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Money growth and consumer price inflation in the euro area: An update

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Martin Mandler
(Deutsche Bundesbank and Justus-Liebig-Universität Gießen)
Michael Scharnagl
(Deutsche Bundesbank)

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Internet http://www.bundesbank.de

Non-technical summary

Research Question

The quantity theory of money predicts a stable long-run one-to-one relationship between money growth and inflation. In this paper, we investigate this relationship for the euro area.

Contribution

We update a previous analysis of ours on the relationship between money growth and inflation in the euro area. We use tools from wavelet analysis to investigate the correlation between money growth and inflation as well as their lead-lag relationship. Wavelet analysis allows us to analyse the relationship between money growth and inflation concerning both possible changes over time and changes across frequencies.

Results

The correlation between long-term fluctuations in the growth rate of the broad monetary aggregate M3 and consumer price inflation has weakened over the 1990s. However, after adjusting M3 growth by subtracting real GDP growth we find a stable and strong correlation with inflation for cycles of 24 years and longer duration. Cycles in adjusted money growth of this length and corresponding cycles in consumer price inflation move roughly 1:1 as predicted by the quantity theory but there is no evidence of money growth leading inflation. By contrast, these fluctuations are coincident.

Nichttechnische Zusammenfassung

Fragestellung

Die Quantitätstheorie des Geldes sagt eine stabile, langfristige 1:1-Beziehung zwischen der Wachstumsrate der Geldmenge und der Inflationsrate voraus. In diesem Papier stellen wir empirische Ergebnisse der Untersuchung dieses Zusammenhangs im Euroraum vor.

Beitrag

Wir aktualisieren eine früher veröffentlichte Studie über die Beziehung zwischen Geldmengenwachstum und Inflation im Euroraum. Wir nutzen Methoden der Wavelet-Analyse, um die Korrelation zwischen Geldmengenwachstum und Inflation und einen möglichen Vorlauf oder Nachlauf der Zeitreihen im Verhältnis zueinander zu untersuchen. Die Wavelet-Analyse ermöglicht es uns, mögliche Veränderungen des Zusammenhangs zwischen den beiden Variablen sowohl über die Zeit als auch über verschiedene Frequenzen zu untersuchen.

Ergebnisse

Die Korrelation zwischen langfristigen Schwankungen der Wachstumsrate der weit gefassten Geldmenge M3 und der Inflationsrate hat sich über die 1990er Jahre abgeschwächt. Eine starke und stabile Korrelation schätzen wir jedoch zwischen dem Geldmengenwachstum abzüglich dem Wachstum des realen BIP („korrigiertes Geldmengenwachstum“) und der Inflationsrate – und zwar für langfristige Schwankungen, die 24 Jahre oder länger andauern. Zwischen diesen langfristigen Schwankungen des korrigierten Geldmengenwachstums und der Inflationsrate zeigt sich die von der Quantitätstheorie vorhergesagte 1:1-Beziehung. Allerdings finden wir keine Hinweise auf einen Vorlauf des Geldmengenwachstums vor der Inflation. Stattdessen sind die gemeinsamen Zyklen in den beiden Variablen ungefähr gleichzeitig.

Money growth and consumer price inflation in the euro area: An update[1]

Martin Mandler

Deutsche Bundesbank
Justus-Liebig-Universität Gießen

Michael Scharnagl

Deutsche Bundesbank

Abstract

We update the wavelet-based analysis of the relationship between money growth and inflation in the euro area in Mandler and Scharnagl (2014). The relationship between headline M3 growth and inflation at low frequencies has weakened over the 1990s. However, we find evidence of stable comovement between money growth adjusted by real GDP growth and consumer price inflation for cycles of 24 years and longer duration. The long-run fluctuations of adjusted money growth and inflation move roughly about 1:1 and are contemporaneous, i.e. there is no lead of money growth. Our analysis of cycles in both variables of 24 years and longer provides information on the relationship between the variables from the late 1980s to the early 2000s.

Keywords: money growth, inflation, euro area, wavelet analysis

JEL-Classification: C30, E31, E40

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[1] Contact address: Wilhelm-Epstein-Strasse 14, D-60431 Frankfurt am Main, Germany. E-Mail: martin.mandler@bundesbank.de, michael.scharnagl@bundesbank.de. This paper represents the authors' personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank or of the Eurosystem.

