RBC Model

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Turbulent Business Cycles¹

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Background

- Turbulence rises in recession
 - Product churn (Aghion, el al 2021; Bernard and Okubo 2016)
 - Productivity ranking churn (Bloom, et al 2018)
- This paper:
 - 1. Documents evidence of macro and reallocation effects of turbulence
 - 2. Studies transmission mechanism
 - 3. Evaluates alternative policy interventions

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What's turbulence?

• Firm-level TFP follows the process

$$z_{j,t+1} = egin{cases} z_{j,t} & ext{with prob} &
ho_t, \ ilde{z} & ext{with prob} & 1-
ho_t, \end{cases}$$

where \tilde{z} is i.i.d. drawn from $\tilde{G}(z)$

• Time-varying turbulence: $1 - \rho_t$

- $\rho_t = 1$: permanent shock
- $\rho_t = 0$: i.i.d. shock
- $1 \rho_t \uparrow \Rightarrow$ high-(low-) productivity firm less likely to remain productive (unproductive) \Rightarrow turbulence \uparrow
- Time-invariant cross-sectional distribution of z

•
$$Pr(z_t = z_j) = \pi_j \rightarrow Pr(z_{t+1} = z_j) = \rho_t \pi_j + (1 - \rho_t) \pi_j = \pi_j$$

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Countercyclical turbulence



Figure: Measured turbulence is countercyclical details

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Turbulence generates recession and reallocation



Figure: One standard deviation turbulence shock reduces real GDP by at least 0.5%: turbulence quantitatively important details

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Reallocation effects of turbulence

Dep. Var.	Employme	nt growth	Capital	Capital growth	
	(1)	(2)	(3)	(4)	
Turb _t * High_TFP _{it}	-0.313***	-0.098**	-0.228***	-0.216***	
- ,	(0.046)	(0.039)	(0.055)	(0.055)	
constant	0.061***	0.059***	0.071****	0.071***	
	(0.002)	(0.002)	(.002)	(.002)	
Firm Fixed Effect	Yes	Yes	Yes	Yes	
Year Fixed Effect	Yes	Yes	Yes	Yes	
Observations	25,955	24,288	25,955	24,288	

• $High_TFP_{it} = 1$ iff firm TFP above median;

Column (2): lagged dummy High_TFP_{jt-1} replacing High_TFP_{jt}

details

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Summary of evidence

- Turbulence rises in recessions
- Increase in turbulence associated with
 - 1. declines in aggregate TFP and firm value,
 - 2. synchronized and persistent declines in aggregate activity,
 - 3. reallocation from high- to low-productivity firms, and
 - 4. reallocation effects working through financial frictions details
- Turbulence is quantitatively important: one std increase in turbulence reduces real GDP by 0.5%

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RBC model: overview

- Heterogeneous firms
 - productive firm: turbulence $\uparrow \rightarrow$ future profit \downarrow more
- Working capital constraint
 - borrow against firm value (equity) to finance N and K
 - firm value: discounted future profit
 - productive firm: borrow $\downarrow \rightarrow N \& K$ share \downarrow (misallocation)
- Misallocation channel of turbulence
 - turbulence $\uparrow \rightarrow$ misallocation $\uparrow \rightarrow$ TFP $\downarrow \rightarrow$ Y \downarrow
 - synchronized declines: C, I, N \downarrow

Firms [1]

• CRS technology, renting capital (k_{jt}) and labor (n_{jt})

$$y_{jt} = A_t z_{jt} k_{jt}^{\alpha} n_{jt}^{1-\alpha}$$
(2)

• z_{jt} : idiosyncratic prod. subject to turbulence shock to ρ_t

$$z_{j,t+1} = \begin{cases} z_{jt} & \text{with prob} \quad \rho_t, \\ \tilde{z} & \text{with prob} \quad 1 - \rho_t, \end{cases}$$
(3)

