde{r}_t^*$.

Case 1. For $v \leq v_F$, we have

$$\widehat{n}_t = (1 - \beta\mu)\left[\frac{\widetilde{r}_t^W}{\Delta_2} + \frac{\widetilde{r}_t^R}{\Delta_2^D}\right]$$

$$\widehat{n}_t^* = (1 - \beta\mu)\left[\frac{\widetilde{r}_t^W}{\Delta_2} - \frac{\widetilde{r}_t^R}{\Delta_2^D}\right]$$

where $\Delta_2 \equiv s(1-\beta\mu)(1-\mu) - \mu k\,(\phi + s) > 0$, and $\Delta_2^D = s_D(1-\beta\mu)(1-\mu) - \mu k\,(\phi + s_D) > 0$, with $\Delta_2 > \Delta_2^D$.[6]

In this case, the home output gap must fall, while the foreign output gap may rise or fall, depending on the size of v.

Case 2. For $v > v_F$, we have $\widetilde{r}_t^W = \widetilde{r}_t^R = \frac{\widetilde{r}_t}{2}$. Then we get:

$$\widehat{n}_t = (1 - \beta\mu)\widetilde{r}_t\left[\frac{1}{\Delta_2} + \frac{1}{\Delta_2^D}\right]$$

$$\widehat{n}_t^* = (1 - \beta\mu)\widetilde{r}_t\left[\frac{1}{\Delta_2} - \frac{1}{\Delta_2^D}\right]$$

Again, the home output gap must fall. But in this case, the foreign output gap will always *rise*, because, from the definitions above, we have $\Delta_2 > \Delta_2^D$.

It is straightforward to show that a negative demand shock causes the output gap in the home economy to fall by more when the economy is in a liquidity trap than under a Taylor rule. As in the literature on the closed economy, a fall in demand during a liquidity trap causes a persistent fall in inflation, which, given no adjustment in the nominal interest rate, causes a rise in the real interest rate, which causes a further fall in demand. So long as $\Delta_2 > 0$, this process converges when output falls by a sufficient amount.

In the open economy, however, there is a further effect at work. The fall in relative home country expected inflation leads to a rise in the home real interest rate, relative to the foreign real interest rate. In case 1 above, neither country's policy interest rate responds. By condition (25), this requires an anticipated terms of trade depreciation for the home country. Since the shock is temporary, an anticipated terms of trade depreciation can only

[6]These terms must be positive in order that the equilibrium be determinate. This puts a limit on the degree of persistence of the demand shock.

be satisfied by an immediate terms of trade appreciation. Thus, the home country terms of trade must *appreciate*. The analogue of condition (26) under a liquidity trap in both countries is thus:

$$\tau_t = 2\frac{\mu}{1-\mu}\pi_t^R$$

Since in this case, $\pi_t^R < 0$, the home country terms of trade appreciates. Thus, in a liquidity trap, relative prices move in the 'wrong direction', leading to a further fall in demand for home goods, following the initial negative demand shock. This appreciation helps to explain why the cross country spillover impact of a negative demand shock may be positive.

In case 2, the appreciation in the terms of trade of the home country is diminished by the increase in the foreign interest rate. The terms of trade response is described as:

$$\tau_t = 2\frac{\mu}{1-\mu}\pi_t^R - \frac{\widetilde{r}_t^*}{1-\mu}$$

The first term is again negative, but the second term is positive. In general, this can go in either direction. But in the quantitative analysis below, we see that, even in the case where the foreign central bank adjusts the policy rate when $\widetilde{r}_t^* > 0$, the home terms of trade still appreciates.

4.3 Fiscal Multiplier

When both countries are in a liquidity trap, fiscal spending policies can be used to reduce output gaps and eliminate deflationary pressures. Consider the world fiscal multiplier in a global liquidity trap. For convenience define $\Delta_3 = s(1-\beta\mu)(1-\mu) - \mu k(s) > \Delta_3^D = s_D(1-\beta\mu)(1-\mu) - \mu k(s_D)$. Also note that $\Delta_3 > \Delta_2$, $\Delta_3^D > \Delta_2^D$ and $\Delta_3 = D\Delta_3^D$.

We can then combine equations (17) and (19) to get:

$$\Delta_2 \widehat{n}_t^W - \Delta_3 \widehat{cg}_t^W = (1-\beta\mu)\widetilde{r}_t^W,$$

so that:

$$\widehat{n}_t^W = \frac{\Delta_3}{\Delta_2}\widehat{cg}_t^W + \frac{(1-\beta\mu)}{\Delta_2}\widetilde{r}_t^W \tag{28}$$

From (28) , we have (noting that output and employment gaps are equivalent in this model) $\frac{d\widehat{y}^W}{d\widehat{g}^W} \approx \frac{\Delta_3}{\Delta_2}(1-c_y)$. But we can define $\frac{d\widehat{y}^W}{d\widehat{g}^W} = \frac{\frac{dY^W}{Y}}{\frac{dG^W}{G}} = \frac{dY^W}{dG^W}\frac{G^W}{Y^W} = \frac{dY^W}{dG^W}(1-c_y)$. Thus, the world government spending multiplier is given by $\frac{dY^W}{dG^W} = \frac{\Delta_3}{\Delta_2}$. Note that by the definition of $\Delta_2$and Δ_3, the world multiplier is independent of the degree of home bias, ν. In the extreme of $\nu = 2$, with complete home bias we would be dealing with two closed economies. However, equation (28) indicates that we can also view $\frac{\Delta_3}{\Delta_2}$ as equal to the multiplier in the closed economy . By definition $\Delta_3 > \Delta_2$ so the world, or closed economy multiplier is greater than one. From the forward looking inflation equation, we can show that an increase in government spending will have a positive effect on inflation. Thus, a persistent increase in government spending will have a positive effect on expected inflation. At the zero lower bound, this will reduce the real interest rate. If an expansion in world government spending reduces world real interest rates, this will stimulate demand beyond the direct spending of government.