1 Introduction

We revisit the analysis in Mandler and Scharnagl (2014) on the relationship between money growth and inflation in the euro area using wavelet-based techniques.[2] We update the sample period from our previous analysis to the second quarter of 2022. The longer data set allows us to extend our analysis to lower frequencies and, for the long cycles in question, up to the early 2000s. Another modification is a changed bootstrap algorithm for significance testing that accounts for possible heteroscedasticity.[3]

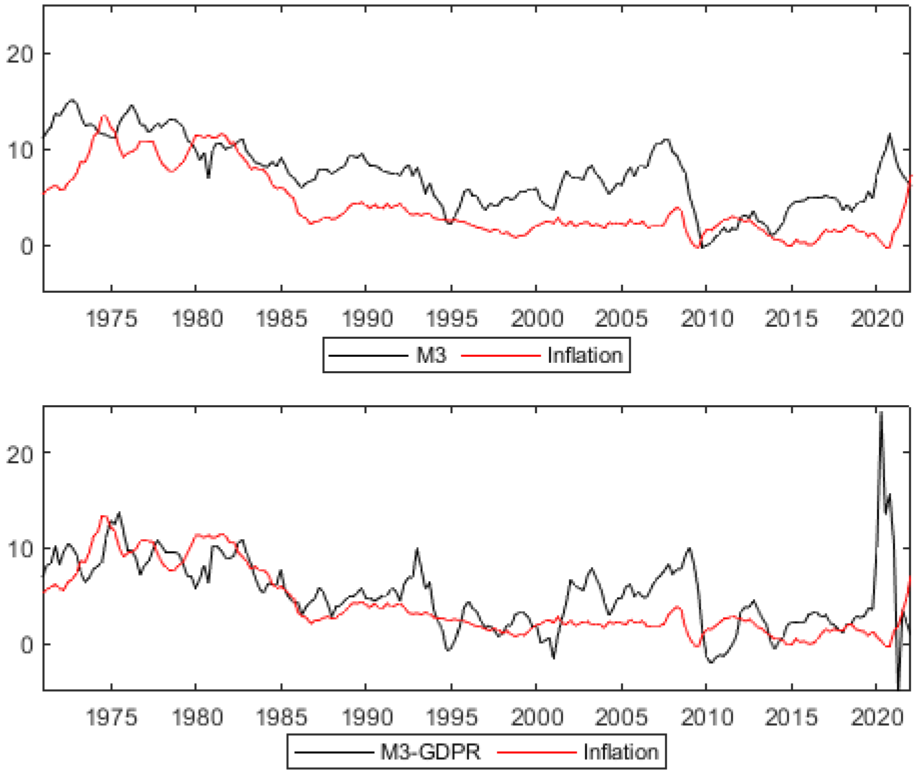

Figure 1: Inflation and annual growth rates of M3 and M3 adjusted by real GDP growth

[2] For an overview on the tools from wavelet analysis used in this application, see Deutsche Bundesbank (2019), pp. 71.
[3] For details, see Section 3 in Mandler and Scharnagl (2022).

2 Methodology

Wavelet analysis a type of spectral analysis that, unlike Fourier analysis, is not restricted to stationary time series. It allows for changes in a time series' spectrum and in the relationship between multiple time series.[4] Wavelet analysis projects time series on flexible periodic base functions with finite support.

A **wavelet** (mother wavelet) is given by

$$\psi_{\tau,s}(t)=\frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right), \tag{1}$$

where ψ is a wavelet function, s is the scale which controls the width of the wavelet and τ controls the location of the wavelet in time. An increase in s stretches the wavelet while changes in τ shift the wavelet in time.

The most widely used wavelet function, which we also use in this analysis, is the **Morlet wavelet**

$$\psi_{\omega_0}(t)=\pi^{-\frac{1}{4}}e^{i\omega_0 t}e^{\frac{t^2}{2}}, \tag{2}$$

with parameter ω_0. The most common parametrization $\omega_0=6$ implies a simple relation between scale and frequency: $\omega(s)\approx 1/s$.

The **continuous wavelet transform** (CWT) of a time series $x(t)$ with respect to the wavelet ψ is a projection of $x(t)$ onto ψ, i.e.

$$W_x(\tau,s)=\int x(t)\frac{1}{\sqrt{|s|}}\psi^*\left(\frac{t-\tau}{s}\right)dt, \tag{3}$$

where * denotes the complex conjugate.

