DEUTSCHE BUNDESBANK
EUROSYSTEM

Technical Paper
A latent weekly GDP indicator for Germany

08/2023

Sercan Eraslan
Magnus Reif

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Internet http://www.bundesbank.de

Non-technical summary

We construct a weekly indicator for real economic activity in Germany—hereafter referred to as WGDP. The WGDP is calculated as the weighted common component within a mixed-frequency dynamic factor model framework that allows to process indicators sampled at different frequencies. Currently, it includes the growth rate of quarterly real gross domestic product (GDP) as well as monthly and weekly indicators related to economic activity. Moreover, the model allows to interpolate quarterly GDP growth in its latent weekly growth rates and provides useful information on the current state of the economic activity in a timely manner.

The WGDP is scaled to the magnitude of the quarterly GDP and matches its realised growth rates due to its construction. As such, the WGDP can be transformed into a level series and interpreted as week-on-week changes of the GDP. For example, a declining WGDP value corresponds to a decrease in overall economic activity compared to the previous week. Moreover, it is also possible to calculate the current value of the WGDP relative to a previous period, such as the average of the previous month or quarter. Therefore, the WGDP is well suited to inform central bankers and policy makers about the current state of the German economy.

We demonstrate that the WGDP follows a plausible path. For instance, the activity decline during the first wave of the COVID-19 pandemic is associated with a decline in the WGDP of approximately 15% compared to the fourth quarter of 2019's average. Moreover, a comparatively milder drop and increase in economic activity during the second lockdown early 2021 are also captured adequately. Finally, using a pseudo real-time analysis, we show that the WGDP estimates are fairly robust with respect to different data vintages and provide a competitive nowcast accuracy with regard to quarterly GDP growth. Thus, the WGDP is suited to assess the current economic environment in real time.

Nichttechnische Zusammenfassung

Wir konstruieren einen wöchentlichen Indikator für die realwirtschafliche Aktivität in Deutschland – nachfolgend WBIP gennant. Das WBIP wird als die gemeinsame, gewichtete Komponente im Rahmen eines gemischt-frequenten dynamischen Faktormodells berechnet, welches es ermöglicht, Information aus Indikatoren verschiedener Frequenz zu berücksichtigen. Neben der Veränderungsrate des vierteljährlichen, realen Bruttoinlandsprodukts (BIP) werden zurzeit mehrere monatliche und wöchentliche Indikatoren mit Bezug zur konjunkturellen Entwicklung verwendet. Damit ist das WBIP in der Lage, das vierteljährliche BIP in seine unbeobachteten, wöchentlichen Wachstumsraten zu interpolieren und zeitnah nützliche Information über die aktuelle Konjunkturlage zu liefern.

Das WBIP ist zum Maßstab des vierteljährlichen BIP skaliert und entspricht per Konstruktion realisierten BIP-Wachstumsraten. So kann das WBIP in eine Niveaureihe transformiert und als wöchentliche Veränderung des BIP interpretiert werden. Beispielsweise entspricht eine Abnahme des Indikators einem Rückgang der gesamtwirtschaftlichen Aktivität im Vergleich zur Vorwoche. Ebenso ist es möglich zu berechnen, in welchem Verhältnis der gegenwärtige Wert des WBIP zu einer vorangehenden Periode steht, beispielsweise zum Durchschnitt des Vormonats oder -quartals. Dementsprechend ist das WBIP sehr gut geeignet, um Zentralbanker und politische Entscheidungsträger über den gegenwärtigen Zustand der deutschen Wirtschaft zu informieren.

Wir zeigen, dass das WBIP einem plausiblen Verlauf folgt. Der Aktivitätseinbruch während der ersten Corona-Welle geht beispielsweise mit einem Rückgang des WBIP um gut 15% gegenüber dem Durchschnitt des vierten Quartals 2019 einher. Zudem werden der vergleichsweise milde Rückgang und die Erholung der realwirtschaftlichen Aktivität während des zweiten Lockdowns in der ersten Hälfte 2021 auch hinreichend gut erfasst. Wir zeigen anhand einer Pseudo-Echtzeit-Analyse, dass die WBIP-Schätzungen recht robust sind und das WBIP damit auch am aktuellen Rand zur Einschätzung der wirtschaftlichen Lage geeignet ist.

A latent weekly GDP indicator for Germany*

Sercan Eraslan†
Deutsche Bundesbank

Magnus Reif†
Deutsche Bundesbank

Abstract

This paper introduces a weekly GDP indicator to track real economic activity in Germany in real-time. We use a mixed-frequency dynamic factor model with quarterly, monthly, and weekly indicators and obtain the weekly GDP indicator as the weighted common component of the mixed-frequency dataset. Our indicator is able to approximate latent week-on-week growth of German GDP. In addition, it enables computing a weekly GDP series in levels, which is also of great interest for central bankers, policy makers, and practitioners interested in analysing the current state of the economy in a timely manner. Finally, we demonstrate the benefits of our indicator for high-frequency tracking of the German economy using a recursive nowcasting exercise.

Keywords: Business cycle, dynamic factor model, economic indicator

JEL classification: C38, C43, E32

*We thank our colleagues at the Macroeconomic Analysis and Projections Division as well as members of the Bundesbank Group on Forecasting, especially Christian Schumacher and Elmar Mertens for their helpful discussions. Moreover, we thank Joshua Chan for sharing his code for sampling missing observations in Gaussian state-space models as well as Fable Data for providing us with its consumer spend data for Germany.
The views expressed in this paper are those of the authors and do not necessarily coincide with the views of the Deutsche Bundesbank or the Eurosystem.