We can also construct a parallel equation for the relative levels of the economies by combining (18) and (20), using

$$\Delta_2^D \widehat{n}_t^R - \Delta_3^D \widehat{cg}_t^R = (1-\beta\mu)\widetilde{r}_t^R,$$

so that:

$$\widehat{n}_t^R = \frac{\Delta_3^D}{\Delta_2^D}\widehat{cg}_t^R + \frac{(1-\beta\mu)}{\Delta_2^D}\widetilde{r}_t^R \tag{29}$$

Note that the *relative* government spending multiplier is also greater than one, $\frac{\Delta_3^D}{\Delta_2^D} > 1$, by the definition of $\Delta_3^D > \Delta_2^D$. That is, a rise in home relative to foreign government spending increases home relative to the foreign output gap more than one for one. The relative multiplier may be worth further examination. Noting that $Ds_D = s$, we have:

$$\begin{aligned}\frac{\Delta_3^D}{\Delta_2^D} &= \frac{D\Delta_3^D}{D\Delta_2^D} = \frac{Ds_D(1-\beta\mu)(1-\mu) - \mu k\left(Ds_D\right)}{Ds_D(1-\beta\mu)(1-\mu) - \mu k\left(D\phi + Ds_D\right)} \\ &= \frac{\Delta_3}{\Delta_2 - (D-1)\mu k\phi}\end{aligned}$$

Notice that the home bias term only enters through D. Remember $D = (\sigma v(2-v) + (1-v)^2) > 1$ so $\frac{dD}{dv} = -2(\sigma-1)(v-1) < 0$. Therefore, the greater is home bias, the smaller will be the relative multiplier. We can also see that the relative multiplier is larger than the world multiplier. A country that is doing a relatively large amount of government spending will have a relatively low interest rate, stimulating demand intertemporally. But the relatively low interest rate will also translate into a terms of trade depreciation, concentrating demand on the output of the country with high government spending. Then the high spending country will receive even higher demand through the expenditure switching effect, thus explaining why the relative multiplier is stronger than the world multiplier.

Now, we can examine how government spending in one country will affect both its own own output gap, and the output gap of its trading partner. Consider the effect of spending by the home economy alone, so that $\widehat{cg}_t^W = \widehat{cg}_t^R = \frac{\widehat{cg}_t}{2}$.

Proposition 1 *In a persistent global liquidity trap ($\mu > 0$), the multiplier of government spending on domestic output will be greater than one. The domestic output multiplier is a decreasing function of home bias; for $v < 2$, the domestic output multiplier is larger than the closed economy multiplier. The multiplier of government spending on foreign output is negative. This foreign multiplier approaches zero as $v \to 2$.*

Proof. . $\widehat{n}_t = \widehat{n}_t^R + \widehat{n}_t^W$. Add both sides of (28) and (29) to get

$$\begin{aligned}\widehat{n}_t^R + \widehat{n}_t^W &= \widehat{n}_t = \frac{\Delta_3}{\Delta_2}\frac{\widehat{cg}_t}{2} + \frac{\Delta_3^D}{\Delta_2^D}\frac{\widehat{cg}_t}{2} + ... \\ &= \left[\frac{1}{2}\frac{\Delta_3}{\Delta_2} + \frac{1}{2}\frac{\Delta_3^D}{\Delta_2^D}\right]\widehat{cg}_t\end{aligned}$$

The multiplier on domestic government spending is the average of the closed economy multipier and the relative multiplier. As the relative multiplier is larger than the closed economy multiplier and both are greater than 1, the multiplier is also greater than one. Also, since the relative multiplier is a declining function of home bias, so is the multiplier on domestic government spending.

Also the foreign output gap is defined as: $\widehat{n}_t^* = \widehat{n}_t^W - \widehat{n}_t^R$. Subtract (29) from (28) to get

$$\begin{aligned}\widehat{n}_t^W - \widehat{n}_t^R &= \widehat{n}_t^* = \frac{\Delta_3}{\Delta_2}\frac{\widehat{cg}_t}{2} - \frac{\Delta_3^D}{\Delta_2^D}\frac{\widehat{cg}_t}{2} + ... \\ &= \frac{1}{2}\left[\frac{\Delta_3}{\Delta_2} - \frac{\Delta_3^D}{\Delta_2^D}\right]\widehat{cg}_t\end{aligned}$$

The multiplier on foreign goverment spending is half the difference between the closed economy multiplier and the relative multiplier. Since the relative multiplier is larger, the multiplier on foreign government spending is negative. Also, since the relative multiplier gets smaller as v gets larger, the multiplier on foreign government spending gets closer to zero as home bias increases. ■

In addition to the standard intertemporal effects, in a liquidity trap, expansionary government spending will weaken the domestic currency, leading to enhanced demand beyond the closed economy multiplier. By the same token, this tends to reduced demand for foreign output through the expenditure switching effect. But the greater is home bias, the less important is the expenditure switching effect. As home bias increases, the domestic multiplier goes to the closed economy multiplier and the foreign multiplier will go to zero.

5 Optimal Monetary and Fiscal Policy

We now turn to the key part of the paper, which is the analysis of the optimal policy response to a liquidity trap shock. We explore optimal *cooperative* monetary and fiscal policy responses, where governments in each country cooperate on policies. While a complete analysis of the determination of fiscal and monetary policy in a global liquidity trap would also require an exploration of the strategic interaction between non-cooperative policy authorities, this raises difficult technical issues (see Benigno and Benigno 2005), and so is left as a topic for future research. Focusing on the cooperative problem is a desirable first approach, since it sets out a benchmark for choosing a policy so as to maximize world welfare in response to a negative demand shock that undermines the normal mechanism of monetary policy[7].

In order to analyze optimal policy, we first need to define an objective function. As shown in Cook and Devereux (2010a), a second order approximation to an equally weighted world social welfare can also be constructed in world averages and world differences.

$$\begin{aligned} V_t = & -(\widehat{n}_t^R)^2 \cdot \frac{A}{2} - (\widehat{n}_t^W)^2 \frac{B}{2} - (\widehat{cg}_t^R)^2 \cdot \frac{F}{2} - (\widehat{cg}_t^W)^2 \cdot \frac{H}{2} - J(\widehat{n}_t^R)(\widehat{cg}_t^R) \\ & -L(\widehat{n}_t^W)(\widehat{cg}_t^W) - \frac{\theta}{4k}(\pi_t^W + \pi_t^R)^2 - \frac{\theta}{4k}(\pi_t^W - \pi_t^R)^2 \end{aligned} \tag{30}$$

where

$$\begin{aligned} A & \equiv \left\{ \frac{(1+\phi c_y)}{c_y^2} + \frac{(\sigma - D)}{D}(1 + \frac{(1-c_y^2)}{c_y^2 D}) \right\} = \frac{(s_{DD}+\phi)}{c_y} \\ s_{DD} & \equiv \frac{(D-1)(1-c_y^2)}{c_y D} + \frac{(\sigma)}{D}(\frac{1+c_y^2(D-1)}{c_y D}) < s_D \\ B & \equiv \frac{(\sigma+\phi c_y)}{c_y^2} = \frac{(s+\phi)}{c_y}, \\ H & \equiv \frac{1}{(1-c_y)}\frac{\sigma}{c_y^2} = \frac{1}{(1-c_y)}\frac{s}{c_y} \quad L \equiv \frac{-\sigma}{c_y^2} = \frac{s}{c_y} \end{aligned}$$

