

**Sectoral holdings of notes and coins in the UK:  
The effect of the crisis commencing in 2007**

Dr. David Barlow  
Lecturer in Economics,  
Newcastle University Business School,  
5 Barrack Rd,  
Newcastle-upon-Tyne,  
UK, NE1 4SE

Tel. +44 0191 208 1664  
Email david.barlow@ncl.ac.uk

**Abstract**

One consequence of the recent global financial and economic crisis for the UK has been that the share of M1 held as notes has increased, contrary to the trend since the mid-1970s. To investigate this, error feedback equations for the holdings of notes and coins by UK households and by UK non-financial corporations are initially estimated for the period up to 2007Q2 (the quarter prior to the rescue of Northern Rock). The cointegrating relationships yield an income elasticity of near to unit with plausible interest (semi) elasticity. When the functional form for the interest rate is semi-log the equations are found to be stable when the period is extended to include the period since the near collapse of Northern Rock (2007Q3 to 2012Q4). But for the double log specification (constant interest rate elasticity) there is evidence of instability, in particular a considerable reduction in the rate at which money balances return to equilibrium. Overall the evidence suggests that if we allow for the non-constant interest rate elasticity the behaviour of the holdings of notes can largely be explained by the extraordinarily low interest rates. Inspection of the components of M1 reveals that the impact of the crisis is much more pronounced for bank deposits than for notes. This effect is particularly strong in the household sector for which there has been a large fall in holdings of interest bearing sight deposits and a rise in non-interest bearing sight deposits greater than the increase in holdings of notes and coins.

## **Introduction**

In the UK the 2007 credit crunch became evident to retail bank customers with the run on Northern Rock in September 2007. The Bank of England, like many other central banks, responded to the intensifying economic financial crisis with marked reductions in its base rate over 2008/9 which since June 2009 has been held at an all-time low of 0.5%. With the policy rate close to the zero bound the Bank was compelled to adopt 'extra-ordinary' monetary policy measures, such as 'quantitative easing'.

During this crisis individuals and corporations have significantly reversed a long term trend by increasing the share of M1 that they hold as notes. Can this increase in holdings of notes be explained by a 'standard' money demand function or are additional variables required, such as was the case to account for the effects of financial innovation in the 1970s and 1980s? Given that this changed behaviour coincides with the unusually low interest rate one issue to consider is the specification of the relationship between money holdings and the rate of interest.

The traditional money demand function takes the 'semi-log' form in which all variables except the interest rate are expressed as logarithms. The response of money to a change in the interest rate is then referred to as semi-elasticity, a response to the absolute change. If agents respond to the relative change in the interest rate then the interest rate should be measured in logarithms (these specifications are referred to as log-log). In the latter specification the interest elasticity of money demand is constant, where-as in the former it is decreasing as the interest rate reduces. Lucas (2000) argues that on theoretical grounds the log-log specification should be preferred, for example such money demand functions can be derived from general equilibrium models in which money balances are held to increase leisure time. Chadha et al. (1998) argue that theoretically consistent preferences and transactions technology would lead to a logarithmic interest rate specification.

At higher rates of interest the two specifications tend to perform equally well. But at low interest rates the performances can vary greatly because the log-log implies infinitely large money balances as the

nominal interest rate approaches zero, where-as the semi-log has a finite satiation point. Janssen (1998) for UK M0 reports a marginal preference for the log-log specification. But note that this applies to a period in which the Treasury rate never fell below 4.3%. In contrast since 2009 the Treasury bill rate has not exceeded 1%<sup>1</sup>.

## Review

Baumol (1952) and Tobin (1956) provide the theoretical underpinnings of money demand analysis by considering the optimal inventory of money held to smooth the lack of synchronisation between income receipts and spending. These theoretical models clearly apply to non-interest bearing money, and show how the balances of such money held for the transactions motive vary positively with income and the cost of converting bonds into the medium of exchange, and negatively with the return on bonds. These models have strong predictions for values of the interest elasticity, income elasticity and conversion cost elasticity. In particular, the models predict economies of scale. There are, however, many reasons to not expect these strong predictions to hold<sup>2</sup>.

