Estimating the German term structure
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Summary

This paper introduces the Deutsche Bundesbank's new procedure for estimating the term structure of interest rates. It describes the basic methodological approaches used (Nelson and Siegel (1987) and Svensson (1994)) and some fundamental concepts which are important for estimating and interpreting such term structures. It also documents the application of the procedure to the prices of German Federal securities on a monthly basis from September 1972 to December 1996. The new procedure meets the requirements of monetary policy analysis as it represents a good compromise between, on the one hand, maximum approximation to the data and, on the other, smoothness of the curve and hence ease of interpretation for monetary policy purposes.

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Estimating the German term structure*)

I. Introduction

From a monetary policy perspective, the term structure of interest rates is interesting because the term structure of (spot) interest rates for different maturities may contain information about the expectations of market participants regarding the future movement of interest rates and other variables, such as inflation. These data can be used by the central bank to complement its own interest rate or inflation forecast, but they are also interesting in their own right. Market players' expectations regarding future trends and events influence their current decisions. These decisions in turn influence future events. Knowledge of market expectations is therefore also useful in order to assess the impact of policy measures and/or to determine their timing. There are various ways of using the term structure of spot interest rates to analyse market expectations. For example, it can be used for deriving the term structure of forward interest rates. Whereas spot interest rates reflect a return on investment viewed over several periods, starting from the present, forward rates measure a return on investment over a given period beginning in the future. From a monetary policy point of view, forward interest rates are particularly interesting as they make it easier to separate the expectations of market participants over the short, medium and long terms than does the term structure of spot interest rates.

This paper explains the Deutsche Bundesbank's new method of estimating term structures; ¹ it replaces the method previously used by the Bundesbank, in which the term structure of

^{*} I would like to thank D. Domanski, H. Herrmann, M. Kremer and P. Schmid for their useful comments. Special thanks are due J. Meier for the technical implementation of the estimation procedure and for very helpful discussions, and to G. Coenen for making valuable suggestions. Any remaining errors are mine alone.

¹ See in this context also the article "Estimating the term structure of interest rates" in the Monthly Report of October 1997 and the annex "Notes on the interpretation of the yield curve" to the article "Interest rate movements and the interest rate pattern since the beginning of the eighties" in the Monthly Report of July 1991, pages 31 to 42, especially pages 40 to 42. The development and implementation of the new programme were facilitated by the experience the author was able to gain using the Bank of England's estimation programme. The author would like to take this opportunity to formally thank the *Money Markets and Instruments Section* of the Bank of England, and particularly Creon Butler, Francis Breedon, Paul Wesson and Sanjay Yadav for their kind help.

interest rates was approximated by an (estimated) yield-to-maturity curve. The direct estimation of the term structure of interest rates enables expectations to be described and analysed more accurately and ensures enhanced international comparability of the results. Moreover, the term structure of spot rates can be used to directly infer the term structure of (implied) forward rates.

The term structure is defined as the relationship between the maturity and the (spot) interest rate of zero-coupon bonds without a default risk. A continuous term structure would be observable directly in the debt securities market if a zero-coupon bond with a negligible default risk were available for every maturity.² In fact, though, only a finite number of bonds is available; their prices define only a finite number of observation points. Moreover, only coupon bonds are observed for Germany for the most part. It is true that the introduction of bond "stripping" in July 1997, by means of which capital and interest can be traded separately, laid the basis in principle for a directly observable term structure of interest rates. However, the liquidity, and thus often the informative value, of the prices of many of these debt securities is relatively low compared with the traditional coupon bonds, at least at present. In any case, coupon bonds have to be used for drawing up historical time series of term structures of interest rates.

The prices of coupon bonds contain the term structure of interest rates only implicitly. Using a valuation model, which establishes a relationship between the observed prices of coupon bonds and the term structure, the latter has to be estimated from these prices. The starting point of the estimates is the definition of theoretical prices. Each coupon bond's individual payment flows are discounted to their present values with the maturity-specific (theoretical or estimated) interest rates and are then aggregated. This produces, in each case, theoretical prices and thus also theoretical yields-to-maturity. With the help of a non-

² An alternative approach would be to base the calculations on the information from swap rates, forward rate agreements, and futures prices. This has the advantage of requiring less calculation, since the interest rates being sought can be calculated relatively easily from the observed interest rates and prices. There are two disadvantages, though. One is that the interest rates and prices must be aggregated on the basis of different instruments, thus violating the principle of homogeneity. The other is that the interest rates and prices being taken into account carry a credit risk (albeit a small one). These problems are avoided by basing the calculations on the prices of (homogeneous) Federal bonds, the credit risk of which is either negligible or zero.

linear optimisation procedure, the (theoretical or estimated) interest rates are varied until the sum of squared deviations of the theoretical or estimated yields from the actually observed yields is minimised.

In order to estimate a continuous term structure, an assumption must be made regarding the functional relationship between interest rates and maturity. A fundamental trade-off exists between the "smoothness" of the estimated curve, on the one hand, and its flexibility, i.e. obtaining a maximum approximation to the observed data, on the other. For the purpose of monetary policy analysis at central banks, parametric methods have become the dominant approach, in particular specification in the form of exponential terms as proposed by Nelson and Siegel (1987) and extended by Svensson (1994). This method is sufficiently flexible to reflect the data constellations observed in the market. These include monotonically rising or falling, U-shaped, inverted U-shaped, and S-shaped curves. Individual "kinks" in the curve are smoothed out, which means the estimation results - unlike in the case of non-parametric methods - are relatively independent of individual observations. For that reason they are less well-suited to identifying abnormalities in individual maturity segments or individual bonds (e.g. "cheap" bonds). However, they provide curve shapes which are relatively independent of outliers and thus easier to interpret in monetary policy terms. Moreover, this specification enables plausible extrapolations to be made for segments extending beyond the maturity range for which data are observed. Specifically, the extrapolated long-term interest rates converge asymptotically towards a value which can be considered a very long-term interest rate. By contrast, non-parametric estimation approaches or approaches which contain terms linked linearly to the maturity (such as the old Bundesbank method) can produce implausible estimation values in long-term extrapolations, such as negative or infinitely high interest rates.

Section II explains the principles of valuing bonds. These explanations are drawn on frequently in Section III, in which the estimation procedure itself is described. In addition, Section III considers the application of this procedure to generate historical interest rate term structures from September 1972 to December 1996 and contains a discussion of the shape of selected spot rate curves and forward rate curves. Section V concludes.

II. Principles of pricing bonds

This section discusses the principles underlying the valuation of fixed-interest (default) risk-free bonds and the important concepts in this context of spot rates, implied forward rates, discount factors, accrued interest rates and yields-to-maturity. For the sake of simplicity, annual frequency is considered here.

II.1 Spot rates

This subsection explains zero-coupon interest (or spot) rates, which define the term structure. Two examples are used. The simplest case involves a bond promising a single payment of N in M years. We shall assume that the interest rate $z_{t,M}$ (expressed as a decimal, e.g. 0.07) at the time t and with a maturity of M years is known. Under these conditions, the price P_t of the bond equals the present value of the payment N discounted at the interest rate $z_{t,M}$:

$$P_{t} = \frac{N}{\left(1 + z_{t,M}\right)^{M}} \,. \tag{1}$$

The interest rate $z_{t,M}$ is also called the spot rate, as it denotes the interest rate for an M period investment at the interest rate valid at the time t, i.e. the rate valid today ("spot"). This bond is also called a zero-coupon bond, since it guarantees only a single payment at maturity and no intermediate payments. Therefore the spot rate is also often referred to as the zero-coupon interest rate.

Another example is a bond which promises a succession of payments of the amount C at the points in time 1, 2, ..., M and a redemption payment of the level N at time M.

³ Discrete compounding has been chosen in these illustrations since it is used also in the estimations detailed in Section III. In the literature, recourse is often taken to the assumption of continuous compounding in order to facilitate the calculations. However, this assumption can substantially alter the results of the estimates.

Assuming that the interest rates $z_{t,m}$ at time t with m maturities (with m = 1, 2, ..., M years) are known, the price of the bond can be written as follows:

$$P_{t} = \frac{C}{(1+z_{t,1})} + \frac{C}{(1+z_{t,2})^{2}} + \dots + \frac{C}{(1+z_{t,M})^{M}} + \frac{N}{(1+z_{t,M})^{M}} = \sum_{m=1}^{M} \frac{C}{(1+z_{t,m})^{m}} + \frac{N}{(1+z_{t,M})^{M}}.$$
 (2)

The spot rates $z_{t,m}$ at which the various payments are discounted, define the *term structure* of interest rates.

II.2 Implied forward rates

The term structure defined by the spot rates contains implications for future interest rates which are described in the term structure of the (implied) forward rates. Whereas spot rates express a rate of return over several periods as seen from the present, (implied) forward rates represent the rate of return starting from a given future point in time over a certain period. They are referred to as "implied" because they are derived from the term structure of the spot rates; in principle, however, they can already be secured today through appropriate spot transactions. In general, the following correlation between forward rates and spot rates applies:

$$(1+z_{t,m})^m = \prod_{\tau=1}^m (1+f_{t,\tau}) , \qquad (3)$$

where $f_{t,\tau}$ denotes the implied forward rate for the future period τ at the point in time t with $\tau=1,...,m$ (for $\tau=1$, spot and forward rates are identical). Consequently, the price of a bond can be represented as follows by taking account of the forward rates:

$$P_{t} = \sum_{m=1}^{M} \frac{C}{\prod_{\tau=1}^{m} (1 + f_{t,\tau})} + \frac{N}{\prod_{\tau=1}^{M} (1 + f_{t,\tau})}.$$
 (4)

Equation (3) yields the following implied relationship between spot rates and (one-year)⁴ forward rates:

$$(1 + z_{t,m})^m = (1 + z_{t,m-1})^{m-1} (1 + f_{t,m}),$$
 (5)

where $f_{t,m}$ denotes the one-year forward rate in the *m*-th year calculated at the time *t*. The spot rate denotes the average rate of return over *m* years, and the forward rate the marginal rate of return in the year *m*. For example, in the case of m=10, the spot rate shows the average rate of return over ten years, and the forward rate the one-year rate from the end of the ninth to the end of the tenth year. Equation (5) permits the following interpretation: the implied forward rate $f_{t,m}$, can also be understood as the marginal rate of return which results from replacing an m-period zero-coupon bond with the rate of return $(1+z_{t,m})^m$ in a bond portfolio by an m-1 period zero-coupon bond whose rate of return is $(1+z_{t,m-1})^{m-1}$.

Let us illustrate this relationship with the help of a numerical example. We shall continue to assume that the spot rate curve is already known. In order to calculate the implied one-year forward rates, we can then use equation (5), according to which:

$$f_{t,m} = \frac{(1+z_{t,m})^m}{(1+z_{t,m-1})^{m-1}} - 1$$
 (5a)

relatively little practical significance, since arbitrage transactions with bonds whose dates of payment differ

⁴ As previously mentioned, for simplicity's sake one-year forward rates are considered here. The estimations detailed in Section III likewise relate to one-year forward rates, but in principle forward rates with any maturity can be calculated. If, for instance, the length of the period for which the forward rate applies approaches the limit of zero, the result is called the instantaneous forward rate. It is true that this rate has

only insignificantly are normally not concluded owing to the associated transaction costs. It is important, however, for theoretical purposes. For example, the procedures for estimating the term structure of interest rates are often formulated as an assumption about the behaviour of the instantaneous forward rate.

Assuming that the four-year spot rate is 4% and the five-year spot rate is 4.5%, and inserting these figures into (5a) gives us the following expression for the one-year forward rate in the fifth year, $f_{1.5}$:

$$f_{t,5} = \frac{(1+0.045)^5}{(1+0.04)^4} - 1 \approx 0.065$$
 (5b)

The (one-year) implied forward rate is (approximately) 6.5 %. Assuming, on the other hand, that the four-year spot rate is 4.5 % and the five-year spot rate is 4 %, the implied forward rate equals 2 %. This illustrates that the forward rate curve is above (below) the spot rate curve if the latter has a rising (falling) slope, respectively. To that extent, the movements of the forward term structure are more pronounced than those of the spot term structure.

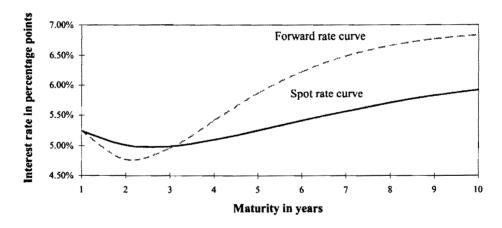


Figure 1: Spot rate and forward rate curve

This is shown in Fig. 1. The forward rate curve is below the spot rate curve in the maturity segment in which the latter shows a downward slope. It cuts the spot rate at its minimum value and is above the spot rate curve in the area in which the latter rises. Furthermore, the figure shows clearly that both curves have the same origin, i.e. that from the present perspective the one-year spot rate is equal to the one-year forward rate. To the extent that the spot rate curve is not completely flat, the curves differ otherwise, and the forward rate curve shows greater variation (in terms of the standard deviation of the rates for the different maturities) across the entire maturity range.