[7]The cooperative approach to fiscal policy in a global liquidity trap is not necessarily unrealistic. In the immediate aftermath of the financial crash of 2008, the G20 group agreed on a joint policy response to the crisis which assigned target levels of fiscal stimulus to each member country.

$$
\begin{aligned}
J &\equiv \left[-\frac{1}{c_y^2} - \frac{(\sigma - D)}{c_y^2 D^2}(1 + (v-1)(D-1)c_y^2)\right] \\
F &\equiv \frac{((1-c_y) + c_y\sigma)}{(1-c_y)c_y^2} + \frac{(\sigma - D)}{c_y^2 D^2}(1 + (v-1)(D-1)c_y^2)
\end{aligned}
$$

In the analysis here, we will concentrate exclusively on optimal discretionary policy responses. This implies that governments cannot commit themselves to future policy actions, so all policies become history independent. As a result, when natural real interest rates are negative, and both countries have current policy rates at the zero bound, monetary policy becomes entirely ineffective, since policy makers cannot commit to expansionary monetary policy in the future, after natural real interest rates have risen again. In some cases, however, policymakers may choose not to set policy interest rates as close as possible to the natural real interest rates, as in policy rules (). We will allow for this possibility in the determination of optimal policy responses.

We begin by focusing only on cooperative discretionary monetary policy, assuming that all fiscal gaps are kept equal to zero. In the next subsection, we relax this restriction, and explore the optimal choice of both fiscal policy and monetary policy together.

5.1 Optimal Monetary Policy

When the fiscal gaps are all set equal to zero, the optimal cooperative policy problem under discretion is described by the Lagrangean:

$$
\begin{aligned}
\max_{\widehat{n}_t^R, \widehat{n}_t^W, \pi_t^W, \pi_t^R, r_t, r_t^*} L_t &= -(\widehat{n}_t^R)^2 \frac{A}{2} - (\widehat{n}_t^W)^2 \frac{B}{2} - \frac{\theta}{4k}(\pi_t^W + \pi_t^R)^2 - \frac{\theta}{4k}(\pi_t^W - \pi_t^R)^2 \\
&\quad + \lambda_{1t}\left[\pi_t^W - k(\phi + s)\widehat{n}_t^W - \beta E_t \pi_{t+1}^W\right] \\
&\quad + \lambda_{2t}\left[\pi_t^R - k(\phi + s_D)\widehat{n}_t^R - \beta E_t \pi_{t+1}^R\right] \\
&\quad + \psi_{1t}\left[s E_t(\widehat{n}_{t+1}^W - \widehat{n}_t^W) - E_t\left(\frac{r_t + r_t^*}{2} - \widetilde{r}_t^W - \pi_{t+1}^W\right)\right] \\
&\quad + \psi_{2t}\left[s_D E_t(\widehat{n}_{t+1}^R - \widehat{n}_t^R) - E_t\left(\frac{r_t - r_t^*}{2} - \frac{\widetilde{r}_t^R}{2} - \pi_{t+1}^R\right)\right] \\
&\quad + \gamma_{1t} r_t + \gamma_{2t} r_t^*
\end{aligned}
$$

The first two constraints are the inflation equations in average and relative terms. The second two constraints are the average and relative 'IS' equations. The final two constraints are the non-negativity constraint on the two policy interest rates. The policy optimum involves the choice of the output gaps,the inflation rates and interest rates to maximize this Lagrangean. The first order conditions of the maximization are:

$$-A\widehat{n}_t^R = \lambda_2 k(\phi + s_D) + s_D\psi_2 \tag{31}$$

$$-B\widehat{n}_t^W = \lambda_1 k(\phi + s) + s\psi_1 \tag{32}$$

$$k\lambda_1 = \theta\pi_t^W \tag{33}$$

$$k\lambda_2 = \theta\pi_t^R \tag{34}$$

$$\psi_{2t} + \psi_{1t} = \gamma_{1t} \tag{35}$$

$$\psi_{1t} - \psi_{2t} = \gamma_{2t} \tag{36}$$

Together with the conditions (**??**)-(20), these equations determine the optimal policy solutions for the variables $n_t^R, n_t^W, \pi_t^R, \pi_t^W, r_t, r_t^*, \lambda_{1t}, \lambda_{2t}, \psi_{1t}, \psi_{2t}, \gamma_{1t}$, and γ_{2t}. Combining (33) and (34) with (31) and (32), we obtain the relationship between world and relative output gaps, inflation rates, and the multipliers ψ_{2t} and ψ_{1t}. Since the underlying demand shock is either a constant (negative) number, or zero, the solution for all variables during the period of the shock will be time invarianet. Hence we can drop the time notation. Thus:

$$-A\widehat{n}^R = \theta\pi^R(\phi + s_D) + s_D\psi_2 \tag{37}$$

$$-B\widehat{n}^W = \theta\pi^W(\phi + s) + s\psi_1 \tag{38}$$

Then, solving the conditions (**??**)-(20) under the assumption that the shock to the natural real interest rates will revert to zero with probability $1 - \mu$ per period, we have:

$$\pi^R(1 - \beta\mu) = k(\phi + s_D)\widehat{n}^R \tag{39}$$

$$s_D(\mu - 1)\widehat{n}^R = \frac{r - r^*}{2} - \widetilde{r}^R - \mu\pi^R \tag{40}$$

$$\pi^W(1 - \beta\mu) = k(\phi + s)\widehat{n}^W \tag{41}$$

$$s(\mu-1)\widehat{n}^W = \frac{r+r^*}{2} - \widetilde{r}^W - \mu\pi^W \tag{42}$$

From (39) and 40), we can derive the partial solution for the relative output gap as:

$$[s_D(1-\mu)(1-\beta\mu) - \mu k(\phi+s_D)]\,\widehat{n}^R = -(\frac{r-r^*}{2} - \widetilde{r}^R)(1-\beta\mu) \tag{43}$$

Likewise, the world output gap that solves (41) and (42) is

$$[s(1-\mu)(1-\beta\mu) - \mu k(\phi+s)]\,\widehat{n}^W = -(\frac{r+r^*}{2} - \widetilde{r}^W)(1-\beta\mu) \tag{44}$$