†Contact addresses: Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main, Germany. Email addresses: sercan.eraslan@bundesbank.de (S. Eraslan), magnus.reif@bundesbank.de (M. Reif).

1 Introduction

Since the outbreak of the coronavirus pandemic, monitoring economic activity at high-frequencies has become a vital task for business cycle analysts and hence a vivid and growing research area. We contribute to this literature by introducing a weekly GDP indicator for Germany, which is expressed in week-on-week growth rates, and hence has a straightforward interpretation both in growth rates and levels. The latter substantially eases interpretation compared to high-frequency indicators that are based on some kind of moving-average of activity growth. Therefore, it is better suited for informing central bankers and policy-makers on the current state of the German economy.

High-frequency economic indicators have been proven useful in times of rapid and strong swings in activity over the past years.[1] For instance, Lewis, Mertens, Stock, and Trivedi (2022) introduce the weekly economic index (WEI) for the US economy by modeling the joint dynamics of a set of weekly indicators. Since the WEI is solely based on weekly indicators, it does not directly provide estimates for the GDP growth, though. Moreover, Lewis et al. (2022) use yearly (52 week) GDP growth rates at weekly frequency. Using the same transformation Baumeister, Leiva-León, and Sims (2022) extract activity indices on the regional level for the US. Eraslan and Götz (2021) extend the WEI into a mixed-frequency setting and construct the weekly activity index (WAI) for Germany based on weekly, monthly, and quarterly data. The WAI is based on 13-week growth rates (of 13-week moving averages) on a weekly basis. Therefore, neither the WEI nor the WAI allow for reconstructing week-on-week GDP growth.

Our modeling approach is closely related to those of Eckert, Kronenberg, Mikosch, and Neuwirth (2020) who construct week-on-week GDP growth for Switzerland. Accordingly, we rely on a mixed-frequency dynamic factor model (DFM) and resort to temporal aggregation constraints akin to those of Mariano and Murasawa (2003). However, our method to obtain weekly GDP growth differs from Eckert et al. (2020), who use data augmentation to estimate missing observations in a mixed-frequency dataset. Conditional on the augmented data and temporal aggregation constraints, Eckert et al. (2020) estimate a latent factor and, using suitable restrictions, ensure that weekly GDP growth coincides with the factor. However, we resort to the more flexible approach of Chan et al. (2023). Its key insight is that the joint density of the missing observations conditional on observed data is Gaussian and can be estimated using the precision sampling approach of Chan and Jeliazkov (2009).

The remainder of this paper is organised as follows. Sections 2 and 3 describe the econometric methodology and the data used for estimation. Section 4 introduces the weekly GDP indicator and provides a brief overview of its in- and out-of-sample properties. Section 5 concludes.

[1] While most studies rely on factor extraction methods to generate weekly indicators for economic activity, high-frequency macroeconomic analyses spread to other models and applications more recently. For example, Chan, Poon, and Zhu (2023) employ a mixed-frequency VAR to derive week-on-week GDP growth rates for the US. Carriero, Clark, and Marcellino (2022) and Ferrara, Mogliani, and Sahuc (2022) use various Bayesian mixed-frequency (quantile) regressions to monitor downside risks to the GDP growth in the US at weekly frequency.

2 Econometric methodology

The weekly GDP indicator is based on a Bayesian mixed-frequency DFM:

$$y_t = \Lambda f_t + u_t, \tag{1}$$

where $y_t = (y_t^{o'}, y_t^{m'})$ is an $n \times 1$ vector at the weekly frequency, with y_t^o and y_t^m denoting $n^o \times 1$ and $n^m \times 1$ vectors of observed and missing data, respectively. Λ contains the time-invariant loadings on the latent common factor f_t and u_t is an $n \times 1$ vector of idiosyncratic components.

The laws of motion of the factor and the idiosyncratic disturbances are described by:

$$(1 - \Phi(L))f_t = \epsilon_t^f, \quad (\epsilon_t^f) \sim \mathcal{N}(0, \sigma_f^2), \tag{2}$$

$$(1 - \Psi_i(L))u_{i_t} = \epsilon_t^i \quad (\epsilon_t^i) \sim \mathcal{N}(0, \sigma_i^2), \quad \text{for: } i = 1, \dots, n, \tag{3}$$

where $\Phi(L)$ and $\Psi_i(L)$ are lag polynominals of order p and q, respectively.

We model all variables at weekly frequency. To link the latent weekly observations of the low-frequency variables to their corresponding actual observations, we employ inter-temporal constraints akin to Mariano and Murasawa (2003). However, since the number of weeks per month/quarter is time-varying, the constraints are also time-varying, which contrasts with mixed-frequency models that only include monthly and quarterly variables.

Let $n_{i,t}^w$ be the number of weeks between the last release date of indicator i and release date t. Moreover, let $z_{i,t}$ be the indicator's observed low-frequency value at t. Then the inter-temporal constraint for indicator i is given by:

$$z_{i,t} = \sum_{s=1}^{2n_{i,t}^w - 1} \left(\mathbf{1}(s \leq n_{i,t}^w) + \mathbf{1}(s > n_{i,t}^w) \frac{2n_{i,t}^w - s}{n_{i,t}^w} \right) y_{i,t-s+1}^m + \varepsilon_{i,t}^z, \qquad \varepsilon_{i,t}^z \sim \mathcal{N}(0, o_i). \tag{4}$$

Note that (4) explicitly allows for measurement error, taking into account that the inter-temporal constraints of Mariano and Murasawa (2003) are based on a log-linear approximation.[2]

The key challenge of this model is to generate draws for the missing observations. Usually, one would use the state space representation of the DFM and resort to simulation smoothing (see, for example, Carter and Kohn, 1994; Durbin and Koopman, 2002). The latter yields draws for the latent states, which can be used to generate draws for the missing observations. However, as shown by Chan et al. (2023), the joint distribution of the missing data conditional on the observed data is both Gaussian and sparse. Hence, the missing observations can be sampled using the efficient precision-based sampler of Chan and Jeliazkov (2009). Subsequently, we briefly outline the algorithm of Chan et al. (2023).