II.3 Discount factors

A key concept behind the estimation of term structures is that of discount factors. They are obtained from a simple transformation of the spot rate. The discount factor $\delta_{t,m}$ is defined as:

$$\delta_{t,m} = \frac{1}{\left(1 + z_{t,m}\right)^m}.$$

Equation (1) can therefore also be written as

$$P_{t} = \left[\frac{1}{\left(1 + z_{t,M}\right)^{M}}\right] N = \delta_{t,M} N. \tag{7}$$

A useful feature of the discount function is that it describes the present value of one unit which is to be paid at some future point in time. The price of an instrument which promises to pay exactly one unit at the point in time M is thus identical to the value of the discount function at this point in time. As explained at the beginning of this paper, a zero-coupon bond is such an instrument; hence the discount factor is also often called the zero-coupon bond price.

Moreover, discount factors can be used to value separately the individual payments which are linked to a coupon bond. This feature is important, as the estimation of the term structure is carried out by decomposing the coupon bond into a notional set of zero-coupon bonds which are then discounted using the respective discount factors. Analogously to equation (2), the price of a coupon bond can be expressed as the sum of the products of the individual payments (coupon payments and redemption) and the corresponding discounting factors:

$$P_{t} = \delta_{t,1} C + \delta_{t,2} C + \dots + \delta_{t,M} C + \delta_{t,M} N = C \sum_{m=1}^{M} \delta_{t,m} + \delta_{t,M} N.$$
 (8)

The algebraic relationship between the three interest rate measures discussed so far is unambiguous. Thus if, for example, the structure of the discount factors is known, the spot

rate curve and the forward rate curve can be calculated by simple transformations (and vice versa). If $\delta_{t,m}$ denotes the discount factor for a single payment after exactly m years, $z_{t,m}$ the average rate of return up to the end of the m-th year, and $f_{t,m}$ the individual rates of return in the years m = 1, 2, ..., M, then the following equations generally apply:

$$1/\delta_{t,1} = (1+z_{t,1}) = (1+f_{t,1})$$

$$1/\delta_{t,2} = (1+z_{t,2})^{2} = (1+f_{t,1})(1+f_{t,2})$$

$$\vdots$$

$$1/\delta_{t,M} = (1+z_{t,m})^{m} = (1+f_{t,1})(1+f_{t,2})...(1+f_{t,m})$$
(9)

If the discount factors are known and/or calculated using the estimations described in Section III, the average and one-year rates of return are easy to calculate. By transforming (8), we obtain the following expression for the spot rate as a function of the discount factor:

$$z_{t,m} = \left(\frac{1}{\delta_{t,m}}\right)^{1/m} - 1. \tag{10}$$

The implied forward rate can accordingly be expressed as follows:

$$f_{t,m} = \frac{\delta_{t,m-1} - \delta_{t,m}}{\delta_{t,m}} \tag{11}$$

These transformations are intended to illustrate the fact that the desired interest rate measures can be calculated by simple transformation from the discount factors.

II.4 Accrued interest

So far we have given a highly simplified description by assuming that the coupon payments are due in exactly one year. But, whenever transactions do not take place precisely at the time of a coupon payment (or issue) of the bond, the valuation of the bond will always reflect the length of time remaining until the next coupon payment date. In keeping with market conventions in Germany, the seller has a right to receive the interest that has accumulated up to then (accrued interest) from the purchaser - interest which indemnifies the seller for having held the security since the last coupon payment for a period for which he will not receive another coupon payment. The accrued interest is calculated by dividing the number of days since the last coupon payment by the total number of days in the year and multiplying this figure by the coupon, with each complete month counted as 30 days and the entire year as 360 days. Although taking accrued interest into account seems trivial, notable valuation differences could result if a different market convention were used (e.g. measuring the year as 365 days as is the case, say, in the UK). If the assumption that the coupon payments are due in exactly one year is abandoned, the valuation equation (2) must be modified as follows:

$$P_{t} + S_{t} = \frac{C}{(1 + z_{t,1-s_{t}})^{1-s_{t}}} + \frac{C}{(1 + z_{t,2-s_{t}})^{2-s_{t}}} + \dots + \frac{C}{(1 + z_{t,M-s_{t}})^{M-s_{t}}} + \frac{N}{(1 + z_{t,M-s_{t}})^{M-s_{t}}}$$

$$= \sum_{m=1}^{M} \frac{C}{(1 + z_{t,m-s_{t}})^{m-s_{t}}} + \frac{N}{(1 + z_{t,M-s_{t}})^{M-s_{t}}}$$
(12)

with $S_t = s_t C$ denoting the accrued interest at the time t, s_t the fraction of the year remaining until the next coupon payment, $s_t = 1 - (ANZ_t / 360)$, ANZ_t the number of days since the last coupon payment, and $z_{t,m-s_t}$ the spot rate for the m-te coupon payment due in $m-s_t$ years. The expression $m-s_t$ is an arbitrary positive real number (e.g. 0.9 years)

II.5 Yields-to-maturity

In practice, the price of a bond is often expressed in the form of yields-to-maturity, since these are easy to interpret and calculate. The price of a bond with the payment profile as in (2) can be expressed in the form of the yield-to-maturity as follows:

$$P_{t} = \frac{C}{(1+r_{tM})} + \frac{C}{(1+r_{tM})^{2}} + \dots + \frac{C}{(1+r_{tM})^{M}} + \frac{N}{(1+r_{tM})^{M}} = \sum_{m=1}^{M} \frac{C}{(1+r_{tM})^{m}} + \frac{N}{(1+r_{tM})^{M}}.$$
 (13)

The yield $r_{t,M}$ for the maturity M represents the average rate of return from holding a bond for M years, precisely assuming that all coupon payments are reinvested during the maturity of the bond (i.e. for m = 1, 2, ..., M) at exactly the same interest rate $r_{t,M}$. For this interpretation, it must therefore be assumed that the spot rate curve, defined in the form of $z_{t,m}$ (with m = 1, 2, ..., M as in (2)) for this security, is always flat and equal to $r_{t,M}$. The yields $r_{t,M}$ for various maturities M define the yield-to-maturity curve.

The relationship between yields-to-maturity, on the one hand, and spot interest rates, forward rates and discount factors, on the other, is not unambiguous for coupon bonds. Although yields-to-maturity can be derived unambiguously from the structure of the discount factors, or the spot rates or forward rates, it is not possible conversely to infer the structure of spot interest rates, forward rates or discount factors unambiguously from the yield-to-maturity structure. This may be illustrated easily through the linkage of interest rates and yields-to-maturity by equating (2) with (13):

$$\sum_{m=1}^{M} \frac{C}{(1+z_{l,m})^m} + \frac{N}{(1+z_{l,M})^M} = \sum_{m=1}^{M} \frac{C}{(1+r_{l,M})^m} + \frac{N}{(1+r_{l,M})^M}$$
(14)

This equation expresses the (non-linear) relationship between the spot interest rate and the yield-to-maturity. The difference between the left and right-hand side of equation (14) is that the right-hand side contains only one yield-to-maturity $r_{i,M}$, whereas on the left-hand side the interest rates are indexed by the running index m, with m=1,2,...,M.

For the sake of clarity, the case of a two-period bond with a redemption payment of 1 is considered here. The time index t is suppressed for simplicity's sake. In this case (14) leads to:

$$\frac{C}{(1+z_1)} + \frac{C}{(1+z_2)^2} + \frac{1}{(1+z_2)^2} = \frac{C}{(1+r_2)} + \frac{C}{(1+r_2)^2} + \frac{1}{(1+r_2)^2}.$$
 (15)

The yield-to-maturity r_2 is a weighted average of the interest rates z_1 and z_2 , with the weights dependent on the payment flow associated with the bond, that is the amount of the coupon and redemption payments and the total number of payments. The equation also shows that the yield-to-maturity and the interest rate are identical only when the spot rate curve is flat, i.e. in the case of $z_1 = z_2$. The deviation of the yields from the interest rates widens with the steepness of the spot rate curve and the size of the coupon C. This dependence of the yield-to-maturity on the coupon is called the coupon effect. If the coupon is not equal to zero for all bonds (i.e. the case of zero-coupon bonds), the spot rate curve and the yield-to-maturity curve diverge systematically.

⁵ See in particular Schaefer (1977).

III. Estimating the term structure

III.1 Functional relationship between interest rates and maturity

The starting point of the estimations of a continuous interest rate term structure is an assumption regarding the functional form of the relationship between the spot rates (and thus also the forward rates and discount factors), on the one hand, and the time to maturity, on the other.

In principle, the observed yields-to-maturity on bonds may be influenced by additional factors. For example, the demand for certain bonds, and thus their price/yield, may be affected by tax considerations. Since in Germany interest income from bonds held by private individuals liable to income tax are taxable, whereas capital gains and holding gains are tax-free (if held for at least six months), it seems plausible that a particularly strong demand exists for bonds with a low coupon (known as the tax-related coupon effect). In order to adjust for this effect, either the income tax rates of all investors would have to be known - which is not possible since the composition of the investor population can change daily - or the estimation would have to be confined to bonds with the same coupon. But this would diminish the available data set so drastically that it would then become difficult to obtain accurate estimation results. The Bundesbank's direct incorporation of the coupons as right-hand variables in the regressions probably serves primarily to correct the mathematical coupon effect (see II.5) and is thus not helpful in solving the present problem. The Bank of England's procedure, in which the coupon effect is expressly modelled, is an alternative. However, the theoretical model on which it is based is very complicated and hinges on a large number of limiting assumptions. Applying the procedure to German data between September 1972 and February 1996 reduces the average yield error by less than two basis points. This suggests that the coupon effect, on average, may not be particularly important in Germany in quantitative terms. For that reason, and since no simple correction procedure is available, this aspect is ignored in the estimations.

Another possible influence is the "convexity" effect. The term convexity refers to the non-linear relationship between the price and interest rate of a bond (see, e.g., equations (1) and (2) in Section II.1). A positive convexity means that the price of a bond rises more sharply

for a given fall in interest rates than it falls given an interest rate rise of the same magnitude. Other things being equal, therefore, a positive convexity of a bond is desirable from the investor's point of view, for it increases the expected return on a portfolio - if interest rates are expected to fluctuate at all. This feature is paid for by a positive price premium or by a yield-to-maturity discount. Examples of securities with a pronounced positive curvature are bonds with a long maturity and, in particular, zero-coupon bonds with a long maturity, by contrast, bonds with fairly early payments, i.e. bonds with a short maturity, show only a low positive curvature. Hence the curvature effect shows up particularly at the long end of the yield curve. It gives rise - other things being equal - to a concave curve, i.e. the curve flattens out at the long end. This effect grows in proportion to the expected volatility of interest rates. Basically, it can be taken into account by using non-parametric estimation approaches. However, these approaches show undesirable features from the point of view of monetary policy analysis, as explained in Section I. An explicit modelling of the curvature effect would exceed the scope of this paper; this potential influence is therefore disregarded in the following estimations.

Nelson and Siegel (1987, hereafter Nelson/Siegel) assume that the relationship between interest rates and time to maturity can be described using exponential terms. They justify the choice of the functional form by its ability to reflect the typically observed shapes of the yield curve. More precisely, they assume that the relationship between the (one-year) forward rate and the maturity m at any time t (the time index is suppressed in the following for simplicity) can be described by the following functional form:

$$f_{m}(\beta) = \beta_{0} + \beta_{1} \exp\left(\frac{-m}{\tau_{1}}\right) + \beta_{2} \frac{m}{\tau_{1}} \exp\left(\frac{-m}{\tau_{1}}\right), \qquad (16)$$

where β denotes the vector of the parameters to be determined β_0 , β_1 , β_2 and τ_1 . By integrating (16) over the interval [0,m] and dividing by m, the following relationship between spot rates and maturities is obtained:

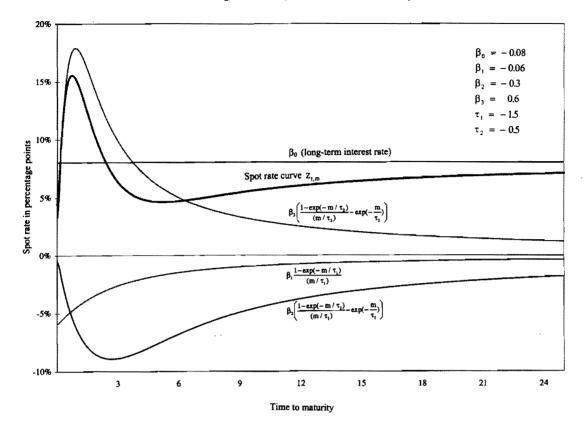
$$z_{m}(\beta) = \beta_{0} + \beta_{1} \frac{1 - \exp(-m/\tau_{1})}{(m/\tau_{1})} + \beta_{2} \left(\frac{1 - \exp(-m/\tau_{1})}{(m/\tau_{1})} - \exp(-\frac{m}{\tau_{1}}) \right).$$
 (17)

In order to study the contribution of the parameters, the limits of the function for m approaching zero and infinity can be calculated. $\lim_{m\to\infty} z_m = \beta_0$ and $\lim_{m\to0} z_m = \beta_0 + \beta_1$ apply. By definition, therefore, the spot rate curve converges asymptotically, just like the forward yield curve, towards the parameter β_0 , which can be interpreted as a long-term interest rate (spot and forward rate). If the maturity approaches zero, the interest rate equals the parameter combination $\beta_0 + \beta_1$, which can be interpreted accordingly as a very short rate (instantaneous interest rate). The parameters β_2 and τ_1 do not have a comparable direct interpretation; they influence the shape of the curve between these limits.