Now using (39) in (31) we get:

$$-\left[A(1-\beta\mu) + \theta k(\phi+s_D)^2\right]\widehat{n}_t^R = s_D\psi_2(1-\beta\mu) \tag{45}$$

and using (43) and (45) we arrive at:

$$\frac{[A(1-\beta\mu)+\theta k(\phi+s_D)^2]}{[s_D(1-\mu)(1-\beta\mu)-\mu k(\phi+s_D)]}(\frac{r_t-r_t^*}{2} - \widetilde{r}_t^R)(1-\beta\mu) = s_D\psi_2(1-\beta\mu) \tag{46}$$

Using the same procedure for the world average measures, we arrive at:

$$\frac{[B(1-\beta\mu)+\theta k(\phi+s)^2]}{[s(1-\mu)(1-\beta\mu)-\mu k(\phi+s)]}(\frac{r_t+r_t^*}{2} - \widetilde{r}_t^W)(1-\beta\mu) = s\psi_1(1-\beta\mu) \tag{47}$$

Using the definitions of A and B above, we may write (46) and (47) respectively as:

$$\frac{\left[\frac{(s_{DD}+\phi)}{c_y}(1-\beta\mu)+\theta k(\phi+s_D)^2\right]}{[s_D(1-\mu)(1-\beta\mu)-\mu k(\phi+s_D)]}(\frac{r_t-r_t^*}{2} - \widetilde{r}_t^R)(1-\beta\mu) = s_D\psi_2(1-\beta\mu) \tag{48}$$

$$\frac{\left[\frac{(s+\phi)}{c_y}(1-\beta\mu)+\theta k(\phi+s)^2\right]}{[s(1-\mu)(1-\beta\mu)-\mu k(\phi+s)]}(\frac{r_t+r_t^*}{2} - \widetilde{r}_t^W)(1-\beta\mu) = s\psi_1(1-\beta\mu) \tag{49}$$

Rewrite these conditions as:

$$\Omega_D(\frac{r_t-r_t^*}{2} - \widetilde{r}_t^R) = \psi_2 \tag{50}$$

$$\Omega(\frac{r_t+r_t^*}{2} - \widetilde{r}_t^W) = \psi_1 \tag{51}$$

where $\Omega_D \equiv \frac{\left[\frac{(s_D D+\phi)}{c_y}(1-\beta\mu)+\theta k(\phi+s_D)^2\right]}{[s_D(1-\mu)(1-\beta\mu)-\mu k(\phi+s_D)]}\frac{1}{s_D}$ and $\Omega \equiv \frac{\left[\frac{(s+\phi)}{c_y}(1-\beta\mu)+\theta k(\phi+s)^2\right]}{[s(1-\mu)(1-\beta\mu)-\mu k(\phi+s)]}\frac{1}{s}$. By the properties already defined above, it must be that $\Omega_D \geq \Omega$, with strict inequality when $v < 2$.

From (50) and (51), we can now characterize the jointly optimal monetary policy in terms of the properties of the policy interest rates r_t and r_t^*. The key question is to see the conditions under which either the home, the foreign, or both non-negativity conditions on interest rates are binding; i.e. what determines when the zero lower bound is reached for each country?

In the absence of the zero lower bound, the optimal policy that closes all gaps would entail $r = \widetilde{r}(\varepsilon, v)$, $r^* = \widetilde{r}^*(\varepsilon, v)$. Recall that ε_H is defined by $\widetilde{r}_t(\varepsilon_H, v) = 0$. Then, for any value of v, it is clear that the unconstrained optimal policy cannot be achieved for a demand shock that satisfies $\varepsilon < \varepsilon_H(v)$, since then either the home country or both countries cannot set policy rates equal to the natural interest rates.

5.2 Characteristics of the optimal policy

We now discuss the characteristics of the optimal policy problem. The critical information may be obtained from conditions (50) and (51), in conjunction with the characteristics of the natural real interest rates () and (). We assume that the shock to the home economy always satisfies $\varepsilon < \varepsilon_H(1)$ (for simplicity, we drop the time notation in this sub-section, because all variables are constant for the duration of a demand shock). This ensures that, if countries are identical, then both would be in a liquidity trap.

Note that, in a closed economy, or in the open economy with fully open trade (i.e. $v = 1$), the policy problem is simple; set the policy rate equal to the natural real interest rate, whenever this is above zero, otherwise set a zero policy rate. When $v > 1$, and countries are hit by differential shocks, however, the policy becomes substantially more complicated.

From (35), the home policy interest rate is zero whenever $\psi_1 + \psi_2 > 0$ and from (36) the foreign rate is zero when $\psi_1 - \psi_2 > 0$. In the case $v = 1$, $\widetilde{r}_t^R = 0$, and $\widetilde{r}_t^W < 0$. Setting $r_t = r_t^* = 0$ in (50) and (51), we find that $\psi_1 > 0$ and $\psi_2 = 0$, so that both constraints are binding. Thus $r_t = r_t^* = 0$ is a solution when $v = 1$.

More generally, define $J(\varepsilon, v) = -\Omega\widetilde{r}_t^W(\varepsilon, v) + \Omega_D\widetilde{r}_t^R(\varepsilon, v)$ as the value of $\psi_1 - \psi_2$ under the assumption that the zero bound constraints bind in both countries. If $J(\varepsilon, v) > 0$, then this conjecture is correct, since then the zero bound constraint binds in the foreign country, and it is easy to show also that $\psi_1 + \psi_2 > 0$, so that constraint binds in both the home country also. But if $J(\varepsilon, v) < 0$, then the ZLB constraint no longer binds for the foreign country (although it still binds for the home country).

Figure () illustrates the properties of the $J(\varepsilon, v)$ function. By the definition of $\widetilde{r}_t^W(\varepsilon, v)$ and $\widetilde{r}_t^R(\varepsilon, \overline{v})$, it must be that $\widetilde{r}_t^R(\varepsilon, 1) = 0$, so that $J(\varepsilon, 1) = -\Omega\widetilde{r}_t^W(\varepsilon, 1) > 0$, while $\widetilde{r}_t^W(\varepsilon, 2) > \widetilde{r}_t^R(\varepsilon, 2)$, so that $J(\varepsilon, 2) = -\Omega\widetilde{r}_t^W(\varepsilon, 2) + \Omega_D\widetilde{r}_t^R(\varepsilon, 2) < 0$, since from the definitions above, we know that $\Omega = \Omega_D$ when $v = 2$. Hence, by continuity, there exists a value $\overline{v}$ defined by the condition $J(\varepsilon, \overline{v}) = -\Omega\widetilde{r}_t^W(\varepsilon, \overline{v}) + \Omega_D\widetilde{r}_t^R(\varepsilon, \overline{v}) = 0$. The Appendix discusses the conditions on $J(\varepsilon, v)$ such that $\overline{v}$ is unique. We assume that these conditions are met[8].