Empirical studies of the demand for notes and coins, either in aggregate or by sector, are much rarer than for broader aggregates such as M1, M3 and M4<sup>3</sup>. Research on M0 is more relevant, as M0 consists of holdings of notes and coins by the household sector and the non-financial corporate sector plus commercial banks operational balances at the Bank of England<sup>4</sup>. Studies of M0 include Hacche (1974), Artis and Lewis (1984), Thomas (1996) and Khadaroo (2003). Janssen (1998) reviews a number of specification issues regarding the demand for M0. Following World War 2 and until the early 1990s the velocity of M0 followed a positive trend. This trend meant that models in which the demand for money depends only upon a scale variable (income or spending) and an interest rate

---

<sup>1</sup> Lucas (2000) uses US M1 data to show plots of the money-income ratio and interest rate that support the log-log specification. However, using more recent data that includes a period of lower interest rates than in Lucas's sample Ireland (2009) presents plots (and econometric evidence) that favour the semi-log specification.

<sup>2</sup> One reason is that for those on low incomes the cost of converting income to bonds and then back into money can exceed the interest earned. Also if the timing of agents' income receipts closely matches their spending pattern they may not hold bonds sufficiently long to make bond holding worthwhile.

<sup>3</sup> For example Hendry and Ericsson (1991) and Ericsson (1998). For sectoral analysis see Drake and Chrystal 1994 and 1997.

<sup>4</sup> Until quite recently, household sector holdings of cash accounted for more than 90% of M0. However, the Bank of England ceased reporting M0 from 2006 in anticipation of a significant rise in the commercial banks' holdings of reserves following a change in the way the Bank remunerated reserves. As a consequence of quantitative easing bank reserves now account for about 80% of the equivalent of M0.

performed poorly. Widely accepted explanations are based upon changes in spending patterns and transactions technology<sup>5</sup> that improve the synchronisation between cash holdings and expenditure that reduced the optimal scale of the cash inventory.

## Data and Estimation

The standard long run money demand function, originating from Cagan (1956), is given by:

$$m_t = \alpha_0 + \alpha_1 R_t, \quad (1)$$

and the log-log money demand function, based on Meltzer (1963), is given by:

$$m_t = \beta_0 + \beta_1 r_t, \quad (2)$$

Where  $m$  is the logarithm of the ratio of money to GDP,  $R$  is the interest rate,  $r$  is the log of the interest rate,  $t$  denotes time, and  $\alpha_1 < 0$  and  $\beta_1 < 0$ .

The data for money are the holdings of notes and coins of UK household and non-financial corporate sectors.  $R$  is the 3 month Treasury bill rate. Figure 1 plots the holdings of notes and coins relative to M1 for both sectors, and shows that around 2007/8 there is a clear reversal of the tendency for this ratio to decline. The initial estimation period is 1990q3 to 2007q2. The latter period is selected as being the quarter prior to Northern Rock's problems becoming public; the period is then extended to 2012Q4 to test for stability. The starting period is more problematic.

For both sectors the velocity of notes exhibits a secular upward trend until the end of the 1980s (see figure 2). A number of papers published over the 1980s and 1990s report that money balances, income and an interest rate require measures of financial innovation to constitute empirically valid money demand functions<sup>6</sup>. However, there is some consensus that financial innovation in cash management moderated greatly from around 1990<sup>7</sup>, as indicated by the income velocity ceasing to rise. Janssen (1998) argues that models of M0 that incorporate measures of financial innovation

---

<sup>5</sup> The increased velocity may also have been due to higher inflation. One reason for this could be lags in the adjustment of the nominal interest rate to expected inflation. A second reason could be the substitution of M0 by durable goods. An additional explanation stems from the positive correlation between inflation and its variability. A number of papers find model performance improves with the addition of inflation. Janssen (1998), for example, incorporates the standard deviation of inflation.

<sup>6</sup> For example, the number of ATMs, the proportion of the population with current accounts and the number of credit cards (see Johnston (1984) and Janssen (1998). Hall et al (1989) and Walton and Westaway (1991) endogenise financial innovation by using a cumulated interest rate. Financial innovation went beyond technology improving access to bank deposits; a further significant innovation was interest bearing sight deposits which increased rapidly over the 1980s from near zero at the start of the decade to almost 80% of sight deposits by the end of the decade.

