# Quality Change, Hedonic Regression and Price Index Construction

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**Abstract:** Quality change in the CPI has two dimensions: quality mix change due to changes in the quantities purchased of existing items and quality change due new items appearing on the market, which have a combination of price-determining characteristics that is new, and "old" items disappearing. Quality mix changes will be treated properly by using a superlative price index number formula. Hedonic regression has become the default method to deal with the impact of new and disappearing items. In this paper, we discuss the various hedonic methods proposed in the literature, and show they can all be viewed as methods that impute "missing prices". We focus on weighted methods with complete information on prices, quantities and characteristics, but we also address the traditional case where quantity information is not available and unweighted methods are used in a sampling context.

**Keywords:** hedonic regression, multilateral price indexes, new and disappearing items, quality change, scanner data.

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# 1. Introduction

A statistical agency traditionally faces the problem of quality change in the Consumer Price Index (CPI) when an item leaves the sample and is replaced by another (new) item from the same product category to keep the sample size fixed. Such replacements may be forced when the replaced item disappears from the market or voluntary if the agency decides to update the sample because the ("old") item's market share has significantly decreased. The agency compares the replaced item and its successor to try and estimate the value of the quality difference and adjust the observed price change for the quality change.

While the above describes usual practices, it does not describe the problem of quality change very well. For example, suppose the statistical agency initially observed all items available in the market but the number of items decreases over time. It would then be impossible to find replacements for the disappearing items. Moreover, even if the population size remains constant but item churn is large, it would be difficult to choose a natural successor for some disappearing item. To understand quality change in the context of price measurement, we should look at the population of items and not at fixed-size samples.

Furthermore, we should not define quality adjustment in terms of trying to link disappearing items to newly selected items. Given a certain price index number formula, quality adjustment is a matter of imputing, or predicting, the "missing" prices or price relatives of unmatched items. In other words, what is required are estimates of what the prices, or price relatives, of disappearing (new) items would have been, had those items been purchased in the current (base) period. With prediction comes modelling, which in this case is hedonic modelling.

Quality adjustment in the CPI relates to broadly comparable items belonging to a product, e.g. different TV models. At this low aggregation level, item churn in the entire population can be very high, especially for high-technology products, such as TVs and other consumer electronics goods. Quality change has a second dimension: quality mix change due to changes in the relative importance of the items purchased. Compositional change will be treated properly by using a superlative index number formula. Thus, the use of a superlative price index combined with imputed values for the "missing prices" of unmatched new and disappearing items seems to be an obvious choice for measuring quality-adjusted price change.

An alternative approach is the time dummy method. Here, the hedonic model is estimated on the pooled data of two or more periods. A disadvantage of this method as compared with hedonic imputation price indexes is that the characteristics parameters are constrained to be fixed over time. Also, the aggregation across items is not explicit but depends on the estimation procedure, in particular on the weights used in the pooled regression. This implies that the estimation of the time dummy hedonic model should be looked at from both an econometrics and an index number point of view. The method has two advantages though: pooling data preserves degrees of freedom and the resulting price index is transitive. Transitivity in this context means that, when data of more than two periods is pooled, the resulting price index will be free from chain drift. Drift can be a serious problem in high-frequency chained weighted indexes.

The main purpose of this paper is to review and compare hedonic imputation and time dummy hedonic methods.[1] Although we focus on weighted methods with complete information on prices, quantities and item characteristics, we also address the traditional case where quantity information is unavailable and unweighted methods are used in a sampling context. The paper is organized as follows.

Section 2 shows why "missing prices" need to be imputed when the universe of broadly comparable items is dynamic, i.e. when there are both matched and unmatched (new and disappearing) items. Our starting point is the single imputation Törnqvist price index. We also discuss why "double" imputation and "full" imputation methods can be useful. Section 3 addresses hedonic regression and the index number implications, with a focus on the frequently used log-linear hedonic model. Hedonic imputation methods and the time dummy method are outlined in detail. Importantly, we explain why the use of expenditure-share weights in both the single regressions (for the imputation methods) and the pooled regressions (for the time dummy approach) makes sense from an index number perspective as well as from an econometrics point of view.

In sections 2 and 3 two time periods were compared. In section 4, many periods will be distinguished and two quality-adjusted price indexes discussed: the multi-period (and expenditure-share weighted) time dummy hedonic index and a hedonic imputation Törnqvist-type GEKS index. The latter is an extension of the well-known multilateral GEKS index in which the "missing prices" are imputed using hedonics. In section 5, we argue that the weighted multi-period time dummy index can alternatively be viewed as an accurate approximation to a "quality-adjusted unit value index".

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[1] The two approaches sometimes yield quite different results. See e.g. Berndt, Griliches and Rappaport (1995), Berndt and Rappaport (2001), Pakes (2003), Diewert (2003), Silver and Heravi (2003), and Silver and Heravi (2007a). Diewert, Silver and Heravi (2009) analysed the difference between "full" imputation hedonic and time dummy indexes.

Section 6 contains an empirical example using scanner data for televisions from a Dutch retailer. The example shows that accounting for new and disappearing items via hedonic quality adjustments can have a significant impact on multilateral price indexes. It also shows that, contrary to what many people seem to believe, hedonic adjustments do not necessarily lead to lower indexes.

Section 7 concludes.

## 2. The dynamic universe and "missing prices"

The set of broadly comparable items belonging to a particular product category usually changes across time: the universe of items is dynamic with new items appearing on the market and "old" items disappearing. The churn rate typically depends on the type of product, and for high-tech goods, it can be quite substantial. This section discusses the index number implications of a dynamic universe and the need for imputing "missing prices". We are comparing two periods, the base period 0 and the comparison period 1, so we are dealing with bilateral indexes. Multilateral methods, which estimate quality-adjusted price indexes simultaneously for more than two periods, will be addressed in section 4.

### 2.1 Weighted indexes

The sets of items available in periods 0 and 1 are denoted by $U^0$ and $U^1$. Importantly, for making price and quantity comparisons between the two periods, we should not look at $U^0$ and $U^1$ in isolation but rather at the *union* $U^{01} = U^0 \cup U^1$.[2] This makes it easier to derive and explain imputation price and quantity indexes. A subset of items is usually purchased in both periods. This matched set (intersection) is denoted by $U_M = U^0 \cap U^1$. The set of disappearing items, i.e. all items which are purchased in period 0 but not in period 1, is denoted by $U_D^0$, while the set of new items, i.e. all items purchased in period 1 but not in period 0, is denoted by $U_N^1$. Note that $U_D^0 \cup U_M = U^0$, $U_N^1 \cup U_M = U^1$, and $U^{01} = U^0 \cup U^1 = U_M \cup U_D^0 \cup U_N^1$. Prices are strictly positive. Quantities purchased are non-negative; in the two period dynamic case, quantities are either positive or zero in one of the periods (or zero in both periods, but that is irrelevant as those items do not belong to the union $U^{01}$).

[2] In the current version of the CPI Manual (ILO et al., 2004), the union is called the double universe. This term can be a bit confusing because at each point in time, there is just a single set (universe) of items that are purchased.

Let $p_i^0$ and $p_i^1$ be the prices and $q_i^0$ and $q_i^1$ the quantities purchased of item *i* in the two periods. Defining the aggregate value ratio on $U^{01}$ is straightforward:

$$V^{01} = \frac{\sum_{i \in U^{01}} p_i^1 q_i^1}{\sum_{i \in U^{01}} p_i^0 q_i^0} = \frac{\sum_{i \in U^1} p_i^1 q_i^1}{\sum_{i \in U^0} p_i^0 q_i^0}. \tag{1}$$

We want the price index $P^{01}$ and the quantity index $Q^{01}$ to satisfy the product test, i.e. to have $V^{01} = P^{01} \times Q^{01}$. Although this paper is on the construction of (quality-adjusted) price indexes, we start with the quantity side to illustrate a couple of important points.[3] Let us define a quantity index on the union $U^{01}$ as follows:

$$Q^{01} = \frac{\sum_{i \in U^{01}} \lambda_{i/b} q_i^1}{\sum_{i \in U^{01}} \lambda_{i/b} q_i^0} = \frac{\sum_{i \in U_M} \lambda_{i/b} q_i^1 + \sum_{i \in U_D^0} \lambda_{i/b} q_i^1 + \sum_{i \in U_N^1} \lambda_{i/b} q_i^1}{\sum_{i \in U_M} \lambda_{i/b} q_i^0 + \sum_{i \in U_D^0} \lambda_{i/b} q_i^0 + \sum_{i \in U_N^1} \lambda_{i/b} q_i^0} = \frac{\sum_{i \in U_M} \lambda_{i/b} q_i^1 + \sum_{i \in U_N^1} \lambda_{i/b} q_i^1}{\sum_{i \in U_M} \lambda_{i/b} q_i^0 + \sum_{i \in U_D^0} \lambda_{i/b} q_i^0}, \tag{2}$$

where the last expression holds because $q_i^1 = 0$ for $i \in U_D^0$ and $q_i^0 = 0$ for $i \in U_N^1$. The $\lambda_{i/b}$ are quality-adjustment/standardization factors. They express the quantity of each item in terms of units of an arbitrary base item *b* $(\lambda_{b/b} = 1)$, enabling us to add up the (standardized) quantities. Just like the value index (1), the quantity index (2) is based on two different sets of items: the period 0 set, $U^0$, in the denominator and the period 1 set, $U^1$, in the numerator.