Taking v such that $1 \leq v \leq \overline{v}$, and setting $r_t = r_t^* = 0$ in (50) and (51) implies that $J(\varepsilon, v) > 0$, which confirms the conjecture that both zero bound constraints are strictly binding, so both policy rates are zero. At $v = \overline{v}$, $J(\varepsilon, \overline{v}) = 0$, and the home constraint is strictly binding while for foreign constraint is just binding. For $\overline{v} < v \leq 2$, $J(\varepsilon, v) < 0$. Then the home country constraint is binding, but the foreign constraint is not binding. To see this, note that when $v > \overline{v}$, the conjecture $r_t = r_t^* = 0$ does not solve (50) and (51), since it leads to the implication that $J(\varepsilon, v) = -\Omega\widetilde{r}_t^W(\varepsilon, 1) + \Omega_D\widetilde{r}_t^R(\varepsilon, 1) < 0$, which implies a contradiction, because given $J(\varepsilon, v) < 0$,the foreign constraint cannot be binding. Then for $v \geq \overline{v}$, given that the foreign constraint is not binding, we set $\gamma_{2t} = 0$ in (36), which implies that $\psi_1 = \psi_2$. Using this condition, we set $r_t = 0$ in (50) and (51), and solve for the equilibrium foreign country interest rate as

$$r_t^* = \widetilde{r}^*(\varepsilon, v) - \frac{(\Omega_D - \Omega)}{\Omega_D + \Omega}\widetilde{r}(\varepsilon, v) > 0 \tag{52}$$

Note that for $v \geq \overline{v}$, this is strictly positive, since from the definition of $J(\varepsilon, v)$, we have $r_t^* = -\frac{2}{\Omega_D + \Omega}J(\varepsilon, v) > 0$, for $v \geq \overline{v}$. Moreover, the critical value $\overline{v}$ must satisfy $\overline{v} < v_F$. This is because, given the definition of the natural interest rates, it must be that $\widetilde{r}_t^W(\varepsilon, v_F) = \frac{\widetilde{r}_t(\varepsilon, v_F)}{2}$, and $\widetilde{r}_t^R(\varepsilon, \overline{v}) = \frac{\widetilde{r}_t(\varepsilon, v_F)}{2}$. Hence $J(\varepsilon, v_F) = -(\Omega - \Omega_D)\frac{\widetilde{r}_t(\varepsilon, v_F)}{2} < 0$, since $\Omega_D > \Omega$. Therefore, the foreign policy rate is strictly positive, for $v \geq \overline{v}$, even in the range $[\overline{v}, v_F]$, for which the foreign natural real interest rate is strictly negative.

We may summarize this discussion in the following proposition

Proposition 2 *For $\varepsilon < \varepsilon_H(1)$, there exists a critical value $\overline{v}$, such that (i) for $1 \leq v \leq \overline{v}$, $r_t = 0$ and $r_t^* = 0$, (ii), for $v > \overline{v}$, $r_t = 0$, and r_t^* is characterized as:*

$$r_t^* = \widetilde{r}^*(\varepsilon_t, v) - \frac{(\Omega_D - \Omega)}{\Omega_D + \Omega}\widetilde{r}(\varepsilon_t, v) > 0,$$

with $r_t^* > \widetilde{r}^*(\varepsilon_t, v)$, and $\overline{v} < v_F$.

Hence, given a shock that is sufficient to drive the home country natural real interest

[8] In extensive simulation we found no case where this condition was violated.

rate to zero for any value of v, the response of the foreign policy interest rate depends on the degree of trade openness. When home bias is relatively small, and trade is very open, both countries will have policy rates at the zero lower bound. But for greater home bias, the foreign country will choose to raise interest rates. Note that in the region $[\overline{v}, v_H]$ the foreign country raises interest rates even though its own natural real interest rate is negative. More generally, the foreign country always chooses an interest rate *strictly higher* than its own natural real interest rate.

This proposition makes it clear that the sense in which the two countries are in a liquidity trap is critically determined not by the fact that their respective natural real interest rates are negative, but by the strength of the shock and the size of the trade flows between the countries. Given the initial negative shock that is large enough, the home country will always choose to have a zero policy interest rate. But the optimal response of the foreign country depends on both v and the size of the shock. For a shock that would be large enough to drive the world natural real interest rate below zero in the fully open world economy (i.e. $\varepsilon < \varepsilon_H(1)$), the foreign country will also set the interest rate at the zero bound, if it is sufficiently open to trade with the home country ($v \leq \overline{v}$). But for $v > \overline{v}$, the foreign country will have a strictly positive interest rate. Note that for $\overline{v} < v < v_F$, the foreign country has a positive interest rate, even though the foreign natural real interest rate is below zero. Moreover, the proposition makes clear that the foreign interest rate is strictly above the foreign natural real interest rate for all $v < 2$.

A similar logic holds for different values of the demand shock, for a given $v > 1$. This leads to a trade-off between the size of home bias and the size of the shock in the assessement of whether a liquidity trap in one country spills over into another country. Figure 1 (missing in this draft) illustrates this. The Figure illustrates a downward sloping locus of points in $v - \varepsilon$ space. Above and to the right of the locus, the foreign country sets a positive policy rate higher than the foreign natural real interest rate. Below and to the left of the locus, the foreign country is constrained by the zero lower bound. Note that the locus become steeper has v increases, because the foreign country is less and less sensitive to foreign demand shocks, the higher is v. Literally, as v approaches 2, the required negative home demand shock that would put the foreign country into a liquidity trap becomes infinitely large.

5.3 Optimal Monetary and Fiscal Policy

We now examine the jointly optimal monetary and fiscal policy that would respond to a home negative demand shock. Again, the cooperative fiscal policy response to a liquidity trap involves maximizing (30) in each period, taking expectations of all future variables as given, subject to the inflation equations for world averages and differences, given by (17) and (19), and subject to the non-negativity constraints on nominal interest rates in each country. Since from the results of the previous section, we know that the non-negatively constraint on the home country policy rate will always bind for the duration of the shock, we only impose the non-negativity condition on the foreign interest rate.