At times of great market uncertainty, the term structure of interest rates can show complex shapes in the very short-term maturity segment. Bearing this in mind, Svensson (1994) increased the flexibility of the Nelson/Siegel estimation approach by incorporating a third term and two additional parameters (β_3, τ_2) . This extended function reads:

$$z_{m}(\beta) = \beta_{0} + \beta_{1} \frac{1 - \exp(-m/\tau_{1})}{(m/\tau_{1})} + \beta_{2} \left(\frac{1 - \exp(-m/\tau_{1})}{(m/\tau_{1})} - \exp(-\frac{m}{\tau_{1}}) \right) + \beta_{3} \left(\frac{1 - \exp(-m/\tau_{2})}{(m/\tau_{2})} - \exp(-\frac{m}{\tau_{2}}) \right)$$
(18)

Figure 2: Decomposition of the Svensson spot rate cure into individual components (simulation results)



Function (18) has the same limit properties as (17), i.e. β_0 can be interpreted as a long-term interest rate and the parameter combination $\beta_0 + \beta_1$ as a short-term interest rate. But owing to the additional term, Svensson's formulation allows the curve to have an additional hump between these two limits.

An example of the contribution of the individual components to the overall shape of the curve is shown in the simulated curves in Fig. 2. It is assumed here that the long-term interest rate (β_0) is 8% and the short-term interest rate $(\beta_0 + \beta_1)$ is 2%. The term structure, drawn as a thick line, moves between these extremes and, overall, is S-shaped. It rises sharply at first, reaching its maximum of around 15.5% at 0.7 years. Then the curve falls, initially rather steeply and then more moderately, reaching a local minimum of approximately 4.6% at a maturity of 5.3 years. Then it rises again continuously, but moderately. The pronounced hump at the beginning of the curve is attributable to the term added by Svensson to the Nelson/Siegel specification with the parameters β_3 and τ_2 .

III.2 Estimating procedure

The objective of the estimation is to determine the parameter vector β . This is estimated separately for every observation point. This means that the estimated parameters may vary over time; for that reason the parameter vector is denoted in the following as β_1 .

In Chapter II it was assumed that the spot rates, forward rates and discount factors are known. In actual fact, though, only the prices and terms of issue of n bonds, i.e. C_i , N_i and M_i are known at any given time, with C_i and N_i being the coupon and redemption payments, respectively, and M_i the time to maturity of the ith bond (with $i=1,2,...,n_i$ and n_i being the number of bonds included in the estimation).⁶ The yield-to-maturity of the bond i, called $r_{i,i}$ hereafter, can be calculated directly from the price $P_{i,i}$ via the relationship in equation (13) using an iterative algorithm, for C_i , N_i and M_i are known and the yield-to-maturity is the only unknown in the equation. This is not possible in the case of the spot rates $z_{i,m}$, forward rates $f_{i,m}$ and discount factors $\delta_{i,m}$ if the time to maturity M_i is greater than one and equations (2), (4) and (8) each contain more than one unknown variable on the right-hand side.⁷ The task is to estimate spot rates, forward rates and discount factors on the basis of the observed C_i , N_i and M_i values.

The estimation requires a theoretical price or a theoretical yield-to-maturity to be defined for each of the bonds included. The starting point of this definition is equation (8), which expresses the price of a bond as the sum of the products of coupon and redemption payments and the associated discount factors. The *theoretical* price of a bond i, $\hat{P}_{t,i} = P_i(\beta_t)$, can be defined analogously as

$$\hat{P}_{t,i} = \hat{\delta}_{t,1} C_i + \hat{\delta}_{t,2} C_i + \dots + \hat{\delta}_{t,M_i} C_i + \hat{\delta}_{t,M} N_i
= \sum_{m=1}^{M_i} \hat{\delta}_{t,m} C_i + \hat{\delta}_{t,M_i} N_i ,$$
(19)

⁶ For the choice of bonds, see Section III.3.1.

⁷ For expositional purposes, but not in the actual calculations we assume that the time to maturity is an integer.

with $\hat{\delta}_{t,m} = \delta_{t,m}(\beta_t)$ being the estimated discount factor, which, just like the theoretical price, is a function of the parameter vector β_t . The theoretical price of bond i is defined as the sum of the present values of the payment flows associated with this bond, with the future payments being discounted to their present values using the (estimated) discount factor. From this theoretical price the theoretical yield of bond i can be calculated, using the Newton-Raphson procedure by solving the following equation:

$$\sum_{m}^{M_{i}} \frac{C_{i}}{\left(1 + \hat{r}_{t,i}\right)^{m}} + \frac{N_{i}}{\left(1 + \hat{r}_{t,i}\right)^{M_{i}}} - \hat{P}_{t,i} = 0$$
(20)

according to $\hat{r}_{t,i}$, where $\hat{r}_{t,i} = r_i(\beta_t)$. The parameter vector β_t is the one which minimises the sum of the squared deviations of the theoretical yields-to-maturity from the observed yields-to-maturity of all the securities included in the estimation. If $\varepsilon_{t,i}$ denotes the deviation in the case of the ith bond, with $\varepsilon_{t,i} = r_{t,i} - \hat{r}_{t,i}(\beta_t)$, the optimisation problem can be written as follows:

$$\underset{\beta_{t}}{\text{Min}} \sum_{i=1}^{n_{t}} \left(\varepsilon_{t,i} \left(\beta_{t} \right) \right)^{2}. \tag{21}$$

Yield errors are minimised rather than price errors, because interest rate estimations are primarily involved here, not price estimations. Although less computational work is involved in minimising price errors, this procedure can lead to relatively large yield deviations for bonds with short maturities. The prices of short-term bonds are relatively insensitive in respect of yields-to-maturity, and so yield errors would be penalised too lightly when estimating the yields-to-maturity of short-term bonds.⁸

⁸ As an alternative, the price errors could also be weighted, with the duration being a possible weighting factor (see, for example, Ricart and Sicsic (1995)). This is defined as the price elasticity regarding (one plus) the yield, and constitutes the weighted average maturity of the respective present values of the coupon payments and redemption of a bond.

The yield errors are a non-linear function of the parameters being sought, which is why a non-linear optimisation procedure is applied. The constraints ensure that $\beta_{t,o}$, $\tau_{t,1}$ and $\tau_{t,2}$, in the case of the Svensson approach, and $\beta_{t,o}$ and $\tau_{t,1}$, in the case of the Nelson/Siegel approach, are greater than zero and that the long-term interest rate is always positive. The starting values for these parameters were set differently. The starting value for $\beta_{t,o}$, which (as explained in Section III.1) can be interpreted as a very long rate, was initialised according to the mean yield-to-maturity of the three bonds with the longest time to maturity. The starting value for the sum of $\beta_{t,1}$ and $\beta_{t,o}$, which can be interpreted as a very short rate, was set at the yield-to-maturity of the bond with the shortest time to maturity. The starting values for $\beta_{t,2}$ and $\beta_{t,3}$ were - 1 and for $\tau_{t,1}$ and $\tau_{t,2}$ (+) 1. Test calculations over the entire observation period showed that, with this selection of starting values, convergence towards a solution was always achieved, and that the distribution of the residuals indicated that the solutions were plausible.

In order to clarify the procedure, its main elements are listed below in the form of individual steps:

- 1. Initialisation of the parameter vector β .
- 2. Calculation of the theoretical interest rates, $\hat{z}_{t,m} = z_{t,m} (\beta_t)$, according to (17) or (18).
- 3. Calculation of the theoretical discount factors $\hat{\delta}_{t,m} = \delta_{t,m} \left(\beta_t \right)$ according to (9).
- 4. Calculation of the theoretical prices $\hat{P}_{t,i} = P_{t,i}(\beta_t)$ according to (19).
- 5. Calculation of the theoretical yields-to-maturity $\hat{\mathbf{r}}_{t,i} = \mathbf{r}_{t,i} \left(\boldsymbol{\beta}_t \right)$ according to (20), using the Newton-Raphson method.

⁹ This is done by specifying the minimum permissible value as 10^{-4} . Further constraints are that the parameter $\beta_{t,0}$, which can be interpreted as a long-term interest rate (as explained in III.1), is no more than 300 basis points above (upper limit) and 300 basis points below (lower limit) the observed yield-to-maturity of the bond with the longest time to maturity. If a negative value is obtained for the lower limit so defined, it is set to zero. For the remaining parameters the values of - 30 (if 10^{-4} was not already specified) and + 30 were set as the lower and upper limits, respectively, Experience showed that once these limits were reached, further iterations did not lead to any major improvement in the target function value. In this context, it should be mentioned that, in addition to the break-off criterion as contained in (21), another one was specified. Specifically, a parameter vector was accepted as a solution provided the root mean squared yield error was less than four basis points.

- 6. Calculation of the target function value $\sum_{i=1}^{n_t} (r_{t,i} \hat{r}_{t,i}(\beta_t))^2$ with $r_{t,i}$ being calculated according to (13) from $P_{t,i}$ using the Newton-Raphson method.
- 7. Checking of the break-off criterion.
- If the break-off criterion is not reached:
 determine a new β, and go back to step 2.

Once the minimum sum of squared errors is obtained and the parameter vector \mathbf{b}_t is estimated, the discount factors, interest rates and forward rates can be determined for any maturity m by simple insertion. Inserting (18) (in the case of the Svensson approach, otherwise (17) when using the Nelson/Siegel approach) into equation (6) gives the following equation for the estimated discount factors $\tilde{\delta}_{t,m}$:

$$\widetilde{\delta}_{t,m} = \left(1 + \widetilde{\beta}_{t,0} + \widetilde{\beta}_{t,1} \frac{1 - \exp(-m/\widetilde{\tau}_{t,1})}{(m/\widetilde{\tau}_{t,1})} + \widetilde{\beta}_{t,2} \left(\frac{1 - \exp(-m/\widetilde{\tau}_{t,1})}{(m/\widetilde{\tau}_{t,1})} - \exp(-\frac{m}{\widetilde{\tau}_{t,1}})\right) + \widetilde{\beta}_{t,3} \left(\frac{1 - \exp(-m/\widetilde{\tau}_{t,2})}{(m/\widetilde{\tau}_{t,2})} - \exp(-\frac{m}{\widetilde{\tau}_{t,2}})\right)\right)^{-1}$$
(22)

The (estimated) spot interest rates and one-year forward rates can be determined, analogously to (10) and (11), from the estimated discount factors as follows:

$$\widetilde{z}_{t,m} = \left(\frac{1}{\widetilde{\delta}_{t,m}}\right)^{1/m} - 1 \tag{23}$$

and

$$\widetilde{f}_{t,m} = \frac{\widetilde{\delta}_{t,m-1} - \widetilde{\delta}_{t,m}}{\widetilde{\delta}_{t,m}}.$$
(24)

III.3 Application to historical data

III.3.1 Original data

A compromise must be found regarding the selection of the data to be included in the estimation. On the one hand, the securities included should be largely homogeneous, something which can be assured by selecting only a very small number of bonds; on the other hand, the estimation results should be fairly reliable for all maturity segments, which is facilitated by including a large number of securities. Bearing this in mind, three categories of listed German Federal securities were included, namely:

bonds of the Federal Republic of Germany ("Bunds"), five-year special Federal bonds ("Bobls"), and Federal Treasury notes ("BSA").

The securities issued by special funds were not included because of the (admittedly small) credit rating differential relative to the securities named. Securities with special terms and conditions were likewise disregarded in the estimations since special conditions, such as a borrower's call option, imply price discounts.¹⁰ In all, end-of-month observations of a total of 309 listed Federal securities were used in order to calculate the historical values of the term structure of interest rates for the period between September 1972 and December 1996.¹¹

Bunds are the Federal Government's traditional financing instrument on the German capital market; throughout the observation period they constituted the biggest group of securities included in the estimation. The (original) maturity of Federal bonds is ten years in most cases, although Bunds with maturities of five, six, seven, eight, twelve, 15 and 30 years

¹⁰ This deviates from the Bundesbank's previous method of estimating yield curves. The latter is based on a relatively large number of securities, including debt instruments issued by special funds or which are subject to special terms and conditions such as a borrower's call option.

¹¹ The observation period was dictated by the availability of the data. No observations were available for May 1982; the estimations for that month were made with the help of a linear interpolation of the estimation values for April and June 1982.

have also been issued. A total of 168 Federal bond issues were included across the entire observation period.

Bobls have been issued since 1979 and have a maturity of five years. They have a significance for the Federal Government's medium-term capital market financing similar to that of Bunds in the longer-term range.