According to basic economic theory, relative prices of broadly comparable items that coexist are measures of their quality differences, at least if markets are competitive. Thus, setting $\lambda_{i/b} = p_i^0 / p_b^0$ would be a way to estimate the quality-adjustment factors. However, base period prices for $i \in U_N^1$ cannot be observed; they are "missing" and must be imputed by $\hat{p}_i^0$. Assuming that the base item *b* belongs to the matched set $U_M$, we find

$$Q_{L,SI}^{01} = \frac{\sum_{i \in U_M} (p_i^0 / p_b^0) q_i^1 + \sum_{i \in U_N^1} (\hat{p}_i^0 / p_b^0) q_i^1}{\sum_{i \in U_M} (p_i^0 / p_b^0) q_i^0 + \sum_{i \in U_D^0} (p_i^0 / p_b^0) q_i^0} = \frac{\sum_{i \in U_M} p_i^0 q_i^1 + \sum_{i \in U_N^1} \hat{p}_i^0 q_i^1}{\sum_{i \in U_M} p_i^0 q_i^0 + \sum_{i \in U_D^0} p_i^0 q_i^0}. \tag{3}$$

We refer to (3) as the (single) imputation Laspeyres quantity index. Note that prices are imputed, not quantities – imputing quantities makes no sense. An alternative choice for the $\lambda_{i/b}$ is $p_i^1 / p_b^1$. Period 1 prices for $i \in U_D^0$ are "missing", and they must be imputed by $\hat{p}_i^1$, yielding the (single) imputation Paasche quantity index

[3] We do this also for later use in section 5 where we discuss the quality-adjusted unit value index. This section draws heavily from de Haan (2015a).

$$Q_{P,SI}^{01} = \frac{\sum_{i \in U_M} (p_i^1 / p_b^1) q_i^1 + \sum_{i \in U_N^1} (p_i^1 / p_b^1) q_i^1}{\sum_{i \in U_M} (p_i^1 / p_b^1) q_i^0 + \sum_{i \in U_D^0} (\hat{p}_i^1 / p_b^1) q_i^0} = \frac{\sum_{i \in U_M} p_i^1 q_i^1 + \sum_{i \in U_N^1} p_i^1 q_i^1}{\sum_{i \in U_M} p_i^1 q_i^0 + \sum_{i \in U_D^0} \hat{p}_i^1 q_i^0} . \tag{4}$$

The corresponding price indexes are obtained by dividing the value index by the imputation quantity indexes (3) or (4). This gives

$$P_{P,SI}^{01} = \frac{\sum_{i \in U_M} p_i^1 q_i^1 + \sum_{i \in U_N^1} p_i^1 q_i^1}{\sum_{i \in U_M} p_i^0 q_i^1 + \sum_{i \in U_N^1} \hat{p}_i^0 q_i^1} = \left[ \sum_{i \in U_M} s_i^1 \left( \frac{p_i^1}{p_i^0} \right)^{-1} + \sum_{i \in U_N^1} s_i^1 \left( \frac{p_i^1}{\hat{p}_i^0} \right) \right]^{-1} , \tag{5}$$

which is the single imputation Paasche price index, where $s_i^1 = p_i^1 q_i^1 / \sum_{i \in U^1} p_i^1 q_i^1$ is the expenditure share of item *i* in period 1, and

$$P_{L,SI}^{01} = \frac{\sum_{i \in U_M} p_i^1 q_i^0 + \sum_{i \in U_D^0} \hat{p}_i^1 q_i^0}{\sum_{i \in U_M} p_i^0 q_i^0 + \sum_{i \in U_D^0} p_i^0 q_i^0} = \sum_{i \in U_M} s_i^0 \left( \frac{p_i^1}{p_i^0} \right) + \sum_{i \in U_D^0} s_i^0 \left( \frac{\hat{p}_i^1}{p_i^0} \right) , \tag{6}$$

which is the single imputation Laspeyres price index, where $s_i^0 = p_i^0 q_i^0 / \sum_{i \in U^{01}} p_i^0 q_i^0$ is the expenditure share of item *i* in period 0. The imputation price indexes are based on a single set of items: $U^1$ for the Paasche index, and $U^0$ for the Laspeyres index.

By taking the geometric mean of (6) and (5), the single imputation Fisher price index is obtained:[4]

$$P_{F,SI}^{01} = \left[ \frac{\sum_{i \in U_M} p_i^1 q_i^0 + \sum_{i \in U_D^0} \hat{p}_i^1 q_i^0}{\sum_{i \in U_M} p_i^0 q_i^0 + \sum_{i \in U_D^0} p_i^0 q_i^0} \right]^{1/2} \left[ \frac{\sum_{i \in U_M} p_i^1 q_i^1 + \sum_{i \in U_N^1} p_i^1 q_i^1}{\sum_{i \in U_M} p_i^0 q_i^1 + \sum_{i \in U_N^1} \hat{p}_i^0 q_i^1} \right]^{1/2}$$

$$= \left[ \sum_{i \in U_M} s_i^0 \left( \frac{p_i^1}{p_i^0} \right) + \sum_{i \in U_D^0} s_i^0 \left( \frac{\hat{p}_i^1}{p_i^0} \right) \right]^{1/2} \left[ \sum_{i \in U_M} s_i^1 \left( \frac{p_i^1}{p_i^0} \right)^{-1} + \sum_{i \in U_N^1} s_i^1 \left( \frac{p_i^1}{\hat{p}_i^0} \right)^{-1} \right]^{-1/2} . \tag{7}$$

Although the single imputation Fisher index can be seen as the default choice, we could use a Törnqvist version instead. The Törnqvist index is also superlative (Diewert, 1976) and, due to its geometric form, is easier to decompose than the Fisher index. The single imputation Törnqvist price index is defined as

$$P_{T,SI}^{01} = \prod_{i \in U_M} \left( \frac{p_i^1}{p_i^0} \right)^{\frac{s_i^0 + s_i^1}{2}} \prod_{i \in U_D^0} \left( \frac{\hat{p}_i^1}{p_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U_N^1} \left( \frac{p_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^1}{2}} , \tag{8}$$

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[4] De Haan (2002) referred to (7) as a generalized Fisher price index.

being the geometric mean of the single imputation geometric Laspeyres price index

$$P_{GL,SI}^{01} = \prod_{i \in U_M} \left( \frac{p_i^1}{p_i^0} \right)^{s_i^0} \prod_{i \in U_D^0} \left( \frac{\hat{p}_i^1}{p_i^0} \right)^{s_i^0} \tag{9}$$

and the single imputation geometric Paasche price index

$$P_{GP,SI}^{01} = \prod_{i \in U_M} \left( \frac{p_i^1}{p_i^0} \right)^{s_i^1} \prod_{i \in U_N^1} \left( \frac{p_i^1}{\hat{p}_i^0} \right)^{s_i^1} . \tag{10}$$

The term "single" imputation relates to the fact that only the "missing" prices are imputed while preserving all of the observed prices, including those for the unmatched items. It would also be possible to use double imputation, where the observed prices of the unmatched items are replaced by predicted values. That is, both the period 0 and period 1 prices in the price relatives for the unmatched items are now predicted values. For example, the double imputation Törnqvist formula is

$$P_{T,DI}^{01} = \prod_{i \in U_M} \left( \frac{p_i^1}{p_i^0} \right)^{\frac{s_i^0 + s_i^1}{2}} \prod_{i \in U_D^0} \left( \frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U_N^1} \left( \frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^1}{2}} . \tag{11}$$

Double imputation may be useful when hedonic regressions (see section 3) are used: the predictions can be biased, and the biases in the numerator and denominator of the price relatives might cancel out; see e.g. Hill and Melser (2008) and Syed (2010).[5]

The single and double imputation Törnqvist price indexes can be written as

$$P_{T,SI}^{01} = \prod_{i \in U_M} \left( \frac{p_i^1}{p_i^0} \right)^{\frac{s_{iM}^0 + s_{iM}^1}{2}} \left[ \frac{\prod_{i \in U_D^0} \left( \frac{\hat{p}_i^1}{p_i^0} \right)^{s_{iD}^0}}{\prod_{i \in U_M} \left( \frac{p_i^1}{p_i^0} \right)^{s_{iM}^0}} \right]^{\frac{s_D^0}{2}} \left[ \frac{\prod_{i \in U_N^1} \left( \frac{p_i^1}{\hat{p}_i^0} \right)^{s_{iN}^1}}{\prod_{i \in U_M} \left( \frac{p_i^1}{p_i^0} \right)^{s_{iM}^1}} \right]^{\frac{s_N^1}{2}} ; \tag{12}$$

$$P_{T,DI}^{01} = \prod_{i \in U_M} \left( \frac{p_i^1}{p_i^0} \right)^{\frac{s_{iM}^0 + s_{iM}^1}{2}} \left[ \frac{\prod_{i \in U_D^0} \left( \frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{s_{iD}^0}}{\prod_{i \in U_M} \left( \frac{p_i^1}{p_i^0} \right)^{s_{iM}^0}} \right]^{\frac{s_D^0}{2}} \left[ \frac{\prod_{i \in U_N^1} \left( \frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{s_{iN}^1}}{\prod_{i \in U_M} \left( \frac{p_i^1}{p_i^0} \right)^{s_{iM}^1}} \right]^{\frac{s_N^1}{2}} , \tag{13}$$

[5] Hill and Melser (2008) review many different imputation price indexes, including indexes where prices in the expenditure shares are replaced by predicted values. In this paper we will not address imputation indexes with model-based expenditure weights.

where $s_{iM}^0 = p_i^0 q_i^0 / \sum_{i \in U_M} p_i^0 q_i^0$, $s_{iM}^1 = p_i^1 q_i^1 / \sum_{i \in U_M} p_i^1 q_i^1$, $s_{iD}^0 = p_i^0 q_i^0 / \sum_{i \in U_D^0} p_i^0 q_i^0$, and $s_{iN}^1 = p_i^1 q_i^1 / \sum_{i \in U_N^1} p_i^1 q_i^1$ are normalized expenditure shares; $s_D^0$ and $s_N^1$ are the aggregate expenditure shares of the disappearing and new items in periods 0 and 1, respectively. The first factor in (12) and (13) is the matched-item Törnqvist price index. The factors between square brackets show that the matched-item Törnqvist index will suffice when the imputation geometric Laspeyres and Paasche indexes for the disappearing and new items are equal to their matched-item counterparts.