Given this, we have the Lagrangean expression:

$$\begin{aligned}
\max_{\widehat{n}_t^R,\widehat{n}_t^W,cg_t^R,cg_t^W,\pi_t^W,\pi_t^R,r_t^*} L_t &= -(\widehat{n}_t^R)^2\cdot\frac{A}{2}-(\widehat{n}_t^W)^2\frac{B}{2}-(c\widehat{g}_t^R)^2\cdot\frac{F}{2}-(c\widehat{g}_t^W)^2\cdot\frac{H}{2} \\
&\quad -J(\widehat{n}_t^R)(\widehat{cg}_t^R)-L(\widehat{n}_t^W)(\widehat{cg}_t^W)-\frac{\theta}{4k}(\pi_t^W+\pi_t^R)^2-\frac{\theta}{4k}(\pi_t^W-\pi_t^R)^2 \\
&\quad +\lambda_{1t}\left[\pi_t^W-k(\phi+s)\widehat{n}_t^W+ks\cdot\widehat{cg}_t^W-\beta E_t\pi_{t+1}^W\right] \\
&\quad +\lambda_{2t}\left[\pi_t^R-k(\phi+s_D)\widehat{n}_t^R+ks_D\widehat{cg}_t^R-\beta E_t\pi_{t+1}^R\right] \\
&\quad +\psi_{1t}\left[sE_t(\widehat{n}_{t+1}^W-\widehat{n}_t^W)-sE_t(\widehat{cg}_{t+1}^W-\widehat{cg}_t^W)-E_t\left(\frac{r_t^*}{2}-\widetilde{r}_t^W-\pi_{t+1}^W\right)\right] \\
&\quad +\psi_{2t}\left[s_DE_t(\widehat{n}_{t+1}^R-\widehat{n}_t^R)-s_DE_t(\widehat{cg}_{t+1}^R-\widehat{cg}_t^R)-E_t\left(-\frac{r_t^*}{2}-\frac{\widetilde{r}_t^R}{2}-\pi_{t+1}^R\right)\right] \\
&\quad +\gamma_t\left[r_t^*\right]
\end{aligned}$$

The first two constraints are the inflation equations in average and relative terms. The second two constraints are the average and relative 'IS' equations. The final constraint is the non-negativity constraint on the foreign policy interest rate. The policy optimum involves the choice of the output gaps, the government spending 'gaps', the inflation rates and the foreign interest rate to maximize this Lagrangean. The first order conditions of the maximization are:

$$-A\widehat{n}_t^R-J(\widehat{cg}_t^R)=\lambda_2k(\phi+s_D)+s_D\psi_2\equiv\Upsilon_2 \tag{53}$$

$$-B\widehat{n}_t^W-L(\widehat{cg}_t^W)=\lambda_1k(\phi+s)+s\psi_1\equiv\Upsilon_1 \tag{54}$$

$$F\widehat{cg}_t^R+J(\widehat{n}_t^R)=ks_D\lambda_2+s_D\psi_2=\Upsilon_2-\phi(k\lambda_1) \tag{55}$$

$$H\widehat{cg}_t^W+L(\widehat{n}_t^W)=ks\lambda_1+s\psi_1=\Upsilon_2-\phi(k\lambda_1) \tag{56}$$

$$k\lambda_1=\theta\pi_t^W \tag{57}$$

$$k\lambda_2=\theta\pi_t^R \tag{58}$$

$$\psi_{2t} - \psi_{1t} + \gamma_t = 0 \tag{59}$$

We analyze these first order conditions in the next series of propositions.

To begin with, assume that the zero bound constraint on r_t^* is binding. In this case, $\gamma_t > 0$, and we may solve (53)-(58) for output gaps, government spending gaps, and inflation rates. First insert (53) and (58) into (55) to get:

$$-(J+F)\widehat{cg}_t^R - (J+A)(\widehat{n}_t^R) = \phi\theta\pi_t^R \tag{60}$$

Then insert (54) and (57) into (56) to get:

$$-(H+L)\widehat{cg}_t^W - (B+L)\widehat{n}_t^W = \phi\theta\pi_t^W \tag{61}$$

Using these conditions, we establish the following:

Proposition 3 *When the zero bound constraint binds in both countries, the optimal discretionary average level of the fiscal gap, $\widehat{cg}_t^W$, will be positive, independent of the degree of home bias, and equal to the optimal closed economy fiscal gap*

Proof. See Appendix ■

To see the intuition behind the proposition, we first take equation (61). Then, using the condition that all expectations terms of variable x_t will satisfy $E_t(x_{t+1}) = \mu x_t$, we can solve (17) and (18) for a second relationship between $\widehat{cg}_t^W$ and $\widehat{n}_t^W$, given by:

$$-\Delta_3\widehat{n}_t^W + \Delta_4\widehat{cg}_t^W = -(1-\beta\mu)\widetilde{r}_t^W \tag{62}$$

Then, combining (61) and (62), we arrive at

$$[(\Delta_3 HL + \Delta_4 BL) + \phi f(1-\mu)s]\widehat{cg}_t^W = -\left[f(\phi+s) + (1-\beta\mu)BL\right]\widetilde{r}_t^W \tag{63}$$

where the expressions $HL>0$ and $BL > 0$ and $f > 0$ are defined in the Appendix. From this, it is clear that when the world average natural rate falls, the world average fiscal gap must increase.

Note that, outside a liquidity trap, it would never be desirable to have a non-zero fiscal gap. But when the policy rate is constrained by the zero lower bound, fiscal spending, by creating anticipated inflation, can reduce real interest rates, stimulate private demand, and reduce the current world output gap. Moreover, since all terms in (63) are independent of the degree of home bias v, the optimal fiscal gap for the world average is the same as that in a closed economy, despite the fact that the initial shock in this exercise emanates from the home country alone and the shock is distributed unequally across the world economy.

The key focus of interest however, is the breakdown of the fiscal spending policies between the home and foreign economy. On the surface, because opening up fiscal gaps is costly in welfare terms, it would seem that the optimal cooperative policy should involve sharing the burden of fiscal adjustment across the two countries. But the appropriate response for fiscal policy depends upon how the policy itself can impact upon expected inflation, and the output gap in each country, which in turn depends on the degree of home bias, and other features of the model, as discussed in section 3 above. Nevertheless, it is easy to establish that the home fiscal gap should always be positive, as stated in the following proposition.

Proposition 4 *In a global liquidity trap, both the world relative fiscal gap and the optimal home fiscal gap will be positive,* $\widehat{cg}_t^R > 0$ *and* $\widehat{cg}_t > 0$

Proof. *See Appendix.* ■

The first part of the proposition is established along the same lines as Proposition (). Putting together the first order conditions for the world relative variables, and the conditions from the model equations (19) and (20), we may derive a relationship between the world relative shock to the natural interest rate and the optimal response of the world relative fiscal spending gap as follows:

$$\left[\Delta_3^D HL + \Delta_4^D JA + f\phi s_D(1-\mu)\right] \widehat{cg}_t^R = -\left[f(\phi + s_D) + (1-\beta\mu)JA\right] \widetilde{r}_t^R \tag{64}$$

where the terms HL, and JA are positive, and defined in the Appendix. Since $\widetilde{r}_t^R > 0$ for a shock that emanates from the home economy, it must be that the relative fiscal gap $\widehat{cg}_t^R$ is positive also. But since from the definition of $\widehat{cg}_t^W$ and $\widehat{cg}_t^R$, we have $\widehat{cg}_t = \widehat{cg}_t^W + \widehat{cg}_t^R$, it then follows that the home fiscal spending gap is positive. Thus, an optimal response to the liquidity trap for the home country is to follow an expansionary fiscal policy.