The **wavelet power spectrum** shows the relative contribution of cycles to the variance of the time series on each scale and at each point in time. Using the CWT it is defined as

$$WPS_x(\tau,s)=\left|W_x(\tau,s)\right|^2. \tag{4}$$

[4] For convenience, this section briefly summarizes the statistics used in the analysis. For an introduction to wavelet analysis, see Aguiar-Conraria and Soares (2014). For estimation, we used the AST-toolbox for MATLAB by Aguiar-Conraria and Soares at https://sites.google.com/site/aguiarconraria/wavelets-and-economics.

The analysis of the relationship between two time series $x(t)$ and $y(t)$ is based on the **cross wavelet transform**

$$W_{xy}(\tau,s)=W_x(\tau,s)W_y^*(\tau,s). \tag{5}$$

Coherency is a measure of local correlation between two time series. It can be computed from the cross wavelet transform and the wavelet power spectra as

$$R_{xy}=\frac{\left|W_{xy}(\tau,s)\right|}{\sqrt{\left|W_x(\tau,s)\right|^2}\sqrt{\left|W_y(\tau,s)\right|^2}}. \tag{6}$$

Using the cross wavelet transform the **phase difference** is defined as

$$\varphi_{xy}(\tau,s)=\tan^{-1}\left[\frac{\Im\left\{W_{xy}(\tau,s)\right\}}{\Re\left\{W_{xy}(\tau,s)\right\}}\right], \tag{7}$$

where $\Im$ is the imaginary and $\Re$ is the real part of wavelet coherency. For a given frequency $\omega(s)$ the phase difference is related to the **time difference** by

$$\Delta_{xy}^{T}=\frac{\varphi_{xy}}{\omega(s)} . \tag{8}$$

The cross wavelet **gain** can be interpreted as the coefficient of a regression of $y(t)$ on $x(t)$ at frequency $\omega(s)$

$$G_{yx}(s)=\frac{\left|W_{xy}(s)\right|}{\left|W_x(s)\right|^2}. \tag{9}$$

3 Results

Figure 2 shows the estimated wavelet coherency of the annual growth rate of money and HICP inflation. The top panel presents the results for headline M3, the bottom panel for the annual growth rate of M3 adjusted for annual real GDP growth, i.e. the annual growth rate of M3 minus the annual growth rate of real GDP. Coherency increases from dark blue (zero) to dark red (one). The curved red lines denote the cone of influence. Results between the start of the sample period and the left red line and between the right red line and the end of the sample period should not be interpreted as they are affected by start- and end-point problems. The black lines surround time-frequency combinations for which coherency is significantly different from zero at the 5%-level.