While this may seem unattractive, in addition we could replace the prices of the matched items by predicted values. Feenstra's (1995) "exact" (hedonic) approach gives rise to the following imputation Törnqvist price index:

$$P_{T,E}^{01} = \prod_{i \in U^0} \left( \frac{\hat{p}_i^1}{p_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U^1} \left( \frac{p_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^1}{2}} . \qquad (14)$$

Replacing all observed prices by predicted values leads to the full imputation Törnqvist index

$$P_{T,FI}^{01} = \prod_{i \in U^0} \left( \frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U^1} \left( \frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^1}{2}} . \qquad (15)$$

The above imputation indexes are all weighted and pertain to the entire universe. Statistical agencies having access to scanner data will be able to construct these indexes, provided of course that the "missing prices" can be measured.

## 2.2 Unweighted (geometric) indexes

In some cases, weighted methods are unnecessary to employ or infeasible. In the case of residential (or commercial) property, for example, quantities purchased are equal to 1 as each property can be considered unique, in particular due to their unique location.[6] For other goods, statistical agencies cannot use weighted methods if they do not have access to EPOS data. This traditional situation still exists today in many countries. In addition, these agencies usually work with sample data rather than population data.[7] The sample

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[6] For more information on the construction of residential property price indexes (RPPIs), see Eurostat (2013).

[7] For sampling approaches to measuring elementary price indexes, see Balk (2005). Statistical agencies are increasingly using prices data from websites of retailers. Depending on the content of the websites and the way in which prices are extracted, the entire universe of items may be observed; see e.g. Griffioen, de Haan and Willenborg (2014).

size is typically fixed in the short run. Let $S^0$ and $S^1$ be the samples of items in periods 0 and 1, with fixed size *n*; $S_M = S^0 \cap S^1$ is the matched sample (with size $n_M$), $S_D^0$ is the subsample of disappearing items and $S_N^1$ is the subsample of new items (both with size $n - n_M$). Setting the expenditure shares in equations (8), (11), (14) and (15) for the two periods equal to $1/n$, the following imputation Jevons price indexes for a fixed-size sample are obtained:

$$P_{J,SI}^{01} = \prod_{i \in S_M} \left( \frac{p_i^1}{p_i^0} \right)^{\frac{1}{n}} \prod_{i \in S_D^0} \left( \frac{\hat{p}_i^1}{p_i^0} \right)^{\frac{1}{2n}} \prod_{i \in S_N^1} \left( \frac{p_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}}; \tag{16}$$

$$P_{J,DI}^{01} = \prod_{i \in S_M} \left( \frac{p_i^1}{p_i^0} \right)^{\frac{1}{n}} \prod_{i \in S_D^0} \left( \frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} \prod_{i \in S_N^1} \left( \frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}}; \tag{17}$$

$$P_{J,E}^{01} = \prod_{i \in U^0} \left( \frac{\hat{p}_i^1}{p_i^0} \right)^{\frac{1}{2n}} \prod_{i \in U^1} \left( \frac{p_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}}. \tag{18}$$

$$P_{J,FI}^{01} = \prod_{i \in U^0} \left( \frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} \prod_{i \in U^1} \left( \frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}}. \tag{19}$$

Statistical agencies often use a slightly different approach, in Europe referred to as “re-pricing”. Any item that disappears from the fixed-size sample is linked to a newly selected item. Suppose item *i* is dropped from the sample and replaced by item *j*; the observed prices are $p_i^0$ and $p_j^1$. To adjust for a difference in quality, $p_j^1$ is multiplied by the ratio $\hat{p}_i^0 / \hat{p}_j^0$ of predicted base period prices. In a sampling context, this approach can easily be generalised to the situation with $n - n_M$ disappearing and new items. This leads to the following Jevons-type price index

$$\begin{aligned} P_{J,R}^{01} &= \prod_{i \in S_M} \left( \frac{p_i^1}{p_i^0} \right)^{\frac{1}{n}} \prod_{i \in S_D^0} \left( \frac{\hat{p}_i^0}{p_i^0} \right)^{\frac{1}{n}} \prod_{i \in S_N^1} \left( \frac{p_i^1}{\hat{p}_i^0} \right)^{\frac{1}{n}} \\ &= P_{J,SI}^{01} \frac{\prod_{i \in S_N^1} \left( \frac{p_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}}}{\prod_{i \in S_D^0} \left( \frac{\hat{p}_i^1}{p_i^0} \right)^{\frac{1}{2n}}}. \end{aligned} \tag{20}$$

The second expression of (20) clearly shows that the “re-pricing index” cannot be called an imputation index. This approach is in fact non-symmetric and difficult to justify; see de Haan (2010) for details.

# 3. Hedonic regression

## 3.1 Background

Quality adjustment assumes that the “missing prices” can be accurately measured. The imputed values should measure what the prices of new (disappearing) items would have been, had they actually been sold in period 0 (period 1). The fact that new items were not available before may mean they were infeasible to produce or sell. Also, if they had been available, the market circumstances would have been different, and so the prices might have been different as well. As any quality adjustment method somehow relies on observed prices to measure the value of quality differences, imputing “missing prices” is not completely without problems (Schultze and Mackie, 2002). But we should not exaggerate the problem, and in any case, the hedonic approach does have an economic-theoretic underpinning, in contrast to some of the implicit methods applied by statistical agencies.

The hedonic hypothesis postulates that a heterogeneous product can be viewed as a combination of its performance characteristics, and that the price is a function of these characteristics. The theoretical foundations of the hedonic hypothesis have been discussed elsewhere, e.g. in Griliches (1990), ILO et al. (2004) and Triplett (2006), so there is no need to go into details here. It is worth quoting Griliches (1990) though, who explained his view – with which we agree – as follows:[8]

> “My own point of view is that what the hedonic approach tries to do is to estimate aspects of the budget constraint facing consumers, allowing thereby the estimation of “missing prices” when quality changes. …. What is being estimated is actually the locus of intersections of the demand curves of different consumers with varying tastes and the supply functions of different producers with possibly varying technologies of production. One is unlikely, therefore, to be able to recover the underlying utility and cost functions from such data alone, except in very special circumstances. Nor can theoretical derivations at the individual level really provide substantive constraints on the estimation of such “market” relations. … Hence my preference for the “estimation of missing prices” interpretation of this approach. Accepting that, one still faces the usual index number problems and ambiguities but at least one is back to the “previous case”.” (Griliches, 1990, p. 188-189)

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[8] Griliches’ (1990) paper contains many useful references to the literature. He expressed his view earlier in Ohta and Griliches (1976), p. 326): “What the hedonic approach attempted was to provide a tool for estimating “missing prices”, prices of particular bundles not observed in the original or later periods. …. Because of its focus on price explanation and its purpose of “predicting” the price of unobserved variants of a commodity in particular periods, the hedonic hypothesis can be viewed as asserting the existence of a reduced-form relationship between prices and the various characteristics of the commodity. That relationship need not be “stable” over time, but changes that occur should have some rhyme and reason to them, otherwise one would suspect that the observed results are a fluke and cannot be used in the extrapolation necessary for the derivation of missing prices.”

As the purpose of our paper is the index number implications of using hedonics, it will suffice to assume that a (reduced-form) relation between price and (the quantities of) characteristics exists and that data is available to estimate this relation.

The general representation of a hedonic model for some period $t$ is

$$p^t = f(z_1, z_2, ..., z_K, t), \tag{21}$$

where $z_k$ is the (quantity of) characteristic $k$ $(k = 1,...,K)$ and $t$ denotes time (period). In (21), the characteristics are assumed independent of time $t$. That is, we assume that for each individual item, the quantities of the characteristics are fixed. For most consumer goods where statistical agencies may wish to apply hedonics for the treatment of quality change, such as high-tech electronics goods, this assumption is reasonable. For existing houses (and other resold durable goods, like second-hand cars), the assumption should be relaxed as housing characteristics can change over time. Anyhow, time-dependency of characteristics would not really change our analysis.

At least three issues should be addressed when performing empirical hedonics: the choice of characteristics, the model's functional form, and the estimation procedure. The importance of the choice of characteristics included should not be understated, and selecting the appropriate set is often a time-consuming part of the empirical work, but in this paper, we simply assume a given set of characteristics. Hence, we focus here on the functional form for the model and the estimation method. The economic theory behind hedonics does not tell us anything about the functional form other than that the shadow prices of the characteristics, which are not directly observable, are implicitly determined by demand and supply factors. Consequently, the shadow prices or, more generally, the coefficients in a parametric model, need not be constant over time. In other words, the functional form for a hedonic model is an empirical matter, and the parameters should preferably not be kept fixed.

Notwithstanding the uncertainty about the true functional form, empirical work has mainly focused on linear models, perhaps because the interpretation and estimation of linear models is straightforward. The two most frequently applied linear models with time-varying intercept terms $\alpha^t$ and characteristics parameters $\beta_k^t$ are

$$p_i^t = \alpha^t + \sum_{k=1}^{K} \beta_k^t z_{ik} + \varepsilon_i^t, \tag{22}$$

and the log-linear or semi-log specification

$$\ln p_i^t = \alpha^t + \sum_{k=1}^{K} \beta_k^t z_{ik} + \varepsilon_i^t, \tag{23}$$

where $\varepsilon_i^t$ is an error term with zero mean (by assumption). The semi-log specification (23) tends to fit the data better than the strictly linear specification (22), at least for most consumer goods. Moreover, the strictly linear specification is more likely to suffer from heteroskedasticity; the absolute errors are likely to be bigger for more expensive items. The logarithmic transformation helps reduce this source of non-constant variance of the errors.[9] For these reasons, in section 3.2 we will use the semi-log hedonic model (23) to show how hedonic imputation methods work.