What is not so clear, however, is the optimal fiscal response for the foreign economy, when both countries are in a liquidity trap. Continue to assume that the $r_t^* = 0$ constraint still binds. Note, that $\widehat{cg}_t^* = \widehat{cg}_t^W - \widehat{cg}_t^R$. Since both $\widehat{cg}_t^W$ and $\widehat{cg}_t^R$ are positive, we cannot obviously sign this expression. Using the solutions from the Appendix, we can write

$$\begin{aligned} &\widehat{cg}_t^W - \widehat{cg}_t^R \\ = \quad & -\frac{[f(\phi + s) + (1-\beta\mu)BL]}{[(\Delta_3 HL + \Delta_4 BL) + \phi f(1-\mu)s]}\widetilde{r}_t^W + \frac{[f(\phi + s_D) + (1-\beta\mu)JA]}{[\Delta_3^D HL + \Delta_4^D JA + f\phi s_D(1-\mu)]}\widetilde{r}_t^R \end{aligned} \tag{65}$$

The first term on the right hand side is positive, while the second term is negative, when the (negative) shock hits the home economy. So in principle, it would seem as if the foreign fiscal expansion (when both countries are in a liquidity trap) could go either way. Note, from (9) and (10) we have the world average and relative natural interest rates as:

$$\widetilde{r}_t^W = \overline{r} + \left(\frac{1}{(\phi + \sigma)} \right) (1 - \mu)\phi c_y \frac{\varepsilon_t}{2} \tag{66}$$

$$\widetilde{r}_t^R = \left(\frac{(\upsilon - 1)}{\Delta} \right) (1 - \mu)\phi c_y \frac{\varepsilon_t}{2} \tag{67}$$

Insert (66) and (67) into (65) to get

$$\widehat{cg}_t^* = -c_r \overline{r} + c_\varepsilon \varepsilon_t \tag{68}$$

where $c_r > 0$ and $c_\varepsilon > 0$ are complicated functions of the underlying coefficients, defined in the Appendix.

In the numerical analysis below, we show that the sign of the foreign fiscal gap in the jointly optimal monetary and fiscal policy is positive. So both the home and foreign country follow an expansionary fiscal package in face of the home liquidity trap. But we can show also that if the foreign monetary authority, instead following an optimal rule, follows the rule (27), then the optimal adjustment of foreign fiscal policy may be *negative*.

6 Numerical analysis of optimal cooperative monetary and fiscal policy

We now provide a numerical illustration of the jointly optimal cooperative monetary and fiscal policy. To evaluate the economy quantitatively, we adopt some parameters from Cook and Devereux (2010a). Let $\beta = 0.99$, so each period is a quarter, and this translates to a value of the steady state interest rate $\overline{r} = 0.01$. The Frisch labor supply elasticity is $\phi = 1$. Price stickiness is $\kappa = 0.85$, so that $k = 0.027$, as in Christiano et al. (2009). Let the share of government in output be 20 percent, $c_y = 0.8$. We assume the inverse of intertemporal demand elasticity σ, is equal to 2. The persistence of the demand shock is set at 0.8 ($\mu = 0.8$) implying an expected length of the slump to be 5 quarters. We set the elasticity of substitution between good varieties within a country, θ, equal to 5. Finally, we set the preference shock in the home country ε so that at $\upsilon = 1$ (the case without any home bias), the natural real interest rate at the quarterly frequency would fall from 1 percent to -1.7 percent, with persistence μ.

Figure 2 illustrates the response of home and foreign output gaps, home and foreign government spending gaps, home and foreign inflation, the foreign country optimal policy rate, as well as the foreign natural real interest rate, and the home country terms of trade, for different values of v, when the optimal fiscal and monetary policy response is chosen. The figure takes account of condition (59), so that, at each value of v, the non-negativity constraint on r_t^* is tested, and if it is not binding, the optimal foreign policy rate is chosen and equal to (52). The first thing to note is that at $v = 1$, then clearly the liquidity trap is binding in both countries, and all variables respond in the same way in the two countries. The output gap falls by over 7 percent in both countries, and this is coupled with a fall in the rate of inflation by equal amounts. Since both countries are affected equally, it must be that the zero lower bound is binding in both countries, and the foreign policy rate is set at zero. The response of policy is illustrated by the positive response of the fiscal gap in each country. Thus, fiscal policy should behave countercyclically, and equally so in each country for a world without home bias in preferences.

Now, as v rises above unity, we know that the impact of the shock on the foreign natural interest rate becomes muted, while the opposite occurs for the home natural interest rate. The negative response of the foreign output gap is then reduced, while that of the home output gap is increased. As v rises more and more, we find, as discussed above, that the foreign output gap may actually increase. A similar dynamic occurs in the response of the inflation rates in the two countries - home inflation becomes more and more negative as v rises, while the negative response of foreign inflation becomes less and less. The optimal response of fiscal policy gaps is illustrated in panel b of the Figure. As v rises, home fiscal policy becomes more aggressive, while the foreign fiscal policy becomes more muted.

Panel d illustrates the optimal response of the foreign country policy rate, alongside the foreign country natural real interest rate. Note that at $v = 1$, the foreign policy rate is stuck at zero, while the natural real interest rate is at -0.017. As v rises, the response of the foreign natural interest rate becomes less and less, as is obvious from the formula (10). Eventually, as v rises to 2, the foreign country would be entirely unaffected by the shock, and the foreign natural interest rate would rise to 0.01, the steady state natural interest rate. But the key feature of panel d is that the foreign country will raise its policy rate above zero for values of $\widetilde{r}_t^* < 0$. That is, the foreign country will choose positive interest rates after point $\overline{v}$ (which is the analogous value from Proposition 2, although here we have both fiscal and monetary policy chosen endogenously), as part of an optimal cooperative policy package, even though, by the usual closed economy logic, it should be still in a liquidity trap, since its natural rate of interest is below zero. Equivalently, and in consistency with Proposition 2 (which held only for zero fiscal gaps), the foreign country will not follow a policy of offsetting the movement in the foreign natural interest rate, to the greatest extent that it can, so long as the policy rate is above the zero bound. Rather, it chooses to raise policy rates, even though $\widetilde{r}_t^* < 0$. In fact, panel d makes clear that, above $\overline{v}$, the foreign country will choose to raise its policy rate *above the steady state natural rate of interest.* Thus, by any definition of the term, the optimal monetary stance for the foreign country, in face of the home liquidity trap, is to tighten its monetary policy.