BSA have a maturity of four years. They were issued until mid-1995 and then discontinued in favour of Bobls. All in all, 17 BSA Treasury notes were included in the estimations. Their inclusion ensures that the short-term segment in the 1990s was densely populated with observations. A list of the individual securities included in the estimation, together with their terms and conditions, is given in Annex 1.

The total number of observations available for the estimation in a single month varies considerably during the period under observation. In 1972, only an average of 17 observations per month is available. In the following years, the number of observations rises evenly until 1984, reaching a total of 103. Thereafter the number of observations fluctuates within a relatively narrow corridor between annual averages of 80 and 110. The distribution of the observations over the entire maturity range varies considerably, too (see also Annex 2). At the beginning of the period under review, the short-term (under two years) and long-term (8 years or more) maturity segments are weakly populated. As a consequence of the issue of five-year Bobls starting in 1979, the observation density in the short-term maturity segment rises markedly in the first half of the 1980s. With the issue of 30-year Federal bonds, observations for the very long end of the maturity range then likewise become available from 1986. Otherwise, in the eighties there is a marked standardisation of the maturities of Bunds at 10 years, with continuous issuing activity, so that the long-term maturity segment of between 8 and 10 years is densely packed. From the mid-eighties onwards, the distribution of observations over the entire range of maturities is relatively even. For this reason, the estimations from 1986 onwards may be regarded as being especially reliable.

III.3.2 "Reliability" of the estimations

The estimation approaches of Nelson/Siegel and Svensson meet the requirements of monetary policy analysis. Both methods generate plausible solutions for every observation point, i.e. also under difficult data constellations. 12 The typically observed curve shapes (monotonically rising or falling) are captured well by both approaches. More complicated forms are represented slightly better by the Svensson specification in that the estimation errors show more favourable features; however, in a graphic representation of the spot rate curves calculated according to the different specifications, no noticeable difference can be perceived with the naked eye.

Fig. 3 shows the deviation between the observed and estimated yields over the entire period under review. The deviation is measured as the mean squared yield error in basis points and reflects (a linear transformation of) the optimisation criterion of the estimations. In particular, the mean yield error declines distinctly with the increasing density of observations in the maturity spectrum from the mid-eighties onwards. The results of the Nelson/Siegel and Svensson methods are very similar on average; the average mean yield error as measured over the entire observation period is 12.43 basis points according to the Nelson/Siegel method and 11.99 basis points in the case of the Svensson method. Over the course of the observation period, the mean yield errors decline and the differences between the two approaches disappear. Taking the average of the nineties, the mean yield error is only 9.04 basis points by the Svensson method and 9.41 using the Nelson/Siegel method; most recently, its average for the year 1996 was 5.23 and 5.30 basis points, respectively. The standard deviations of the mean yield error, measured over the entire observation period, are 5.07 basis points in the case of the Svensson procedure and 5.23 in the case of the Nelson/Siegel method. In the course of the nineties, the results were more stable; for

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¹² Only in a very few months at the beginning of the seventies and at the end of the seventies/beginning of the eighties were there implausibly high values for short-term interest rates (less than one year) in situations in which the short-term maturity segment was thinly represented, or not represented at all, and the term structure otherwise showed a monotonically rising curve. Since interest rates of one year and above seemed plausible even in these situations, and barely changed after the introduction of further restrictions (such as constraining the origin of the curve to a weighted yield of securities with short times to maturity or to a money market rate), such restrictions were not added.

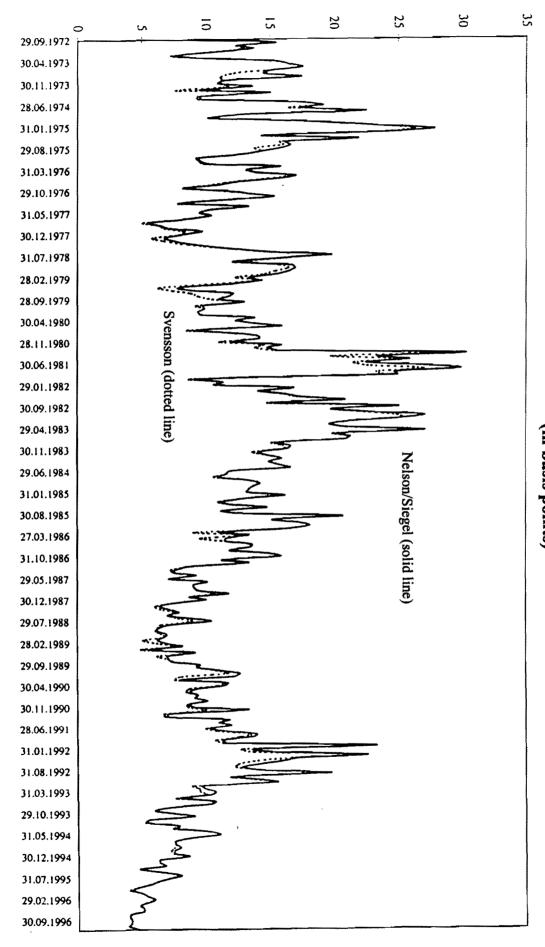


Figure 3: Mean yield-to-maturity error in the estimations from September 1972 to December 1996 (in basis points)

example, the standard deviation in the case of the Svensson method dropped to below 4 basis points. In summary it can be said that, measured in terms of the mean yield error (and its standard deviation), the Svensson method is to be preferred on balance. Because of its greater number of parameters, it is more flexible and can show an extra "hump", as explained in III.1.¹³ Overall, the goodness of the fit produced by the estimations using both approaches, measured by the squared yield error, can be rated as high compared with the Deutsche Bundesbank's traditional estimations of the yield curve.¹⁴

III.3.3 Selected statistics

This section contains a few short remarks regarding selected statistics of the estimated spot and forward rate curves. The following observations are worth emphasising. The spot rate curve rises on average over the whole observation period. It does so continuously across the entire range of maturities, but at declining rates. The (average) forward rate curve, which, so to speak, "exaggerates" the movements of the (average) spot rate curve, initially rises more sharply than the latter, and then flattens out very markedly in the segment of long-term maturities, in which the spot rate curve shows only a decelerating rise (see Fig. 4). The interest rates in the short-term segments of both the spot and forward rate curves fluctuate more strongly over time and reach more pronouced extreme values than in the long-term segments; Table 1 shows that they have a greater standard deviation and a more pronounced difference between minimum and maximum values. The tests of the degree of integration of the interest rates (Augmented Dickey Fuller (ADF)) suggest that the levels of the interest rates are non-stationary and that the first differences are stationary. The null hypothesis of non-stationarity cannot be rejected for the majority of spot and forward interest rates; however, this hypothesis can be rejected for their first differences at the 1 % significance level. Finally, Table 1 shows that the estimations of the spot and forward rate curves are robust on average in respect of the chosen functional approach (Nelson/Siegel or Svensson). The divergencies in the (average) spot or forward rates estimated by means of

¹³ This proves advantageous in some circumstances. In situations in which the two additional parameters are not necessary owing to the data constellation, identification problems may arise. In such situations one can then switch to the Nelson/Siegel estimation method, since the two additional parameters make no additional contribution towards explaining the data.

¹⁴ See also Schich (1996).

Figure 4: Average spot rate and forward rate curve from September 1972 to December 1996

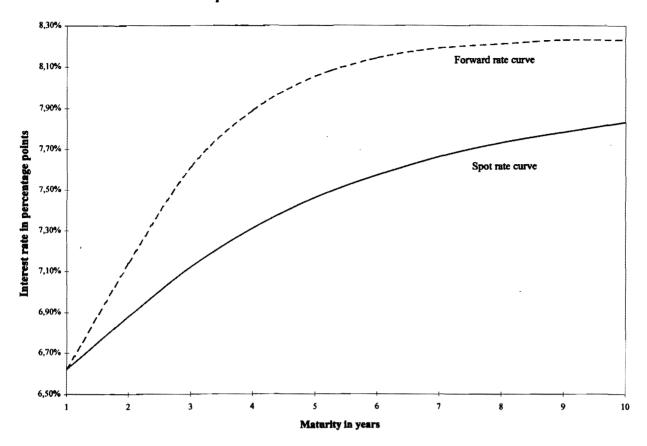


Figure 5: Average spot rate curve and forward rate curve for selected periods

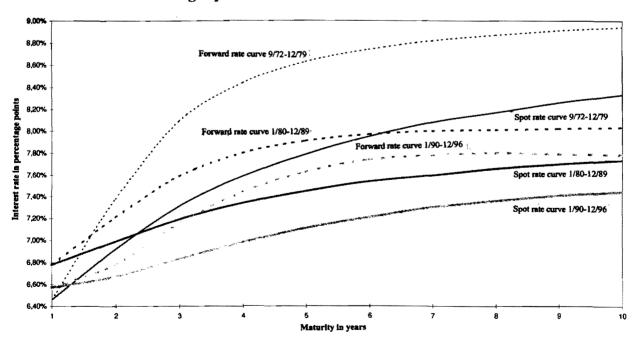


Table 1:

Selected spot and forward rate statistics from September 1972 to December 1996

Spot rates according to Nelson/Slegel										
	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
Mean	6.62	6.88	7.12	7.31	7.46	7.57	7.66	7.73	7.78	7.83
Maximum	13.01	12.54	12.11	11.73	11.39	11.09	10.82	10.99	11.16	11.30
Minimum	3.19	3.55	4.03	4.49	4.88	5.21	5.47	5.61	5.73	5.79
Std. dev.	2.22	1.94	1.75	1.60	1.50	1.41	1.35	1.31	1.27	1.25
ADF (N,12,C) ^{a)}	- 2.79	- 2.48	- 2.46	- 2.55	- 2.63	- 2.72	- 2.80	- 2.84	- 2.87*	- 2.90*
ADF (D,12,C) ^{a)}	- 3.80**	- 4.06**	- 4.22**	- 4.27**	- 4.24**	-4.18**	- 4.13**	- 4.07**	- 4.03**	- 4.00**
Forward rates according to Nelson/Siegel										
Mean	6.62	7.14	7.61	7.89	8.05	8.14	8.19	8.21	8.23	8.23
Maximum	13.01	12.06	11.27	10.97	11.36	11.77	12.08	12.32	12.49	12.61
Minimum	3.19	3.90	4.83	5.29	5.75	6.12	6.16	6.17	6.17	6.17
Std. dev.	2.22	1.74	1.46	1.30	1.21	1.17	1.15	1.15	1.16	1.16
ADF (N,12,C) ^{a)}	- 2.79	- 2.36	- 2.50	- 2.70	- 2.86	- 2.92*	- 2.91*	- 2.87*	- 2.83	- 2.80
ADF (D,12,C) ^{a)}	- 3.80**	- 4.42**	- 4.49**	- 4.22**	- 4.01**	- 3.97**	- 4.02**	- 4.11**	- 4.19**	- 4.27**
Spot rates according to Svensson										
Mean	6.62	6.88	7.12	7.31	7.46	7.57	7.66	7.73	7.78	7.82
Maximum	13.01	12.35	12.02	11.76	11.49	11.20	10.90	11.01	11.19	11.34
Minimum	3.19	3.55	4.03	4.49	4.88	5.22	5.47	5.61	5.66	5.59
Std. dev.	2.21	1.94	1.75	1.61	1.50	1.42	1.36	1.31	1.27	1.23
ADF (N,12,C) ^{a)}	- 2.77	- 2.49	- 2.46	- 2.52	- 2.63	- 2.74	- 2.82	- 2.86	- 2.86	- 2.83
ADF (D,12,C) ^{a)}	- 3.85**	- 4.05**	- 4.22**	- 4.24**	- 4.19**	- 4.16**	- 4.12**	- 4.11**	- 4.14**	- 4.21**
Forward rates according to Svensson										
Mean	6.62	7.15	7.61	7.88	8.05	8.14	8.19	8.21	8.21	8.20
Maximum	13.01	11.87	11.37	10.99	11.30	11.86	12.24	12.48	12.63	12.71
Minimum	3.19	3.90	4.83	5.29	5.75	6.12	6.11	5.76	5.34	4.92
Std. dev.	2.21	1.74	1.46	1.31	1.23	1.18	1.15	1.14	1.13	1.14
ADF (N,12,C) ^{a)}	- 2.77	- 2.40	- 2.43	- 2.75	- 3.00*	- 3.06*	- 3.00*	- 2.88*	- 2.76	- 2.66
$ADF(D,12,C)^{a)}$	- 3.85**	- 4.62**	- 4.45**	- 4.14**	- 3.95**	- 3.96**	- 4.19**	- 4.58**	- 4.99**	- 5.33**

a Augmented Dickey-Fuller test of the null hypothesis of non-stationarity. Tests were performed in levels (L) and first differences (D). The number in parentheses denotes the number of lagged endogenous variables. The estimations were performed using a constant (C) but without a time trend. A rejection of the null hypothesis at the significance level of 5 % is marked by a * and at a significance level of 1 % by ***, the critical values being used in line with McKinnon (1991).

the two approaches never exceed three basis points; therefore, the statistics of the two approaches are not presented separately.