The use of Ordinary Least Squares (OLS) regression to estimate linear models can provide reasonable results; under the classical model assumptions, OLS produces unbiased parameter estimates with minimum variance. Yet, these assumptions are quite restrictive, and so testing whether they are satisfied is good practice. An important issue is whether the economic importance of items must be reflected when running hedonic regressions. If this is the case, some form of Weighted Least Squares (WLS) rather than OLS would be required. We return to the issue of weighted vs. unweighted regressions in sections 3.2 and 3.3.

### 3.2 Hedonic imputation methods

We will again look at the bilateral case where period 1 is compared with the base period 0. Suppose first that *i*) the statistical agency has no information on quantities purchased and aims at a quality-adjusted Jevons index, and *ii*) the semi-log hedonic model (23) is appropriate and the classical assumptions are satisfied, in particular a constant variance and zero covariance of the error terms, so that OLS regression and performing standard tests do not give any problems. The parameter estimates from running OLS regressions in each period separately are denoted by $\hat{\alpha}^t$ and $\hat{\beta}_k^t$ $(t = 0,1)$, and the predicted prices are[10]

$$\hat{p}_i^0 = \exp\left[\hat{\alpha}^0 + \sum_{k=1}^{K} \hat{\beta}_k^0 z_{ik}\right]; \tag{24}$$

$$\hat{p}_i^1 = \exp\left[\hat{\alpha}^1 + \sum_{k=1}^{K} \hat{\beta}_k^1 z_{ik}\right]. \tag{25}$$

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[9] For more on the choice between (untransformed) price or the logarithm of price in hedonic models, see Diewert (2003a).

[10] Due to the non-linear transformation, the time dummy indexes are not unbiased. Kennedy (1981) and Van Garderen and Shah (2002) discussed bias-correction terms. Usually, the bias will be small and can be ignored. We will not make any bias adjustments.

Substituting (24) and (25) into (14) yields the hedonic single imputation Jevons price index. The three other imputation methods, (15) and (16) and (17) are a bit more interesting because substitution of (24) and (25) shows how changes in the parameter estimates and the average characteristics affect the indexes. For example, for the double imputation index (15) we find

$$P_{J,DI}^{01} = \left[ \prod_{i \in S_M} \left( \frac{p_i^1}{p_i^0} \right)^{\frac{1}{n_M}} \right]^{f_M} \left[ \exp\left(\hat{\alpha}^1 - \hat{\alpha}^0\right) \exp\left\{ \sum_{k=1}^{K} \left(\hat{\beta}^1 - \hat{\beta}^0\right) \left( \frac{\bar{z}_{Dk}^0 + \bar{z}_{Nk}^1}{2} \right) \right\} \right]^{1-f_M}, \tag{26}$$

where $\bar{z}_{Dk}^0 = \sum_{i \in S_D^0} z_{ik} /(n - n_M)$ and $\bar{z}_{Nk}^1 = \sum_{i \in S_N^1} z_{ik} /(n - n_M)$ denote the (unweighted) average characteristics for the disappearing and new items, respectively; $f_M = n / n_M$ is the fraction of matched items. It can be seen that the double imputation price index is a weighted geometric average of the matched-item Jevons price index, given by the first factor between square brackets, and a quality-adjusted price index for the unmatched items.

It can easily be verified that the "exact" hedonic imputation Jevons index (16) is equal to the full imputation index (17), using the OLS (orthogonality) property that the regression residuals $e_i^t = \ln(p_i^t / \hat{p}_i^t)$ $(t = 0,1)$ sum to zero in each period. Substituting (24) and (25) into (17) yields

$$P_{J,FI}^{01} = \exp\left(\hat{\alpha}^1 - \hat{\alpha}^0\right) \exp\left\{ \sum_{k=1}^{K} \left(\hat{\beta}_k^1 - \hat{\beta}_k^0\right) \left( \frac{\bar{z}_k^0 + \bar{z}_k^1}{2} \right) \right\}, \tag{27}$$

where $\bar{z}_k^0 = \sum_{i \in S^0} z_{ik} / n$ and $\bar{z}_k^1 = \sum_{i \in S^1} z_{ik} / n$ are the average characteristics in the full samples of periods 0 and 1. $P_{J,FI}^{01}$ can be viewed as a (symmetric) characteristics price index (Triplett, 2004).

When all the estimated characteristics parameters stay the same, equations (26) and (27) simplify to

$$P_{J,DI}^{01} = \left[ \prod_{i \in S_M} \left( \frac{p_i^1}{p_i^0} \right)^{\frac{1}{n_M}} \right]^{f_M} \left[\exp\left(\hat{\alpha}^1 - \hat{\alpha}^0\right)\right]^{1-f_M}; \tag{28}$$

$$P_{J,FI}^{01} = \exp\left(\hat{\alpha}^1 - \hat{\alpha}^0\right). \tag{29}$$

But if a statistical test indicates that the characteristics parameters do not significantly change over time, then the data may be pooled across the two periods to estimate a time dummy hedonic model. The bilateral time dummy hedonic index is discussed in section

3.3, where we will also show the similarity between this index and the full imputation hedonic (or characteristics) index.

Suppose next that the agency has access to scanner data, so that sampling is not required. The number of items, $N^t$, now varies over time and is typically much larger than the number of sampled items in the traditional situation. The question is: should the items be weighted in the hedonic regressions according to their economic importance? At first glance, the answer is no: WLS regression would unnecessarily raise the standard errors of the coefficients if the errors have constant variance. We will argue, however, that if the prices are unit values rather than price quotes at a single point in time, then the errors are likely to be heteroskedastic and WLS regression is useful.

While we might expect the "law of one price" to hold for a homogeneous item, in reality there can be disturbances, and the prices of individual transactions of the item can slightly differ (Balk, 1998). Let $p_{ij}^t$ be the price of transaction *j* for item *i* in period *t*. The semi-log hedonic model at the individual transaction level is

$$\ln p_{ij}^t = \alpha^t + \sum_{k=1}^{K} \beta_k^t z_{ik} + \varepsilon_{ij}^t, \tag{30}$$

where the errors $\varepsilon_{ij}^t$ have a zero mean and constant variance. Notice that we assume the parameters fixed within each period (month or quarter) *t*, hence that they change in a discrete way.

If individual transaction data was available, model (30) could be estimated (by OLS regression). Statistical agencies using scanner data typically do not have access to data at the individual transaction level, however; they work with data which has been aggregated across all the transactions within a certain time period and use unit values as prices at the item level. This is not a disadvantage, of course, because the unit value is the appropriate concept of price for a homogeneous item (ILO et al., 2004). The unit value, or average transaction price, in period *t* is defined as

$$\bar{p}_i^t = \frac{\sum_{j=1}^{q_i^t} p_{ij}^t}{q_i^t}, \tag{31}$$

where $q_i^t$ is the total number of transactions of *i* in period *t*., i.e. the quantity purchased, as before. Because of the logarithmic model specification, we will use the "geometric unit value"

$$\breve{p}_i^t = \prod_{j=1}^{q_i^t} \left(p_{ij}^t\right)^{\frac{1}{q_i^t}} \tag{32}$$

rather than the (ordinary) arithmetic unit value (31) to illustrate the heteroscedasticity problem.

Taking the logarithm of (32) and then substituting the "true" hedonic model (28) into the result yields

$$\ln \breve{p}_i^t = \frac{\sum_{j=1}^{q_i^t} \ln p_{ij}^t}{q_i^t} = \alpha^t + \sum_{k=1}^{K} \beta_k^t z_{ik} + \bar{\varepsilon}_i^t, \quad (33)$$

where $\bar{\varepsilon}_i^t = \sum_{j=1}^{q_i^t} \varepsilon_{ij}^t / q_i^t$. So $\text{var}(\bar{\varepsilon}_i^t) = (1/q_i^t)^2 \sum_{j=1}^{q_i^t} \text{var}(\varepsilon_{ij}^t) = \text{var}(\varepsilon_{ij}^t)/q_i^t$ (assuming zero covariances), indicating that heteroskedasticity occurs when model (23) is estimated by OLS regression, unless the quantities purchased of the various items are equal. The use of WLS regression with $q_i^t$ as weights corrects the problem. Quantity shares $q_i^t / \sum_{i=1}^{N^t} q_i^t$ can be used instead so that the regression weights add up to 1; this normalization does not affect the parameter estimates.

The use of quantity shares is a bit strange in the sense that adding up quantities of heterogeneous items should generally not be done. Instead, we could use expenditure shares $s_i^t = \bar{p}_i^t q_i^t / \sum_{i=1}^{N^t} \bar{p}_i^t q_i^t$ as weights. If the differences in the unit values of the items are relatively small, then the regression results do not change a lot. Also, there are good reasons for expenditure-share weighting in a pooled time dummy hedonic regression, as will be explained in section 3.3. Most importantly, if ordinary unit values rather than their geometric counterparts are used as independent variables, expenditure shares are actually the appropriate weights to adjust for heteroskedasticity. Details can be found in the Appendix.