Thus, an optimal cooperative policy response to a liquidity trap can be characterized by expansionary fiscal policy in all countries, but contractionary monetary policy in the least affected country. This seemingly paradoxical result is related to the results of section (4) above. As v rises, the home economy is significantly more affected by the negative demand shock. An optimal policy response is to raise world demand, and to re-orient world demand towards the home country. The raising of the foreign policy rate is associated with an appreciation of the foreign currency, which generates an additional expenditure switching of demand towards the home country. Since the impact of the home country shock on foreign output is positive in any case, when v is sufficiently greater than unity, the rise in the foreign policy rate has the additional benefit that it helps to minimize the response of the foreign output gap to the home country shock. The Figure shows that the tightening of the policy rate in the foreign country as v rises reduces the degree to which the home terms of trade appreciates in response to the initial savings shock, and for sufficiently high v the home country terms of trade will actually depreciate. Thus, the key benefit of the foreign monetary response is to shape the response of the terms of trade.

We note that, when an optimal foreign monetary policy is used, the foreign country has a very small fiscal gap. Since $\dot{g}_t^* > 0$, it is optimal for the foreign country to follow an expansionary fiscal policy. But quantitatively, the size of the fiscal expansion is much less than that of the home country.

Figure 3 contrasts the optimal policy to an alternative possibility for foreign monetary policy. Here we assume that the foreign country follows the same monetary strategy as the home country, setting the policy rate equal to zero when the natural real interest rate is negative, and adjusting the policy rate to the natural real interest rate when it is above zero. Thus, we assume that the foreign monetary authority follows the rule (27). The Figure shows that the response of fiscal policy is substantially different when $v > 1$ and the foreign economy follows this (non-optimal) monetary rule. The key feature of this policy is that it is excessively expansionary for the foreign economy, relative to the optimal rule. As v rises more and more, the foreign economy experiences a boom, which is countered by a *contractionary* fiscal policy. At the same time, the outcome of expansionary monetary and contractionary fiscal policy in the foreign country leads to an excessive contraction in the home economy, which then requires a much greater fiscal expansion than would take place under the optimal policy. The foreign economy experiences inflation, while the deflation in the home economy is greater than it would be under the optimal policy. In addition the terms of trade appreciates much more for the home economy that it would under the optimal policy.

7 Conclusions

To be added

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Figure 2: Optimal Policy

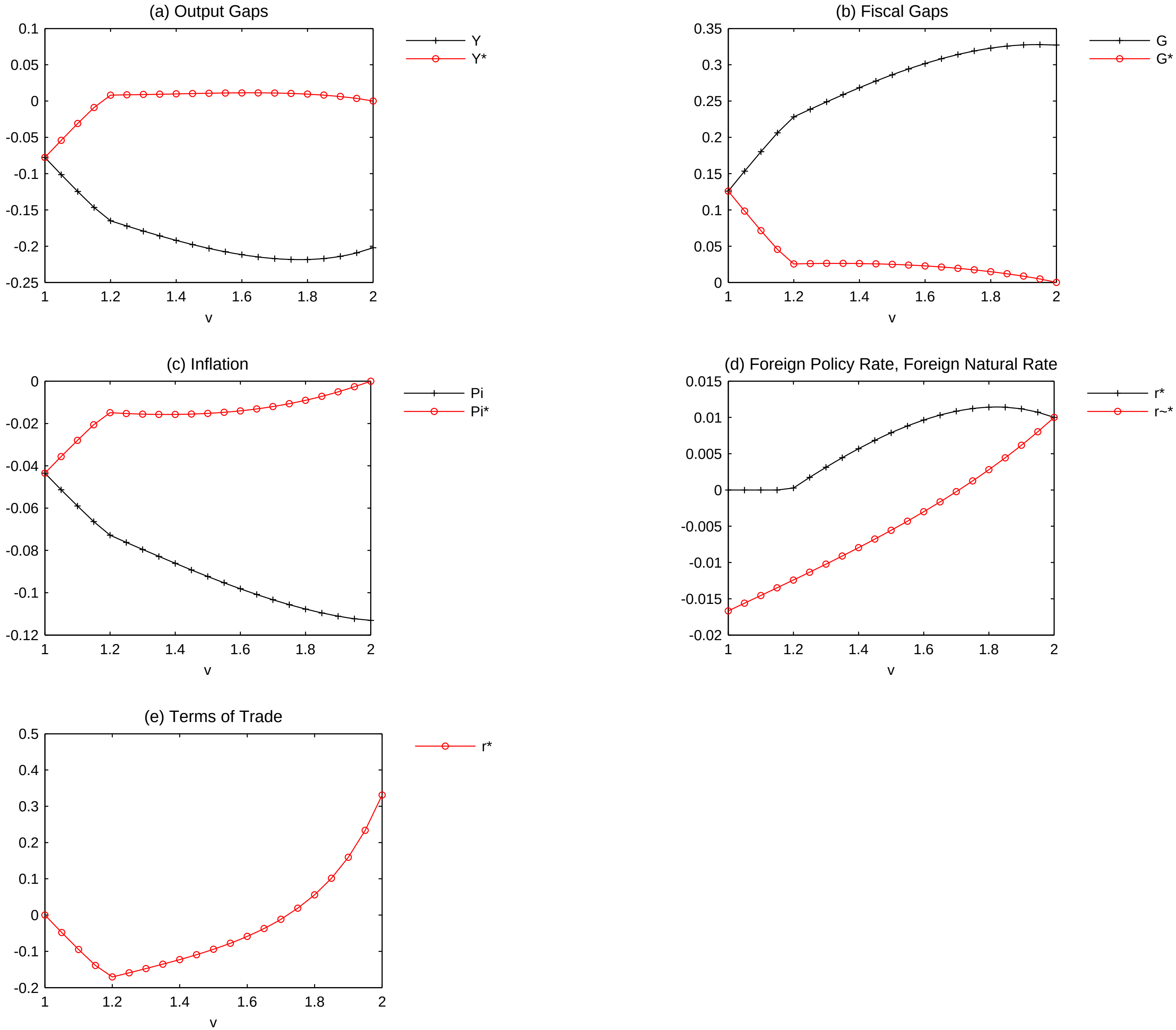

Figure 3: Optimal and Constrained Policy