The average spot rate and forward rate curves may serve as reference values for assessing individual selected curves. However, much caution is required as the period used to calculate the average has a decisive influence on the results. For example, Table 2 shows that the spot and forward rate curves, taking the average of the nineties and eighties, were less steep than in the seventies. Compared with the seventies, the average spot and forward rate curves in the eighties and nineties are flatter and also lower. Whereas, for instance, the average spread between the ten-year and the one-year (spot) interest rate in the seventies was 187 basis points, it was only 95 basis points in the eighties and no more than 113 in the nineties. The differences in the average forward rate curves are even more pronounced. Fig. 5 shows that all three curves (seventies, eighties, and nineties) originated in the region of 6.4 to 6.8 percentage points but that the slopes were very different. In the seventies, the curve rose by 250 basis points to 8.94 percentage points (for a ten-year maturity). That contrasts with a rise of only 125 basis points to 8.03 percentage points in the eighties, and of only 121 basis points to 7.78 percentage points in the nineties. This comparison indicates that the average rise in the spot and forward rate curves declined during the period under review. Since, therefore, the spot and forward rate curves, measured as an average, are very much dependent on the chosen time period, a comparison of selected curves with historical averages should be treated with extreme caution.

Table 2:

Selected spot and forward rate statistics from September 1972 to December 1996 (according to Nelson/Siegel)

Spot rates										
	l year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
1972.9-96.12										
Mean	6.62	6.88	7.12	7.31	7.46	7.57	7.66	7.73	7.78	7.83
Maximum	13.01	12.54	12.11	11.73	11.39	11.09	10.82	10.99	11.16	11.30
Minimum	3.19	3.55	4.03	4.49	4.88	5.21	5.47	5.61	5.73	5.79
Std. dev.	2.22	1.94	1.75	1.60	1.50	1.41	1.35	1.31	1.27	1.25
1972.9-79.12										[
Mean	6.46	6.92	7.31	7.59	7.79	7.95	8.08	8.17	8.26	8.33
Maximum	10.46	10.20	10.08	10.28	10.47	10.62	10.80	10.99	11.16	11.30
Minimum	3.66	4.16	4.57	4.87	5.12	5.34	5.52	5.66	5.78	5.87
Std. dev.	2.14	1.77	1.60	1.51	1.48	1.46	1.45	1.45	1.45	1.46
1980.1-89.12										
Mean	6.78	6.99	7.19	7.34	7.45	7.54	7.60	7.66	7.70	7.73
Maximum	13.01	12.54	12.11	11.73	11.39	11.09	10.82	10.60	10.45	10.37
Minimum	3.53	3.98	4.45	4.80	5.11	5.37	5.58	5.69	5.75	5.79
Std. dev.	2.34	2.07	1.85	1.68	1.54	1.44	1.35	1.28	1.23	1.18
1990.1-96.12										
Mean	6.57	6.67	6.83	6.98	7.11	7.21	7,30	7.36	7.41	7.44
Maximum	9.45	9.12	9.09	9.18	9.24	9.27	9.28	9.28	9.26	9.23
Minimum	3.19	3.55	4.03	4.49	4.88	5.21	5.47	5.61	5.73	5.84
Std. dev.	2.12	1.93	1,73	1,54	1.38	1.24	1.13	1.03	0.96	0.89
	Forwar	rd rates								
	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
1972.9-96.12	1 year	2 700.5	Jours	Tyours	Jours	o years	, years	o jours	7 700.5	10) 5 4 10
Mean	6.62	7.14	7.61	7.89	8.05	8.14	8.19	8.21	8.23	8.23
Maximum	13.01	12.06	11.27	10.97	11.36	11.77	12.08	12.32	12.49	12.61
Minimum	3.19	3.90	4.83	5.29	5.75	6.12	6.16	6.17	6.17	6.17
Std. dev.	2.22	1.74	1.46	1.30	1.21	1.17	1.15	1.15	1.16	1.16
1972.9-79.12		1								
Mean	6.46	7.38	8.09	8.44	8.63	8,74	8.82	8.87	8.91	8.94
Maximum	10.46	10.12	10.61	10.97	11.36	11.77	12.08	12.32	12.49	12.61
Minimum	3.66	4.63	5.26	5.76	6.15	6.38	6.53	6.60	6.60	6.60
Std. dev.	2.14	1.51	1.38	1.40	1.44	1.47	1.49	1.51	1.53	1.54
1980.1-89.12										
Mean	6.78	7.21	7.59	7.,80	7.91	7.97	8.00	8.01	8.02	8.03
Maximum	13.01	12.06	11.27	10.60	10.27	10.27	10.27	10.27	10.27	10.27
Minimum	3.53	4.39	5.22	5.89	6.10	6.14	6.16	6.17	6.17	6.17
Std. dev.	2.34	1.84	1.46	1.21	1.06	0.98	0.93	0.91	0.90	0.89
1990.1-96.12										
Mean	6.57	6.78	7.15	7.44	7.63	7.74	7.78	7.80	7.79	7.78
Maximum	9.45	9.13	9.34	9.45	9.48	9.45	9.37	9.25	9.11	9.10
Minimum	3.19	3.90	4.83	5.29	5.75	6.12	6.40	6.59	6.69	6.72
Std. dev.	2.12	1.79	1.40	1.09	0.87	0.74	0.66	0.61	0.57	0.55

To sum up, the following regularities can be ascertained, which can be termed stylised facts of the German term structure of spot and forward interest rates:

- The spot and forward rate curves show positive slopes on average, with the latter being more pronounced. Hence rising curves are normal, in that they occur most frequently; flat and inverted curves, by contrast, are relatively rarely observed.
- 2. The spot and forward rate curves normally show a *monotonic* (positive or negative) rise. U-shaped and S-shaped curves appear extremely rarely and only over relatively short time periods.
- 3. Short-term interest and forward rates fluctuate more sharply and reach higher maximum and lower minimum values than long-term interest and forward rates. One possible interpretation of this is that changes in the slope of the curve are influenced notably by the movement of short-term interest rates.

III.4 Interpretation of a selected example

This section presents a simplified analysis designed to explain the difference between the spot rate curve and forward rate curve. The spot rate curve and forward rate curve, in principle, reflect the same information at any given time regarding interest rate expectations. But they present this information in different ways. Spot interest rates are a measure of the rate of return which is associated with buying future payment flows at present values. Forward interest rates, by contrast, are a measure of the rate of return associated with the purchase at a future date of payment flows which are even further in the future. The relationship between interest rates and forward rates is therefore similar to that between average costs and marginal costs, a familiar concept in microeconomics; the identity is exact, in the case of continuous compounding, for the instantaneous forward rate. Whereas interest rates are a measure of average rates of return over a given period, forward rates measure marginal rates of return.

Let us assume that the pure expectations hypothesis is valid and no risk or forward premiums exist. It then follows that a financial investment should generate the same expected return for a given period, regardless of whether the investment takes the form of a succession of short-term bond purchases or a single investment in a longer-term bond. The one-year (implied) forward interest rate for n years then corresponds to the expected oneyear (spot) interest rate for n years. Whereas, under these conditions, the slope of the spot rate curve, measured as the difference between the interest rates of different maturities, is a measure of the expected average changes in short-term interest rates during the relevant period, the shape of the forward rate curve describes the expected future time-path of oneyear (spot) interest rates. This is particularly interesting for monetary policy makers since it enables them to separate expectations better over the short, medium and long term than they can using the spot rate curve. Monetary policy measures often only take effect following relatively long and variable time-lags; therefore, the medium and long-term maturity segments are especially interesting for monetary policy makers. Furthermore, interpreting the spot rate curve rather than the forward rate curve in the longer-range segments can lead to inaccurate conclusions, especially if, for example, short-term expectations (e.g. expectations of interest rate reductions) are different from medium and long-term expectations (e.g. expectations of interest rate increases). This problem can be avoided by taking account of the forward rate curve, since short-term expectations are already filtered out of the medium-term segment of this curve.

The spot and forward rate curves at the beginning of 1994 can serve as an example, ¹⁵ firstly, of a fundamental reassessment of future interest rate and inflation rate prospects between two dates and, secondly, of the fact that expectations may diverge from one another over different time horizons at one point in time. The assessment of market participants on January 31, 1994, was apparently still marked by the expectations of further decreases in interest rates, at least in the short term - even if, over the longer term, an increase in interest rates was expected. This is suggested, among other things, by information from *Consensus Forecast*; in January and February 1994 lower interest rates

were expected over a three-month forecasting horizon than for a forecasting horizon of twelve months. The expectations that interest rates would fall were then dashed by the decision taken by the Board of Governors on February 4, 1994 to increase the Federal funds rate for the first time in five years; this measure also prompted a reassessment of interest rate prospects on an international scale. Market players then significantly upgraded their interest rate expectations over both the short and long run.

Fig. 6 shows the spot rate curve and the curve of the (one-year) implied forward rates at the end of January and the end of May 1994. Both curves have the same origin, ¹⁶ but thereafter the forward rate curve is below the spot rate curve, up to three years' maturity, and above it from three years onwards. The S shape of the forward rate curve permits the following interpretation: over the short term, up to about two years, falling interest rates are expected; beyond that period, rising interest rates are once again expected, which in the end are significantly above the level of current interest rates.

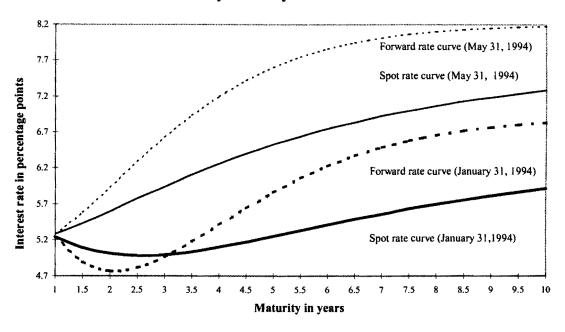
The forward rate curve thus permits a better differentiation of short, medium and long term interest rate expectations. The fundamental reassessment by market players of the outlook for short-term interest rates between January and May 1994 is graphically illustrated by the change in the forward rate curves. Whereas in January interest rates were still expected to fall over the short term, interest rate expectations in May pointed upwards, including for the short term. These changes in short-term expectations are not shown as clearly by the two spot rate curves. Thus, in principle, the forward rate curves permit a more precise analysis of expectations. Nevertheless, caution is warranted in interpreting these findings. The problems (not dealt with here) associated with interpreting term structures remain, such as the existence of time-variable risk and forward premiums.

¹⁵ Svensson (1994) provides a more comprehensive analysis of spot and forward rate curves at the beginning of 1994.

¹⁶ This reflects the fact that the current one-year (spot) rate and the (one-year) implied forward rate are identical, which can easily be verified by setting m = 1 in equation (5).

Figure 6: Spot and forward rate curves,

January and May 1994



IV. Conclusions

The direct estimation of the term structure of interest rates explained here replaces the method previously used by the Bundesbank, in which the term structure was approximated by an estimated yield-to-maturites curve. The latter is problematical in that the term structure and the yield-to-maturity curve, in the case of coupon bonds, are systematically different, and that this difference, over time, can change depending on the level of the coupons of the bonds included in the estimation. This can hamper the monetary policy interpretation of the curve shapes.

The methods of directly estimating term structures of interest rates described here are well suited to monetary policy analysis. They are a good compromise between describing the data as precisely as possible, on the one hand, and obtaining a smooth curve, on the other. For one thing, they capture the information relevant to monetary policy analysis and condense it in a manner which makes it relatively easy for monetary policy makers to interpret. For another, they are sufficiently flexibile to accommodate the constellations of

the yields on German Federal securities observed in the market. In particular, Svensson's extension (1994) of the estimation procedure originally proposed by Nelson and Siegel (1987) allows the price and yield constellations observed to be represented with a sufficient degree of precision. However, the greater flexibility of Svensson's approach has a perceptible impact only in the case of somewhat more complex curve shapes (e.g. S-shaped curves). The Svensson approach should be favoured since it provides a somewhat better statistical description of the data and, at the same time, is just as robust as the Nelson/Siegel approach. In addition, the Svensson approach ensures better international comparability of the estimation results, as it is used by many central banks.

The direct estimation of term structures of interest rates presented here makes it possible, moreover, to directly calculate the structure of (implied) forward interest rates. This forward rate curve is particularly well suited to monetary policy analysis, as it permits a better separation of the expectations contained in the term structure over the short, medium and long term than the spot rate curve.