From now on, we assume that the prices $p_i^t$ are ordinary unit values $(\bar{p}_i^t)$. The parameter estimates from running expenditure-share weighed regressions of model (23) in the two periods are denoted by $\tilde{\alpha}^t$ and $\tilde{\beta}_k^t$ $(t = 0,1)$. The WLS-based predicted prices are

$$\tilde{p}_i^0 = \exp\left[\tilde{\alpha}^0 + \sum_{k=1}^{K} \tilde{\beta}_k^0 z_{ik}\right]; \quad (34)$$

$$\tilde{p}_i^1 = \exp\left[\tilde{\alpha}^1 + \sum_{k=1}^{K} \tilde{\beta}_k^1 z_{ik}\right]. \quad (35)$$

Substituting (34) and (35) into (8), (11), (12) and (15) gives the hedonic single, double, "exact" and full imputation Törnqvist indexes. Using the orthogonality property that the weighted sum of the regression residuals $u_i^t = \ln(p_i^t / \tilde{p}_i^t)$ $(t = 0,1)$ is equal to zero in each period $(\sum_{i \in U^t} s_i^t u_i^t = 0)$, it can be shown that the "exact" hedonic Törnqvist index coincides with the full imputation Törnqvist index (Diewert, Silver and Heravi, 2009; de Haan, 2010). Again, the full imputation index can be written as a characteristics price index:

$$P_{T,FI}^{01} = \exp\left[ (\tilde{\alpha}^1 - \tilde{\alpha}^0) + \sum_{k=1}^{K} (\tilde{\beta}_k^1 - \tilde{\beta}_k^0) \tilde{z}_k^{01} \right], \tag{36}$$

where $\tilde{z}_k^{01} = (\tilde{z}_k^0 + \tilde{z}_k^1)/2$, in which $\tilde{z}_k^0 = \sum_{i \in U^0} s_i^0 z_{ik}$ and $\tilde{z}_k^1 = \sum_{i \in U^1} s_i^1 z_{ik}$ denote the expenditure-share weighted averages of characteristics in period 0 and 1. Similar to what we found for the unweighted case, if $\tilde{\beta}_k^1 = \tilde{\beta}_k^0$ for all *k*, the index then simplifies to

$$P_{T,FI}^{01} = \exp\left(\tilde{\alpha}^1 - \tilde{\alpha}^0\right). \tag{37}$$

Another way to write $P_{T,FI}^{01}$ is

$$P_{T,FI}^{01} = \frac{\prod_{i \in U^1} (p_i^1)^{s_i^1}}{\prod_{i \in U^0} (p_i^0)^{s_i^0}} \exp\left[ \sum_{k=1}^{K} \tilde{\beta}_k^{01} (\tilde{z}_k^0 - \tilde{z}_k^1) \right], \tag{38}$$

where $\tilde{\beta}_k^{01} = (\tilde{\beta}_k^0 + \tilde{\beta}_k^1)/2$.

Expression (38) will be used to compare the full imputation index with the time dummy index, to which we now turn.

### 3.3 Weighted bilateral time dummy hedonic method

Let us now consider the following log-linear hedonic model that can be estimated on the pooled data for periods 0 and 1:

$$\ln p_i^t = \delta^0 + \delta^1 D_i^1 + \sum_{k=1}^{K} \beta_k z_{ik} + \varepsilon_i^t, \tag{39}$$

where the time dummy variable $D_i^1$ has the value 1 if the observation pertains to period 1 and 0 if it pertains to period 1. The characteristics parameters $\beta_k$ are constrained to be fixed across time. The parameter estimates obtained from running an expenditure-share weighted regression on the pooled data of $U^0$ and $U^1$ are denoted by $\tilde{\delta}^0$, $\tilde{\delta}^1$ and $\tilde{\beta}_k$. The weighted time dummy hedonic (TDH) index is given by

$$P_{TDH}^{01} = \exp(\tilde{\delta}^1) = \frac{\tilde{p}_i^1}{\tilde{p}_i^0}, \tag{40}$$

where the predicted prices are now given by

$$\tilde{p}_i^0 = \exp\left[\tilde{\delta}^0 + \sum_{k=1}^{K} \tilde{\beta}_k z_{ik}\right]; \tag{41}$$

$$\tilde{p}_i^1 = \exp\left[\tilde{\delta}^0 + \tilde{\delta}^1 + \sum_{k=1}^{K} \tilde{\beta}_k z_{ik}\right]. \tag{42}$$

Using again the WLS property that the weighted regression residuals sum to zero in each period, it can easily be shown that

$$P_{TDH}^{01} = \prod_{i\in U^0}\left(\frac{\tilde{p}_i^1}{p_i^0}\right)^{s_i^0} = \prod_{i\in U^1}\left(\frac{p_i^1}{\tilde{p}_i^0}\right)^{s_i^1} = \prod_{i\in U^0}\left(\frac{\tilde{p}_i^1}{p_i^0}\right)^{\frac{s_i^0}{2}} \prod_{i\in U^1}\left(\frac{p_i^1}{\tilde{p}_i^0}\right)^{\frac{s_i^1}{2}}. \tag{43}$$

So, the weighted TDH index can be written as an "exact" hedonic Törnqvist index like (14), but with the predicted prices coming from a pooled regression rather than single regressions.

Another way to write the weighted TDH index is

$$P_{TDH}^{01} = \frac{\prod_{i\in U^1}(p_i^1)^{s_i^1}}{\prod_{i\in U^0}(p_i^0)^{s_i^0}} \exp\left[\sum_{k=1}^{K} \tilde{\beta}_k (\tilde{z}_k^0 - \tilde{z}_k^1)\right]. \tag{44}$$

The exponentiated factor in (44) adjusts the ratio of weighted geometric average prices for changes in the weighted average characteristics. From equations (44) and (38), it follows that

$$P_{TDH}^{01} = \exp\left[\sum_{k=1}^{K} (\tilde{\beta}_k - \tilde{\beta}_k^{01})(\tilde{z}_k^0 - \tilde{z}_k^1)\right] P_{T,FI}^{01}. \tag{45}$$

Thus, if $\tilde{z}_k^1 = \tilde{z}_k^0$ for all *k*, i.e. if the expenditure-share weighted average characteristics do not change over time, the weighted time dummy index and full imputation Törnqvist indexes will both be equal to the ratio of expenditure-share weighted geometric average prices. The two indexes also coincide if $\tilde{\beta}_k = \tilde{\beta}_k^{01}$ for all *k*. Diewert, Silver and Heravi (2009) show that this condition holds when $\tilde{\beta}_k^1 = \tilde{\beta}_k^0$ for all *k*, indicating that separate weighted regressions in each period produce the same characteristics coefficients, or if the expenditure-share weighted characteristics variance-covariance matrix is the same in the two periods.

For the bilateral case, two alternative types of expenditure-share weighting have been proposed. Diewert (2005) proposed using the average shares in the two periods for the matched items, i.e. $(s_i^0 + s_i^1)/2$, rather than the shares in the single periods. This is useful because, if the universe is static, the result would be the matched-item Törnqvist index. Following up on this, de Haan (2004a) suggested to choose half the expenditure shares for the unmatched new and disappearing in the periods they are available, i.e. $s_i^0/2$ for $i \in U_D^0$, $s_i^1/2$ for $i \in U_N^1$, and $(s_i^0 + s_i^1)/2$ for $i \in U_M$. He showed that the resulting index is a single imputation Törnqvist price index like (8), but with predicted prices based on the pooled regression.

## 4. Multilateral hedonic methods

When compiling a time series for multiple periods *t*, say $t = 0,..,T$, we could apply the quality-adjustment methods discussed in sections 2 and 3 above and construct bilateral price indexes which compare each period $t = 1,..,T$ directly with the base period 0. The problem is that, due to item churn, the number of matches in the data decreases, and the hedonic indexes are increasingly based on modelling. A solution would be to construct a time series by chaining period-on-period hedonic indexes. This was recommended in the CPI Manual (ILO et al., 2004). However, there is now ample empirical evidence that high-frequency chaining of weighted indexes can lead to huge drift, particularly when storable goods go on sale; see e.g. Feenstra and Shapiro (2003) and Ivancic (2007).

Multilateral methods provide a solution to the chain-drift problem. Multilateral indexes are transitive, hence independent of the choice of base period and free from chain drift. In this section, we will discuss two quality-adjusted multilateral methods: the weighted multi-period time dummy hedonic method and the recently developed (single) imputation GEKS method.

### 4.1 Weighted multilateral time dummy hedonic method

The multilateral TDH approach pools the observations of multiple periods $t = 0,..,T$ instead of just two periods and runs a regression of the multi-period log-linear hedonic model

$$\ln p_i^t = \delta^0 + \sum_{t=1}^{T} \delta^t D_i^t + \sum_{k=1}^{K} \beta_k z_{ik} + \varepsilon_i^t , \qquad (46)$$

where the time dummy variable $D_i^t$ has the value 1 if the observation pertains to period $t$ $(t = 0,..,T)$ and 0 otherwise. The TDH index is found by exponentiating the estimated parameter $\delta^t$. We again assume that expenditure shares are used as weights. Obviously, in a multi-period context, the alternative weighting schemes mentioned above for the bilateral case cannot be applied here. The estimated parameters are denoted by $\hat{\delta}^0$, $\hat{\delta}^t$

$(t=1,..,T)$ and $\widehat{\beta}_k$ $(k=1,..,K)$. Note that the characteristics parameters are now fixed during multiple periods $t=0,..,T$.[11]

The weighted TDH index, i.e. $P_{TDH}^{0t} = \exp(\widehat{\delta}^t) = \widehat{p}_i^t / \widehat{p}_i^0$, can be written as

$$P_{TDH}^{0t} = \prod_{i \in U^0} \left( \frac{\widehat{p}_i^t}{p_i^0} \right)^{s_i^0} = \prod_{i \in U^t} \left( \frac{p_i^t}{\widehat{p}_i^0} \right)^{s_i^t} = \prod_{i \in U^0} \left( \frac{\widehat{p}_i^t}{p_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U^t} \left( \frac{p_i^t}{\widehat{p}_i^0} \right)^{\frac{s_i^t}{2}} \tag{47}$$

or as

$$P_{TDH}^{0t} = \frac{\prod_{i \in U^t} (p_i^t)^{s_i^t}}{\prod_{i \in U^0} (p_i^0)^{s_i^0}} \exp\left[ \sum_{k=1}^{K} \widehat{\beta}_k (\tilde{z}_k^0 - \tilde{z}_k^t) \right]. \tag{48}$$

Equations (47) and (48) are similar to equations (43) and (44) for the weighted bilateral case.