<sup>7</sup> See for example Grant *et al* (2004)

performed less well once they were applied to data that included the 1990s and that there is evidence that the shift to a lower inflation environment caused a shift in the demand for M0<sup>8</sup>. To avoid the estimates depending upon how these shifts are modelled the estimation period starts at 1990q3<sup>9</sup>.

The estimation strategy is to first determine the orders of integration of the data. Then use Johansen's (1988) maximum likelihood procedure to test if equations (1) and (2) constitute cointegrating relationships. If a single cointegrating vector cannot be rejected then the values of  $\alpha_1$  and  $\beta_1$  are calculated using ARDL equations, such as 3 and 4 below, estimated over the period to 2007Q2.

$$\Delta M_t = \theta_0 + \sum_{j=1}^n \theta_{1,j} \Delta M_{t-j} + \sum_{j=1}^n \delta_{2,j} \Delta R_{t-j} + \sum_{j=1}^n \delta_{3,j} \Delta Y_{t-j} + \delta_4 (M - Y)_{t-1} + \theta_5 R_{t-1} + \varepsilon_t \quad (3)$$

$$\Delta M_t = \gamma_0 + \sum_{j=1}^n \gamma_{1,j} \Delta M_{t-j} + \sum_{j=1}^n \gamma_{2,j} \Delta r_{t-j} + \sum_{j=1}^n \gamma_{3,j} \Delta Y_{t-j} + \gamma_4 (M - Y)_{t-1} + \gamma_5 r_{t-1} + \sigma_t \quad (4)$$

Where  $M$  is the logarithm of holdings of notes and coins deflated by the consumer price deflator,  $Y$  is the logarithm of real GDP,  $R$  is the 3 month Treasury bill rate<sup>10</sup>,  $r$  is the logarithm of  $R$ ,  $\Delta$  is the first difference operator,  $\sigma$  and  $\varepsilon$  are random errors, and  $t$  denotes time. The long run interest rate semi-elasticity ( $\alpha_1$ ) is calculated as  $\delta_5/\delta_4$ , and the long run interest rate elasticity ( $\beta_1$ ) as  $\gamma_5/\gamma_4$ . Notice that though the long run income elasticity is constrained to unity this does not apply to the short run income elasticity<sup>11</sup>. The estimated long run parameters are then used to calculate the corresponding cointegrating residuals, EC1 and EC2, to use to obtain parsimonious equations from the following general equations that permit contemporaneous responses.

$$\Delta M_t = \theta_0 + \sum_{j=1}^n \theta_{1,j} \Delta M_{t-j} + \sum_{j=0}^n \delta_{2,j} \Delta R_{t-j} + \sum_{j=0}^n \delta_{3,j} \Delta Y_{t-j} + \delta_4 EC1_{t-1} + \varepsilon_t \quad (5)$$

$$\Delta M_t = \gamma_0 + \sum_{j=1}^n \gamma_{1,j} \Delta M_{t-j} + \sum_{j=1}^n \gamma_{2,j} \Delta r_{t-j} + \sum_{j=1}^n \gamma_{3,j} \Delta Y_{t-j} + \gamma_4 EC2_{t-1} + \sigma_t \quad (6)$$

---

<sup>8</sup> Khadaroo (2003) models M0 using a smooth transition equation in which the transition variable is the quarterly change in the interest rate one year previously.

<sup>9</sup> If the estimation period starts earlier there is strong evidence of serial correlation and non-cointegration for whichever interest rate specification is adopted. The study by Chadha *et al.* (1998) using annual UK data for 1870 to 1994, illustrates the problem of using long runs of monetary data as they report that evidence for cointegration between M0, real GDP and the opportunity cost is 'not overwhelming'.

<sup>10</sup> Data for balances of notes and coins and for the Treasury bill rate are from the Bank of England. Data for the GDP deflator and GDP are from the US Federal Reserve Economic Data. The estimating equations also include quarterly dummies.