Bellman equation:

$$V_t(z_{jt}, \tau_{jt}) = \max_{k_{jt}, n_{jt}} \tau_{jt} A_t z_{jt} k_{jt}^{\alpha} n_{jt}^{1-\alpha} - R_t k_{jt} - W_t n_{jt} + \mathbb{E}M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1})$$

s.t. working capital constraints

$$R_t k_{jt} - W_t n_{jt} \le \theta \mathbb{E} M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}) \equiv \theta B_{jt}$$
(4)

where $\tau_j \sim F(\tau)$: i.i.d. distortion (Hsieh-Klenow 2009; Buera-Shin 2013)

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Production decisions



- Active firms: $\{\tau, z\}$ above threshold curve $\tau_t^* \propto \frac{R_t^{\alpha} W_t^{1-\alpha}}{A_t z_t}$ details
- Higher average turbulence (lower p
): high z firms less likely to remain productive, lowering wages and shifting down threshold curve

Equilibrium

• Household's problem:

$$\max_{C_t, N_t, K_{t+1}} \mathsf{E} \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \psi \frac{N_t^{1+\gamma}}{1+\gamma} \right\}$$
(5)

• s.t. budget constraint

$$C_t + K_{t+1} = (R_t + 1 - \delta)K_t + W_t N_t + D_t + T_t$$
 (6)

• Factor market clearing

$$N_t = \sum_j \pi_j n_{jt} \equiv \sum_j \pi_j \frac{(1-\alpha)\theta B_{jt}}{W_t} \left[1 - F(\tau_{jt}^*) \right]$$
(7)

$$K_t = \sum_j \pi_j k_{jt} \equiv \sum_j \pi_j \frac{\alpha \theta B_{jt}}{R_t} \left[1 - F(\tau_{jt}^*) \right]$$
(8)

Goods market clearing

$$Y_t = C_t + K_{t+1} - (1 - \delta)K_t \tag{9}$$

IRF: turbulence shock



Figure: Impulse responses to one std turbulence shock

calibration

Counterfactual: turbulence shock



Figure: Counterfactual: "Quasi-fixed" borrowing limit (red lines)

$$R_t k_{jt} + W_t n_{jt} \le \theta E_t M_{t+1} \left[\rho_t \overline{V}_j^{ss} + (1 - \rho_t) \sum_{i=1}^J \pi_i \overline{V}_i^{ss} \right]$$

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Turbulence vs micro-level uncertainty shock

• Uncertainty shock:

$$\ln(\sigma_{\tau,t}) = (1 - \rho_{\sigma}) \ln(\sigma_{\tau}) + \rho_{\sigma} \ln(\sigma_{\tau,t-1}) + \sigma_{\sigma} \varepsilon_t^{\sigma},$$

- Similar to uncertainty, turbulence increases conditional vol of productivity
 - conditional variance of z_{jt+1}

$$\operatorname{var}(z_{jt+1}) = (1 - \rho_t)^2 \operatorname{var}(\tilde{z}) \tag{10}$$

- strictly decreasing in ρ_t for all j
- Different from uncertainty, turbulence preserves ex ante productivity distribution

$$G_{t+1}(z) = \Pr(z_{t+1} \le z)$$

= $\Pr(z_t \le z)\rho_t + \Pr(\tilde{z} \le z)(1-\rho_t)$
= $G_t(z)\rho_t + \tilde{G}(z)(1-\rho_t) = G_t(z)$

IRF: uncertainty shock



Figure: Impulse responses to micro uncertainty shock $\sigma_{\tau t}$

Uncertainty expands right tail of τ distribution, raising average subsidy for active firms and increasing labor demand

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Model vs data



Figure: Impulse response to turbulence shock: Model vs. data One std turbulence shock reduces aggregate output about 0.5%, both in annual model (blue line) and in data (black line)

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Two types of policy interventions

• Policy I: Borrowing subsidy

s.t.
$$(1 - \omega_{1t})(R_t k_{jt} + W_t n_{jt}) \le \theta B_{jt}$$
 (11)

Policy II: Credit easing

s.t.
$$R_t k_{jt} + W_t n_{jt} \le \theta (1 + \omega_{2t}) B_{jt}$$
 (12)