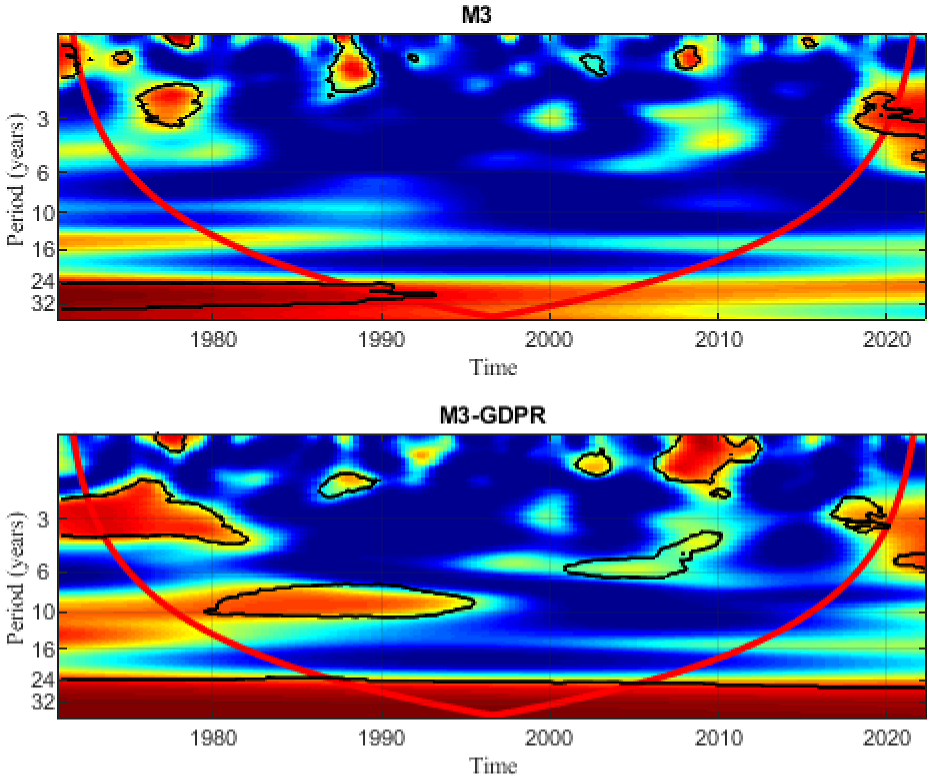

Figure 2: Wavelet coherency of money growth and inflation

The extension of the sample period by almost ten years has two main consequences: First, it widens the area of time-frequency combinations inside the red lines for which coherency estimates can be interpreted. Second, it allows us to extend our analysis to lower frequencies, more specifically to cycle lengths of about 40 years.

A stable long-run comovement between money growth and inflation implies high correlation, i.e. high coherency, at low frequencies. The top panel shows no evidence for a stable long-run relationship between money growth and inflation: Initially, coherency is close to one and significantly positive for cycles lasting 24 years or longer but declines over the 1990s and becomes insignificant. By contrast, we find stable and significant coherency for M3 adjusted by real GDP growth for cycles lasting 24 years and longer in the bottom panel of Figure 2. The figure shows that the upper bound of this frequency interval shifts slowly over time towards longer cycles. In Mandler and Scharnagl (2014), cycles of these lengths were already close to or outside of the curved red lines and therefore could not be interpreted. Furthermore, the change in the bootstrap algorithm from our previous analysis results in low frequency coherency estimates being significantly different from zero at smaller point estimates.[5]

[5] In Mandler and Scharnagl (2014) coherency was insignificant for 24-year cycles for both headline M3 and adjusted M3.

There is also evidence for significant comovement of cycles with a length of about ten years in adjusted money growth and inflation in the 1980s and the early 1990s but these common cycles are not stable over time and coherency declines substantially as time progresses.

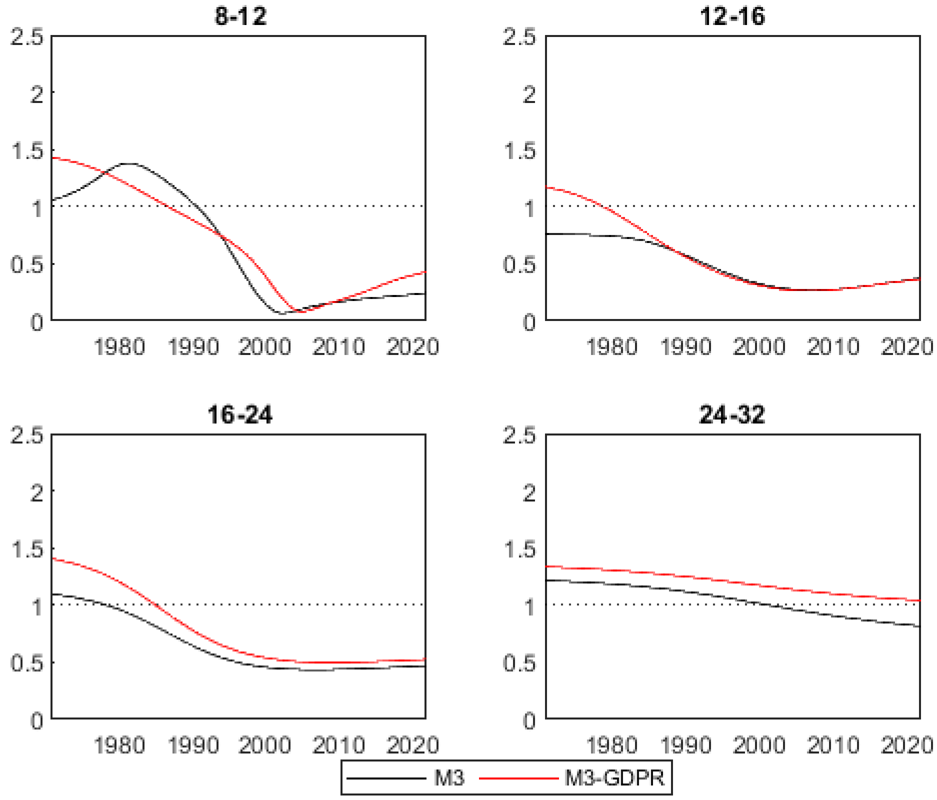

Figure 3: Frequency-dependent cross-spectral gain of money growth and inflation

Figure 3 shows estimates of the cross–spectral gain. The cross-spectral gain can be interpreted as a frequency-dependent coefficient of a regression of inflation on money growth. The different subplots show the gain for various cycle lengths in years. The black lines are the estimates for headline M3 growth, the red lines for adjusted M3 growth. We focus on the bottom right panel, since the coherency analysis indicated significant and stable correlation between cycles in adjusted money growth and inflation with length of 24 years or more.[6] From the late 1980s to the early 2000s, i.e. for the years between the red lines in Figure 2, the gain is close to one, consistent with the quantity theory.