An alternative way of writing the index is (de Haan and Krsinich, 2014b)

$$P_{TDH}^{0t} = \frac{\prod_{i \in U^t} (\widehat{p}_i^{*t})^{s_i^t}}{\prod_{i \in U^0} (\widehat{p}_i^{*0})^{s_i^0}}. \tag{49}$$

where $\widehat{p}_i^{*0} = p_i^0 / \exp(\sum_{k=1}^{K} \widehat{\beta}_k z_{ik})$ and $\widehat{p}_i^{*t} = p_i^t / \exp(\sum_{k=1}^{K} \widehat{\beta}_k z_{ik})$ are estimated quality-adjusted prices. Thus, expression (49) writes the index as the ratio of expenditure-share weighted geometric averages of estimated quality-adjusted prices. Because the use of chained superlative price indexes has been advocated in the CPI Manual, it will be useful to show what drives the difference between the weighted TDH index and the chained matched-item Törnqvist price index. Due to transitivity, (49) can be written as a period-on-period chained index:

$$P_{TDH}^{0t} = \prod_{\tau=1}^{t} \left[ \frac{\prod_{i \in U^\tau} (\widehat{p}_i^{*\tau})^{s_i^\tau}}{\prod_{i \in U^{\tau-1}} (\widehat{p}_i^{*\tau-1})^{s_i^{\tau-1}}} \right]. \tag{50}$$

[11] This is obviously a rather strong assumption. Von Auer and Brennan (2006) proposed an estimation procedure for a hedonic model where the characteristics parameters vary between some of the adjacent periods while they are assumed fixed between other adjacent periods. They criticised the period-on-period chained time dummy approach for being inconsistent in that if characteristics parameters are assumed fixed for all adjacent periods, they are actually assumed fixed across the whole sample period (which is a valid point). Their approach does obviously not lead to transitive price indexes, and so we will not discuss it here.

Let us focus on a single chain link, i.e. the bracketed factor in (50). Each chain link can be decomposed into four components, as follows[12]

$$\frac{P_{TDH}^{0t}}{P_{TDH}^{0,t-1}} = \prod_{i \in U_M^{t-1,t}} \left( \frac{p_i^t}{p_i^{t-1}} \right)^{\frac{s_{iM}^{t-1}+s_{iM}^{t}}{2}} \left[ \frac{\prod_{i \in U_N^{t-1,t}} (\hat{p}_i^{*t})^{s_{iN}^t}}{\prod_{i \in U_M^{t-1,t}} (\hat{p}_i^{*t})^{s_{iM}^t}} \right]^{s_N^t} \left[ \frac{\prod_{i \in U_D^{t-1,t}} (\hat{p}_i^{*t-1})^{s_{iD}^{t-1}}}{\prod_{i \in U_M^{t-1,t}} (\hat{p}_i^{*t-1})^{s_{iM}^{t-1}}} \right]^{-s_D^{t-1}} \frac{\prod_{i \in U_M^{t-1,t}} (\hat{p}_i^{*t})^{\frac{s_{iM}^t - s_{iM}^{t-1}}{2}}}{\prod_{i \in U_M^{t-1,t}} (\hat{p}_i^{*t-1})^{\frac{s_{iM}^{t-1} - s_{iM}^{t}}{2}}} . \quad (51)$$

In (51), $U_M^{t-1,t} = U^{t-1} \cap U^t$ is the set of (matched) items purchased in both period *t*-1 and period *t*, $U_D^{t-1,t}$ is the set of (disappearing) items purchased in period *t*-1, and possibly in other periods as well, but not in period *t*, and $U_D^{t-1,t}$ is the set of (new) items purchased in period *t*, and perhaps also in other periods, but not in period *t*-1; $s_M^{t-1} = \sum_{i \in U_M^{t-1,t}} s_i^{t-1}$ and $s_M^t = \sum_{i \in U_M^{t-1,t}} s_i^t$ denote the aggregate expenditure shares of the matched items in periods $t-1$ and *t*, and $s_D^{t-1} = \sum_{i \in U_D^{t-1,t}} s_i^{t-1} = 1 - s_M^{t-1}$ and $s_N^t = \sum_{i \in U_N^{t-1,t}} s_i^t = 1 - s_M^t$ denote the aggregate expenditure shares of the disappearing items and new items. The item-specific expenditure shares in equation (51) have been normalized: $s_{iM}^{t-1} = s_i^{t-1} / s_M^{t-1}$ and $s_{iM}^t = s_i^t / s_M^t$ are the matched items' normalized shares in *t*-1 and *t*, and $s_{iD}^{t-1} = s_i^{t-1} / s_D^{t-1}$ and $s_{iN}^t = s_i^t / s_N^t$ are the normalized shares for the unmatched (new and disappearing) items, with $\sum_{i \in S_M^{t-1,t}} s_{iM}^{t-1} = \sum_{i \in S_M^{t-1,t}} s_{iM}^t = \sum_{i \in U_D^{t-1,t}} s_{iD}^{t-1} = \sum_{i \in U_N^{t-1,t}} s_{iN}^t = 1$.

The first component of (51), $\prod_{i \in S_M^{t-1,t}} (p_i^t / p_i^{t-1})^{(s_{iM}^{t-1} + s_{Mi}^t)/2}$, is the adjacent-period matched item Törnqvist index. The second and third components describe the effects of disappearing and new items, respectively. When there are no new or disappearing items between periods $t-1$ and *t*, $s_N^t = s_D^{t-1} = 0$ and the chain link is equal to the product of the matched-model Törnqvist index and the fourth component of (51). Since the TDH index is transitive but the Törnqvist index is not, we can view the fourth component as a factor that eliminates potential drift in the chained Törnqvist index.

### 4.2 Quality-adjusted GEKS method

Multilateral index number methods have typically been applied to compare price levels across countries. The resulting indexes are transitive. Transitivity is desirable for spatial price comparisons since the results then are independent of the choice of base country. For details on the many methods, see e.g. Balk (1996, 2001), chapter 7 in Balk (2008), and Diewert (1999). Multilateral spatial index number methods can be easily adapted to

[12] For details, see de Haan and Hendriks (2013). They derived the decomposition for the weighted Time-Product Dummy index (which will be explained in section 6), but it equally holds for the weighted TDH index.

the intertemporal context so that the resulting indexes are independent of the choice of base period, hence free from chain drift.

The (intertemporal) GEKS (Gini, 1931; Eltetö and Köves; 1964; Szulc, 1964) method is based on "transitivizing" a set of bilateral price indexes between all pairs of time periods across the sample period (or window) $t = 0,..,T$.[13] In its original form, the GEKS index is not quality-adjusted because it relies on matched-item bilateral indexes. Following de Haan and Krsinich (2014a), we will discuss a generalization of the GEKS index that is quality adjusted.

The GEKS index between periods 0 and *t* is calculated as the geometric mean of the ratios of the matched-item bilateral price indexes $P^{0,l}$ and $P^{t,l}$, where each period *l* is taken as the base. If the bilateral indexes satisfy the time reversal test (requiring that when the base period and the comparison period are reversed, the index should be the reciprocal of the original index), the GEKS index can be written as (Ivancic, Diewert and Fox, 2011; De Haan and Van der Grient, 2011)

$$P_{GEKS}^{0t} = \prod_{l=0}^{T} \left[ \frac{P^{0,l}}{P^{t,l}} \right]^{\frac{1}{T+1}} = \prod_{l=0}^{T} \left[ P^{0,l} \times P^{l,t} \right]^{\frac{1}{T+1}} . \qquad (52)$$

In its original form, the GEKS index (52) makes use of bilateral Fisher indexes, but we will use Törnqvist indexes instead for reasons given below.[14]

The choice of window length remains a point of concern. Ivancic, Diewert and Fox (2011) argue that a 13-month (or 5-quarter) window is optimal as it is the shortest window that can deal with strongly seasonal goods. Enlarging the window would lead to a loss of characteristicity in that recent price movements will be increasingly affected by prices and price changes in the distant past.[15]

To explicitly adjust for quality change, i.e. to incorporate new and disappearing items, de Haan and Krsinich (2014a) proposed to use bilateral hedonic imputation price

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[13] The other multilateral methods attain transitivity in some other way. These methods include the Geary-Khamis method (Geary, 1958; Khamis, 1972) and the Country-Product Dummy method (Summers, 1973).

[14] This was first proposed by also Caves, Christensen and Diewert (1982), and the approach is therefore also known as the CCD method.

[15] It is possible to formulate weighted GEKS indexes that take into account the reliability of the bilateral price indexes; see e.g. Rao (1999) (2001). Melser (2016) proposed a weighted GEKS approach in which the weights are dependent on the degree of matching of the items, for example in terms of expenditure shares. Here, the choice of window length is not such a big issue since less reliable bilateral indexes will be down-weighted. This potentially enables the use of a longer window.

indexes rather than matched-item price indexes as inputs in the GEKS procedure. More specifically, they proposed using de Haan's (2004a) suggestion mentioned in section 3.3 to estimate bilateral single imputation Törnqvist price indexes by estimating weighted bilateral time dummy hedonic indexes with a specific set of expenditure share weights. That is, to construct a time series going from 0 to *T*, weighted time dummy hedonic regressions are run on the pooled data of the (two) periods 0 and *l* and the periods (two) periods *l* and *t*, where $l = 0,..,T$ and $t = 0,..,T$. The weights used in, for example, the bilateral regressions between 0 and *l* are $(s_i^0 + s_i^l)/2$ for $i \in U_M^{0l}$, $s_i^0/2$ for $i \in U_D^{0(l)}$, and $s_i^l/2$ for $i \in U_N^{l(0)}$, where $U_M^{0l}$ denotes the subset of items which are purchased in both period 0 and period *l*, $U_D^{0(l)}$ is the subset of items that is purchased in period 0 but not in period *l*, and $U_N^{l(0)}$ is the subset of items that is purchased in period *l* but not in period 0.