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Annex 1: List of Federal securities included in the estimation

Securities								
Identification	Date	Issue volume				Copon		Date of
Code	of issue 1	(DM million)	Design	natio	on	date 2	Maturity	expiry
W 110024	2.10.1963	300	6		Bund 63 V	1.10.	12	1.10.1975
W 110027	1.10.1964	400	6	%	Bund 64 III	1.10.	10	1.10.1974
W 110028	4. 1.1965	400	6	%	Bund 65	1. 2.	12	1. 2.1977
W 110031	21.10.1965	200	7	%	Bund 65 III	1.11.	10	1.11.1975
W 110032	25. 1.1966	250	7	- %	Bund 66	F/A	10	1. 2.1976
W 110033	25. 4.1967	250	6.5	%	Bund 67 I	M/N	12	1. 5.1979
W 110034	13. 7.1967	300	6.5	%	Bund 67 II	F/A	10	1. 8.1977
W 110035	3. 4.1968	300	6.5	%	Bund 68 I	A/O	10	1. 4.1978
W 110036	10. 7.1968	400	6.5	%	Bund 68 II	1. 7.	12	1. 7.1980
W 110037	18.11.1969	400	7	%	Bund 69	J/D	10	1.12.1979
W 110038	29. 4.1970	410	8	%	Bund 70	M/N	10	1. 5.1980
W 110039	11. 8.1970	410	8.5	%	Bund 70 I	F/A	10	1. 8.1980
W 110040	1.12.1970	260	8.5	%	Bund 70 II	J/D	10	1.12.1980
W 110041	19. 1.1971	550	7.5	%	Bund 71	1. 2.	10	1. 2.1981
W 110042	1.12.1971	400	7.75	%	Bund 71	1.12.	10	1.12.1981
W 110043	26. 7.1972	450	8	%	Bund 72	1. 8.	10	1. 8.1982
	27.10.1972	400	8	%	Bund 72 II	1.11.	10	1.11.1982
W 110045	15. 2.1973	500	8.5	%	Bund 73	1. 3.	12	1. 3.1985
W 110046	12. 3.1973	1500	8.5	%	Bund 73 II	1. 4.	8	1. 4.1981
W 110048	7. 9.1973	500	10	%	Bund 73 II	1.10.	7	1.10.1980
i i	30.10.1973	600	10	%	Bund 73 IV	1.11.	7	1.11.1980
	13.11.1973	600	9.5	%		1.12.	8	1.12.1981
1	18. 1.1974	600	9.5	%	Bund 74	1. 2.	8	1. 2.1982
1	31. 5.1974	500	10	%	Bund 74	1. 6.	7	1. 6.1981
1	22. 8.1974	600	10	%	Bund 74 II	1. 9.	6	1. 9.1980
1 1	21.10.1974	500	10	%	Bund 74 III	1.11.	6	1.11.1980
	11.12.1974	600	9.5	%		1.12.	8	1.12.1982
3	10. 1.1975	660	9.25	%		1. 2.	8	1. 2.1983
W 110058	1. 4.1975	660	8.5	%	Bund 75 (83)	1.4.	8	1. 4.1983
i 1	21. 5.1975	660	8.25		Bund 75 (83)	1. 6.	8	1. 6.1983
W 110060	9. 6.1975	660	8		Bund 75 (83)	1. 6.	8	1. 6.1983
1 1	26. 6.1975	660	8		Bund 75 (83) II	1. 7.	8	1. 7.1983
	30.12.1975	660	8		Bund 75 (82) III	1. 1.	6	1. 1.1982
1 3	11. 2.1976	250	7.5		Bund 76 (81)	1. 2.	5	1. 2.1981
1 .	11. 2.1976	450	8		Bund 76 (84)	1. 2.	8	1. 2.1984
	22. 3.1976	660	7.5		` ′	1. 4.	8	1. 4.1984
	12. 7.1976	800	8	%	Bund 76 II (81)	1. 8.	5	1. 8.1981
1 1	16. 8.1976	500	8	%	Bund 76 III (81)	1. 9.	5	1. 9.1981
1	16. 8.1976	300	8.25	%	Bund 76 (84)	1.9.	8	1. 9.1984
1 1	29. 9.1976	700	8	%	` ′	1.10.	9	1.10.1984
	25.11.1976	1000	7.25	%	Bund 76 (86)	1.12.	10	1.12.1986
	29.12.1976	750	7 25	%		1.1.	6	1. 1.1983
1	29.12.1976	750 700	7.25	%	Bund 76 II (87)	1. 1.	10	1. 1.1987
W 110073	2. 3.1977	700	7		Bund 77 (84)	1.3.	7	1. 3.1984
W 110074	4. 4.1977	850	6.75	-	Bund 77 (87)	1. 4.	10	1. 4.1987
W 110075	1. 7.1977	900	6.5		Bund 77 (87)	1. 7.	10	1. 7.1987
1 1	15. 9.1977 29 12 1977	900 800	6	% %	` ′	1.10.	10	1.10.1987
W 110077	29.1 <u>2.1977</u>	800	5.5	%	Bund 78 (84)	1. 1.	6	1. 1.1984

y								
W 110078	29.12.1977	900	6	%	Bund 78 (88)	1. 1.	10	1. 1.1988
W 110079	14. 2.1978	800	5.5	%	Bund 78 II (86)	1. 3.	8	1. 3.1986
W 110080	14. 2.1978	500	6	%	Bund 78 II (93)	1. 3.	15	1. 3.1993
W 110081	13. 4.1978	500	5	%	Bund 78 (84)	1.5.	6	1. 5.1984
W 110082	13. 4.1978	500	5.25	%	Bund 78 (86)	1. 5.	8	1. 5.1986
W 110083	13. 4.1978	500	5.75	%	Bund 78 (90)	1. 5.	12	1. 5.1990
W 110084	4. 9.1978	900	6	%	Bund 78 III (84)	1. 9.	6	1. 9.1984
W 110085	4. 9.1978	500	6.5	%	Bund 78 (88)	1. 9.	10	1. 9.1988
W 110086	27.11.1978	700	6	%	Bund 78 IV (84)	1.12.	6	1.12.1984
W 110087	27.11.1978	500	6.5	%	Bund 78 II (88)	1.12.	10	1.12.1988
W 110088	28.12.1978	1000	6.25	%	Bund 79 (85)	1. 1.	6	1. 1.1985
W 110089	28.12.1978	350	6.5	%	Bund 79 (87)	1. 1.	8	1. 1.1987
W 110090	28.12.1978	350	6.75	%	Bund 79 (89)	1. 1.	10	1. 1.1989
W 110091	5. 4.1979	600	7	%	Bund 79 (85)	1. 4.	6	1. 4.1985
W 110092	5. 4.1979	600	7.25	%	Bund 79 (89)	1. 4.	10	1. 4.1989
W 110093	10. 5.1979	750	7.25	%	Bund 79 II (85)	1. 6.	6	1. 6.1985
W 110094	10. 5.1979	750	7.5	%	Bund 79 (89)	1. 6.	10	1. 6.1989
W 110094 W 110095	7. 6.1979	600	7.75	-%	Bund 79 (85)	1. 7.	6	1. 7.1985
W 110096	7. 6.1979	900	8	%	Bund 79 (89)	1. 7.	10	1. 7.1989
W 110090 W 110097	5. 7.1979	1600	8	%	Bund 79 II (89)	1. 7.	10	1. 8.1989
W 110097	15. 8.1979	1600	7.5	%	Bund 79 II (89)	1. 8. 1. 9.	10	1. 9.1989
W 110099	15.10.1979	1500	7.75	%	Bund 79 II (89)	1.11.	10	1.11.1989
W 110099	28.12.1979	1500	7.75	%	Bund 80 (90)	1.11.	10	1. 1.1990
		1100		%		1. 4.	10	1. 4.1990
W 113400 W 113401	1. 4.1980		10		Bund 80 (90)	1. 7.	10	1. 7.1990
	9. 6.1980	1500	8.25	%	Bund 80 (90)		l	1.11.1990
W 113402	24.10.1980	1500	8.25	%	Bund 80 II (90)	1.11.	10	
W 113403	13. 1.1981	1500	9	%	Bund 81 (91)	1. 2.	10	1. 2.1991
W 113404	27. 3.1981	1500	10	%	Bund 81 (89)	1. 4.	8	1. 4.1989
W 113405	1. 7.1981	1500	10.5	%	Bund 81 (91)	1. 7.	10	1. 7.1991 1. 9.1991
W 113406	3. 9.1981	1600	10.75	%	Bund 81 (91)	1. 9.	10	1.12.1991
W 113407	12.11.1981	1600	10	%	Bund 81 II (91)	1.12.	10	
W 113408	4. 1.1982	1600	9.75	%	Bund 82 (92)	1. 1.	10 10	1. 1.1992 1. 3.1992
W 113409	24. 2.1982	1600	9.75	%	` '	1.3.		1
W 113410	11. 3.1982	1600	9.50		Bund 82 (92)	1.4.	10	1. 4.1992
W 113411	13. 4.1982	1600	9		Bund 82 (92)	1.5.	10	1. 5.1992
W 113412	17. 5.1982	1600	8.50		Bund 82 (92)	1. 6.	10	1. 6.1992
W 113413	28. 7.1982	1600	9	%	` ′	1. 8.	10	1. 8.1992
W 113414	24. 8.1982	1600	8.75	%	` ′	1. 9.	10	1. 9.1992
W 113415	27.10.1982	1600	7.75	%	` ′	1.11.	8	1.11.1990
W 113416	7.12.1982	1600	7.75		Bund 82 II (92)	1.12.	10	1.12.1992
W 113417	3. 1.1983	1600	7.50		Bund 83 (93)	1. 1.	10	1. 1.1993
W 113418	1. 3.1983	1600	7.50		Bund 83 II (93)	1. 3.	10	1. 3.1993
W 113419	1. 6.1983	1600	7.5	%	` ′	1. 6.	8	1. 6.1991
W 113420	7. 6.1983	1600	8.25	%	` ′	1. 6.	8	1. 6.1993
W 113421	7. 7.1983	1600	8		Bund 83 (93)	1. 7.	10	1. 7.1993
W 113422	5. 8.1983	1600	8.25		Bund 83 II (93)	1. 8.	10	1. 8.1993
W 113423	3.10.1983	1600	8.25		Bund 83 III (93)	1.10.	10	1.10.1993
W 113424	1.11.1983	1600	8.25	%	Bund 83 IV (93)	1.11.	10	1.11.1993
W 113425	7.12.1983	1600	8.25	%	Bund 83 V (93)	1.12.	10	1.12.1993
W 113426	2. 1.1984	2000	8.25	%	` ,	1. 1.	10	1. 1.1994
W 113427	2. 2.1984	2000	8.25		Bund 84 II (94)	1. 2.	10	1. 2.1994
W 113428	1. 3.1984	2000	8		Bund 84 (94)	18. 3.	10	18. 3.1994
W 113429	6. 6.1984	2000	8.25	%	Bund 84 III (94)	20. 6.	10	20. 6.1994