- Both policies incur resource costs (gov't inefficiency); both financed by lump-sum taxes
- Policy interventions triggered by turbulence shock, with same persistence as shock

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Figure: Stabilizing effects of policy interventions

Both policies effective for stabilizing output fluctuations
 Borrowing subsidy exacerbates misallocation; credit easing improves it

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Conclusion

- Firm-level evidence shows that countercyclical turbulence has important macro and reallocation effects
 - Financial frictions are important in reallocation effects
- RBC model with firm heterogeneity and financial frictions highlights reallocation channel for transmitting turbulence over business cycles
- Credit policies can stabilize turbulence-drive recessions, but implications for reallocation depend on policy
 - Borrowing subsidies amplify misallocation whereas credit easing mitigates it.

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Measuring turbulence

 Construct firm-level TFP following literature (Syverson, 2004; Foster, et al 2008), using data from Compustat and NBER-CES (1958-2016)

$$tfp_{ijt} = y_{ijt} - \alpha_{it}k_{ijt} - (1 - \alpha_{it})n_{ijt}$$
(13)

- Calculate Spearman rank correlation of firm-level TFP between year t and $t + 1 \Rightarrow \rho_t$
- Turbulence measured by $1 \rho_t$

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Macro effects of turbulence

Local projections (Jorda, 2005)

 $x_{t+h} - x_{t-1} = \beta_0^h + \beta_1^h turb_t + \beta_2^h turb_{t-1} + \beta_3^h dx_{t-1} + \epsilon_{t+h},$ (14)

- x_t denotes macro variable of interest (log level of GDP, C, I, H, IQR_n , and IQR_k); $turb_t$ denotes turbulence in log units $(log(1 \rho_t))$
- β_1^h measures IRFs to turbulence shock at horizon h (years)
- Sample: Annual time series from 1958 to 2015

Reallocation effects of turbulence [1]

• Estimate the regression:

$$x_{jt} = \beta_0 + \beta_1 Turb_t * High_TFP_{jt} + \mu_j + \eta_t + \epsilon_{jt}, \quad (15)$$

- 1. x_{jt} : growth of N (or K) in firm j at year t;
- 2. Turb_t: turbulence at year t;
- 3. *High_TFP_{jt}*: binary dummy of high TFP firms (> median)
- 4. Robustness: quartile dummies replacing binary dummy
- β₁: additional effects of turbulence on high-productivity firms
 - $\beta_1 < 0$: high-productivity firms more sensitive to turbulence

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Reallocation effects of turbulence: robustness

Dep. Var.	Employme	ent growth	Capital growth	
	(1)	(2)	(3)	(4)
Turb _t * z _{2,it}	-0.186***	-0.061	-0.182***	-0.127***
	(0.054)	(0.044)	(0.052)	(0.049)
Turb _t * z _{3.it}	-0.338***	-0.098*	-0.283***	-0.256* ^{**}
	(0.064)	(0.047)	(0.072)	(0.070)
Turb _t * z _{4.it}	-Ò.658* ^{**}	-Ò.242* ^{**}	-Ò.512* ^{**}	-Ò.412* ^{**}
	(0.093)	(0.062)	(0.092)	(0.084)
constant	0.076***	0.055****	0.085****	0.081* ^{**}
	(0.005)	(0.003)	(0.005)	(0.005)
Firm Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	24,288	24,288	24,288	24,288

- $z_{2,jt} = 1$, $z_{3,jt} = 1$, $z_{4,jt} = 1$ when firm j's TFP ranking is in 50th, 75th and 100th quartile;
- Column (2): lagged dummy $z_{2,jt-1}, z_{3,jt-1}, z_{4,jt-1}$ replacing $z_{2,jt}, z_{3,jt}, z_{4,jt}$

Reallocation effects: role of financial friction

Dep. Var.	IQR of Employment		IQR of	Capital
	(1)	(2)	(3)	(4)
High_FF _{it}	0.682**	0.435	0.962**	0.724*
	(0.315)	(0.340)	(0.368)	(0.406)
Turb _t * High_FF _{it}	-5.789**	-3.805	-8.151**	-6.256*
	(2.863)	(3.055)	(3.397)	(3.634)
constant	2.008****	2.036**'*	2.15** [*]	2.174** [*]
	(0.043)	(0.045)	(0.040)	(0.043)
Industry Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	2,505	2,421	2,505	2,421