First, we cast the model into state space form:

[2]Imposing an exact constraint amounts to drawing from a degenerate Gaussian distribution, which is computationally much more demanding than drawing from a standard Gaussian distribution as in the case of an approximate constraint.

$$y_t = H\alpha_t + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, R), \tag{5}$$
$$\alpha_t = F\alpha_{t-1} + \varepsilon_t^{\alpha}, \qquad \varepsilon_t \sim \mathcal{N}(0, Q), \tag{6}$$

where α_t is the state vector. H and F are matrices of covariates, and R and Q are the covariance matrices of the observation and transition equation, respectively. Let $\mathbf{y} = (y_1', \ldots, y_T')' \in \mathbb{R}^{Tn}$, $\mathbf{y}^o = (y_1^{o\prime}, \ldots, y_T^{o\prime})' \in \mathbb{R}^{N^o}$, and $\mathbf{y}^m = (y_1^{m\prime}, \ldots, y_T^{m\prime})' \in \mathbb{R}^{N^m}$ and re-write $\mathbf{y}$ in terms of $\mathbf{y}^o$ and $\mathbf{y}^m$:

$$\mathbf{y} = \mathbf{S}^0\mathbf{y}^o + \mathbf{S}^m\mathbf{y}^m, \tag{7}$$

where $\mathbf{S^o}$ and $\mathbf{S^m}$ are selection matrices of size $Tn \times N^o$ and $Tn \times N^m$. Using (7), we stack (5) over t:

$$\mathbf{S}^0\mathbf{y}^o + \mathbf{S}^m\mathbf{y}^m = \mathbf{H}\boldsymbol{\alpha} + \boldsymbol{\varepsilon}, \tag{8}$$

and obtain the joint conditional distribution of the missing data given the observed data:

$$p(\mathbf{y}^m | \mathbf{y}^0, \boldsymbol{\alpha}, \boldsymbol{\theta}) \sim \mathcal{N}(\boldsymbol{\mu}_{\boldsymbol{y}^m}, \mathbf{K}_{\mathbf{y}^m}^{-1}), \tag{9}$$

where $\boldsymbol{\theta}$ contains the model parameters and other latent variables. The precision and mean are given by:

$$\mathbf{K}_{\mathbf{y}^m}^{-1} = \mathbf{S}^{m\prime}\mathbf{R}^{-1}\mathbf{S}^m, \tag{10}$$
$$\boldsymbol{\mu}_{\mathbf{y}^m} = \mathbf{K}_{\mathbf{y}^m}^{-1}\mathbf{S}^{m\prime}\mathbf{R}^{-1}(\mathbf{H}\boldsymbol{\alpha} - \mathbf{S}^m\mathbf{y}^o). \tag{11}$$

While we can use (9) to generate draws for the missing data, the latter is not mapped to the observed low-frequency variables. To do so, we introduce the following linear system:

$$\mathbf{z} = \mathbf{M}\mathbf{y}^m + \boldsymbol{\varepsilon}^z, \qquad \boldsymbol{\varepsilon}^z \sim \mathcal{N}(\mathbf{0}_M, \mathbf{O}), \tag{12}$$

where $\mathbf{M}$ stacks the intra-temporal constraint (4) over t and $\mathbf{O}$ is a fixed diagonal covariance matrix that determines the magnitude of the measurement error. We set the diagonal elements of $\mathbf{O}$ to 10^{-8} to mimic an exact relationsship. The key idea for mapping the missing data to the observed low-frequency variables is to update the conditional distribution of $\mathbf{y}^m$ given the additional information specified by (12). Thus, we can interpret (9) as the prior, (12) as the likelihood and Bayesian calculus yields:

$$p(\mathbf{y}^m | \mathbf{y}^0, \boldsymbol{\alpha}, \boldsymbol{\theta}, \mathbf{z}) \sim \mathcal{N}(\overline{\boldsymbol{\mu}}_{\mathbf{y}^m}, \overline{\mathbf{K}}_{\mathbf{y}^m}^{-1}), \tag{13}$$

where

$$\overline{\mathbf{K}}_{\mathbf{y}^m}^{-1} = \mathbf{K}_{\mathbf{y}^m} + \mathbf{M}'\mathbf{O}^{-1}\mathbf{M}, \tag{14}$$

$$\overline{\boldsymbol{\mu}}_{\mathbf{y}^m} = \overline{\mathbf{K}}_{\mathbf{y}^m}^{-1}(\mathbf{S}^{m\prime}\mathbf{R} + \mathbf{M}'\mathbf{O}^{-1}\mathbf{z}). \tag{15}$$

Conditional on the missing observations, the remaining coefficients can be drawn using standard approaches (see, for example, Bai and Wang, 2015). Specifically, we sample the factor loadings, the autoregressive coefficients of the idiosyncratic components, and the autoregressive coefficients of the factor using Bayesian linear regression techniques. To uniquely identify the model, we fix the loading of GDP to unity (see Bai and Wang, 2015).