				_				
W 113430	4. 7.1984	2000	8.25	%	Bund 84 IV (94)	20. 7.	10	20. 7.1994
W 113431	1. 8.1984	2000	8.25	%	Bund 84 V (94)	22. 8.	10	20. 8.1994
W 113432	9.10.1984	2000	7.50	%	Bund 84 (94)	20.10.	10	20.10.1994
W 113433	5.12.1984	2000	7	%	Bund 84 (94)	20.12.	10	20.12.1994
W 113434	2. 1.1985	2000	7	%	Bund 85 (95)	20. 1.	10	20. 1.1995
W 113435	29. 1.1985	2000	7.25	%	Bund 85 (95)	20. 2.	10	20. 2.1995
W 113436	26. 2.1985	2000	7.625	%	Bund 85 (95)	20. 3.	10	20. 3.1995
W 113437	26. 3.1985	2500	7.50	%	Bund 85 (95)	20. 4.	10	20. 4.1995
W 113438	22. 4.1985	2500	'7.25	%	Bund 85 II (95)	22. 5.	10	22. 5.1995
W 113439	3. 6.1985	2500	7	%	Bund 85 II (95)	20. 6.	10	20. 6.1995
W 113440	29. 7.1985	2500	6.75	%	Bund 85 (95)	20. 7.	10	20. 7.1995
W 113441	9.10.1985	2500	6.50	%	Bund 85 (95)	20.10.	10	20.10.1995
W 113442	2. 1.1986	3000	6.375	%	Bund 86 (96)	22. 1.	10	22. 1.1996
W 113443	5. 2.1986	3000	6.375	%	Bund 86 II (96)	20. 2.	10	20. 2.1996
W 113444	5. 3.1986	3000	6	%	Bund 86 (98)	20. 4.	10	20. 4.1998
W 113445	28. 5.1986	3000	5.75	%	Bund 86 (96)	20. 6.	10	20. 6.1996
W 113446	28. 5.1986	7000	6	%	Bund 86 II (16)	20. 6.	30	20. 6.2016
W 113447	14. 7.1986	4000	5.75	%	Bund 86 II (96)	22. 7.	10	22. 7.1996
W 113448	2. 9.1986	3000	5.50	%	Bund 86 (96)	20. 9.	10	20. 9.1996
W 113449	2. 9.1986	1000	5.625	%	Bund 86 (16)	20. 9.	30	20. 9.2016
W 113450	6.10.1986	4000	6	%	Bund 86 III (98)	20.10.	12	20.10.1998
W 113451	17.11.1986	4000	6.50	%	Bund 86 (96)	20.10.	10	20.12.1996
W 113452	2. 1.1987	4000	6.125	%	Bund 87 (97)	20. 1.	10	20. 1.1997
W 113453	6. 2.1987	4000	5.75	%	Bund 87 (97)	20. 2.	10	20. 2.1997
W 113454	10. 3.1987	4000	6	%	Bund 87 (97)	20. 3.	10	20. 3.1997
W 113455	27. 5.1987	4000	5.50	%	Bund 87 (97)	20. 5.	10	20. 5.1997
W 113456	14. 7.1987	4000	6.125	%	Bund II 87 (97)	21. 7.	10	21. 7.1997
W 113457	27. 8.1987	3200	6.375	%	Bund 87 (97)	20. 8.	10	20. 8.1997
W 113458	7.10.1987	4000	6.75	%	Bund 87 (97)	22. 9.	10	22. 9.1997
W 113459	11.11.1987	2000	6.375	%	Bund 87 II (97)	20.10.	10	20.10.1997
W 113460	4. 1.1988	5000	6.375	%	Bund 88 (98)	20. 1.	10	20. 1.1998
W 113461	4. 2.1988	4000	6.25	%	Bund 88 (98)	20. 2.	10	20. 2.1998
W 113462	22. 3.1988	4000	6.125	%	` '	20. 3.	10	20. 3.1998
W 113463	30. 5.1988	4000	6.50	%	Bund 88 (98)	20. 5.	10	20. 5.1998
W 113464	8. 8.1988	4000	6.75		Bund 88 (98)	20. 7.	10	20. 7.1998
W 113465	7. 9.1988	4000	6.75	%	Bund 88 II (98)	20. 8.	10	20. 8.1998
W 113466	31.10.1988	4000	6.375	%	Bund 88 II (98)	20.11.	10	20.11.1998
W 113467	1.12.1988	4000	6.375		Bund 88 III (98)	21.12.	10	21.12.1998
W 113468	2. 1.1989	5000	6.50		Bund 89 (99)	2. 1.	10	2. 1.1999
W 113469	1. 2.1989	4000	6.75		Bund 89 (99)	20. 1.	10	20. 1.1999
W 113470	6. 3.1989	4000	7		Bund 89 (99)	22. 2.	10	22. 2.1999
W 113471	10. 5.1989	4000	7		Bund 89 II (99)	20. 4.	10	20. 4.1999
W 113472	5. 7.1989	4000	6.75		Bund 89 II (99)	21. 6.	10	21. 6.1999
W 113473	7. 9.1989	4000	7		Bund 89 III (99)	20. 9.	10	20. 9.1999
W 113474	11.10.1989	4000	7		Bund 89 IV (99)	20.10.	10	20.10.1999
W 113475	6.11.1989	4000	'7.125	_	Bund 89 (99)	20.12.	10	20.12.1999
W 113476	2. 1.1990	5000	7.25		Bund 90 (2000)	20. 1.	10	20. 1.2000
W 113477	6. 2.1990	4000	7.75		Bund 90 (2000)	21. 2.	10	21. 2.2000
W 113479	22. 5.1990	6000	8.75		Bund 90 (2000)	22. 5.	10	22. 5.2000
W 113480	31. 7.1990	8000	8.50	%	` ,	21. 8.	10	21. 8.2000
W 113481	8.10.1990	8000	9	%	` '	20.10.	10	20.10.2000
W 113482	4.12.1990	8000	8.875	%	, ,	20.12.	10	20.12.2000
W 113483	27.12.1990	10000	9	%	Bund 91 (2001)	221.	10	22. 1.2001

W 113484	7. 5.1991	10000	8.375	%	Bund 91 (2001)	21. 5.	10	21. 5.2001
W 113485	8.10.1991	18000	8.25	%	Bund 91 (2001)	20. 9.	10	20. 9.2001
W 113486	9. 7.1992	15000	8	%	Bund 92 (2002)	22. 7.	10	22. 7.2002
W 113487	6.10.1992	10000	7.25	%	(" ")	21.10.	10	21.10.2002
W 113488	29.12.1992	16000	7.125	_	Bund 92 (2002)	20.12.	10	20.12.2002
W 113489	4. 5.1993	10000	6.75	%	Bund 93 (2003)	22. 4.	10	22. 4.2003
W 113490	3. 8.1993	16000	6.50	%	Bund 93 (2003)	15. 7.	10	15. 7.2003
W 113491	12.10.1993	12000	6	%	Bund 93 (2003)	15. 9.	10	15. 9.2003
W 113492	28.12.1993	10000	6.25	%	Bund 93 (2024)	4. 1.	30	4. 1.2024
W 113493	19. 7.1994	10000	6.75	%	Bund 94 (2004)	15. 7.	10	15. 7.2004
W 113495	8.11.1994	10000	7.50	%	Bund 94 (2004)	11.11.	10	11.11.2004
W 113496	28.12.1994	17000	7.375		Bund 95 (2005)	3. 1.	10	3. 1.2005
W 113497	12. 5.1995	18000	6.875	%	` ′	12. 5.	10	12. 5.2005
W 113498	14.10.1995	20000	6.50	%		14.10.	10	14.10.2005
W 113499	5. 1.1996	25000	6.00	%	Bund 96 (2006)	5. 1.	10	5. 1.2006
W 113500	16. 2.1996	12000	6.00	%	Bund 96 II (2016)	16. 2.	20	16. 2.2016
W 113501	26. 4.1996	14000	6.25	%	Bund 96 (2006)	26. 4.	10	26. 4.2006
W 113653	22. 6.1988	n.a.	6	%		22. 6.	4	22. 6.1992
W 113657	22.11.1989	n.a.	7.625	%	BSA	22.11.	4	22.11.1993
W 113659	22.05.1991	n.a.	8.625	%	BSA	22. 5.	4	22. 5.1995
W 113660	20.07.1991	n.a.	8.875	%	BSA	20. 7.	4	20. 7.1995
W 113661	20.12.1991	n.a.	8.75	%	BSA	20.12.	4	20.12.1995
W 113662	22.01.1992	n.a.	8.125	%	BSA	22. 1.	4	22. 1.1996
W 113663	22.08.1992	n.a.	8.5	%	BSA	20. 8.	4	20. 8.1996
W 113664	21.11.1992	<u>n</u> .a.	7.125	%	BSA	21.11.	4	21.11.1996
W 113665	20.02.1993	n.a.	6.5	%	BSA	20. 2.	4	20. 2.1997
W 113666	20.05.1993	n.a.	6.375	%	BSA	20. 5.	4	20. 5.1997
W 113667	20.08.1993	n.a.	5.75	%	BSA	20. 8.	4	20. 8.1997
W 113668	20.11.1993	n.a.	5.25		BSA	20.11.	4	20.11.1997
W 113669	25.02.1994	n.a.	5.25		BSA	25. 2.	4	25. 2.1998
W 113671	14.08.1994	n.a.	6.375		BSA	14. 8.	4	14. 8.1998
W 113672	2.12.1994	n.a.	6.875		BSA	2.12.	4	2.12.1998
W 113673	24. 2.1995	n.a.	6.875		BSA	24. 2.	4	24. 2.1999
W 113674	28. 5.1995	n.a.	5.75		BSA	28. 5	4	28. 5.1999
W 113675	18. 9.1996	10000	3.50		BSA	18. 9.	2	18. 9.1998
W 114001	3.12.1979	1100	7.75		Bobl	1.12.	5	1.12.1984
W 114002	5. 2.1980	250	8.0		Bobl	1. 2.	5	1. 2.1985
W 114003	25. 2.1980	100	8.25		Bobl	1. 2.	5	1. 2.1985
W 114004	14. 3.1980	2800	9.25		Bobl	1. 3.	5	1. 3.1985
W 114005	22. 4.1980	1300	8.75		Bobl	1. 4.	5	1. 4.1985
W 114006	28. 5.1980	550	8.25		Bobl	1. 5.	5	1. 5.1985
W 114007	19. 6.1980	1250	8.0		Bobl	1. 6.	5	1. 6.1985
W 114008	30. 7.1980	350	7.5		Bobl	1. 8.	5	1. 8.1985
W 114009	1.10.1980	1300	8.25		Bobl	1.10.	5	1.10.1985
W 114010	11.12.1980	200	8.75		Bobl	1. 1.	5	1. 1.1986
W 114011	6. 1.1981	2500	9.0		Bobl	1. 1.	5	1. 1.1986
W 114012	18. 2.1981	200	9.75		Bobl	1. 3.	5	1. 3.1986
W 114013	27. 2.1981	2200	10.0		Bobl Bobl	1. 3.	5	1. 3.1986
W 114014	23. 3.1981	120	9.5		Bobl	1. 4.	5	1. 4.1986
W 114015	3. 4.1981	900	10.00		Bobl	1. 4.	5	1. 4.1986
W 114016	21. 5.1981	1400	10.50		Bobl	1. 6.	5	1. 6.1986
W 114017	28. 7.1981	1500	11.00		Bobl	1. 8.	5	1. 8.1986
W 114018	23. 9.1981	900	10.50	7∕0	Bobl	1.10.	5	1.10.1986

								,
W 114019	8.10.1981	400	10.00		Bobl	1.10.	5	1.10.1986
W 114020	14.10.1981	100	9.50		Bobl	1.11.	5	1.11.1986
W 114021	30.10.1981	550	10.25	%	Bobl	1.11.	5	1.11.1986
W 114022	6.11.1981	350	10.00	%	Bobl	1.11.	5	1.11.1986
W 114023	16.11.1981	850	9.50	%	Bobl	1.12.	5	1.12.1986
W 114024	12. 1.1982	1800	9.75	%	Bobl	1. 1.	5	1. 1.1987
W 114025	5. 3.1982	1000	9.50	%	Bobl	1. 3.	5	1. 3.1987
W 114026	19. 3.1982	1000	9.25	%	Bobl	1. 4.	5	1. 4.1987
W 114027	5. 4.1982	450	9.00	%	Bobl	1. 4.	5	1. 4.1987
W 114028	28. 4.1982	600	8.50	%	Bobl	1. 5.	5	1. 5.1987
W 114029	29. 6.1982	1150	9.50	%	Bobl	1. 7.	5	1. 7.1987
W 114030	26. 7.1982	1800	9.00	%	Bobl	1. 8.	5	1. 8.1987
W 114031	23. 8.1982	1100	8.75	%	Bobl	1. 9.	5	1. 9.1987
W 114032	22. 9.1982	1500	8.25	%	Bobl	1.10.	5	1.10.1987
W 114033	14.10.1982	1300	8.00	%	Bobl	1.11.	5	1.11.1987
W 114034	25.10.1982	2200	7.75	%	Bobl	1.11.	5	1.11.1987
W 114035	7.12.1982	2200	7.50	%	Bobl	1.12.	5	1.12.1987
W 114036	11. 1.1983	2850	7.25	%	Bobl	1. 1.	5	1. 1.1988
W 114037	9. 3.1983	600	6.75	%	Bobl	1. 3.	5	1. 3.1988
W 114038	22. 4.1983	400	7.00	%	Bobl	1. 5.	5	1. 5.1988
W 114039	18. 5.1983	100	7.25	%		1. 6.	5	1. 6.1988
W 114040	27. 5.1983	200	7.50	%	Bobl	1. 6.	5	1. 6.1988
W 114041	10. 6.1983	3600	8.00	%		1. 6.	5	1. 6.1988
W 114042	1. 9.1983	2200	8.00	%	Bobl	1. 9.	5	1. 9.1988
W 114043	30.11.1983	500	8.00	%		1.12.	5	1.12.1988
W 114044	16.12.1983	1000	8.25	%		1.12.	5	1.12.1988
W 114045	30.12.1983	3800	8.00	%		1. 1.	5	1. 1.1989
W 114046	6. 3.1984	300	7.50	%	Bobl	20. 3.	5	20. 3.1989
W 114047	27. 3.1984	2400	7.75	%		20. 3.	5	20. 3.1989
W 114048	5. 7.1984	3900	7.75	%	Bobl	20. 7.	5	20. 7.1989
W 114049	14. 9.1984	2000	7.50	%	Bobl	20. 9.	5	20. 9.1989
W 114050	9.10.1984	800	7.25	%		20.10.	5	20.10.1989
W 114051	23.10.1984	1650	'7.00		Bobl	20.10.	5	20.10.1989
W 114 <u>052</u>	6.12.1984	1400	6.75		Bobl	20.12.	5	20.12.1989
W 114053	15. 2.1985	4500	7.25		Bobl	20. 3.	5	20. 3.1990
W 114054	16. 4.1985	2900	7.00		Bobl	20. 4.	5	20. 4.1990
W 114055	22. 5.1985	2700	6.75		Bobl	20. 6.	5	20. 6.1990
W 114056	23. 7.1985	2500	6.50	%		20. 7.	5	20. 7.1990
W 114057	19. 8.1985	1100	6.25		Bobl	20. 9.	5	20. 9.1990
W 114058	1.11.1985	3100	6.75		Bobl	20.11.	5	20.11.1990
W 114059	8.11.1985	1100	6.50		Bobl	20.12.	5	20.12.1990
W 114060	20.12.1985	3800	6.25		Bobl	21. 1.	5	21. 1.1991
W 114061	25. 2.1986	2800	6.00		Bobl	22. 4.	5	22. 4.1991
W 114062	13. 3.1986	2800	5.75		Bobl	22. 4.	5	22. 4.1991
W 114063	4. 4.1986	5000	5.50		Bobl	21. 5.	5	21. 5.1991
W 114064	22. 7.1986	1500	5.50		Bobl	20. 8.	5	20. 8.1991
W 114065	2. 9.1986	500	5.25		Bobl	20. 9.	5	20. 9.1991
W 114066	27.10.1986	3300	5.75		Bobl	21.10.	5	21.10.1991
W 114067	4.12.1986	5200	5.50		Bobl	20.12.	5	20.12.1991
W 114068	4. 2.1987	4500	5.25	_	Bobl	20. 2.	5	20. 2.1992
W 114069	30. 4.1987	1800	5.00		Bobl	21. 4.	5	21. 4.1992
W 114070	7. 7.1987	500	5.25		Bobl	20. 7.	5	20. 7.1992
W 114071	7. 8.1987	3500	5.50		Bobl	20. 8.	5	20. 8.1992
77 23 70/1	1 0.1707							