- $High_FF_{it} = 1$ if industry *i*'s external financing dependence is above the median.
- Column (2): lagged dummy High_FF_{it-1} replacing High_FF_{it}

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Firms [2]

- At each productivity z_{jt} , firms are active iff $\tau_{jt} \geq \tau_{jt}^*$
- Break-even threshold

$$\tau_{jt}^* = \frac{R_t^{\alpha} W_t^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} A_t z_{jt}}$$
(16)

Labor demand

$$n_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{(1-\alpha)\theta B_{jt}}{W_t}, & \text{if } \tau_{jt} \ge \tau_{jt}^* \\ 0, & \text{otherwise} \end{cases}$$
(17)

• Capital demand

$$k_{t}(z_{jt}, \tau_{jt}) = \begin{cases} \frac{\alpha \theta B_{jt}}{R_{t}}, & \text{if } \tau_{jt} \ge \tau_{jt}^{*} \\ 0, & \text{otherwise} \end{cases}$$
(18)

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Misallocation effect of turbulence in steady state

Proposition 1

Given the steady-state factor prices R and W, an increase in average turbulence reduces the share of labor hours allocated to high-productivity firms.

$$rac{\partial \eta_{ji}}{\partial ar{
ho}} > 0,$$

where $\eta_{ji} \equiv \frac{N_j}{N_i}$ denotes relative labor hours allocated to firms with $z_j > z_i$.

• Higher turbulence reduces expected value of high-productivity firms, tightening their credit constraints and reallocating N to low-productivity firms

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Calibration [1]

- Turbulence shock $1 \rho_t$
 - Calibrated based on Spearman correlation of firm-level TFP in annual data 1960-2015, converted to quarterly
 - TFP in data is revenue based, corres. to $TFP_{jt} = z_{jt}\tau_{jt}$ in model
 - τ_{jt} is i.i.d. \Rightarrow Spearman correlation of true productivity z_{jt} same as that of TFP_{jt}
- Idiosyncratic production distortion τ_{jt}
 - Average dispersion $\sigma_{\tau} = 0.6$ to match IQR of employment (17) in 1960-2015 data
 - Mean value of normalized such that $\mathbb{E} au_{jt} = 1$
- Calibrate process for z_{jt} based on $tfp_{jt} = \log(z_{jt}) + \log(\tau_{jt})$
 - Measured TFP has std $\sigma_{tfp} = 0.607$
 - $\log(z_{jt})$ and $\log(\tau_{jt})$ independent $\Rightarrow \sigma_z = \sqrt{\sigma_{tfp}^2 \sigma_{\tau}^2} = 0.05$

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Calibration [2]

	Parameter Description	Value	Target
β	Discount factor	0.99	Annual real rate of 4% per year
ά	Capital share	0.34	Ave. cost share of capital
δ	Capital depreciation rate	0.025	Annual depreciation rate of 10%
γ	Inverse Frisch elasticity	5	Frisch elasticity of 0.2
$\dot{\theta}$	Loan to value ratio	0.35	Working K to equity (Compustat)
$\bar{\rho}$	Ave persistence	0.974	Compustat and NBER-CES
ρ_{ρ}	AR(1) of turbulence	0.882	Compustat and NBER-CES
σ_0	std of turbulence shock	0.124	Compustat and NBER-CES
μ_{τ}	Average distortion	-0.18	Compustat and NBER-CES
σ_{τ}	std of distortion	0.60	Compustat and NBER-CES
σ_z	std of firm-level TFP	0.05	Compustat and NBER-CES

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IRF: TFP shock



Figure: Impulse responses to negative aggregate TFP shock.

1. Decline in TFP shrinks set of active firms; 2. Lower prod. firms more likely to go inactive, improving allocation efficiency (cleansing effect)