The resulting single imputation Törnqvist GEKS price index is quality-adjusted, transitive and preserves all of the matches in the data across the sample period $0,...,T$. Moreover, as it is (implicitly) based on single imputations, all the observed prices are preserved. An advantage of this approach compared with the multilateral time dummy hedonic approach is that, due to the use of superlative (imputation) price indexes, it is grounded in standard index number theory. For example, in a matched-item context, the GEKS index satisfies the identity test (requiring a price index to be equal to 1 when the prices of all items in period *t* are equal to those in period 0), whereas the time dummy index violates this test. A practical disadvantage is its complexity and the fact that many bilateral time dummy regressions must be run. The multilateral time dummy index is much easier to construct.

The two methods rely on the assumption that the characteristics parameters are fixed across the entire sample period $t = 0,..,T$. As mentioned earlier, it is not possible to relax this assumption in a multilateral context.

## 5. Quality-adjusted unit value indexes

Von Auer (2014) showed that many conventional matched-item price indexes belong to, what he calls, the family of generalized unit value indexes. In the matched-item, static universe context, with $U^0 = U^t = U$, a generalized unit value index between periods 0 and $t$ $(t = 0,...,T)$ is defined as the ratio of the value index, $V^{0t}$, and a quantity index, $Q^{0t}$, given by

$$Q^{0t} = \frac{\sum_{i \in U} \lambda_{i/b} q_i^t}{\sum_{i \in U} \lambda_{i/b} q_i^0}. \tag{53}$$

That is, the quantity index is defined as the ratio of standardized quantities, where the $\lambda_{i/b}$ express the quantities purchased of each item *i* in terms of units of an arbitrary base item *b*, as was also done in section 2.1. The resulting generalized unit value index is

$$P_{GUV}^{0t} = \frac{V^{0t}}{Q^{0t}} = \frac{\sum_{i \in U} p_i^t q_i^t \Big/ \sum_{i \in U} p_i^0 q_i^0}{\sum_{i \in U} \lambda_{i/b} q_i^t \Big/ \sum_{i \in U} \lambda_{i/b} q_i^0} = \frac{\left[ \sum_{i \in U} s_i^t (p_i^{*t})^{-1} \right]^{-1}}{\left[ \sum_{i \in U} s_i^0 (p_i^{*0})^{-1} \right]^{-1}}. \tag{54}$$

If $\lambda_{i/b} = 1$ for all *i*, meaning that all items are essentially equivalent from the consumers' point of view, then the generalized unit value index simplifies to the ordinary unit value index.

As mentioned before, standard economic theory suggests that we could measure the $\lambda_{i/b}$ by relative prices. For $\lambda_{i/b} = p_i^0 / p_b^0$, the index turns into a Paasche-type price index for a direct comparison between period 0 and period *t*; the corresponding quantity measure is a Laspeyres-type quantity index. Because the $\lambda_{i/b}$, which can be viewed as relative "reference prices", are fixed across the sample period, the generalized unit value index is transitive, in contrast to the Paasche price index itself. For later use, in the last expression on the right-hand side of equation (54) the generalized unit value index is written as a ratio of harmonic averages of quality-adjusted prices $p_i^{*0} = p_i^0 / \lambda_{i/b}$ and $p_i^{*t} = p_i^t / \lambda_{i/b}$.

The generalized unit value approach can be easily adapted to a dynamic universe of items. Following Dalén (2001) and de Haan (2004b), we will now call it a quality-adjusted rather than generalized unit value approach since new and disappearing items are explicitly included. Assuming again constant quality-adjustment factors $\lambda_{i/b}$, the quantity index defined on (the union of) $U^0$ and $U^t$ is

$$Q^{0t} = \frac{\sum_{i \in U^t} \lambda_{i/b} q_i^t}{\sum_{i \in U^0} \lambda_{i/b} q_i^0} = \frac{\sum_{i \in U_M^{0t}} \lambda_{i/b} q_i^t + \sum_{i \in U_N^{t(0)}} \lambda_{i/b} q_i^t}{\sum_{i \in U_M^{0t}} \lambda_{i/b} q_i^0 + \sum_{i \in U_D^{0(t)}} \lambda_{i/b} q_i^0}, \tag{55}$$

where $U_M^{0t}$ is the subset of items which are purchased in both period 0 and period *t*, $U_D^{0(t)}$ is the subset of items that is purchased in period 0 but not in period *t*, and $U_N^{t(0)}$ is the subset of items that is purchased in period *t* but not in period 0. Notice that equation (55) is the multi-period counterpart to equation (2) for the two-period case. A single imputation Laspeyres-type quantity index would be found by substituting $\lambda_{i/b} = p_i^0 / p_b^0$ for $i \in U_M^{0t}$ and $U_D^{0(t)}$, and $\lambda_{i/b} = \hat{p}_i^0 / p_b^0$ for $i \in U_N^{t(0)}$ (assuming, as before, that the base item $b$ belongs to the matched set $U_M^{0t}$). Double, "exact" and full imputation approaches are of course also possible.

The quality-adjusted unit value index becomes

$$P_{QAUV}^{0t} = \frac{V^{0t}}{Q^{0t}} = \frac{\sum_{i \in U^t} p_i^t q_i^t \Big/ \sum_{i \in U^0} p_i^0 q_i^0}{\sum_{i \in U^t} \lambda_{i/b} q_i^t \Big/ \sum_{i \in U^0} \lambda_{i/b} q_i^0} = \frac{\left[ \sum_{i \in U^t} s_i^t (p_i^{*t})^{-1} \right]^{-1}}{\left[ \sum_{i \in U^0} s_i^0 (p_i^{*0})^{-1} \right]^{-1}}. \tag{56}$$

Since the $\lambda_{i/b}$ are kept fixed across the entire sample period, the quality-adjusted unit value index is transitive. Another useful property is that, just like the generalized unit value index, the index simplifies to the ordinary unit value index for $\lambda_{i/b} = 1$ for all $i$, i.e. when the product is perfectly homogeneous. A disadvantage of the quality-adjusted unit value approach, which it shares with the weighted TDH approach, is that the index violates the identity test (in the static universe context). However, it can be argued that, because standardization ensures full homogeneity across items, the quality-adjusted unit value is the appropriate concept of price so that axioms or tests are not relevant at this elementary level of aggregation.

The quality-adjustment/standardization factors $\lambda_{i/b}$ in (56) are fixed across time and hence can be estimated using the expenditure-share weighted TDH method. This is equivalent to using $\widehat{p}_i^{*0} = p_i^0 / \exp(\sum_{k=1}^K \widehat{\beta}_k z_{ik})$ and $\widehat{p}_i^{*t} = p_i^t / \exp(\sum_{k=1}^K \widehat{\beta}_k z_{ik})$, defined below equation (49), as estimates of the quality-adjusted prices $p_i^{*0}$ and $p_i^{*t}$ in (56). So the (estimated) quality-adjusted unit value index becomes

$$P_{QAUV}^{0t} = \frac{\left[ \sum_{i \in U^t} s_i^t (\widehat{p}_i^{*t})^{-1} \right]^{-1}}{\left[ \sum_{i \in U^0} s_i^0 (\widehat{p}_i^{*0})^{-1} \right]^{-1}}. \tag{57}$$

Equation (57) is similar to (49), the only difference being that the quality-adjusted unit value index is the ratio of expenditure-share weighted harmonic rather than geometric averages of the estimated quality-adjusted prices.

From Jensen's inequality we know that weighted harmonic means are smaller than the corresponding geometric means unless there is no variability in the data and the two means coincide. This points towards the dispersion of the quality-adjusted prices or, equivalently, the dispersion of the regression residuals, as the driver of the difference between the two indexes. Based on Taylor linearization, de Haan and Krsinich (2014b) derived the following result:

$$P_{QAUV}^{0t} \cong P_{TDH}^{0t}\left[\frac{1+\frac{1}{2}(\sigma^{0})^{2}}{1+\frac{1}{2}(\sigma^{t})^{2}}\right]. \tag{58}$$

where $(\sigma^{0})^{2}=\sum_{i\in U^{0}} s_{i}^{0}(u_{i}^{0})^{2}$ and $(\sigma^{t})^{2}=\sum_{i\in U^{t}} s_{i}^{t}(u_{i}^{t})^{2}$ denote the weighted variances of the residuals from the WLS regression in periods 0 and *t*. Thus, the variance of the regression residuals or, equivalently, the dispersion of the quality-adjusted prices, is the main driver of the difference between the two indexes.[16]

Expression (58) indicates that the quality-adjusted unit value index will sit below (above) the time dummy index when the variance of the residuals increases (decreases) over time. Due to the logarithmic functional form for the hedonic model, this type of heteroskedasticity is unlikely to occur. In a hedonic model with price rather than log of price as the dependent variable, the absolute errors tend to grow over time when there is inflation. The logarithmic transformation neutralizes this tendency (Diewert, 2004), and we therefore expect the two indexes to have similar trends and volatility.[17] In fact the expenditure-share weighted time dummy index can be seen as an approximation, and indeed an accurate one, to a quality-adjusted unit value index.