We impose, in most cases, at best mildly informative priors. For the factor equation, we use a Minnesota-style prior with a prior mean set to 0.9 for the first lag and to zero for the remaining lags—as proposed by D'Agostino, Giannone, Lenza, and Modugno (2016). For the idiosyncratic components, the prior mean is set to zero for each lag. In both cases, the prior variance equals δ/l^2, where $\delta = 0.05$ and l refers to the lag of respective coefficients. The latter is somewhat more informative than in studies using U.S. data (see, for example, Antolin-Diaz, Drechsel, and Petrella, 2017), owing to our rather short dataset. For the factor loadings, we use independent Gaussian priors, centered at 1 for variables that are a priori positively correlated with the business cycle and -1 otherwise. The prior variance is set to unity, to make the prior rather uninformative.

3 Data

Our dataset extends the one used by Eraslan and Götz (2021), which has been proven to be adequate for extracting a business cycle index for the German economy. Accordingly, the dataset covers the period from January 2004—the first week for which at least one high-frequency indicator is available—to October 2023. Table 1 provides an overview about the indicators, their frequency, and the release pattern.

Our dataset consists of 14 variables, four of them sampled at daily frequency, two at weekly, seven at monthly, and one at quarterly frequency. Thereby, indicators at daily frequency enter the estimation as weekly averages. Moreover, we use calender-, seasonally- and price-adjusted series for indicators, exihibiting such patterns.[3]

Along the lines of Eraslan and Götz (2021), our dataset strives to cover a broad range of real economic activity in Germany. For instance, consumer behaviour is partly captured by consumer spending[4] and pedestrian frequency[5]. Moreover, relative Google searches for "unemployment" and "short-term work" aim at monitoring the domestic labour market.[6] We measure high-frequency activity in manufacturing sector and trade with the indicator toll[7], while the global activity is measured by the total number of flights.[8] Finally, the monthly indicators industrial production, construction, services production, exports, sales

[3]See Ollech (2023) for more details on the procedure for seasonal adjustment of high-frequency economic indicators.

[4]Source of unadjusted figures: Fable Data.

[5]Source of unadjusted figures: hystreet.com and Federal Statistical Office.

[6]Source of unadjusted figures: Google Trends.

[7]Sources of unadjusted figures: Federal Logistics and Mobility Office and Federal Statistical Office.

[8]Source of unadjusted figures: flightradar24.

Table 1: Dataset

Indicator	Description	Frequency	Release pattern
Consumer spending	Real credit card turnover	daily	Wednesdays $(t-8)$
Pedestrian frequency	Pedestrian frequency in selected shopping districts in large German cities	daily	daily $(t-1)$
Google trends unemployment	Relative search frequency for Google search term unemployment	weekly	Mondays $(t-2)$
Google trends short-time work	Relative search frequency for Google search term short-time work	weekly	Mondays $(t-2)$
Toll	Daily truck toll mileage index	daily	Thursdays $(t-9)^{*}$
Flights	Total number of worldwide flights	daily	daily $(t-1)$
Industrial production	Real output in manufacturing sector	monthly	monthly $(\approx t-35)$
Construction	Real output in construction sector	monthly	monthly $(\approx t-35)$
Service production	Real output in service sector	monthly	monthly $(\approx t-60)$
Exports	Real exports according to monthly trade statistics	monthly	monthly $(\approx t-35)$
Retail sales	Real turnover in retail trade	monthly	monthly $(\approx t-30)$
Wholesales	Real turnover in wholesales	monthly	monthly $(\approx t-45)$
Gastronomy	Real turnover in food services	monthly	monthly $(\approx t-45)$
GDP	Real gross domestic product	quarterly	quarterly $(\approx t-30)^{**}$

Notes: First and second column displays the indicator and provide a brief description. Third column depicts the frequency in which the data is downloaded. Fourth column shows the release pattern of the indicators. *As of 19. Oct. 2020, the Federal Office for Goods Transport stopped delivering data on truck toll on a daily basis. As of 22. Oct., they are provided on a weekly basis on Thursdays leading to a change in publication lag from 5 to 9 days. **As of end of Jul. 2020, the Federal Statistical Office changed the release date of the flash estimate for GDP from approx. 45 to approx. 30 days after the end of the quarter.

in retail, wholesale, and gastronomy cover related parts of the economy, while quarterly GDP is included as the target variable.[9]