<sup>11</sup> If the Johansen test is applied to a vector containing money, income and the interest rate or log (interest rate) rather than the money income ratio then a single cointegrating vector cannot be rejected and the parameter on income is very close to unit. This is in marked contrast to studies that cover the 1970s and 1980s in which the unit value is only found when proxies for financial innovation are included.

## Results

Using both the Augmented Dickey-Fuller test and the Phillips-Peron test a single unit root could not be rejected for  $m$ ,  $R$ ,  $r$ ,  $M$  and  $Y$ . For the Johansen procedure, information criteria suggested using 2 lags of the differenced terms for the household sector and 4 lags for the corporate sector. For both sectors a single cointegrating vector could not be rejected. From the estimates of equations (3) and (4)  $\alpha_1$  is calculated to be -4.07 and -5.8 for the household and corporate sectors respectively<sup>12</sup>, and  $\beta_1$  is calculated to be -0.24 and -0.31 for the households and corporates respectively<sup>13</sup>.

For each sector when the parsimonious equations are estimated to 2007Q2, it is difficult to distinguish between the interest rate specifications. All of the equations pass the diagnostic tests for serial correlation, heteroscedasticity, the reset test and the Cusum test for stability. Furthermore, the sign on each cointegrating residual is consistent with cointegration, and each is strongly significant from zero. However, when the period is extended to 2012Q4 major differences become evident. For the corporate sector, the log-log specification fails the Chow forecast test, and a number of parameters values change markedly. In both sectors, when the log-log specification is applied there is a large reduction (in absolute value) of the response to the cointegrating residuals. But note that for both sectors the semi-log specification does not fail the Chow test and that the parameter estimates hardly change when the sample is extended<sup>14</sup>.

## Conclusion

So long as the interest rate relationship is modelled in the semi-log form there is no sign of instability in the demand for notes and coins in the UK. The increased share of M1 held as notes over the economic crisis is a consequence of the extremely low interest rate reducing the interest rate elasticity. With regard to the composition of M1 an even bigger change has been the decline in the share of sight deposits paying interest, particularly for household deposits for which the share decreases from about 90% to about 75%. Part of the reason for the proportion of M1 held as interest notes increasing is that

---

<sup>12</sup> These values for the semi-elasticity are very similar to those reported by Khadaroo (2003) for M0, who also finds a unit income elasticity. Khadaroo also includes a time trend to control for the effects financial innovation.

<sup>13</sup> The results of the unit root tests and Johansen estimation are available from the author on request.

<sup>14</sup> There is no evidence of significant intercept shifts for the semi-log model in either sector in the quarters of the Northern Rock or Lehmann crises, or in the immediate subsequent quarters. In contrast for the log-log model the addition of a dummy variable for the period since the collapse of Lehman brothers restores the parameter estimates to close to their 1990Q3-07Q2 values.

for many individuals, and to a lesser degree corporates, the benefits of holding transactions balances within banks decreased. Once the effect of the extremely low interest rate is taken into account, there is no evidence that the Northern Rock 'shock' led to people maintaining their transactions balances outside of the banking sector.