[6] We do not extend cycle length in the bottom right panel to 40 year, the upper limit of our analysis. The reason for this is that for 40 years long cycles we cannot perform a meaningful analysis of time variation as all the data is required for estimation at a single point in time.

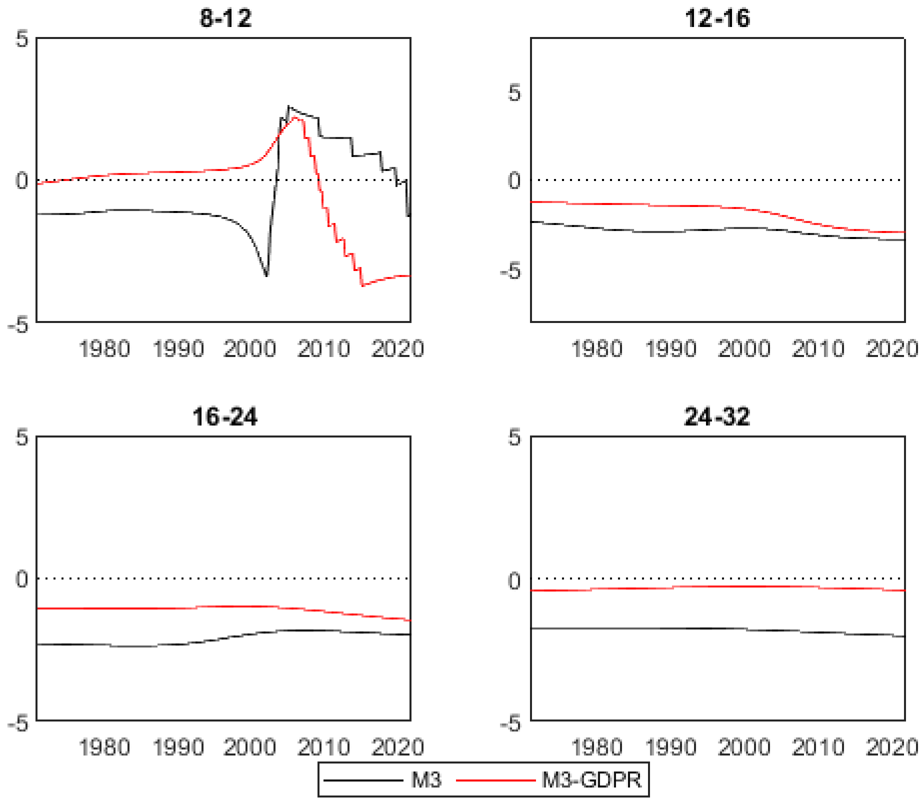

Figure 4: Frequency-dependent time difference between cycles in money growth and inflation

Figure 4 presents the estimated time difference between cycles in money growth and inflation for different cycle length intervals. The vertical axes show the time-difference. Negative values indicate a lead in money growth. The point estimates for 24-32 year cycles in adjusted money growth and inflation show a small lead of about a quarter which, given estimation uncertainty, is indistinguishable from contemporaneous cycles in both variables. The point estimates for headline money growth indicate a lead of money growth of about two years. However, Figure 2 shows that the comovement between M3 growth and inflation becomes insignificant around the start of the European Monetary Union (EMU), which cautions against putting much weight on a lead of headline M3 in the second half of the sample. The significant comovement between ten-year cycles in adjusted money growth and inflation in the 1980s and early 1990s in Figure 2 implies a roughly contemporaneous relationship between both variables, as well (Figure 4, top left panel).