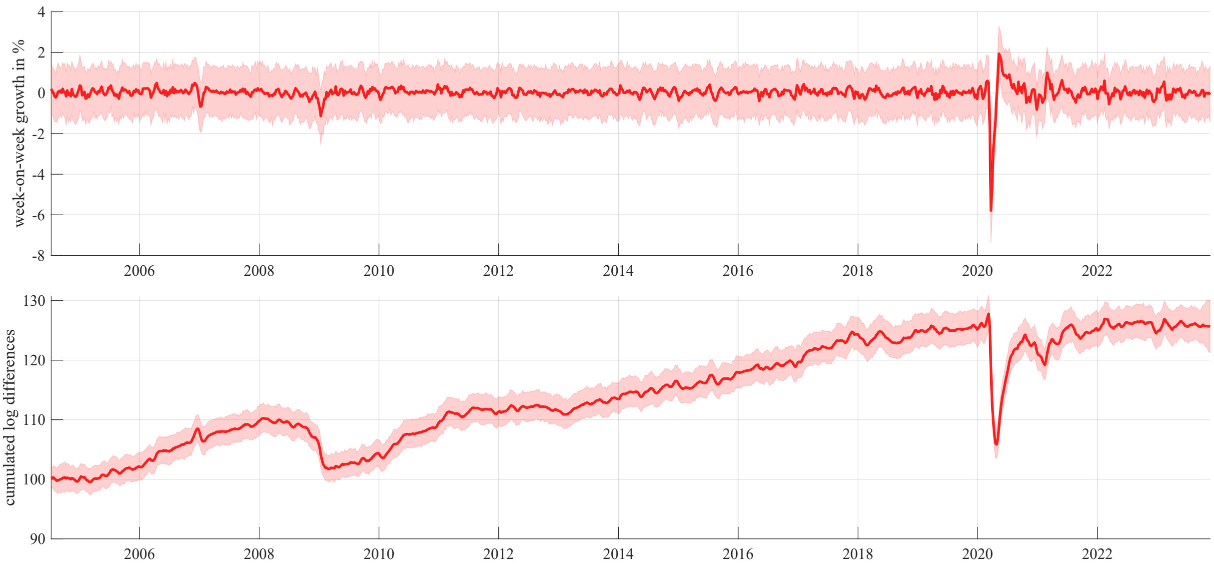
Note that we do not include survey indicators in our dataset. While it is well-known that these indicators often exhibit significant predictive power for either certain parts of the economy or the aggregate economy (see, for example, Lehmann and Reif, 2021), they do not directly measure economic activity but sentiment. However, we aim at constructing a measure of real economic activity. Thus, we restrict ourselves to using only indicators that either represent activity or are at least closely related to activity.

[9]The data for indicators industrial production, construction, services production, exports, retail sales, whole sales, sales in gastronomy, and GDP are retrieved from the time series database of the Deutsche Bundesbank.

4 Weekly GDP indicator

The weekly GDP indicator (WGDP) is obtained as the weighted common component of the weekly growth rates of a mixed-frequency dataset (see Section 2). We scale WGDP to the magnitude of actual GDP growth by de-standardizing the estimated week-on-week GDP growth rates and adding the average growth rate of quarterly GDP divided by the appropriate number of weeks. Finally, we accumulate weekly (log) growth rates and take the exponential to construct an index representation of the WGDP. Figure 1 illustrates WGDP in both growth rates and levels for the entire sample.

Figure 1: Weekly GDP in growth rates and levels

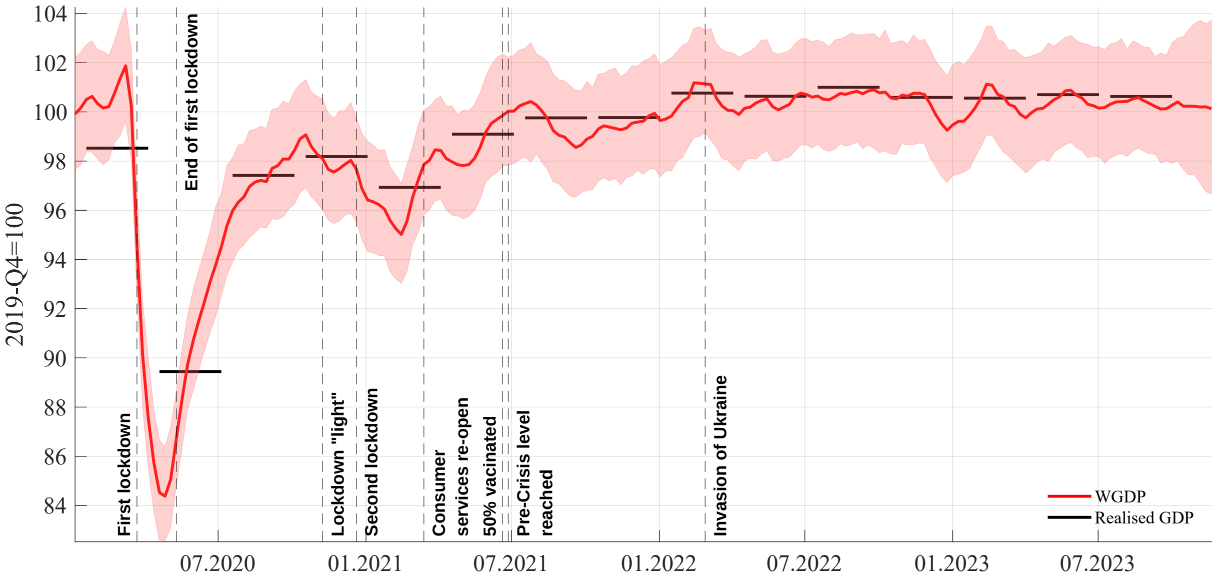


Notes: Posterior mean of weekly GDP in week-on-week growth rates (top) and in levels (bottom) at the weekly frequency. Shaded areas refer to 90% equal-tailed point-wise posterior probability bands.

The top panel of Figure 1 shows the latent week-on-week GDP growth rates. Most of the time, week-on-week GDP growth does not exhibit pronounced fluctuations with the exception of two periods: the Great Recession and in the course of the coronavirus pandemic. During the Great Recession at the turn of the year 2008/2009, weekly GDP declines by around 0.5% to 1% for several consecutive weeks. However, this episode is dwarfed by the sharp drop during the early phase of the coronavirus pandemic in Germany: weekly GDP consecutively declines by 0.5%, 3%, 7%, 3%, and 1% in the four weeks of March 2020 and in the first week of April 2020.