Figure 1 Ratio of notes in M1

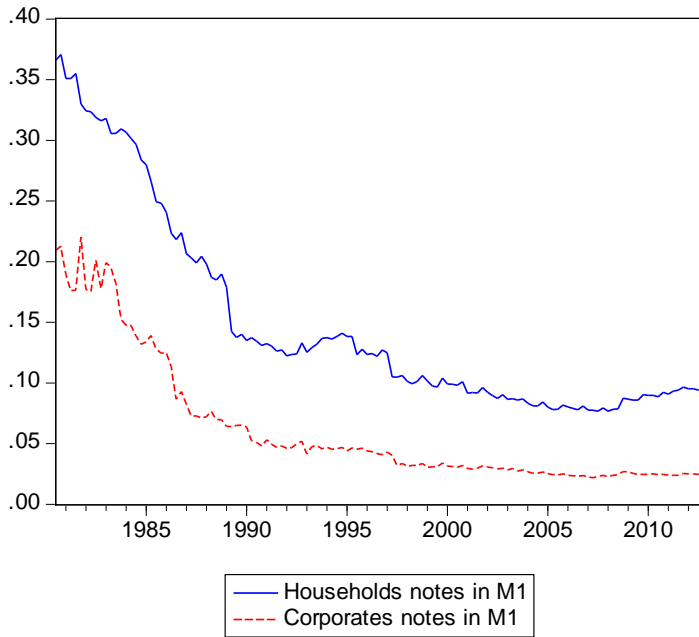


Figure 2, income velocity of circulation, 1963Q3=1

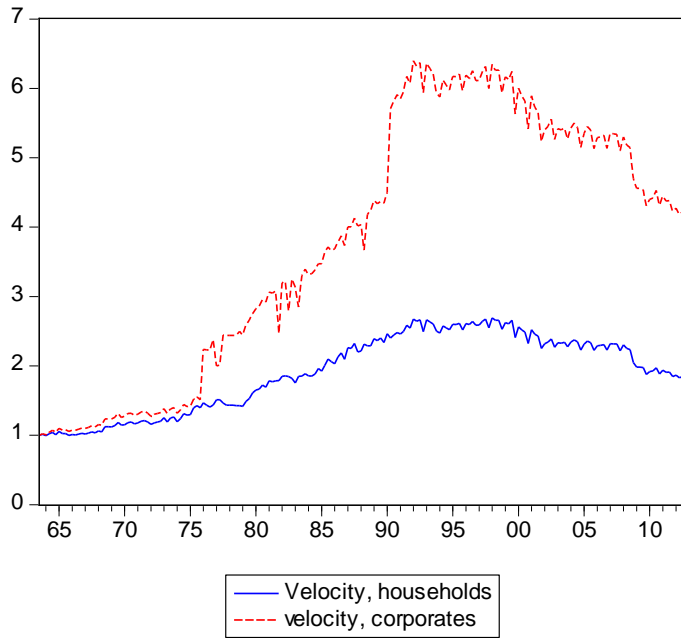




Table 1, Parsimonious Estimates for Household Sector

Households	Semi-log	Semi-log	log-log	log-log
period	90Q3- 07Q4	90Q3- 12Q4	90Q3- 07Q4	90Q3- 12Q4
intercept	-1.87 (0.40)	-1.73 (0.37)	-2.13 (0.67)	-0.46 (0.23)
$cv_{t-1}$	-0.12 (0.03)	-0.11 (0.02)	-0.13 (0.04)	-0.03 (0.014)
$\Delta M_{t-1}$	-0.31 (0.11)	-0.27 (0.09)	-0.38 (0.11)	-0.31 (0.10)
$\Delta Y_{t-1}$			1.07 (0.57)	1.12 (0.34)
$\Delta r_t$			-0.06 (0.03)	-0.05 (0.015)
adj. $R^2$	0.72	0.72	0.74	0.69
LM(5)	0.19	0.28	0.52	0.29
Arch	0.19	0.2	0.38	0.77
Reset	0.22	0.2	0.14	0.53
Chow	0.92		0.28	
n. obs	68	90	68	90

Notes.

Numbers in brackets are standard errors,  $cv$  is the cointegrating residual calculated from the estimates of equations 3 or 4.

$\Delta$  is the first difference operator,  $M$  is the log of notes and coins deflated by the GDP deflator,  $Y$  is log of real GDP,  $r$  is the log of the interest rate, LM(5) is the Godfrey test for serial correlation of up to the 5<sup>th</sup> order, Arch is the test for first order auto-regressive conditional heteroscedasticity, Reset is the first order Ramsey reset test, Chow is the Chow forecast test applied to the period 2007Q3-2012Q4 (LM5, Arch, Reset and Chow are reported as probability values), and n. obs is the number of observations.