W 114072	23. 9.1987	600	5.75 % Bobl	21. 9.	5	21. 9.1992
W 114073	15.10.1987	3600	6.00 % Bobl	20.10.	5	20.10.1992
W 114074	12.11.1987	2600	5.50 % Bobl	20.11.	5	20.11.1992
W 114075	29. 1.1988	3600	5.25 % Bobl	22. 2.	5	22. 2.1993
W 114076	2. 3.1988	1000	5.00 % Bobl	22. 3.	5	22. 3.1993
W 114077	10. 5.1988	500	5.25 % Bobl	21. 5.	5	21. 5.1993
W 114078	30. 5.1988	3000	5.50 % Bobl	21. 5.	5	21. 5.1993
W 114079	1. 8.1988	5000	6.00 % Bobl	20. 8.	5	20. 8.1993
W 114080	3.10.1988	2400	5.75 % Bobl	20.10.	5	20.10.1993
W 114081	31.10.1988	750	5.50 % Bobl	22.11.	5	22.11.1993
W 114082	5. 1.1989	1000	6.00 % Bobl	5. 1.	5	5. 1.1994
W 114083	8. 2.1989	4000	6.50 % Bobl	20. 1.	5	20. 1.1994
W 114084	5. 5.1989	5000	6.75 % Bobl	20. 4.	5	20. 4.1994
W 114085	14. 7.1989	5000	6.75 % Bobl	20. 7.	5	20. 7.1994
W 114086	20. 9.1989	5000	7.00 % Bobl	20. 9.	5.	20. 9.1994
W 114087	13.11.1989	5000_	7.25 % Bobl	20.12.	5	20.12.1994
W 114088	4. 1.1990	5000	7.50 % Bobl	20. 1.	5	20. 1.1995
W 114089	9. 2.1990	5000	8.00 % Bobl	20. 2.	5	20. 2.1995
W 114090	22. 2.1990	6000	8.50 % Bobl	20. 3.	5	20. 3.1995
W 114091	30. 4.1990	10000	8.75 % Bobl	22. 5.	5	22. 5.1995
W 114092	6. 7.1990	8000	8.75 % Bobl	20. 7.	5	20. 7.1995
W 114093	12.10.1990	12000	9.00 % Bobl	20.10.	5	20.10.1995
W 114094	12.12.1990	10000	8.875 % Bobl	22. 1.	5	22. 1.1996
W 114095	11. 2.1991	12000	8.625 % Bobl	20. 2.	5	22. 2.1996
W 114096	12. 4.1991	10000	8.50 % Bobl	22. 4.	5	22. 4.1996
W 114097	27. 9.1991	8000	8.5 % Bobl	20. 9.	5	20. 9.1996
W 114098	6. 1.1992	8000	8.375 % Bobl	20. 1.	5	20. 1.1997
W 114099	28. 2.1992	8000	8.00 % Bobl	20. 3.	5	20. 3.1997
W 114100	26. 6.1992	10000	8.25 % Bobl	21. 7.	5	21. 7.1997
W 114101	15. 9.1992	8000	8.00 % Bobl	22. 9.	5	22. 9.1997
W 114102	6.10.1992	7000	7.50 % Bobl	20.10.	5	20.10.1997
W 114103	20.10.1992	7000	7.25 % Bobl	20.10.	5	20.10.1997
W 114104	6. 1.1993	10000	7.00 % Bobl	22.12.	5	22.12.1997
W 114105	15. 2.1993	10000	6.625 % Bobl	20. 1.	5	20. 1.1998
W 114106	2. 3.1993	8000	6.00 % Bobl	20. 2.	5	20. 2.1998
W 114107	2. 6.1993	6000	6.375 % Bobl	20. 5.	5	20. 5.1998
W 114108	30. 8.1993	5000	5.75 % Bobl	20. 8.	5	20. 8.1998
W 114109	15.11.1993	5000	5.25 % Bobl	20.10.	5	20.10.1998
W 114110	14. 3.1994	4000	5.375 % Bobl	22. 2.	5	22. 2.1999
W 114111	16. 6.1994	4000	6.125 % Bobl	20. 5.	5	20. 5.1999
W 114112	15. 9.1994	6000	6.75 % Bobl	15. 9.	5	15. 9.1999
W 114113	13. 1.1995	5000	7.00 % Bobl	13. 1.	5	13. 1.2000
W 114114	23. 3.1995	7000	6.50 % Bobl	15. 3.	5	15. 3.2000
W 114115	10. 5.1995	10000	5.875 % Bobl	15. 5.	5	15. 5.2000
W 114116	22. 8.1995	10000	5.75 % Bobl	22. 8.	5	22. 8.2000
W 114117	21.11.1995	10000	5.125 % Bobl	21.11.	5	21.11.2000
W 114118	21. 2.1996	10000	5.25 % Bobl	21. 2.	5	21. 2.2001
W 114119	21. 5.1996	8000	5.00 % Bobl	21. 5.	5	21. 5.2001
W 114120	20. 8.1996	13000	5.00 % Bobl	20. 8.	5	20. 8.2001

¹ In the case of five-year special Treasury bonds (Bobls), the frist issue date is indicated.

² F/A: February/August, M/N = May/November, A/O = April/October, J/D = June/December.

Annex 2: Number of bonds classed by remaining time-to-maturity

M>10	0	_		0	0	0	7	2	_	-	-	0	0	0	4	4	2	2	2	2	2	2	3	ю	3
9 <m<10< td=""><td>2</td><td>0</td><td>0</td><td></td><td>0</td><td>ю</td><td>_</td><td>9</td><td>4</td><td>4</td><td>. 0</td><td>7</td><td>9</td><td>7</td><td>5</td><td>7</td><td>10</td><td>00</td><td>9</td><td>3</td><td>2</td><td>3</td><td>2</td><td></td><td>3</td></m<10<>	2	0	0		0	ю	_	9	4	4	. 0	7	9	7	5	7	10	00	9	3	2	3	2		3
6>M>8	2	2	0	0	2	7	4	3	7	4	5	∞	6	∞	∞	9	∞	10	7	9	3	т.	3	2	3
7 <m<8< td=""><td>4</td><td>3</td><td>ĸ</td><td>5</td><td>ю</td><td>2</td><td>4</td><td>4</td><td>3</td><td>∞</td><td>5</td><td>9</td><td>∞</td><td>6</td><td>∞</td><td>90</td><td>9</td><td>∞</td><td>_</td><td>7</td><td>9</td><td>ю</td><td>8</td><td>ю</td><td>2</td></m<8<>	4	3	ĸ	5	ю	2	4	4	3	∞	5	9	∞	6	∞	90	9	∞	_	7	9	ю	8	ю	2
6 <m<7< td=""><td>2</td><td>9</td><td>8</td><td>4</td><td>S</td><td>4</td><td>2</td><td>5</td><td>4</td><td>3</td><td>œ</td><td>'n</td><td>9</td><td>œ</td><td>6</td><td>∞</td><td>∞</td><td>9</td><td>∞</td><td>=</td><td>7</td><td>9</td><td>ю</td><td>2</td><td>3</td></m<7<>	2	9	8	4	S	4	2	5	4	3	œ	'n	9	œ	6	∞	∞	9	∞	=	7	9	ю	2	3
5 <m<6< td=""><td>-</td><td>2</td><td>∞</td><td>5</td><td>4</td><td>8</td><td>9</td><td>\$</td><td>S</td><td>4</td><td>8</td><td>∞</td><td>S</td><td>9</td><td>∞</td><td>6</td><td></td><td>∞</td><td>9</td><td>∞</td><td></td><td>7</td><td>9</td><td>ю</td><td>3</td></m<6<>	-	2	∞	5	4	8	9	\$	S	4	8	∞	S	9	∞	6		∞	9	∞		7	9	ю	3
4 <m<5< td=""><td>2</td><td>grand.</td><td>7</td><td>∞</td><td>6</td><td>۸</td><td>9</td><td>œ</td><td>13</td><td>91</td><td>14</td><td>6</td><td>4</td><td>=</td><td>12</td><td>4</td><td>15</td><td>12</td><td>41</td><td>6</td><td>13</td><td>91</td><td>6</td><td>10</td><td>9</td></m<5<>	2	grand.	7	∞	6	۸	9	œ	13	91	14	6	4	=	12	4	15	12	41	6	13	91	6	10	9
3 <m4< td=""><td></td><td>7</td><td>_</td><td>7</td><td>∞</td><td>6</td><td>8</td><td>9</td><td>6</td><td>15</td><td>8</td><td>16</td><td>12</td><td>15</td><td>13</td><td>13</td><td>15</td><td>16</td><td>13</td><td>14</td><td>13</td><td>16</td><td>61</td><td>12</td><td>=</td></m4<>		7	_	7	∞	6	8	9	6	15	8	16	12	15	13	13	15	16	13	14	13	16	61	12	=
2 <m<3< td=""><td>2</td><td></td><td>2</td><td></td><td>7</td><td>∞</td><td>6</td><td>2</td><td>9</td><td>6</td><td>15</td><td><u>8</u></td><td>16</td><td>12</td><td>15</td><td>13</td><td>14</td><td>15</td><td>16</td><td>13</td><td>17</td><td>13</td><td>61</td><td>19</td><td>12</td></m<3<>	2		2		7	∞	6	2	9	6	15	<u>8</u>	16	12	15	13	14	15	16	13	17	13	61	19	12
1 <m<2< td=""><td>_</td><td>7</td><td></td><td>2</td><td>_</td><td>2</td><td>∞</td><td>6</td><td>\$</td><td>9</td><td>6</td><td>15</td><td><u>&</u></td><td>16</td><td>12</td><td>15</td><td>12</td><td>14</td><td>15</td><td>16</td><td>13</td><td>17</td><td>13</td><td>19</td><td>21</td></m<2<>	_	7		2	_	2	∞	6	\$	9	6	15	<u>&</u>	16	12	15	12	14	15	16	13	17	13	19	21
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Total	17	21	25	28	35	40	49	61	63	74	88	86	103	105	106	108	801	109	110	101	901	86	16	85	80
Date	12/29/72	12/28/73	12/30/74	12/30/75	12/30/76	12/30/77	12/29/78	12/28/79	12/30/80	12/30/81	12/30/82	12/30/83	12/27/84	12/30/85	12/30/86	12/30/87	12/29/88	12/28/89	12/28/90	12/30/91	12/30/92	12/30/93	12/30/94	12/29/95	12/30/96

M = Remaining time-to-maturity in years

The following papers have so far been published:

May	1995	The Circulation of	
		Deutsche Mark Abroad	Franz Seitz
June	1995	Methodology and technique	
		for determining structural	
		budget deficits	Gerhard Ziebarth
July	1995	The information content of derivatives	
		for monetary policy - Implied vola-	
		tilities and probabilities	Holger Neuhaus
August	1995	Das Produktionspotential	
		in Ostdeutschland *	Thomas Westermann
January	1996	Sectoral Disaggregation	
		of German M3	Vicky Read
March	1996	Monetary aggregates with special	
		reference to structural changes in the	
		financial markets	Michael Scharnagl
March	1996	The impact of interest rates on	
		private consumption in Germany	Hermann-Josef Hansen
May	1996	Market Reaction to Changes	
		in German Official Interest Rates	Daniel C. Hardy
May	1996	The role of wealth	
•		in money demand	Dieter Gerdesmeier

^{*} Available in German only.

August	1996	Intergenerational redistribution through the public sector – Methodology of generational accounting and its empirical application to Germany	Stephan Boll
		application to Germany	Stephan Bon
August	1996	The impact of the exchange rate	
		on Germany's balance of trade	Jörg Clostermann
October	1996	Alternative specifications of the	
		German term structure and its informa-	
		tion content regarding inflation	Sebastian T. Schich
November	1996	Enterprises' financing structure and	
		their response to monetary policy stimuli	
		An analysis based on the	
		Deutsche Bundesbank's	
		corporate balance sheet statistics	Elmar Stöss
January	1997	Reserve Requirements	
·		and Economic Stabilization	Ulrich Bindseil
June	1997	Direct investment	
-		and Germany as a business location	Thomas Jost
July	1997	Price Stability versus	
oury	100,	Low Inflation in Germany	Karl-Heinz Tödter
		An Analysis of Costs and Benefits	Gerhard Ziebarth
October	1997	Estimating the German	
_ 3.000.		term structure	Sebastian T. Schich