The bottom panel of Figure 1 provides an index representation of the weekly GDP indicator. In this representation, periods with marked fluctuations in economic activity become distinctly visible. Overall, we obtain a plausible pattern. Given the extraordinary nature of the coronavirus pandemic, Figure 2 zooms in the period from January 2020 to October 2023. The lockdown-related drop during the early phase of the coronavirus pandemic implies, on the weekly frequency, a decline of GDP by about 15% compared to the 2019Q4 average at the end of March. Starting at the end of the first lockdown in the first week of May, GDP swiftly recovers. However, it only reaches about 98% of the pre-crisis level in Autumn 2020. In the first week of November, the so-called "lockdown light" becomes effective and GDP again starts to decline, reaching 96% of the pre-crisis level. From the first week of March 2021 onward, GDP again recovers, reflecting, *inter alia*, the re-opening of consumer services. However, real economic activity reaches its pre-crisis level not before early 2022—just on the onset of the Russian invasion of the Ukraine. Since then economic activity remains weak and wavers around its pre-pandemic level.

Figure 2: Weekly GDP from 2020 onwards



Notes: Posterior mean of weekly GDP (solid line) along with 90% equal-tailed point-wise posterior probability bands (shaded area), and realized quarterly GDP growth (dashes).

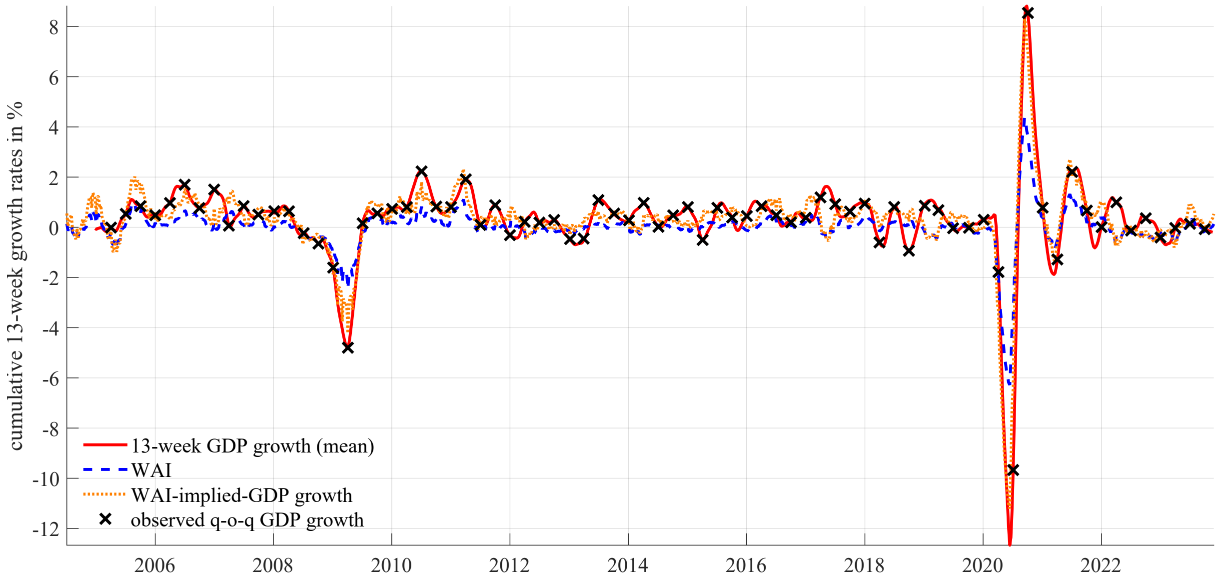
For the sake of comparison with the WAI, we convert weekly GDP growth into (rolling) quarterly growth at weekly frequency, using the intertemporal aggregation constraint from (4). Figure 3 depicts these quarterly growth rates (solid line), which can be interpreted as the cumulative GDP growth of the last 13 weeks over the preceding 13 weeks period. In addition, it includes the WAI (dashed line), WAI-implied-GDP-growth (dotted line), and actual quarterly GDP growth rates (indicated by $\times$). Overall, 13-week GDP growth and WAI-implied-GDP-growth are highly correlated. The former, however, seems to exhibit slightly stronger swings.[10] Finally, both indicators successfully capture strong fluctuations in economic activity associated with the Great Recession and the coronavirus pandemic.

We now shift our focus from in-sample properties to out-of-sample features of the weekly GDP indicator. To this end, we conduct a pseudo real-time estimation exercise using revised data and estimate the model recursively from January 2020 until October 2023 on a weekly basis.[11] First, we examine the stability of the model over historical

[10]Note that the quarterly GDP growth rates calculated from the weekly GDP indicator are identical to the realized quarterly GDP growth rates due to the temporal aggregation. In contrast, the WAI is not subject to such an assumption and it is also based on a smaller dataset. These differences may partly explain the slight divergences in both series.

[11]We re-construct the data availability in the historical vintages, but cannot take potential data revisions into account. Limited availability of real-time data restricts us from conducting a true real-time analysis.