Table 2, Parsimonious Estimates for Corporate Sector

corporate	Semi-log	Semi-log	log-log	log-log
period	90Q3- 07Q4	90Q3- 12Q4	90Q3- 07Q4	90Q3- 12Q4
intercept	-2.88 (0.40)	-2.63 (0.47)	-4.20 (0.61)	-0.37 (0.22)
$cv_{t-1}$	-0.16 (0.02)	-0.15 (0.02)	-0.22 (0.03)	-0.02 (0.01)
$\Delta M_{t-1}$	-0.20 (0.07)	-0.17 (0.07)	-0.16 (0.07)	-0.01 (0.07)
$\Delta M_{t-3}$	-0.30 (0.07)	-0.27 (0.06)	-0.26 (0.07)	-0.11 (0.07)
$\Delta \Delta R_t$	-1.37 (0.46)	-0.97 (0.35)		
$\Delta \Delta r_t$			-0.10 (0.03)	-0.09 (0.03)
adj. $R^2$	0.79	0.77	0.78	0.64
LM(5)	0.12	0.12	0.15	0.09
Arch	0.26	0.59	0.16	0.32
Reset	0.69	0.42	0.74	0.9
Chow	0.66		0.002	
n. obs	68	90	68	90

Notes.

See table 1, plus  $R$  is the interest rate.

## References

- Artis, M J and Lewis, M K (1984), 'How unstable is the demand for money in the United Kingdom', *Economica*, 51, 473-6.
- Baumol, W J (1952), 'The transactions demand for cash: An inventory theoretic approach', *Quarterly Journal of Economics*, 66, 545-556.
- Cagan, Phillip (1956), 'The Monetary Dynamics of Hyperinflation', In *Studies in the Quantity Theory of Money*, ed. Milton Friedman, 25-117, Chicago, Chicago University Press.
- Chadha, Jagit, S, Andrew G. Haldane and Norbert G. J. Janssen (1998), Shoe leather costs reconsidered, *Economic Journal*, 108 (March), 363-382.
- Drake, Leigh and K. Alec Chrystal (1994) 'Company-Sector Money Demand: New Evidence on the Existence of a Stable Long-Run Relationship for the United Kingdom' *Journal of Money Credit and Banking*, 26, 3, 479-494
- Drake, Leigh and K. Alec Chrystal (1997) 'Personal Sector Money Demand in the UK' *Oxford Economic Papers*, 49, 188-206
- Ericsson, Neil R. (1998) 'Empirical Modelling of Money Demand' *Empirical Economics*, 23 295-315.
- Grant, Kathryn, Gertjan Vlieghe, Andrew Brigden (2004), Assessing the stability of narrow money demand in the United Kingdom, *Bank of England Quarterly Bulletin*, summer, 131-41.
- Ireland, Peter N. (2009) 'On the Welfare Cost of Inflation and the Recent Behavior of Money Demand', *American Economic Review*, 99, 3, 1040-1052
- Hacche, G (1974), The demand for money in the United Kingdom: Experience since 1971, *Bank of England Quarterly Bulletin*. 14. 284-301.
- Hall, S G, Henry, S G and Wilcox, J B (1989), 'The long-run determination of UK monetary aggregates', *Bank of England Discussion Paper*, 41.
- Hendry, David F and Ericsson, Neil R. (1991) 'Modelling the demand for narrow money in the United Kingdom and the United States, *European Economic Review*, 35 833-886.
- Janssen, Norbert (1998), The demand for M0 in the United Kingdom Reconsidered: Some specification issues, *Bank of England, working paper* no. 83
- Johansen, Soren (1988), Statistical Analysis of Cointegration Vectors', *Journal of Economic Dynamics and Control*, 12(2/3), 231-54.
- Johnston, R. B. (1984), The demand for non-interest bearing money in the UK, *HM Treasury working paper* no. 28.
- Khadaroo, A J (2003), 'A smooth transition regression equation of the demand for UK M0', *Applied Economics Letters*, 10, 769-773.

Lucas, Robert E. Jr. (2000) 'Inflation and Welfare', *Econometrica*, 68 (March), 2, 247-274

Meltzer, Allan H. (1963), The Demand for Money: The Evidence from Time Series, *Journal of Political Economy*, 71(3), 219-46.

Thomas, R (1996) 'Understanding broad money', *Bank of England Quarterly Bulletin*, 163-79.

Tobin, J. (1956) 'The interest elasticity of transactions demand for cash', *Review of Economics and Statistics*, 38, 241-7.

Walton, D. and Westaway, P. (1991), Endogenous financial innovation and the demand for M0, in Taylor, M (ed), *Money and Financial Markets*, Basil Blackwell, Oxford.