## 6. An empirical example for TVs

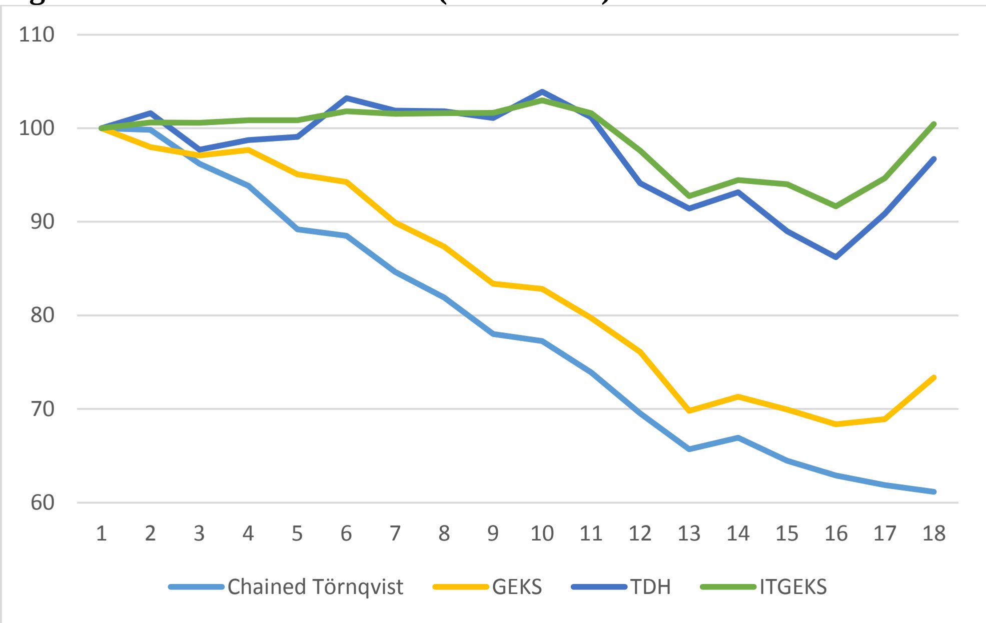
For an empirical illustration, we use 18 months of scanner data on TVs sold by a Dutch retailer. Items are identified by EAN (European Article Number), the European version of GTIN. Figure 1 shows four different price indexes: the chained Törnqvist index, the (Törnqvist-type) GEKS index, the hedonic imputation (Törnqvist-type) GEKS index, and the weighted TDH index. As expected, the chained Törnqvist appears to suffer from downward drift. The GEKS procedure adjusts for the chain drift, and the GEKS index indeed sits above the chained Törnqvist index.

[16] For a discussion on the difference between unweighted price indexes at the elementary level in terms of price dispersion and product heterogeneity, see Silver and Heravi (2007b).

[17] Empirical work by de Haan and Krsinich (2014b) on scanner data for several consumer electronics goods showed that the differences between the two types of index were negligible. Given this result, we decided not to estimate the quality-adjusted unit value index in the empirical section 6 below.

The most striking result is the huge difference the inclusion of unmatched items makes. According to the hedonic imputation GEKS index, quality-adjusted TV prices remained virtually constant across the 18-month period, with a temporary dip between months 12 and 17. The weighted time dummy hedonic index exhibits a similar, albeit slightly more volatile, pattern. Decomposition (51) suggests that the difference between the weighted time dummy index and the chained Törnqvist price index stems from the (expenditure-share weighted) average quality-adjusted prices of new items being above those of the matched items or the average quality-adjusted prices of disappearing items being relatively low. The pricing strategy followed by the retailer, or the manufacturers, is apparently characterized by price skimming and inventory cleaning (or dumping); see also Silver and Heravi (2005).

**Figure 1: Price indexes for TVs (EAN based)**




All the explanatory variables in the multilateral time dummy hedonic model (and the bilateral time dummy models for the imputation Törnqvist GEKS price index) are dummy variables. The following attributes were used: brand (6 categories, including an “inferior brands” category), screen type (2 categories), screen size (7 categories), screen curvature (2 categories), resolution (3 categories), energy class (4 categories), Dlna (2 categories; yes/no), 3D (2 categories; yes/no), Internet (2 categories; yes/no), video on demand (2 categories; yes/no), processor type (4 categories), and satellite receiver (2 categories; yes/no). R squared for the multilateral time dummy model was 0.942, which is extraordinarily high.

Instead of identifying items by EAN, they can be identified by the characteristics they have. In the latter case, goods with the same (quantities of) characteristics but with different EANs will be treated as a single homogeneous item. We cross-classified all the categorical variables, giving $6 \times 2 \times 7 \times 2 \times 3 \times 4 \times 2 \times 2 \times 2 \times 2 \times 4 \times 2 = 258{,}048$ potential combinations. Many, if not most, combinations are not feasible to produce or sell. Our data set contains xxxx different combinations. Prices are now calculated as unit values across all the EANs belonging to a particular combination of characteristics. Figure 2 shows the four types of price indexes when items are identified in this way. As might be expected, the weighted TDH index is hardly affected. The same goes for the imputation Törnqvist GEKS index, even though the original GEKS index does change quite a bit; the downward bias in the characteristics-based GEKS index is smaller than in the EAN-based version.

**Figure 2: Price indexes for TVs (characteristics based)**

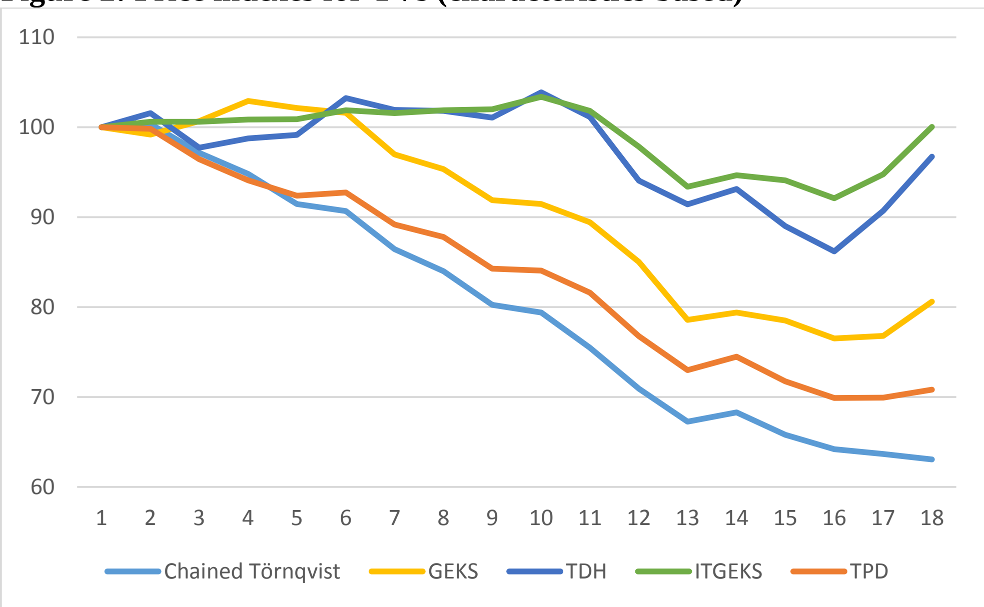



Figure 2 shows a fifth price index, the expenditure-share weighted Time Product Dummy (TPD) index. This multilateral price index is the intertemporal counterpart to the Country Product Dummy index proposed by Summers (1973) for price comparisons across countries. The TPD index is constructed using a time dummy regression model which, instead of characteristics, includes dummy variables for the different items found in the data set as the only explanatory variables. That is, the hedonic effects $\sum_{k=1}^{K} \beta_k z_{ik}$ in equation (46) for the multilateral time dummy hedonic model are replaced by item fixed effects $\gamma_i$, yielding the pseudo hedonic model

$$\ln p_i^t = \delta^0 + \sum_{t=1}^{T} \delta^t D_i^t + \sum_{i=1}^{N-1} \gamma_i D_i^t + \varepsilon_i^t , \qquad (59)$$

where $N$ is again the total number of items in the data set; the dummy for item $N$ is left out to identify the model as we included an intercept term.[18] After estimating model (59) on the pooled data by expenditure-share weighted regression, the TPD index is found by exponentiating the estimated time dummy parameters, i.e. $P_{TPD}^{0t} = \exp(\hat{\delta}_{(TPD)}^t)$, similar to the TDH approach. As can be seen from Figure 1, this model does not perform well for our TV data: the TPD index is significantly lower than the TDH index.

A similar result was found by De Haan, Hendriks and Scholz (2016) for men's T-shirts.[19] They explained it as follows. When identifying items by their characteristics, the TPD model is equivalent to the "saturated" TDH model which includes all first and higher order interaction terms observed in the data along with the main effects, as was shown by Krsinich (2016). In a hedonic model, one would only include main effects – like we did in the empirical example – and perhaps some first-order interaction terms. The TPD model thus includes many irrelevant variables (interaction terms) and suffers from overfitting; it fits the outliers, and so unduly raises R squared. In other words, the TPD approach "distorts the regression residuals towards zero". In terms of equation (51) this means that the effect of new and disappearing items is understated as compared with the TDH index. Under a price skimming and inventory cleaning pricing strategy, and with prices of matched items continually falling, this leads to a strongly decreasing TPD index.

## 7. Conclusions

For products exhibiting significant item churn and quality change, and because period-on-period chaining of weighted indexes potentially leads to drift, multilateral hedonic regression methods may be considered to measure quality-adjusted price change. In this paper, we discussed three such methods: the single hedonic imputation Törnqvist GEKS method, the weighted time dummy hedonic method, and the quality-adjusted unit value method (based on predicted prices from the weighted time dummy hedonic method).

[18] As far as we know, Balk (1981) was the first to use this approach for constructing price indexes over time. Other early applications are Kokoski et al. (1999) and Aizcorbe, Corrado and Doms (2003). Note that they all used unweighted regressions.

[19] Greenlees and McClelland (2010) compared a (rolling-year) GEKS index with a TDH index for misses' tops and similarly found a strong downward bias in the GEKS index.

The three methods assume time fixity of the characteristics parameters, which is quite restrictive. A practical disadvantage of multilateral methods is revision of previously estimated index numbers when extending the sample period, something which statistical agencies generally do not accept. A rolling-window approach addresses this issue and will also mitigate the time-fixity issue because because the parameters are continuously updated.

## Appendix: expenditure shares as weights in time dummy regressions

(Work in progress)

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