Figure 3: Weekly GDP, WAI, WAI-implied-GDP-growth, and actual GDP growth

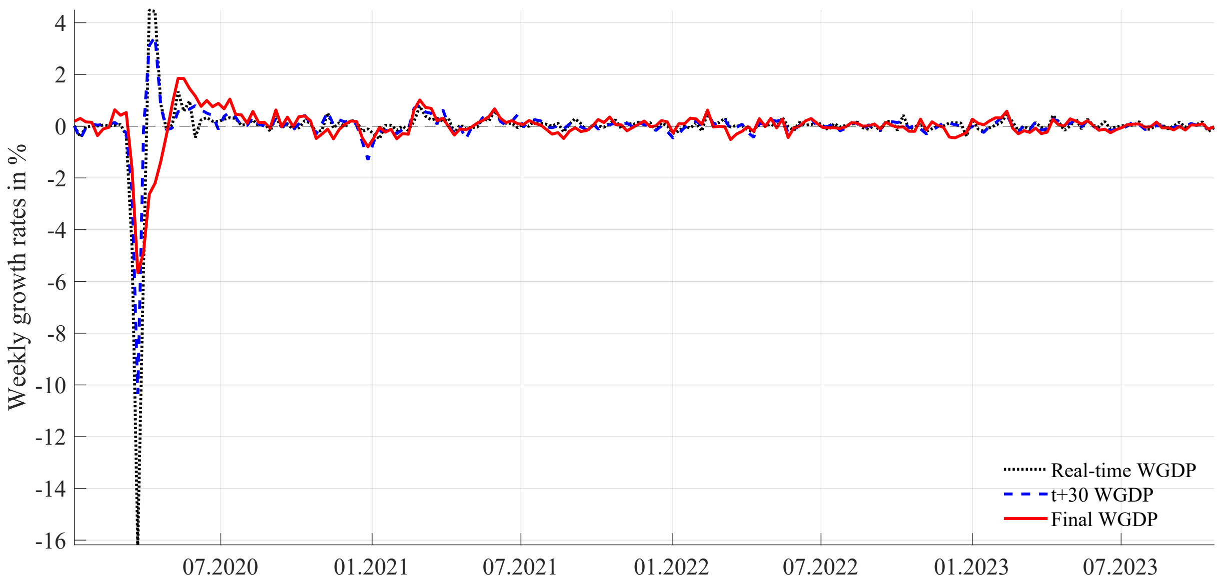


Notes: Posterior mean of weekly GDP in 13-week cumulative growth rates (solid line), WAI (dashed line), WAI-implied-GDP-growth (dotted line), and actual quarterly GDP growth ($\times$).

vintages. Figure 4 plots week-on-week GDP growth rates based on various information sets. The dotted line denotes the weekly GDP indicator in real-time, i.e. taking into account only information that is available at the reference period. The dashed line displays the indicator based on the data availability around 30 days after the reference period. The solid line refers to the weekly GDP indicator based on the final vintage data. While there are remarkable differences in weekly GDP based on these different data vintages during the early phase of the coronavirus pandemic, the difference are fairly negligible in the remaining period of the considered sample. Overall, the model appears to be stable and thus is well-suited for real-time monitoring of real economic activity at the weekly frequency.

Figure 5 depicts quarterly GDP growth nowcasts based on the weekly GDP indicator for the period 2020-2023 in pseudo real-time. Accordingly, the weekly GDP indicator is depicted in 13-week cumulative growth rates (solid lines), which are generated in the last week of the target quarter. As such, the final value of each vintage can be interpreted as nowcasts (denoted by $\times$) for quarterly GDP growth. The dashes indicate revised quarterly GDP growth. At the end of 2020Q1 (corresponding to week 5.4.2020), the model generates a nowcast of -2.6% for 2020-Q1 and thus a forecast error of -0.9 pp. Subsequently, quarterly GDP growth nowcasts are -6.8% and +5.2% for 2020Q2 and 2020Q3, which implies significantly higher forecast errors. However, these forecasts are slightly better than those of the WAI (WAI-implied-GDP-growth rates: -4% and +6.2% for 2020Q2 and 2020Q3). Considering the extraordinary nature of the coronavirus pandemic and its impact on the economy, the WGDP is provides useful and timely information on the current state of the Germany economy in 2020. Thereafter, the WGDP nowcasts are quite close to the actual GDP growth rates; the nowcasts from the WGDP have a mean absolute forecast error (MAFE) of 0.5 pp for the period 2020Q4–2023Q1, whereas the WAI-implied-GDP

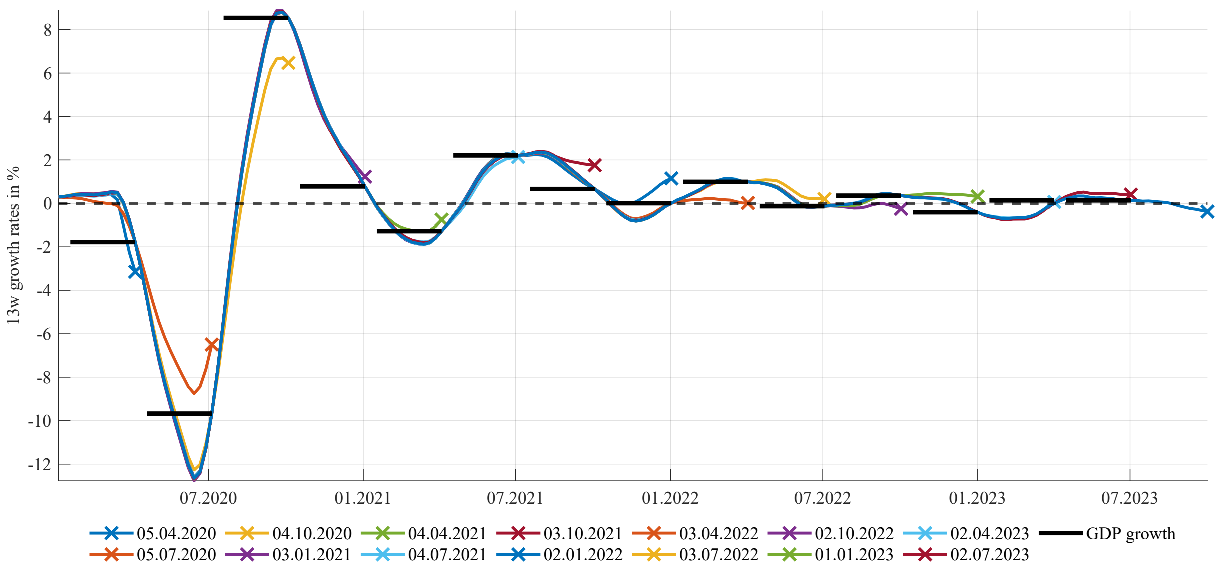
Figure 4: Revisions of the weekly GDP indicator