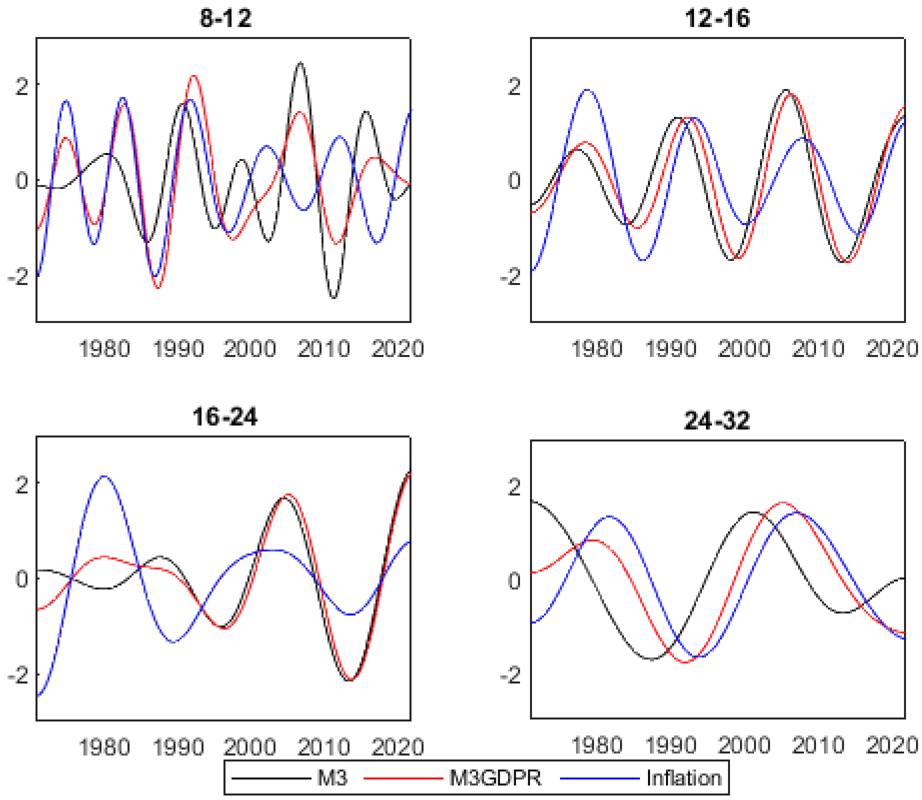

Figure 5: Filtered time series

Figure 5 shows components of money growth and inflation obtained from inverting the estimated wavelet representation at various frequency intervals. While the two money growth series are relatively similar for the 12-16 year and the 16-24 year bands they differ markedly for the longer fluctuations. For cycles lasting 24-32 years the subtraction of real GDP growth shifts the money growth series to the right much closer to the filtered inflation series.

4 Discussion

We update our previous analysis on possible comovement in money growth and inflation in the euro area by extending the sample period by about ten years to mid-2022. We find evidence for stable comovement between money growth adjusted by real GDP growth and consumer price inflation at the lower bound of the frequency range in our analysis, i.e. for cycles exceeding 24 years. By contrast, comovement between headline M3 growth and inflation have become weaker since the end of the 1990s and coherency has not remained significant in the EMU period. We show that there is roughly a 1:1 relationship between the long-lasting fluctuations in adjusted money growth and inflation and that those cycles are almost contemporaneous, i.e. there is no evidence for a marked lead of adjusted money growth and inflation.

The empirical literature has shown the low-frequency relationship between money growth and inflation to weaken in a regime of low and stable inflation rates (e.g. De Grauwe and Polan, 2005; Benati, 2009; Sargent and Surico, 2011; Teles, Uhlig and Valle e Azevedo, 2016; Guo, Kulish and Nicolini, 2021). The length of our sample period implies that our wavelet-based analysis can provide reliable evidence on the low-frequency relationship between money growth and inflation within a time window around the middle of our sample period. More specifically, we require of a sufficiently wide window of observations around each point in time for which we apply the wavelet transform to the data and the width of the required window increases as we move to lower frequencies. Our evidence concerning the 24+ year cycles thus applies only to the years between the late 1980s and the early 2000s in which many euro area countries transitioned from a regime of higher inflation rates to relatively low inflation rates in the EMU period. The question, whether the comovement between adjusted money growth and inflation has remained stable afterwards currently cannot be investigated with our wavelet-based techniques and require additional data going forward.

Even if the comovement between adjusted money growth and inflation has remained stable until now, the information content of long-run money growth for monetary policy is limited. First, the almost contemporaneous relationship implies that low-frequency fluctuations in money growth convey little advance information about inflation and in particular about turning points in the long-run inflation cycle. Second, deriving information from 24-year and longer cycles in adjusted money growth for inflation requires real-time estimation of this long-run component in adjusted money growth which is very imprecise.

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