Notes: Posterior means of week-on-week GDP growth rates calculated at various data vintages.

growth rates have a MAFE of 0.6 pp for the same period. Hence, also in the aftermath of the pandemic, the WGDP has a superior performance compared to the WAI.

Figure 5: Pseudo real-time GDP nowcasts



Notes: The graph shows 13-week cumulative growth rates (solid lines) and GDP nowcasts (×) at various data vintages as well as actual GDP growth rates (black dashes) for related quarters.

5 Concluding remarks

We introduce a latent weekly GDP indicator to track aggregate economic activity in Germany at a high frequency. For this purpose, we utilise a novel Bayesian dynamic factor model, which can efficiently deal with missing observations resulting from the mixed-frequency dataset. Accordingly, our indicator is calculated as the common component of weekly, monthly, and quarterly indicators. It is generated in week-on-week growth rates and scaled to the magnitude of the realised quarterly GDP growth rate.

The weekly GDP indicator builds upon the recent literature on high-frequency economic indicators addressing some of its limitations. In comparison with the WAI for Germany developed by Eraslan and Götz (2021), the weekly GDP is estimated using Bayesian estimation techniques and is based upon a larger dataset that covers broader parts of the German economy. Moreover, the indicator is calculated in week-on-week growth instead of a 13-week moving average representation used by the WAI. Therefore, the proposed weekly GDP indicator has a straightforward interpretation both in growth rates and levels. Moreover, the new indicator is constructed in a mixed-frequency dynamic factor model instead of a static factor extraction algorithm employed to build the WAI. The latter is particularly appealing because it allows to compute up-to-date short-term forecasts for the German GDP on a weekly basis.

We show that the model provides a reasonable characterization of the German business cycle. A pseudo real-time exercise moreover shows that the WGDP estimates are fairly stable and provide competitive GDP growth nowcasts. Thus, the model output is able to supply useful information for central bankers, policy makers and business cycle analysts on the real economic activity in Germany on a weekly basis.

References

Antolin-Diaz, J., T. Drechsel, and I. Petrella (2017). Tracking the Slowdown in Long-Run GDP Growth. *Review of Economics and Statistics 99*(2), 343–356.

Bai, J. and P. Wang (2015). Identification and Bayesian Estimation of Dynamic Factor Models. *Journal of Business & Economic Statistics 33*(2), 221–240.

Baumeister, C., D. Leiva-León, and E. Sims (2022). Tracking Weekly State-Level Economic Conditions. *The Review of Economics and Statistics forthcoming*.

Carriero, A., T. E. Clark, and M. Marcellino (2022). Nowcasting tail risk to economic activity at a weekly frequency. *Journal of Applied Econometrics 37*(5), 843–866.

Carter, C. K. and R. Kohn (1994). On Gibbs Sampling for State Space Models. *Biometrika 81*(3), 541–553.

Chan, J. C. C. and I. Jeliazkov (2009). Efficient simulation and integrated likelihood estimation in state space models. *International Journal of Mathematical Modelling and Numerical Optimisation 1*(1-2), 101–120.

Chan, J. C. C., A. Poon, and D. Zhu (2023). High-dimensional conditionally Gaussian state space models with missing data. *Journal of Econometrics 236*(1), 105468.

D'Agostino, A., D. Giannone, M. Lenza, and M. Modugno (2016). Nowcasting Business Cycle: A Bayesian Approach to Dynamic Heterogenous Factor Models. In S. J. K. Eric Hillebrand (Ed.), *Dynamic Factor Models*, Volume (Advances in Econometrics, Vol. 35), pp. 569–594. Emerald Group Publishing Limited.

Durbin, J. and S. J. Koopman (2002). A simple and efficient simulation smoother for state space time series analysis. *Biometrika 89*(3), 603–616.

Eckert, F., P. Kronenberg, H. Mikosch, and S. Neuwirth (2020). Tracking economic activity with alternative high-frequency data. *KOF Working Papers 488*.

Eraslan, S. and T. Götz (2021). An unconventional weekly economic activity index for Germany. *Economics Letters 204*, 109881.

Ferrara, L., M. Mogliani, and J.-G. Sahuc (2022). High-frequency monitoring of growth at risk. *International Journal of Forecasting 38*(2), 582–595.

Lehmann, R. and M. Reif (2021). Predicting the German economy: Headline survey indices under test. *Journal of Business Cycle Research 17*(2), 215–232.

Lewis, D. J., K. Mertens, J. H. Stock, and M. Trivedi (2022). Measuring real activity using a weekly economic index. *Journal of Applied Econometrics 37*(4), 667–687.

Mariano, R. S. and Y. Murasawa (2003). A new coincident index of business cycles based on monthly and quarterly series. *Journal of Applied Econometrics 18*(4), 427–443.

Ollech, D. (2023). Economic analysis using higher-frequency time series: challenges for seasonal adjustment. *Empirical Economics 64*(3), 1375–1398.